EE 503 PS 2 Charlie Andrus

~~2.6, 2.64, 2.22~~, 2.69, 2.89, 2-6, 2-7

2.6 A, B, C names in a hat. draw w/o replacement
A draws then B then C

(a) $\Omega = \{ABC, ACB, CBA, CAB, BCA, BAC\}$

(b) {Al draws his name} = A = {ABC, ACB}
B = {CBA, ABC}   C = {ABC, BAC}

(c) {no one draws his name} = {CAB, BCA}

(d) {everyone draws his own name} = {ABC}

(e) {one or more draws his own name} = {ABC, ACB, CBA, BAC}

2.64 w/o replacement

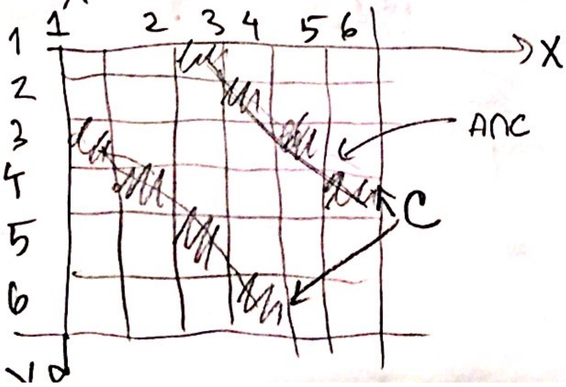
$$P(B \cap C \mid A) = \frac{P(B \cap C \cap A)}{P(A)} = \frac{1/6}{2/6} = \frac{1}{2}$$

$$P(C \mid A \cap B) = \frac{P(C \cap A \cap B)}{P(A \cap B)} = \frac{1/6}{1/6} = 1$$

2.22 one die tossed twice

(a) all events equally likely w/ 36 different events

X = 1st toss   Y = second toss



$P(X = x) = 1/6$

$P(Y = y) = 1/6$

$P(X = x, Y = y) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$

(b) $P(A) = ?$ $P(B) = ?$ $P(C) = ?$ $P(A \cap B') = ?$ $P(A \cap C) = ?$

$A = \overline{X < Y} = X \geq Y$

$P(A) = \frac{15}{36} + \frac{6}{36}$

$P(A) = P(X > Y) + P(X = Y) = \frac{21}{36}$

$P(B) = P(X = 6) = \frac{1}{6}$

$P(C) = P(|X - Y| = 2) = \frac{8}{36}$ (see graph in A)

$P(A \cap B') = P(X \geq Y, X \neq 6) = \frac{15}{36}$

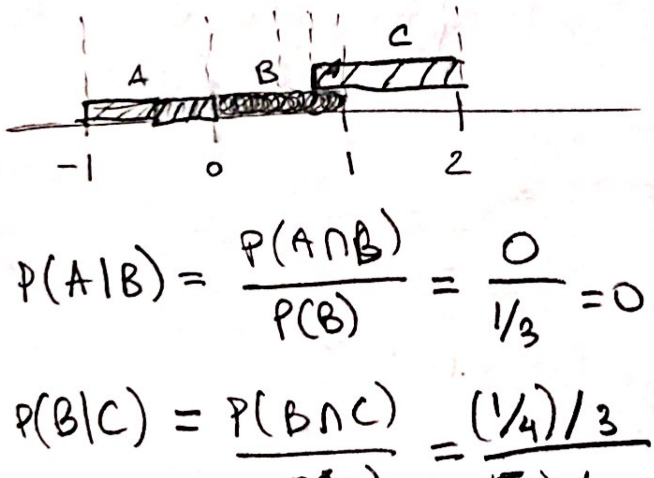
$P(A \cap C) = \frac{4}{36}$ (see graph)

$\boxed{2.69}$ conditional prob.

x in interval $[-1, 2]$ $\quad A = \{x < 0\}$

$B = \{|x - 0.5| < 0.5\}$

$C = \{x > 0.75\}$



$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{1/3} = 0$

$B = \{x - 0.5 < 0.5,\ x - 0.5 > -0.5\}$
$= \{x < 1,\ x > 0\}$

$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{(1/4)/3}{(5/4)/3} = \frac{1}{5}$

$P(A|C') = \frac{P(A \cap C')}{P(C')} = \frac{P(A)}{P(C')} \leftarrow A \subset C'$ $= \frac{1/\cancel{3}}{(7/4)/\cancel{3}} = \frac{4}{7}$

$P(B|C') = \frac{P(B \cap C')}{P(C')} = \frac{(3/4)/3}{(7/4)/3} = \frac{3}{7}$

$\boxed{2.89}$ 2.14 assume A, B, C indep

(a) $\{\text{one of 3 occurs}\} = P(A\bar{B}\bar{C}) + P(\bar{A}B\bar{C}) + P(\bar{A}\bar{B}C)$

$= P(A)P(B')P(C') + P(A')P(B)P(C') + P(A')P(B')P(C)$

(b) $\{\text{2 events occur}\} = P(\bar{A}BC) + P(A\bar{B}C) + P(AB\bar{C})$

$= P(A')P(B)P(C) + P(A)P(B')P(C) + P(A)P(B)P(C')$

(c) $\{\text{1+ events occur}\} = 1 - P(A')P(B')P(C')$ or $(a)+(b)+P(A)P(B)P(C)$

prefer to write shorter answers

(d) $\{\text{2+ events occur}\} = P(A')P(B)P(C) + P(A)P(B')P(C) + P(A)P(B)P(C') + P(A)P(B)P(C)$

(e) $= P(A')P(B')P(C') = \{\text{no events occur}\}$

$\boxed{2\text{-}6}$ $P(A|C) = 1/2$ $P(A|D) = 1/5$ $P(C) = 1/5$ $P(D) = 2/5$

$C \cap D = \Phi$

$$P(A|(C+D)) = \frac{P(A \cap (C \cup D))}{P(C \cup D)} = \frac{P(AC) + P(AD)}{P(C+D)} = \frac{P(AC)+P(AD)}{P(C)+P(D)}$$

$$P(A|C) = \frac{P(AC)}{P(C)} \qquad P(A|D) = \frac{P(AD)}{P(D)}$$

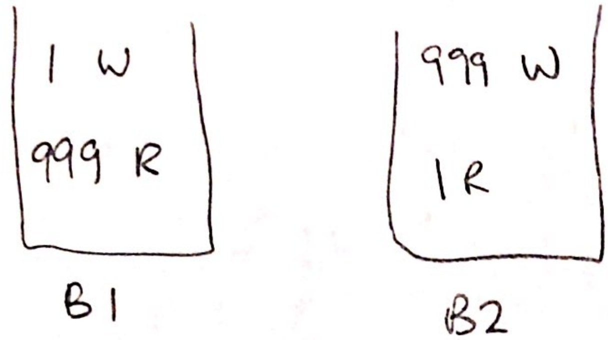
$$P(A|C)\cdot P(C) = P(AC) \qquad P(A|D)\cdot P(D) = P(AD)$$

$$P(A|(C+D)) = \frac{P(AC)+P(AD)}{P(C)+P(D)} = \boxed{\frac{P(A|C)\cdot P(C) + P(A|D)\cdot P(A)}{P(C)+P(D)}} =$$

$$\frac{9/10}{3} = \frac{\frac{1}{5}\left(\frac{1}{2}+\frac{2}{5}\right)}{\left(\frac{1}{5}\right)[1+2]} = \frac{(1/2)(1/5)+(1/5)(2/5)}{(1/5)+(2/5)} = \frac{1}{3}\left(\frac{9}{10}\right) = \boxed{\frac{3}{10}}$$

sol'n

2-7



$$Pr(B_1|R) = \frac{Pr(R|B_1)P(B_1)}{Pr(R|B_1)P(B_1) + P(R|U_2)P(B_2)}$$

$$= \frac{(999/1000)\left(\cancel{\tfrac{1}{2}}\right)}{\left(\frac{999}{1000}\right)\left(\cancel{\tfrac{1}{2}}\right) + \left(\frac{1}{1000}\right)\left(\cancel{\tfrac{1}{2}}\right)}$$

$$P(B_1|R) = \frac{(999/1000)}{\frac{999}{1000} + \frac{1}{1000}} = \frac{999/1000}{1} = \frac{999}{1000}$$