Predictive Analytics and Probability

Veterans Analytics Course

September 16-17, 2020

Provided by: CANA Advisors

The Monty Hall Problem

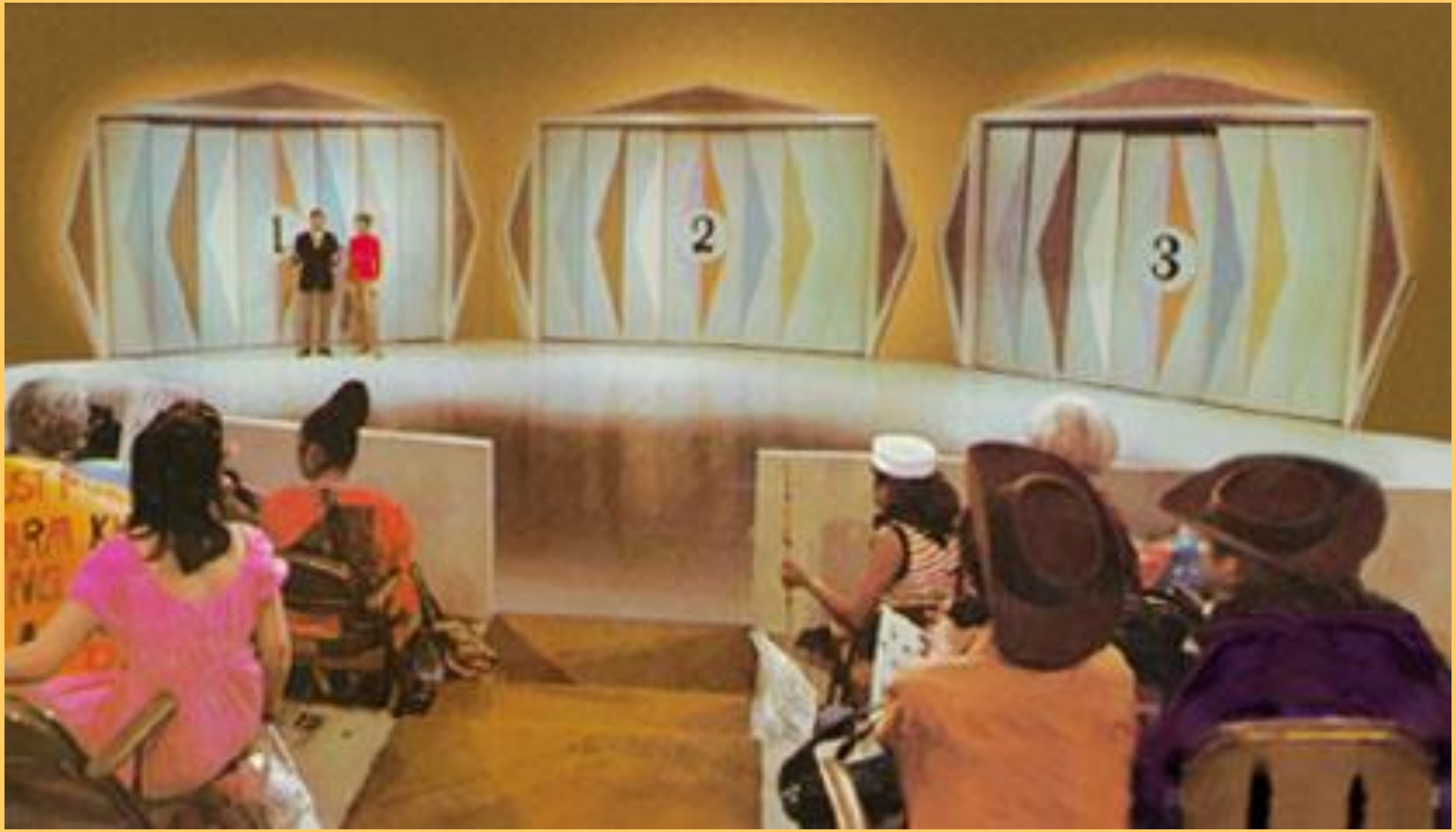
About *Let's Make a Deal*

- *Let's Make a Deal* was a game show hosted by Monty Hall and Carol Merril. It originally ran from 1963 to 1977 on network TV
- The highlight of the show was the "Big Deal," where contestants would trade previous winnings for the chance to choose one of three doors and take whatever was behind it--maybe a car, maybe livestock

You are now a contestant on **Let's Make a Deal**

Let’s Make a Deal

- You will choose one of three doors
- The grand prize is behind one of the doors
- The other doors hide silly consolation gifts which Monty called “zonks”



Let’s Make a Deal

You choose a door

Monty reveals a zonk behind one of the other doors

He then gives you the option of switching doors or sticking with your original choice….

Let’s Make a Deal

You choose a door

Monty reveals a zonk behind one of the other doors

He then gives you the option of switching doors or sticking with your original choice….

Do you want to switch doors?

Conditional Probability

We can determine these probabilities using Bayes' Theorem

$$p(A \mid B) = \frac{p(A \cap B)}{p(B)}$$

In words: The probability of event A given event B is the probability of both A and B divided by the probability of B

Conditional Probability

In the following argument:

- Assume that:
  - we originally chose door #1.
  - Monty opened door #2.
- Notation
  - Let "#1" denote the event that the prize is behind door #1, and similarly for doors #2 and #3.
  - Let "opened #2" denote the event that Monty has opened door #2.
- Our aim is to compute p(#1 | opened #2) and p(#3 | opened #2).

Conditional Probability

Rules:

1. $p(A \mid B) = \dfrac{p(A \cap B)}{p(B)}$

2. $p(A \cap B) = p(B) \times p(A \mid B)$

$$p(\#1 \mid \text{opened}\,\#2) = \frac{p(\#1 \cap \text{opened}\,\#2)}{p(\text{opened}\,\#2)}$$

$$p(\#3 \mid \text{opened}\,\#2) = \frac{p(\#3 \cap \text{opened}\,\#2)}{p(\text{opened}\,\#2)}$$

$$p(\#1 \cap \text{opened}\,\#2) = p(\text{opened}\,\#2 \mid \#1) \times p(\#1)$$ (By rule 2.)

$$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$ (If the prize is behind door #1, Monty can open either #2 or #3.)

$$p(\#3 \cap \text{opened}\,\#2) = p(\text{opened}\,\#2 \mid \#3) \times p(\#3)$$ (By rule 2.)

$$= 1 \times \frac{1}{3} = \frac{1}{3}$$ (If the prize is behind door #3, Monty **must** open door #2.)

Conditional Probability

$$p(\#1 \mid \text{opened } \#2) = \frac{p(\#1 \cap \text{opened } \#2)}{p(\text{opened } \#2)} = \frac{1/6}{p(\text{opened } \#2)}$$

$$p(\#3 \mid \text{opened } \#2) = \frac{p(\#3 \cap \text{opened } \#2)}{p(\text{opened } \#2)} = \frac{1/3}{p(\text{opened } \#2)}$$

$$p(\text{opened } \#2) = p(\text{opened } \#2 \cap \#1) + p(\text{opened } \#2 \cap \#2) + p(\text{opened } \#2 \cap \#3)$$
$$= \frac{1}{6} + 0 + \frac{1}{3} = \frac{1}{2}$$

So:

$$p(\#1 \mid \text{opened } \#2) = \frac{1/6}{1/2} = \frac{1}{3}$$ and $$p(\#3 \mid \text{opened } \#2) = \frac{1/3}{1/2} = \frac{2}{3}$$

Conclusions

- Switching increases your chances of winning to 2/3
- A similar result holds for *n* doors
- This strategy works only if we assume that Monty behaves predictably, offering a chance to switch every time
- On *Let's Make a Deal*, Monty would play mind games with contestants, sometimes offering them money not to open the selected door
- Play the game and check out the statistics at http://math.ucsd.edu/~crypto/Monty/monty.html
- *Let's Make a Deal* graphics courtesy of letsmakeadeal.com