Middle East Technical University
Electrical and Electronics Engineering Department

EE430 Computer Assignment 1

Given: March 14, 2018
Due: March 29, 2018 at 23:55

In this assignment, there are questions about DTFT, DFT and DCT.

Before starting the homework, please read the notes at the end.

Question 1

Consider $X(e^{j\omega})$ as follows,

$$X(e^{j\omega}) = \begin{cases} 1, & |\omega| < \frac{\pi}{6} \\ 0, & \text{otherwise} \end{cases}.$$

The inverse-DTFT of $X(e^{j\omega})$ is given by $x[n] = \frac{\sin(\frac{\pi}{6}n)}{\pi n}$.

a) Plot $X(e^{j\omega})$ vs ω for $\omega \in [-\pi, \pi)$ using "`plot`" command. In order to take enough samples, generate the vector $\bar{\boldsymbol{\omega}} = [\,-\pi : \frac{2\pi}{1024} : \ \pi - \frac{2\pi}{1024}]$ and plot $X(e^{j\omega})$ versus $\bar{\boldsymbol{\omega}}$.

b) Now, generate a finite length portion of $x[n]$ as follows,

$$x_N[n] = \begin{cases} x[n], & -\frac{N}{2} \leq n < \frac{N}{2} \\ 0, & \text{otherwise} \end{cases}$$

where N is even. Note that evaluating $x[n]$ at $n = 0$ may result `NaN` in MATLAB. If you face this problem, you can correct $x[0]$ properly. Now, define $y_N[n] \triangleq x_N\left[n - \frac{N}{2}\right]$. Hence, $y_N[n]$ is nonzero for $n = 0,1, \dots, N-1$. Take the N-point DFT of $y_N[n]$, i.e., $Y_N[k]$, for $N = 8,\ 32,\ 128$. Plot $|Y_N[k]|$ versus the frequency bins in the vector $\bar{\boldsymbol{k}} = [-\pi : \frac{2\pi}{N} : \ \pi - \frac{2\pi}{N}]$ on the same graph which you used for plotting $X(e^{j\omega})$. Now, use "`stem`" command.

c) Compare $|Y_N[k]|$ to the samples of $X(e^{j\omega})$ for different N. Comment on the results considering windowing.

Question 2

Consider the sequence

$$x[n] = \begin{cases} 2, & n = 0 \\ 1, & 0 < n < 7 \\ 3, & n = 7 \\ 0, & \text{otherwise} \end{cases}$$

The DTFT of $x[n]$ is

$$X(e^{j\omega}) = 1 + 2e^{-j7w} + \frac{1 - e^{-j8w}}{1 - e^{-jw}}.$$

a) Take $N = 12$ samples of $X(e^{j\omega})$ uniformly in the interval $\omega \in [0, 2\pi)$ and place them into a vector. That is, define the vector

$$\boldsymbol{X}_{12} \triangleq [X_{12}[0] \quad X_{12}[1] \quad \dots \quad X_{12}[11]]^{\mathrm{T}}$$

where $X_{12}[k] \triangleq X(e^{j2\pi k/12})$ for $k \in \{0, 1, \dots, 11\}$. Note that evaluating $\boldsymbol{X}_{12}[0]$ may result `NaN`. Correct it properly.

What is the 12-point inverse-DFT of $\boldsymbol{X}_{12}$ in terms of the samples $x[n], n \in \mathbb{Z}$? How do the elements of $\boldsymbol{X}_{12}$ relate to the 12-point DFT of $[x[0] \quad x[1] \quad \dots \quad x[11]]^{\mathrm{T}}$?

Let the vector $\boldsymbol{x}_{12}$ be the inverse-DFT of $\boldsymbol{X}_{12}$ and check your answers and interpret. We expect the result to be real. Why? With numerical errors, you may get a small imaginary part. Be certain that your imaginary values are indeed small and use the function "`real`" to get the real part only.

b) Take $N = 8$ samples of $X(e^{j\omega})$ uniformly in the interval $\omega \in [0, 2\pi)$ and place them into a vector. That is, define the vector

$$\boldsymbol{X}_{8} \triangleq [X_{8}[0] \quad X_{8}[1] \quad \dots \quad X_{8}[7]]^{\mathrm{T}}$$

where $X_8[k] \triangleq X(e^{j2\pi k/8})$ for $k \in \{0, 1, \dots, 7\}$. What is the 8-point inverse-DFT of $\boldsymbol{X}_8$ in terms of the samples $x[n], n \in \mathbb{Z}$? Name this vector as $\boldsymbol{x}_8$. How do $\boldsymbol{x}_8$ and $\boldsymbol{x}_{12}$ compare?

c) Now take $N = 5$ samples of $X(e^{j\omega})$ uniformly in the interval $\omega \in [0, 2\pi)$ and place them into a vector. That is, define the vector

$$\boldsymbol{X}_{5} \triangleq [X_{5}[0] \quad X_{5}[1] \quad \dots \quad X_{5}[4]]^{\mathrm{T}}$$

where $X_5[k] \triangleq X(e^{j2\pi k/5})$ for $k \in \{0, 1, \dots, 4\}$. What is the 5-point inverse-DFT of $\boldsymbol{X}_5$ in terms of the samples $x[n], n \in \mathbb{Z}$? Interpret the values of this vector.

d) Now, consider the sequence $z[n] = (0.5)^n u[n]$. The DTFT of $z[n]$ is

$$Z(e^{j\omega}) = \frac{1}{1 - 0.5e^{-jw}}.$$

Generate a finite portion of $z[n]$ as follows

$$z_N[n] = \begin{cases} (0.5)^n, & 0 \le n < N \\ 0, & otherwise \end{cases}.$$

Take $N = 5$ samples of $Z(e^{j\omega})$ uniformly in the interval $\omega \in [0, 2\pi)$ and place them into a vector. That is, define the vector

$$\boldsymbol{Z}_5 \triangleq [Z_5[0] \quad Z_5[1] \quad \dots \quad Z_5[4]]^{\mathrm{T}}$$

where $Z_5[k] \triangleq Z(e^{j2\pi k/5})$ for $k \in \{0, 1, \dots, 4\}$. What is the 5-point inverse-DFT of $\boldsymbol{Z}_5$ in terms of the samples $z[n], n \in \mathbb{Z}$? Interpret the values of this vector. Can you write a closed-form expression for the inverse-DFT of $\boldsymbol{Z}_5$? If yes, write the expression for it. If no, explain the reason.

Question 3

Let the sequence $x[n]$ be defined as

$$x[n] = \begin{cases} \cos(\frac{\omega_2 - \omega_1}{2N} n^2 + \omega_1 n), & 0 \le n < N \\ 0, & \text{otherwise}. \end{cases}$$

where $x[n]$ is a chirp signal whose instantaneous frequency changes from ω_1 to ω_2 with time. Let the DFT and DCT of the vector $\boldsymbol{x} = [x[0] \quad x[1] \quad \dots \quad x[N-1]]^{\mathrm{T}}$ be denoted as $\boldsymbol{X^F} = [X_0^F \quad X_1^F \quad \dots \quad X_{N-1}^F]^{\mathrm{T}}$ and $\boldsymbol{X^C} = [X_0^C \quad X_1^C \quad \dots \quad X_{N-1}^C]^{\mathrm{T}}$, respectively.

a) Take $N = 64$, $\omega_1 = 0$, and $\omega_2 = \frac{\pi}{6}$. Plot $x[n]$.

b) Compute $\boldsymbol{X^F}$ using "`fft`" command in MATLAB. Plot the magnitude and phase of $\boldsymbol{X^F}$.

c) Compute $\boldsymbol{X^C}$ using "`dct`" command in MATLAB. Plot $\boldsymbol{X^C}$.

d) Let $\boldsymbol{X_M^F}$ and $\boldsymbol{X_M^C}$ be the vectors that are constructed by setting M least values of $\boldsymbol{X^F}$ and $\boldsymbol{X^C}$ in magnitude to zero. Assume that $x_M^F[n]$ and $x_M^C[n]$ are the sequences that are inverse transformations of $\boldsymbol{X_M^F}$ and $\boldsymbol{X_M^C}$, respectively. For $M = 21$ and $M = 51$, plot $x_M^F[n]$, $x_M^C[n]$, and $x[n]$ together. What is your observation?

e) Define the approximation error as

$$e_M \triangleq \sum_{n=0}^{N-1} |x[n] - x_M[n]|^2$$

and plot it with respect to $M = \{1,3,5,7,\dots,63\}$ for both $x_M^F[n]$ and $x_M^C[n]$ on the same figure. Comment on the result.

f) Repeat the above parts for $\omega_1 = 0$ and $\omega_2 = \frac{\pi}{2}$. Compare it with the previous part and comment on the results.

Question 4

Consider $x[n] = 2\cos\left(\frac{\pi}{14}n - \frac{\pi}{10}\right) + 3\sin\left(\frac{\pi}{5}n + \frac{\pi}{2}\right)$.

a) Obtain 140-point DFT of $x[n]$ for $n = 0,\dots,139$. Plot the magnitude and phase of DFT and explain your observations. At which frequency bins the peaks of the magnitude of DFT occur? What are the magnitudes at these frequency bins? What are the phases of DFT at these frequency bins? What are the magnitudes at the other frequency bins? Comment on your observations.

b) Obtain 139-point DFT of $x[n]$ for $n = 0,\dots,138$. Plot the magnitude and phase of DFT and explain your observations. Compare the magnitude of DFT with the one in previous part. Comment on your observations.

c) Now, consider the signal $x[n]$ you obtained in part (a) for $n = 0,\dots,139$. Obtain 142-point DFT of $x[n]$ by padding 2 zeros at the end of it. Plot the magnitude and phase of DFT and explain your observations. Compare the magnitude of DFT with the one in part (a). Comment on your observations.

d) Consider the signal $x[n]$ you obtained in part (a) for $n = 0,\dots,139$. Obtain 280-point DFT of $x[n]$ by padding 140 zeros at the end of it. Plot the magnitude and phase of DFT and explain your observations. Compare the magnitude of DFT with the one in part (a). Comment on your observations. Now, in a second figure window, plot only the odd indexed samples $(1,3,5,\dots,279)$ of magnitude of DFT which correspond to the frequency bins $k = 0, 2, 4, \dots, 278$. Is this new plot the same as the magnitude characteristics in part (a)? If yes, how can you explain this?

Notes

1) Returning <u>all</u> of the homework assignments and attempting each question is compulsory for this course.
2) This homework will be evaluated by Özlem Tuğfe Demir (deozlem@metu.edu.tr, E-102). Any specific question about the homework is to be addressed to her.
3) Submission of the homework will be through the ODTUClass system. Unexpected problems happen. Therefore, do not wait until the last minute to submit.
4) You will submit m-files and a word or a pdf document all zipped in a <u>single zip-file</u>. The m-files will contain working code. In addition to the m-files, provide a word or a pdf document (preferred format is pdf: you can easily convert a word file into a pdf) to give your comments, observations and other material.
5) Before submitting the homework, be sure that all the m-files work in a clear workspace.
6) Clarity and the structure of the code will also be graded. The evaluator must be able to easily read and understand what your code does. Place comments if you think they are necessary.
7) Format and appearance of your figures/(numeric outputs)/(text outputs) will also be graded. Do not forget figure titles, legends, labels, etc. Please take some time for the consideration of those issues. Do not just randomly give an unknown plot. Do not just randomly throw some unknown values to the command prompt.
8) Do not hesitate to contact Özlem Tuğfe Demir any time you need.