[I225] Statistical Signal Processing(E) Office Hour 5

1. The stochastic process X(t) is white noise satisfying E{X(T)} = 0 and $R_{XX}(\tau) = 12\delta_{(\tau)}$. ow consider a system in which the input X(t) and the output Y(t) satisfy the following relationship: $Y''(t) - Y'(t) - 2Y(t) = X(t)$. In this case, find the autocorrelation function $R_{YY}(\tau)$ and the power spectral density $S_{YY}(\omega)$ of the output Y(t).

2. Given the power spectral density

$$S_X(\omega) = \frac{\omega^4 + 64}{\omega^4 + 10\omega^2 + 9}$$

determine the innovation filter for the process x(t).

3. Given the observation vector and design matrix:

$$s = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix},\ H = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Use the least squares method to find the optimal parameter vector $\hat{\boldsymbol{\theta}} \in \mathbb{R}^{\mathbf{2}}$ that minimizes the squared error:

$$J(\theta) = \| s - H\theta \|^2$$

4. Let the observation matrix H: and the observed vector s be:

$$H = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, s = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

(1) Compute the least squares solution $\hat{\theta}$ for minimizing$\| s - H\theta \|^2$.

(2) Compute the residual vector $\epsilon = s - H\hat{\theta}$.

(3) Verify that ϵ is orthogonal to both column vectors of H i.e., show $\mathrm{h}_1^T\epsilon = 0$ and $\mathrm{h}_2^T\epsilon = 0$

5. Suppose the observed signal is given by:

$$X[n] = \theta + W[n], n = 0,1,\ldots,N-1$$

where:

θ is a random constant following a uniform distribution:

$$\theta \sim U(-1,1)$$

$W[n]$ are independent Gaussian noises with mean 0 and variance $\sigma^2 = 0.25$

θ and $W[n]$ are independent.

Let $N = 4$, and assume the observed values are:

$$X = [1.2, 0.8, 1.0, 0.6]^T$$

Compute the minimum mean square error (MMSE) estimate of θ.