Digital Signal Processing

Lab 2

Frequency zooming by the FFT

Today's assignment

Consider (again) the ecg signal available in data_ecg.mat, and sampled @ F = 125Hz

Explore its frequency content by using the:

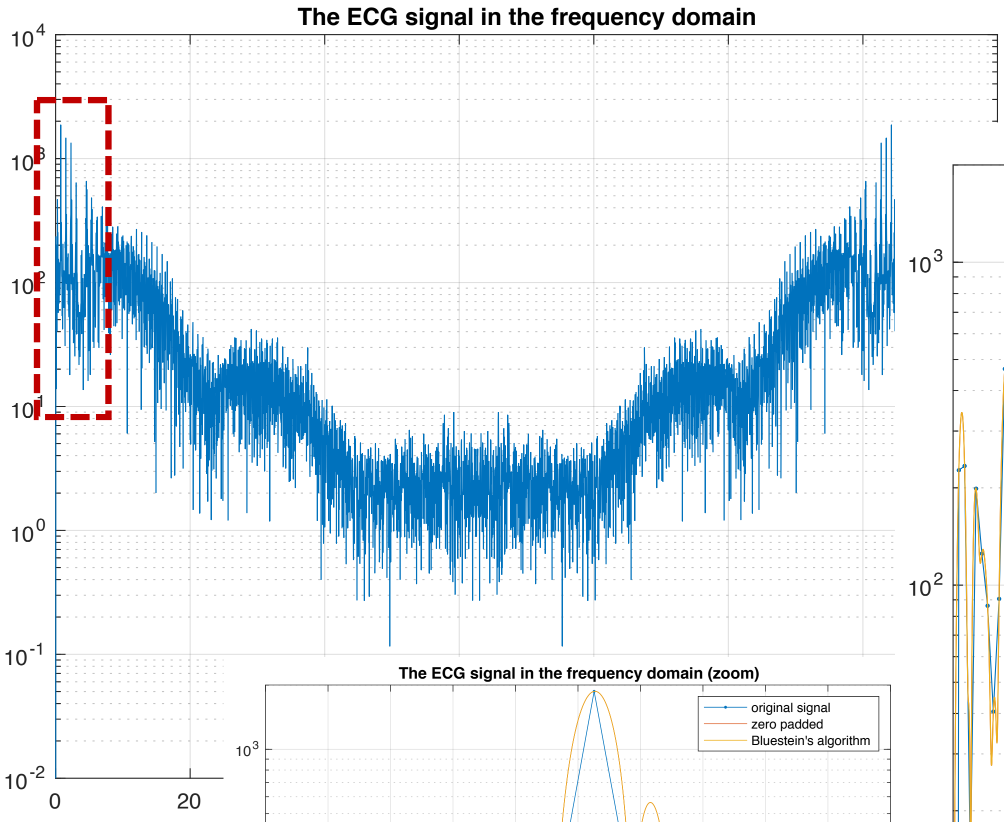
1. efficient DFT map via fft
2. zero-padding approach to increase the number of samples in frequency
3. Bluestein's algorithm to zoom on a specific frequency interval

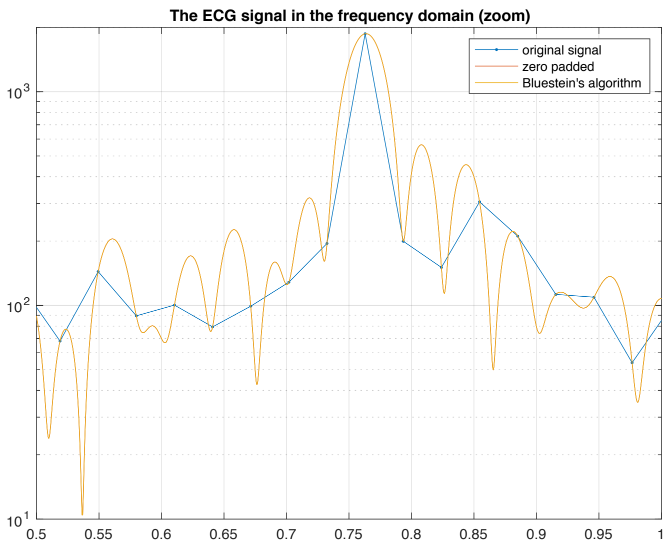
solution given for these 2

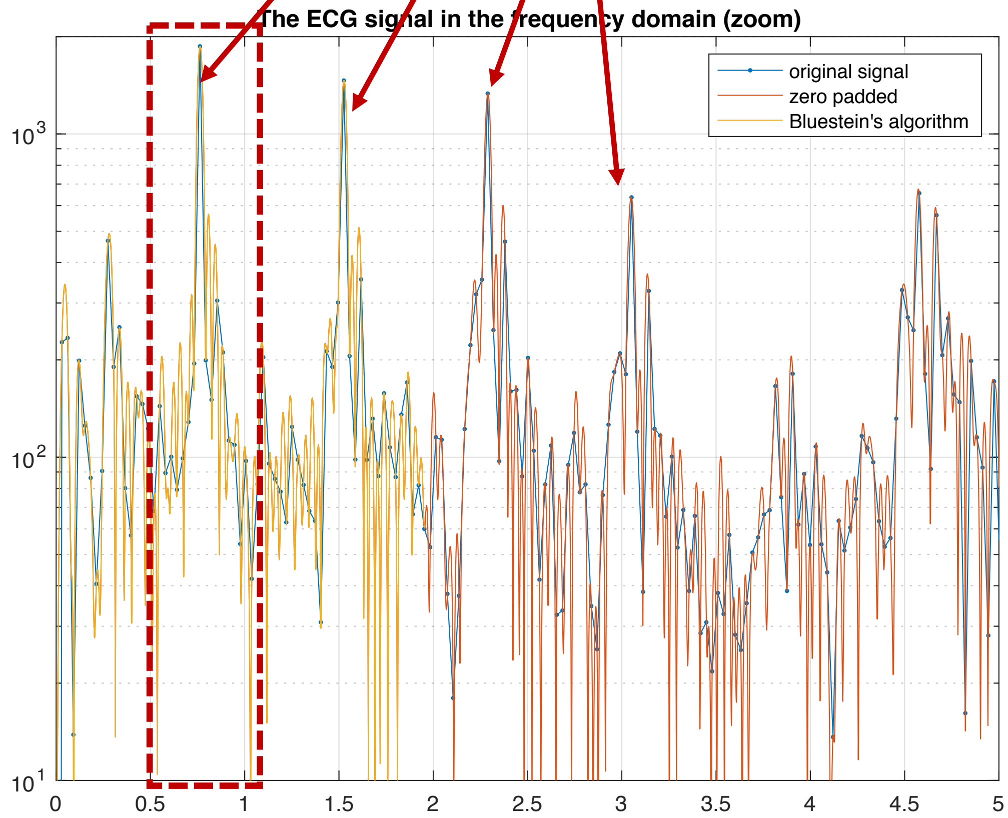
Check that they all give the same result !

Outcome

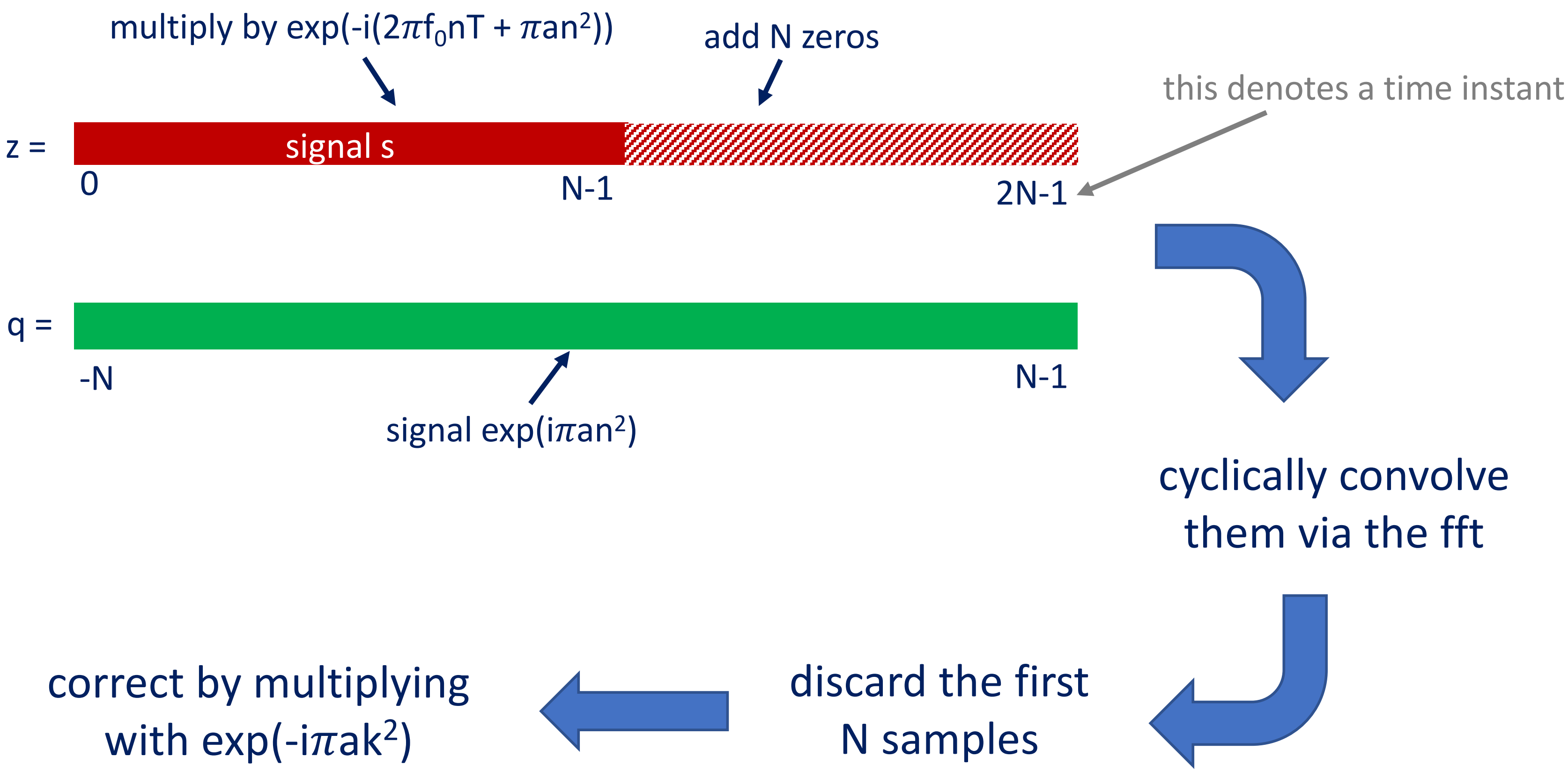
peaks linked to the (approximate) periodicity of the ECG signal in the time domain; can be used to estimate the period







Bluestein’s FFT



Today's assignment #2

Consider (again) the ecg signal

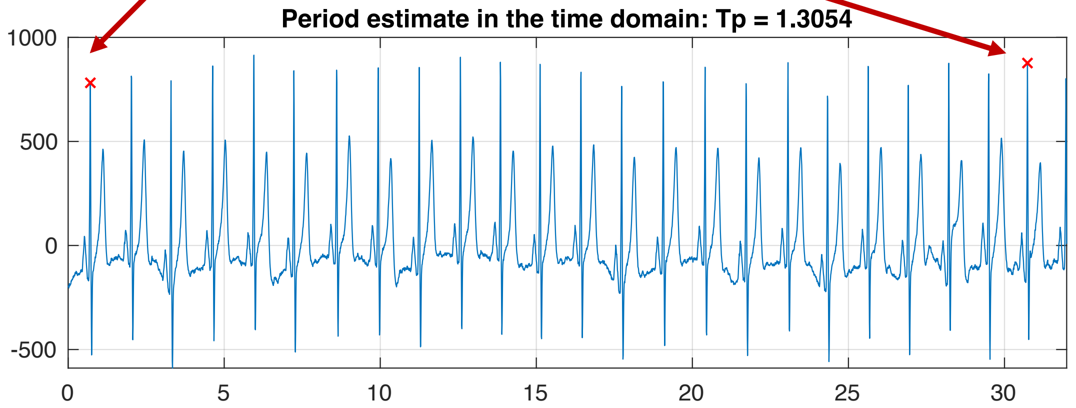
Try estimating its (average) period T_p by exploiting the information available :

1. in the temporal domain
   (e.g., find peaks distances)
2. in the frequency domain
   (e.g., find peak position)

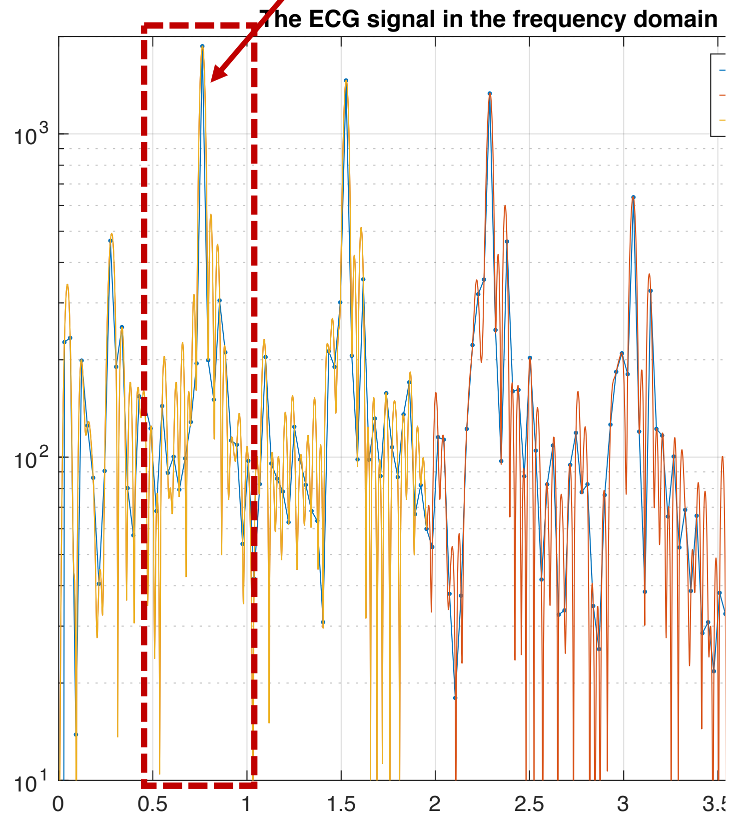
Do the estimates correspond ?
What if we apply a window ?

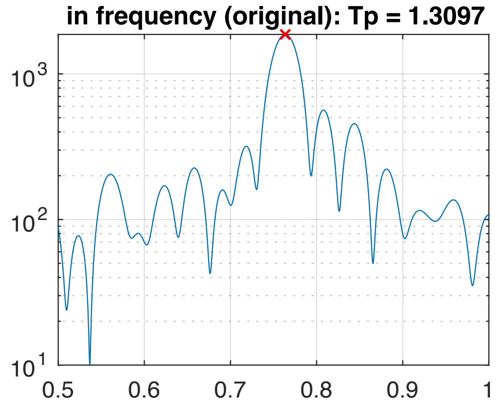
Outcome

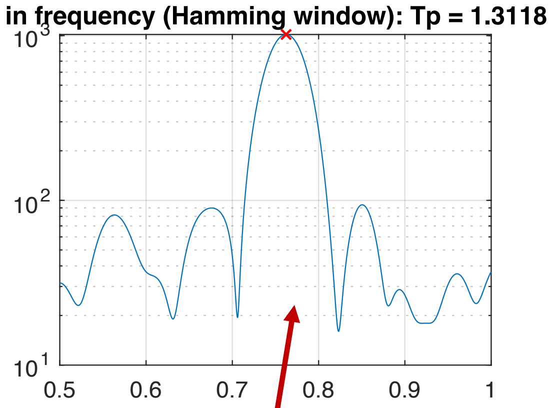
the distance between these two peaks is K times Tp



the frequency of this peak is $f_0 = 1/Tp$







windowing slightly changes the result

Windowing effect

Simple signal = complex exponential with linear phase

time → frequency

$s(nT) = \exp(-i2\pi f_0 nT)$ → $S(f) = \mathrm{rep}_{1/T}\,\delta(f+f_0)$

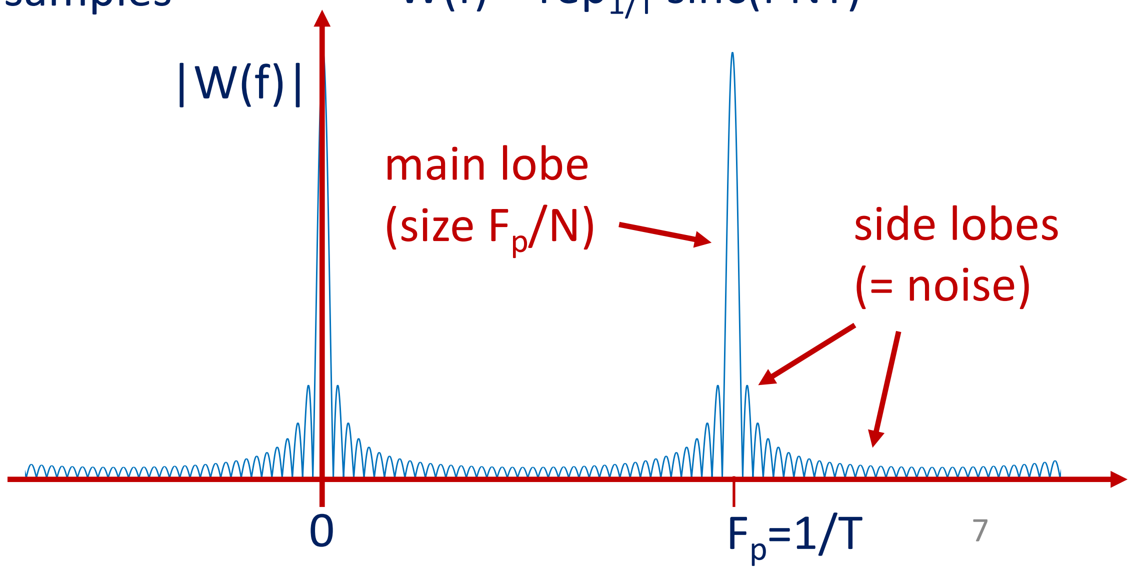
$s(nT) = \exp(-i2\pi f_0 nT)\, w(nT)$ → $S(f) = W(f+f_0)$

window

$w(nT)$ = rectangle, N samples

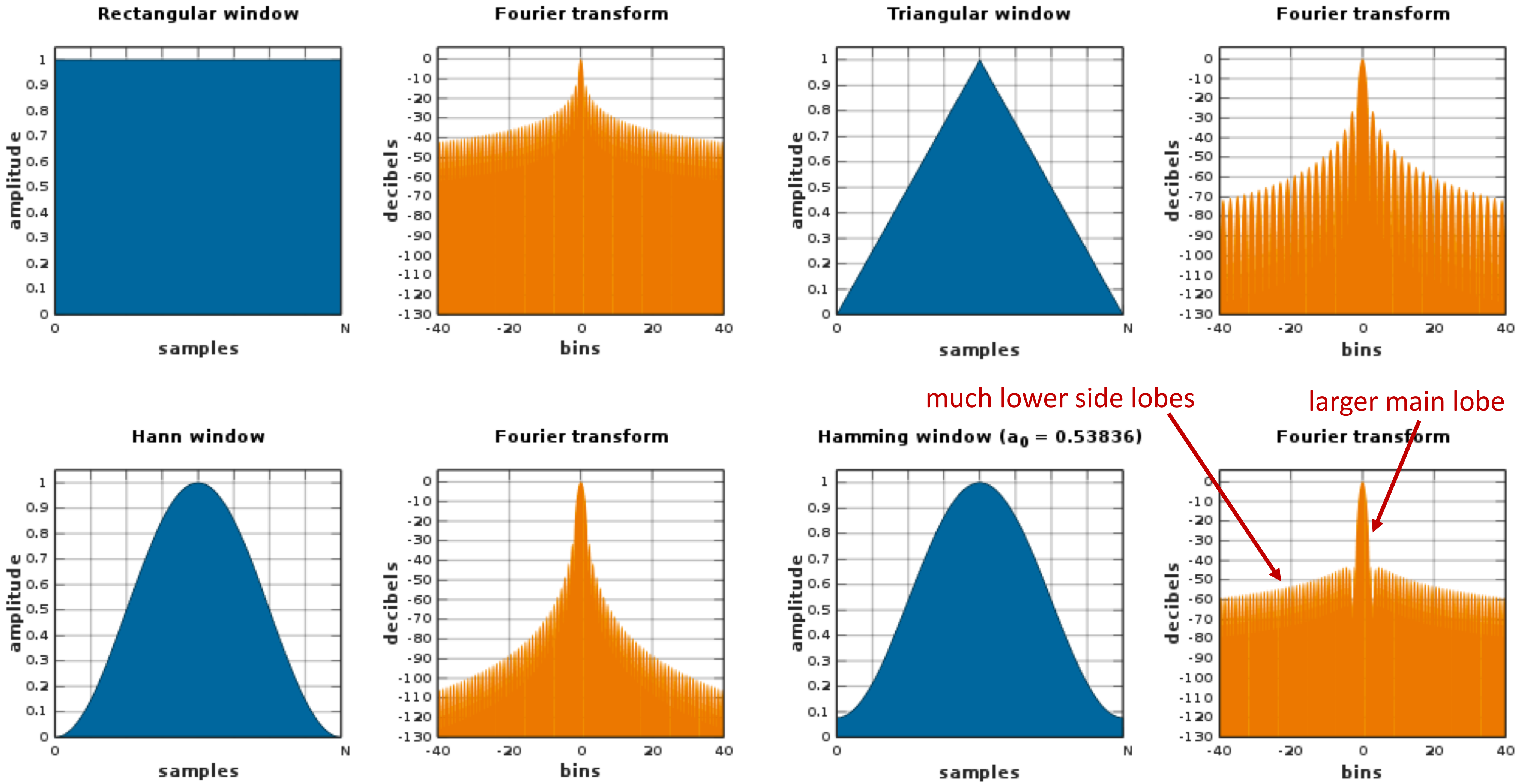
DTFT of the window

$W(f) = \mathrm{rep}_{1/T}\,\mathrm{sinc}(f\,NT)$



Alternative windows

en.wikipedia.org/wiki/Window_function#Hann_and_Hamming_windows



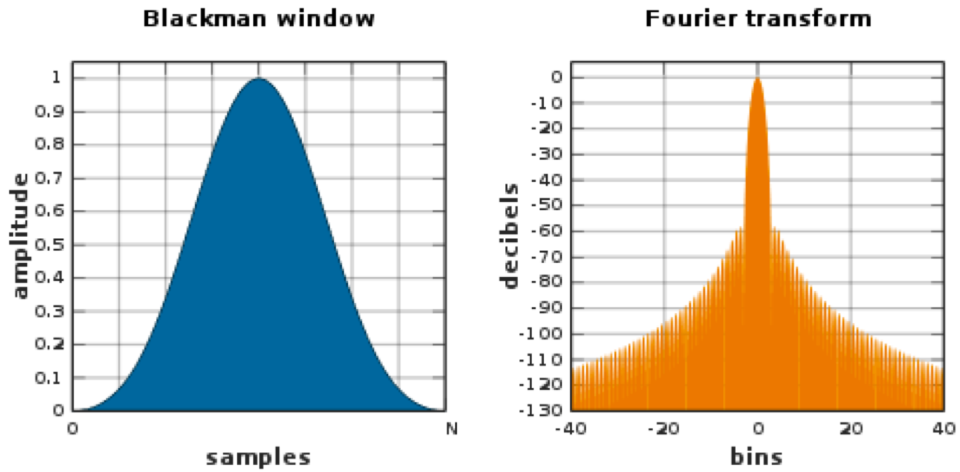
$0.5+0.5\cos(2\pi n/N),\ |n| \le N/2$

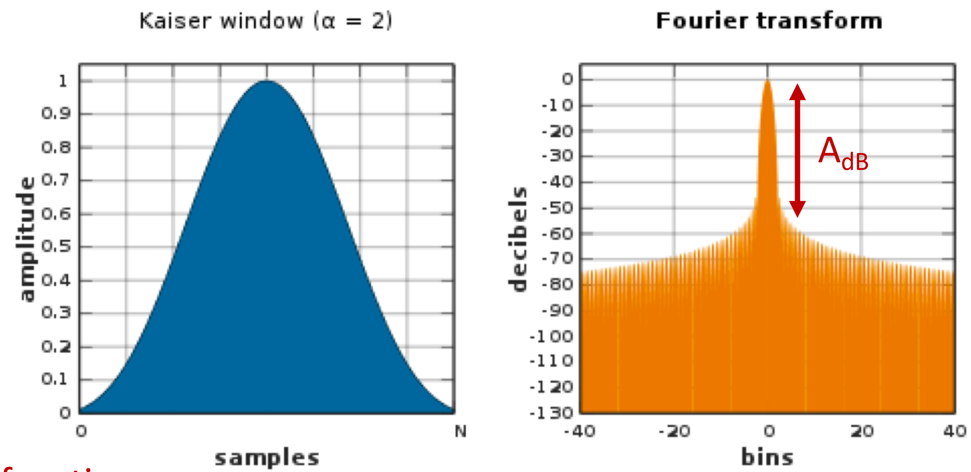
$0.54+0.46\cos(2\pi n/N),\ |n| \le N/2$

Alternative windows

en.wikipedia.org/wiki/Window_function#Hann_and_Hamming_windows

0.42+0.5cos(2πn/N)+0.08cos(4πn/N), |n|≤ N/2





Bessel function of the first kind

$$w(n) = \begin{cases} \dfrac{I_0\left(\beta\sqrt{1-\left(\frac{2n}{N}\right)^2}\right)}{I_0(\beta)}, & |n| \le \frac{N}{2}, \\ 0, & \text{elsewhere} \end{cases}$$

choose β according to the desired stop-band attenuation A_{dB}

$$\beta = \begin{cases} 0.1102\,(A_{dB} - 8.7)\,, & \text{for } A_{dB} > 50 \text{ dB} \\ 0.5842\,(A_{dB} - 21)^{0.4} + 0.07886(A_{dB} - 21)\,, & \text{for } 21 \text{ dB} < A_{dB} \le 50 \text{ dB} \\ 0\,, & \text{for } A_{dB} \le 21 \text{ dB}\,. \end{cases}$$

Questions ?