# [I225] Statistical Signal Processing(E) Office Hour 3

**1.** Consider a discrete-time linear time-invariant (LTI) system with the following impulse response: $\mathrm{h[n]} = \delta[\mathrm{n}] + 0.6\ \delta[\mathrm{n}] + 0.3\ \delta[\mathrm{n} - 4]$

The input signal is given by: $\mathrm{x[n]} = \{1, -1, 2, 0, -2, 1\}$

Compute the output signal $\mathrm{y[n]} = \mathrm{x[n]} * \mathrm{h[n]}$ using convolution.

**2.** Consider a discrete-time LTI system with an impulse response:

$$\mathrm{h[n]} = \{1, 0.5, -0.25\}$$

where the value at n=0 is the first element. The input to this system is a zero-mean white noise process $\omega[\mathrm{n}]$ with variance $\sigma_w^2$. Recall that for zero-mean white noise:

- $\mathbb{E}\big[\omega[\mathrm{n}]\big] = 0$ for all n
- The autocorrelation function $R_{\omega\omega}[k] = \mathbb{E}\big[\omega[n]\omega[n-k]\big] = \sigma_\omega^2 \delta[k]$

The output of the LTI system is:

$$y[n] = (\omega * h)[n] = \sum_{m=-\infty}^{\infty} h[m]\omega[n-m]$$

a) Determine the autocorrelation function of the output signal:

$$R_{yy}[k] = \mathbb{E}\big[y[n]y[n-k]\big]$$

b) Calculate the specific form for $R_{yy}[k]$ using the given impulse response

$\mathrm{h[n]} = \{1, 0.5, -0.25\}$

c) Compute the power spectral density (PSD) of the output $S_{yy}[\omega]$, using the output signal that:

$$S_{yy}(\omega) = \sum_{k=-\infty}^{\infty} R_{yy}[k]e^{-j\omega k}$$

**3.** A stationary input process $\mathrm{X(t)}$ is passed through a linear system with the following transfer function:

$$\mathrm{H}(\omega) = \frac{1}{(\omega - 2)^2 + 1}$$

Assume that the total input power is: $\mathbb{E}\{\mathrm{X}^2(\mathrm{t})\} = 20$, Find the input power spectrum $S_{XX}(\omega)$ hat maximizes the output power $\mathbb{E}\{\mathrm{Y}^2(\mathrm{t})\}$, where the output $\mathrm{Y(t)}$ is the result of filtering $\mathrm{X(t)}$ through the system $\mathrm{H}(\omega)$.

**4.** A linear system is described by the following first-order differential equation driven by white noise input $X(t)$:

$$\frac{dZ(t)}{dt} + 2bZ(t) = X(t)$$

Where $b > 0$ is a positive constant. The input $X(t)$ is a white noise process with autocorrelation function

$$R_{xx}(\tau) = r\delta(\tau)$$

Where $r$ is the power spectral density (PSD) level.

**Find:**

a) The transfer function $H(\omega)$ of the system.

b) The power spectral density $S_{zz}(\omega)$ of the output $Z(t)$.

c) The autocorrelation function $R_{zz}(\tau)$ of the output $Z(t)$.

**5.** Suppose a temperature sensor is used to record ambient temperatures each minute, and the recorded values are modeled as

$$Y[n] \sim \mathcal{N}(\mu, \sigma^2), n = 0, 1, \ldots., N-1$$

Where $\mu$ is the true average ambient temperature, and the measurements are i.i.d. Gaussian. Consider the estimator:

$$\hat{\mu} = \frac{1}{N}\sum_{n=0}^{N-1} Y[n]$$

a) What is the bias of $\hat{\mu}$

b) Compute the variance of $\hat{\mu}$ and check whether it is a consistent estimator.