THREE-DIMENSIONAL VIEWING
(Sections 12-1 and 12-2 in *Computer Graphics*)

- Projections
    - parallel projections
    - perspective projections
- Viewing Transformation
    - specifying the view plane
    - view volumes
    - clipping

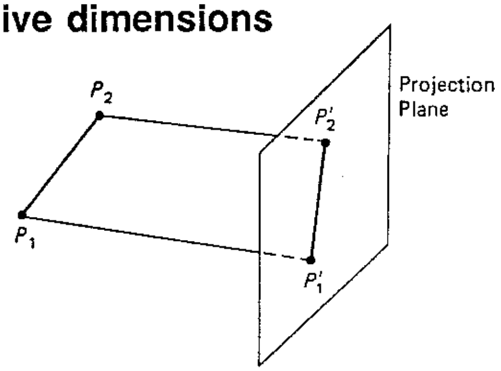
Introduction

- two-dimensional viewing
  - clip
  - map from the window to the viewport
  - convert from normalized device coordinates to physical device coordinates

- three-dimensional viewing
  - from where shall we view the scene?
    - inside?
    - outside?
      - above?
      - below?
      - from the side?
  - how shall we project the scene onto the two-dimensional viewing surface?

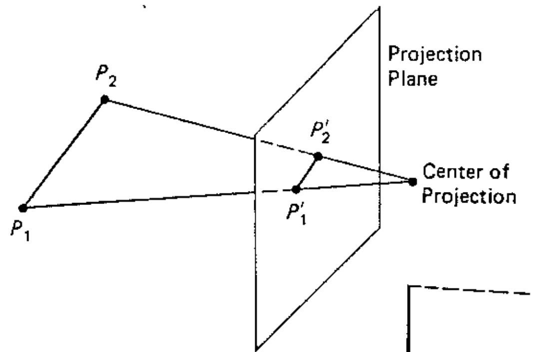
Projections

- parallel projections
  - points on an object are projected to the viewing surface along parallel lines
  - preserves relative dimensions

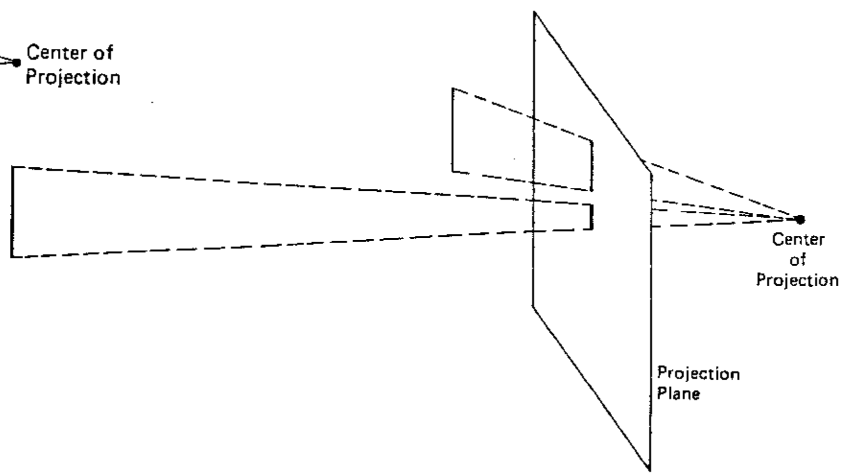


(a)
Parallel Projection

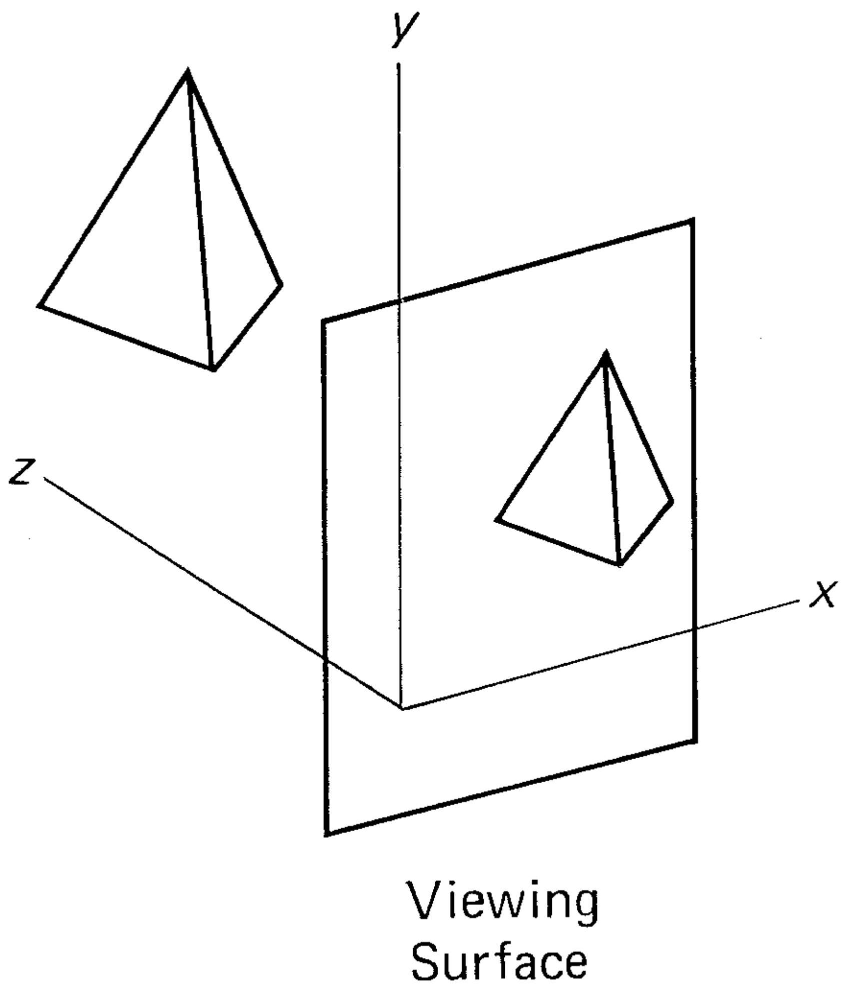
- perspective projections
  - points on an object are projected to the viewing surface along lines that converge to a center of projection
  - produces realistic views, but does not preserve relative dimensions
  - distant lines are foreshortened



(b)
Perspective Projection

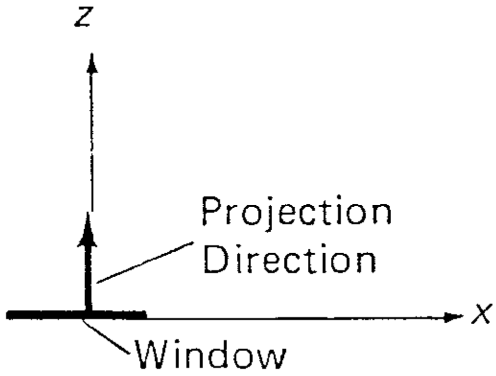


- we'll assume that the view projection plane is at z = 0 in left-handed coordinates

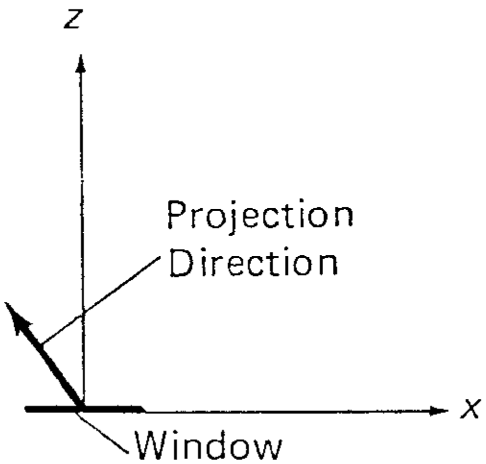


parallel projections

- orthographic projection: direction of projection is perpendicular to the projection plane

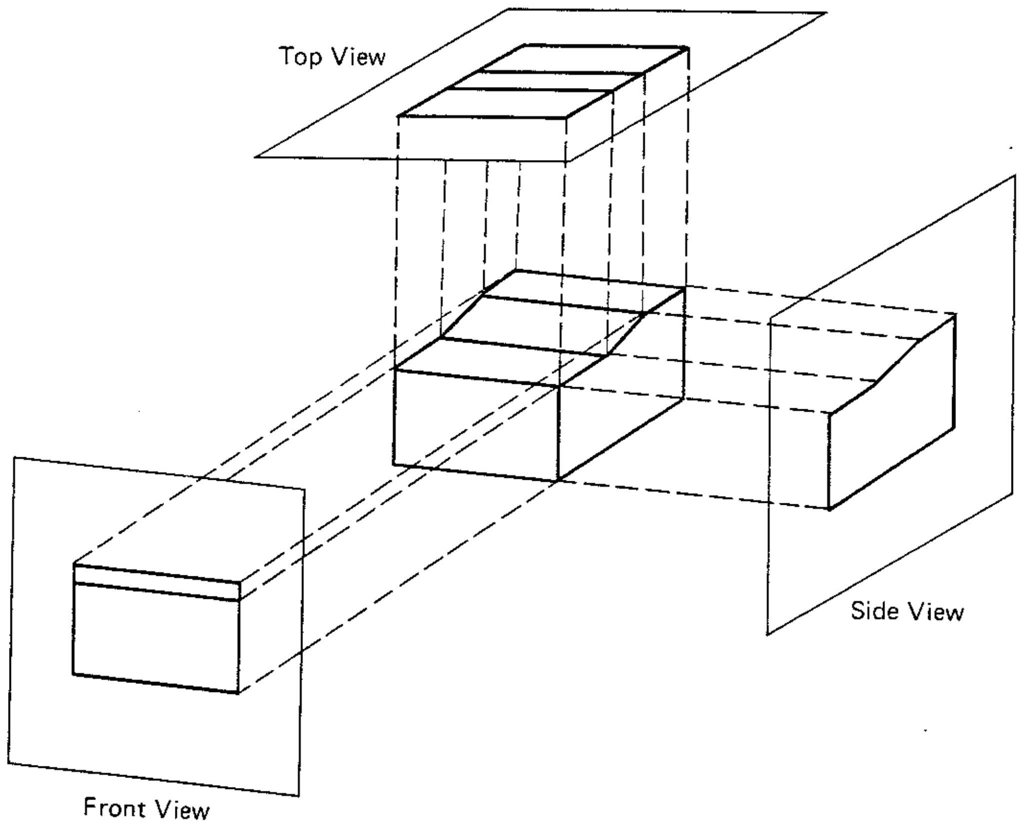


- oblique projection is not perpendicular to the projection plane



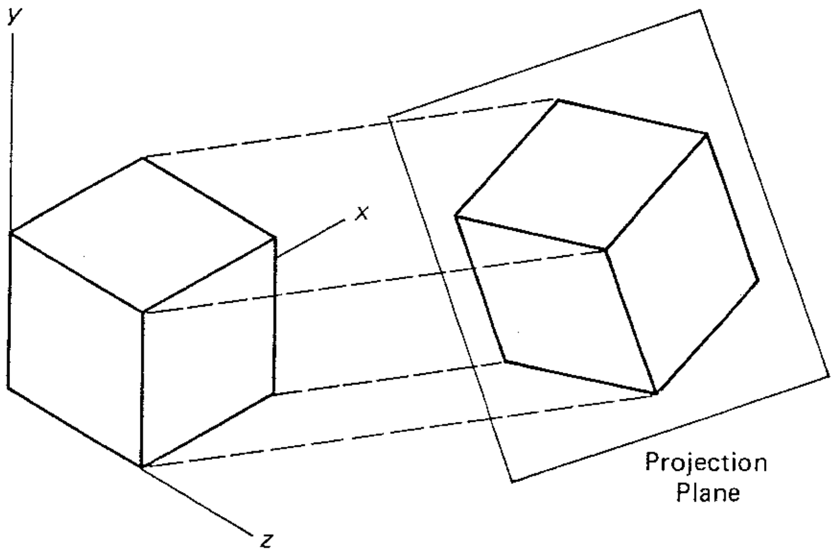
orthographic projections

- common projections
  - "elevations"
    - front
    - side
    - rear
  - "plan"
    - top



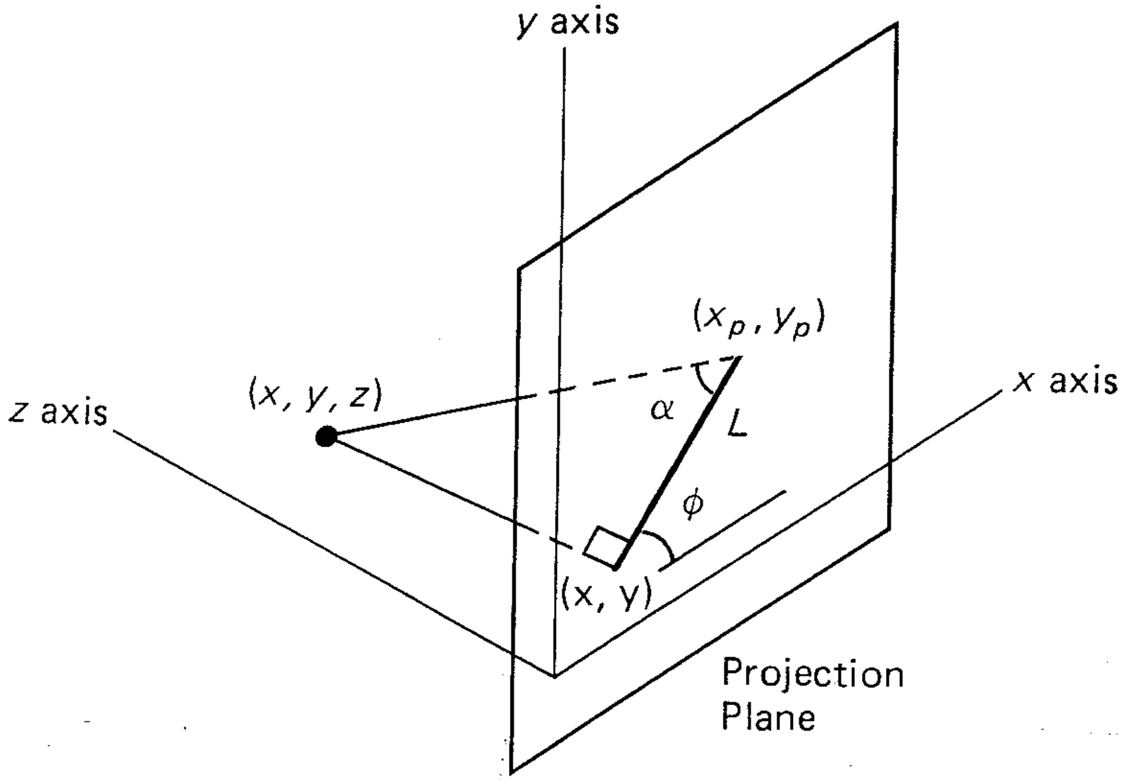
orthographic projections, continued

- axonometric projections
  - projection is not parallel to a principal axis
  - isometric: the projection-plane normal makes equal angles with each principal axis (all three principal axes are foreshortened equally, retaining relative proportions)



- transformation equations
  - orthographic parallel projection
    - $x_p = x$
    - $y_p = y$
    - $z_p = 0$

oblique projections



$$x_p = x + z(L \cos\phi)$$
$$y_p = y + z(L \sin\phi)$$

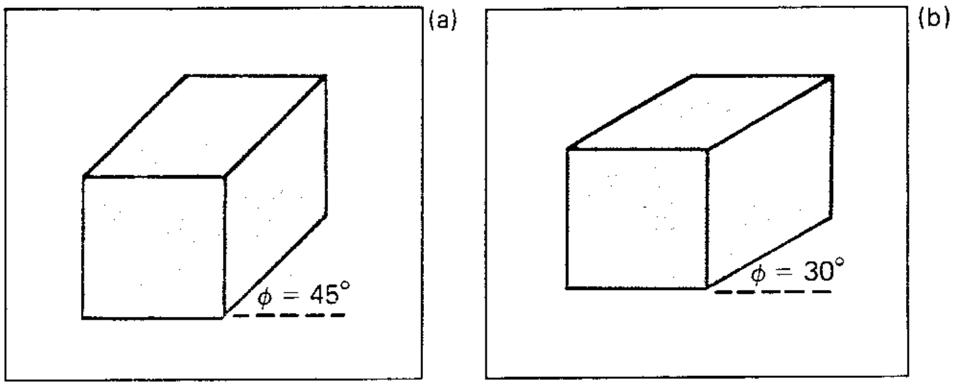
oblique projections, continued

$$P_{\text{parallel}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ L\ \cos\phi & L\ \sin\phi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

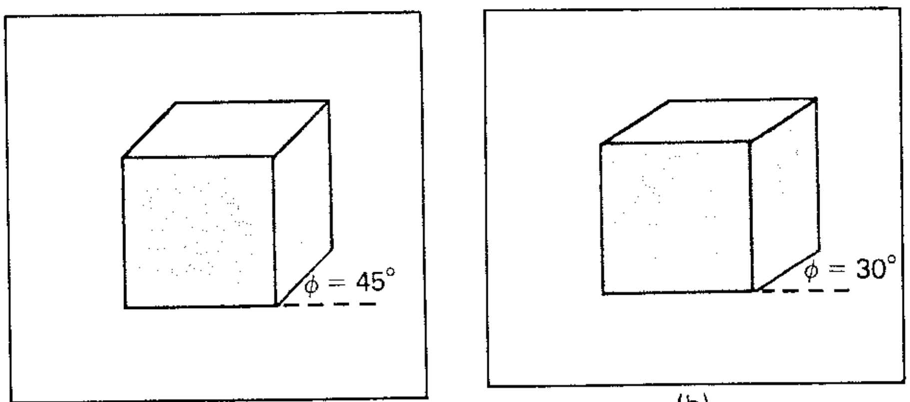
- **parallel orthographic projection**
  L = 0
- **parallel oblique projection**
  L ≠ 0

common parallel oblique projections

- cavalier: lines perpendicular to the projection plane are preserved in length



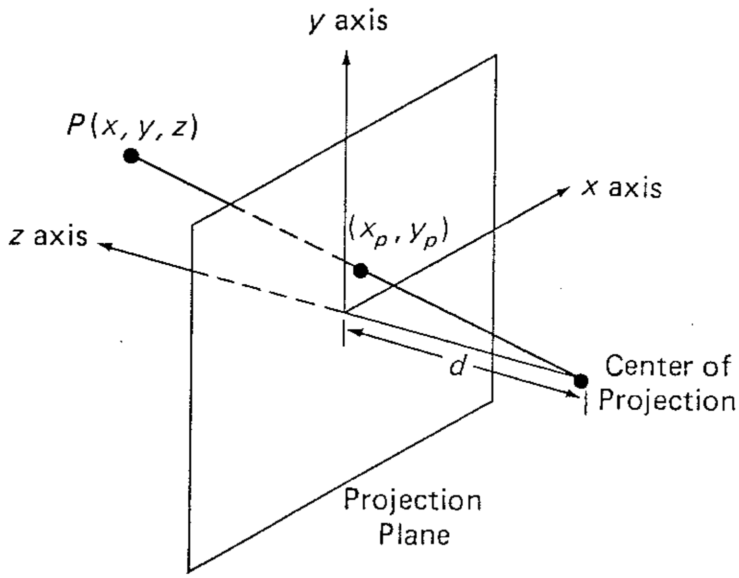
- cabinet: lines perpendicular to the projection plane have one-half their length



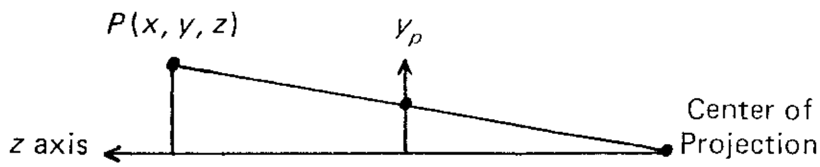
(a) (b)

perspective projections

- project points along projection lines that meet at the center of projection



- by similar triangles



$$y_p = y\left(\frac{d}{z + d}\right) = y\left(\frac{1}{z/d + 1}\right)$$

$$z_p = 0$$

perspective projections, continued

- in homogeneous coordinates

$$[x_h \; y_h \; z_h \; w] = [x \; y \; z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$w = \frac{z}{d} + 1$$

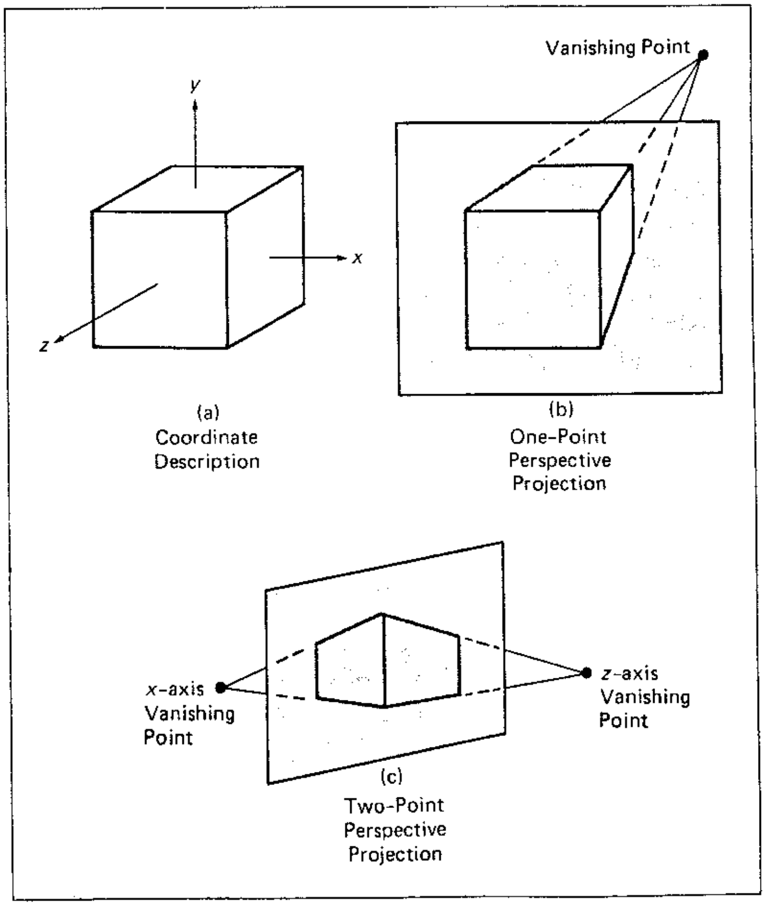
and

$$[x_p \; y_p \; z_p \; 1] = [x_h/w \; y_h/w \; z_h/w \; 1]$$

- the homogeneous coordinate must become 1
- in general, w is different for each coordinate

vanishing points

- parallel lines (which are not parallel to the projection plane) appear to converge at a vanishing point
- parallel lines which are parallel to a principal axis converge at a principal vanishing point
- the orientation of the projection plane determines the number of principal vanishing points, producing one-point, two-point, or three-point projections



(a) Coordinate Description

(b) One-Point Perspective Projection

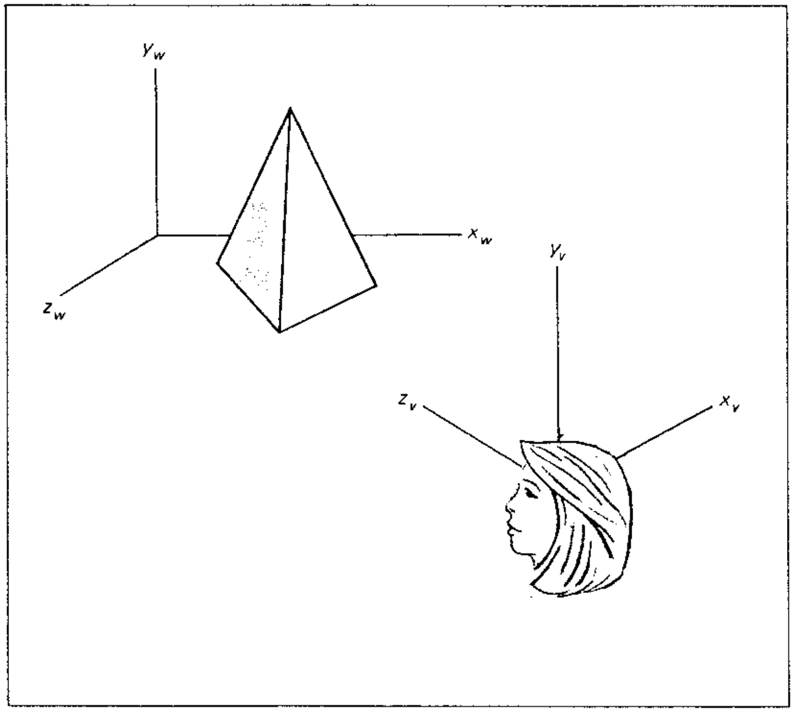
(c) Two-Point Perspective Projection

Viewing Transformation

- a three-dimensional scene can be viewed from any position in three-dimensional space, with any viewing direction and with the view plane in any orientation and of any size

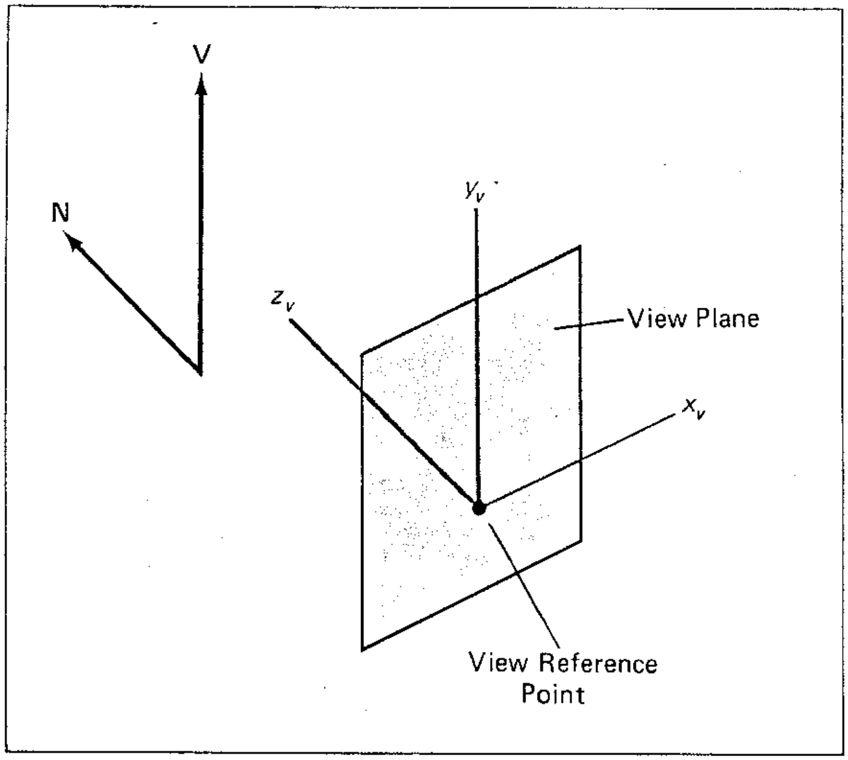
specifying the view plane

- define the view plane in the viewing (or eye) coordinate system



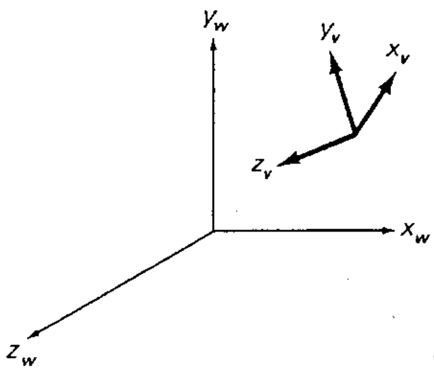
establishing viewing coordinates

- pick a view reference point to be the origin of viewing coordinates
- pick a view up vector relative to the viewpoint along the y axis in viewing coordinates
- use a left-handed coordinate system so that objects further away have larger z values

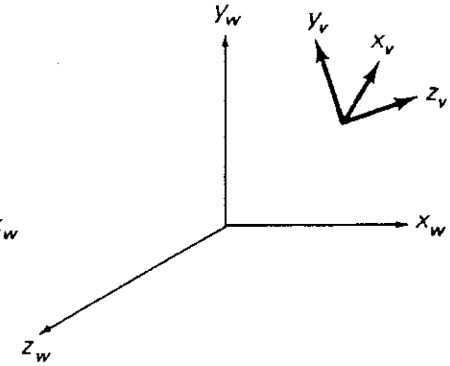


transform to the viewing coordinate system

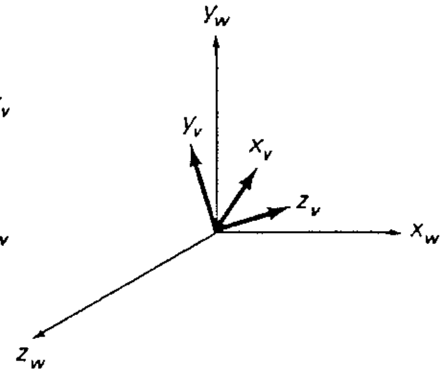
1. Reflect relative to the *xy* plane, reversing the sign of each *z* coordinate.
   This changes the left-handed viewing coordinate system to a right-handed system.
2. Translate the view reference point to the origin of the world coordinate system.
3. Rotate about the world coordinate *x* axis to bring the viewing coordinate *z* axis into the *xz* plane of the world coordinate system.
4. Rotate about the world coordinate *y* axis until the *z* axes of both systems are aligned.
5. Rotate about the world coordinate *z* axis to align the viewing and world *y* axes.



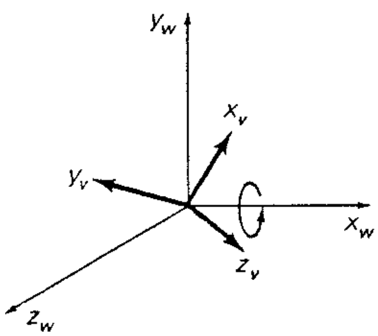
(a)
Original Orientation
of World and Viewing
Coordinate Systems



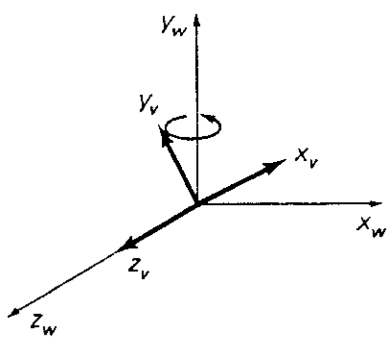
(b)
Invert Viewing
z Axis



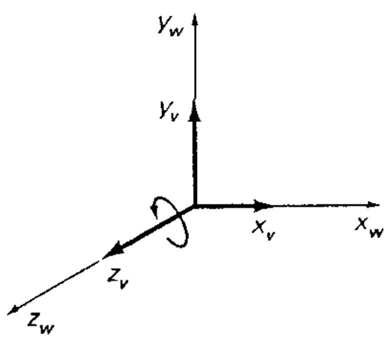
(c)
Translate Viewing
Origin to World Origin



(d)
Rotate About World
x Axis to Bring
Viewing z Axis into
the xz Plane of
the World System



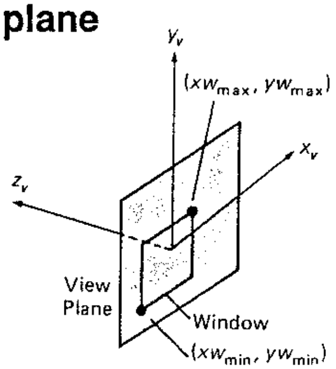
(e)
Rotate About
the World y
Axis to Align
the Two z Axes



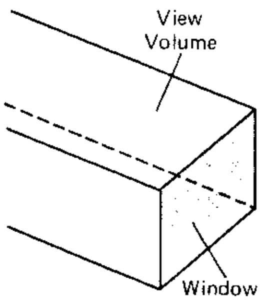
(f)
Rotate About
the World z
Axis to Align
the Two Viewing
Systems

view volumes

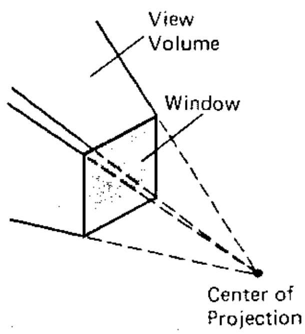
- a projection window determines how much of a scene is visible
  - defined by minimum and maximum values for x and y on the view plane



- the projection window defines a view volume
  - a parallel projection produces an infinite parallelepiped

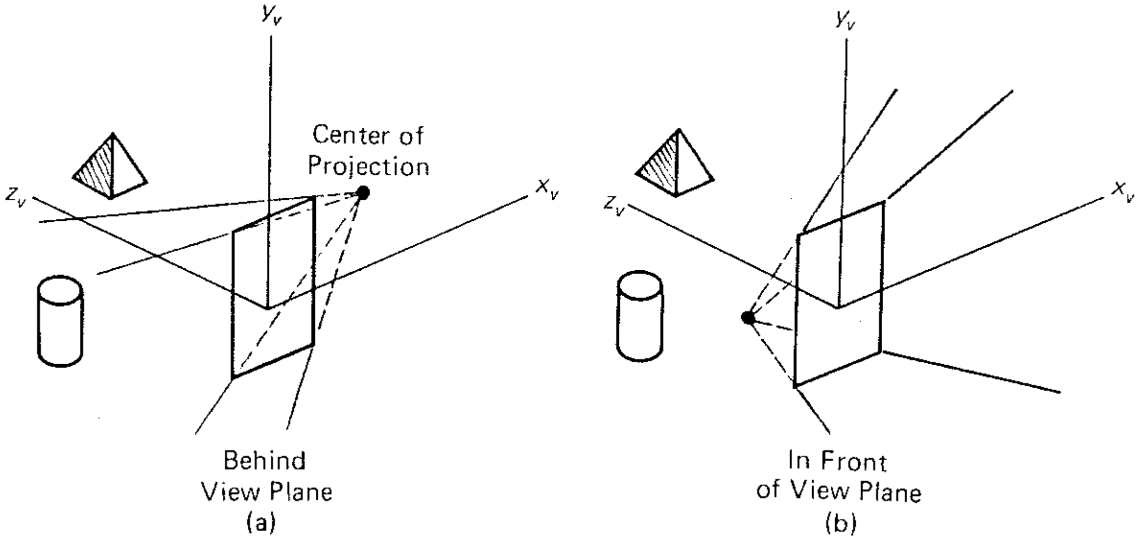


- a perspective projection produces a frustrum
  - (truncated pyramid) with the apex at the center of projection

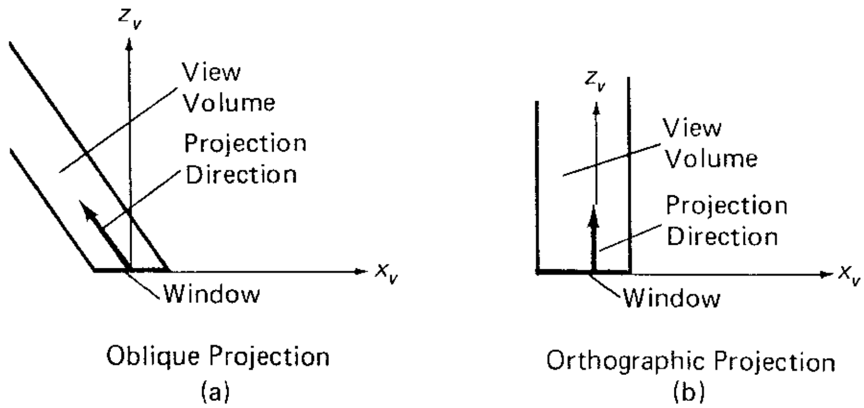


view volumes, continued

- the center of projection can be anywhere
- perspective projections can be orthographic or oblique to the view of plane

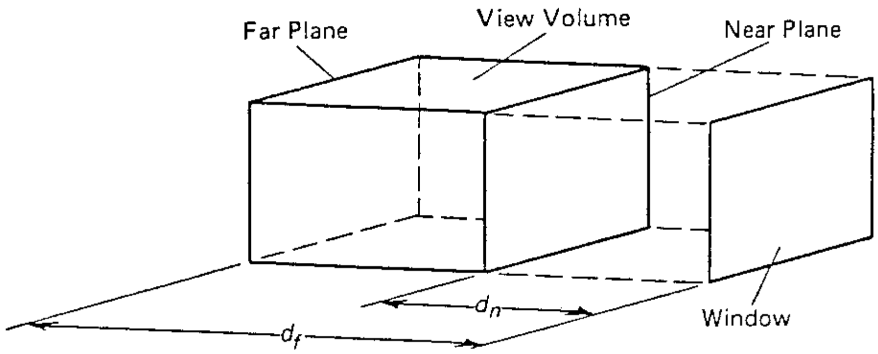


- parallel projections can be orthographic and oblique to the view plane

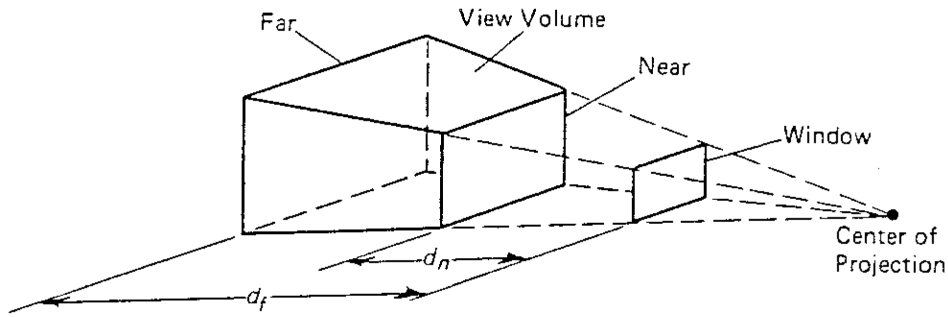


view volumes, continued

- a near plane and a far plane produce a finite view volume



(a)
Parallel Projection



(b)
Perspective Projection

clipping

- **save only points, line segments, and polygons within the view volume for projection onto the view plane**
- extend two-dimensional clipping methods
  test vertices (x, y, z) against view volume boundaries
  ($Ax + By + Cz + D = 0$)
  - > 0: outside
  - < 0: inside
  - $= 0$: on the boundary

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