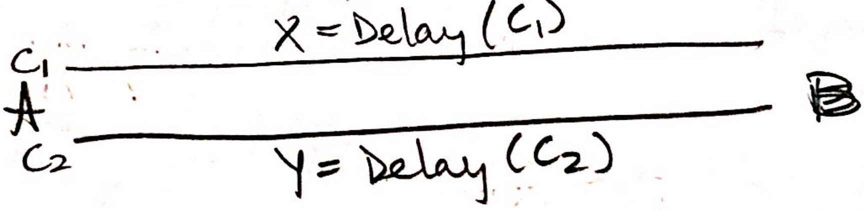
PS 6 EE 503 Charlie Andrée

(1)



X, Y are indep, both $\sim U[0, 1\text{hr}]$

rec'vd : over one channel

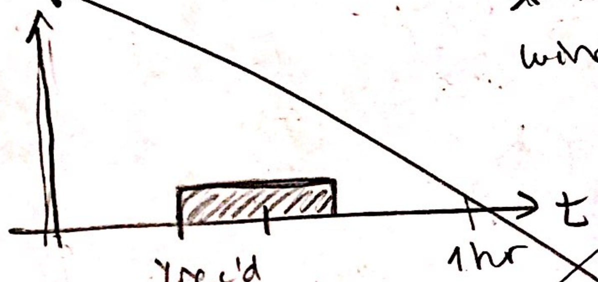
verified : rec'vd over both

(a) $Pr(X < 15 \text{ or } Y < 15) = \frac{1}{4} + \frac{1}{4} - \left(\frac{1}{4} \cdot \frac{1}{4}\right) = 0.4375$

(b) Pr(received, but not verified within 15 mins)

$X\bar{Y} + \bar{X}Y = \frac{1}{4}\frac{3}{4} + \frac{1}{4}\frac{3}{4} = 0.375$

(c) $Pr(X < Y + 15)$



X equally likely to occur in remaining window

$$f_X(x \mid Y < x) = \begin{cases} \dfrac{15}{60-Y} & Y < 45 \\ 1 & Y > 45 \end{cases}$$

$Pr(X < Y + 15) = Pr(Y < X + 15)$ → either can arrive first. going to solve for 1 channel then multiply by 2.

$$Pr(X < Y + 15) = 1 \cdot Pr(Y > 45) + \frac{15}{60 - Y} \cdot Pr(Y < 45)$$

$$= 1 \cdot \frac{1}{4} + \frac{15}{60 - Y} \cdot \frac{3}{4} \qquad Y \approx E[Y] = 3$$

$$= \frac{1}{4} + \frac{15}{30} \cdot \frac{3}{4}$$

ⓒ $Pr(|X-Y| \leq 15)$

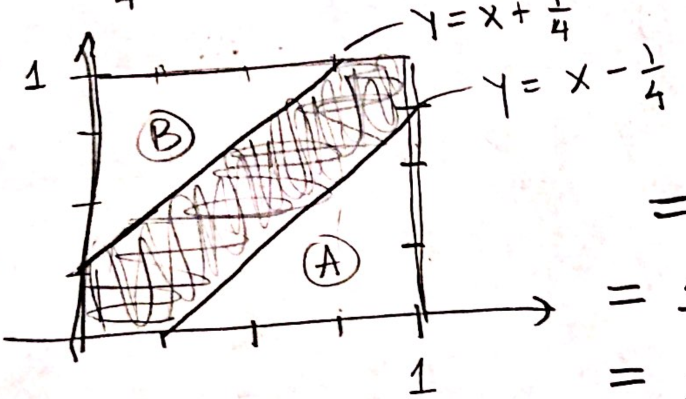
$X = Y = \begin{cases} \frac{1}{60} & 0 < x, y < 60 \\ 0 & ew \end{cases}$

$= \begin{cases} 1 & 0 < x, y < 1 \\ 0 & ew \end{cases}$

$Pr(|X-Y| \leq \frac{1}{4})$

$X - Y \leq \frac{1}{4}$ $\qquad$ $X - Y \geq -\frac{1}{4}$

$X - \frac{1}{4} \leq Y$ $\qquad$ $X + \frac{1}{4} \geq Y$



$= 1 - P(A) - P(B)$

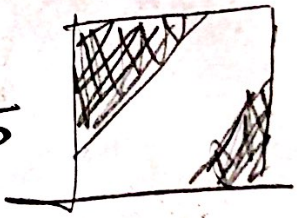
$= 1 - \text{Area}(A) - \text{Area}(B)$

$= 1 - \left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right) - \left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right)$

$P(|X-Y| \leq 15) = 0.4375$

ⓓ $Pr(|X-Y| > 15) = 1 - Pr(|X-Y| \leq 15)$

$= 1 - 0.4375 = 0.5625$



ⓔ $Pr(X < 45 \cap Y < 45) = P(X<45) \cdot P(Y<45)$

$= \frac{3}{4} \cdot \frac{3}{4} = 0.5625$

d & e bothe equal the same chance of the attendant being present for reception & verification

6-2 5.27

$$f_{XY}(x,y) = kx(1-x)y \quad \begin{matrix} 0<x<1 \\ 0<y<1 \end{matrix}$$

(a) $k = ?$

$$1 = \int_0^1\int k x(1-x) y \, dy\, dx$$

$$k \int_0^1 x(1-x)dx \int_0^1 y\, dy$$

$$1 = k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \left[\frac{y^2}{2}\right]_0^1$$

$$1 = k\left[\frac{1}{2} - \frac{1}{3}\right]\left[\frac{1}{2}\right]$$

$$1 = k \cdot \frac{1}{12} \implies k = 12$$

(b) CDF $F_{XY}(x,y)$

$$\int_0^x \int_0^y 12\alpha(1-\alpha) z \, dz\, d\alpha$$

$$\int_0^x 12\alpha(1-\alpha)\left[\frac{z^2}{2}\right]_0^y d\alpha$$

$$\int_0^x 6(\alpha - \alpha^2)\, y^2\, d\alpha$$

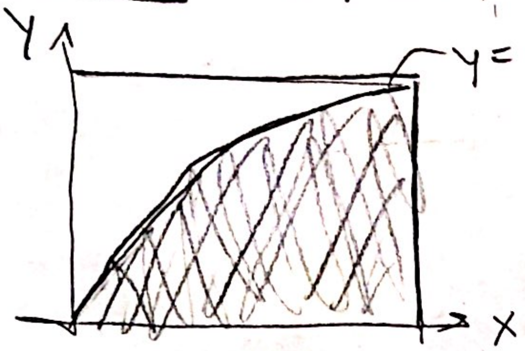
$$6y^2\left[\frac{\alpha^2}{2} - \frac{\alpha^3}{3}\right]_0^x = 6y^2\left(\frac{x^2}{2} - \frac{x^3}{3}\right) = F_{XY}(x,y)$$

(c) $f_X(x) = \int_0^1 12(x-x^2)y\, dy$ $\qquad f_Y(y) = \int_0^1 12(x-x^2)y\, dx$

$$= 6(x-x^2)y^2\Big]_{y=0}^{y=1} \qquad = 12y \cdot \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$f_X(x) = 6x(1-x) \qquad f_Y(y) = 12y \cdot \frac{1}{6} = 2y$$

$\boxed{5.27\ d}$ $P(Y < X^{1/2})$, $P(X < Y)$



$$P(Y < \sqrt{x}) = \int_0^1 \int_0^{\sqrt{x}} 12x(1-x)\,y\,dy\,dx$$

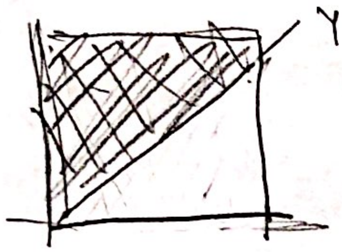
$$= \int_0^1 12x(1-x) \cdot \frac{\sqrt{x}^2}{2}\,dx$$

$$= \int_0^1 6x(1-x)\,x\,dx$$

$$= \int_0^1 6(x^2 - x^3)\,dx$$

$$P(Y < \sqrt{x}) = 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 6 \cdot \frac{1}{12} = \frac{1}{2}$$

$P(X < Y) = ?$



$$= \int_0^1 \int_{y=0}^{y=x} 12x(1-x)\,y\,dy\,dx$$

$$= \int_0^1 6x(1-x)\,x^2\,dx$$

$$= \int_0^1 6(x^3 - x^4)\,dx$$

$$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = \frac{6}{20} = P[(X<Y)^c]$$

realized I mixed up my y bounds.

$$P(X<Y) = 1 - \frac{6}{20} = \frac{14}{20} = \frac{7}{10}$$ ✓

5.46 | X, Y indep in 5.27?

$f_X(x) = 6x(1-x)$ $f_Y(y) = 2y$

$f_X(x) \cdot f_Y(y) = 12xy(1-x) = 12x(1-x)y$ ✓

they are independent :)

6-4 | 5.66

5.65 for X, Y from 5.27 correlation & covariance, indicate indep, orthog, uncorrelated

$cov(X,Y) = E[XY] - E[X]E[Y]$

$E[XY] = \int_0^1\int_0^1 12x^2(1-x)y^2 \, dy\,dx$

$= \int_0^1 4x^2(1-x)\,dx$

$= 4\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

$E[XY] = \frac{4}{12}$

$E[X] = \int_0^1 6(x^2 - x^3)\,dx$

$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]$

$E[X] = \frac{1}{2}$

$E[Y] = \int_0^1 2y^2\,dy$

$E[Y] \left[\frac{2}{3}y^3\right]_0^1 = \frac{2}{3}$

$cov(X,Y) = \frac{4}{12} - \frac{1}{2}\left(\frac{2}{3}\right) = 0$

uncorrelated independent [From 6-3]

$\text{correlation} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = 0 = \rho$

6-5 $\mu_x = 1 \quad \sigma_x = 0.6$

Find following in terms of $Q(x) = 1 - \phi(x)$

(a) $P(x > 0) = Q\left(\frac{0-1}{0.6}\right) = Q\left(\frac{-1}{0.6}\right)$

(b) $Q\left(\frac{0.2-1}{0.6}\right) - Q\left(\frac{1.8-1}{0.6}\right)$

$= Q\left(\frac{-0.8}{0.6}\right) - Q\left(\frac{0.8}{0.6}\right)$

(c) $f_{X|A}(x|A) = \begin{cases} \frac{f_X(x)}{P(A)} & 0.5 < x < 1.5 \\ 0 & ew \end{cases}$

$P(A) = Q\left(\frac{0.5-1}{0.6}\right) - Q\left(\frac{1.5-1}{0.6}\right)$

$= Q\left(\frac{-0.5}{0.6}\right) - Q\left(\frac{0.5}{0.6}\right)$

$P(A) = Q\left(-\frac{5}{6}\right) - Q\left(\frac{5}{6}\right)$

$f_{X|A}(x|A) = \begin{cases} \frac{\exp\left[-\frac{(x-1)^2}{2(0.6)^2}\right]}{\sqrt{2\pi}\,0.6\left(Q\left(\frac{-5}{6}\right) - Q\left(\frac{5}{6}\right)\right)}, & \frac{1}{2} < x < \frac{3}{2} \\ 0, & ew \end{cases}$