

Programming Language Concepts

Syntax and Parsing

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Introduction

- **Syntax**: the form and structure of a program.
- **Semantics**: meaning of a program
- Language definitions are used by:
 - Programmers
 - Implementors of the language processors
 - Language designers

Definitions

- A **sentence** is a string of characters over some alphabet
- A **language** is a set of sentences
- A **lexeme** is the lowest level syntactic unit of the language (i.e. `++`, `int`, `total`)
- A **token** is a category of lexemes (i.e. *identifier*)

Definitions

- **syntax recognition**: read input strings of the language and verify the input belonging to the language
- **syntax generation**: generate sentences of the language (i.e. from a given data structure)
- Compilers and interpreters recognize syntax and convert it into machine understandable form.

Backus-Naur Form and CFGs

- CFG's introduced by Noam Chomsky (mid 1950s)
- Programming languages are usually in **context free language** class
- BNF introduced by John Bakus and modified by Peter Naur for describing Algol language
- BNF is equivalent to CFGs. It is a **meta-language** that describes other languages
- Extended BNF improves readability of BNF

A Grammar Rule

$\langle \text{while_stmt} \rangle \rightarrow \text{while } (\langle \text{logic_expr} \rangle) \langle \text{stmt} \rangle$

- LHS is a non-terminal denoting an intermediate phrase
- LHS can be defined (rewritten) as the RHS sequence which can contain terminals (lexems and tokens) of the language and other non-terminals
- Non-terminals are denoted as strings enclosed in angle brackets.
- $::=$ may be used in BNF notation instead of the arrow
- $|$ is used to combine multiple rules with same LHS in a single rule

$$\begin{array}{lcl} \langle \text{lgc_cons} \rangle ::= \text{true} & \equiv & \langle \text{lgc_cons} \rangle ::= \text{true} \mid \text{false} \\ \langle \text{lgc_cons} \rangle ::= \text{false} & & \end{array}$$

Context Free Grammar

- A **grammar** G is defined as $G = (N, \Sigma, R, S)$:
 - N , finite set of non terminals
 - Σ , finite set of terminals
 - R is a set of grammar rules. A relation from N to $(N \cup \Sigma)^*$.
 - $S \in N$ the start symbol
- Application of a rule maps one sentential form into the other by replacing a non-terminal element in sentential form with its right handside sequence in the rule, $u \mapsto v$.
- Language of a grammar $L(G) = \{w \mid w \in \Sigma^*, S \xrightarrow{*} w\}$

- Recursive or list like structures can be represented using recursion

$$\langle \text{expr_list} \rangle \rightarrow \langle \text{expr} \rangle , \langle \text{expr_list} \rangle$$

$$\langle \text{btree} \rangle \rightarrow \langle \text{head} \rangle (\langle \text{btree} \rangle , \langle \text{btree} \rangle)$$

- A **derivation** starts with a starting non-terminal and rules are applied repeatedly to end with a sentence containing only terminal symbols.
- **leftmost derivation**: always leftmost non-terminal is chosen for replacement
- **rightmost derivation**: always rightmost non-terminal is chosen for replacement
- Same sentence can be derived using leftmost, rightmost, or other derivations.

Sample Grammar

$$\begin{aligned}\langle \text{stmt} \rangle &\rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle \\ \langle \text{expr} \rangle &\rightarrow \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \mid \langle \text{id} \rangle \\ \langle \text{op} \rangle &\rightarrow + \mid * \\ \langle \text{id} \rangle &\rightarrow a \mid b \mid c\end{aligned}$$

- Leftmost derivation of $a = a * b$:

$$\begin{aligned}\langle \text{stmt} \rangle &\mapsto \langle \text{id} \rangle = \langle \text{expr} \rangle \mapsto a = \langle \text{expr} \rangle \\ &\mapsto a = \langle \text{id} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \mapsto a = a \langle \text{op} \rangle \langle \text{expr} \rangle \\ &\mapsto a = a * \langle \text{expr} \rangle \mapsto a = a * \langle \text{id} \rangle \mapsto a = a * b\end{aligned}$$

- Rightmost derivation of $a = a * b$:

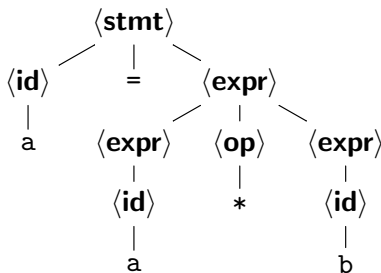
$$\begin{aligned}\langle \text{stmt} \rangle &\mapsto \langle \text{id} \rangle = \langle \text{expr} \rangle \mapsto \langle \text{id} \rangle = \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \\ &\mapsto \langle \text{id} \rangle = \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{id} \rangle \mapsto \langle \text{id} \rangle = \langle \text{expr} \rangle \langle \text{op} \rangle b \\ &\mapsto \langle \text{id} \rangle = \langle \text{expr} \rangle * b \mapsto \langle \text{id} \rangle = \langle \text{id} \rangle * b \\ &\mapsto \langle \text{id} \rangle = a * b \mapsto a = a * b\end{aligned}$$

Parse Tree

- Steps of a derivation gives the structure of the sentence. This structure can be represented as a tree.
- All non-terminals used in derivation are **intermediate nodes**. Each grammar rule replaces the non-terminal node with its children. Root node is the start symbol.
- Terminal nodes are the **leaf nodes**.
- preorder traversal of leaf nodes gives the resulting sentence.
- leftmost and rightmost derivations can be retrieved by traversal of the tree.

Parse Tree Example

a = a * b

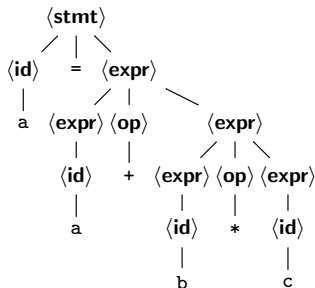


Parse Tree Generation

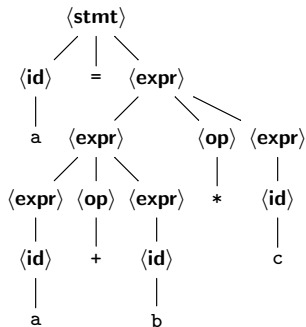
- A parse tree gives the structure of the program so semantics of the program is related to this structure.
- For example local scopes, evaluation order of expressions etc.
- During compilation, parse trees might be required for code generation, semantic analysis and optimization phases.
- After a parse tree generated, it can be traversed to do various tasks of compilation.
- The processing of parse tree takes too long, so creation of parse trees is usually avoided.
- Approaches like [syntax directed translation](#) combines parsing with code generation, semantic analysis etc..

Ambiguous Grammars

- Consider $a = a + b * c$ in our grammar:



VS



- Both can be derived by the grammar!

- A grammar is called **ambiguous** if same sentence can be derived by following different set of rules, thus resulting in a different parse tree
- If structure changes semantic meaning of the program, ambiguity is a serious problem.
- Even if not, which one is the result?
- i.e. Precedence of operators affects the value of the expression.
- Programming languages enforces precedence rules to resolve ambiguity.
- Solution:
 - 1 design grammar not to be ambiguous, or
 - 2 during parsing, choose rules to generate the correct parse tree

Precedence and Grammar

- Operators with different precedence levels should be treated differently
- Higher precedence operations should be deep in the parse tree
→ their rules should be applied later.
- Lower precedence operations should be closer to root → applied earlier in derivation.
- For each precedence level, define a non-terminal
- One rewritten on the other based on the precedence lower to higher

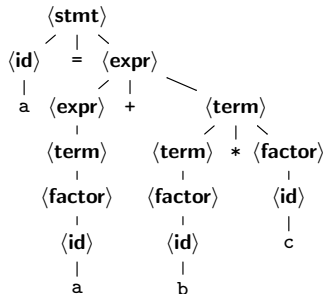
Rewritten Grammar

$$\langle \text{stmt} \rangle \rightarrow \langle \text{id} \rangle = \langle \text{expr} \rangle$$

$$\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \mid \langle \text{term} \rangle$$

$$\langle \text{term} \rangle \rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle \mid \langle \text{factor} \rangle$$

$$\langle \text{factor} \rangle \rightarrow \langle \text{id} \rangle \mid (\langle \text{expr} \rangle)$$

$$\langle \text{id} \rangle \rightarrow a \mid b \mid c$$


- `⟨term⟩` and `⟨expr⟩` has different precedence.
- Once inside of `⟨term⟩`, there is no way to derive `+`
- Only one parse possible

Associativity

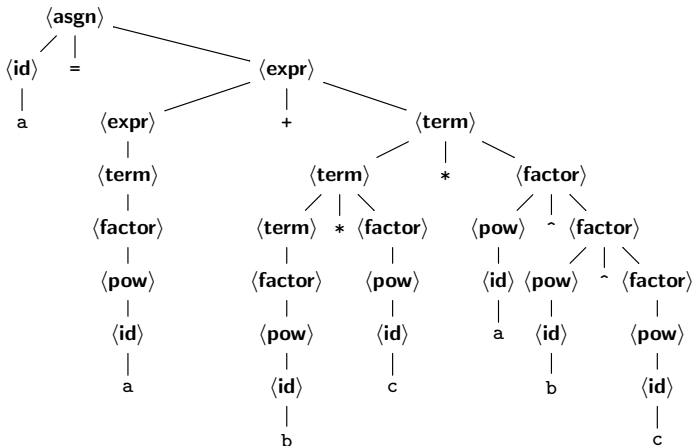
- Associativity of operators is another issue
 $a - b - c \equiv (a - b) - c \quad \text{or} \quad a - (b - c)$
- Recursion of grammar defines how tree is constructed for operators in the same level.
- If left recursive, later operators in the sentence will be closer to root, if right recursive earlier operators will be closer to root
- **left recursion** implies left associativity, **right recursion** implies right associativity.
- Consider $a + b + c$ in these grammars:

$$\begin{array}{l} \langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle + \langle \text{id} \rangle \mid \langle \text{id} \rangle \\ \langle \text{id} \rangle \rightarrow a \mid b \mid c \end{array} \quad \text{VS} \quad \begin{array}{l} \langle \text{expr} \rangle \rightarrow \langle \text{id} \rangle + \langle \text{expr} \rangle \mid \langle \text{id} \rangle \\ \langle \text{id} \rangle \rightarrow a \mid b \mid c \end{array}$$

Sample Grammar

$$\begin{aligned}
 \langle \text{asgn} \rangle &\rightarrow \langle \text{id} \rangle = \langle \text{asgn} \rangle \mid \langle \text{id} \rangle = \langle \text{expr} \rangle \\
 \langle \text{expr} \rangle &\rightarrow \langle \text{expr} \rangle + \langle \text{term} \rangle \mid \langle \text{term} \rangle \\
 \langle \text{term} \rangle &\rightarrow \langle \text{term} \rangle * \langle \text{factor} \rangle \mid \langle \text{factor} \rangle \\
 \langle \text{factor} \rangle &\rightarrow \langle \text{pow} \rangle ^ \langle \text{factor} \rangle \mid \langle \text{pow} \rangle \\
 \langle \text{pow} \rangle &\rightarrow \langle \text{id} \rangle \mid (\langle \text{expr} \rangle) \\
 \langle \text{id} \rangle &\rightarrow a \mid b \mid c
 \end{aligned}$$

- $\langle \text{asgn} \rangle$ is right recursive like right associative C assignments.
- $\langle \text{expr} \rangle$ and $\langle \text{term} \rangle$ are left recursive, $*$ and $+$ left associative
- $\langle \text{factor} \rangle$ is right recursive for power operation $^$ to be right associative.
- precedence order is $(\dots) \prec ^ \prec * \prec + \prec =$

$$a = a + b * c * a \wedge b \wedge c$$


if-then-else ambiguity

- Following grammar is ambiguous:

$$\langle \text{stmt} \rangle \rightarrow \langle \text{if-stmt} \rangle$$

$$\langle \text{if-stmt} \rangle \rightarrow \text{if } \langle \text{logic-expr} \rangle \text{ then } \langle \text{stmt} \rangle \mid$$

$$\text{if } \langle \text{logic-expr} \rangle \text{ then } \langle \text{stmt} \rangle \text{ else } \langle \text{stmt} \rangle$$

- Consider if a then if b then x=1 else x=0:

if-then-else ambiguity

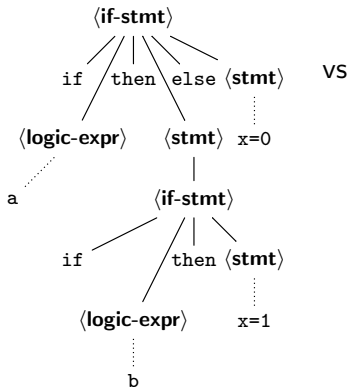
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if-then-else ambiguity

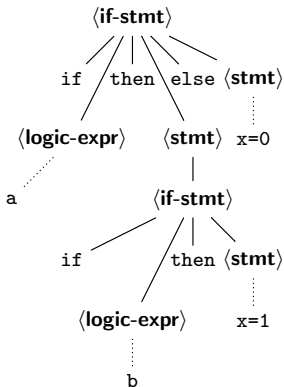
- Following grammar is ambiguous:

$$\langle \text{stmt} \rangle \rightarrow \langle \text{if-stmt} \rangle$$

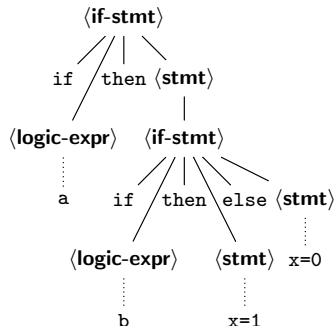
$$\langle \text{if-stmt} \rangle \rightarrow \text{if } \langle \text{logic-expr} \rangle \text{ then } \langle \text{stmt} \rangle \mid$$

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- Consider if a then if b then x=1 else x=0:



VS



Solution

- Distinguish categories of statements. if's matched with else and unmatched:

```

<stmt> → <matched> | <unmatched>
<matched> → if <logic-expr> then <matched> else <matched> |
            <other-stmt>
<unmatched> → if <logic-expr> then <stmt> |
              if <logic-expr> then <matched> else <unmatched>

```

- if a then if b then x=1 else x=0:

Solution

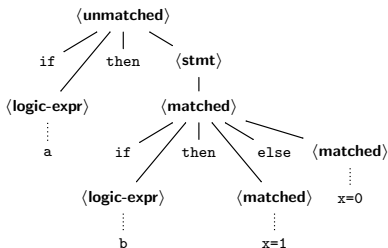
- Distinguish categories of statements. if's matched with else and unmatched:

$$\langle \text{stmt} \rangle \rightarrow \langle \text{matched} \rangle \mid \langle \text{unmatched} \rangle$$

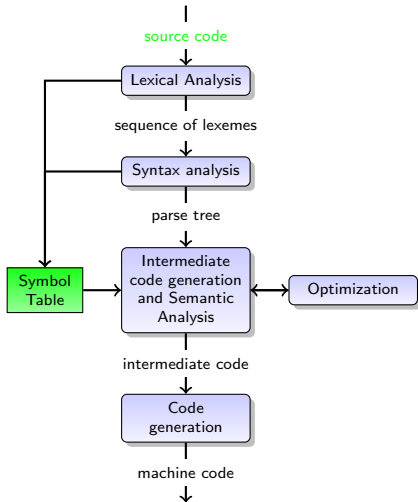
$$\langle \text{matched} \rangle \rightarrow \text{if } \langle \text{logic-expr} \rangle \text{ then } \langle \text{matched} \rangle \text{ else } \langle \text{matched} \rangle \mid \langle \text{other-stmt} \rangle$$

$$\langle \text{unmatched} \rangle \rightarrow \text{if } \langle \text{logic-expr} \rangle \text{ then } \langle \text{stmt} \rangle \mid \text{if } \langle \text{logic-expr} \rangle \text{ then } \langle \text{matched} \rangle \text{ else } \langle \text{unmatched} \rangle$$

- if a then if b then x=1 else x=0:

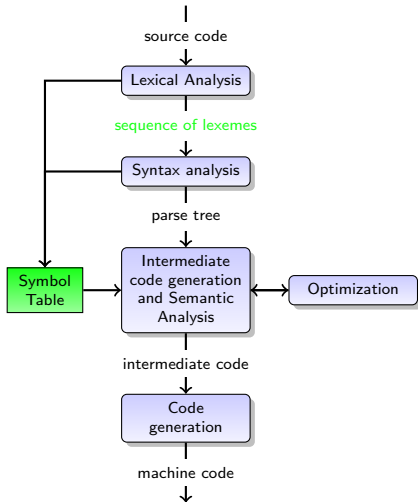


Compilation



```
while (counter < 12341) {  
    f() ;  
    counter += 12;  
}
```

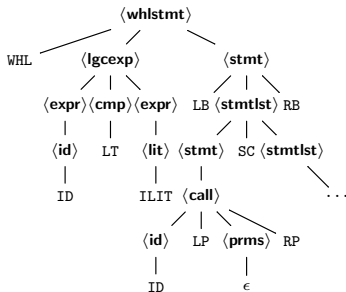
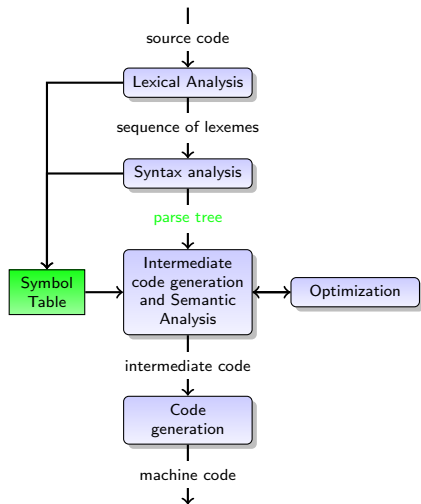
Compilation



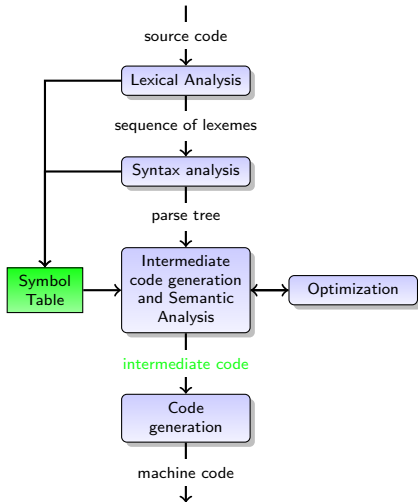
```

WHL LP ID LT ILIT RP LB
      ID LP RP SC
      ID PLEQ ILIT SC
RB
  
```

Compilation



Compilation

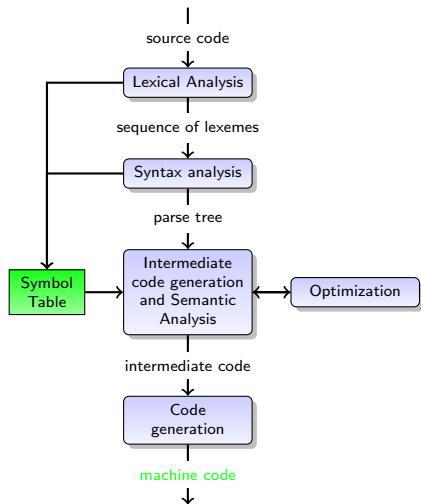


```

.L3:      movl    $0, %eax
          call   f
          addl   $12, -4(%rbp)

.L2:      cmpl   $12340, -4(%rbp)
          jle    .L3
          leave  .L3
          ret
  
```

Compilation



```

001 f 0001 0006 0000
0034 0021 0000 0000
0004 0000 001b 0009
0000 0000 02f4 0008
0008 0001 0004 0008
0025 0001 0003 0000
0058 0000 0000 0000
0004 0000 002b 0008
  
```

Lexical Analysis

- **input**: sequence of characters, source code.
- **output**: sequence of lexemes
- Worst case complexity of parsing is $\mathcal{O}(n^3)$. Depending on algorithm type, recursion type and number of grammar rules, this might change. n is the length of the string.
- Regular language processing complexity is $\mathcal{O}(n)$. Grammars can be defined in terms of lexemes.
- # of chars vs # of lexemes?
- Lexical analysis convert character sequences into lexemes.
Identifiers registered on symbol table

Parsing

- **input**: sequence of lexemes (output of lexical analysis) or characters.
- **output**: parse tree, intermediate code, translated code, or sometimes only if document is valid or not.
- Two main classes of parser:
 - Top down parsing
 - Bottom up parsing

Top-down Parsing

- Start from the starting non-terminal, apply grammar rules to reach the input sentence

$$\begin{aligned}
 \langle assign \rangle &\mapsto a = \langle expr \rangle \mapsto a = \langle expr \rangle + \langle term \rangle \mapsto \\
 &a = \langle term \rangle + \langle term \rangle \mapsto a = \langle fact \rangle + \langle term \rangle \mapsto \\
 &a = a + \langle term \rangle \mapsto a = a + \langle term \rangle * \langle fact \rangle \mapsto \\
 &a = a + \langle fact \rangle * \langle fact \rangle \mapsto a = a + b * \langle fact \rangle \mapsto \\
 &a = a + b * a
 \end{aligned}$$

- Simplest form gives leftmost derivation of a grammar processing input from left to right.
- Left recursion in grammar is a problem. Elimination of left recursion needed.
- **Deterministic parsing:** Look at input symbols to choose next rule to apply.
- **recursive descent parsers, LL family parsers** are top-down parsers

Recursive Descent Parser

```

typedef enum {ident, number, lparen, rparen, times,
             slash, plus, minus} Symbol;
int accept(Symbol s) { if (sym == s) { next(); return 1; }
                      return 0;
}
void factor(void) {
    if (accept(ident)) ;
    else if (accept(number)) ;
    else if (accept(lparen)) { expression(); expect(rparen);}
    else { error("factor: syntax error at ", currsym); next(); }
}
void term(void) {
    factor();
    while (accept(times) || accept(slash))
        factor();
}
void expression(void) {
    term();
    while (accept(plus) || accept(minus))
        term();
}

```

- Each non-terminal realized as a parsing function
- Parsing functions calls the right handside functions in sequence
- Rule choices are based on the current input symbol. accept checks a terminal and consumes if matches.
- Cannot handle direct or indirect left recursion. A function has to call itself before anything else.
- Hand coded, not flexible.

LL Parsers

- First L is 'left to right input processing', second is 'leftmost derivation'
- Checks next N input symbols to decide on which rule to apply: $LL(N)$ parsing.
- For example $LL(1)$ checks the next input symbol only.
- $LL(N)$ parsing table: A table for $V \times \Sigma^N \mapsto R$
- for expanding a nonterminal $NT \in V$, looking at this table and the next N input symbols, $LL(N)$ parser chooses the grammar rule $r \in R$ to apply in the next step.

- Grammar and lookup table for a LL(1) parser:

$$1 \quad S \rightarrow E$$

$$2 \quad S \rightarrow -E$$

$$3 \quad E \rightarrow N + E$$

$$4 \quad E \rightarrow (E)$$

$$5 \quad N \rightarrow a$$

$$6 \quad N \rightarrow b$$

	a	b	-	(
S	1	1	2	1
E	3	3		4
N	5	6		

- What if we add $E \rightarrow N$ to grammar?
- You need an LL(2) grammar. What if N is recursive?

- Grammar and lookup table for a LL(1) parser:

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- What if we add $E \rightarrow N$ to grammar?
- You need an LL(2) grammar. What if N is recursive?
see LL(*) parser

Bottom-up Parsing

- Start from input sentence and merge parts of sentential form matching RHS of a rule into LHS at each step. Try to reach the starting non-terminal. reach the input sentence

$$\begin{aligned}
 a = a + b * a &\mapsto a = \langle fact \rangle + b * a \mapsto a = \langle term \rangle + b * a \mapsto \\
 a = \langle expr \rangle + b * a &\mapsto a = \langle expr \rangle + \langle fact \rangle * a \mapsto \\
 a = \langle expr \rangle + \langle term \rangle * a &\mapsto a = \langle expr \rangle + \langle term \rangle * \langle fact \rangle \mapsto \\
 a = \langle expr \rangle + \langle term \rangle &\mapsto a = \langle expr \rangle \mapsto \langle assign \rangle
 \end{aligned}$$

- Simplest form gives rightmost derivation of a grammar (in reverse) processing input from left to right.
- Shift-reduce parsers are bottom-up:
 - **shift**: take a symbol from input and push to stack.
 - **reduce**: match and pop a RHS from stack and reduce into LHS.

Shift-Reduce Parser in Prolog

```

% Grammar is E-> E-T/E+T/T  T -> a/b
rule(e,[e,-,t]).
rule(e,[e,+,t]).
rule(e,[t]).
rule(t,[a]).
rule(t,[b]).

parse([], [S]) :- S = e .  % starting symbol alone in the stack
% reduce: find RHS of a rule on stack, reduce it to LHS
parse(Input, Stack) :- match(LHS, Stack, Remainder),
                        parse(Input, [LHS|Remainder]).

% shift: nonterminals are removed from input added on stack
parse([H|Input], Stack) :- member(X, [a,b,-,+]),
                           parse(Input, [H|Stack]).

% check if RSH of a rule is a prefix of Stack (reversed).
match(LHS, List, L) :- rule(LHS, RHS),  reverse(RHS, NRHS),
                      prefix(NRHS, List, L).

```


- Shift reduce parser tries all non-deterministic shift combinations to get all parses.
- For deterministic parsing states based on input lookahead or precedence required
- Deterministic bottom up parsers: LALR, SLR(1).