# [I225] Statistical Signal Processing(E) Office Hour 6

**1.** There are four boxes (Box 1 to Box 4), and each box contains a certain number of lottery tickets as described below:

Box 2: 500 tickets (50% of them are winning tickets)

Box 3: 200 tickets (20% of them are winning tickets)

Box 4: 1000 tickets (5% of them are winning tickets)

Now, one of the four boxes is chosen at random, and then one ticket is drawn at random from the selected box.

(a) What is the probability of drawing a winning ticket?

(b) Given that the ticket drawn is a winning ticket, what is the probability that it was drawn from Box 2?

**2.** A complex stochastic process Z(t) is defined as:

$$Z(t) = 3X(t) - iY(t)$$

Where:X(t) and Y(t) are real-valued random processes. i is the imaginary unit.

You are given the following information:

$$E[X(t)] = 1, E[Y(t)] = 2$$
$$Var[X(t)] = 3, Var[Y(t)] = 4$$
$$Cov\big(X(t), Y(t)\big) = 0$$

The autocorrelation functions:

$$R_{XX}(t_1, t_2) = 3 + \sin\left(\omega_0(t_1 - t_2)\right)$$

$$R_{YY}(t_1, t_2) = 4 \cdot \cos\big(\omega_0(t_1 - t_2)\big)$$

(a) Calculate the mean function of $Z(t)$

(b) Calculate the autocovariance function of $Z(t)$

(c) Calculate the autocorrelation function of $Z(t)$

(d) Calculate the correlation coefficient between $Z(t_1)$ and $\overline{Z(t_2)}$

**3.** A call center records the number of calls received during 5 different one-hour intervals: Data: [1, 2, 3, 2, 2]

Assume the number of calls received in one hour follows a Poisson distribution with rate $\lambda$ (calls per hour), and each hour is independent.

Answer the following:

(a) Write the likelihood function for this data given the parameter $\lambda$

(b) Write the log-likelihood function.

(c) Find the maximum likelihood estimate $\hat{\lambda}$ for the rate of calls per hour.

**4.** Consider a continuous-time signal $x(t) = cos(2\pi \cdot 50t)$.

(a) What is the regular frequency $f$ of this signal in Hertz (Hz)?

(b) What is the angular frequency $\omega$ of this signal in radians per second (rad/s)?

(Hint: You might find Euler's formula helpful: $\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$ )

(d) Now, find the Fourier Transform $X(\omega)$of $x(t)$.

**5.** Consider a modulated signal defined as:

$$x(t) = \cos(2\pi \cdot 10t) + \cos(2\pi \cdot 40t)$$

(a) What are the regular frequencies $f_1$ and $f_2$ (in Hz) of the two cosine components?

(b) What are the angular frequencies $\omega_1$ and $\omega_2$ (in rad/s) of these components?

(c) Sketch (or describe) the frequency-domain representation $X(f)$ of $x(t)$.

(d) Express the Fourier transform $X(f)$ using delta functions.

(e) Repeat (d) using angular frequency $\omega$ and write $X(\omega)$.