

### 13.1 Introduction

- Stat-arb analysts sift through volumes of historical data with the objective of identifying a pervasive statistical relationship.
  - ➔ Price levels between two different securities
  - ➔ Price level of one security and the volatility of another
- At least 90-percent confidence level

### 13.2 Mathematical Foundations

1. Identify the universe of liquid securities
  - ➔ Desired trading frequency unit에서 최소한 한 번은 거래되어야 한다.
2. Measure the difference between prices of every two securities

$$\Delta S_{ij,t} = S_{i,t} - S_{j,t}, \quad t \in [1, T]$$

(price level of  $i$  security at  $t : S_{i,t}$ )

- ➔ Intra-day data는 time-seasonality가 존재해서 CLT 적용을 위해, frequency unit으로 30개 데이터가 아닌, 30일 데이터를 준비해야 한다. 또한, Robust inference 위해서, 500 daily observation (2 years) 권장한다.
3. Select the most stable relationship – security pairs that move together

$$\min_{i,j} \sum_{t=1}^T (\Delta S_{ij,t})^2$$

➔ Cointegration 사용해서 구한다.

- cointegration : 두 시계열 간 회귀방정식 변수의 장기 균형 관계를 나타내며, 시계열 데이터가 장기적으로 얼마나 같은 추세를 지니는지 확인할 때 사용됨

4. Basic distributional properties

- Mean

$$E[\Delta S_t] = \frac{1}{T} \sum_{t=1}^T \Delta S_t$$

- Standard Deviation

$$\sigma[\Delta S_t] = \frac{1}{T-1} \sum_{t=1}^T (\Delta S_t - E[\Delta S_t])^2$$

5. Monitor and act (particular time  $\tau$  )

- Sell security  $i$  and buy security  $j$

$$\Delta S_\tau = S_{i,\tau} - S_{j,\tau} > E[\Delta S_\tau] + 2\sigma[\Delta S_\tau]$$

- Buy security  $i$  and sell security  $j$

$$\Delta S_\tau = S_{i,\tau} - S_{j,\tau} < E[\Delta S_\tau] - 2\sigma[\Delta S_\tau]$$

6. Clear position when price arrive particular level

- 통계적 이상 가격을 찾기 위해서, 위와 비슷하게, 다른 변수 or 지표 가지고도 stat-arb 진행해 볼 수 있다. (correlation, fundamental relationship)
- For detecting statistical anomalies in price levels, statistical arbitrage can be applied to other variable with similar way (correlation, fundamentals)
- Without economical logic, detected statistical relationships are random and spurious and have little predictive staying power. (black-box)
- Issues embedded in stat-arb
  1. Positive probability of bankruptcy
  2. Transaction cost
  3. Spread size
  4. Position size of chosen stocks for pair trading

### 13.3 Practical Applications of Statistical Arbitrage

- High-Frequency statistical arbitrage based on economic models has ex-ante longer staying power

➔ Ex) Fundamental arbitrage model : using fundamental economic principles

- Examples of securities pairs

1. Same industry

➔ Respond similarly to changes in the broad market

2. Shares of the same class issued by same company but trading on different exchange

➔ Different conditions among different exchanges result in price deviation

3. Passive indexes

➔ Temporary price deviations between an index fund and its constituents

#### i. Foreign Exchange

- **Triangular arbitrage**

➔ Make 'Synthetic exchange' for stat-arb

$$EUR/CAD_{Synthetic,bid} = EUR/USD_{Market,bid} \times USD/CAD_{Market,bid}$$

➔ Compare synthetic exchange rate with real exchange rate for stat-arb

- **Uncovered interest parity arbitrage (UIP)** : 유위험 이자율 평형 차익거래

➔ UIP gives the best prediction of changes in fx rates at hft and daily rates (Chaboud and Wright(2005))

$$1 + i_{t,USD} = (1 + i_{t,CHF}^*) \frac{E_t[S_{t+1,CHF/USD}]}{S_{t,CHF/USD}}$$

■ Regression form

$$\ln(S_{t+1,CHF/USD}) - \ln(S_{t,CHF/USD}) = \alpha + \beta(\ln(1 + i_{t,USD}^*) - \ln(1 + i_{t,CHF}^*)) + \varepsilon_{t+1}$$

#### ii. Equities

- Popular pair trading strategies

: different equity classes of the same issuer, market-neutral, liquid arbitrage, large-to-small information spillovers

- **Arbitrage different equity classes of the same issuer**

➔ Hypothesis : Expectation that stocks corresponding to two common equity classes issued by the same company trade within a relatively constant price range.

➔ But 1) Voting rights, and 2) number of shares outstanding make price difference

- Premium for superior voting right compared with inferior voting right

: 해당 국가나 거래소의 투명성, 규제 등등을 고려해 다르게 분포한다. (대체로 투명성이 적을수록 superior voting right에 대한 프리미엄이 커짐)

- Higher number of shares outstanding are usually more liquid, which are valued more highly

: more liquid class of shares is likely to incorporate market information significantly faster than the less liquid share class, creating potential for information arbitrage.

➔ Dual-class share (차등의결권 주식)

: 1주당 의결권이 서로 다른 주식

- Dual-class share를 관리하는 회사 수가 적어, 전략 이용에 제한이 있다.

- 적은 유동성을 가진 class가 존재하게 되는데, 그 주식들은 daily trade volume이 매우 적어, 전략 수용에 제한이 있다.

- **Market-Neutral Arbitrage**

➔ CAPM에서 비슷한 베타를 가진 pair를 만들고 같은 양의 long & short을 넣는다.

➔ 그렇게 되면 알파의 차이가 곧 수익이 된다. (long higher  $\alpha$ , short lower  $\alpha$ )

$$\Delta\hat{\beta} = \hat{\beta}_i - \hat{\beta}_j, \quad \widehat{\sigma_{\Delta\beta}} = \sqrt{\frac{\sigma_{\beta i}^2}{n_i} + \frac{\sigma_{\beta j}^2}{n_j}} \quad \rightarrow \quad t_{\beta} = \frac{\Delta\hat{\beta}}{\widehat{\sigma_{\Delta\beta}}}$$

→ 조건

$$t_{\beta} \in [\Delta\hat{\beta} - \widehat{\sigma}_{\Delta\beta}, \Delta\hat{\beta} + \widehat{\sigma}_{\Delta\beta}], \quad \Delta\hat{\alpha} > TC, \quad |t_{\alpha}| > [\Delta\hat{\alpha} + 2\widehat{\sigma}_{\Delta\alpha}]$$

→ CAPM에서 변수를 주어서 다른 펀더멘털 팩터로도 해볼 수 있다. (Fama-French..)

#### - Liquidity Arbitrage

→ 전통적 자산가격이론에 따르면, 유동성이 적으면 투자하기 불편하고 리스크가 커진다. 그로 인한 프리미엄이 존재했다. 그러나 프리미엄이 존재한다고 해서, 수익률이 절대적으로 '양수'라는 것이 아니다.

→ Pastor and Stamburgh (2003)

: If the market-wide liquidity is priced into individual asset returns, then market illiquidity arbitrage strategies may well deliver consistent positive abnormal returns on the risk-adjusted basis.

$$r_{i,t+1}^e = \theta + \beta r_{i,t} + \gamma \text{sign}(r_{i,t}^e) \cdot v_{i,t} + \tau_{t+1}$$

■ Sign은 direction을 의미하며, 거래량과 곱해져 있어서 거래량이 buy / sell order에 치중되어 있는지 표현하는 지표이다.

#### - Large-to-Small Information Spillovers

→ 시가총액이 작은 회사의 주식은 정보에 덜 민감하다.

→ 왜냐하면, 기관들이 더 정보에 민감하게 반응하는데, small firm의 주식일수록 그들에게 좋은 투자 대상이 아니므로,,,

→ Huge bid-ask spread(비유동적)를 가진 smaller-cap은 거래량이 뛰는 기간에 모멘텀을 보여주었고, Large-cap은 그런게 적었다.

### iii. Futures

#### - Basis trading

→ Arbitrage using difference between the spot and futures prices

→ Hypothesis : difference between the spot and futures prices reverts to its mean or median values.

- **Futures/Equity Arbitrage**

- ➔ Kawaller, Koch and Koch(1993)

- : Futures markets have been shown to adjust more quickly than spot markets

- ➔ Stoll and Whaley (1990)

- : both S&P500 and money market index futures led stock market returns by 5 to 10 minutes.

- Because of development in trading system, the price deviation between spot and futures decrease more rapidly, which is 1- to 2-second arbitrage.

- iv. Indexes and ETFs

- **Law of one price**에 의해서 펀드는 구성성분의 가격 총합과 같아야 한다.

- Arbitrage opportunity : relative mispricing of indexes and their underlying components

### 13.4 Conclusion

- Statistical arbitrage based on solid economic theories is likely to have longer staying power than strategies based purely on statistical phenomena.