Hastily-written notes for Sebastian, revised

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1 Fitting PCN spectra

The electric field at the photodiode can be approximated as the sum of two components: though I have tried my best to suppress it, there is some residual "flat" response unrelated to the signal coming off the PCN; to this we add the response of the PCN. We end up with a field that looks like

$$E(\Delta) = \alpha + \beta \frac{\gamma}{\Delta - \mathrm{i}\gamma}, \tag{1}$$

where α is the amplitude of the "flat" response, β the amplitude of the PCN response, which has linewidth γ and varies depending on the detuning from resonance, Δ. At optical frequencies we measure intensities, not the field directly, meaning our signal is proportional to

$$I(\Delta) \propto |E(\Delta)|^2 = \left(\alpha + \beta \frac{\gamma}{\Delta - \mathrm{i}\gamma}\right)^* \left(\alpha + \beta \frac{\gamma}{\Delta - \mathrm{i}\gamma}\right) \tag{2}$$

where * represents the complex conjugate. γ and Δ are real quantities, but unfortunately α and β are in general complex. Only the relative phase between them matters, so we'll just set the phase of α to zero and make the substitution $\beta \to \beta e^{\mathrm{i}\phi}$, now with $\{\alpha, \beta, \gamma, \Delta, \phi\} \in \mathbf{R}$. This gives us

$$I(\Delta) \propto \alpha^2 + \beta^2 \frac{\gamma^2}{\Delta^2 + \gamma^2} + \alpha\beta e^{\mathrm{i}\phi} \frac{\gamma}{\Delta - \mathrm{i}\gamma} + \alpha\beta e^{-\mathrm{i}\phi} \frac{\gamma}{\Delta + \mathrm{i}\gamma} \tag{3}$$

where the simplification of those last two terms into sine and cosine components is left as an exercise for the doctoral student (I have the expression somewhere, just not within arm's reach). In the case $\alpha = 0$ this reduces to a Lorentzian lineshape, as should be expected.

2 Additional complexities

In the setup we actually have there is an unfortunate etalon upstream of the sample. When we need to take this into account we can treat the model in section 1 as a transfer function:

$$E_{\text{out}} = \left(\alpha + \beta \frac{\gamma}{\Delta - \mathrm{i}\gamma}\right) E_{\text{in}} \tag{4}$$

and modify E_{in} appropriately. The canonical way to treat an etalon is to consider the infinite series

$$E_{\text{out}} = t^2 E_{\text{in}} \left(1 + r^2 e^{2\mathrm{i}kL} + r^4 e^{4\mathrm{i}kL} + \ldots\right), \tag{5}$$

where t and r are the amplitude (complex) reflection and transmission coefficients[1], L is the length of the etalon and k the propagation coefficient within the etalon medium (to a good approximation $k = n\omega/c$, where n is the bulk refractive index). For the generic case, just assume $kL \equiv \Phi$, i.e., some accumulated phase. What is the closed form of this expression? It ends up being a geometric series.

[1] Actually, each interface has four coefficients resulting from possibly different transmissions and reflections on either side of the interface, but we aren't going for perfect accuracy: just trying to capture behavior.

3 A short introduction to phase modulation as it applies to our experiment

As I mentioned the other day, I am too lazy to calibrate a Mach–Zehnder interferometer to calibrate scale in our experiment, opting instead to use a phase modulator to put frequency sidebands on the laser. Mathematically, that is equivalent to saying I am creating an electric field that looks like

$$E(\omega, t) = A \exp\left(\mathrm{i}(\omega t + \phi(t))\right) \tag{6}$$

where as always we are using a complex quantity to represent a real field with a phase. Here I've left $\phi(t)$ deliberately vague, but let's consider a phase modulated by a fixed frequency: $\phi(t) \to a\cos(\Omega t)$. In fact, to save time, let's go one step further and assume that there is a phase offset for this modulation: $\phi(t) \to a\cos(\Omega t + \Phi)$. This results in the potentially intimidating expression

$$E(\omega, t) = A \exp\left(\mathrm{i}\omega t\right) \exp\left(\mathrm{i}a\cos(\Omega t + \Phi)\right). \tag{7}$$

As children we would look at that second exponential term and say "Welp, that sucks. Maybe I can assume small Ω and Taylor expand or something?" And yes, you *could* try that. As grown-ups (and some of us proud Germans), all we need remember is that oscillators on top of oscillators can probably be treated using the Jacobi–Anger expansion:

$$\exp\left(\mathrm{i}a\cos(\Omega t + \Phi)\right) = \sum_{n=-\infty}^{\infty} \mathrm{i}^n J_n(a) \exp\left(\mathrm{i}n\left(\Omega t + \Phi\right)\right) \tag{8}$$

which shows that phase modulation results in an infinite series of frequencies spaced by the RF modulation Ω, potentially with a non-trivial phase relationship. Here we will only concern ourselves with the $n = 0$ and $n = 1$ terms, which in this case are

$$S_0 = J_0(a) \tag{9}$$

$$\begin{aligned} S_1 &= \mathrm{i}J_1(a)\exp(\mathrm{i}(\Omega t + \Phi)) \\ &= J_1(a)\exp\left(\mathrm{i}\left(\Omega t + \Phi + \frac{\pi}{2}\right)\right) \end{aligned} \tag{10}$$

This reveals an interesting property about the sidebands, namely the $n = \pm 1$ sidebands have a fixed phase relationship to each other (turns out they are phase separated by π—check for yourself), but not necessarily to the carrier, as Φ does not appear in the S_0 term.

You could dump this expression for $E(\omega, \Omega, \Phi, t)$ into the expressions on the first page and determine what the field at the photodetector looks like.