Student ID : ____________________

Name : ________________________

1. A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women.
   (a) Find the probability that the committee consists of 2 men and 3 women.
   (b) Find the probability that the committee consists of all women.

2. Derive the Poisson distribution of mean λ and variance λ from the Binominal distribution in the limit of $n \to \infty$, $p \to 0$, and $np = \lambda$.

$$f_\lambda(k) = \frac{e^{-\lambda}\lambda^k}{k!}.$$

Hint: Use Stirling's formula $n! \approx \sqrt{2\pi n}\, n^n e^{-n}$.

3. Please derive the mean and variance of Binominal distribution, Poisson distribution, Uniform distribution, Exponential distribution, Gamma distribution, and Normal distribution functions from their probability density functions. The answers of them are shown in the bellow table. Please confirm these by yourself.

Name of the distribution and range of the parameters	Probability density function	Mean and Variance	Characteristic function
Bernoulli distribution $B(1;p)$ $(0 < p < 1, q = 1-p)$	$p^k q^{1-k}$ $k = 0, 1$	p, pq	$pe^{jt} + q$
Binominal distribution $B(n;p)$ (n: Integer, $0 < p < 1, q = 1-p$)	$\binom{n}{k} p^k q^{n-k}$ $k = 0, 1, 2, \cdots, n$	np, npq	$(pe^{jt} + q)^n$
Poisson distribution $Po(\lambda)$ $(\lambda > 0)$	$e^{-\lambda}\frac{\lambda^k}{k!}$ $k = 1, 2, \cdots$	λ, λ	$\exp[\lambda(e^{jt} - 1]$
Uniform distribution $U(a,b)$ $(-\infty < a < b < \infty)$	$\frac{1}{b-a}$ $a \leq x \leq b$	$(a+b)/2$ $(b-a)^2/12$	$\frac{e^{jbt} - e^{jat}}{j(b-a)t}$
Exponential distribution $Ex(\alpha)$ $(\alpha > 0)$	$\alpha e^{-\alpha x}$ $x \geq 0$	$1/\alpha, 1/\alpha^2$	$(1 - \frac{jt}{\alpha})^{-1}$
Gamma distribution $G(\alpha, \nu)$ $(\alpha, \nu > 0)$	$\frac{1}{\Gamma(\nu)}\alpha^\nu x^{\nu-1} e^{-\alpha x}$ $x \geq 0$	ν/α ν/α^2	$(1 - \frac{jt}{\alpha})^{-\nu}$
Normal distribution $N(\mu, \sigma^2)$ $(-\infty < \mu < \infty, \sigma > 0)$	$\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ $-\infty < x < \infty$	μ σ^2	$\exp[j\mu t - \frac{\sigma^2}{2}t^2]$

4. Compute the following expectation value

$$E\{uv\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} uvf(u,v)dudv$$

when the joint density function is given as a two-dimensional Gaussian,

$$f(u,v) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2}Q(u,v)\right\}$$

where

$$Q(u,v) = \frac{1}{1-\rho^2}\left\{\left(\frac{u}{\sigma_1}\right)^2 - 2\rho\left(\frac{u}{\sigma_1}\right)\left(\frac{v}{\sigma_2}\right) + \left(\frac{v}{\sigma_2}\right)^2\right\}$$

5. Prove the DeMoivre-Laplace theorem using Stirling's formula.

$$\binom{n}{k} p^k q^{n-k} \approx \frac{1}{\sqrt{2\pi npq}} e^{-\frac{(k-np)^2}{2npq}}$$

Note: Dead-line for submission: 22 April 2025.