EE496 : COMPUTATIONAL INTELLINGENCE
EA01 : EVOLUTIONARY ALG. INTRODUCTION

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Solving optimization problems

Definition (Optimization problem)

An optimization problem $(\Omega, f, \succ)$ is given by a (search) space Ω, an evaluation function $f : \Omega \rightarrow R$, that assigns a quality assessment to all candidate solutions, as well as a (comparison) relation $\succ \; \in \{<,>\}$.

Then, the set of global optima $H \subseteq \Omega$ is defined as

$H = \{ x \in \Omega \mid \; x' \in \Omega : f(x) \succcurlyeq f(x') \}$

Note:

- global maximum: $f(x) \geq f(x')$
- global minimum: $f(x) \leq f(x')$

Given: an optimization problem $(\Omega, f, \succ)$

Wanted: an element $x \in \Omega$ which optimizes the function f in the whole search space

Fundamental approaches

The undamental approaches for solving optimization problems are:

Analytical solution:

- efficient, but rarely applicable

Exhausting exploration:

- very inefficient, so only usable in small search spaces

Random search:

- always usable, but mostly inefficient

Guided search:

- Precondition: similar elements in Ω have similar function values

Biological basics

EA are grounded on **theory of biological evolution** [Darwin, 1859].

Variation: new variants are continuously created by mutation and genetic recombination (sexual reproduction)

Inheritance : variations are genetically passed to the next generation

Fundamental principles:

- Beneficial traits (features, properties) resulting from random variation are favored by natural selection
- Better chances of reproduction of individuals with beneficial traits

Fundamental terms and meaning I

notion	biology	computer science
individual	living organism	solution candidate
chromosome	DNA-histone-protein-strand	sequence of comp. objects
	describes „construction plan“ or (some of the traits) of an individual in encoded form	
	usually multiple chromosomes per individual	usually only one chromosome per individual
gene	part of a chromosome	computational object
	is the fundamental unit of inheritance which determines a (partial) characteristic of an individual	
allele (allelomorph)	form or „value“ of gene	value of comp. object
	in each chromosome at most one form/value of a gene	
locus	position of a gene	position of comp. object
	at each position in chromosome exactly one gene	

Fundamental terms and meaning II

notion	biology	computer science
phenotype	physical appearance of a living organism	implementation of a solution candidate
genotype	genetic constitution of a living organism	encoding of a solution candidate
population	set of living organism	bag/multiset of chromosomes
generation	population at a point in time	
reproduction	creating offspring of one or multiple (usually two) (parent) organisms	creating (child) chromosomes from one or multiple (parent) chromosomes
fitness	aptitude/conformity of a living organism	aptitude/quality of a solution candidate
	determines chances of survival and reproduction	

Ingredients of an evolutionary algorithm I

Encoding for the solution candidates

- highly problem-specific
- no general rules
- attention should be paid to when choosing an encoding

A method to create an **initial population**

- commonly created by simple generation of random sequences
- depending on the chosen encoding: more complex methods needed

Evaluation function (fitness function) to evaluate the individuals

- represents environment and assess quality of individuals
- often: identical to the function to optimize
- may also contain additional elements (e.g. constraints)

Ingredients of an evolutionary algorithm II

Selection method on the basis of the fitness function

- chooses parental individuals to create offspring
- selects individuals transferred to the next generation without change

A set of **genetic operators** to modify chromosomes

- **Mutation** — randomly changes of individual genes
- **Crossover** — recombination of chromosomes
  - better: “crossing over” (meiosis-process, cell division phase)
  - chromosomes are dissipated and assembled cross-over

Various parameters (population size, mutation probability, etc.)

Ingredients of an evolutionary algorithm III

Termination criterion

- user-specified number of generations have been created
- no improvement (of the best solution candidate) for a user-specified number of generations
- user-specified minimum solution quality has been obtained

Decoding function

- For each optimization problem a different representations of solution candidates is used
- EA seperates space Ω (so called phenotype) from representation of the solution candidate in individual (so called genotype $\mathcal{G}$)
- Mutation and Recombination is defined on $\mathcal{G}$
- Fitness function f is defined on Ω
- For evaluation of fitness of an individual : the genotype representing an individual in $\mathcal{G}$ should be transformed to phenotype in Ω

Definition (Decoding function)

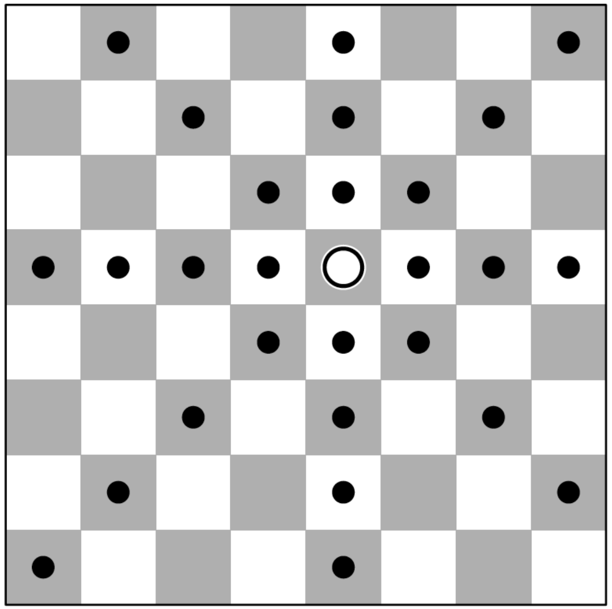
- A decoding function dec : $\mathcal{G} \rightarrow \Omega$ is a transformation of a genotype to the phenotype Ω.

[Weicker, 2007]

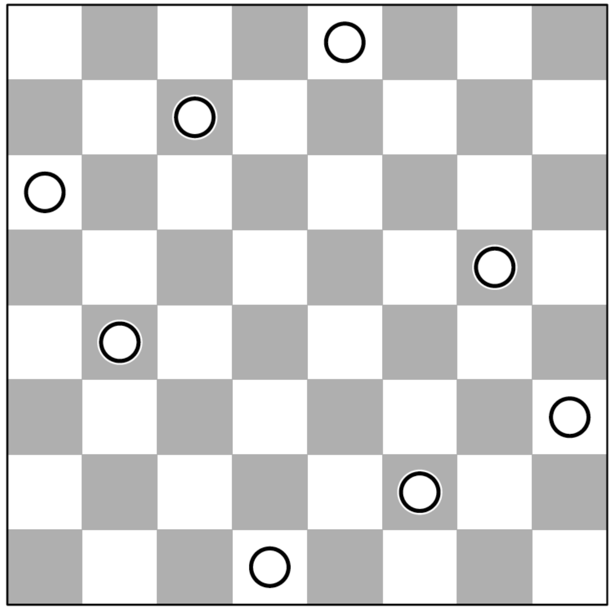
Example: The n-Queens Problem

place n queens onto a n x n chessboard in such a way that no row (rank) , no column (file) and no diagonal contains more than one queen

or: place queens in such a way that no queen is in the way of another queen



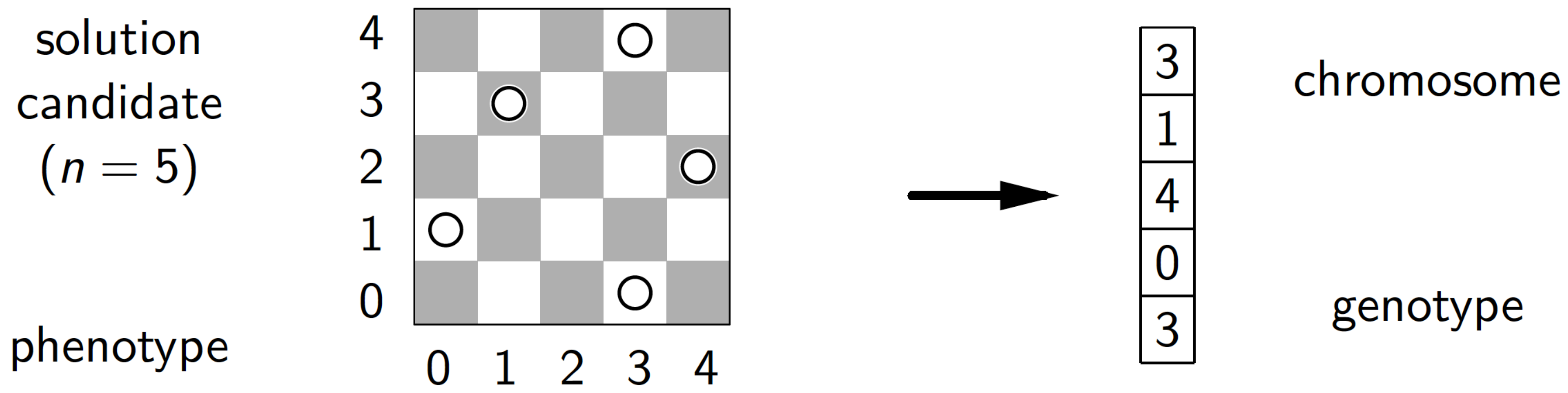
Draw options of a queen



A solution for n-queen problem

EA: Encoding

- Representation: 1 solution candidate = 1 chromosome with n genes
- Each gene: one row of the board with n possibles alleles
- value of the gene: position of the queen in corresponding row



solution candidates with more than 1 queen is not permitted for any row

⇒ smaller search space

Individual

An **individual A** contains in general:

1. genotype A.G $\in \mathcal{G}$ of an individual A
2. additional information or strategy parameters A.S $\in \mathcal{Z}$

- e.g. parameter settings for genetic operators
- space $\mathcal{Z}$ of all possible additional information
- A.S as well as A.G are modifiable by operators

3. quality or fitness A.F $\in$ R

Definition (individual)

An individual A is a tuple (A.G, A.S, A.F) containing the solution candidate (genotype A.G $\in \mathcal{G}$), the optional additional information A.S $\in \mathcal{Z}$ and the quality assessment A.F = f (dec(A.G)) $\in$ R.

genetic operators

Let ξ be a state chosen randomly within all possible states of a random number generator

Definition (genetic operators)

A **mutation operator (**which is applied on a $\mathcal{G}$-encoded optimization problem and $\mathcal{Z}$) is defined by the mapping

$$\mathrm{Mut}^{\xi} : \mathcal{G} \times \mathcal{Z} \rightarrow \mathcal{G} \times \mathcal{Z} .$$

A **recombination** operator with r ≥ 2 parents and s ≥ 1
offspring (r, s ∈ N, i.e. natural numbers) is defined by the mapping

$$\mathrm{Rec}^{\xi} : (\mathcal{G} \times \mathcal{Z})^{r} \rightarrow (\mathcal{G} \times \mathcal{Z})^{s} .$$

Selection operator

Selection operator

- Input: population of r individuals, whereas s are chosen
- selection is not creating new individuals
- selection defines indices of individuals considering their fitness

Definition (Selection operator)

A selection operator Sel is applied on a population

$P = < A^{(1)}, . . . , A^{(r)} >$:

$Sel^{\xi} : (\mathcal{G} \times \mathcal{Z} \times R)^r \rightarrow (\mathcal{G} \times \mathcal{Z} \times R)^s$

$$\left\langle A^{(i)} \right\rangle_{1 \leq i \leq r} \mapsto \left\langle A^{(\mathrm{IS}^{\xi}(c_1,\ldots,c_r)_k)} \right\rangle_{1 \leq k \leq s}$$

with $A^{(i)} = (a_i , b_i , c_i)$.

- The underlying index-selection for indivuduals has the shape

  $IS^{\xi} : R^r \rightarrow \{1, . . . , r\}^s$.

i.e. it considers the fitnesses of r individuals and selects s individuals from the set (some of them may be selected more than once)

Selection operator

Simple example for a selection operator

- Parental population consists of individuals $A^{(1)}, A^{(2)}, \ldots, A^{(5)}$
- Related quality assessments of the individuals are given by

  1. A(1).F = 2.5 *
  2. A(2).F = 1.9
  3. A(3).F = 3.7 *
  4. A(4).F = 4.1 *
  5. A(5).F = 2.4

- selection chooses with $IS^{\xi} : R^5 \rightarrow \{1, \ldots, 5\}^3$ indices 4, 3 and 1, respectively individuals $A^{(4)}, A^{(3)}$ and $A^{(1)}$

- another selection could could choose indices 4, 3 and 4 again, so respectively individuals $A^{(4)}, A^{(3)}$ and $A^{(4)}$

genotype decoding for phenot... mutation recombination parent selection Natural selection

Definition

A *simple evolutionary algorithm* on an optimization problem $(\Omega, f, \succ)$ is an 8-tuple $(\mathcal{G}, \text{dec}, \text{Mut}, \text{Rek}, \text{IS}_{\text{Parents}}, \text{IS}_{\text{Environment}}, \mu, \lambda)$. Here, μ describes the amount of individuals of the parental population and λ defines the offspring per generation. In addition, it holds

$$\text{Rek} : (\mathcal{G} \times \mathcal{Z})^k \to (\mathcal{G} \times \mathcal{Z})^{k'},$$

(for crossover k=2 (two parents) k'=2 (two offsprings))

$$\text{IS}_{\text{parents}} : \mathbb{R}^\mu \to (1, \ldots, \mu)^{\frac{k}{k'} \cdot \lambda} \qquad \text{with } \frac{k}{k'} \cdot \lambda \in \mathbb{N},$$

$$\text{IS}_{\text{Environment}} : \mathbb{R}^{\mu+\lambda} \to (\underbrace{1, \ldots, \mu + \lambda}_{})^\mu.$$

New generation

whoever fits The envirenment has higher chance to be selected

prev generation $1 \ldots \mu$ $\cup$ offsprings $\mu+1 \ldots \mu+\lambda$

Generic Evolutionary Algorithm

Algorithm: General Scheme of an Evolutionary Algorithm

Input: optimization problem (Ω, f ,$\succ$)

    t ← 0
    pop(t) ← create the initial population of size μ
    evaluate pop(t)
    while not termination criterion {
        pop $_1$ ← select parents of offsprings with size λ from pop(t)
        pop$_2$ ← create offspring by recombination of pop1
        pop3 ← mutate individuals in pop$_2$
        evaluate pop$_3$
        t ← t + 1
        pop(t) ← select μ individuals from pop3 ∪ pop(t − 1)
    }
    return best individual of pop(t)

Genetic vs. Evolutionary algorithm

Genetic algorithm:

- Encoding: Sequence of ones and zeros
  ⇒ Chromosome is Bitstring (word on alphabet {0, 1})

Evolutionary algorithm:

- Encoding: problem-related
  (Sequence of letters, graphs, formulas, etc.)
- genetic operators: defined in relation to encoding and problem