I225E Statistical Signal Processing

1. Course Introduction

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JAIST
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Statistical Signal Processing

Objective

Basic understanding of random variables, probability theory, stochastic process, statistical signal processing, and model estimation theory.

Contents

Definition of stochastic process, stochastic process and system, correlation analysis, spectral analysis, maximum likelihood estimation, least squares estimation, Bayesian estimation, Wiener filter, Kalman filter.

Lecture Slides

https://candyolivia.github.io/teaching/FY2025/I225E/

Material

- Review of probability theory
- Basics of stochastic process
- Stochastic process and systems
- Spectral analysis

Modeling part
1st exam

- Estimation theory
- Maximum likelihood estimation
- Least squares algorithm
- Bayesian estimation
- Linear model
- Signal processing (Wiener filter, Kalman filter)

Inference part
2nd exam

Lecture Schedule (1)

1. Course introduction **(15 Apr 2025)** ← **Today!**
2. Review of probability theory **(15 Apr 2025)** ← **Today!**
3. Basics of stochastic process **(17 Apr 2025)** **Report 1**
4. Stochastic process and system I **(22 Apr 2025)**
   <Tutorial hour 1>
5. Stochastic process and systems II **(24 Apr 2025)**
6. Spectral analysis I **(1 May 2025)** **Report 2**
7. Spectral analysis II **(7 May 2025)** **<Tutorial hour 2>**
8. Estimation theory **(8 May 2025)** **Report 3**

Lecture Schedule (2)

9. Maximum likelihood estimation **(13 May 2025)**
   <Tutorial hour 3>
10. Mid-term Examination **(15 May 2025)**
11. Least squares estimation **(20 May 2025)** **<Tutorial hour 4>**
12. Bayesian estimation **(22 May 2025)**
13. Linear model **(27 May 2025)** **<Tutorial hour 5> Report 4**
14. Signal processing I **(29 May 2025)**
15. Signal processing II **(3 June 2025)** **<Tutorial hour 6>**
16. Final Examination **(5 June 2025)**

Reports and exams

- **Viewpoint of evaluation:**

  Comprehension of basic theory of stochastic processing and statistical signal processing and their applications in information science.

- **Evaluation criteria:**

  Reports (20%): Four times (each 5%)

  Examinations (80%): Two times (each 40%)

  Bring only your own “handwritten” notebooks

  (NO printed handouts, calculator, textbook, computer, or smartphone).

Related Courses

- I114 Fundamental Mathematics for Information Science
- I119 Statistics for Data Analytics
- I212 Analysis for Information Science
- I213 Discrete Signal Processing
- I225 Statistical Signal Processing
- I439 Speech Information Processing

COURSE INTRODUCTION

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1. Introduction

- **What is random variable?**

  All possible outcomes that may result from a trial are known a priori. However, it is impossible to predict which outcome to occur.

  (ex) dice throwing, coin tossing, etc.

- **What is Probability Theory?**

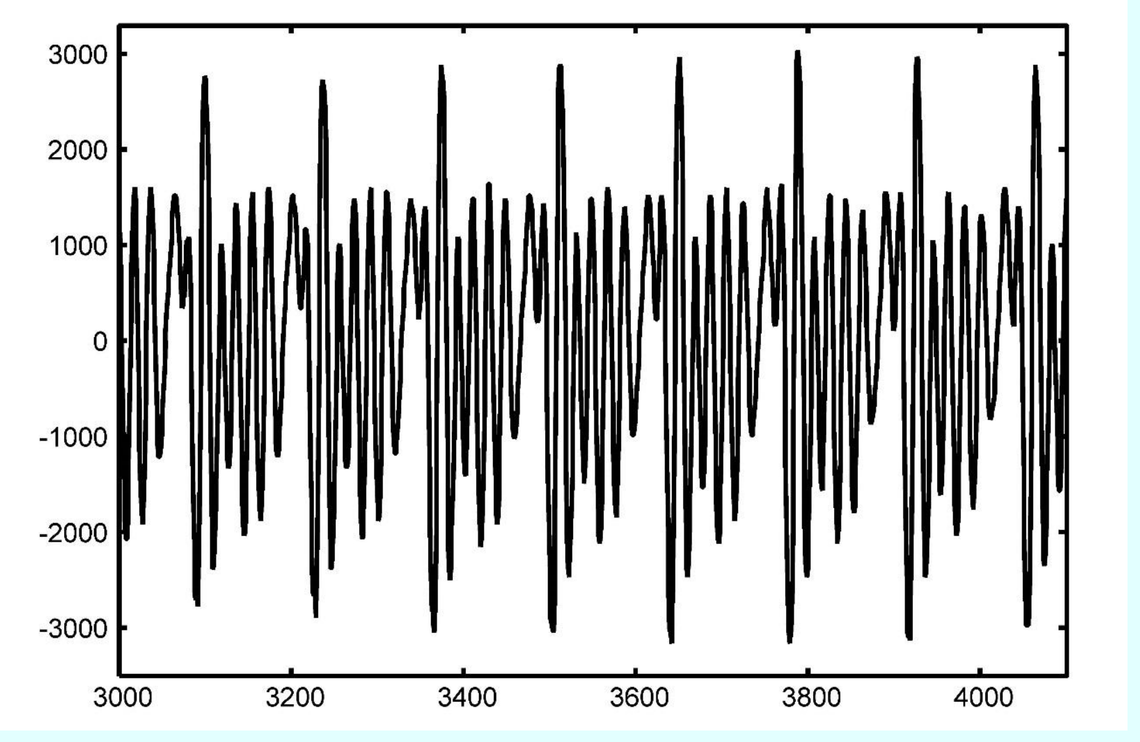
  Consider mathematical models for stochastic phenomena, to which only probabilistic predictions can be made.

  One of mathematical areas.

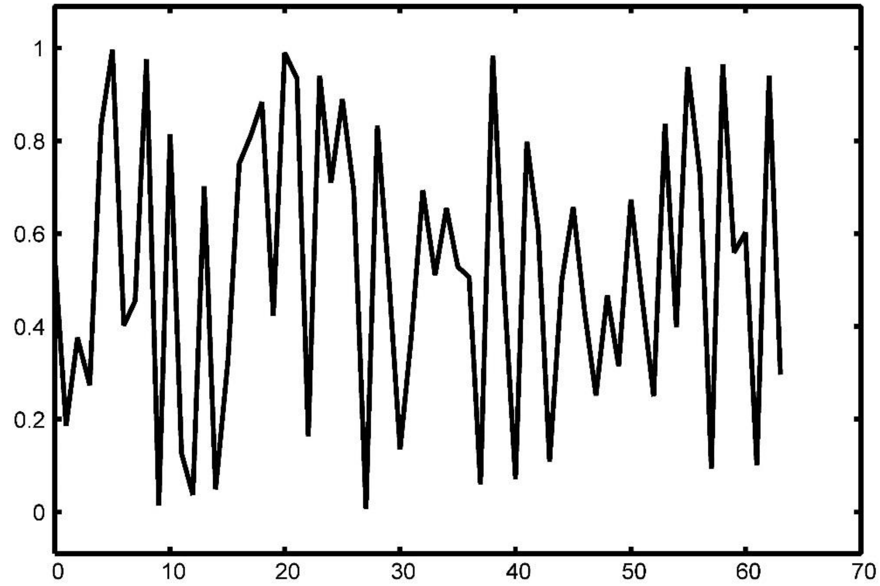
  Foundations by Pascal, Fermat, Huygens already in the 17^{th} century.

What is Stochastic Process?

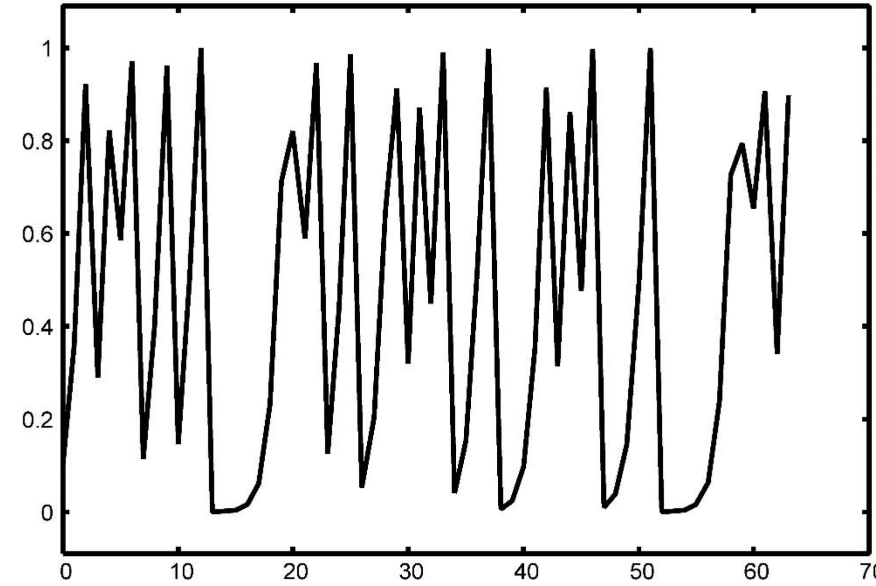
Dynamic phenomena that *change* in time in accordance with probabilistic rule. Stock exchange market, price change. Brownian motion (random behavior of particles).



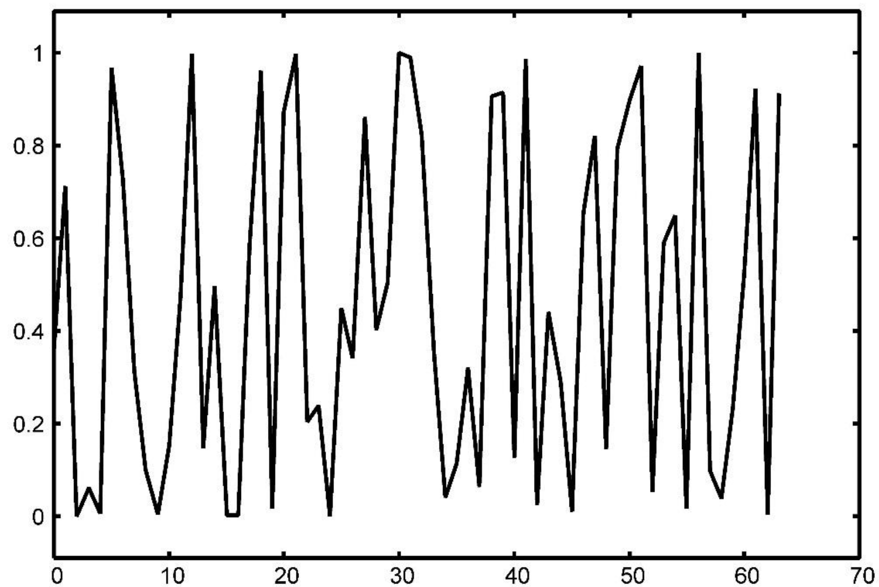
Time series 1



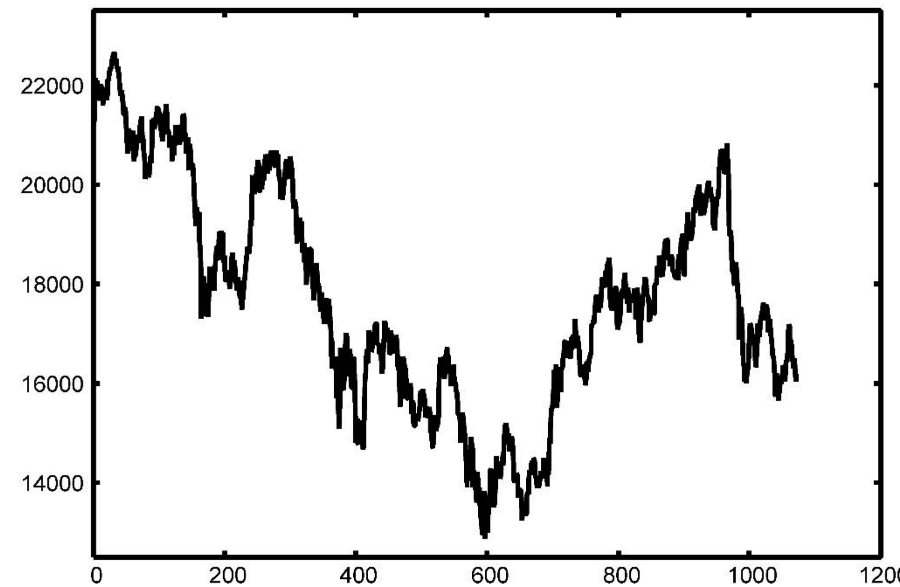
Time series 2



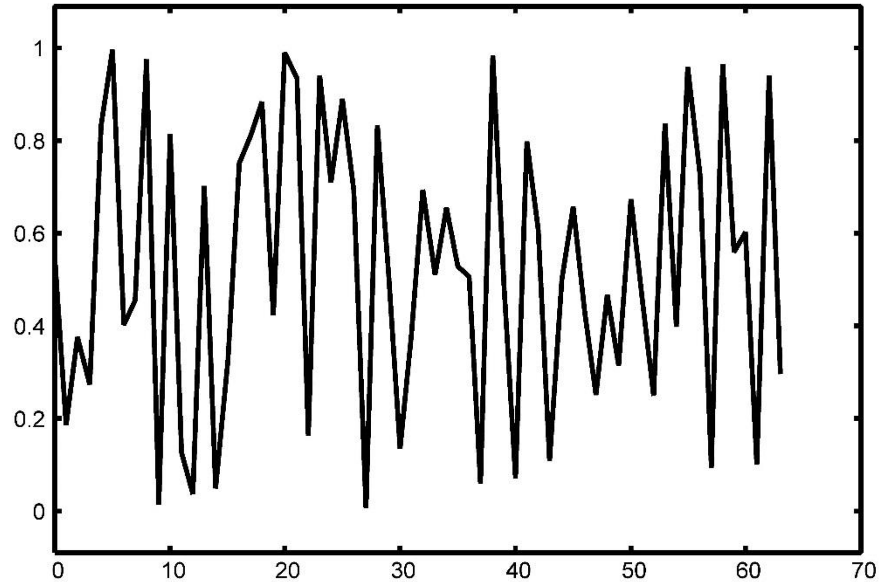
Time series 3



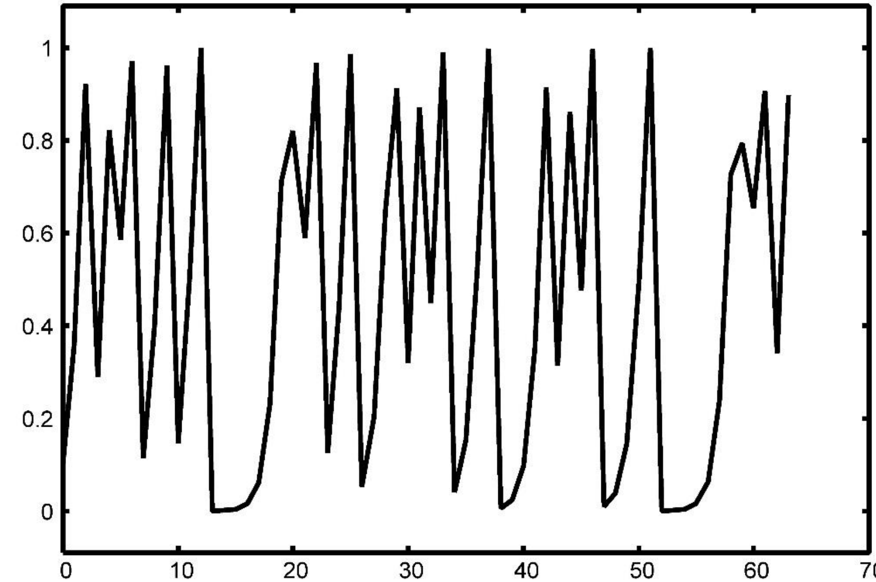
Time series 4



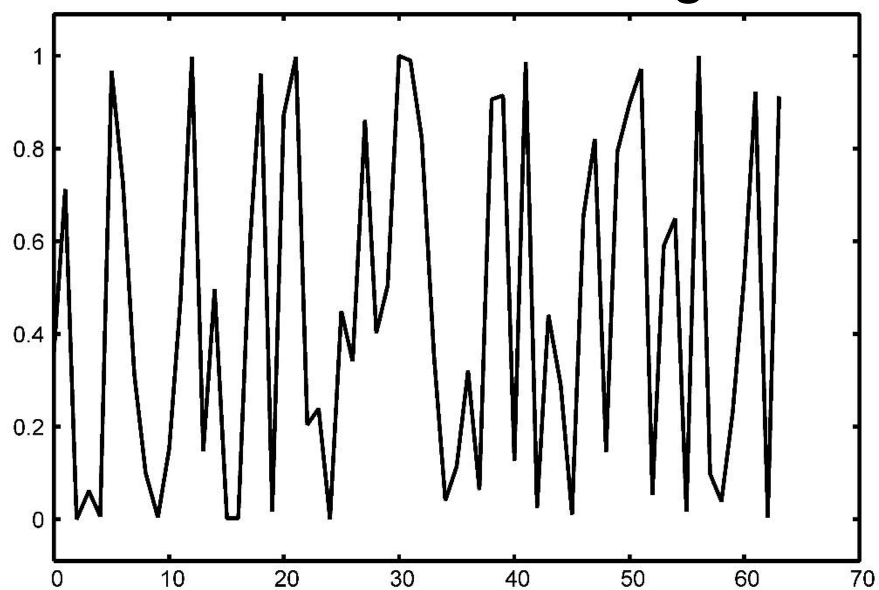
Random numbers



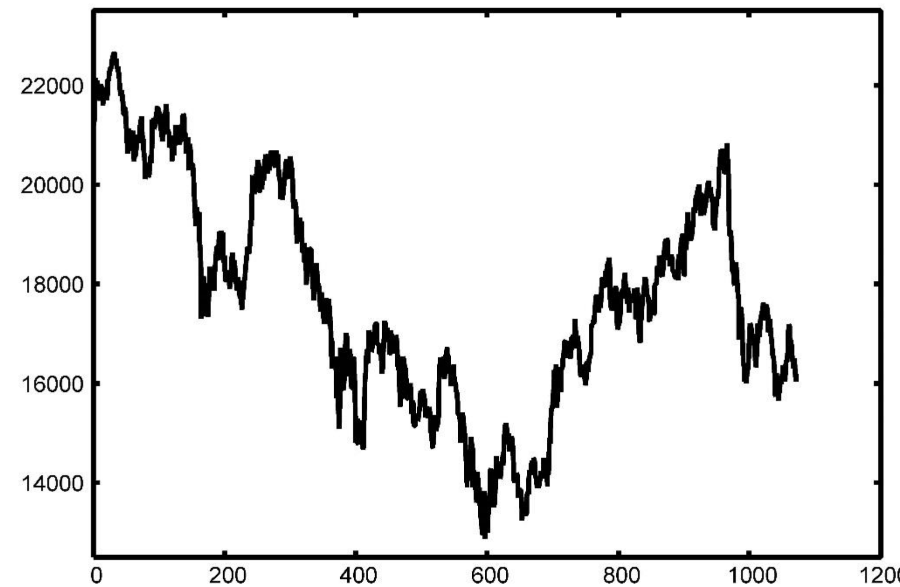
Logistic map



Random shuffling



Nikkei 225 (stock market)



Logistic map

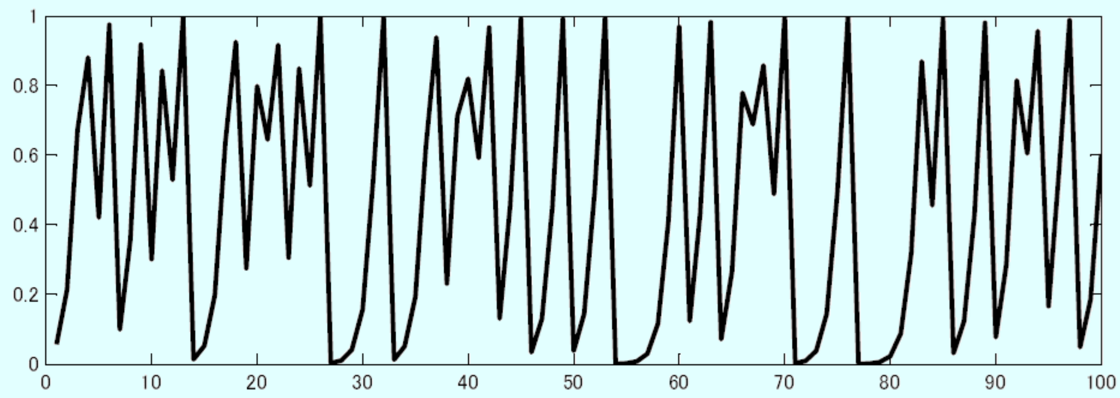
$$x_{n+1} = ax_n\left(1 - x_n\right)$$

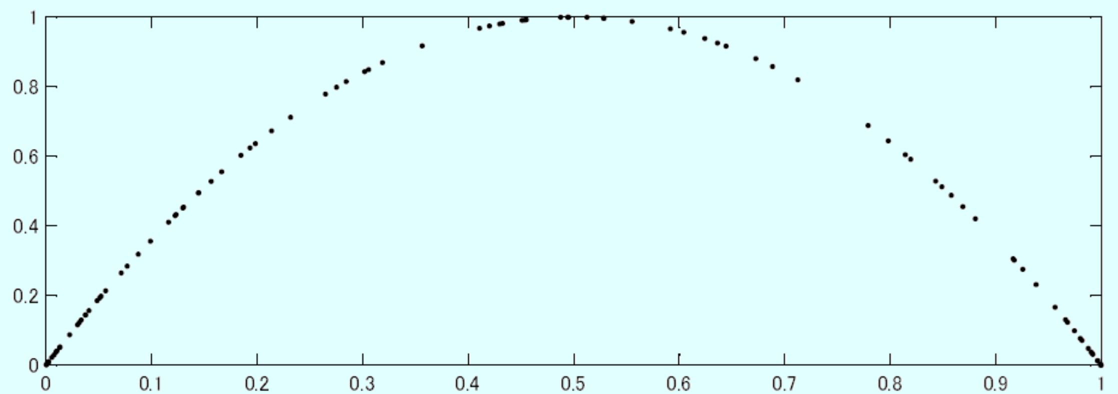
```
a=4;

N = 100;
x = zeros(1, N);
x(1) = rand;

for n=1:N-1
   x(n+1) = a*x(n)*(1-x(n));
end

subplot(211);
plot(x, 'k');
subplot(212);
plot(x(1:end-1), x(2:end), 'k.');
```



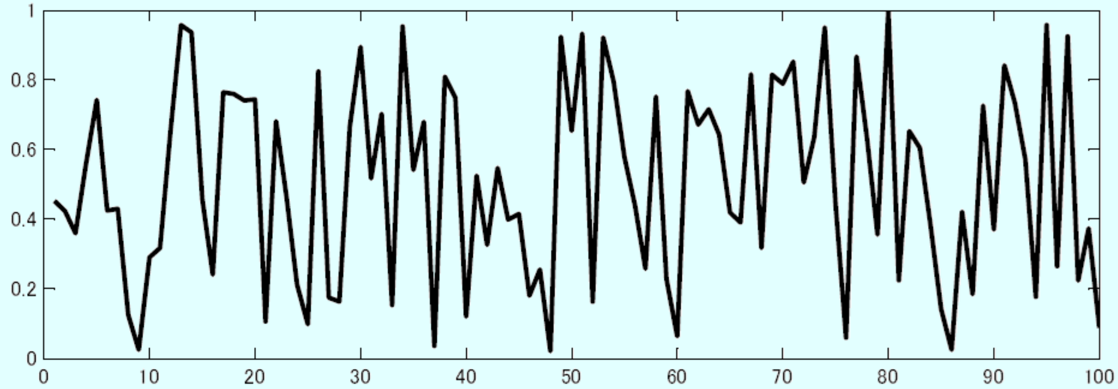


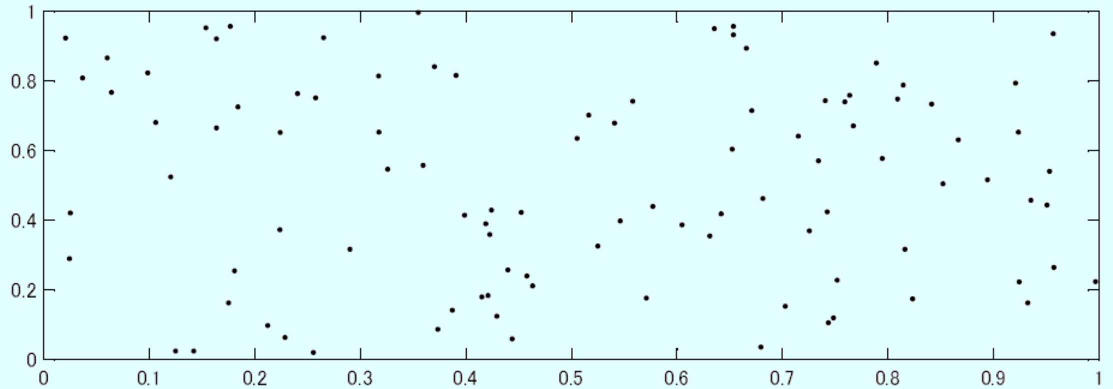
Random numbers

```
N = 100;

x = rand(1,N);

subplot(211);
plot(x, 'k', 'linewidth', 2);
subplot(212);
plot(x(1:end-1), x(2:end), 'k.');
```



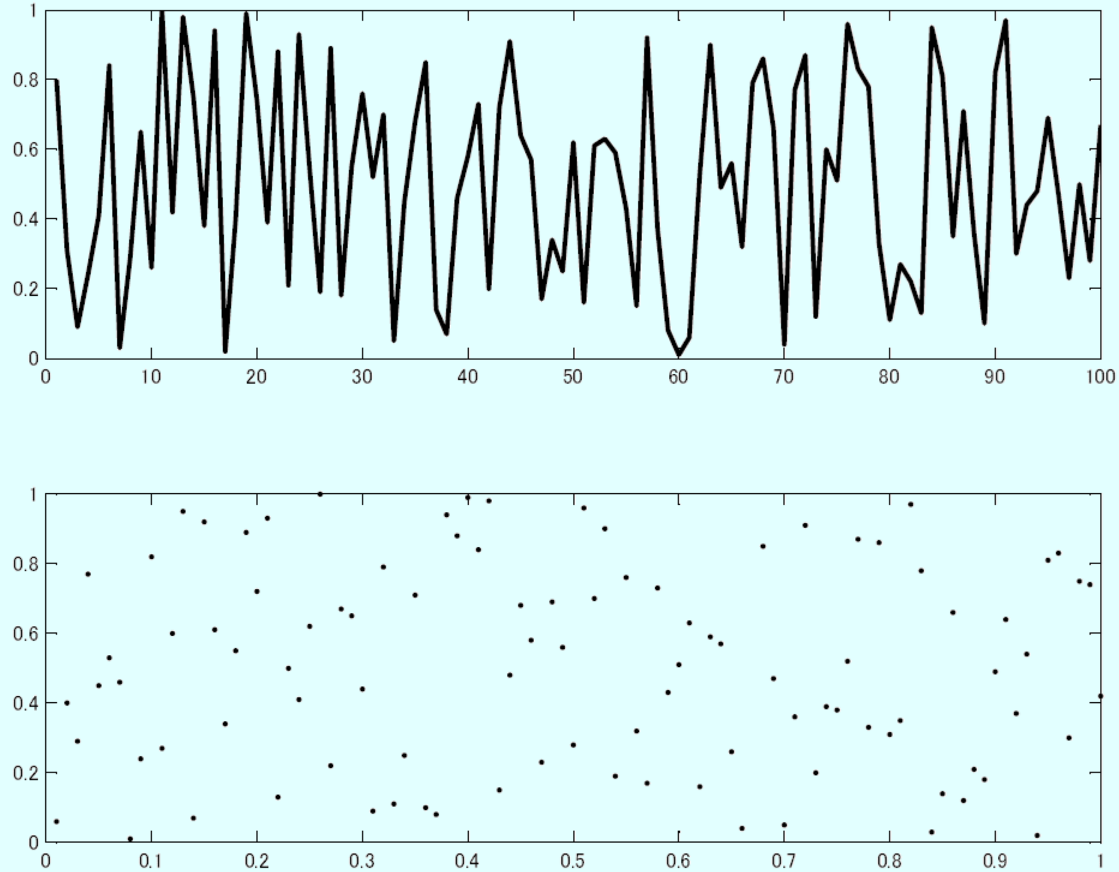


Random shuffling

```
N = 100;

x = randperm(N)/N;

subplot(211);
plot(x, 'k');
subplot(212);
plot(x(1:end-1), x(2:end), 'k.');
```



Random variables and stochastic process

- Random variable
  - History independence; independence of previous values

$$X \sim P(X)$$

- Stochastic process
  - History dependence; current value depends on previous values.

$$X_t \sim P(X_t | X_{t-1}, X_{t-2}, \cdots)$$

  - Markov process

$$X_t \sim P(X_t | X_{t-1})$$

Application of Statistical Signal Processing

- **Time Series Analysis**
  - Modeling
  - System identification (parameter estimation)
  - State estimation
  - Prediction
  - Control
- **Statistics**
  - Basic statistical analysis
  - Statistical estimation
  - Statistical test

Other applications

- Information theory
- Filter theory
- Secure communication
- Data coding
- Speech/Image restoration
- Machine learning

Time Series Analysis

- **Characterization of Time Series $x(t)$**
  - Mean (First-order Moment), Trend
  - Autocorrelation (Second-order Moment)
  - Spectrum
- **Estimation of System Dynamics**
  - Differential Equation, Linear System
- **State Estimation**
  - Past (Smoothing, Noise Reduction)
  - Present (Filtering)
  - Future (Prediction)

- **Prediction (Future)**
- **Filtering (Present)**
- **Smoothing (Past)**

  Noise Reduction