Student ID :
Name :

Problem 1. Among 50 balls, 10 of them are red. In a blind condition, we pick up a ball from a box that contains the 50 balls. After each trial, we do not bring the ball back to the box. **A** is denoted as an event, in which we pick up a red ball in the initial trial. **B** is denoted as an event, in which we pick up a red ball in the second trial. Judge whether event **A** and event **B** are independent or not.

Problem 2. There is a test for checking whether an adult is infected with disease C. Probability of getting a positive reaction for adult who is infected with disease C is 70 %, whereas probability of getting a positive reaction for adult who is not infected with disease C is 20 %. Among the entire adults, 10 % of the population is infected with disease C. Suppose now that one adult got a positive reaction. How much is the probability that this adult is really infected with the disease C?

Problem 3. Consider a random variable X, which is distributed exponentially as $\sim Ex(2)$. Derive a probability density function $f_Y(y)$ of a random variable defined as $Y = \sqrt{X}$.

Problem 4. Using two random variables X and Y , which are independent from each other and distributed as $\sim N(1, 4)$ and $\sim N(2, 1)$, respectively, define a random variable as $Z = X + Y$. Note that the characteristic function of $N(\mu, \sigma 2)$ is given as $\varphi(t) = \exp\left(i\mu t - \frac{\sigma^2 t^2}{2}\right)$.

(1) Derive the characteristic function $\varphi_Z(t)$ for the random variable Z.

(2) By using $\varphi_Z(t)$, compute the mean and the variance of Z.

Problem 5. Using two random variables X and Y , which are independent from each other and distributed as $\sim Ex(\alpha)$ ($\alpha > 0$), define a random variable as $Z = X+2Y$. Note that the characteristic function of $Ex(\alpha)$ is given as $\varphi(t) = \left(1 - \frac{it}{\alpha}\right)^{-1}$.

(1) Derive the characteristic function $\varphi_Z(t)$ for the random variable Z.

(2) By using $\varphi_Z(t)$, compute the mean and the variance of Z.

Problem 6. Consider a random variable X, which takes a value of s with probability 1/2 and a value of $-s$ with probability 1/2 (s represents a positive constant). Then the characteristic function of X is given as $\varphi(t) = \frac{1}{2}\left\{e^{ist} + e^{-ist}\right\}$. Suppose we have n independent random variables $X_1, X_2, \cdots, X_n$, which have the same distribution function as X and define a new variable as $S = \sum_{k=1}^{n} X_k$.

(1) Derive the characteristic function $\varphi_S(t)$ of S.

(2) By using $\varphi_S(t)$, compute the mean and the variance of S.

Note:

- Please provide your answer in **clear handwriting**
- Dead-line for submission: **8 May 2025**

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