Middle East Technical University
Electrical and Electronics Engineering Department

EE430 Computer Assignment 3

Given: April 27, 2018
Due: May 12, 2018 at 23:55

In this assignment, there are questions about the polyphase decomposition, decimation, upsampling, discrete-time processing of continuous time signals, and quantization.

Before starting the homework, please read the notes at the end.

Question 1

This question is about decimation and polyphase decomposition. Consider the system shown in Fig.1 where $x[n]$ is filtered by $H(e^{jw})$ and then decimated by a factor of 3. Let $h[n]$ denote the inverse-DTFT of $H(e^{jw})$. Assume that $h[n]$ is zero outside the interval $0 \leq n \leq N-1$.

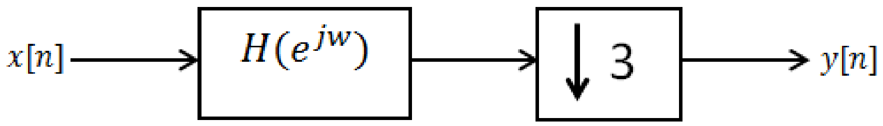


Fig.1. Discrete-time system for Question 1.

Notice that the two of every three samples of filter output are discarded by the decimator. It is therefore wasteful to compute all the outputs of the filter. The above system can be arranged for a more computational efficient structure. Consider the polyphase decomposition of $H(e^{jw})$ which is given by

$$H\left(e^{jw}\right) = \sum_{m=0}^{2} e^{-jwm} H_m(e^{j3w})$$

where $H_m(e^{jw})$ is the DTFT of the sequence $h[3n+m]$, $n = 0, \ldots, \frac{N}{3} - 1$.

Consider the following subsystem.

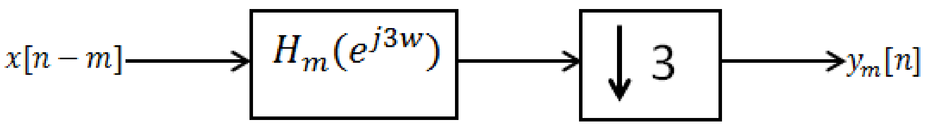


Fig.2. The mth subsystem.

We obtain

$$y[n] = \sum_{m=0}^{2} y_m[n]$$

a) Theoretically show that the following subsystem in Fig.3 is equivalent to the above subsystem in Fig.2.

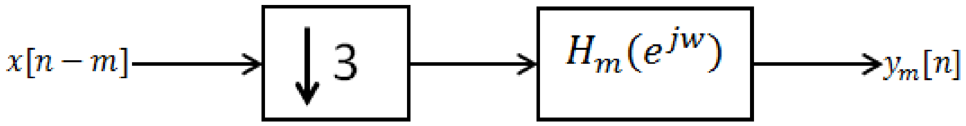


Fig.3. Equivalent mth subsystem.

b) The subsystem in Fig.3 has the advantage that discarded samples do not take place in filter computations anymore. Now, consider the following input and filter, respectively.

$$x[n] = \begin{cases} \cos(2\pi n/7) + 2\cos(2\pi n/5), & 0 \leq n < 96 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1 + (0.97)^n \cos(2\pi n/7), & 0 \leq n < 96 \\ 0, & \text{otherwise} \end{cases}$$

Compute $y[n]$ by directly implementing Fig.1. Plot $y[n]$.

c) Now, compute $y[n]$ by implementing subsystems in Fig.3. Plot $y[n]$ on the same figure you use in part (b). Check that the outputs are the same except for small numerical errors.

Question 2

This question is about upsampling and polyphase decomposition. Consider the system shown in Fig.4 where $x[n]$ is expanded by a factor of 3 and then filtered by $H(e^{jw})$. Let $h[n]$ denote the inverse-DTFT of $H(e^{jw})$. Assume that $h[n]$ is zero outside the interval $0 \leq n \leq N-1$.

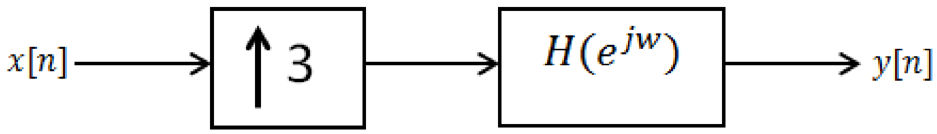


Fig.4. Discrete-time system for Question 2.

Notice that the inserted zeros by the expander go into the filter $H(e^{jw})$ and multipliers of the filter operate at a higher rate resulting in wasted computation. The above system can be arranged for a more computational efficient structure. Consider the polyphase decomposition of $H(e^{jw})$ which is given by

$$H(e^{jw}) = \sum_{m=0}^{2} e^{-jwm} H_m(e^{j3w})$$

where $H_m(e^{jw})$ is the DTFT of the sequence $h[3n+m], n = 0, \dots, \frac{N}{3} - 1$.

Consider the following subsystem.

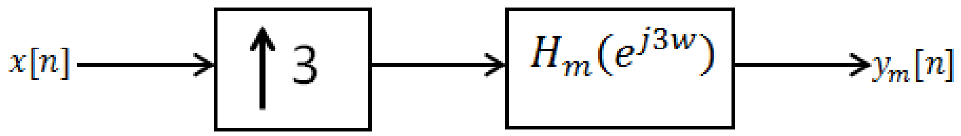


Fig.5. The mth subsystem.

We obtain

$$y[n] = \sum_{m=0}^{2} y_m[n-m]$$

a) Theoretically show that the following subsystem in Fig.6 is equivalent to the above subsystem in Fig.5.

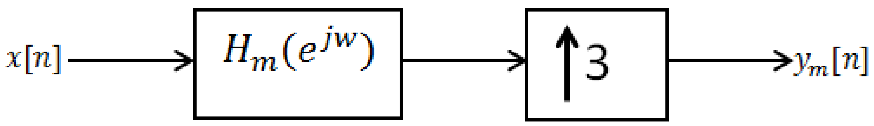


Fig.6. Equivalent mth subsystem.

b) The subsystem in Fig.6 has the advantage that zeros do not take place in filter computations and operation rate of multipliers are not increased anymore. Now, consider the following input and filter, respectively.

$$x[n] = \begin{cases} \cos(2\pi n/7) + 2\cos(2\pi n/5), & 0 \le n < 96 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 1 + (0.97)^n \cos(2\pi n/7), & 0 \le n < 96 \\ 0, & \text{otherwise} \end{cases}$$

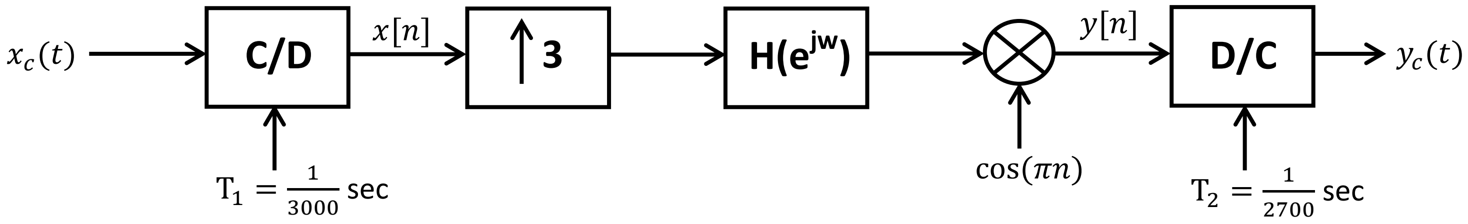
Compute $y[n]$ by directly implementing Fig.4. Plot $y[n]$.

c) Compute $y[n]$ by implementing subsystems in Fig.6. Plot $y[n]$ on the same figure you use in part (b). Check that the outputs are the same except for small numerical errors.

Question 3

This question is about discrete-time processing of continuous-time signals.

Consider the following system.



$H(e^{jw})$ is given by,

$$H(e^{j\omega}) = \begin{cases} e^{-j100w}, & |\omega| < \dfrac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} .$$

a) Theoretically find the response of the above system, i.e., $y_c(t)$ to the input $x_c(t) = \cos(1000\pi t + \frac{\pi}{3})$.

b) In the "`input_cosine.wav`" file which you can download from ODTUClass, the input signal given in part (a) is recorded with $T_1 = \frac{1}{3000}$ sec sampling period corresponding to the first block of the above system. Using "`audioread`" command in MATLAB obtain samples of it, i.e.,

```
[x, fs]=audioread('input_cosine.wav')
```

Here, the samples in "`x`" vector correspond to $x[n]$ in the above system.

c) Implement the remaining part of the system in MATLAB. In order to obtain an approximate magnitude characteristics of $H(e^{jw})$, design a 200th order optimum equiripple FIR filter using the Parks-McClellan algorithm. In order to implement the Parks-MacClellan algorithm, you can use "`firpm`" command in MATLAB as follows. "`h`" is the time domain coefficients of the filter.

```
freq=[0 2/3 2/3+0.001 1];
mag=[1 1 0 0];

h=firpm(200, freq, mag);
```

Note that $0 - \pi$ range is normalized to $0 - 1$ and $2/3$ corresponds to $\frac{2\pi}{3}$.

d) Plot the magnitude and phase characteristics of the designed filter in part (c). Note that the phase response is given by $\angle H(e^{j\omega}) = -100w$ as in our original system.

e) Using "`sound`" command, convert $y[n]$ into the continuous-time audio signal with sample frequency 2700 Hz and listen to it.

f) Generate $y[n]$ according to the theoretical result you found in part (a). Using "`sound`" command, convert it into the continuous-time audio signal with sample frequency 2700 Hz and listen to it. Compare it with part (e). Comment on the result.

Question 4

This question is about quantization.

Consider the following uniform quantizer.

$$x[n] \longrightarrow \boxed{Q(\cdot)} \longrightarrow y[n]$$

For a given number of quantization levels L, and for a given input range $[-x_{\mathrm{m}}, x_{\mathrm{m}}]$, let the quantization map be defined as

$$Q(x) = \begin{cases} b_1, & x \le a_1 \\ b_k, & a_{k-1} < x \le a_k \text{ for } k = 2, \dots, L-1 \\ b_L, & a_{L-1} < x \,. \end{cases}$$

where

$$a_k = -x_{\mathrm{m}} + \frac{2x_{\mathrm{m}}}{L} k\,, \qquad k = 0, 1, \dots, L$$

are the quantization interval boundaries and

$$b_k = -x_{\mathrm{m}} - \frac{x_m}{L} + \frac{2x_{\mathrm{m}}}{L} k\,, \qquad k = 1, \dots, L$$

are the quantization levels.

a) Download the file "`input_sound.wav`" from ODTUClass. Using "`audioread`" command obtain samples of it and sampling rate, i.e.,

```
[x, fs]=audioread('input_sound.wav')
```

b) Set x_{m} as the maximum value of the absolute value of the samples in "x" vector. Quantize the samples of audio signal using the above quantization map with $L = 2, 8, 32, 64$. Using "sound" command, convert $y[n]$ into the continuous-time audio signal with sample frequency `fs` Hz (original sampling rate of the audio signal) and listen to it for each L. Comment on the results.

c) The signal to noise ratio (SNR) of the quantizer can be estimated as

$$\widehat{\mathrm{SNR}} = 10 \log_{10}\left(\frac{\hat{\sigma}_x^2}{\hat{\sigma}_e^2}\right) \quad (\mathrm{dB})$$

where $\hat{\sigma}_x^2 = \frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2$ is the average power estimate of $x[n]$ and $\hat{\sigma}_e^2 = \frac{1}{N}\sum_{n=0}^{N-1}|e[n]|^2$ is the average power estimate of the quantization error $e[n] = Q(x[n]) - x[n]$. Here, N is the number of samples. Set $L = 2^B$ for $B = 1, 2, \dots, 8$. For each L, compute $\widehat{\mathrm{SNR}}$. Plot $\widehat{\mathrm{SNR}}$ vs B and comment on the results.

Notes

1) Returning <u>all</u> of the homework assignments and attempting each question is compulsory for this course.
2) This homework will be evaluated by Özlem Tuğfe Demir (deozlem@metu.edu.tr, E-102). Any specific question about the homework is to be addressed to her.
3) Submission of the homework will be through the ODTUClass system. Unexpected problems happen. Therefore, do not wait until the last minute to submit.
4) You will submit m-files and a word or a pdf document all zipped in a <u>single zip-file</u>. The m-files will contain working code. In addition to the m-files, provide a word or a pdf document (preferred format is pdf: you can easily convert a word file into a pdf) to give your comments, observations and other material.
5) Before submitting the homework, be sure that all the m-files work in a clear workspace.
6) Clarity and the structure of the code will also be graded. The evaluator must be able to easily read and understand what your code does. Place comments if you think they are necessary.
7) Format and appearance of your figures/(numeric outputs)/(text outputs) will also be graded. Do not forget figure titles, legends, labels, etc. Please take some time for the consideration of those issues. Do not just randomly give an unknown plot. Do not just randomly throw some unknown values to the command prompt.
8) Do not hesitate to contact Özlem Tuğfe Demir any time you need.