EE 503: Midterm 1
Wed. Feb. 20, 2019 10:00am-11:55am
Location 1: OHE 122 (Last name: Gao - Zuo)
Location 2: GFS 222 (Last name: Abedsoltan - Gajjar)

A. *Instructions*

- Put your name on the top of this page.
- The exam time is 1 hour and 55 minutes.
- No notes, books, phones, or other electronic equipment.
- Feel free to leave your answer as a fully defined sum, such as $\sum_{n=1}^{\infty} n^2 e^{-n}$, without simplifying.
- Justify your work. A correct answer with little or no explanation will receive zero credit.

B. *Formulas and reminders*

1) Set unions and intersections:
   - $(\cap_{n=1}^{\infty}\mathcal{A}_n)^c = \cup_{n=1}^{\infty}\mathcal{A}_n^c$.
   - $(\cup_{n=1}^{\infty}\mathcal{A}_n)^c = \cap_{n=1}^{\infty}\mathcal{A}_n^c$.
   - $\mathcal{A} \cap (\cup_{n=1}^{\infty}\mathcal{B}_n) = \cup_{n=1}^{\infty}(\mathcal{A} \cap \mathcal{B}_n)$.
2) Axioms of Probability:
   - $P[\mathcal{A}] \geq 0$ for all events $\mathcal{A}$.
   - $P[\mathcal{S}] = 1$.
   - If $\{\mathcal{A}_n\}_{n=1}^{\infty}$ are mutually exclusive, then $P[\cup_{n=1}^{\infty}\mathcal{A}_n] = \sum_{n=1}^{\infty} P[\mathcal{A}_n]$.
3) Consequences of the axioms:
   - $P[\mathcal{A}] + P[\mathcal{A}^c] = 1$, $0 \leq P[\mathcal{A}] \leq 1$.
4) Total conditioning: Suppose events $\{\mathcal{A}_n\}_{n=1}^{\infty}$ partition $\mathcal{S}$. Then:
   - $P[\mathcal{B}] = \sum_{n=1}^{\infty} P[\mathcal{B}|\mathcal{A}_n]P[\mathcal{A}_n]$.
   - $\mathbb{E}[X] = \sum_{n=1}^{\infty} \mathbb{E}[X|\mathcal{A}_n] P[\mathcal{A}_n]$.
5) Conditional probability and Baye's rule:
   - $P[\mathcal{A}|\mathcal{B}] = \frac{P[\mathcal{A}\cap\mathcal{B}]}{P[\mathcal{B}]} = \frac{P[\mathcal{B}|\mathcal{A}]P[\mathcal{A}]}{P[\mathcal{B}]}$.
6) Cumulative distribution function (CDF) and its properties:
   - $F_X(x) = P[X \leq x]$ for all $x \in \mathbb{R}$.
   - $\lim_{x\to\infty} F_X(x) = 1$, $\lim_{x\to-\infty} F_X(x) = 0$, $F_X(x)$ is nondecreasing in x.
   - $P[X = x] = F_X(x) - F_X(x^-) =$ height of "jump" at point x.
   - $F_{X|\mathcal{A}}(x) = P[X \leq x|\mathcal{A}]$ for all $x \in \mathbb{R}$.
7) Expectations:
   - $\mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x)dx$, $\mathbb{E}[h(X)] = \int_{-\infty}^{\infty} h(x) f_X(x)dx$, $Var(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right]$.
   - Discrete case: $\mathbb{E}[X] = \sum_{x\in\mathcal{S}_X} xP[X = x]$, $\mathbb{E}[h(X)] = \sum_{x\in\mathcal{S}_X} h(x)P[X = x]$.
8) PDF transformations:
   - $Y = aX + b$ with $a \neq 0$. Then $f_Y(y) = \frac{f_X(\frac{y-b}{a})}{|a|}$.
   - $Y = h(X)$ with $\{x_1, ..., x_n\}$ the points that satisfy $h(x_i) = y$. Then $f_Y(y) = \sum_{i=1}^{n} \frac{f_X(x_i)}{|h'(x_i)|}$.
9) Gaussian PDF: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$ for $x \in \mathbb{R}$. $\mathbb{E}[X] = m, Var(X) = \sigma^2$.
10) Exponential PDF: $f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. $\mathbb{E}[X] = 1/\lambda, Var(X) = 1/\lambda^2$.