[I225] Statistical Signal Processing(E) Office Hour 1

1. There are four boxes (Box 1 to Box 4), and each box contains a certain number of lottery tickets as described below:

- Box 1: 2000 tickets (5% of them are winning tickets)
- Box 2: 500 tickets (40% of them are winning tickets)
- Box 3: 1000 tickets (10% of them are winning tickets)
- Box 4: 1000 tickets (10% of them are winning tickets)

Now, one of the four boxes is chosen at random, and then one ticket is drawn at random from the selected box.

(a) What is the probability of drawing a winning ticket?

(b) Given that the ticket drawn is a winning ticket, what is the probability that it was drawn from Box 2?

2. A box contains 12 new table tennis balls. For each match, 3 balls are randomly selected without replacement, used, and then returned to the box after the match. Let a ball be considered "used" once it has been drawn at least once.

(1) Let X be the number of new (unused) balls drawn in the second match. Find the probability distribution of X.

(2) Suppose that all 3 balls drawn in the third match are new (i.e., have not been drawn in either of the previous two matches). Given this condition, what is the probability that all 3 balls drawn in the second match were also new?

3. A point (X, Y) is randomly and uniformly selected from the unit square region where $0 \le X \le 1$ and $0 \le Y \le 1$.

(1) Find the joint probability density function (PDF) of X and Y.

(2) Find the probability that the point lies within the triangle $T = \{(x, y) \mid 0 \le y \le x \le 1\}$.

(3) Find the conditional probability that $X > 0.5$ given that $Y < 0.5$.

4. Two people agree to meet at the east gate of a park sometime between 9:00 AM and 10:00 AM. The one who arrives first agrees to wait for at most 20 minutes (i.e., 1/3 hour). If the other person has not arrived by then, the first person will leave. What is the probability that the two people successfully meet?

5. Let X be a random variable such that $X \sim N(\mu, \sigma^2)$. Derive the characteristic function of X, and use it to compute the mean and variance of X.