[I225] Statistical Signal Processing(E) Office Hour 2

1. There are 80 identical machines working independently. Each machine has a probability of 0.01 to fail at any given moment. A failed machine can be repaired by a single maintenance worker. Two maintenance strategies are considered:

a) Strategy A: 4 workers, each assigned to 20 specific machines.

b) Strategy B: 3 workers jointly responsible for all 80 machines.

Compare the probability that a machine cannot be repaired in time under each strategy.

2. A total of $\boldsymbol{r}$ **balls** are to be placed into $\boldsymbol{n}$ **distinct boxes**. Consider the following four different cases:

a) Balls are distinguishable, and no limit on the number of balls per box.

How many different arrangements are possible?

b) Balls are distinguishable, and each box can contain at most one ball.

How many such arrangements exist?

c) Balls are indistinguishable, and no limit on the number of balls per box.

How many different combinations are possible?

d) Balls are indistinguishable, and each box can contain at most one ball.

How many such distributions are possible?

Please give the answer for each case using combinatorial notation such as n^r.

3. A complex stochastic process Z(t) is defined as:

$$Z(t) = 2X(t) - iY(t)$$

Where:X(t) and Y(t) are real-valued random processes. i is the imaginary unit.

You are given the following information:

$$E[X(t)] = 0, E[Y(t)] = 1$$
$$Var[X(t)] = 2, Var[Y(t)] = 6$$
$$Cov\big(X(t), Y(t)\big) = 0$$

The autocorrelation functions:

$$R_{XX}(t_1, t_2) = 2 \cdot e^{-\frac{|t_1 - t_2|}{3}}$$

$$R_{YY}(t_1, t_2) = 6 \cdot \cos\big(\pi(t_1 - t_2)\big)$$

a) Calculate the mean function of $Z(t)$

b) Calculate the autocovariance function of $Z(t)$

c) Calculate the autocorrelation function of $Z(t)$

d) Calculate the correlation coefficient between $Z(t_1)$ and $Z(t_2)$

4. Suppose you have a transformation that maps a point (u, v) in the uv-plane to a point (x, y) in the xy-plane, defined by the following equations:

$$x(u, v) = sin(u) + v^2$$
$$y(u, v) = uv + cos(v)$$

Calculate the Jacobian matrix of the output variables (x, y) with respect to the input variables (u, v).

5. Let the original audio signal be a simple sequence representing a short sound:

$$x[n] = \{1, -2, 0, 4\}$$

Let the impulse response be:

$$h[n] = \{3, 0, -1\}$$

Calculate the output signal $y[n]$, which is the convolution of the input signal $x[n]$ with the impulse response $h[n]$.

The linear convolution for discrete-time signals is defined as:

$$y[n] = (x * h)[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$