PS10

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① $f_x(x) = \begin{cases} A, & 1 < x < 3 \\ 0, & ew \end{cases}$ $\quad F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{2} & 1 < x < 3 \\ 1 & x > 3 \end{cases}$

ⓐ $A = ?$ $\quad \int_1^3 A\,dx = Ax\Big]_1^3 = 2A = 1$

$A = 1/2$

ⓑ select 4 bars. Pr(exactly 2 of 4 weigh more than 150 grams)

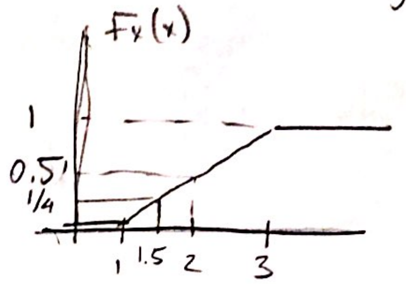
$= Pr$(2 of 4 weigh less than 150 grams) =

$= \binom{4}{2} \left[Pr(\text{weigh less than } 150)\right]^2 \left[Pr(\text{weigh more than } 150)\right]^2$

$= 6\,[F_x(1.5)]^2\,[1 - F_x(1.5)]^2$

$= 6\,[\tfrac{1}{4}]^2\,[3/4]^2$

$= 0.2109$



ⓒ Pr(total weight of 100 bars > 20,010 grams)

$W = \sum_{i=1}^{100} X_i > 20{,}010$ $\quad \mu_X = \frac{3+1}{2} \cdot 100 = 200$ $\quad \sigma_X^2 = \frac{(3-1)^2}{12} = \frac{4}{12}$

$\sigma_X = \sqrt{\frac{4}{12}}$

$= 1 - Pr(W < 20{,}010)$

$= 1 - \Phi\left(\frac{20{,}010 - 100(200)}{\sqrt{\frac{4}{12}} \cdot 10}\right)$

$= 1 - \Phi\left(\frac{1}{\sqrt{4/12}}\right) = 1 - \Phi(1.7321) \approx 0.0416$

10-2 continuity correction comparison.

$Pr(499 < S_H < 502) = ?$

$np = 0.5(1000) = 500$

$\sqrt{np(1-p)} = \sqrt{500(0.5)} = \sqrt{250} \approx 15.811$

without correction:

$$= \phi\left(\frac{502-500}{15.811}\right) - \phi\left(\frac{499-500}{15.811}\right)$$

$$\approx 0.0755$$

with correction $= \phi\left(\frac{502+\frac{1}{2}-500}{15.811}\right) - \phi\left(\frac{499-500-\frac{1}{2}}{15.811}\right)$

$$\approx 0.1006$$

so w/ correction > w/o correction.

10-3 text problem 7.11 w/o CLT.

# of packet arrivals $= X_i \sim$ poisson (α_i) at any port. Router w/ k ports.

total packet arrivals $= \sum_{i=1}^{k} X_i = R$

pmf of R: $f_R(r) = ?$

$G_R(z) = e^{\alpha_1(z-1)} e^{\alpha_2(z-1)} \cdots e^{\alpha_k(z-1)}$

$G_R(z) = e^{(z-1)(\alpha_1+\alpha_2+\cdots\alpha_k)} \longrightarrow f_R(r) = \sim \text{poisson}\left(\sum_{i=1}^{k} \alpha_i\right)$

so $\alpha_R = \sum_{i=1}^{k} \alpha_i \longrightarrow P_R(k) = \frac{\alpha_R^{k}}{k!} e^{-\alpha_R}$

10-4 | $X \in 0, 1, 2 \ldots$ $P(X<0)=0$

$$\Phi_x(\omega) = \frac{1}{\lambda} \ln\left(\frac{1}{1-qe^{j\omega}}\right) \qquad 0<q<1$$

ⓐ $\lambda = ?$

$$\Phi_x(\omega)\Big|_{w=0} = 1$$

$$\frac{1}{\lambda} \ln \frac{1}{1-q} = 1$$

$$\boxed{\ln \frac{1}{1-q} = \lambda}$$

$$\cancel{\frac{1}{1-q} = e^{\lambda}}$$

$$\cancel{1 = (1-q)e^{\lambda}}$$

forgot what I was solving for.

b) $E[X] =$ $jE[X] = \frac{1}{\lambda} \frac{d}{dw} \Phi_X(w)\Big|_{w=0}$

$$= \frac{1}{\lambda} \frac{\left[\frac{jqe^{j\omega}}{(1-qe^{j\omega})^2}\right]}{\frac{1}{1-qe^{j\omega}}} = \frac{1}{\lambda} \frac{jqe^{j\omega}}{(1-qe^{j\omega})^2} \cdot \cancel{1-qe^{j\omega}}$$

$$= \frac{1}{\lambda} \frac{jq\cancel{e^{j\omega}} \to 1}{1-q\cancel{e^{j\omega}} \to 1}\Bigg|_{w=0} \Rightarrow \frac{1}{\lambda} \frac{jq}{1-q} = j\frac{q}{1-q}\frac{1}{\lambda}$$

So $E[X] = \left[\frac{q}{1-q}\right]\left[\frac{1}{\ln\left(\frac{1}{1-q}\right)}\right]$

ⓒ $P(X=2) = ?$ $G_x(z) = \frac{1}{\lambda} \ln\left(\frac{1}{1-qz}\right)$

$$P_x(2) = \frac{1}{\lambda} \frac{d^2}{dz^2} \ln\left(\frac{1}{1-qz}\right)\Bigg|_{z=0}$$

$$\frac{d}{dz} \ln \frac{1}{1-qz} = \frac{\frac{q}{(1-qz)^2}}{\left(\frac{1}{1-qz}\right)} = \frac{q}{1-qz}$$

$$\frac{d}{dz}\left(\frac{q}{1-qz}\right) = \frac{q^2}{(1-qz)^2}\Bigg|_{z=0} = \frac{q^2}{1} = q^2$$

10-5) text 7.30 error $\sim U[-0.5, 0.5]$

$Pr(\text{sum error} > 4)$ $N = 64$

$Pr(S_n > 4)$ $\mu_x = 0$ $\sigma_x^2 = \frac{1}{12}$

$= 1 - Pr(S_n < 4)$ $\sigma_x = \sqrt{\frac{1}{12}}$

$= 1 - \Phi\left(\frac{4 - 0 \cdot 64}{\sqrt{\frac{1}{12}} \cdot 8}\right) = Q\left[\frac{1}{2\sqrt{\frac{1}{12}}}\right]$

$= 1 - \Phi\left(\frac{1}{2\sqrt{\frac{1}{12}}}\right) \approx 0.0416$