Student ID : ________________

Name : ____________________

1. Derive the autocorrelation function $R_{zz}(\tau)$ and the power spectrum $S_{ZZ}(\omega)$ of the following stochastic process

$$Z(t) = aX(t) + bY(t),$$

where a and b represent constant parameters, $X(t)$ stands for stationary white noise with $E\{X(t)\} = 0$ and $R_{XX}(\tau) = q\delta(\tau)$, and $Y(t)$ stands for stationary noise with $E\{Y(t)\} = 0$ and $R_{YY}(\tau) = \exp(j\tau)$. The processes $X(t)$ and $Y(t)$ are supposed to be independent from each other.

2. Consider the following memoryless system

$$Y(t) = g[X(t)] = \exp(\beta X(t)),$$

where β is a positive constant ($\beta > 0$). Derive the probability density function $f_Y(y,t)$ of the output $Y(t)$ with respect to input $X(t)$, whose probability density function is given as $f_X(x,t) = t\exp(-tx)$, ($x \geq 0$); 0 ($x < 0$).

3. Consider the following differentiator

$$Y(t) = L[X(t)] = X'(t).$$

Derive the autocorrelation function $R_{YY}(\tau)$ of the output $Y(t)$ with respect to input $X(t)$, whose autocorrelation function is given as $R_{XX}(\tau) = \cos \beta\tau$.

4. With respect to the following linear differential equation with white noise input $X(t)$

$$Y''(t) + 2Y'(t) - 3Y(t) = X(t),$$

derives the autocorrelation function $R_{YY}(\tau)$ and the power spectrum $S_{YY}(\omega)$. The input $X(t)$ is considered to be white noise with $E\{X(t)\} = 0$ and $R_{XX}(\tau) = 48\delta(\tau)$.

Note:

- Please provide your answer in **clear handwriting**
- Dead-line for submission: **27 May 2025**