Utilising metasurface antennas to generate random beam patterns for signal direction detection (DoA) based on compressive sensing algorithm

Master’s Thesis
Caner Kadıoğlu
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HFE Höchstfrequenz elektronik
RWTH AACHEN UNIVERSITY

Outline

Structure of Presentation

- My Background
- Introduction
- Design of Leaky-Wave Metasurface Antenna
- Design of Compressive Sensing Algorithm
- Simulation Results
- Conclusion
- Q&A

My Background

Caner Kadıoğlu

- Ankara, Türkiye
  - In Turkish the letter ‘C’ is pronounced as ‘J’

- B. Sc. in Electrical and Electronics Engineering – Bilkent University, Ankara

- Aselsan A.Ş., Ankara
  - Leading electronic defense company in Turkey
  - Last role: RF/Microwave Design Engineer, Engineer II

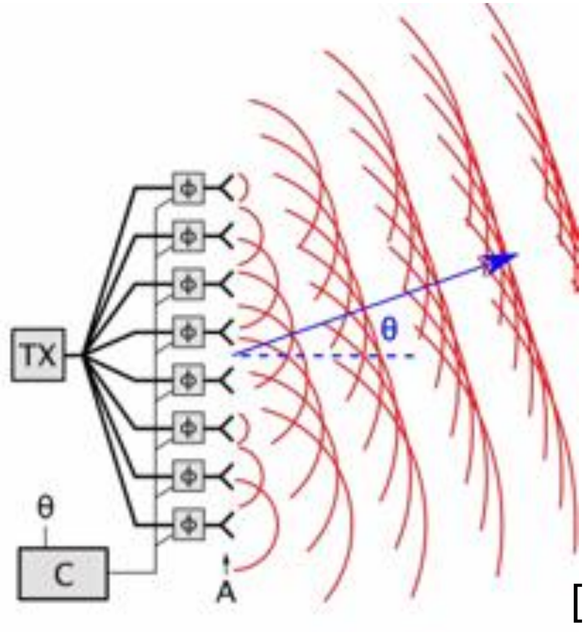
- M. Sc. Electrical Engineering, Information Technology and Computer Engineering – RWTH Aachen, DE

- Today

Introduction

Direction of Arrival (DoA) Estimation

- What is Direction of Arrival (DoA)?
  - The direction which a propagating wave impinges on the receiver
  - Crucial in many applications: wireless communication, radar, imaging, navigation
  - Objective is to find the direction of an incident signal
- Electronic Systems
  - Conventionally, DoA estimation is an array signal processing problem
  - Consists of a dense grid of array elements
    - Phased array architecture is a multi-channel system → Beamforming
    - Requires many phase-shifters & power amplifiers
    - High complexity, high power consumption → Costly and lossy
  - Uses sophisticated computational methods (MUSIC, ESPRIT)
- Compressive sensing (CS)
  - Gained interest in computational millimeter-wave radar imaging
  - Simplifies the process using **frequency-diversity technique** with **metasurface antennas**



[1]

[1] Amedeo Capozzoli, Claudio Curcio, Francesco DâAgostino, and Angelo Liseno. "A Review of the Antenna Field Regions". In: *Electronics* 13.11 (2024). issn: 2079-9292. doi: 10.3390/electronics13112194.

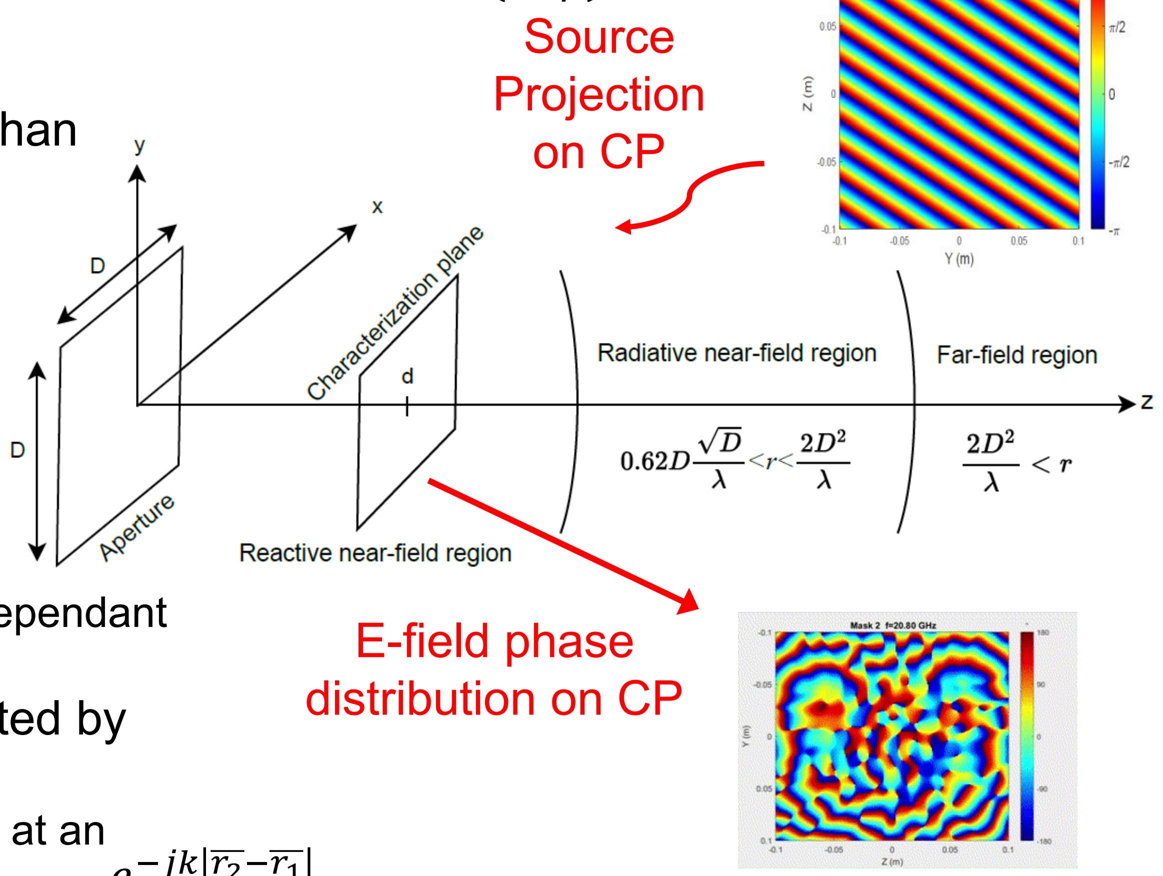
Introduction

Compressive Sensing

- Encodes incoming signals through a single channel rather than multiple channels
  - Eliminates array-based multi-channels
  - Decreases complexity → more cost-effective
  - Increased acquisition speed
  - Demands for pattern diversity and orthogonality
- Radiation regions in an antenna
  - CS occurs in the near-field region
  - Fields are predominantly reactive, non-radiating, and distance dependant
- Radiation from each source point on the aperture is calculated by Green's Function
  - Impulse response for source signals where the resulting fields at an arbitrary point can be determined
  - Aperture fields are transformed into near-field

$$G(\overline{r_1}, \overline{r_2}, \omega) = \frac{e^{-jk|\overline{r_2}-\overline{r_1}|}}{|\overline{r_2}-\overline{r_1}|}$$

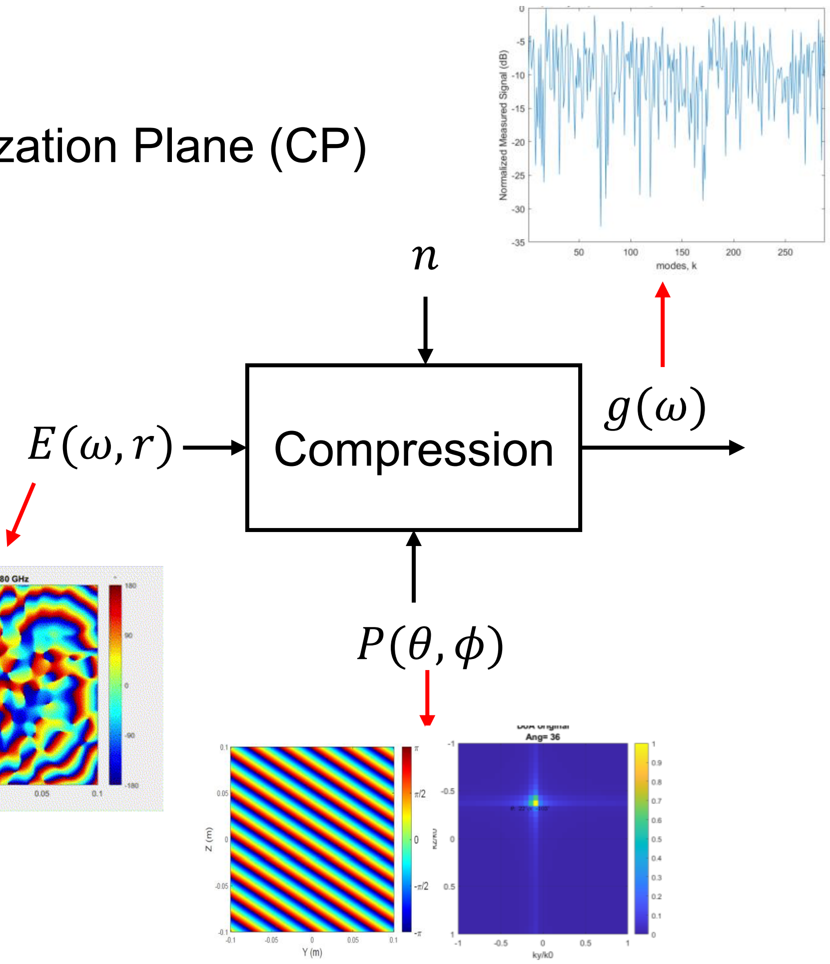
$$P(\theta, \varphi) = e^{-j\vec{k}\cdot\vec{r}}$$

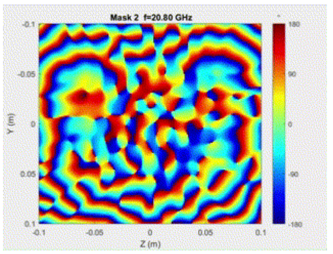


Introduction

Compressive Sensing

- Each source point on the aperture contributes to the field on a Characterization Plane (CP)
  - Frequency dependent complex E-fields are the transfer functions
- Compression occurs in the CP through the transfer functions
  - Measured signal: $g(\omega) = \int_r E(r,\omega)P(r)dr + n$
- Leverages the **frequency-diverse aperture** technique
  - Spatio-temporally varying radiation patterns
  - Single channel $\rightarrow$ Frequency-sweep $\rightarrow$ Pattern variation
- Further techniques are used to obtain pattern diversity
  - **Dynamic aperture**
    - Beamforming with various mask configurations as array inputs
  - **Randomly placed patches**
    - To obtain unique radiation pattern within identical row elements of an array

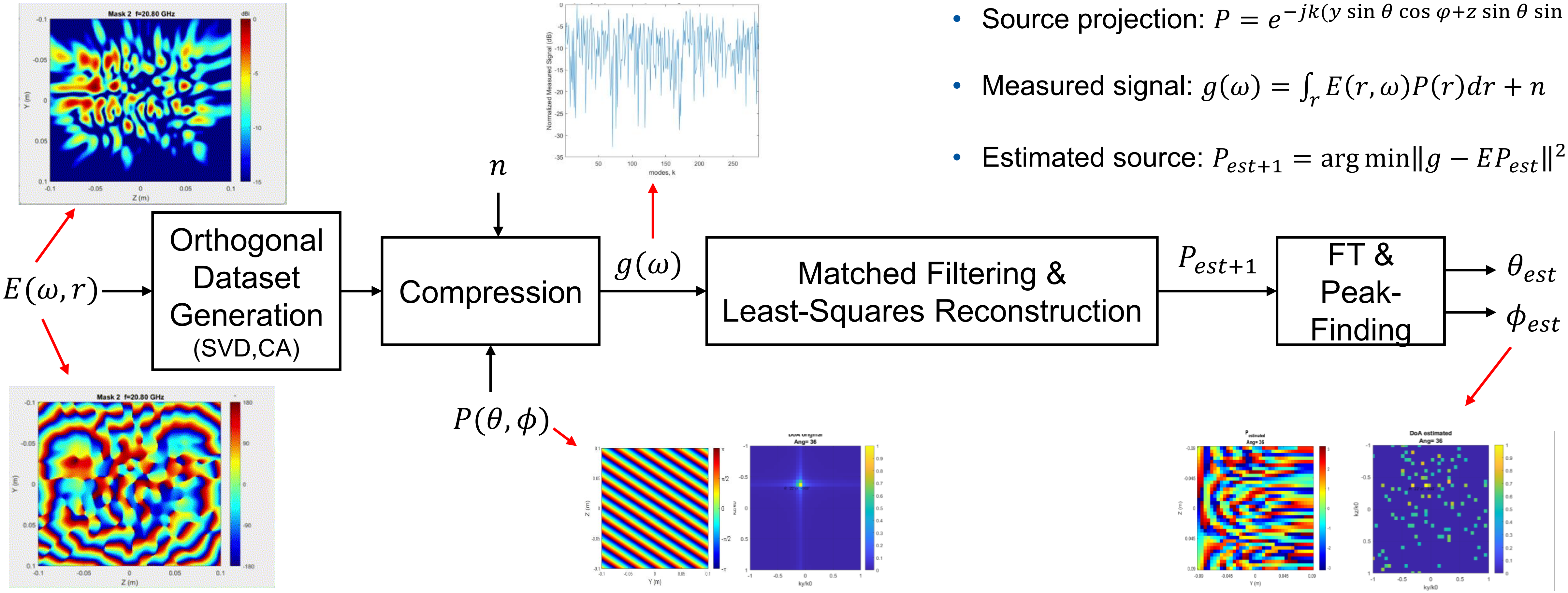




Introduction

Direction of Arrival (DoA) Estimation: Forward Model

- Transfer function: $E(\omega, r)$
- Source projection: $P = e^{-jk(y \sin\theta \cos\varphi + z \sin\theta \sin\varphi)}$
- Measured signal: $g(\omega) = \int_r E(r,\omega)P(r)dr + n$
- Estimated source: $P_{est+1} = \arg\min \| g - EP_{est} \|^2$



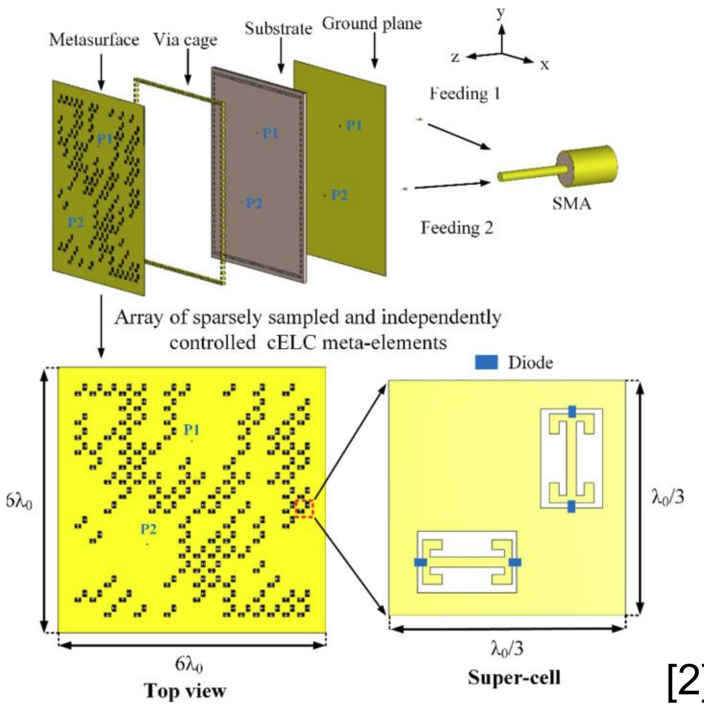
Introduction

Metasurface Antenna

- Planar electrical structure loaded with meta-elements called **unit cells**

- Metasurface antennas are periodically distributed electrical thin structures comprised of subwavelength metamaterials
  - Periodically disturbed along the surface
  - Electrically small in subwavelength levels
  - Dual-polarized meta-elements to exploit different polarizations
  - Meta-element are randomly distributed

- Objective is to generate random beam patterns
  - Single channel feed (single-pixel)
  - Individually accessible array meta-elements



[2]

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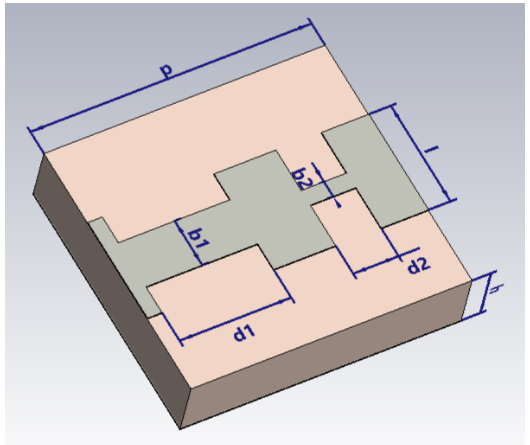
Design of Leaky-Wave Metasurface Antenna

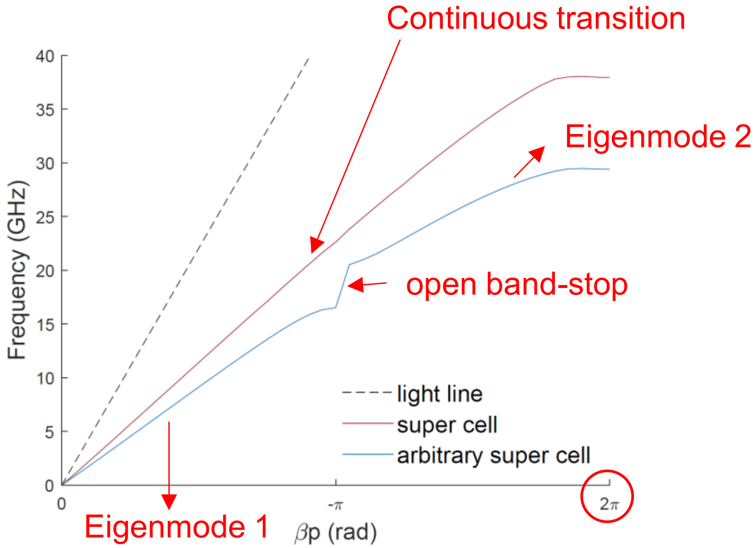
Super Cell

- The dispersion curve is periodic with 2π for one unit cell
  - With multiple unit cells the periodicity is reduced accordingly
  - Separation distance of space harmonics is reduced
  - More modes enter the fast-wave region $\rightarrow$ multiple beams

- Optimized dimensions of the super cell
  - Same length a $\rightarrow$ total periodicity p
  - Different modulation width and gap, b and d to introduce variation in the surface impedance

- Open band-stop problem
  - No propagation modes are supported $\rightarrow$ reduced gain and radiation
  - Continuous dispersion characteristics in the frequency domain





Design of Leaky-Wave Metasurface Antenna

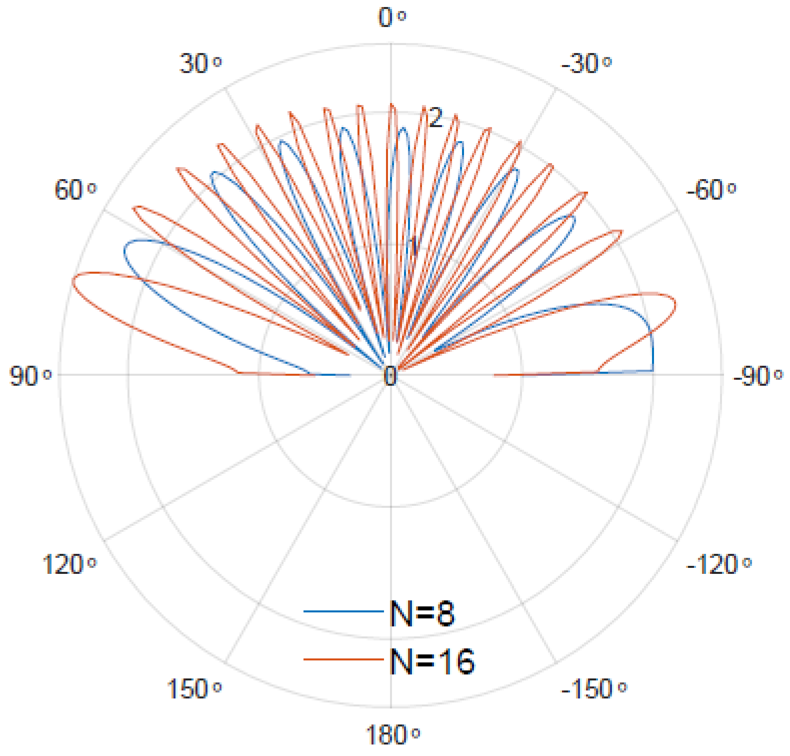
1D Leaky-Wave Antenna

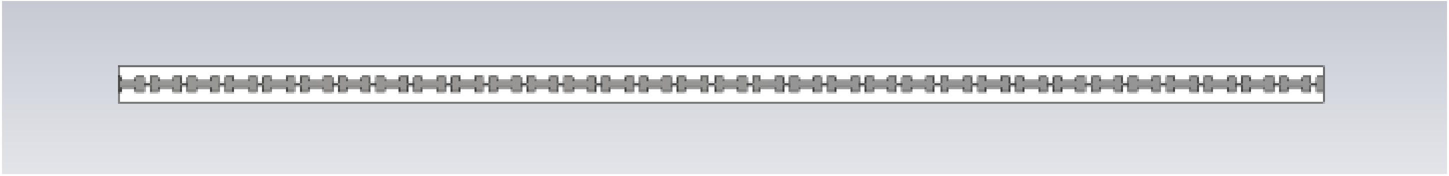
- Radiation pattern changes with increase of elements
  - Number of beams generated increases
  - Narrower beam-width obtained
  - Slight improvement of gain

- Trade-off
  - Computation time
  - Size restrictions

- 16 elements is found to be adequate

- <u>Elevation radiation pattern</u> → Longitudinal axis



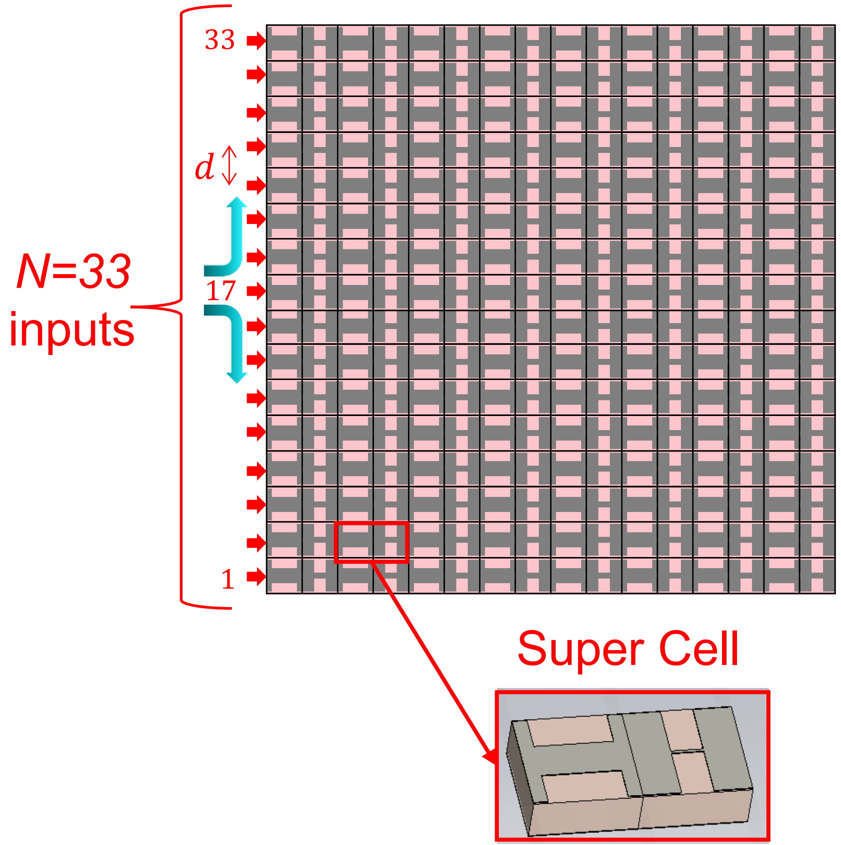


1D Leaky-Wave Antenna Array

- Generate a planar array with the parallel arrangement of 1D LWAs

- Parameters to consider are:
  - Form factor $\longrightarrow$ Feasible antenna size
  - Number of elements $\longrightarrow$ Narrow beamwidth
  - Element spacing $\longrightarrow$ Grating lobes occur for $\mathrm{d} > \frac{\lambda}{2}$
  - Linear phase shift $\longrightarrow$ Steering angle
  - <u>Feeding point</u>

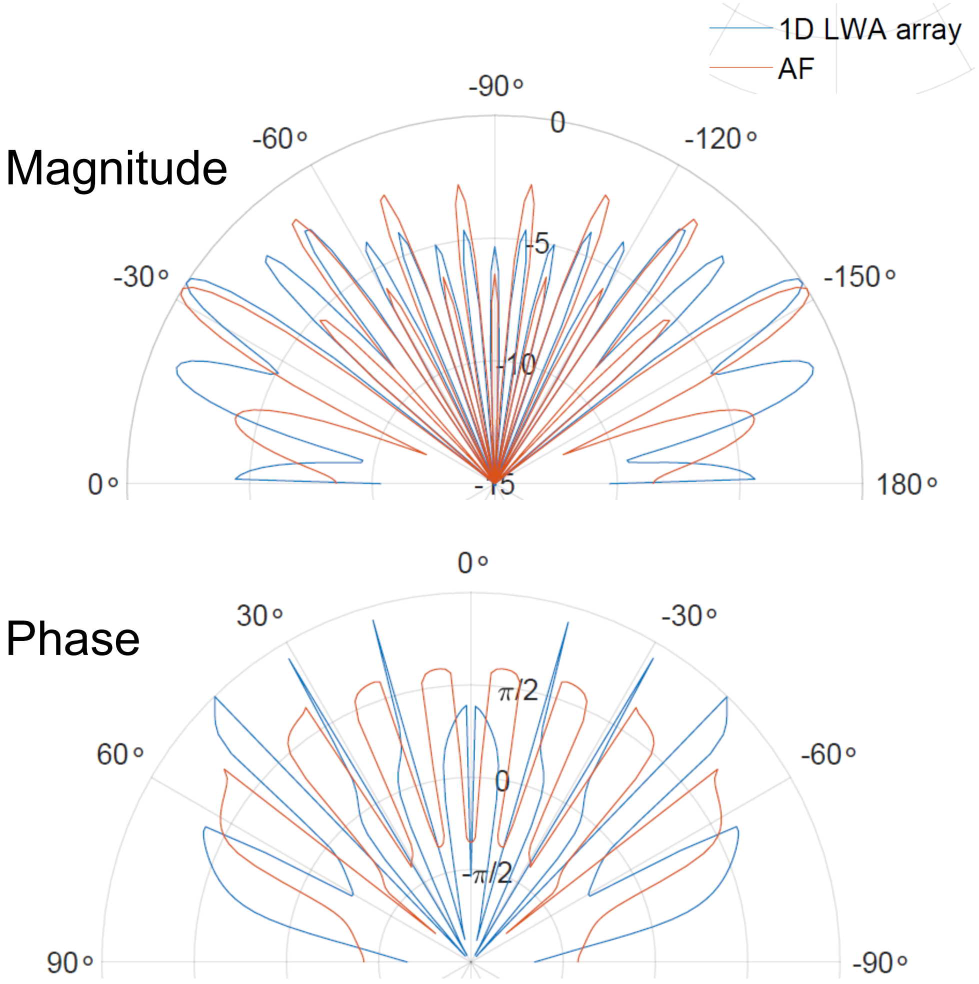
- Feeding point
  - Center feeding splits the aperture into two
  - Two main beams are generated propagating in opposite directions
  - Total array factor $\longrightarrow$ Superposition of the symmetric array factors
  - Number of elements are halved



Design of Leaky-Wave Metasurface Antenna

1D Leaky-Wave Antenna Array – Optimum Parameters

- Optimum value for α
  - Measure the self-correlation of array factor radiation pattern
  - Correlation between the AF pattern and the circularly shifted copy
  - Calculated for angles from $0°$ to $180°$ with $1°$ step
  - Obtain the lowest average mutual coherence
  - $N = 33$, $d = 3.33$ mm, $n_c = 17 \rightarrow \alpha = 154°$ at 22.5 GHz
- Azimuth radiation pattern $\rightarrow$ Transversal axis
  - Perpendicular to the elevation pattern
- Simulation results align with the analytic
  - Good spread of the magnitude and phase

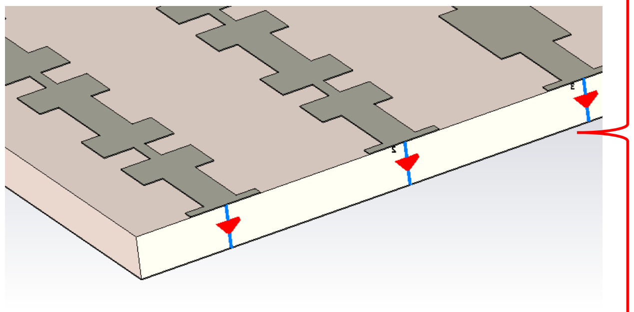


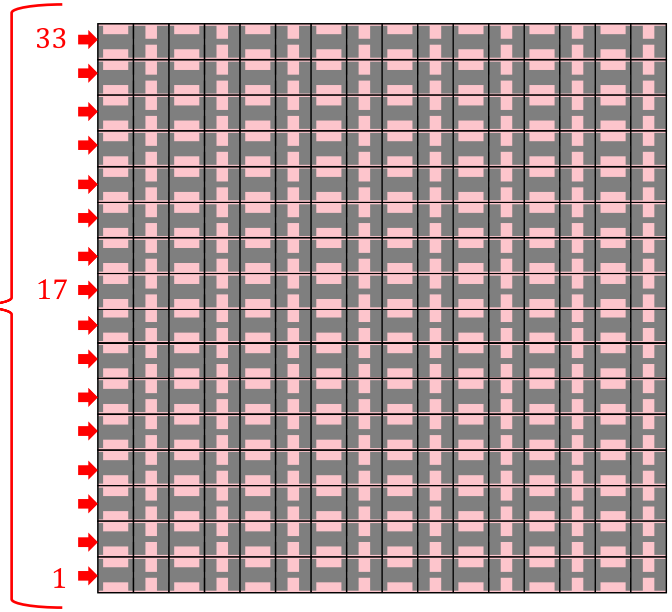
Design of Leaky-Wave Metasurface Antenna

Dynamic Aperture Array

- Objective is to create pattern diversity in the azimuth axis
- Dynamically control the inputs with electronically tunable switches
  - Ex. PIN diodes
  - Forward bias: *on*, reverse bias: *off*
  - Discrete ports are used as substitute for diodes
- Each *on/off* configuration → Mask
  - One mode = one frequency from one mask
  - Table of masks with Hex representation
- AF reduces to sparse AF
  - Calculated from the series expansion formula

Mask No.	Hexadecimal representation
1	F-F-F-F-1-F-F-F-F
2	2-7-B-4-1-D-6-A-B
3	3-4-6-3-1-6-2-2-B
4	8-A-8-9-1-D-A-7-4
5	B-9-D-A-0-F-7-D-4
6	3-4-6-3-1-6-2-A-F
7	7-3-9-5-0-6-3-F-7
8	2-A-E-2-1-C-5-C-5

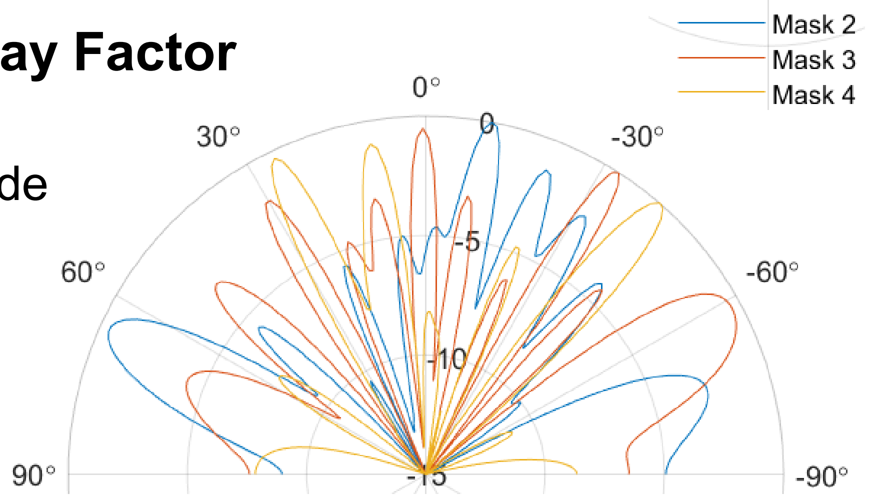




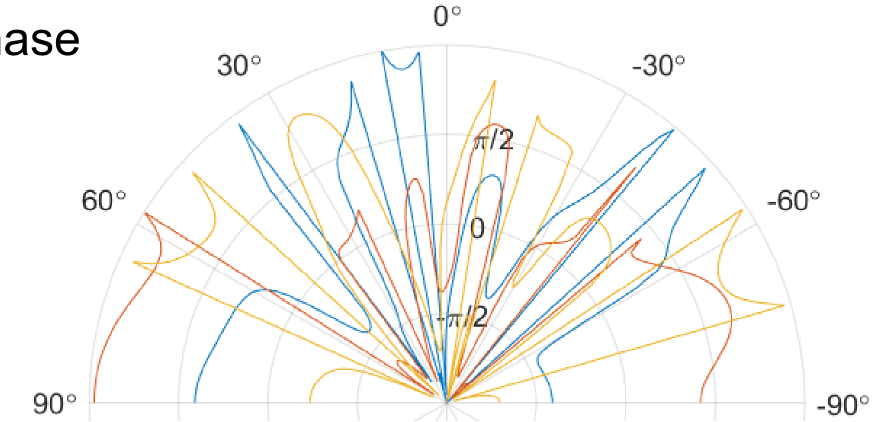
Design of Leaky-Wave Metasurface Antenna

Sparse Array Factor

Magnitude



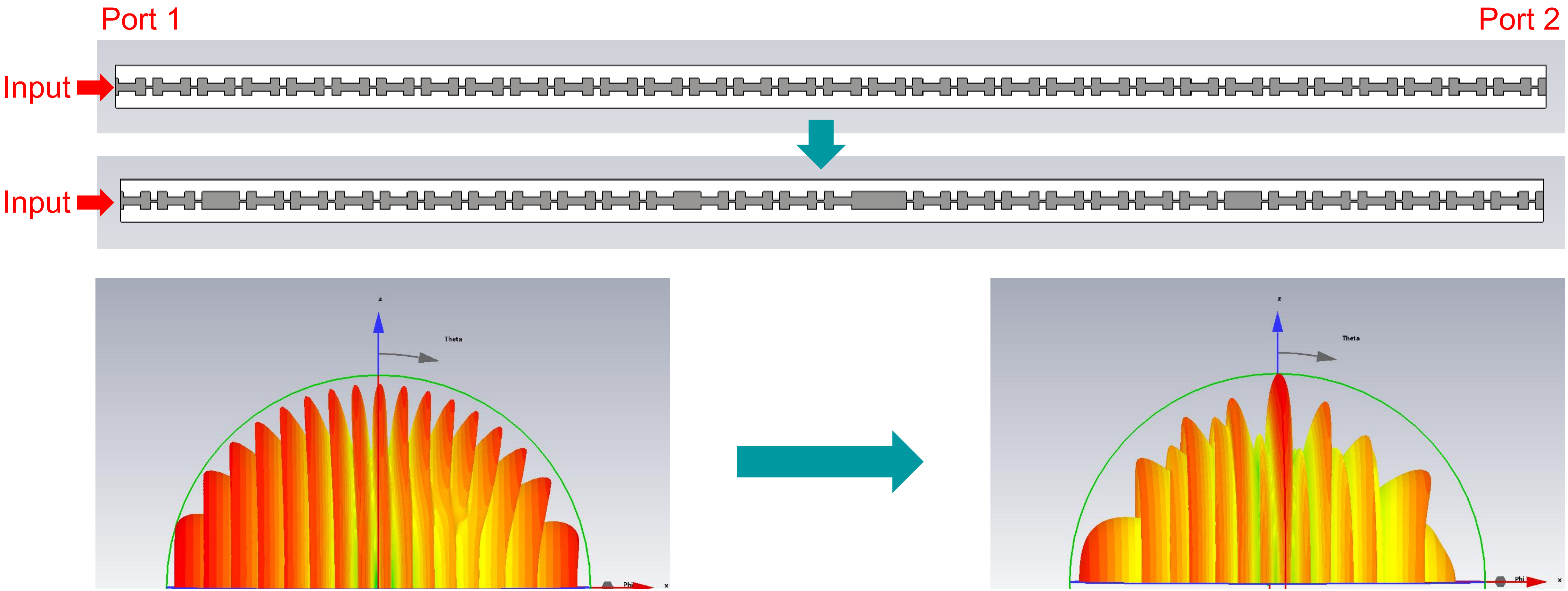
Phase



Mask No.	Hexadecimal	Binary
1	F-F-F-F-1-F-F-F-F	1111-1111-1111-1111-1-1111-1111-1111-1111
2	2-7-B-4-1-D-6-A-B	0010-0111-1011-0100-1-1111-0110-1010-1011
3	3-4-6-3-1-6-2-2-B	0011-0100-0110-0011-1-0110-0010-0010-1101
4	8-A-8-9-1-D-A-7-4	1000-1010-1000-1001-1-1101-1010-0111-0100
5	B-9-D-A-0-F-7-D-4	1101-1001-1101-1010-0-1111-0111-1101-0100
6	6-6-3-F-0-D-8-C-5	0110-0110-0011-1111-0-1101-1000-1100-0101
7	7-3-9-5-0-6-3-F-7	0111-0011-1001-0101-0-1010-0011-1111-0111
8	2-A-E-2-1-C-5-C-5	0010-1010-1110-0010-1-1100-0101-1100-0101
9	4-0-5-5-0-9-B-8-9	0100-0000-0101-0101-0-1001-1011-1000-1001
10	D-B-A-4-0-4-2-C-9	1101-1011-1010-0100-0-0100-0010-1100-1001
11	F-9-2-F-1-2-E-A-6	1111-1001-0010-1111-1-0010-1110-1010-0110
12	B-6-E-6-1-E-0-F-3	1011-0110-1110-0110-1-1110-0000-1111-0011
13	A-B-8-1-1-3-2-E-6	1010-1011-1000-0001-1-0011-0010-1110-0110
14	1-B-7-1-0-9-7-3-C	0001-1011-0111-0001-0-1001-0111-0011-1100
15	2-E-8-1-0-F-7-7-D	0010-1110-1000-0001-0-1111-0111-0111-1101
16	D-C-0-B-0-8-9-8-F	1101-1100-0000-1011-0-1000-1001-1000-1111
17	7-4-C-8-1-F-9-0-2	0111-0100-1100-1000-1-1111-1001-0000-0010
18	8-4-D-9-0-0-B-5-1	1000-0100-1101-1001-0-0000-1011-0101-0001
19	7-7-5-B-0-F-1-4-C	0111-0111-0101-1011-0-1111-0001-0100-1100
20	4-1-B-9-1-A-D-4-6	0100-0001-1011-1001-1-1010-1101-0100-0110
21	C-4-C-7-1-1-1-2-2	1100-0100-1100-0111-1-0001-0001-0010-0010
22	8-7-F-3-1-C-1-7-B	1000-0111-1111-0011-1-1100-0001-0111-1011
23	7-3-1-B-0-0-9-8-6	0111-0011-0001-1011-0-0000-1001-1000-0110
24	D-B-2-3-0-8-E-C-D	1101-1011-0010-0011-0-1000-1110-1100-1101
25	7-9-7-5-0-3-5-5-8	0111-1001-0111-0101-0-0011-0101-0101-1000

Design of Leaky-Wave Metasurface Antenna

Random Placement of Patches: Generate pattern diversity in the elevation axis

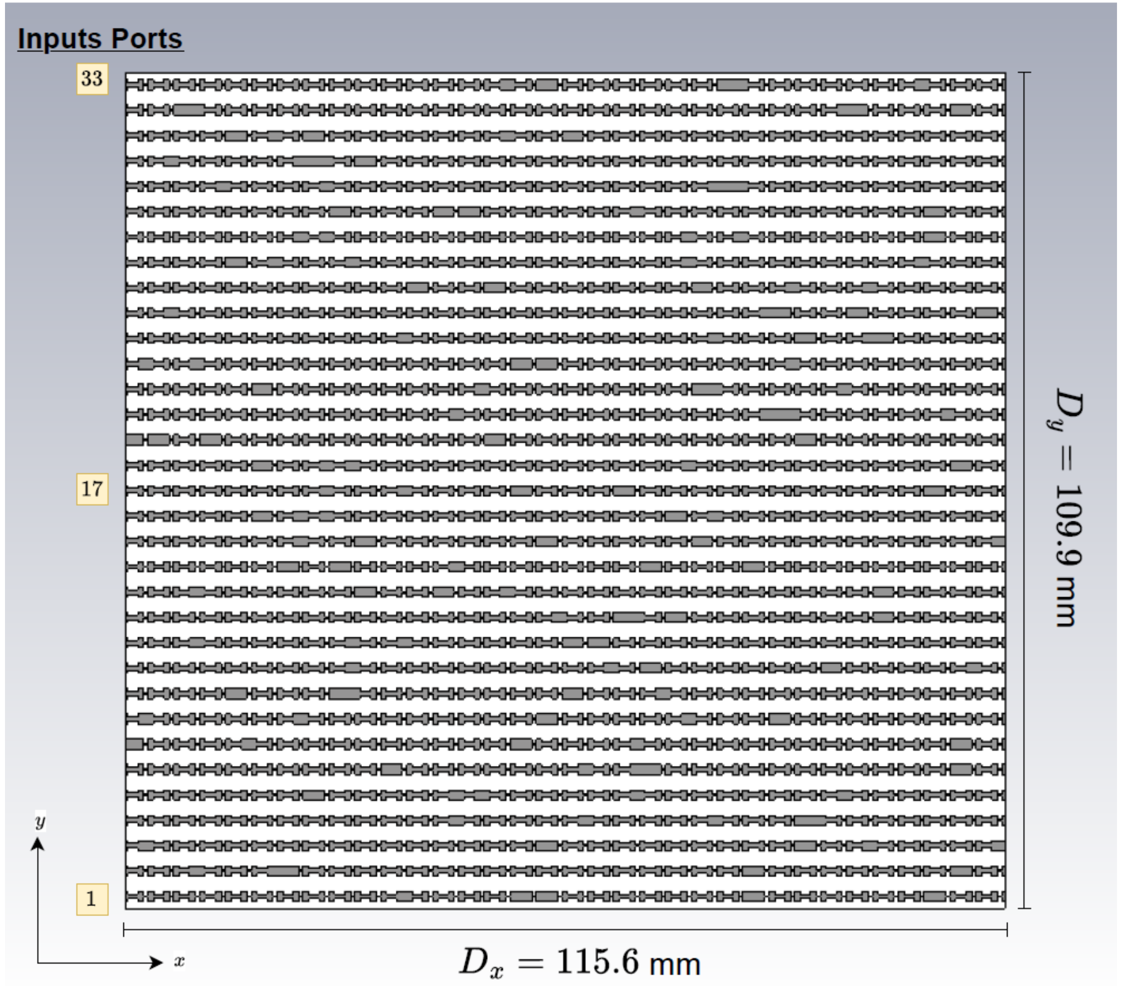


Summary of Proposed Antenna

- Substrate and conductor
  - Rogers RO4350B
  - $\epsilon_r = 3.66$
  - $\tan\delta = 0.0037$
  - $h = 0.762\ \mathrm{mm}$
  - $t = 0.017\ \mathrm{mm}$

- Physical Dimensions
  - $N = 33$ rows, $n_c = 17$
  - $d = 3.33\ mm,\ f_0 = 22.5\ GHz$
  - $D_x \approx D_y \approx 110\ mm$
  - $\alpha = 154°$

- 5 uniquely placed patches on each row
  - Unique rows $\rightarrow$ Unique patterns $\rightarrow$ Pattern diversity in array



Design of Compressive Sensing Algorithm

Generation of Orthogonal Dataset

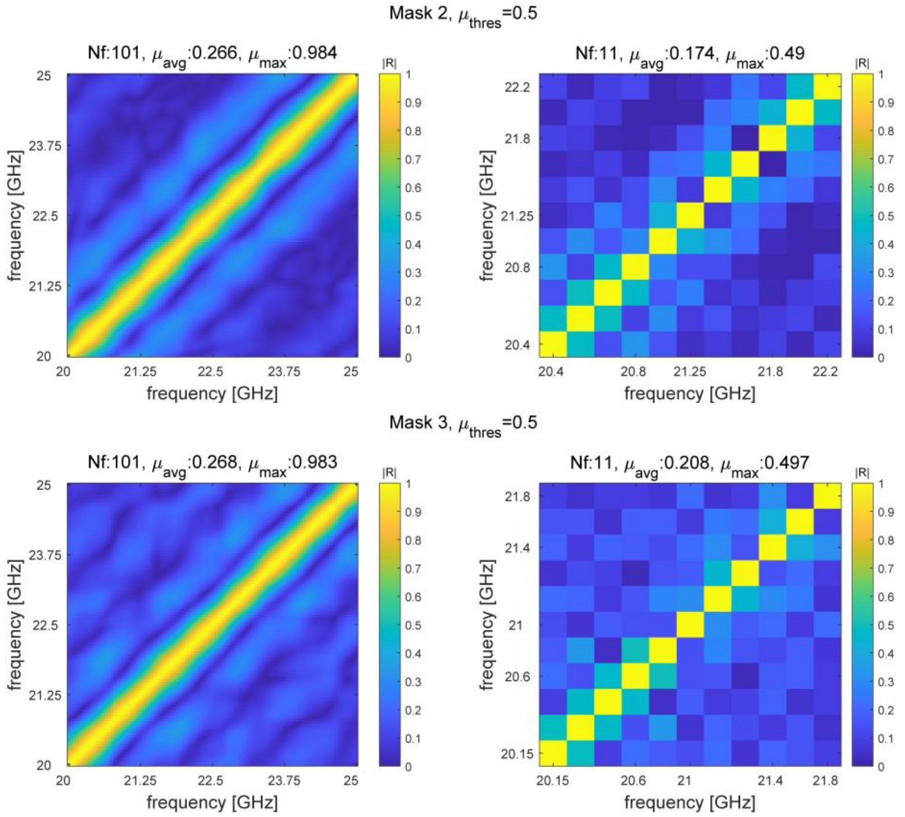
- For each mask, perform SVD and CA of patterns

- Determine threshold for mutual coherence
  - Ex: $\mu_{thres} = 0.5$

- Keep modes lower than the threshold and discard the rest
  - Remaining becomes the orthogonal modes
  - Reduces total number of modes
  - One mode = one frequency from one mask

- Perform SVD and CA again for validation

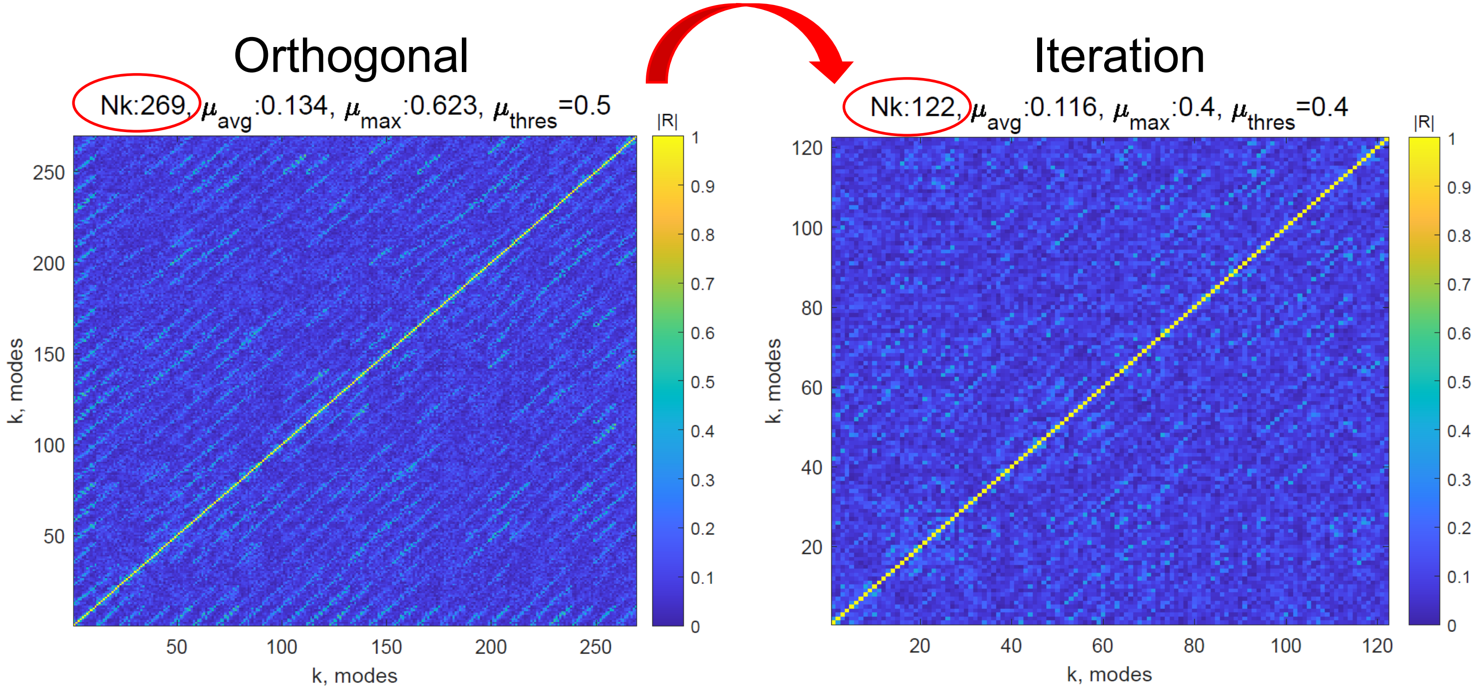
- Selection of Modes: Masks

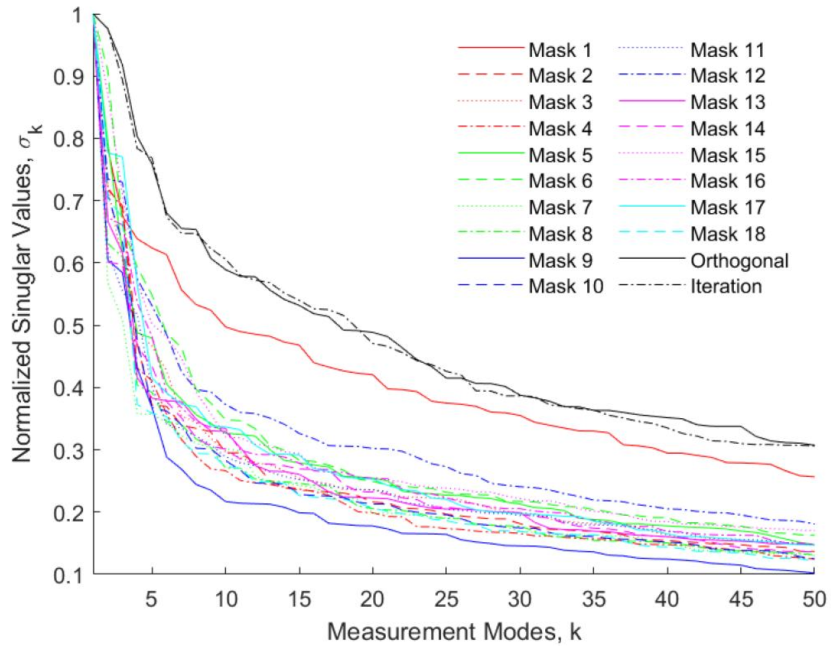


Design of Compressive Sensing Algorithm

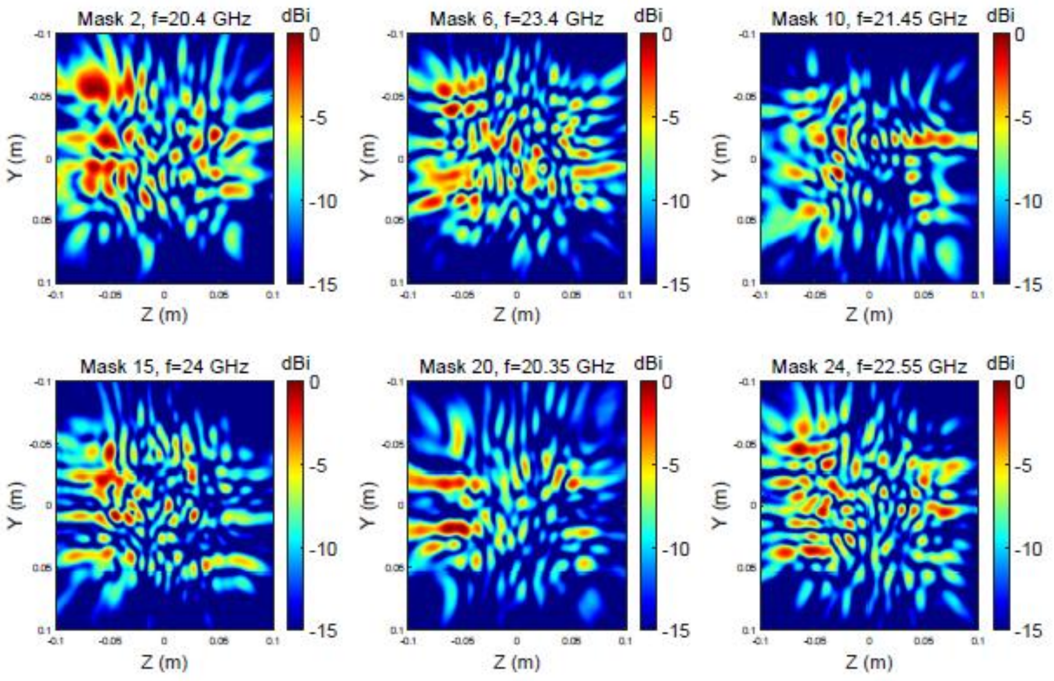
Generation of Orthogonal Dataset

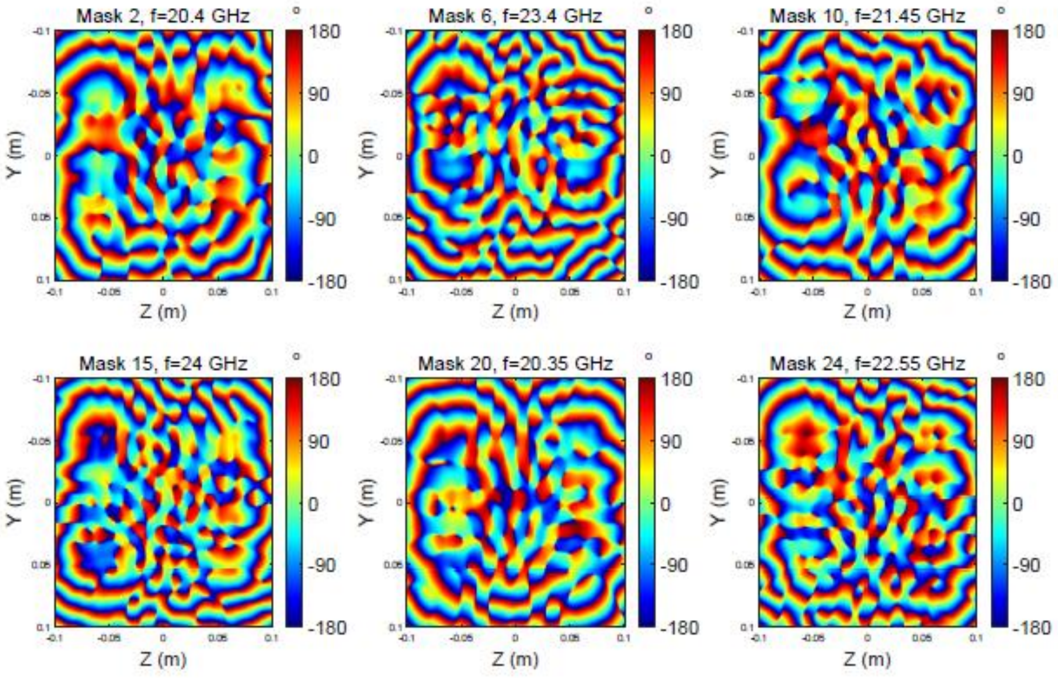
- Iteration of orthogonal dataset with new threshold
  - Ex: $\mu_{thres} = 0.4$
- Similar level of singular values but with <u>reduced number of modes</u>





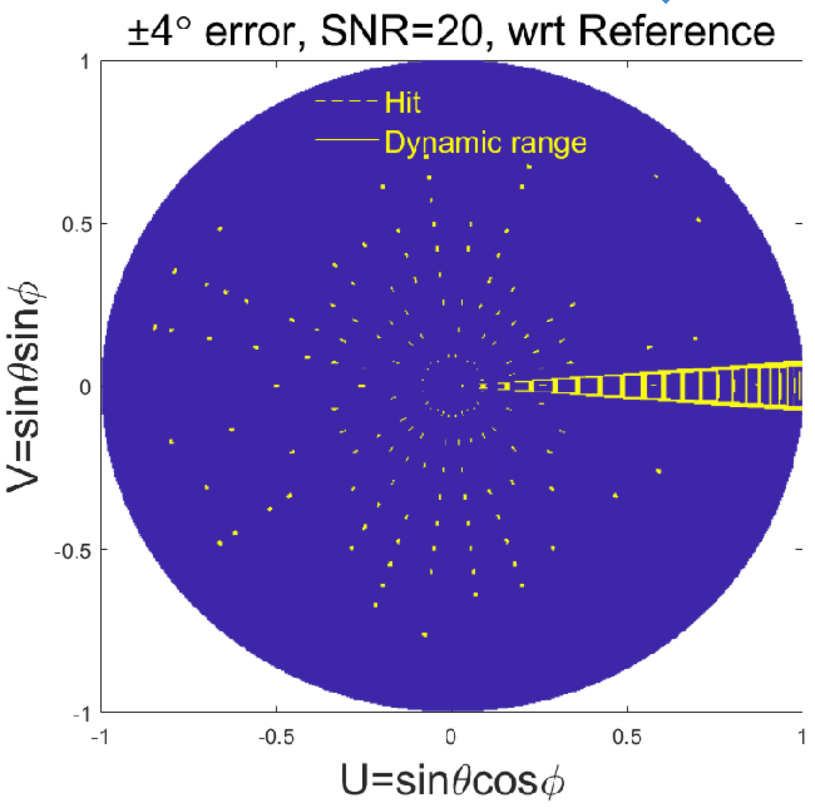
Generation of Orthogonal Dataset

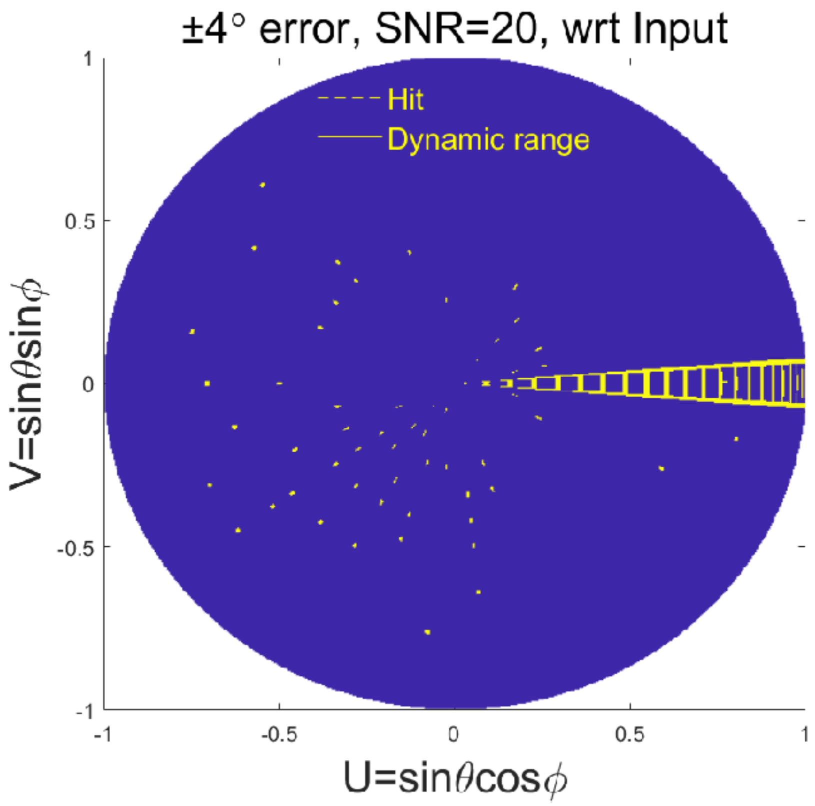




Simulation Results

Sweep of Angles





Input (θ, ϕ)	Reference (θ, ϕ)	Estimated (θ, ϕ)
(10, 30)	(5, 18)	(5, 18)
(18, 75)	(15, 84)	(15, 84)
(26, 120)	(22, 122)	(22, 122)
(34, 165)	(30, 170)	(30, 170)
(42, -150)	(39, -150)	(39, -150)
(50, -105)	(43, -107)	(43, -107)
(58, -60)	(49, -61)	(54, -63)
(66, -15)	(54, -18)	(54, -18)

Conclusion

Summary

- Workflow: Compressive Sensing, Leaky-Wave Antennas, Direction-of-Arrival Estimation

- Design of Leaky-Wave Metasurface Antenna
  - Constructed a LWA array from unit cells to super cells to array
  - Adapted dynamic aperture structure alongside the frequency-diversity concept
  - Introduced randomly placed patches

- Designed the Compressive Sensing Algorithm
  - Selection of modes with CA and SVD
  - Iteration to refine the number of selected modes

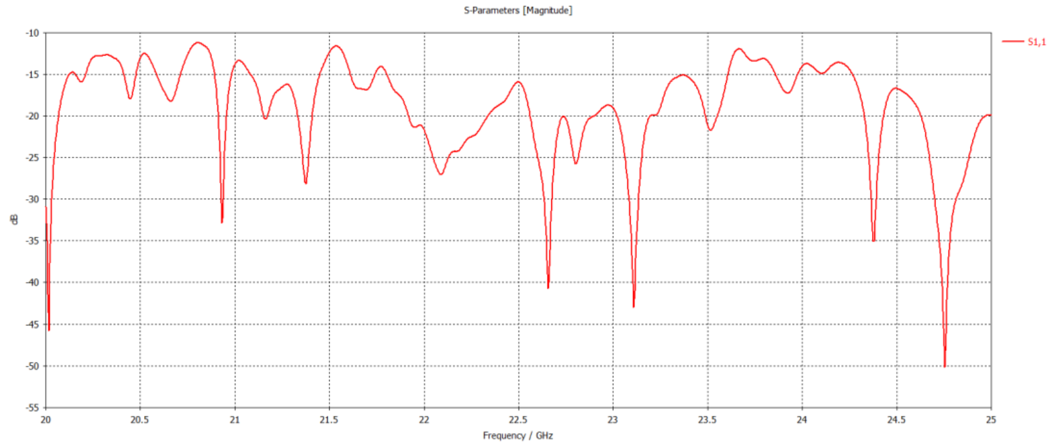
- Used dataset of orthogonal patterns to retrieve an estimate of DoA
  - Compared estimate to reference
  - Detected angles in UV plot and dynamic range
  - Performance with variation in SNR

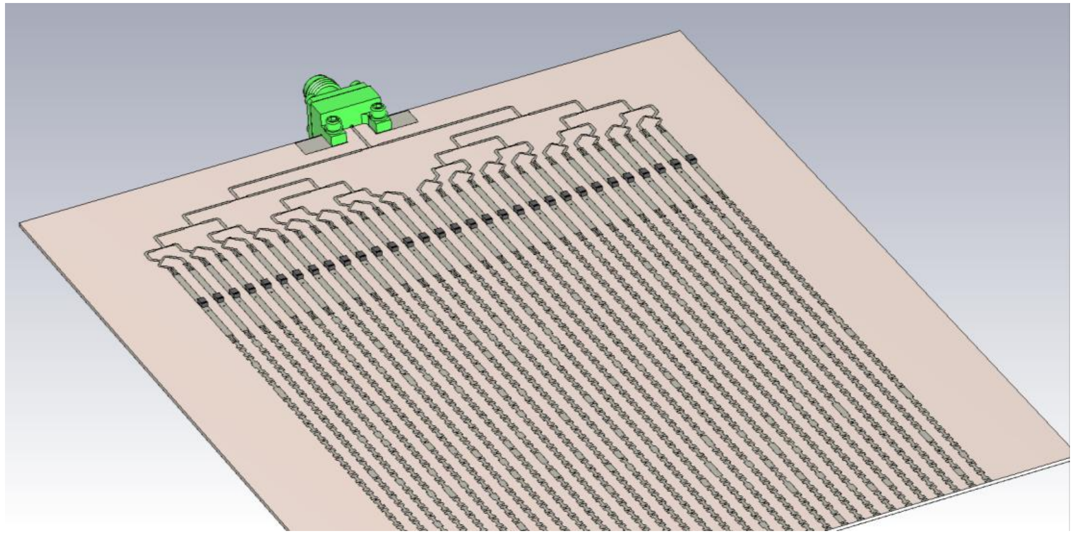
Conclusion

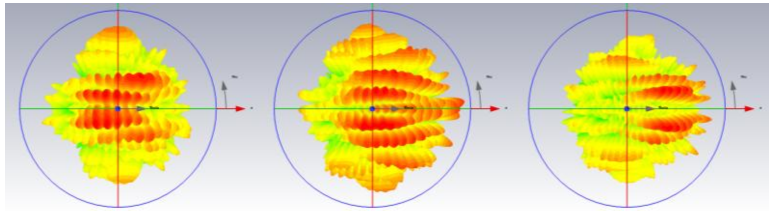
Final State

- Continuation of thesis
  - Design of corporate feeding network with phase shifts
    - 32 way divider
    - Reduced the array from 33 to 32 elements
  - Component selection: Diodes
    - Including design of diode bias circuit
  - Connector footprint and placement

- Tasks due
  - Fabrication and assembly
  - Near-field, far-field and return loss measurements
  - Real-world scenario







Thank you!

Questions?

References

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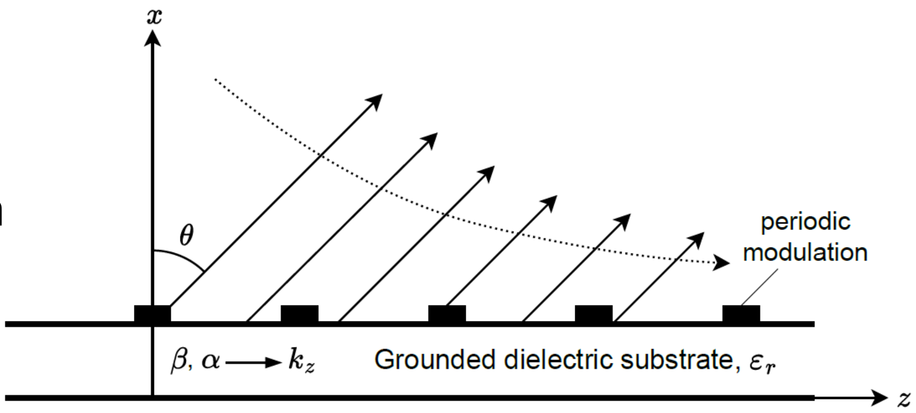
Extra Slide – Design of Leaky-Wave Metasurface Antenna

Metasurface Antenna: Leaky-Wave Antenna (LWA)

- Longitudinal and transversal propagation in z- and x- directions
- Propagation wavenumber in the longitudinal direction: $k_z = \beta - j\alpha$
  - Real propagation constant, β
  - Real attenuation constant, $\alpha > 0 \rightarrow$ Leakage in k_z
- Periodic modulation of LWA causes radiation in the transverse direction
  - $k_x = \beta_x - j\alpha_x$
- From Helmholtz equation, fields in free-space are characterized by:
  - $k_0^2 = k_z^2 + k_x^2$ with real $k_0 = \frac{2\pi f}{c} \rightarrow$ imaginary parts must cancel
  - $Im\{k_z^2 + k_x^2\} = \alpha\beta + \alpha_x\beta_x = 0 \rightarrow \alpha_x < 0$ suggests exponential increase
  - Attenuation in longitudinal direction accounts for radiation in transversal direction
- Periodicity is fundamental for **Floquet's Theorem** and **Dispersion Diagram**



Extra Slide – Design of Leaky-Wave Metasurface Antenna

Floquet's Theorem

- The fields can be expressed as a series expansion due to periodicity
  - Wave number of the space harmonics: $k_{z0} + \frac{2\pi n}{p}$

$$E(x,z) = \sum_{-\infty}^{\infty} A_n(x) e^{-j\left(k_{z0}+\frac{2\pi n}{p}\right)z}$$

- Infinite space harmonics can be generated
  - Identical attenuation constant $\rightarrow$ k_{zn} simplifies to nth space harmonic: β_n

$$k_{zn} = k_{z0} + \frac{2\pi n}{p} = \beta_0 - j\alpha + \frac{2\pi n}{p}$$

- Space harmonics have independent phase constants
  - Radiation occurs independent from other harmonics

$$\Rightarrow \beta_n = \beta_0 + \frac{2\pi n}{p}$$

- Fast-wave condition is required for radiation $\rightarrow$ **Dispersion Diagram**

Extra Slide – Design of Leaky-Wave Metasurface Antenna

Dispersion Diagram

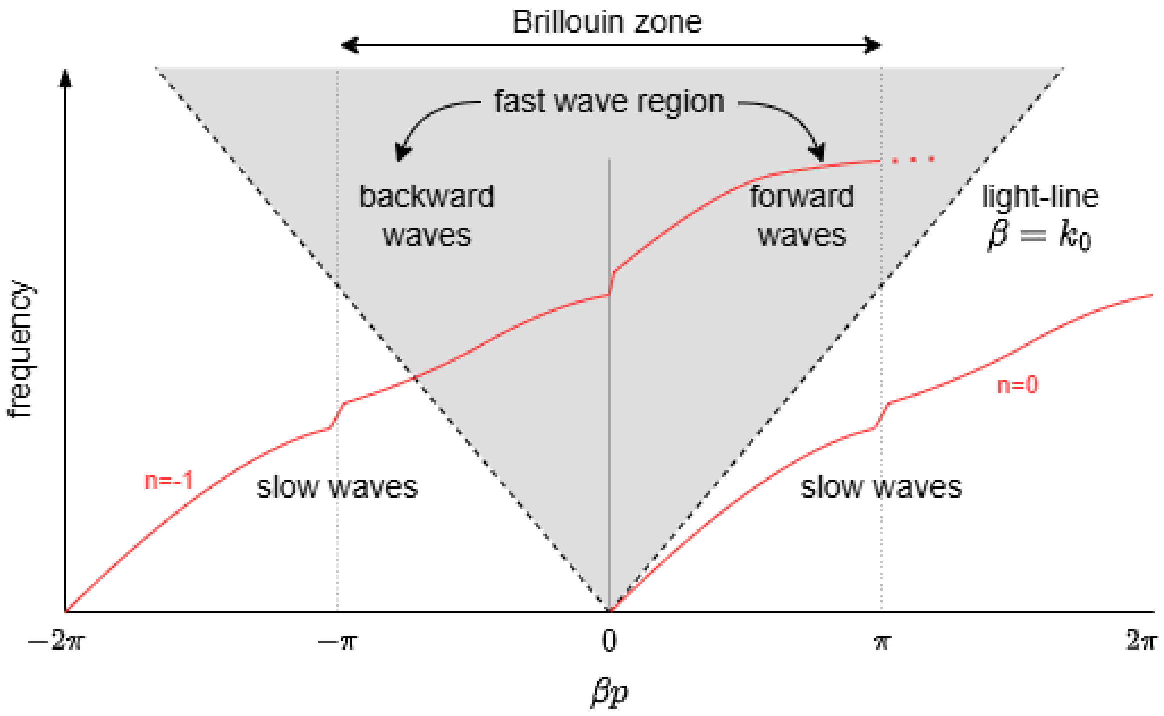
- Tool in determining the radiation property of periodic structures
  - periodic with $\frac{2\pi}{p}$ due to Floquet harmonics
  - Confined within Brillouin zone: $-\frac{\pi}{p} < \beta_n < \frac{\pi}{p}$

- Light line separates the fast and slow wave regions

- Radiation in periodic LWA occurs for fast-waves condition
  - Phase constants of space harmonics: $\beta > k_0$
  - N space harmonics $\rightarrow$ N radiated beams

- n[th] space harmonic: $\beta_n = \beta_0 + \frac{2\pi n}{p}$

- Beam angle of nth space harmonic: $\theta = \arcsin\left(\frac{\beta_n}{k_0}\right)$
  - Scan angle increases with increasing $\beta_n \rightarrow$ Frequency dependent spatial scanning



Extra Slide – Design of Leaky-Wave Metasurface Antenna

Unit Cell

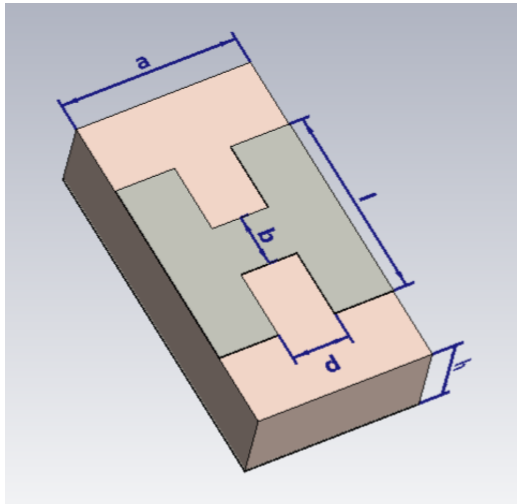
- “H”-shaped surface modulation
  - Periodic modulation of a transmission line
  - Squarely Modulated Reactance Surface (SquMRS) [3]

- Parameters: periodicity, modulation gap, modulation width/depth
  - Modified to obtain the desired dispersion curve

- Most effect on the dispersion curve: b
  - Lower slope $\rightarrow$ Higher scanning rate

- Modulation period p is equal to length a

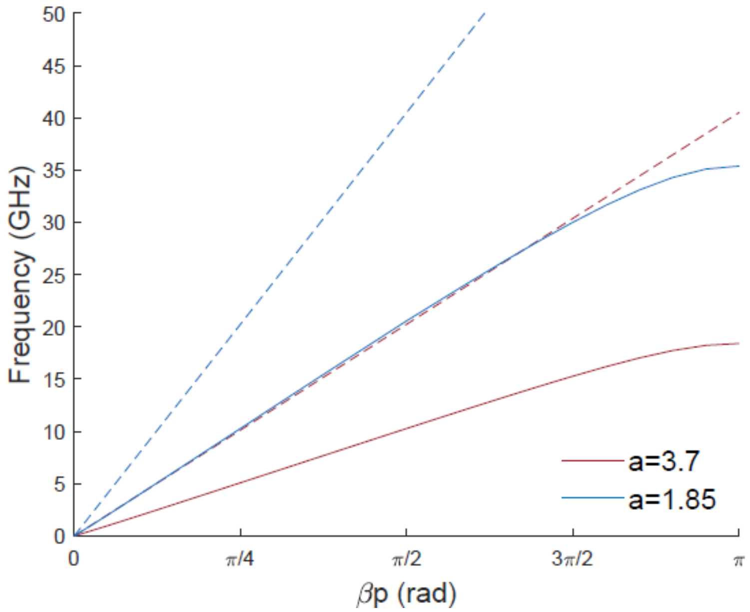
- Dimension l determines the line impedance

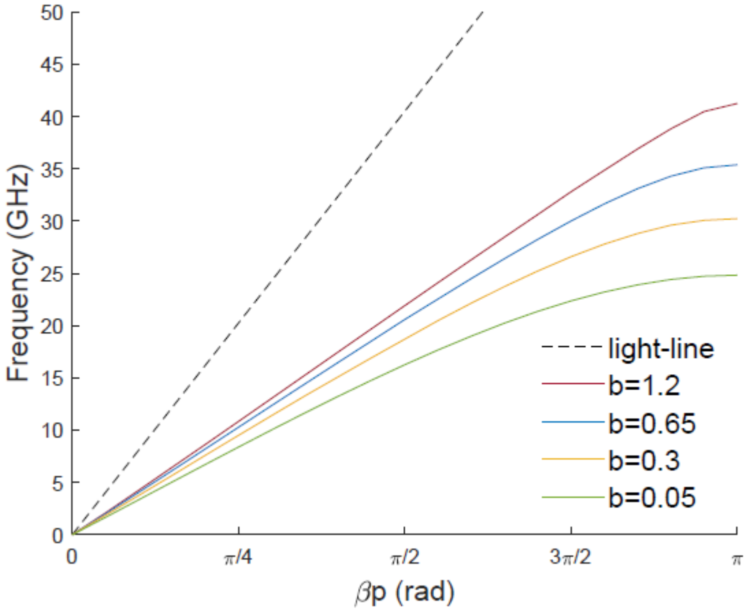


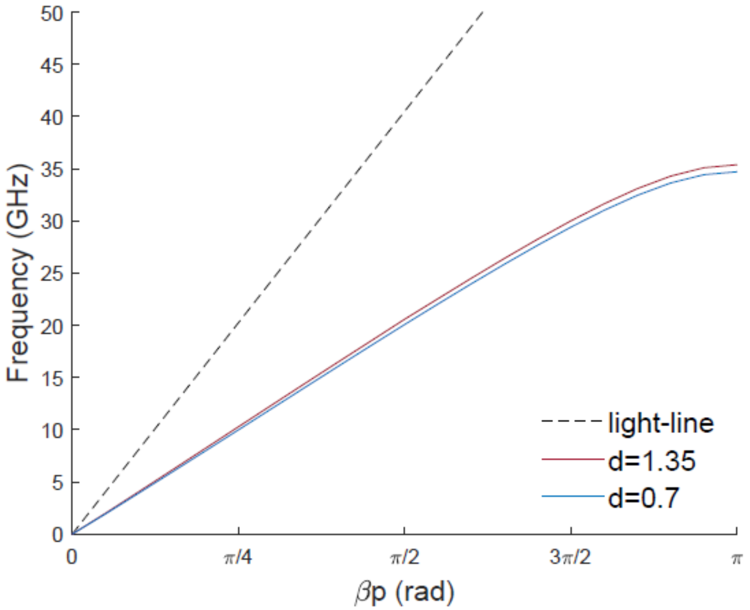
Parameter	Description	Default values (mm)
a	Length of the unit cell	1.85
b	Modulation width	0.65
d	Modulation gap	1.35
l	Width of transmission line	1.825
h	Substrate thickness	0.762
p	Modulation period	1.85

Extra Slide – Design of Leaky-Wave Metasurface Antenna

Unit Cell – Dispersion Curve



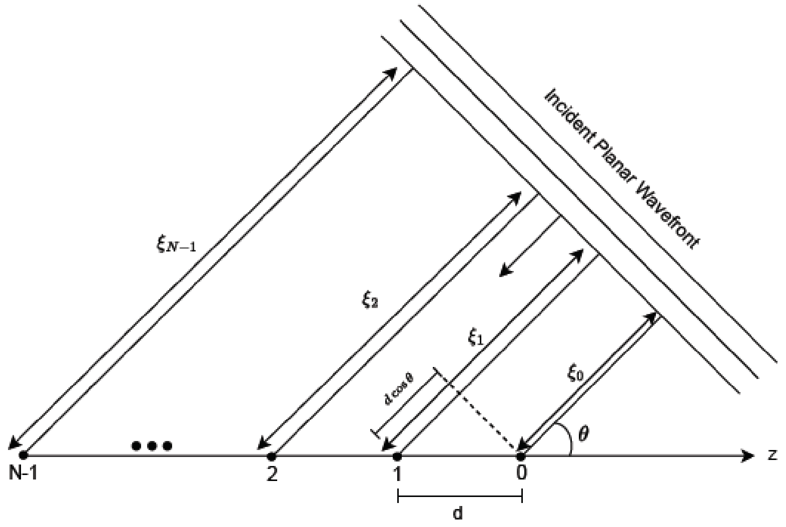


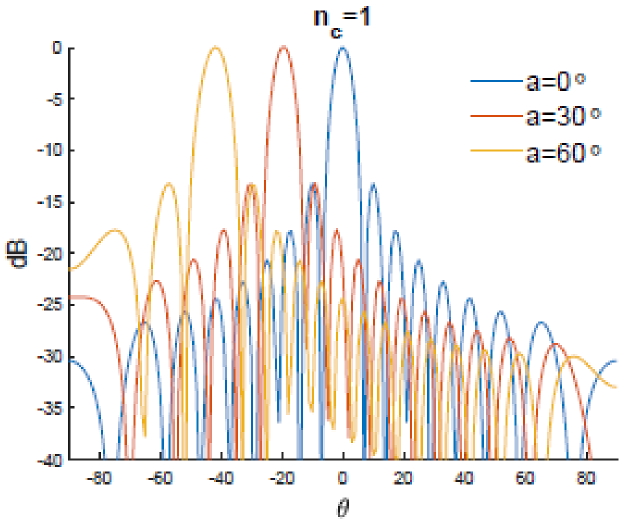


Array Approach

- Total radiation pattern of an antenna array consists of the element factor (EF) and array factor (AF)
  - Array elements must be identical
  - Array factor is due to the arrangement of the elements
  - Effective radiation pattern = $AF \times EF$
- Uniform Linear Array
  - Equally spaced isotropic radiating elements: d
  - Relative phase of uniformly spaced elements: ψ
- Beam-steering with linearly increasing phase shift: α
  - Excess pass due to approach angle: $\theta \rightarrow$ spatial phase delay

$$AF = \sum_{n=0}^{N-1} I_n e^{jn\underbrace{(\beta d \cos\theta + \alpha)}_{\psi}}$$





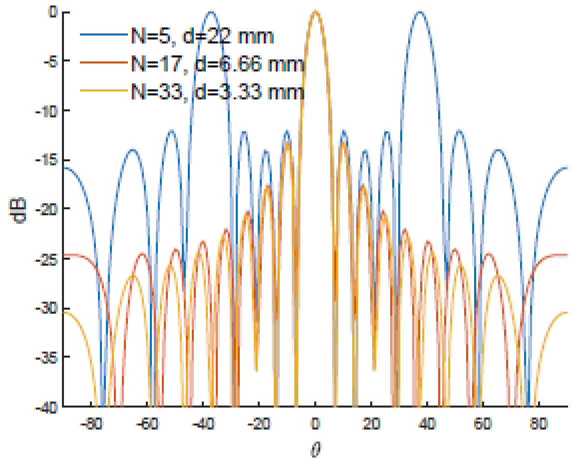
Extra Slide – Design of Leaky-Wave Metasurface Antenna

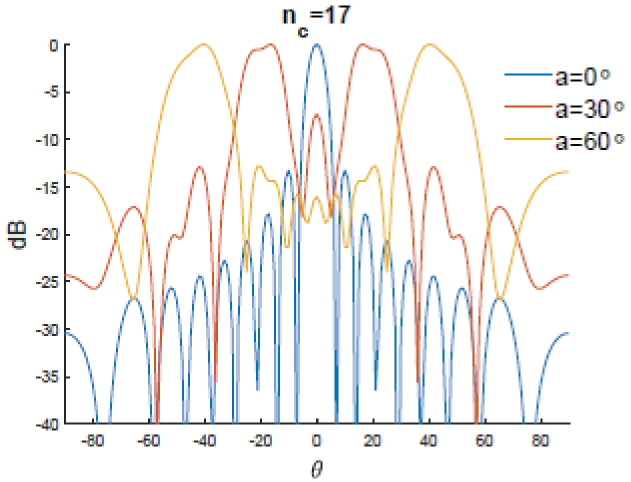
1D Leaky-Wave Antenna Array

- Identical form factor for all cases
  - Antenna size is constant
  - Number of elements and spacing is adjusted accordingly
  - Wavelength is calculated at center frequency, $f_0 = 22.5\ GHz$

- Case 1: $\mathrm{d} > \frac{\lambda}{2}$ and $N = 5 \rightarrow$ Grating lobes
- Case 2: $\mathrm{d} = \frac{\lambda}{2}$ and $N = 17 \rightarrow$ Nyquist limit
- Case 3: $\mathrm{d} = \frac{\lambda}{4}$ and $N = 33 \rightarrow$ Dynamic Aperture

- Magnitude spread which spans the space is desired
  - Beneficial during modulation
  - Tune the remaining parameter α
  - Some values fixes the beam angle at an angle

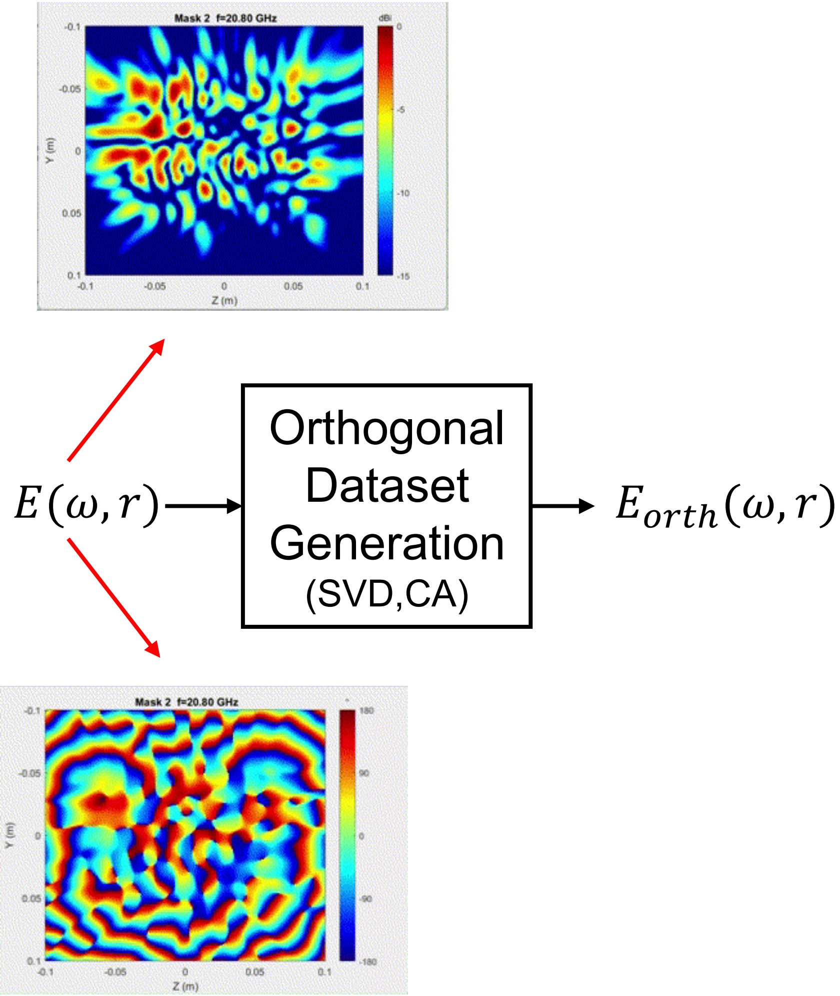




Extra Slide – Design of Compressive Sensing Algorithm

Orthogonality Assessment

- Objective is to generate a set of orthogonal radiation patterns
- Mutual Incoherence Property
  - Lower coherence among patterns → Better reconstruction capability
- Two methods to analyze the orthogonality between radiation patterns
  - **Singular Value Decomposition (SVD)**
  - **Coherence Analysis (CA)**



Extra Slide – Design of Compressive Sensing Algorithm

Orthogonality Assessment: SVD

- SVD of 3-D data by using Mastorakis (1996) algorithm [5]
  - Unfold 3-D matrix into 2-D by means of transformation
  - Perform SVD twice

- Higher singular values means more orthogonality

- Steps briefly:

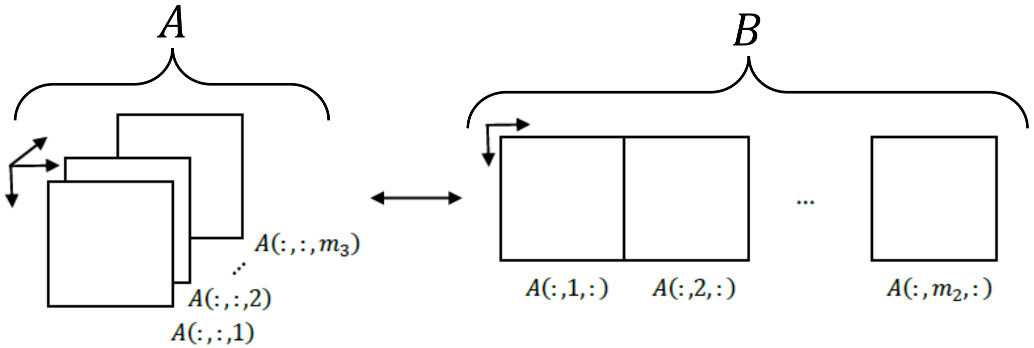
  1) $B = U_B \Sigma_B V_B^*$, singular values in Σ_B are σ_B

  2) $B^{sB} = V^{sB} \Sigma^{sB} W^{sB*}$, singular values in σ_τ

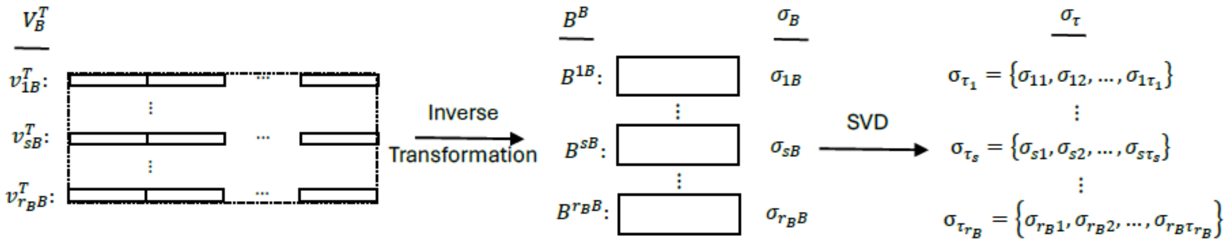
  3) combine singular values $\sigma = \sigma_B \sigma_{tau}$

  4) Arrange in decreasing order

1)



2)



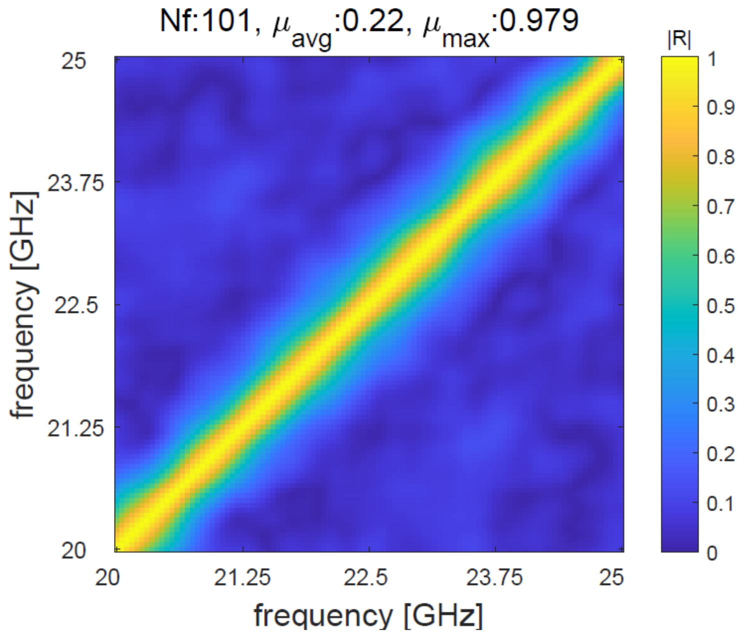
3)

$$
\begin{aligned}
\sigma &= \sigma_B \, \sigma_\tau \\
&= \{\sigma_{1B}, \ \ldots, \ \sigma_{sB}, \ \ldots, \ \sigma_{r_B B}\} \, \{\sigma_{\tau 1}, \ \ldots, \ \sigma_{\tau_s}, \ \ldots, \ \sigma_{\tau_{r_B}}\} \\
&= \{\sigma_{1B}\sigma_{\tau_1}, \ \ldots, \ \sigma_{sB}\sigma_{\tau_s}, \ \ldots, \ \sigma_{r_B B} \, \sigma_{\tau_{r_B}}\}
\end{aligned}
$$

Extra Slide – Design of Compressive Sensing Algorithm

Orthogonality Assessment: CA

- Correlation plot of 3-D data with dimensions (θ, φ, f)
- Simply angle-by-angle dot-product of patterns
  - Normalised → Phase information determines orthogonality
- Smaller correlation coefficients → better reconstruction
- Maximum Mutual Coherence, μ_{max}
  - Strongest similarity between any two matrices
- Average Mutual Coherence, μ_{avg}
  - More closely related to the overall CS performance [6]
  - Strongest of the average between any two matrices is taken as the worst case

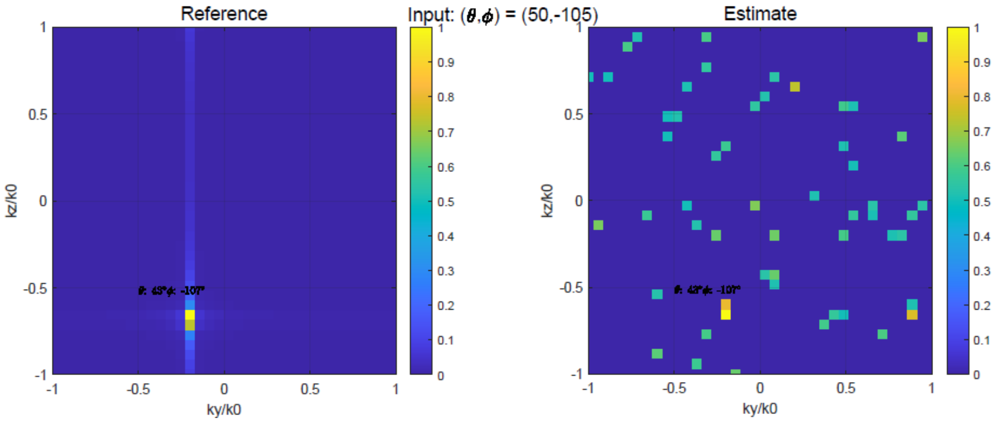


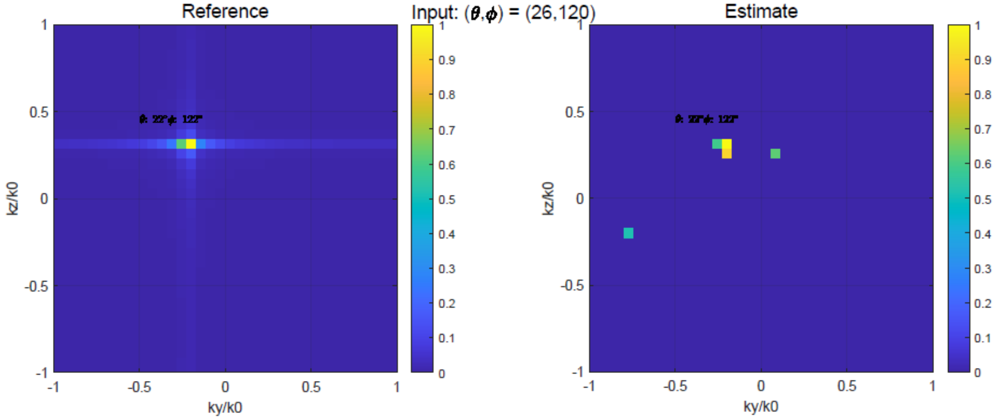
$$\mu_{max}(A) = \max_{i \neq j} \frac{|A_i^T A_j|}{\|A_i\|_2 \|A_j\|_2}$$

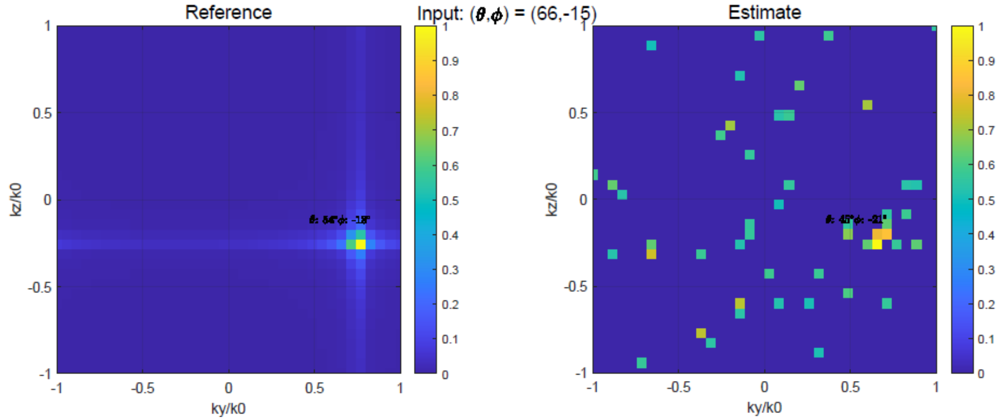
$$\mu_{avg}(A) = \frac{\sum_{\forall(i \neq j)} |A_i^T A_j|}{N_t}$$

Extra Slide – Simulation Results

Example Angle Pairs (θ, ϕ)







Input (θ, ϕ)	Reference (θ, ϕ)	Estimated (θ, ϕ)
(10, 30)	(5, 18)	(5, 18)
(18, 75)	(15, 84)	(15, 84)
(26, 120)	(22, 122)	(22, 122)
(34, 165)	(30, 170)	(30, 170)
(42, -150)	(39, -150)	(39, -150)
(50, -105)	(43, -107)	(43, -107)
(58, -60)	(49, -61)	(54, -63)
(66, -15)	(54, -18)	(54, -18)

Extra Slide – Simulation Results

Sweep with SNR