EE 503 PS#3

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3-1 $S = ?$

(a)

carlos

	1st flip H	T
2nd flip H	HH	TH
T	HT	TT

all w/ equal prob $= \frac{1}{4}$

Michael

	H	T
H	HH	TH
T	HT	TT

you can combine these 2 because they're sequential so 16 total events w/ prob $= \frac{1}{16}$ each.

Implied independence from knowledge of coin flipping.

$$P(C = HH) \cdot P(M = HH) = P(C = HH \cap M = HH)$$

(b) S_x where X is max(# Heads from C, M)

michael \ carlos	0	1	2
0	[0,0]	1,0	2,0
1	0,1	1,1	2,1
2	0,2	1,2	2,2

$\xrightarrow{S_x}$

0	1	2
1	1	2
2	2	2

$$S_x = \{0, 1, 2\}$$

(c) $Pr(X = 0) = \left(\frac{1}{4} \cdot \frac{1}{4}\right) = \frac{1}{16}$

$$Pr(x=1) = P([1,0]) + P[1,1] + P([0,1])$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{2}{8} + \frac{1}{4} = \frac{1}{2}$$

$$Pr(x=2) = P([2,0]) + P([2,1]) + P[2,2] + P([1,2]) + P([0,2])$$

$$= \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4}$$

$$= 1/16 + 1/8 + 1/16 + 1/8 + 1/16 = 7/16$$

3-2 | 5 card hand

(a) total # of possible hands = ${}_{52}C_5 = \frac{52!}{(52-5)!5!} = 2,598,960$

(b) $Pr(\text{5 hearts}) = \frac{{}_{13}C_5}{{}_{52}C_5} = \frac{\left(\frac{13!}{5!\,8!}\right)}{2598960} = 0.000495 = 0.0495\%$

(c) $Pr(\text{all same suit}) = Pr(\text{5 hearts}) + Pr(\text{5 clubs}) + Pr(\text{5 spades}) + Pr(\text{5 diamonds})$

$= 4 \cdot Pr(\text{5 hearts}) = 4 \cdot 0.000495 = 0.00198$

3-3 | (a) transmission exists b/t 1 & 2:

$= A + (B \cdot (C + D))$

(b) $Pr(\text{transmission b/t 1 \& 2}) = Pr(A) + Pr(BC) + Pr(BD) - Pr(ABC) - Pr(ABD) - Pr(BCD) + Pr(ABCD)$

$Pr(T) = a + bc + bd - abc - abd - bcd + abcd$

(c) given transmission exists, Pr that B works:

$$Pr(B|T) = \frac{Pr(T|B) \cdot Pr(B)}{Pr(T|B) \cdot Pr(B) + Pr(T|\bar{B}) \cdot Pr(\bar{B})}$$

$$= \frac{(a + c + d - ac - ad - cd + acd) \cdot b}{(a + c + d - ac - ad - cd + acd) \cdot b + (a \cdot b)}$$

or

$$Pr(B|T) = \frac{Pr(B \cap T)}{Pr(T)} = \frac{Pr(AB + BC + BD)}{P(T)}$$

3-4 | 3.46

15 toppings, choose 4 → no order

w/ repeats : # combos = $\binom{n-1+k}{k}$ (w/ replacement)

$n = 15$

$k = 4$

$$= \frac{(n-1+k)!}{(n-1+k-k)!\,k!} = \frac{(n-1+k)!}{(n-1)!\,k!}$$

$$= \frac{18!}{14!\,4!} = 3060 \text{ pizzas}$$

w/o repeats : # combos = $\binom{n}{k} = \frac{15!}{11!\,4!} = 1365$ pizzas

3-5 | Pr(4 people choosing 4 diff hotels) = Pr(A)

⇒ no order, no replacement $\cancel{Pr(A) = \frac{4\cdot\binom{3}{3}}{\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1}}}$

$\cancel{Pr(A) = \frac{4}{4\cdot 3\cdot 2\cdot 1} = \frac{1}{2}}$

to check

$$\boxed{Pr(A) = 1\cdot\frac{3}{4}\cdot\frac{2}{4}\cdot\frac{1}{4} = \frac{6}{4^3} = \frac{6}{64}}$$

3-6 | $P(\text{error}) = p = 10^{-2}$

ⓐ P(accepted) = P(no errors) + P(1 error)

$P(\text{accepted}) = (1-p)^{100} + (1-p)^{99}\cdot p\binom{100}{1} = 0.7358$

ⓑ Pr(M transmissions needed) = $P(\text{not Accepted})^{M-1}\cdot P(\text{accepted})$

$$= (1-0.7358)^{M-1}\cdot(0.7358)$$

3-D

$P(\text{drawer}) = 0.9 \quad P(\text{table}) = 0.06$

$P(\text{backpack}) = 0.03 \quad P(\text{usc}) = 0.01$

$P(\text{missing find}) = 0.1 \quad P(\text{find} \mid \text{look in right spot}) = 0.9 = P(F)$

ⓐ $P(T \mid \text{not found in drawer}) = \dfrac{P(\text{table})}{P(D)\cdot P(M) + P(B) + P(USC) + P(T)}$

$$= \frac{0.06}{(0.1)(0.9) + 0.03 + 0.01 + 0.06}$$

$$= 0.3159$$

ⓑ $P(B \mid \bar{T} \wedge \bar{D}) = \dfrac{0.03}{(0.1)(0.9 + 0.06) + 0.03 + 0.01}$

$$= 0.2206$$

ⓒ $P(U \mid \bar{D}, \bar{D}, \bar{T}, \bar{T}, B) = \dfrac{0.01}{(0.1)^2(0.9 + 0.06) + 0.03(0.1) + 0.01}$

$$= 0.4425$$