SUMMER SCHOOL 2020

MACHINE LEARNING DAY 2

DEEP LEARNING

Session II: Linear regression

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Categories of ML problems

	Supervised	Unsupervised	Reinforcement
Discrete	Classification	Clustering	Action space agent
Continuous	Regression	Dimensionality reduction	Action space agent

Categories of ML problems

	Supervised	Unsupervised	Reinforcement
Discrete	Classification	Clustering	**Action space agent**
Continuous	Regression	Dimensionality reduction	**Action space agent**

Reinforcement learning

Discrete	Action space agent
Continuous	Action space agent

Reinforcement

Goal	Train an agent to achieve a goal through state/action/reward
Examples	Path finder, decision making problem

Categories of ML problems

	Supervised	Unsupervised	Reinforcement
Discrete	Classification	Clustering	Action space agent
Continuous	Regression	Dimensionality reduction	Action space agent

Unsupervised learning

Discrete	**Clustering**
Continuous	**Dimensionality reduction**

Unsupervised

Input	Data x
Goal	learn some underlying hidden structure of the data
Examples	Clustering, dimensionality reduction, feature learning, density estimation

Categories of ML problems

	Supervised	Unsupervised	Reinforcement
Discrete	**Classification**	Clustering	Action space agent
Continuous	**Regression**	Dimensionality reduction	Action space agent

Supervised learning

Discrete

Classification

Continuous

Regression

Supervised

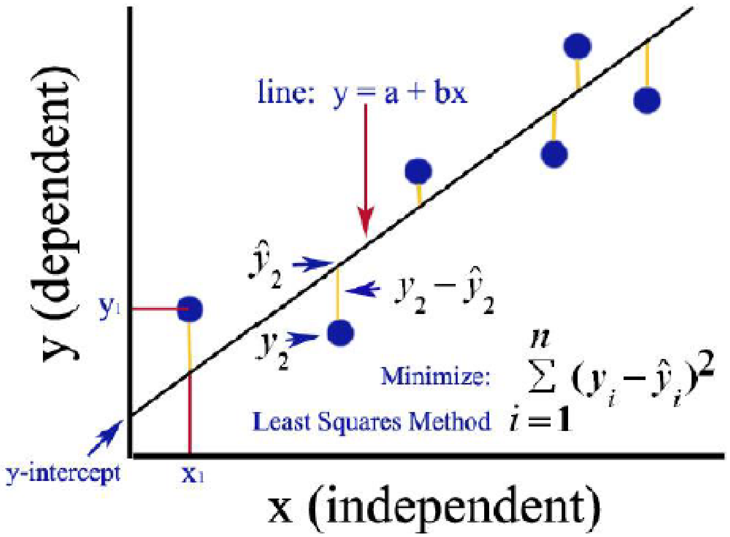
Input	Data (x, y) x: data, y: label
Goal	learn a function to map x to y
Examples	classification, regression, object detection semantic segmentation, image captioning

Categories of ML problems

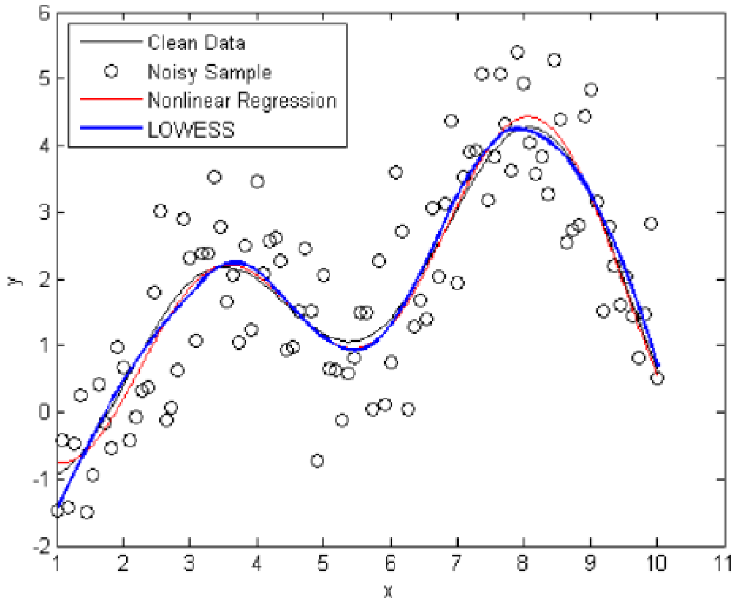
	Supervised	Unsupervised	Reinforcement
Discrete	Classification	Clustering	Action space agent
Continuous	Regression	Dimensionality reduction	Action space agent

Regression problem

Fit the prediction function f(x) to the training data to predict continuous real value



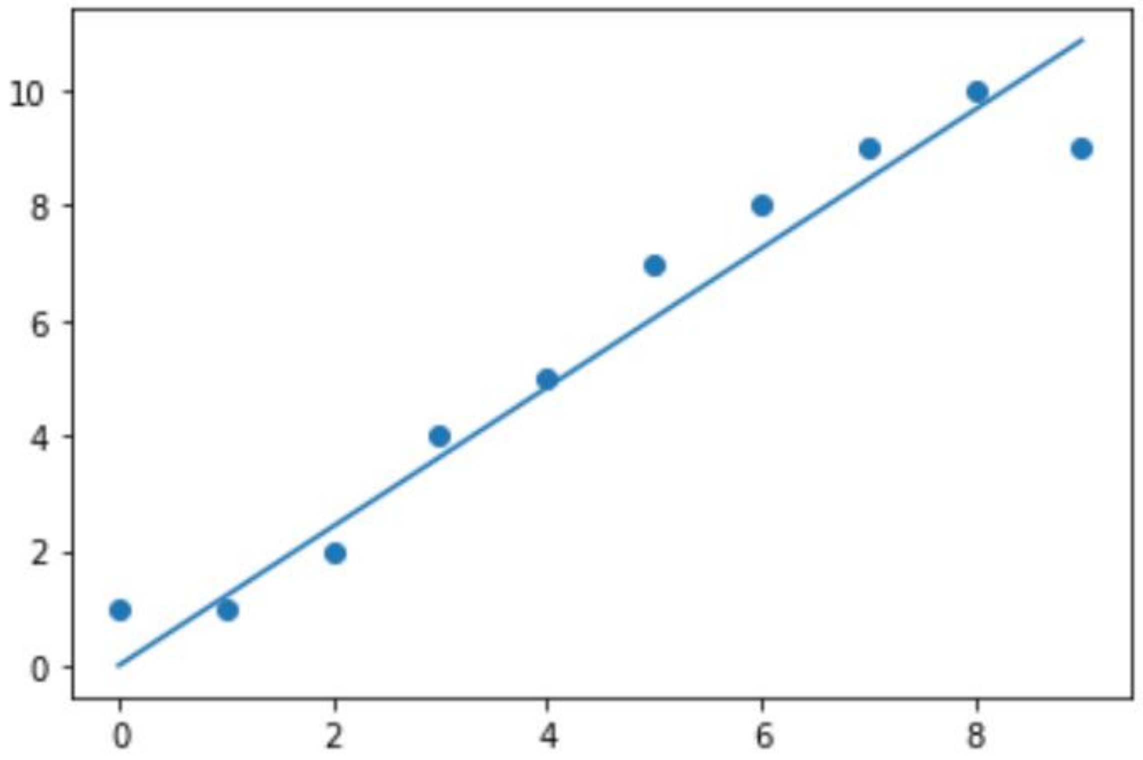
Linear regression



Nonlinear regression

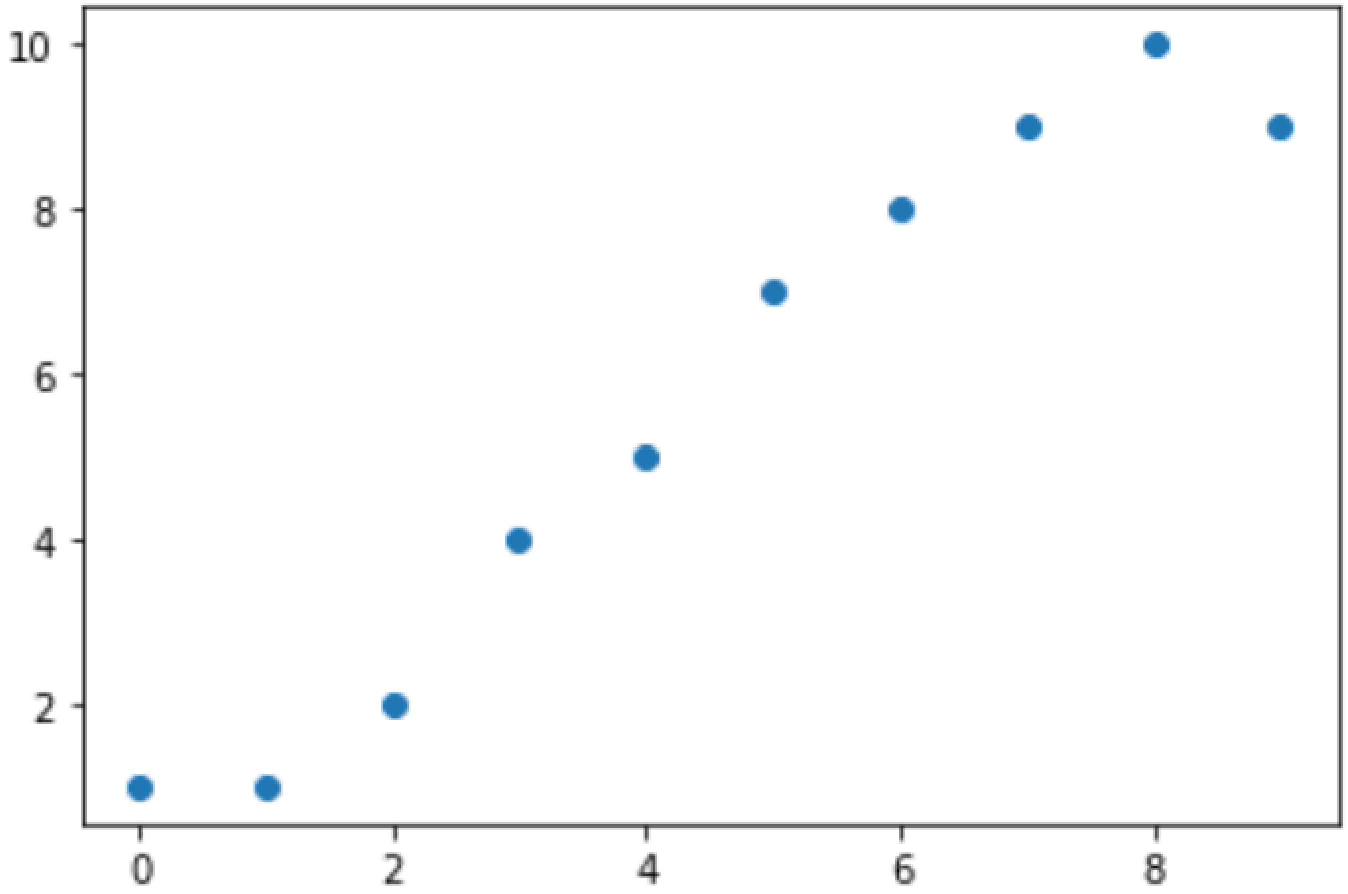
Linear regression : single variable

x	y
0	1
1	1
2	2
3	4
4	5
5	7
6	8
7	9
8	10
9	9

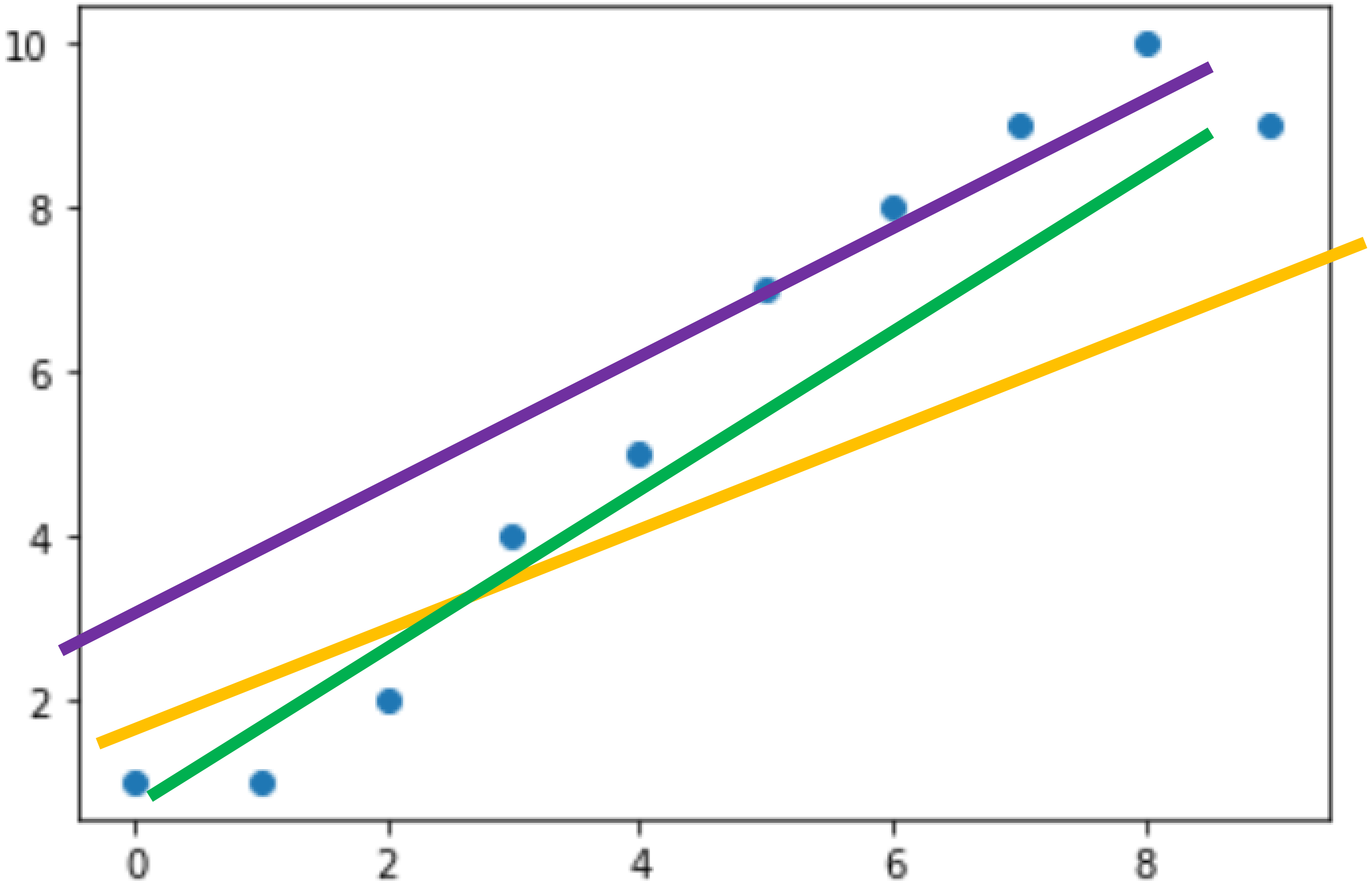


Data preparation: Input (x, y)

x	y
0	1
1	1
2	2
3	4
4	5
5	7
6	8
7	9
8	10
9	9

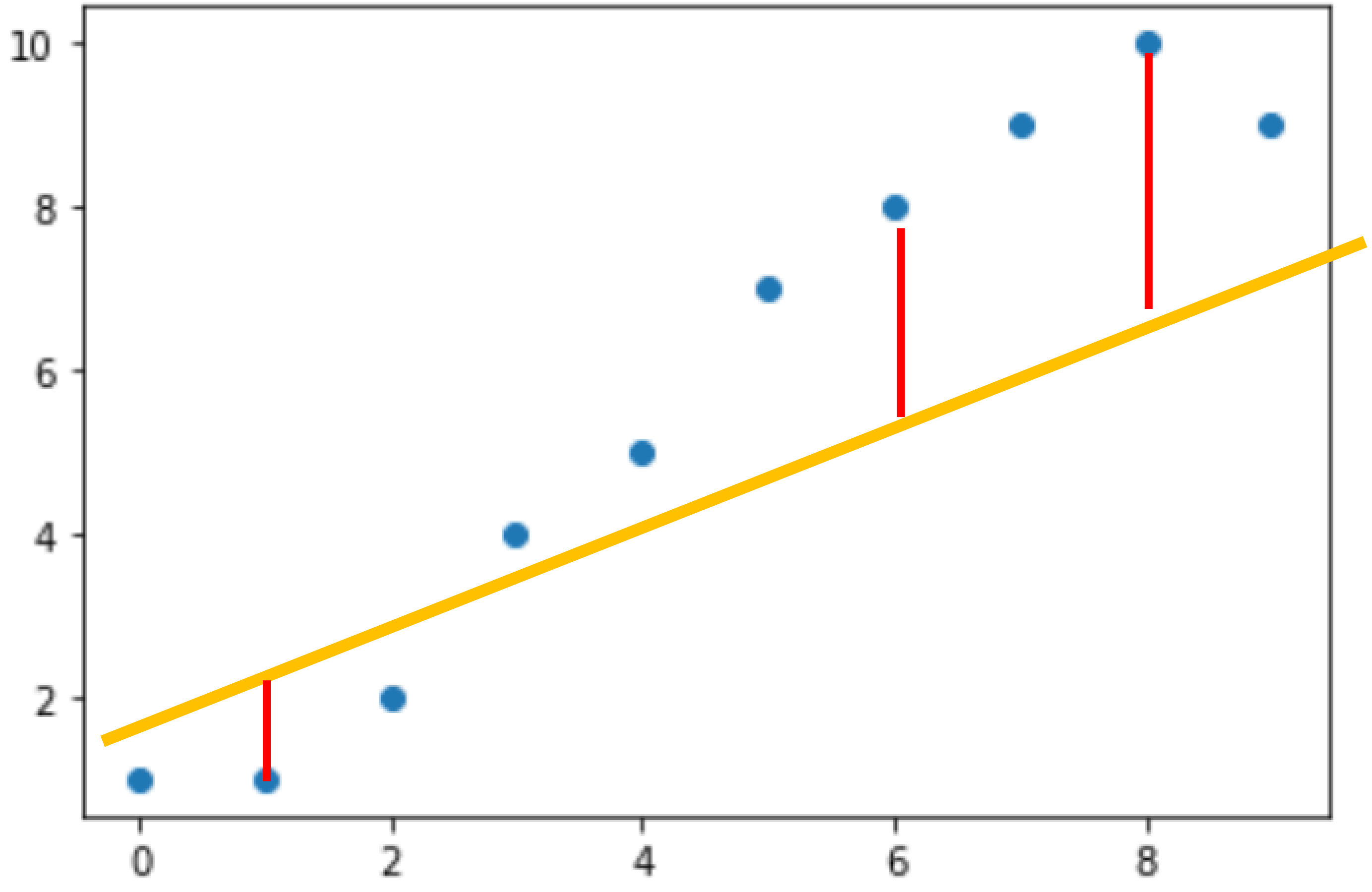


Model (Hypothesis)



$$H(x) = Wx + b$$

Which model is better?



How well fit the line to data?

$$\underbrace{H(x)}_{\text{Predicted}} - \underbrace{y}_{\text{True}}$$

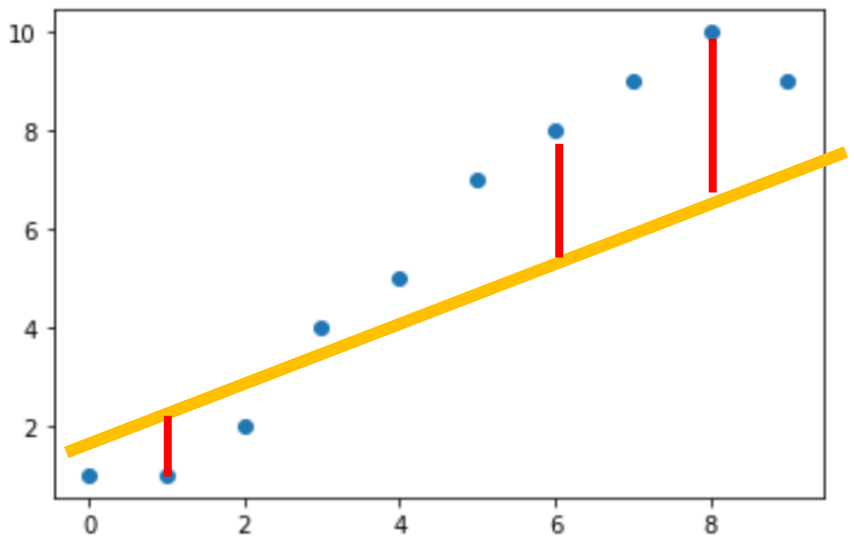
Cost function

Model

$$H(x) = Wx + b$$

Mean Square Error

$$cost = \frac{1}{m}\sum_{i=1}^{m}(H(x_i) - y_i)^2$$



m is the number of data.

Now we can see the cost function as a function of W and b.

$$cost(W, b) = \frac{1}{m}\sum_{i=1}^{m}\left((Wx_i + b) - y_i\right)^2$$

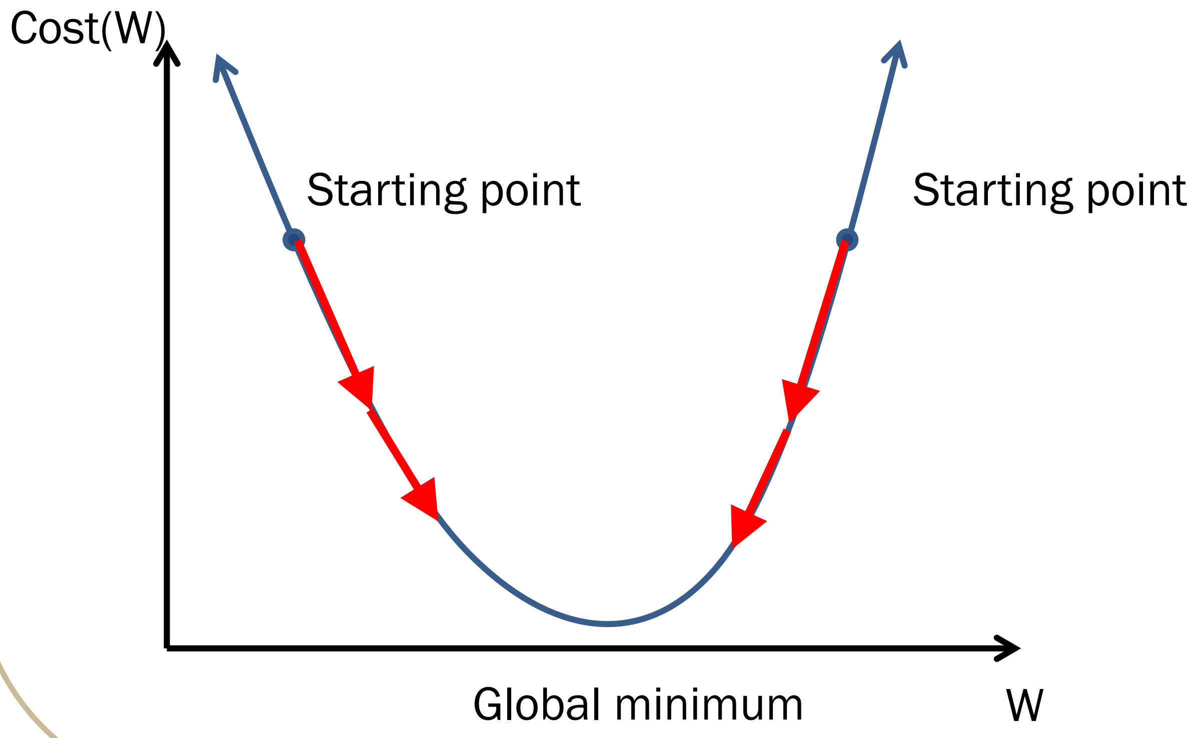
Cost function: what we want?

We want to minimize the cost!

$$cost(W, b) = \frac{1}{m}\sum_{i=1}^{m}\left((Wx_i + b) - y_i\right)^2$$

Gradient decent algorithm

Let's consider a simple case with W only.



$$cost(W) = \frac{1}{2m}\sum_{i=1}^{m}(Wx_i - y_i)^2$$

$$\frac{d}{dW}cost(W) = \frac{1}{m}\sum_{i=1}^{m}(Wx_i - y_i)x_i$$

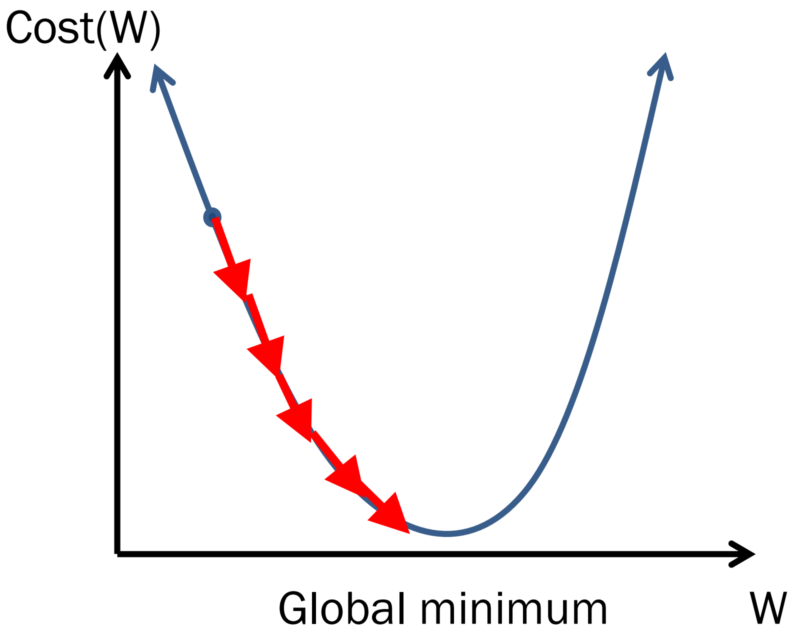
$$W := W - \alpha\frac{d}{dW}cost(W)$$

Learning rate

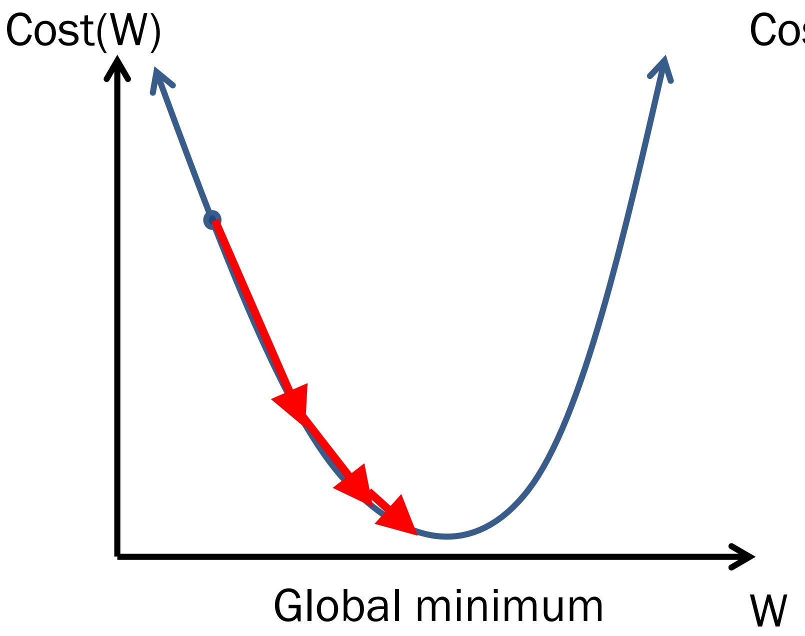
Learning rate

$$W := W - \alpha \frac{d}{dW} cost(W)$$

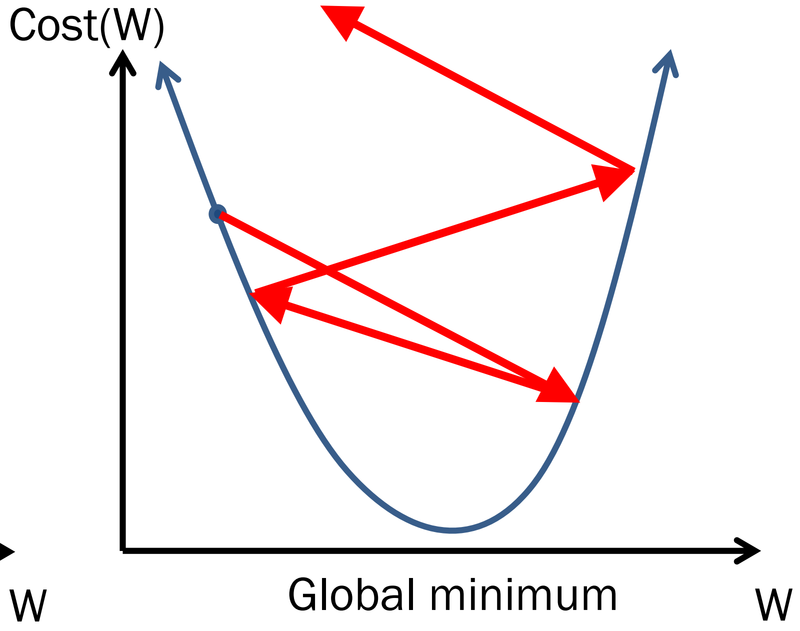
Learning rate



A small learning rate
requires many steps



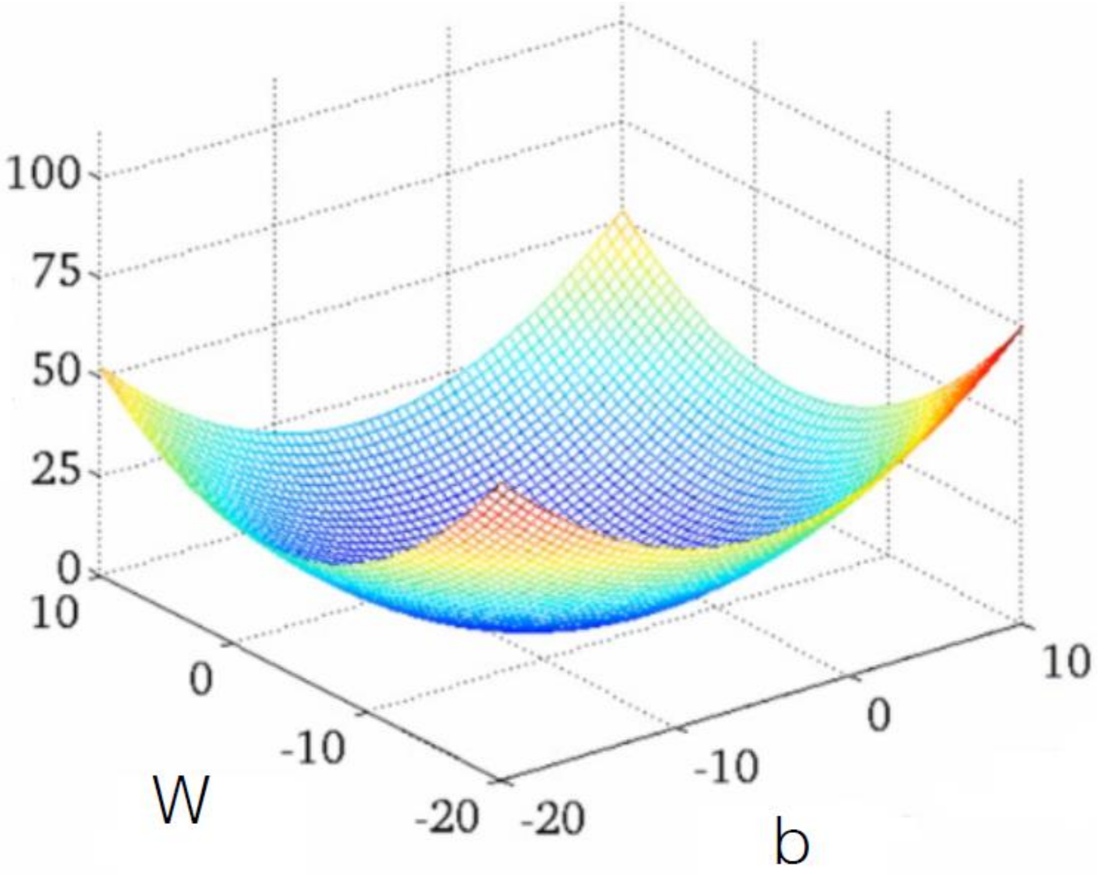
The optimal learning rate
(possibly adaptive value)



Too large learning rate
causes divergence

Cost (loss) function

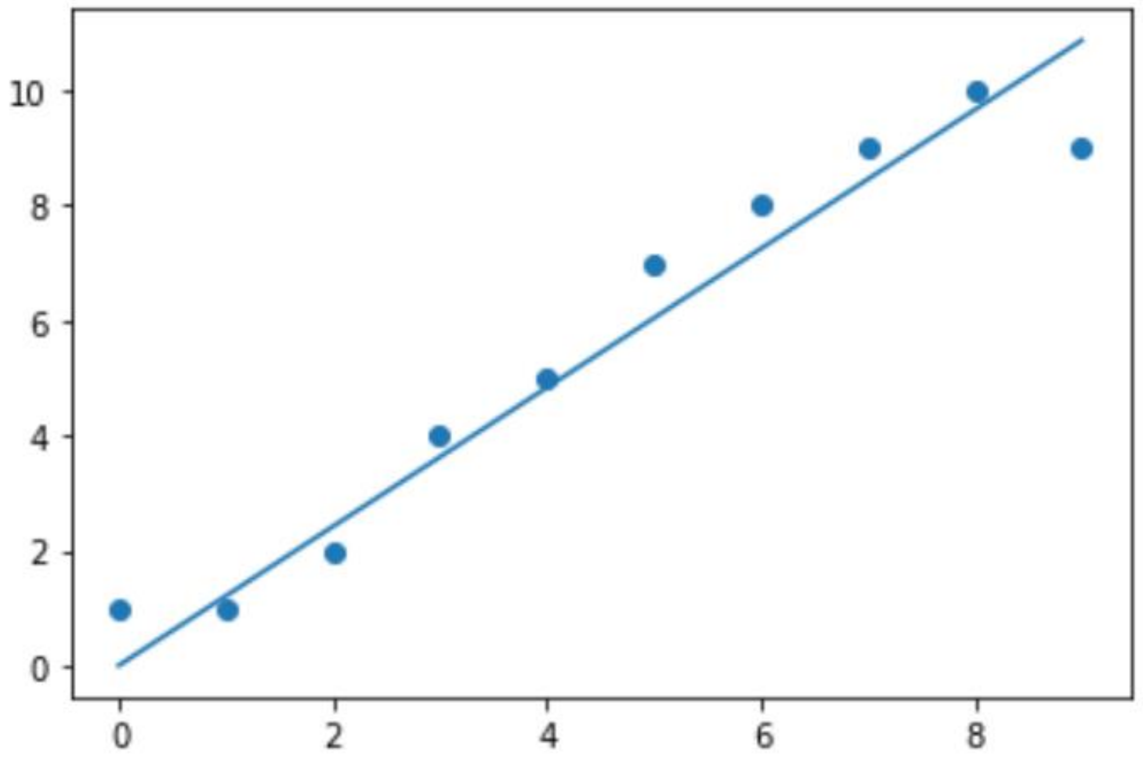
$$cost(W,b) = \frac{1}{m}\sum_{i=1}^{m}\left((Wx_i + b) - y_i\right)^2$$



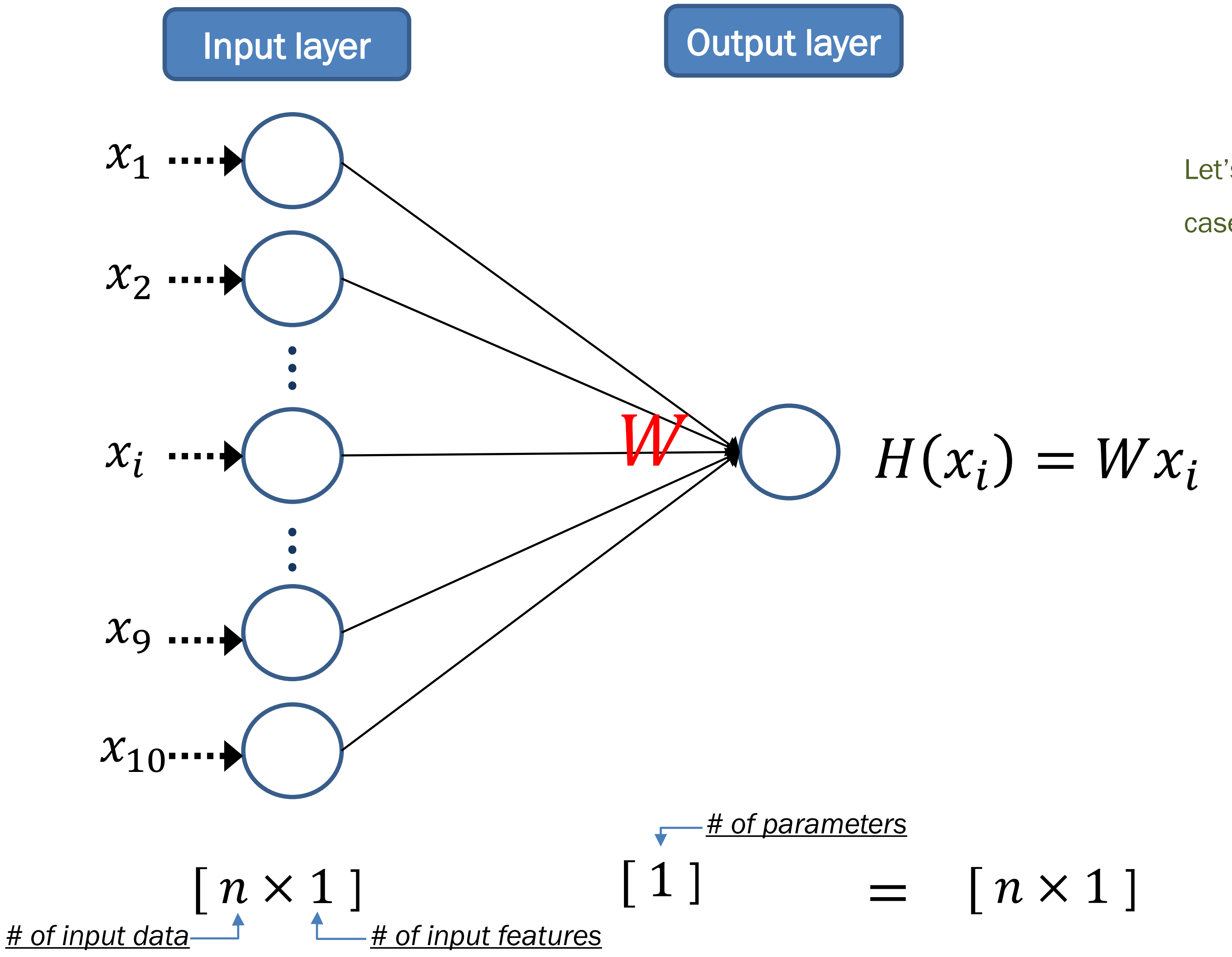
http://www.holehouse.org/mlclass

Lab 2A: Linear regression (single variable)

x	y
0	1
1	1
2	2
3	4
4	5
5	7
6	8
7	9
8	10
9	9



Layout



Let's consider a simple case with W only.

Coding modules

Data Preparation

Model define

Cost function + optimizer

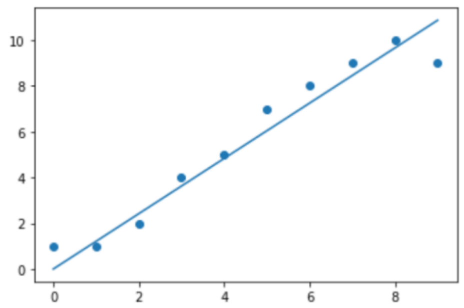
Model Test

x	y
0	1
1	1
2	2
3	4
4	5
5	7
6	8
7	9
8	10
9	9

$$H(x) = Wx + b$$

$$cost(W) = \frac{1}{m}\sum_{i=1}^{m}(H(x_i) - y_i)^2$$

$$W := W - \alpha \frac{d}{dW} cost(W)$$



Lab 2A: Linear regression - vanilla

Github:

https://github.com/isaacye/SS2020_ML_Day2

What you may want to try :

1. Check the model define
2. Check the result by changing the starting point
3. Check the result by changing learning rate

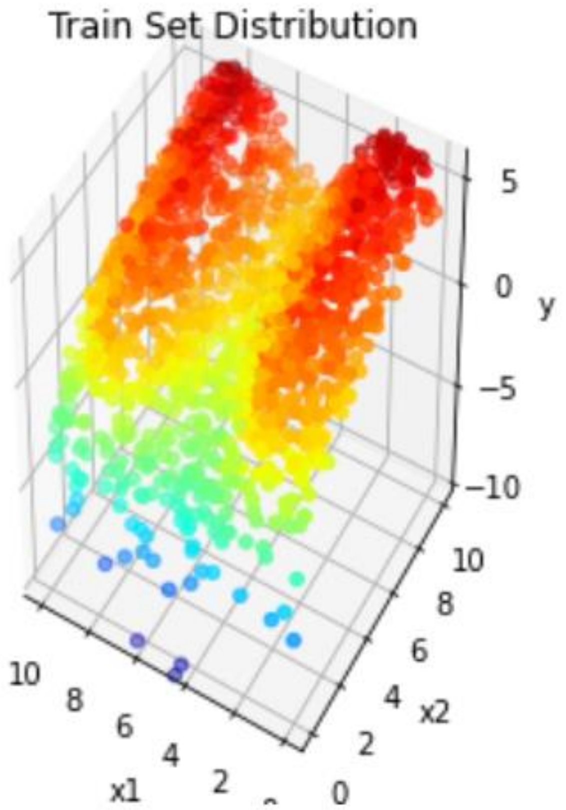
Break room

Linear regression: multivariable

Data preparation: Input (x_1,x_2, y)

x_1	x_2	y
3.91870851	2.32626914	0.73817558
2.59194437	6.00656071	4.3940048
6.46991632	3.57514815	0.61488728
:	:	:
4.56486433	2.14296641	3.95964088
1.29483514	1.67730041	3.48018992

```
num_data = 2400
x1 = np.random.rand(num_data) *10
x2 = np.random.rand(num_data) *10
e = np.random.normal(0, 0.5, num_data)
X= np.array([x1,x2]).T  # T for transpose from (2, 2400) to (2400, 2)
y=2*np.sin(x1) + np.log(0.5*x2**2)+e
```



Model (Hypothesis)

$$H(x_1, x_2) = w_1 x_1 + w_2 x_2 + b$$

For many input data with n number of features, it is can be written as

$$H(x_{i1}, x_{i2}, x_{i3}, \dots, x_{in-1}, x_{in}) = w_1 x_{i1} + w_2 x_2 + w_3 x_3 + \cdots + w_n x_n + b$$

Expression in matrix

$$H(x_{i1}, x_{i2}) = w_1 x_{i1} + w_2 x_{i2} + b$$

$$\begin{pmatrix} x_{i1} & x_{i2} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + b = w_1 x_{i1} + w_2 x_{i2} + b$$

$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + \begin{pmatrix} b \\ b \\ \vdots \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + b \\ x_{21}w_1 + x_{22}w_2 + b \\ \vdots \end{pmatrix}$$

$$[n \times 2] \qquad [2 \times 1] \qquad [n \times 1] \qquad n \times 1$$

$$H(X) = XW + b$$

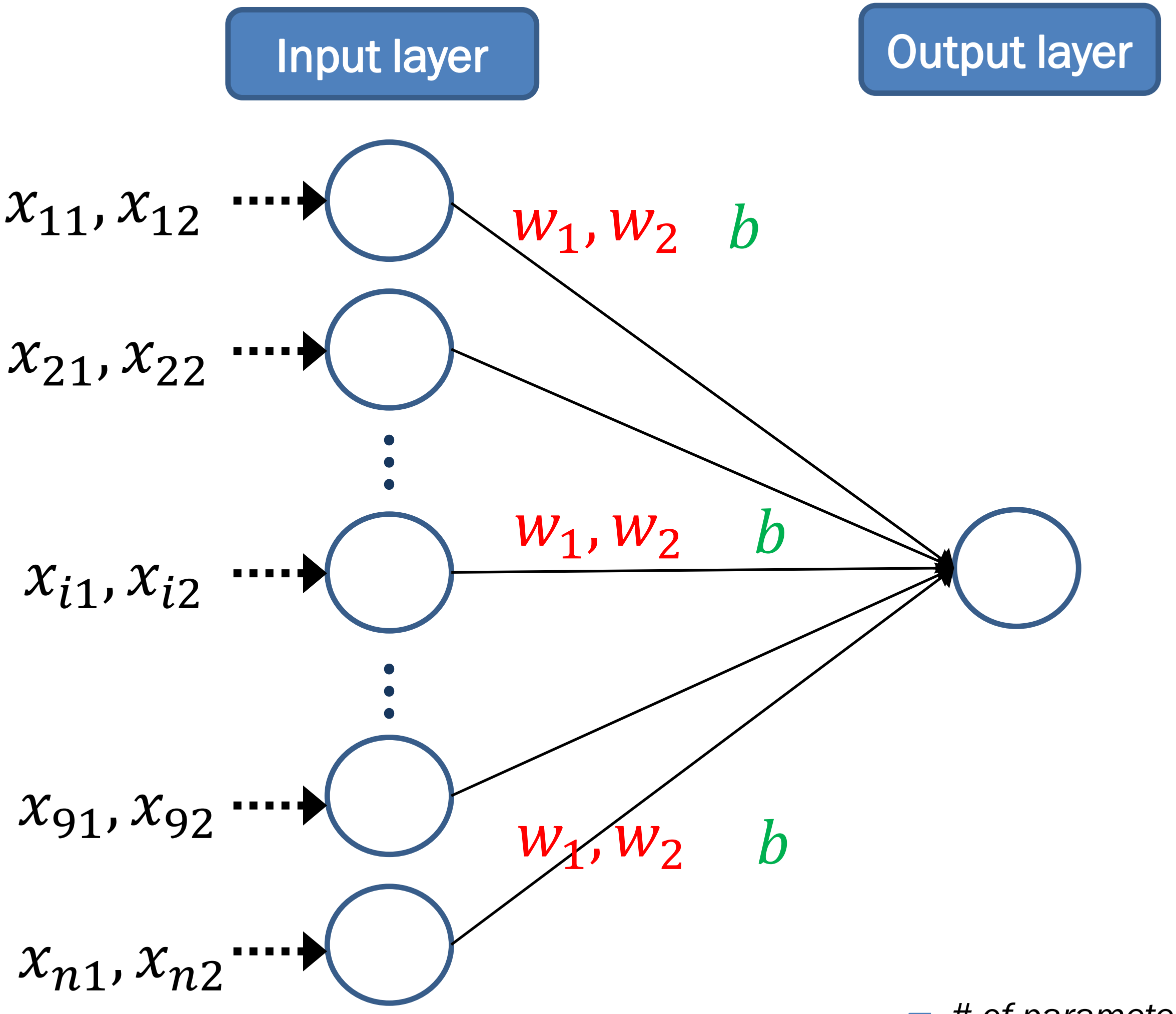
Cost function

$$H(X) = XW + b$$

$$cost = \frac{1}{m}\sum_{i=1}^{m}(H(x_{i1}, x_{i2}) - y_i)^2$$

We want to minimize the cost as well!

Layout



$$H(x_{i1}, x_{i2}) = XW + b$$

# of parameters

$$[\, n \times 2 \,] \quad [\, 2 \times 1 \,] \quad = \quad [\, n \times 1 \,]$$

# of input data *# of input features*

Coding modules

Data Preparation

Model define

Cost function + optimizer

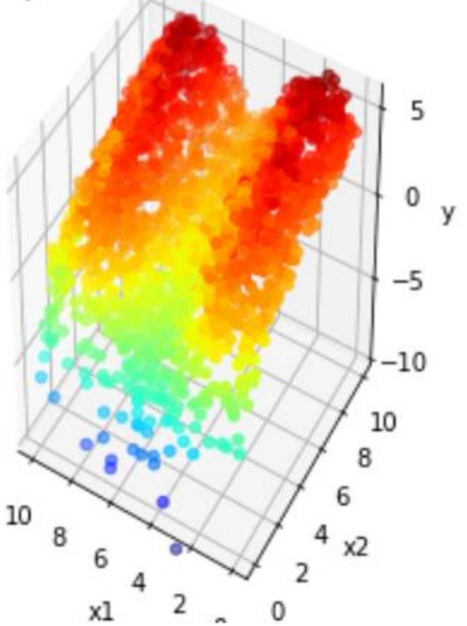
Model Test

Data preparation

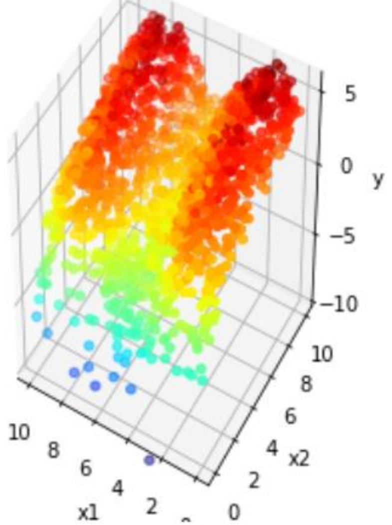
Data Preparation

x_1	x_2	y
3.91870851	2.32626914	0.73817558
2.59194437	6.00656071	4.3940048
6.46991632	3.57514815	0.61488728
:	:	:
4.56486433	2.14296641	3.95964088
1.29483514	1.67730041	3.48018992

Input Data Set Distribution

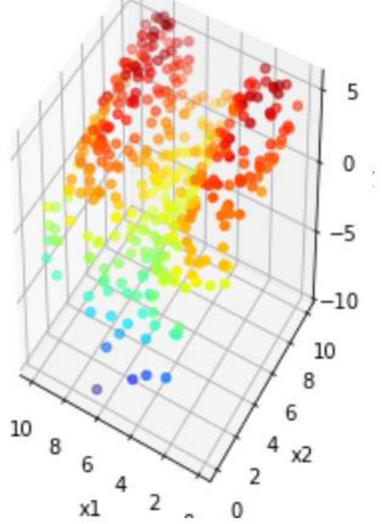


Train Set Distribution



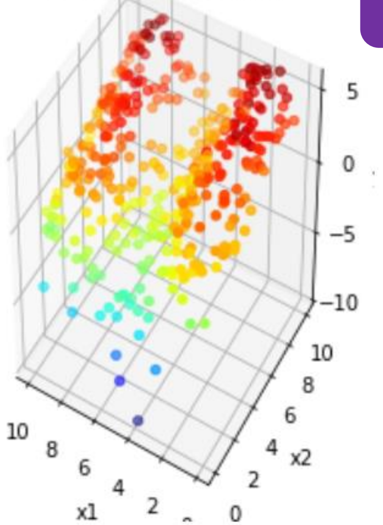
Train set

Validation Set Distribution



Validation set

Test Set Distribution



Testing set

In the code

```
train_X, train_y = X[:1600, :], y[:1600]
val_X, val_y = X[1600:2000, :], y[1600:2000]
test_X, test_y = X[2000:, :], y[2000:]
```

Model define

In the code

```
import torch
import torch.nn as nn

class LinearModel(nn.Module):
  def __init__(self):
    super(LinearModel, self).__init__()
    self.linear = nn.Linear(in_features=2, out_features=1, bias=True)

  def forward(self, x):
    return self.linear(x)
```

Linear model in PyTorch

Linear

CLASS `torch.nn.Linear(in_features, out_features, bias=True)` [SOURCE]

Applies a linear transformation to the incoming data: $y = xA^T + b$

Parameters

- **in_features** – size of each input sample
- **out_features** – size of each output sample
- **bias** – If set to `False`, the layer will not learn an additive bias. Default: `True`

Shape:

- Input: $(N, *, H_{in})$ where $*$ means any number of additional dimensions and $H_{in} = \text{in\_features}$
- Output: $(N, *, H_{out})$ where all but the last dimension are the same shape as the input and $H_{out} = \text{out\_features}$.

Cost (loss) function + Optimizer

Cost function + optimizer

Loss function

In the code

```
reg_loss = nn.MSELoss()
```

MSELoss

CLASS `torch.nn.MSELoss(size_average=None, reduce=None, reduction='mean')` [SOURCE]

Creates a criterion that measures the mean squared error (squared L2 norm) between each element in the input x and target y .

Optimizer

In the code

```
lr = 0.005
optimizer = optim.SGD(model.parameters(), lr =lr)
```

CLASS `torch.optim.SGD(params, lr=<required parameter>, momentum=0, dampening=0, weight_decay=0, nesterov=False)` [SOURCE]

Implements stochastic gradient descent (optionally with momentum).

Model test

Model Test

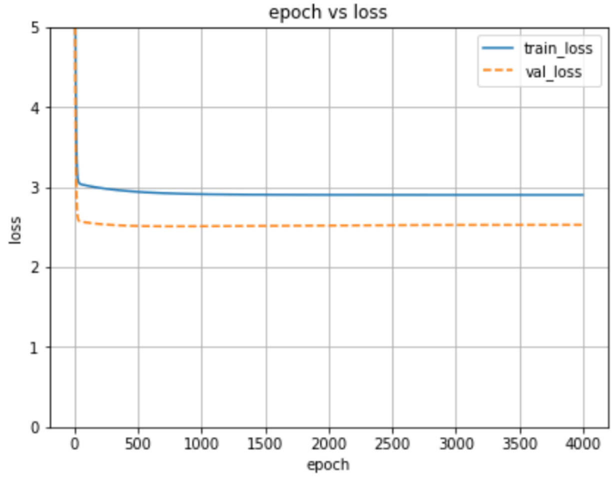
EPOCH=4000

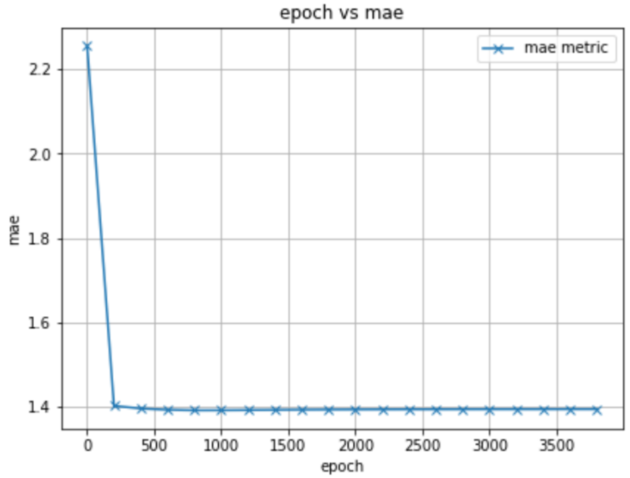
Train set

Validation set

Every 200 epoch

Test set





Lab 2B: Linear regression – Linear model

1. Check the model define (linear model)
2. Check the result by varying learning rate
3. Check the result w/ w/o bias
4. Check the result with different number of Epoch

Break room

Session break:

Please come back by 2 PM