

Continuous-Time Derivatives Pricing

Express Certificate on Siemens AG Stock

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I – IV. Designing Certificate

This assignment analyzes a hypothetically issued express certificate on Siemens AG, launched 15 April 2024 with maturity on 10 February 2030. It includes five annual observation dates from July 2025 and offers a fixed 100 EUR repayment plus rising annual coupons from 7 EUR up to 42 EUR if held to maturity.

At each observation date, Siemens' stock is checked against a step-down threshold, starting at 100% of the initial price and decreasing 5% yearly to 80%. If the stock closes above the threshold, early redemption occurs with repayment and coupon. At maturity, if the stock stays above 60% of its initial price, investors receive full repayment plus coupons. If it drops below 60%, they bear the full proportional loss.

Certificate payoff function for $t = 1, \dots, 6$:

$$\text{payoff} = \begin{cases} 100 + t \cdot c, & \text{if } S_t \geq S_0 \cdot (1 - 0.05 \cdot (t-1)) \\ & t = 1, \dots, 5 \\ 100 + t \cdot c, & \text{if } S_t \geq 0.6 \cdot S_0, \quad t = 6 \\ 100 \cdot \frac{S_t}{S_0}, & \text{if } S_t < 0.6 \cdot S_0, \quad t = 6 \end{cases}$$

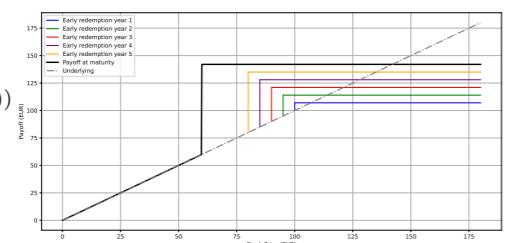


Figure 1: Payoff structure of the Express Certificate

The product targets private investors with solid financial knowledge and moderate risk appetite, who aim for long-term growth. It suits investors expecting sideways or slight upward moves in Siemens, a stock with moderate volatility. As there is no capital protection, investors must tolerate full losses if the stock ends below 60% at maturity. Figure 1 shows that the product outperforms the underlying between 60 and 142 EUR.

Express certificates make up about a third of the German structured products market and over half of non-capital-protected issues. For DAX-based names like Siemens, typical volumes reach 50 million EUR. A realistic estimate for this product is 25 million EUR, driven by Siemens' brand and retail appeal. Demand rises in low-rate environments but tends to decline in volatile or bearish markets due to rising risk aversion.

V. Valuation

In this section, I value a real product (ISIN: DE000DK1BKQ3) issued by Deka, which follows the same payoff logic as the one modeled. Due to its path-dependent nature, I implement monte carlo simulation framework based on geometric brownian motion, as the Black-Scholes formula is only applicable to closed-form solutions. I simulate 1,000 stock price paths from the emission date to maturity. The certificate value is then computed daily as the discounted average payoff across all paths, which also accounts for early redemption and barrier conditions.

The risk-free rate, which equals 2.28%, was derived using the Svensson method, applied to the Bundesbank yield data. Since the express certificate has a 6-year maturity, I extracted the 6-year spot rate from the fitted yield curve to discount future cash flows. Volatility was estimated from the standard deviation of daily log returns. I compared two modeling approaches: a fixed volatility, based on 1-year (31.63%) and 2-year (27.77%) historical windows, and a rolling volatility, where I computed daily updated volatilities using a 252-day and 504-day moving window. This allowed the model to better account for the evolving market conditions.

Figure 2 shows that simulated prices based on 2-year rolling volatility align most closely with market prices, while the 2-year fixed volatility tends to underestimate the certificate's value. To evaluate the model fit, I calculated error metrics such as bias, MAE, RMSE, and quantiles of the absolute error. As shown in Table 1, the 2-year rolling model performed best overall, and delivered the lowest bias and RMSE.

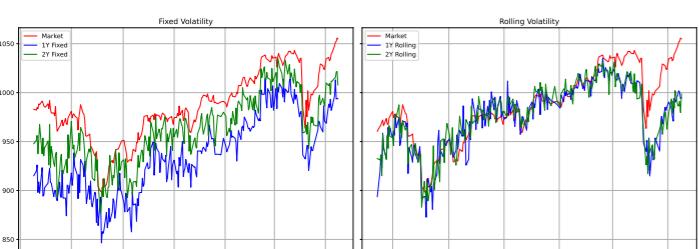


Figure 2: Simulated vs. Actual Certificate Price

MAE RMSE Q(25%) Q(50%) Q(75%)

Rolling Vol.

	MAE	RMSE	Q(25%)	Q(50%)	Q(75%)
1Y Rolling	17.25	24.66	4.81	10.66	24.17
2Y Rolling	16.86	24.05	5.48	10.28	20.04

Fixed Vol.

	MAE	RMSE	Q(25%)	Q(50%)	Q(75%)
1Y Fixed	50.98	53.17	40.81	47.87	59.66
2Y Fixed	23.76	26.85	14.58	21.69	30.87

Table 1: Valuation error metrics under different volatility assumptions

VI. Sensitivity Analysis

To perform sensitivity analysis, I look at the Greeks Delta Δ , Vega V , and Theta θ to understand how the certificate reacts to changes in key inputs like stock price, volatility, and time. **Delta** shows how the certificate's value responds to changes in the stock price. It reaches its maximum just above the barrier level, where even small upward movements in the stock can trigger early redemption and significantly increase the expected payoff. As the product gets closer to maturity, this response becomes steeper and more pronounced. **Vega** measures sensitivity to volatility. Near the barrier, Vega drops sharply and becomes negative, reflecting that higher volatility increases the likelihood of breaching the barrier and ending up with a lower payoff. As the stock price moves away from the barrier, Vega gradually increases again, but stays close to or below zero. This asymmetric pattern becomes more visible closer to maturity. **Theta** reflects time decay, how the value of the certificate changes as time passes. Around the barrier, small changes in time can have a larger impact on value, since the probability of redemption or loss is sensitive to remaining maturity. Away from the barrier, Theta tends to be more stable and closer to zero. Overall, the analysis shows that the region around the barrier is the most sensitive for all three risk factors, and that this sensitivity increases as maturity approaches.

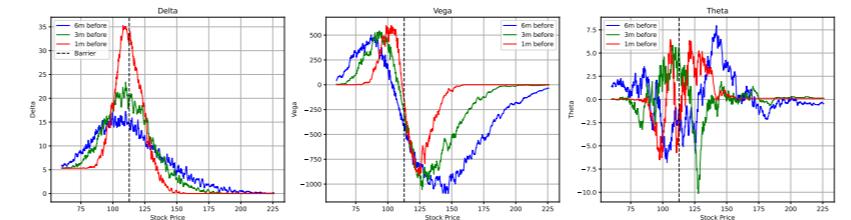


Figure 3: Greek Analysis

VII. Replication

To replicate the certificate's payoff, I constructed a dynamic portfolio using daily Siemens stock prices and corresponding Delta values. The portfolio is rebalanced daily, adjusting the equity and cash positions. The resulting replicating portfolio closely tracks the simulated certificate value, which validates the approach.

I also examined the equity fraction as a function of the stock price one year before maturity. When the stock is far above the barrier, early redemption is very likely, so Delta drops close to zero; the portfolio holds mainly the risk-free asset. Around the barrier, the equity fraction jumps up, meaning the portfolio needs to hold more stock to match the certificate's value changes. This reflects how sensitive the product becomes near the barrier. Below it, the fraction drops quickly, showing the certificate behaves more like a bond, with less exposure to the market. This shows how the product adjusts its risk depending on the stock price.

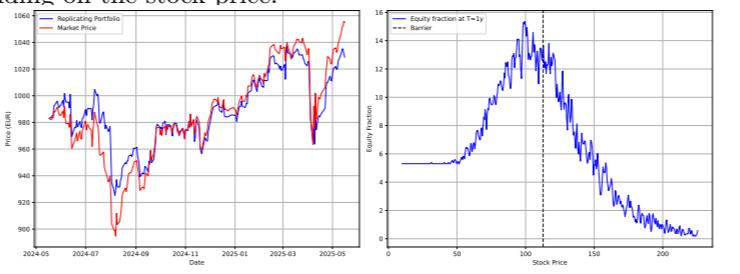


Figure 4: Replicating portfolio

VIII-IX. Performance Analysis

To assess the impact of downside protection, I simulate 10,000 one-year price paths of the S&P 500 using historical estimates for return and volatility. I first evaluate the portfolio without any protection, and then compare it to a version that uses a protective put option strategy, where a fixed portion of the initial wealth is spent on insurance. Put option

prices are calculated using the Black–Scholes formula, with a price of 674.74 € for a strike of 10,000.

The plots below show the distribution of discounted final portfolio values with and without protection. The unhedged strategy keeps full exposure to market gains, but also faces large losses in bad scenarios. The hedged version, on the other hand, shows a narrower distribution and smaller downside risk, which demonstrates how the insurance helps reduce losses.

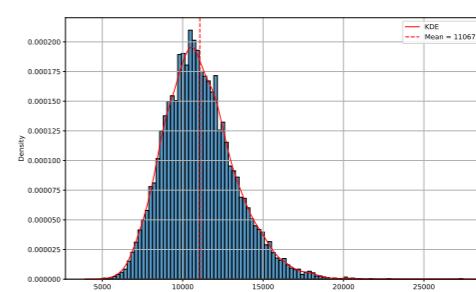


Figure 5: Unhedged Portfolio Distribution

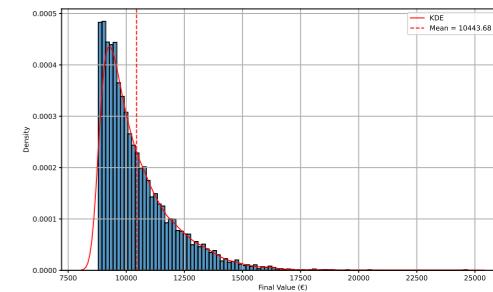


Figure 6: Hedged Portfolio Distribution (Strike = 10,000, Weight = 10%)

Strategy	Mean	Std Dev	VaR 95%	CVaR 95%	VaR 99%	CVaR 99%	Sharpe
Unhedged	10.67%	21.90%	-21.89%	-27.86%	-31.66%	-34.87%	0.487
Hedged (5%)	7.55%	17.85%	-11.24%	-12.49%	-13.28%	-14.23%	0.423
Hedged (10%)	4.44%	15.03%	-11.06%	-11.54%	-11.84%	-12.95%	0.295
Hedged (20%)	-1.80%	15.65%	-20.23%	-21.02%	-21.46%	-21.15%	-0.115

Table 2: Performance and risk metrics (Strike = 10,000)

To investigate the impact of different levels of protection, I vary both the strike price (9,000, 10,000, 11,000) and the share of capital invested in puts (from 5% to 20%). For each setup, I calculate key performance metrics including mean return, standard deviation, Sharpe ratio, Value-at-Risk, and Conditional Value-at-Risk. I focus on the strategy with a 10,000 strike and a 10% put allocation, which offers what I consider the best balance between reducing risk and keeping some return. The results are shown in Table 2. While the unhedged portfolio delivers a higher average return of 10.67%, the hedged version brings the 5% Conditional Value-at-Risk down from -27.86% to -11.54%, while still earning a moderate return of 4.44%. This shows how using options in a targeted way can reduce extreme losses while keeping upside potential.

X. Stress Scenario Analysis

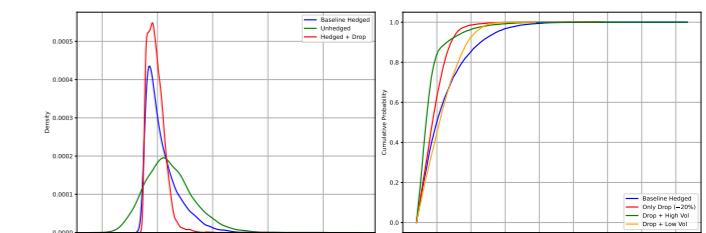


Figure 7: Stress Analysis

Stress scenarios show how hedging performance shifts under adverse conditions. A 20% mid-year drop cuts the mean return from 4.44% (baseline) to -1.42%, while CVaR remains stable due to put protection. Volatility has a subtler impact: higher volatility lowers the Sharpe ratio (0.295 to 0.276), while lower volatility slightly reduces downside risk. The combined stress (drop + high vol) leads to the worst outcome (-3.79% return, -0.398 Sharpe), showing how market shocks and pricing assumptions jointly affect hedge performance. Results are shown in Table 3.

Scenario	Mean	Std Dev	VaR 95%	CVaR 95%	VaR 99%	CVaR 99%	Sharpe
Baseline	4.44%	15.03%	-11.06%	-11.54%	-11.84%	-12.95%	0.295
Only Drop (-20%)	-1.42%	7.72%	-11.04%	-11.52%	-11.81%	-12.84%	-0.184
Only Low Vol (-5pp)	3.21%	11.55%	-10.66%	-11.38%	-11.78%	-12.27%	0.278
Only High Vol (+5pp)	5.28%	19.15%	-11.40%	-11.71%	-11.91%	-12.26%	0.276
Drop + Low Vol	3.05%	10.50%	-10.82%	-11.43%	-11.78%	-12.29%	0.290
Drop + High Vol	-3.79%	9.52%	-11.43%	-11.73%	-11.91%	-12.39%	-0.398

Table 3: Performance and risk metrics for stress scenarios (Strike = 10,000, Weight = 10%)