

# Programming Language Concepts

## Logic Programming Paradigm

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# Outline

- 1 Introduction
- 2 Prolog basics
- 3 Prolog Terms
- 4 Unification
- 5 Backtracking
- 6 List Processing
- 7 Arithmetical Operations
- 8 List Examples
- 9 Cut

# Logic Programming Paradigm

- Based on logic and [declarative programming](#)
- 60's and early 70's
- Prolog (**P**rogramming in **l**ogic, 1972) is the most well known representative of the paradigm.
- Prolog is based on [Horn clauses](#) and [SLD resolution](#)
- Mostly developed in [fifth generation computer systems project](#)
- Specially designed for theorem proof and artificial intelligence but allows general purpose computation.
- Some other languages in paradigm: ALF, Frill, Gödel, Mercury, Oz, Ciao,  $\lambda$ Prolog, datalog, and CLP languages

# Constraint Logic Programming

- Clause: disjunction of universally quantified literals,

$$\forall(L_1 \vee L_2 \vee \dots \vee L_n)$$

- A logic program clause is a clause with exactly one positive literal

$$\begin{aligned} \forall(A \vee \neg A_1 \vee \neg A_2 \dots \vee \neg A_n) &\equiv \\ \forall(A \Leftarrow A_1 \wedge A_2 \dots \wedge A_n) \end{aligned}$$

- A goal clause: no positive literal

$$\forall(\neg A_1 \vee \neg A_2 \dots \vee \neg A_n)$$

- Proof by refutation, try to unsatisfy the clauses with a goal clause  $G$ . Find  $\exists(G)$ .
- Linear resolution for definite programs with constraints and selected atom.

# What does Prolog look like?

```
father(ahmet, ayse).  
father(hasan, ahmet).  
mother(fatma, ayse).  
mother(hatice, fatma).  
parent(X,Y) :- father(X,Y).  
parent(X,Y) :- mother(X,Y).  
grandparent(X,Y) :- parent(X,Z), parent(Z,Y).
```

- CLP on first order terms. (Horn clauses).
- **Unification**. Bidirectional.
- **Backtracking**. Proof search based on trial of all matching clauses.

# Prolog Terms

- Every valid phrase in prolog is a **Term** and instead of strict type checking, **unification** is used. A term can be one of the following:
  - Atoms:
    - 1 Strings with starting with a small letter and followed by any letter, digit and `_`. `[a-z][a-zA-Z_0-9]*`
    - 2 Strings consisting of only punctuation symbols as:  
`[+~*/\^<>=~:~.?@#$!&]+`
    - 3 `[]`, `{}`, `;`, `!` are only treated as atoms in these forms (alone and only spaces in between).
    - 4 Any string enclosed in single or back quotes. Quotes are not part of the atom.
  - Numbers
    - 1 Any integer `[0-9]+`
    - 2 Any floating point value `[0-9]+.[0-9]+`
    - 3 Any scientific notation value `[0-9]+.[0-9]+e[0-9]+`

## ■ Variables:

- 1 Strings with starting with a capital letter or `_` and consist of `[_A-Z][a-zA-Z_0-9]*`
- 2 `_` alone is the universal match symbol. Not variable

## ■ Structures:

- starts with an `atom` head. No number, no variable
- has one or more arguments enclosed in paranthesis, separated by comma
- no space between structure head and paranthesis.
- arguments can be any valid prolog term, including other structures.



Term	Atom	Num.	Var.	Struct.	not a term
hELLO					
Hello					
_abc					
'A_and_B'					
"hello"					
:->>					
:-P					
--					
1e1					
1.0e1					
0.2					
.2					
3.					
0000123					
x(4)					
++(a,b)					
R(3)					
2(4)					
a(a(a,a(a(a(a))))))					
a(X,Y,.(X),2,3)					

Term	Atom	Num.	Var.	Struct.	not a term
hELLO	✓				
Hello			✓		
_abc			✓		
'A_and_B'	✓				
"hello"				✓	
:->>	✓				
:-P					✓
--			✓		
1e1		✓			
1.0e1		✓			
0.2		✓			
.2					✓
3.					✓
0000123		✓			
x(4)				✓	
++(a,b)				✓	
R(3)					✓
2(4)					✓
a(a(a,a(a(a(a))))))			✓		
a(X,Y,.(X),2,3)				✓	

# Syntax Elements

- A Prolog program consists of **clauses** or **predicates**.
- **Unit clauses** are structure or atom terms followed by a dot.  
`father(ayse, ahmet).`
- Unit clauses are considered as facts, no implication.
- Non-unit clauses consists of a **head clause** and a **body**  
`grand(X, Y) :- parent(X,Z), parent(Z,X).`
- In order to prove head clause, body should be proven.
- Body consist of structures seperated by comma, semi-colon and optionally combined with parantheses.  
`uncle(X,Y) :- (brother(X,Z), father(Z,Y)) ; (brother(X,Z), mother(Z,Y)).`
- Comma stands for conjunction ( $\wedge$ ), semicolon stands for disjunction ( $\vee$ ).
- Structures in the body are **goal clauses** to be proven.

# Prolog Lists

- `[1,2,3]` is parsed and interpreted as `.(1,.(2,.(3,[])))`
- `[Head | Tail]` form is interpreted as `.(Head , Tail)`
- `[]` denotes empty list
- `[1,2,3 | R]` is interpreted as `.(1, .(2, .(3, R)))`
- strings in double quotes like `"abc"` are interpreted as list of ASCII numbers as `[97, 98, 99]`.
- As Prolog structures can contain arbitrary terms, lists are heterogeneous as `[1, 2.1, a(b,c), [a,b,c], "hello"]` is a valid list.

# Unification

- In functional languages, caller arguments are pattern-matched against the function definition. This operation is also called **unification**. All constructors and values in caller are matched against the patterns in the definition. The variables in definition are **instantiated** with the values in the caller.
- Unification in Prolog is bi-directional. Both the defining clause and goal clause have variables instantiated.

`same(X,X).`

`goal: same(ali,Y).`

`X = ali, Y = X, Y = ali`

- Result of a unification can result in some variable instantiations as:

`X = ali, Y = ali  $\Rightarrow$  true`

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 $x = h_x(x_1, x_2, \dots, x_n)$ ,  $y = h_y(y_1, y_2, \dots, y_n)$  and  
 $h_x = h_y$  and  $\forall x_i = y_i$ ,  $i = 1, \dots, n$ . Head and all coressponding  
 elements are unified.

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 of instantiations, unification is successfull with  $x = y$  is added  
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- 3 If  $x$  is a variable and  $x = y$  is compatible with the current set  
 of instantiations, unification is successfull with  $x = y$  is added  
 to current set of instantiations.
- 4 Otherwise, unification fails.

<code>a = b</code>	false
<code>'abc' = abc</code>	true
<code>X = 12</code>	true $\Leftarrow$ X=12
<code>a(1,X) = a(Y,2)</code>	true $\Leftarrow$ X=2, Y=1
<code>a(1,X) = a(Y,Y)</code>	true $\Leftarrow$ X=Y=1
<code>a(1,X,X) = a(Y,Y,2)</code>	false (Y=1, X=Y, X=2)
<code>a(1,_) = a(1)</code>	false (different arities)
<code>X = a(X)</code>	true $\Leftarrow$ X = a(X) (cannot display but succesfull)
<code>a(c(X,d),c(a,Y),X) = a(Y,Z,t)</code>	true $\Leftarrow$ X = t, Y = c(t,d) , Z = c(a,c(t,d))
<code>a(c(X,d),c(a,Y)) = a(Y,X)</code>	true $\Leftarrow$ X = c(a,Y), Y = c(X,d)

# Prolog Program

- A Prolog program can be written by putting all alternatives as a separate head clause with same name and arity.
- You define **relations** instead of functions returning a value.
- For example, not a membership test function but a **member relation**.
- Membership relation for a list can be defined verbally as:  
“ $x$  is member of list  $lst$  if either  $x$  is the first element of the  $lst$  or it is the member of the remaining list”
- Each alternative is another **member(X,LST)** clause.  
`member(X, [First | Remaining]) :- X = First.`  
`member(X, [First | Remaining]) :- member(X, Remaining).`
- Shorter form:  
`member(X, [X | _]).`  
`member(X, [_ | Remaining]) :- member(X, Remaining).`

# Prolog Interpreter

- **Gnu Prolog** or **Sicstus Prolog** are free alternatives.
- entering '[filename].' in interpreter loads the clauses from filename.pl.
- '?- ' prompt asks user to enter goal clauses like:  
?- member(b, [a,b,c]).
- Prolog checks if this goal can be proven with the current program and replies **yes**, **no**
- If there are alternatives, it prompts **true ?** and asks for continuation. pressing enter will terminate, ; will try other alternatives.

```

~$ swipl
Welcome to SWI-Prolog (Multi-threaded, 64 bits, Version 7.2.3)
...
Please visit http://www.swi-prolog.org for details.

?- [testmember].                % load testmember.pl
true.

?- member(b,[a,b,c]).
true                             % hit enter
?- member(d,[a,b,c]).
false.

?- member(b,[a,b,c]).
true ;                           % hit ; , try alternatives
false.                          % no other alternatives true
?- member(b,[a,b,b]).
true ;                           % hit ; , try alternatives
true ;                          % one more alternative, no other
false.

?- member(X,[a,b,c]).           % ask who is member of [a,b,c]?
X = a ;
X = b ;
X = c ;
false.

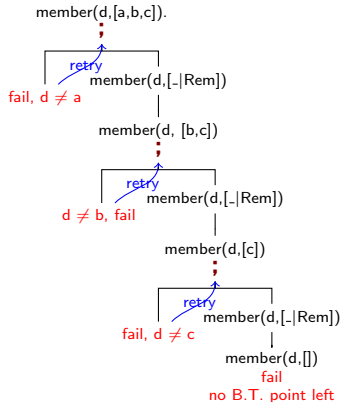
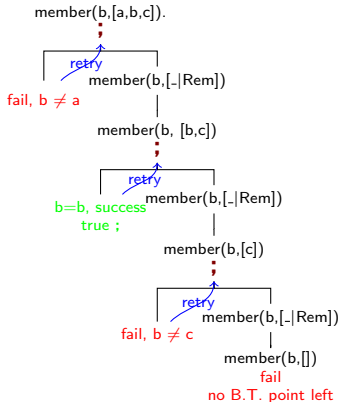
```

# Backtracking

- Backtracking is the search procedure of Prolog and makes it a universal programming language.
- Each alternative head clause that can be unified with goal clause is a backtracking point.
- similarly each operand of ';' is a backtracking point.
- Prolog saves the current state in backtracking points. On failure, tries the next backtracking branch.
- On success, if user hits ';' in prompt, it resumes search from the next backtracking point.

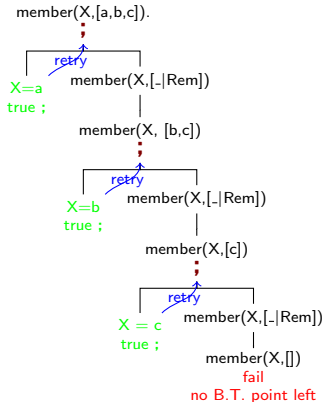
`member(X, [X | _]).`

`member(X, [_ | Rem]) :- member(X, Rem).`



```
member(X, [X | _]).
```

```
member(X, [_ | Rem]) :- member(X, Rem).
```

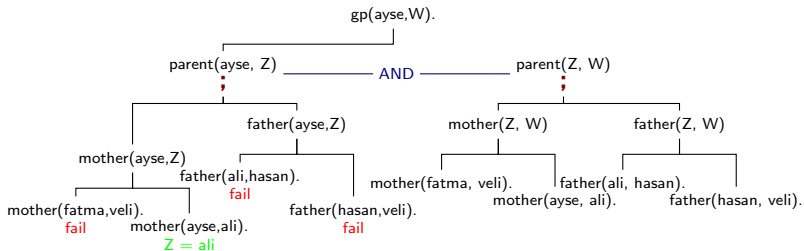




```

mother(fatma,veli).    father ( ali , hasan).    parent(X,Y) :- mother(X,Y).
mother(ayse, ali).     father (hasan, veli).    parent(X,Y) :- father(X,Y).
gp(X,Y) :- parent(X,Z), parent(Z,Y).

```

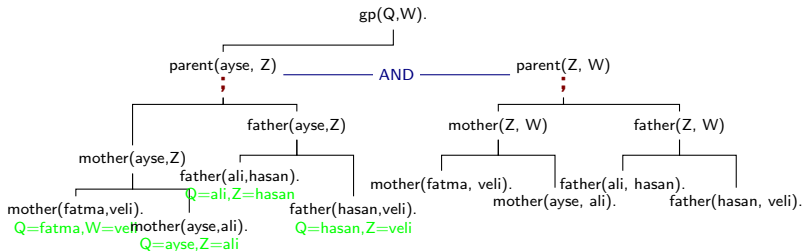


Only solution from left branch is  $Z=ali$ , applied to right branch. `father ( ali , hasan)` matches. Result is  $W = hasan$ .

```

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mother(ayse, ali).     father (hasan , veli).    parent(X,Y) :- father(X,Y).
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```



For each solution in left parent branch it backtracks and test solution from right parent branch, keeping the instantiated variables.  $Z=ali$  and  $Z=hasan$  returns success. Results are:  $Q=ayse, W=hasan$  and  $Q=ali, W=veli$

# List Processing

## ■ Appending lists.

```
append([], LST, LST).
append([H | Rem], LST, Res) :- append(Rem, LST, Res).
```

## ■ `append(X,Y,[a,b,c,d])` works as well.

## ■ Reverse:

```
reverse([], []).           % reverse of empty list
reverse([H|Rem], Rev) :- reverse(Rem, RR), .
```

## ■ Efficient reverse:

```
reverse2([], L, L).        % no element left, result is stack
reverse2([H | Rem], P, L) :- reverse2(Rem, [H | P], L). % insert
reverse(LST, LSTREV) :- reverse2(LST, [], LSTREV).
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reverse(LST, LSTREV) :- reverse2(LST, [], LSTREV).
```

# Arithmetical Operations

- $X = 3 * 5$  is equivalent to  $X = *(3,5)$  and does not make any calculation.
- A special operator 'is' evaluates the expressions:  
 $X \text{ is } 3 * 5$  will instantiate  $X$  to 15.
- `is` requires right handside to be fully instantiated (no variables without a value) and evaluates it, the resulting number is unified with LHS.
- ' $2+X \text{ is } 5$ ' is equivalent to unification of  $+(2,X)$  to 5, which fails.
- Comparison operators also evaluate their both operands which should be fully instantiated:  
 $< , > , = < , > = , =: = , = \backslash =$
- Some of the arithmetic operators (in evaluation context):  
 $+, +, *, / , //$  (int.div.) `mod` .
- Also mathematical functions can be used:  
`sin , cos , ... , exp , log , log10 , abs , round , ceil , ...`

# Build-in Predicates

## ■ Testing term type:

`var(T)`, `nonvar(T)`, `atom(T)`, `number(T)`, `atomic(T)`, `ground(T)`.

## ■ Equivalence which does not cause instantiation:

`X == X` strict, `X \== Y` strict not eq., `X \= Y` not unifiable.

## ■ Bidirectional list to term conversion:

`X =.. [+ ,b,c] → X = +(b,c)`

`f(a,b,c) =.. X → X = [f,a,b,c]`

`X =.. [t] → X = t`

## ■ List predicates:

`member/2`, `length/2`, `append/3`, `select/3`, `union/3`, `reverse/2`

## ■ Displaying all clauses with given name and arity:

`listing(father/2)` `listing(reverse/_)`.

## ■ Find all solutions in a list:

`findall(X, father(ali,X), L)`, `setof(X, father(ali,X), L)`.

# Functional to Logical

- A function can be converted into a relation by adding a result argument. Result can be propagated from recursive calls in this argument.
- Haskell:

```
length [] = 0  
length (_,r) = (length r) + 1
```

- Prolog:

```
length([], Res) :- Res is 0.  
length([_|R], Res) :- length(R, RLen), Res is RLen + 1.
```



# Examples: List

- Relations may work different ways. Give all partitions of a list:

`append(P1, P2, [1,2,3]).`

`P1=[],P2=[1,2,3]; P1=[1],P2=[2,3]; P1=[1,2],P2=[3]; P1=[1,2,3],P2=[].`

- Selecting/removing element from a list1 results in list2:

```
select(E, [E|R],R).
select(E, [H|R], [H|R2]) :- select(E, R, R2).
```

`select(3,[1,2,3], L)`

`L=[1,2]`

- For all elements of list1, give remaining list as well:

`select(A,[1,2,3], L)`

`A=1,L=[2,3]; A=2,L=[1,3]; A=3,L=[1,2]`

- Inserting element to all positions of list1 results in list2:

`select(a,[1,2,3], L)`

`L=[a,1,2,3]; L=[1,a,2,3]; L=[1,2,a,3]; L=[1,2,3,a]`

## ■ Subset:

```
subset([], []).
subset(Rsub, [_|R]) :- subset(Rsub, R).
subset([H|Rsub], [H|R]) :- subset(Rsub, R).
```

## ■ Permutations:

```
% insert H to all positions in the remainder permutations
perm([], []).
% get first element H, get perms of rest
% insert H in every position of rest perm
perm([H|R], HINS) :- perm(R, RP), select(H, HINS, RP).
```

## ■ N combinations:

```
combin(_, [], 0). % 0 combination is empty
% all N combin of remain. is also in comb.
combin([_|R], Res, N) :- N > 0, combin(R, Res, N).
% N-1 combin of remain. add H
combin([H|R], [H|Res], N) :- N > 0, M is N-1,
                             combin(R, Res, M).
```

## ■ N permutations:

```
permut(_, [], 0). % 0 permutation is empty
% for all elements H of L, permute remaining.
permut(L, [H|RP], N) :- N > 0, M is N-1,
                        select(H, L, Rem), permut(Rem, RP, M).
```

- $\backslash+( P )$  or `not(P)` is **negation as failure** operator. Successful only if the argument clause fails (cannot be proven).
- Set intersection:

```
inter([],_,[]).
inter([H|R],S, [H|Res] ) :- member(H,S), inter(R,S,Res).
inter([H|R],S, Res ) :- not(member(H,S)), inter(R,S,Res).
```

- Set union:

```
union([],S,S).
union([H|R],S, Res ) :- member(H,S), union(R,S,Res).
union([H|R],S, [H|Res] ) :- not(member(H,S)), union(R,S,Res).
```

# Cut

■ `f(x) = if x > 10 then 5 else if x > 5 then 3 else 1`

```
f(X, Y) :- X > 10, Y = 5.
```

```
f(X, Y) :- X =< 10, X > 5, Y = 3.
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- **Cut** symbol, '!' prunes search tree and change behaviour.

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- Cut is always successfull with side effect of deleting all backtracking points from head clause so far. Only current solution is kept.
- Rewrite set intersection with a **cut**:

```
inter([], _, []).
```

```
inter([H|R], S, [H|Res]) :- member(H, S), !, inter(R, S, Res).
```

```
inter([H|R], S, Res) :- inter(R, S, Res).
```

- $\neg(P)$ , not operator can be implemented by a cut.

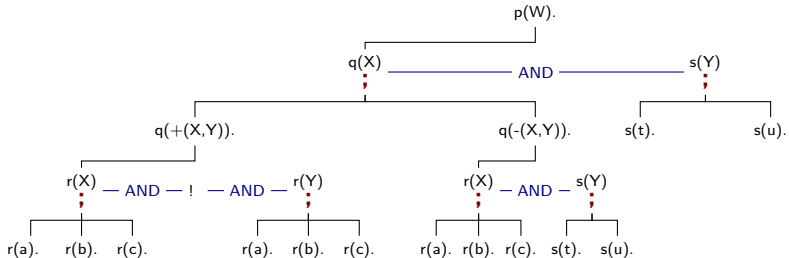
```
not(P) :- P , !, fail.  
not(P).
```

- This is called **negation as failure** semantics, not **logical negation**. In **logical negation** you may expect `not(member(X,[a,b,c]))` to instantiate  $X$  to complement set of  $[a,b,c]$ . However it simply fails.
- When a **cut** does not change the program semantics, set of values returned, it is called a **green cut**.

```

p(* (X,Y)) :- q(X), s(Y).      r(a).      s(t).
q(+ (X,Y)) :- r(X), !, r(Y).    r(b).      s(u).
q(- (X,Y)) :- r(X), s(Y).      r(c).

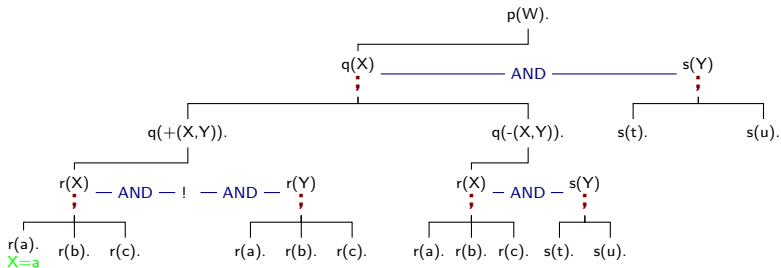
```



$p(* (X, Y)) \text{ :- } q(X), s(Y).$        $r(a).$        $s(t).$

$q(+ (X, Y)) \text{ :- } r(X), !, r(Y).$        $r(b).$        $s(u).$

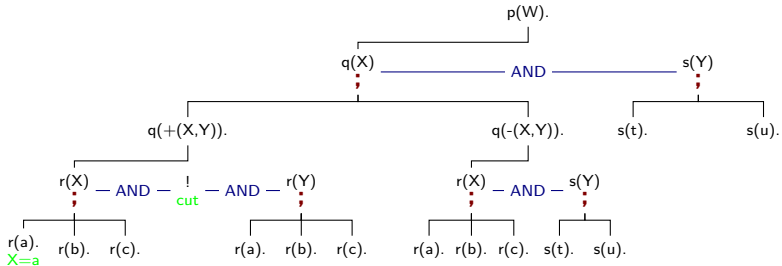
$q(- (X, Y)) \text{ :- } r(X), s(Y).$        $r(c).$



$p(* (X,Y)) \text{ :- } q(X), s(Y).$        $r(a).$        $s(t).$

$q(+ (X,Y)) \text{ :- } r(X), !, r(Y).$        $r(b).$        $s(u).$

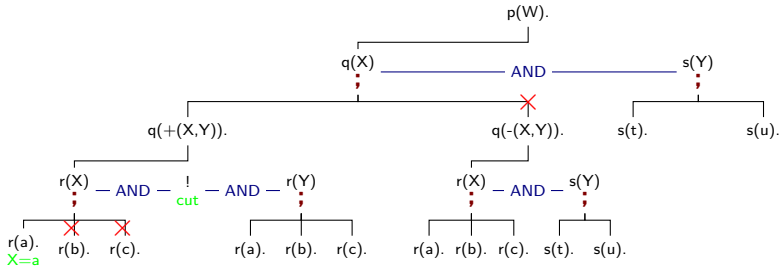
$q(- (X,Y)) \text{ :- } r(X), s(Y).$        $r(c).$



$p(* (X, Y)) \text{ :- } q(X), s(Y).$        $r(a).$        $s(t).$

$q(+ (X, Y)) \text{ :- } r(X), !, r(Y).$        $r(b).$        $s(u).$

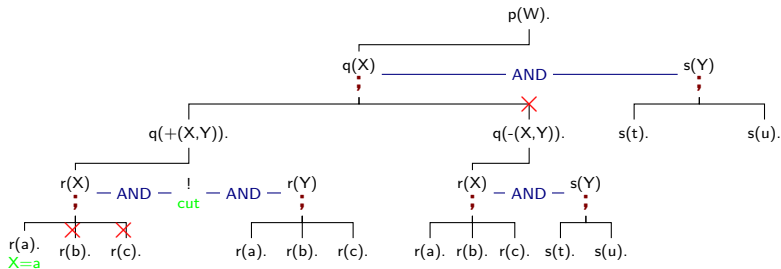
$q(- (X, Y)) \text{ :- } r(X), s(Y).$        $r(c).$



$p(* (X,Y)) \text{ :- } q(X), s(Y).$        $r(a).$        $s(t).$

$q(+ (X,Y)) \text{ :- } r(X), !, r(Y).$        $r(b).$        $s(u).$

$q(- (X,Y)) \text{ :- } r(X), s(Y).$        $r(c).$



- When cut is hit, current solution is kept and all backtracking points from head clause to cut are pruned. Backtracking points introduced later still produces alternatives.
- $r(Y)$  produces 3 alternatives,  $s(Y)$  at top right produces 2. Prolog finds 6 solutions in total.
- Without a cut:  $3 \times 3 = 9$  from  $q(+ (X,Y))$ ,  $3 \times 2 = 6$  from  $q(- (X,Y))$  give 15 solutions for  $q(X)$ . 15 times 2 solutions for  $s(Y)$  gives 30 solutions.

The following program generates 90 alternatives. Putting **cut** in marked positions one at a time changes this behaviour.

```
p( +(X,Y,Z) ) :- ○ q(X) ,○ r(Y),○ s(Z) ○. % 15*3*2 = 90

% 15 for q(X)
q( -(X,Y) ) :-○ r(X),○ r(Y)○ . % 3 * 3 = 9
q( -(X,Y) ) :-○ r(X),○ s(Y)○ . % 3 * 2 = 6

r(a). % 3 from r(X)
r(b).
r(c).

s(t). % 2 from s(X)
s(u).
```

Number of solutions per cut will be:



The following program generates 90 alternatives. Putting **cut** in marked positions one at a time changes this behaviour.

```
p( +(X,Y,Z) ) :- ○ q(X) ,○ r(Y) ,○ s(Z) ○. % 15*3*2 = 90

% 15 for q(X)
q( -(X,Y) ) :-○ r(X) ,○ r(Y)○ . % 3 * 3 = 9
q( -(X,Y) ) :-○ r(X) ,○ s(Y)○ . % 3 * 2 = 6

r(a). % 3 from r(X)
r(b).
r(c).

s(t). % 2 from s(X)
s(u).
```

Number of solutions per cut will be:

90, 6, 2, 1,  
54, 18, 6,  
90, 66, 60

# Example: Binary Search Tree

- we can represent a tree as a Prolog structure. Each node contains a key value pair in `p(k,v)`.

```
insert(e, K,V, tree(p(K,V),e,e)).
```

```
insert(tree(p(K,_),L,R), K, V, tree(p(K,V),L,R)) :- !.
```

```
insert(tree(p(H,HV),L,R), K, V, tree(p(H,HV),LRes,R)) :-  
    K < H,!, insert(L, K, V, LRes).
```

```
insert(tree(p(H,HV),L,R), K, V, tree(p(H,HV),L,RRes)) :-  
    insert(R, K, V, RRes).
```

```
insertlist(R,[],R).
```

```
insertlist(R,[K,V|L], RR) :- insert(R,K,V,RTemp),  
                             insertlist(RTemp,L,RR).
```

```
search(tree(p(K,V),_,_), K, V) :- ! .
```

```
search(tree(p(H,_),L,_), K, V) :- K < H, !, search(L,K,V).
```

```
search(tree(p(_,_),_,R), K, V) :- search(R,K,V).
```

```
?- insertlist(e,[4,veli,1,ali,6,hasan,2,ayse,5,fatma],R).
```

```
R = tree(p(4, veli), tree(p(1, ali), e, tree(p(2, ayse),e,e)),  
         tree(p(6, hasan), tree(p(5, fatma),e,e),e))
```

# N-Queens

- Place  $N$  queens on a  $N$  by  $N$  board with no queen is threatening others.

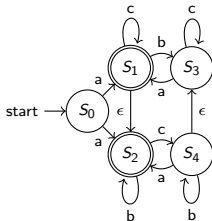
```
% give a list [N,N-1,...,0]
fill(N,[N|R]) :- M is N-1, M > 0, !, fill(M,R).
fill(L, [L]).

% get N queens, N columns and place them with state []
% queen id is assume to be the row
queen(N,R) :- fill(N,Queens), fill(1,N,Positions),
               place(Queens,Positions,[],R).

% get first queen, pick a position, check it is compatible with
% existing state, place rest with new state
place([],[],L,L).
place([Q|QRest],PList,In,RResult) :- select(P,PList,PRest),
                                     compatible(Q,P,In), place(QRest,PRest,[Q-P|In],RResult).

% compatibility, test not same column and diagonal.
compatible(_,_,[]).
compatible(Q,P,[QT-PT|R]) :- P \= PT, QDel is abs(Q-QT),
                             PDel is abs(P-PT), QDel \= PDel, compatible(Q,P,R).
```

# Example: NDFA



- Defined by a 5-tuple  $(Q, \Sigma, \Delta, q_0, F)$ :  
 $Q$  set of states,  $\Sigma$  input symbols,  
 $\Delta : Q \times \Sigma \rightarrow \mathcal{P}(Q)$  set of transitions,  
 $q_0 \in Q$  start state,  $F \subseteq Q$  final states.
- In prolog, we can define all those relations:
  - 1 define all transitions as `trans(s0, a, s1)`.
  - 2 define all empty transitions as `empty(s1,s2)`.
  - 3 define all accepting states as `final(s1)`.
  - 4 define starting state as `start(s0)`.
- NDFA parser using backtracking power of Prolog is easy:

```

parse(State, []) :- final(State).
parse(State, [H|R]) :- trans(State, New, H), parse(State, R).
parse(State, Inp) :- empty(State, New), parse(State, Inp).
parse(Inp) :- start(S), parse(S, Inp).
  
```

# Example: Numbers game

- Given list of numbers, i.e. [1,3,5,9,30], find arithmetic operations to calculate the given number, i.e.  $331 = (9-3+5)*30+1$
- `findit (numlist, number, expression)`
- Pick two numbers from list, pick an operator, compose an expression and put it back on the remaining list and try with new list. When the expression evaluates to the number. it is a success.

```
% if top element evaluates to number, it is the result
findit([A|_],V,A) :- V is A.
% pick two values, pick one operand, check if valid,
% compose an expression, test with new expression
findit(L,V,T) :- select(A,L,AR), select(B,AR,Rest),
    member(O,[+,-,/,*]), check(O,A,B), E =.. [O,A,B],
    findit([E|Rest],V,T).

check(+,_,_).
check(*,_,_).
check(-,A,B) :- A>B.
check(/,A,B) :- 0 is A mod B.
```

Implementation creates duplicate results because of symmetry in '+' and '\*'. In symmetric operators use combination, `taketwo` which will get `a`, `b` once, not `b,a`.

```
taketwo(A, B, [A|R] , Rem ) :- select(B, R, Rem).
taketwo(A, B, [H|R], [H|Rem]) :- taketwo(A, B, R, Rem).

% if top element evaluates to number, it is the result
findit([A|_],V,A) :- V is A.
% asymmetric ops.
findit(L,V,T) :- select(A,L,AR), select(B,AR,Rest),
    member(O,[-,//]), check(O,A,B), E =.. [O,A,B],
    findit([E|Rest],V,T).
% symmetric ops.
findit(L,V,T) :- taketwo(A,B,L,Rest),
    member(O,[+,*]), E =.. [O,A,B],
    findit([E|Rest],V,T).

check(-,A,B) :- A>B.
check(//,A,B) :- 0 is A mod B.
```

# Example: Symbolic Differentiation

- Given an expression containing the variable, find the derivative of the expression with respect to that variable.
- `diff(exp, var, diffexp).`
- `diff(3*x*x+2x+1, x, D)` results in `D = 6*x+2..`

```
diff(A, A, 1) :- !.      % x -> 1
diff(A, _, 0) :- number(A). % c -> 0
diff(-A, X, -C) :- diff(A, X, C). % -f(x) = - f(x)
diff(A+B, C, D+E) :- diff(A, C, D), diff(B, C, E).
diff(A-B, C, D-E) :- diff(A, C, D), diff(B, C, E).
diff(A*B, C, A*D) :- number(A), diff(B, C, D), !.
diff(A*B, C, A*D+B*E) :- diff(A, C, E), diff(B, C, D).
diff(A/B, C, D) :- diff(A*B^(-1), C, D).
diff(A^B, C, B*A^B1*D) :- number(B), diff(A, C, D), B1 is B -
diff(log(A), B, C*A^ -1) :- diff(A, B, C).

?- diff(1/((x^2+1)*(x-1)), x, R).
R = 1* (-1* ((x^2+1)* (x-1)) ^ (-1-1)*
      ((x^2+1)* (1-0)+ (x-1)* (2*x^ (2-1)*1+0))).
```

A simplifier is needed.

```

constant(X) :- number(X),!.
constant(A):- A =.. [_ ,T1,T2] , constant(T1),constant(T2).
constant(A) :- A =.. [_ ,T1] , constant(T1).
s(X,Y) :- constant(X),!,Y is X.
s(A*1,Y) :- simp(A,Y),!.
s(1*A,Y) :- simp(A,Y),!.
s(-1*A,-Y) :- simp(A,Y),!.
s(A+0,Y) :- simp(A,Y),!.
s(0+A,Y) :- simp(A,Y),!.
s(A-0,Y) :- simp(A,Y),!.
s(0-A,-Y) :- simp(A,Y),!.
s(A^1,AE) :- simp(A,AE),!.
s(1^_,1) :- !.
s(_^0,1) :- !.
s(X,X).

simp(A*B,R) :- simp(A,AE),simp(B,BE),s(AE*BE,R),!.
simp(A+B,R) :- simp(A,AE),simp(B,BE),s(AE+BE,R),!.
simp(A-B,R) :- simp(A,AE),simp(B,BE),s(AE-BE,R),!.
simp(A^B,R) :- simp(A,AE),simp(B,BE),s(AE^BE,R),!.
simp(X,X).
derivative(X,R) :- diff(X,Z), simp(Z,R).

?- derivative(1/((x^2+1)*(x-1)), x, R).
R = - ((x^2+1)*(x-1))^-2* (x^2+1+ (x-1)* (2*x)).

```