Computation and Analysis of Multiple Structural Change Models

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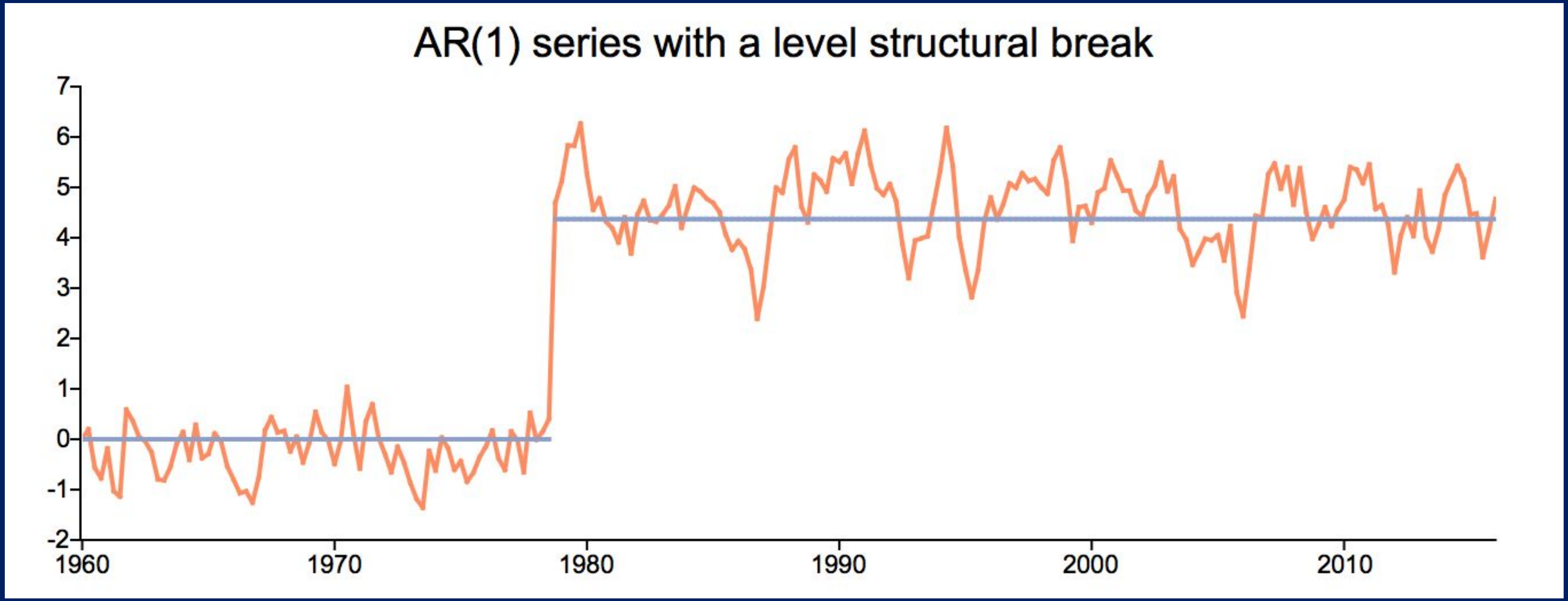
Studied and replicated by: **Verdiyev Jamal**

What is a structural break?

- Change in monetary policy
- Financial crisis
- Institutional change

Initial **standard** regression model **doesn't work anymore**



Multiple structural breaks model

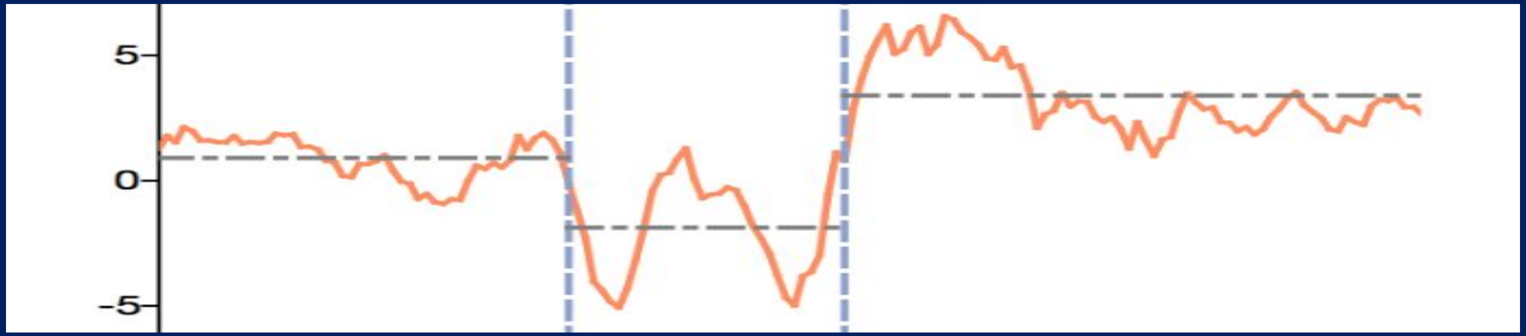
$$y_t = x_t'\beta + z_t'\delta_j + u_t \quad t = T_{j-1} + 1, \dots, T_j$$
$$j = 1, \dots, m + 1$$

Unknown parameters:

- Number of breaks m
- Dates of breaks $T_1, \dots, T_m$
- Coefficients $\beta, \delta_1, \dots, \delta_{m+1}$

Features:

- β the same
- δ_j exact for j regime
- Coefficients $\beta, \delta_1, \dots, \delta_{m+1}$



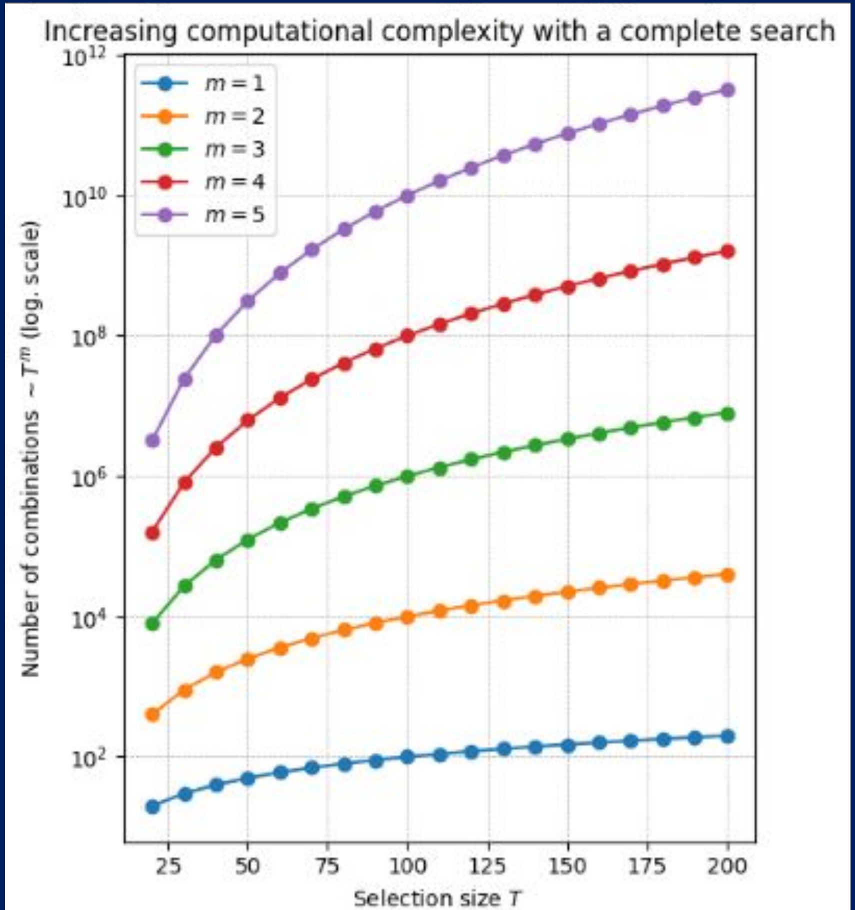
Why is it hard?

$$T = 100, m = 3$$

$> 150\,000$ combos

$$T = 200, m = 5$$

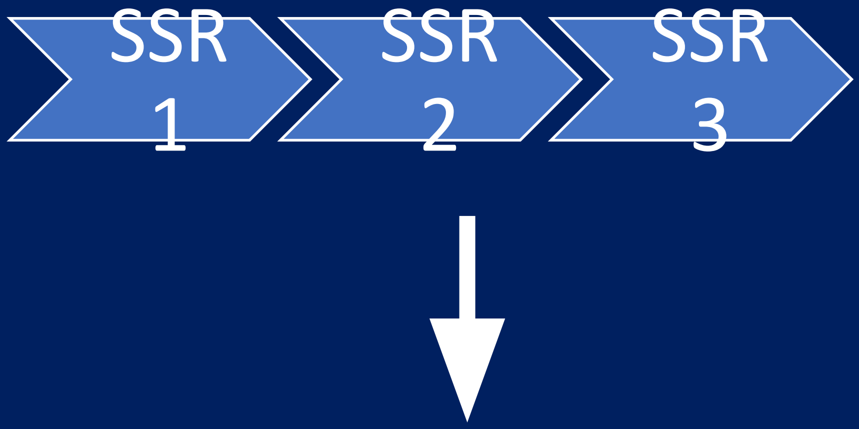
billions of combos



We need a method to find global SSR minimum without full search

Main idea of the paper

$$\mathrm{SSR_{total}} = \sum_{j=1}^{m+1} \mathrm{SSR}\left(T_{j-1} + 1, T_j\right)$$



$$O(T^m) \rightarrow O(T^2)$$

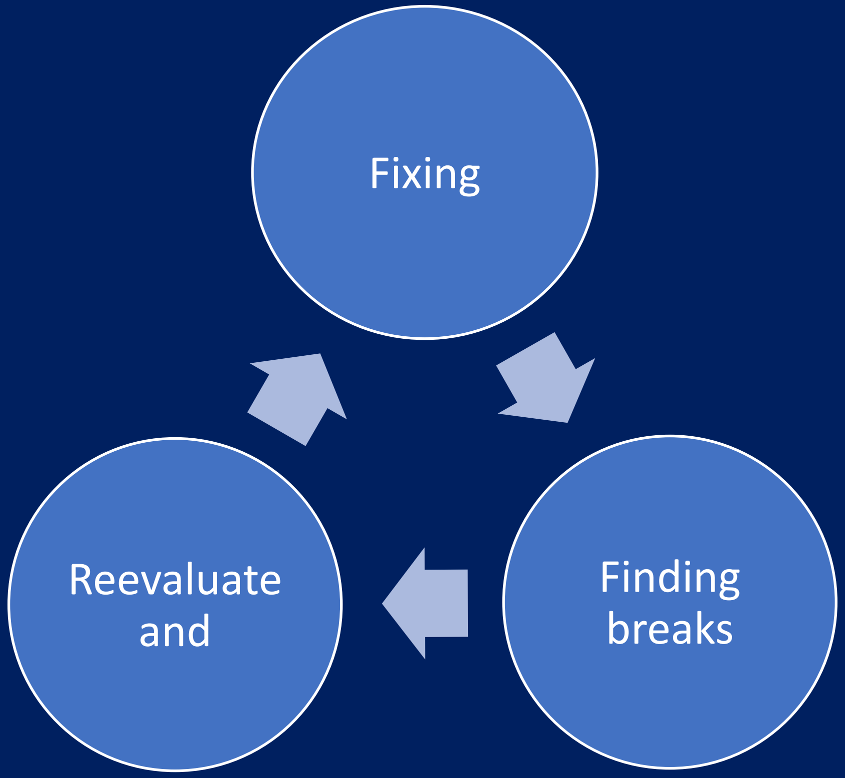
Pre-calculation

- Calculate SSR(i,j) for all valid segments [i,j] ⟶ fill in the triangular matrix.

Recursive search for the minimum

- Use dynamic programming to find the partition with the minimum total SSR

Partial Structural Change: Estimation via Iterative Procedure



Repeat until SSR stops diminishing

$$y_t = x_t'\beta + z_t'\delta_j + u_t$$

The results of this algorithm are:

- Optimal breaking points $\widehat{T_1^{(k)}}, \ldots, \widehat{T_m^{(k)}}$
- Estimates of common parameters $\hat{\beta}$
- Estimates of regime-specific $\widehat{\delta_1}, \ldots, \widehat{\delta_{m+1}}$

Which minimize SSR!

Estimating Break Dates and Confidence Intervals

We found a break in 1973 – how sure we are about it?

$$P\left(\left|\hat{T} - T^0\right| \leq c_{0.95}\right) \approx 0.95$$

“Advanced” CIs
Based on functionals of Brownian motion (simulated critical values)

Bai (1997) — «Estimation of a Change Point in Multiple Regression Models»
Bai & Perron (1998) — «Estimating and Testing Linear Models with Multiple Structural Changes»

Testing for Structural Breaks

Key test statistics:

- **supF(k)**

  H_0: no breaks vs H_1: exactly k breaks

  Maximizes the F-statistics over all admissible break points

  Requires choosing k in advance

- **Double maximum test**

$$\boldsymbol{WDmax} \text{ — } weighted\ version\ of\ UDmax\ with\ equal\ marginal\ p - values$$

$$\textbf{UDmax} = \max_{1 \le m \le M} \sup F(m))$$

Choosing the Number of Breaks: Empirical Results

Results of choosing the number of breaks on synthetic data.
BIC and LWZ recommend 2 breaks — the sequential test confirms that adding a third break is not significant.

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STATISTICS FOR CHOOSING THE NUMBER OF BREAKS
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Information Criteria (looking for minimum BIC/LWZ):
 k_breaks        SSR        BIC        LWZ  supF(0_vs_k)
        0 973.178558 235.956104 238.808624      0.000000
        1 487.544603 169.398696 175.103737    100.604189
        2 267.281945 112.122421 120.679983    132.050935
        3 256.215807 112.401899 123.811982     92.343134
        4 254.391447 116.300603 130.563206     69.225143
        5 263.012004 124.367855 141.482978     52.382518

Recommendation by BIC: 2 breaks
Recommendation by LWZ: 2 breaks

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Sequential test (supF(l+1|l)):
Shows whether to add another break.
l_vs_l+1     F_stat
  0 vs 1 100.604189
  1 vs 2  82.408357
  2 vs 3   4.275878
  3 vs 4   0.702804
  4 vs 5  -3.179300

Final dates for k=2: ['1972:Q3', '1980:Q3']
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Practical recommendations

Trimming

Autocorrelation

Heteroscedasticity

The US *Ex-post* Real Interest Rate: Simulation

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REPRODUCTION RESULTS (Number of breaks = 3)
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Found break indices: [20, 46, 78]
Found break dates:
 - 1966:Q1
 - 1972:Q3
 - 1980:Q3

(for reference) Results from the Bai & Perron (2003) paper:
 ['1966:Q4', '1972:Q3', '1980:Q3']
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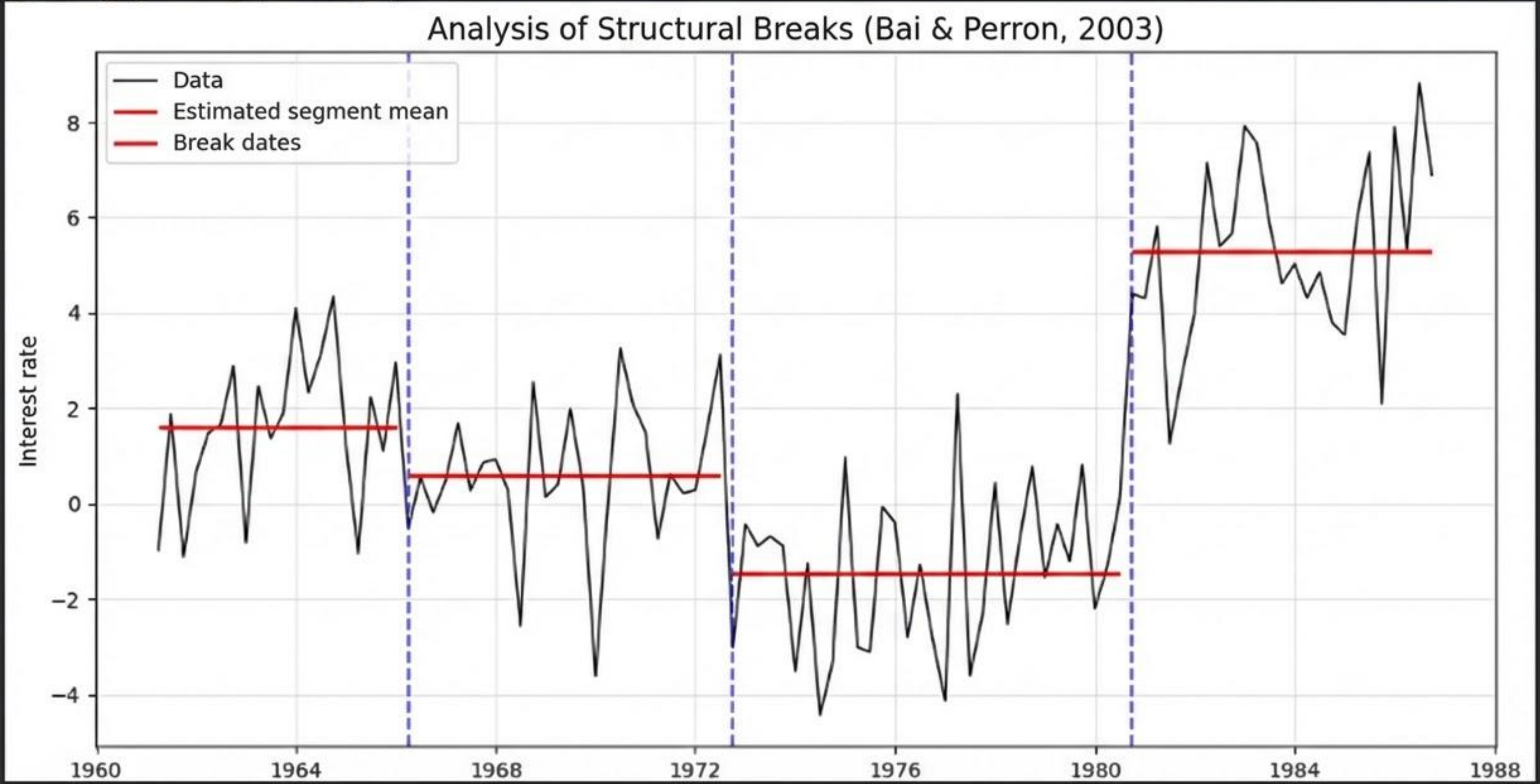

Conclusion

- Structural breaks are the norm, not the exception, in economic data.
- Bai & Perron provided a complete and implementable toolkit for analyzing them.
- Their method has become a practical standard in modern econometrics.