PS1 Due 09/05 Charlie André

2.2, 2.9, 1-3, 2.14, 2.23, 1-6, 1-7

2.2 F_x = First toss = X S_x = second toss = x

ⓐ $\Omega = \{F_1S_1, F_1S_2, F_1S_3 \ldots\ldots F_6S_6\}$

ⓑ $\bar{A} = \{F < S\}$

$A = \{F \geq S\} = 1 - \{F < S\}$

A = { [1,1], [2,1], [2,2], [3,1], [3,2], [3,3], [4,1], [4,2], [4,3], [4,4], [5,1], [5,2], [5,3], [5,4], [5,5], [6,1], [6,2], [6,3], [6,4], [6,5], [6,6] }

where [1st toss, 2nd toss]

ⓒ $B = \{F_6, S_x\}$ = { [6,1], [6,2], [6,3], [6,4], [6,5], [6,6] }

ⓓ $B \subset A$ so if B is true, A is true too.
Therefore B implies A

ⓔ $A \cap B^c$ is the set of all outcomes where (roll 1 $\geq$ roll 2) and (roll 1 $\neq$ 6)

$A \cap B^c$ = { [1,1], [2,1], [2,2], [3,1], [3,2], [3,3], [4,1], [4,2] [4,3], [4,4], [5,1], [5,2] [5,3] [5,4] [5,5] }

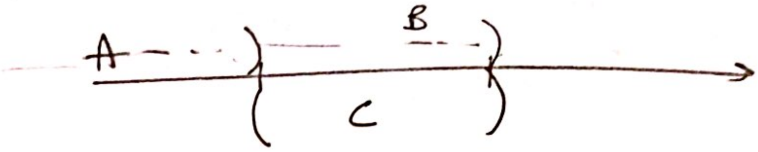
ⓕ C = { # of dots differs by 2 }

$A \cap C$ = { [3,1], [4,2], [5,3], [6,4] } 2.9 $S = R = (-\infty, \infty)$

$A = (-\infty, r]$ $B = (-\infty, s]$ $r \le s$

$C = (r, s]$ in terms of A, B

$\boxed{C = A' \cdot B}$



$B = A \cup C \longrightarrow$ show

$B = A \cup (A' \cdot B)$

$B = A \cup A'B \longrightarrow$

$B = A \cup B \longrightarrow A \subset B$

$B = B$ ✓

1-3 (a) show $(A+B)(A'+B') = (AB') + (A'B)$

$\cancel{AA'}^{0} + AB' + BA' + \cancel{BB'}^{0} = AB' + A'B$ ✓

Because $ZZ' = 0$

Intersection of an event and its compliment $= 0$

(b) $(ABC)' = (A'BC) + (AB'C) + (ABC') + (A'B'C) + (A'BC') + (AB'C')$
$+ \cancel{(A'B'C')}$

$(ABC)' = A' + B' + C' = A'(B'C' + B'C + BC' + BC) + B'(AC + A'C + AC' + A'C')$
$+ C'(AB + A'B + AB' + A'B')$

$A' + B' + C' = A'S + B' \cdot S + C' \cdot S$

$= A' + B' + C'$ ✓

(c) $A + BC = (A+B)(A+C) = AA + AC + BC + AB$

$= A(S + C + B) + BC$

$A + BC = A + BC$

1 (a) {one of three occurs}:

$= AB'C' + A'BC' + A'B'C$

(b) {2 events occur} $= A'BC + AB'C + ABC'$

(c) {one or more events occur} = (a) + (b) + ABC

$AB'C' + A'BC' + A'B'C + A'BC + AB'C + ABC' + ABC$

(d) {2+ events occur} = (b) + ABC = (c) − (a) =

$A'BC + AB'C + ABC' + ABC$

(e) {no events occur} $= A'B'C'$

$\boxed{2.23}$ Assume M.E $\quad S = \{a, b, c, d\}$

$P(\{c,d\}) = \frac{3}{8} \quad P(\{b,c\}) = \frac{6}{8} \quad P(\{d\}) = \frac{1}{8}$

$P(\{d\}) + P(\{c\}) = P(\{c,d\})$

$\frac{1}{8} + P(\{c\}) = \frac{3}{8} \longrightarrow P(\{c\}) = \frac{2}{8} = \frac{1}{4}$

$P(\{b,c\}) = P(\{b\}) + P(\{c\})$

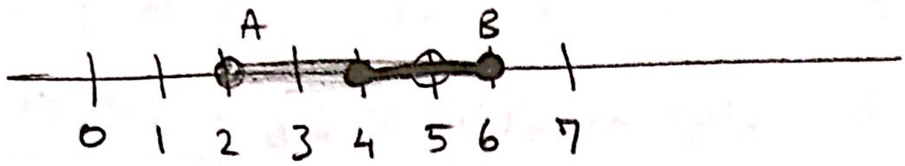
$\frac{6}{8} = P(\{b\}) + \frac{1}{4} \longrightarrow P(\{b\}) = \frac{1}{2}$

$P(S) = 1 = P(\{a,b,c,d\}) = P(\{a\}) + P(\{b\}) + P(\{c\}) + P(\{d\})$

$1 = P(\{a\}) + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

$P(\{a\}) = \frac{1}{8}$

1-6 $A=\{2\le x\le 5\}$ $B=\{4\le x\le 6\}$



$A+B=\{2\le x\le 6\}$

$A\cdot B=\{4\le x\le 5\}$

$(A+B)(AB)'=\{2\le x\le 3 \cup 5< x\le 6\}$

1-7 Show:

ⓐ $(A'+B')'+(A'+B)'=A$

$A''B''+A''B'=A$ de Morgan's

$AB+AB'=A$

$A(B+B')=A$

$A\cdot S=A$ ✓

ⓑ $(A+B)(AB)'=(AB')+(BA')$

$(A+B)(A'+B')=AB'+BA'$

$\cancel{AA'}^{0}+AB'+BA'+\cancel{BB'}^{0}=AB'+BA'$

$AB'+BA'=AB'+BA'$ ✓