OUTPUT PRIMITIVES
(Chapter 3 in *Computer Graphics*)

- Points and Lines
- Line-drawing Algorithms
- Antialiasing Lines
- Line Command
- Fill Areas
- Circle-generating Algorithms
- Other Curves
- Character Generation
- Instruction Sets for Display Processors

Points and Lines

- plotting points
  - illuminate a phosphor dot on a CRT
    - for a random-scan CRT, deflect and activate the electron beam
    - for a raster-scan CRT, set a bit or byte at the appropriate location in the frame buffer
- plotting lines
  - specify the endpoint coordinates of each line segment
  - fill the straight line path between the endpoints
    - analog devices (random-scan CRTs, etc.) produce excellent lines
    - digital devices (raster-scan CRTs, plasma panels, pen plotters) use pixels near the true line, resulting in the "jaggies"

Line-drawing Algorithms

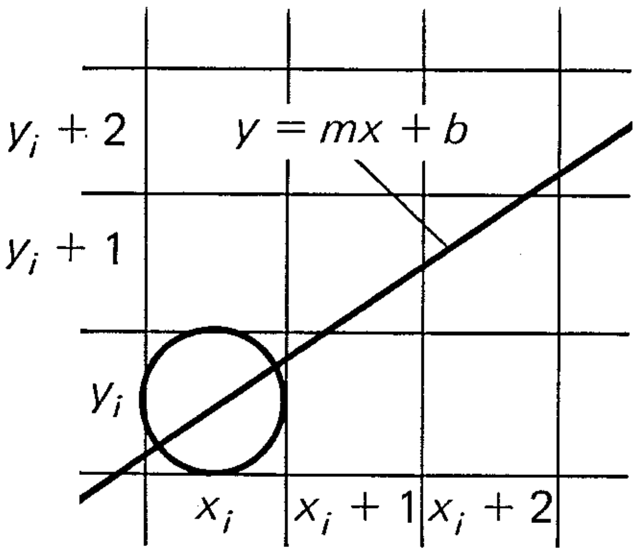
- DDA (Digital Differential Analyzer) algorithms
  - simple DDA
  - symmetric DDA
- Bresenham's algorithm
  - more efficient than DDA algorithms
    - adds
    - shifts
    - compares

Simple DDA

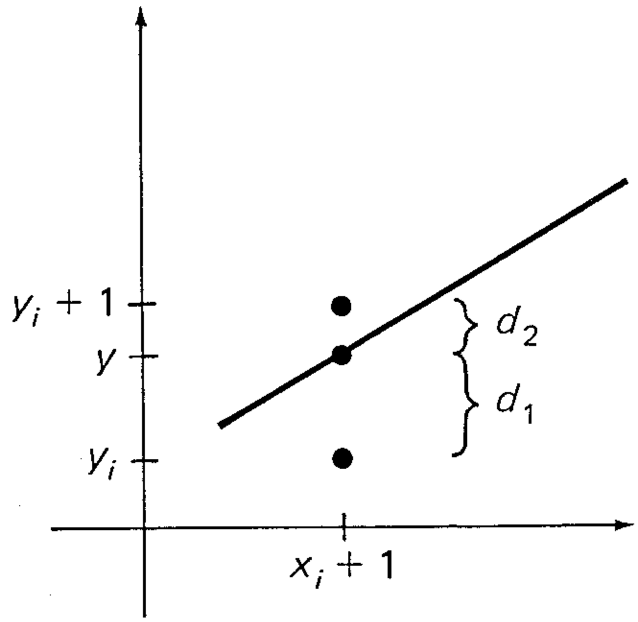
```
procedure dda (x1, y1, x2, y2 : integer);
   var
      dx, dy, steps, k : integer;
      x_increment, y_increment, x, y, : real;
   begin
      dx : = x2 - x1;
      dy : = y2 - y1;
      if abs(dx) > abs(dy) then steps : = abs(dx)
         else steps : = abs(dy);
      x_increment: = dx / steps;
      y_increment: = dy / steps;
      x : = x1 + 0.5; y : = y1 + 0.5;
      set_pixel (trunc(x), trunc (y));
      for k : = 1 to steps do begin
         x : = x + x_increment;
         y : = y + y_increment;
         set_pixel (trunc(x), trunc(y));
      end {for k}
   end; {dda}
```

Symmetric DDA

pick a line length estimate which is some power of 2 and greater than or equal to max (abs(dx), abs(dy))



y_i + 2
y = mx + b
y_i + 1
y_i
x_i
x_i + 1
x_i + 2



y_i + 1
y
y_i
d_2
d_1
x_i + 1

1. Input line end points. Store left endpoint in (x_1, y_1). Store right endpoint in (x_2, y_2).

2. The first point to be selected for display is the left endpoint (x_1, y_1).

3. Calculate $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$, and $p_1 = 2\Delta y - \Delta x$. If $p_1 < 0$, the next point to be set is $(x_1 + 1, y_1)$. Otherwise, the next point is $(x_1 + 1, y_1 + 1)$.

4. Continue to increment the x coordinate by unit steps. At position $x_i + 1$ the coordinate to be selected, y_{i+1}, is either y_i or $y_i + 1$, depending on whether $p_i < 0$ or $p_i \geq 0$. The calculations for each parameter p depend on the last one. If $p_i < 0$, the form for the next parameter is
$$p_{i+1} = p_i + 2\Delta y$$
But it $p_i \geq 0$, the next parameter is
$$p_{i+1} = p_i + 2(\Delta y - \Delta x)$$
Then, if $p_{i+1} < 0$, the next y coordinate to be selected is y_{i+1}. Otherwise select $y_{i+1} + 1$. (Coordinate y_{i+1} was determined to be either y_i or y_{i+1} by the parameter p_i in step 3.)

5. Repeat the procedures in step 4 until the x coordinate reaches x_2.

```
procedure bres_line (x1, y1, x2, y2: integer);
    var
        dx, dy, x, y, x_end, p, const1, const2 : integer;
    begin
        dx : = abs(x1 - x2);
        dy : = abs(y1 - y2);
        p : = 2 * dy - dx;
        const1 ; = 2 * dy;
        const2 : = 2 * (dy - dx);
        {determine which point to use as start, which as end}
        if xl > x2 then begin
            x : = x2; y : = y2;
            x_end : = x1
          end {if x1> x2}
        else begin
            x : = x1;y : = y1
            x_end : = x2
         end;{if x1< =x2}
        set_pixel (x, y);
        while x < x_end do begin
            x : = x + 1;
            if p < 0 then p : = p + const1
            else begin
                    y : = y + 1;
                    p : = p + const2
                 end: {else begin}
             set_pixel (x, y,)
            end {while x < x_end}
        end; {bres_line}
```

extensions for slopes > 1

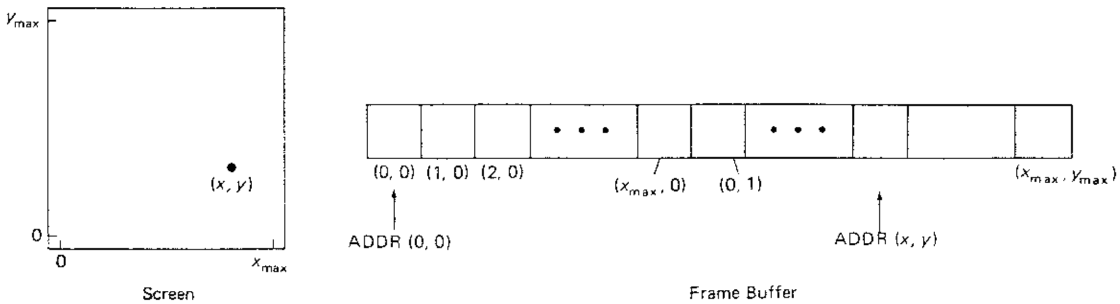
- interchange the roles of x and y

extensions for negative slopes

- decrement rather than increment

loading the frame buffer

- use the set_pixel procedure
- $ADDR(x,y) = ADDR(0,0) + y(x_{max}+1) + x$



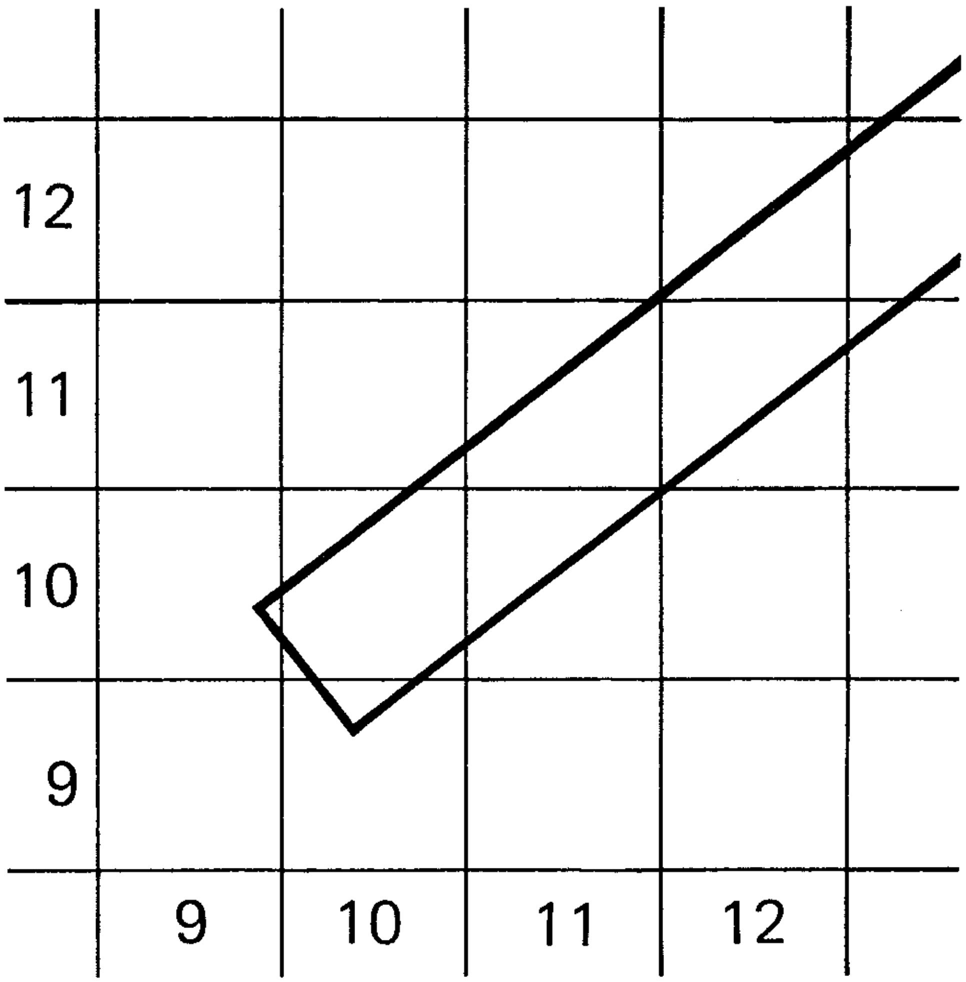
- simplification
  - $ADDR(x + 1,y) = ADDR(x,y) + 1$
  - $ADDR(x + 1, y + 1) = ADDR(x,y) + x_{max} + 2$

Antialiasing Lines

- aliasing: rounding to discrete integer pixel positions
- antialiasing: adjusting pixel intensities to remove the stairstep appearance

use of sampling theory

- pixels have measurable diameters
- lines have measurable thickness
- make intensity proportional to overlap



pixel phasing

- adjust pixel positions
  - 1/4 of a pixel diameter
  - 1/2 of a pixel diameter
- alter the sizes of individual pixels

Line Command

- plot
  - line segments
  - points (= very short line segments)
- polyline (n,x,y)
  - a single point, a single line segment or a series of connected line segments
  - n = number of vertices
    x = array of n coordinate values
    y = array of n coordinate values

Line Command, continued

- plotting a single point
  ```
  x[1] := 150;
  y[1] := 100;
  polyline (1,x,y);
  ```
- plotting a single line segment
  ```
  x[1] := 50;
  y[1] := 100;
  x[2] := 250;
  y[2] := 25;
  polyline (2,x,y);
  ```
- absolute coordinates
- relative coordinates
- current position

Fill Areas

- fill_area (n,x,y)
- a closed polygon defined by
  - (x[1],y[1]) to (x[2],y[2])
  - (x[2],y[2]) to (x[3],y[3])

    ⋮

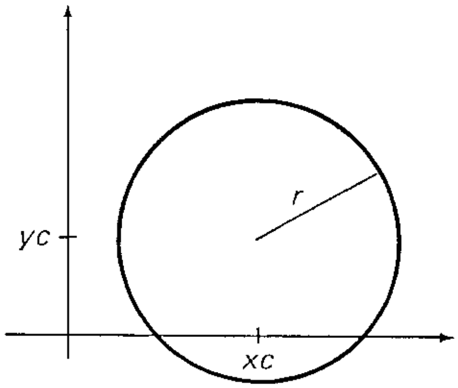
  - (x[n],y[n]) to x[1],y[1])
- fill with
  - border only
  - background
  - color
  - pattern

Circle-generating Algorithms

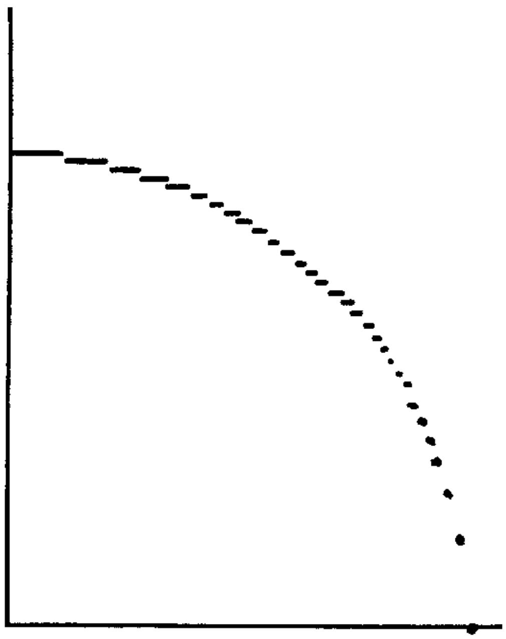
- DDA algorithm
- Bresenham algorithm

points on the circumference of a circle

- $(x - xc)^2 + (y - yc)^2 = r^2$



- $y = yc + [r^2 - (x - xc)^2]^{1/2}$
  - considerable computation
  - nonuniform spacing

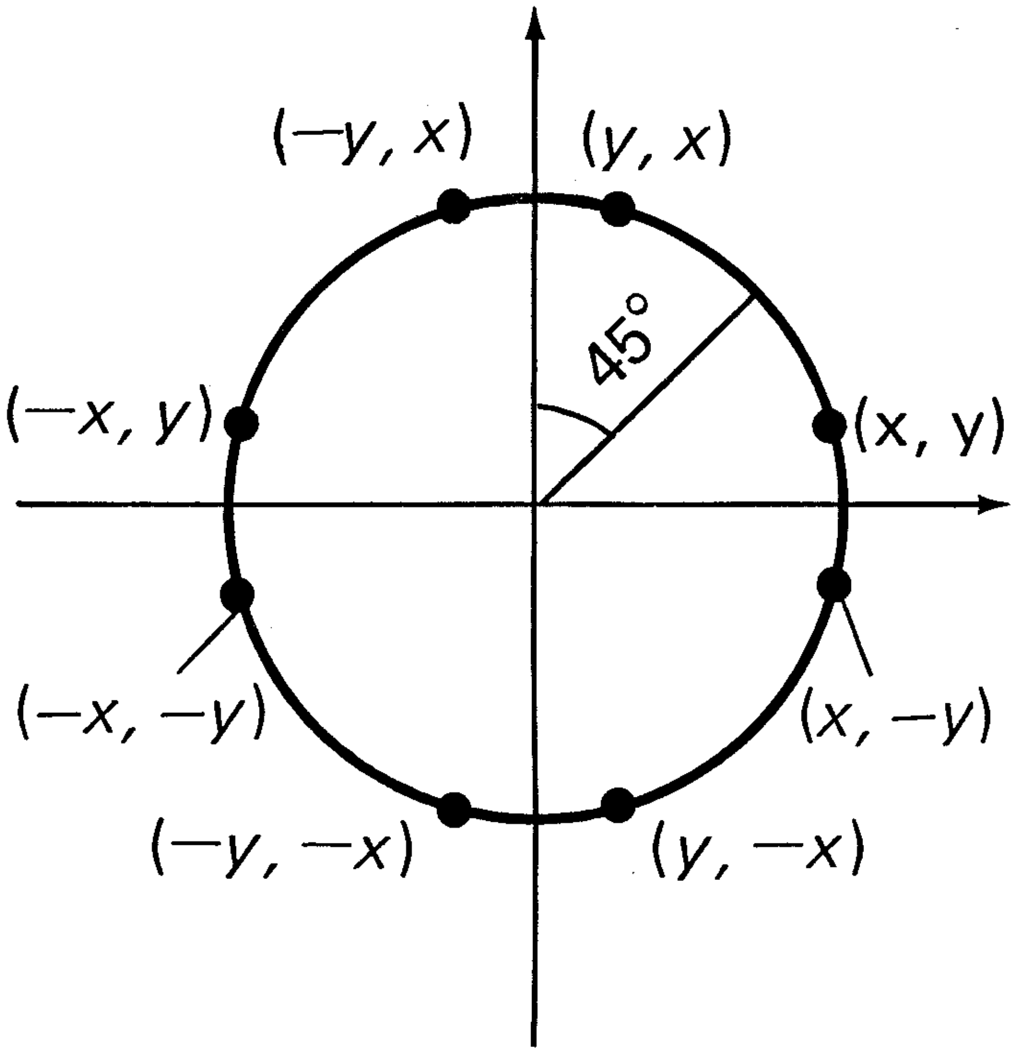


alternatives

- parametric polar form
  - $x = xc + r\cos(\Theta)$
  - $y = yc + r\sin(\Theta)$
- DDA
  - $dy/dx = -x/y$
  - $x' = x + (y)(\text{epsilon})$
  - $y' = y - (x')(\text{epsilon})$

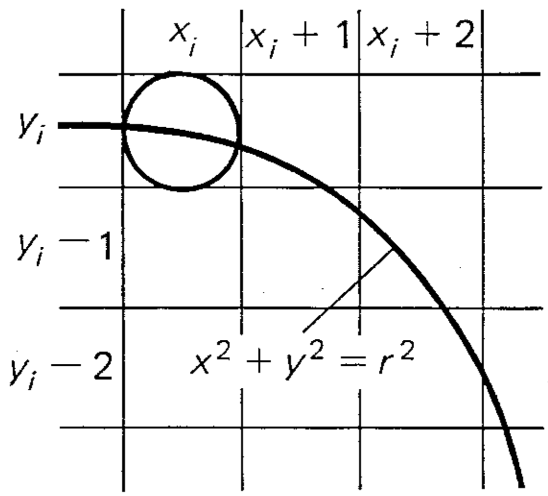
improvement

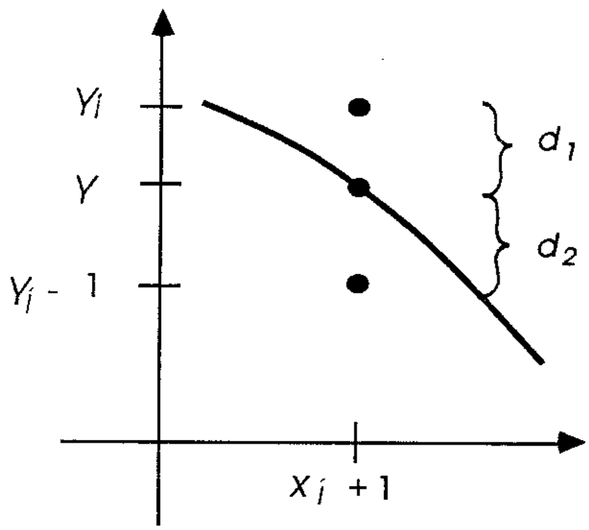
- calculate points from x = 0 to x = y
- take advantage of symmetry



Bresenham's circle algorithm

- determine which of two pixels is nearer the true boundary of the circle





- more efficient than other algorithms
  - adds
  - subtracts
  - shifts
  - compares

1. Select the first position for display as
$$(x_1,\ y_1) = (0,\ r)$$

2. Calculate the first parameter as
$$p_1 = 3 - 2r$$
If $p_1 < 0$, the next position is $(x_1 + 1,\ y_1)$. Otherwise, the next position is $(x_1 + 1,\ y_1 - 1)$.

3. Continue to increment the x coordinate by unit steps, and calculate each succeeding parameter p from the preceding one. If for the previous parameter we found that $p_i < 0$, then
$$p_{i+1} = p_i + 4x_i + 6.$$
Otherwise (for $p_i \geq 0$),
$$p_{i+1} = p_i + 4(x_i - y_i) + 10$$
Then, if $p_{i+1} < 0$, the next point selected is $(x_i + 2,\ y_{i+1})$. Otherwise, the next point is $(x_i + 2,\ y_{i+1} - 1)$. The y coordinate is $y_{i+1} = y_i$, if $p_i < 0$ or $y_{i+1} = y_i - 1$, if $p_i \geq 0$.

4. Repeat the procedures in step 3 until the x and y coordinates are equal.

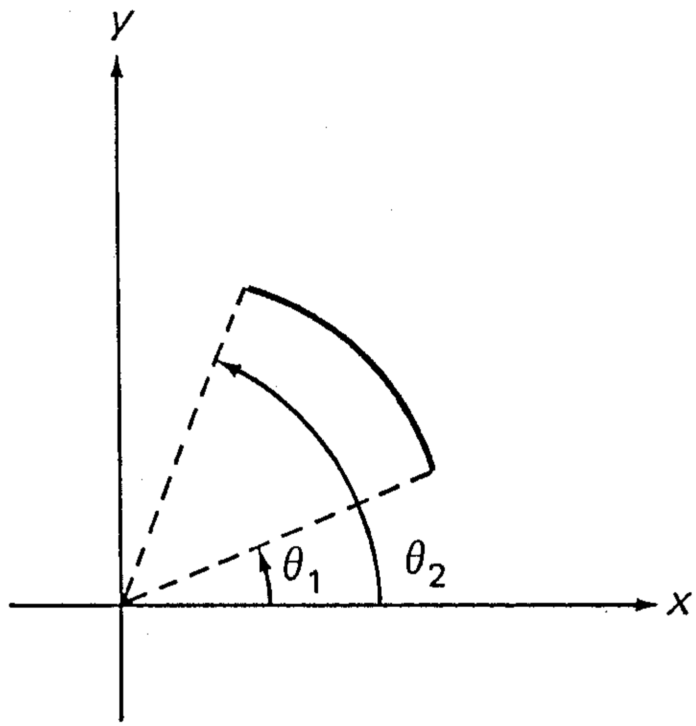
```
procedure bres_circle(x_center, y_center, radius : integer);
    var
      p,  x,  y  :  integer

    procedure  plot_circle_points:
      begin
        set_pixel  (x_center + x,  y_center + y);
        set_pixel  (x_center – x,  y_center + y);
        set_pixel  (x_center + x,  y_center – y);
        set_pixel  (x_center – x,  y_center – y);
        set_pixel  (x_center + y,  y_center + x);
        set_pixel  (x_center – y,  y_center + x);
        set_pixel  (x_center + y,  y_center – x);
        set_pixel  (x_center – y,  y_center – x);
      end;  {plot_circle_points}


    begin {bres-circle}
      x  : = 0;
      y  : = radius;
      p  : = 3 - 2 * radius;
      while x < y do begin
        plot_circle_points;
        if  p<0 then p  : = p + 4 * x + 6
        else  begin
            p  : = p + 4 * ( x – y) + 10
            y  : = y – 1
          end;  {if p not < 0}
        x  : = x + 1
        end;  {while x < y}
      if  x = y  then plot_circle_points;
    end;  {bres_circle}
```

plotting ellipses and circles

- ellipse (xc, yc, r_1, r_2)
- for circles, $r_1 = r_2$

plotting elliptical arcs and circular arcs

- ellipse (xc, yc, r_1, r_2, Θ_1, Θ_2)
  - Θ_1 = angle between x-axis and start of arc
  - Θ_2 = angle between x-axis and end of arc



Other Curves

- procedures are similar to those for circles and ellipses
- common curves
  - sine functions
  - exponential functions
  - polynomials
  - probability distributions
  - spline functions
- symmetry considerations can improve efficiency
- points along curves can be connected using short line segments or using other curve-fitting techniques

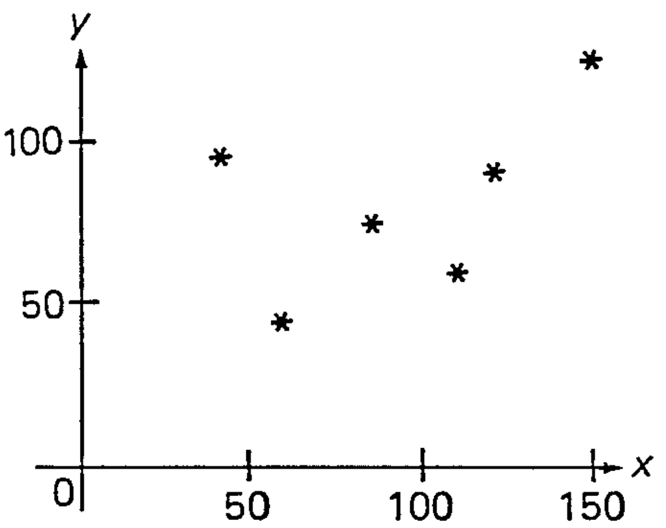
Character Generation

- rectangular grid patterns
  - 5-by-7 to 9-by-14
  - stored in read-only memory
  - copied into the frame buffer

1	1	1	1	1	1	0	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
0	1	1	1	1	1	0	0
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0

Output Commands

- text (x, y, string)
  - example: text (100, 450, "Winter in Rochester")
- polymarker (n, x, y)
  - example: polymarker (6, x, y)



x	y
41	94
59	43
85	74
110	59
121	89
149	122

Instruction Sets for Display Processors

- dependent on the type of device in use
- raster-scan systems
  - registers for coordinate values
    - endpoints
    - circle and ellipse parameters
    - positions for character strings and markers
  - fields
    - opcode: type of operation
    - address: register or memory location
  - instructions for loading the frame buffer
  - intensities are read from the frame buffer
- random-scan systems
  - instructions are used by the refresh process
  - load and execute the first instruction
  - increment the instruction counter
  - load and execute the next instruction

    •
    •
    •
  - reset the instruction counter and repeat

OUTPUT PRIMITIVES

- Points and Lines
- Line-drawing Algorithms
- Antialiasing Lines
- Line Command
- Fill Areas
- Circle-generating Algorithms
- Other Curves
- Character Generation
- Instruction Sets for Display Processors