NEURAL NETWORKS
MACHINE LEARNING

ANN ee543

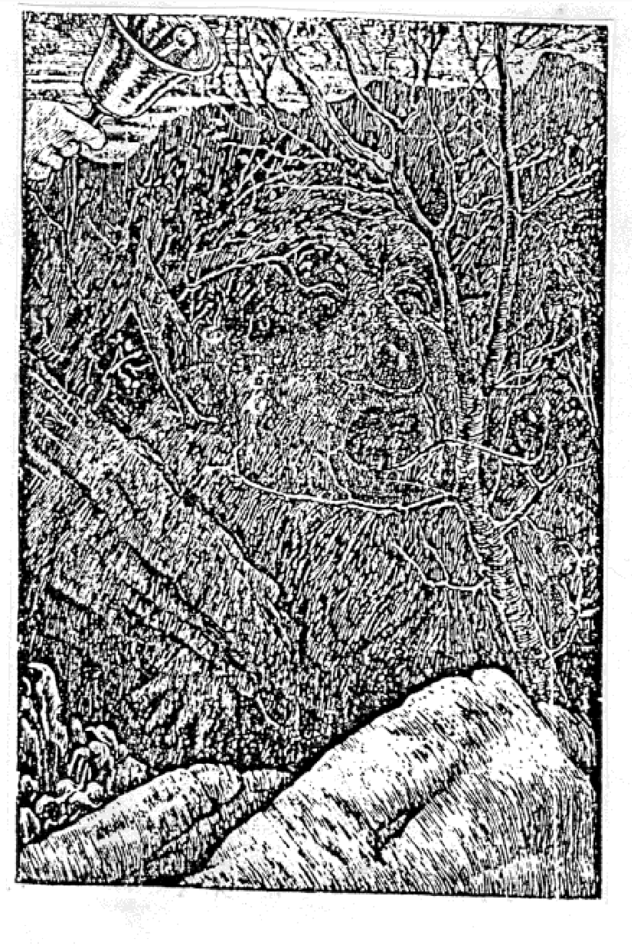
CHAPTER I

From Biological to Artificial Neuron Model

CHAPTER I : *From Biological to Artificial Neuron Model*

What you see in the picture?

CHAPTER I : *From Biological to Artificial Neuron Model*



- Is there any conventional computer at present with the capability of perceiving both the trees and Baker's transparent head in this picture at the same time?
- Most probably, the answer is no.
- Although such a visual perception is an easy task for human being, we are faced with difficulties when sequential computers are to be programmed to perform visual operations.

Conventional Computers

- In a conventional computer, usually there exist a single processor implementing a sequence of arithmetic and logical operations, nowadays at speeds about 10^9 operations per second.

- However this type of devices have ability neither to adapt their structure nor to learn in the way that human being does.

What traditional computers can't do?

- There is a large number of tasks for which it is proved to be virtually impossible to device an algorithm or sequence of arithmetic and/or logical operations.

- For example, in spite of many attempts, a machine has not yet been produced which will automatically recognize words spoken by any speaker and will translate from one language to another, or identify objects in visual scenes as human does.

CHAPTER I : *From Biological to Artificial Neuron Model*

Computers versus Brain

- What makes such a difference between brain and conventional computers seems to be neither because of the processing speed of the computers nor because of their processing ability.
    - Today's processors have a speed 10^6 times faster than the basic processing elements of the brain called ***neuron***.
    - When the abilities are compared, the neurons are much simpler.
- The difference is mainly due to the structural and operational trend.
    - While in a conventional computer the instructions are executed sequentially in a complicated and fast processor,
    - the brain is a massively_parallel interconnection of relatively simple and slow processing elements.

CHAPTER I : *From Biological to Artificial Neuron Model*

1.1. Biological Neuron Human nervous system

- It is claimed that the human central nervous system is comprised of about $1,3 \times 10^{10}$ neurons and that about 1×10^{10} of them takes place in the brain.

- At any time, some of these neurons are firing and the power dissipation due this electrical activity is estimated to be in the order of 10 watts.

- Monitoring the activity in the brain has shown that, even when asleep, 5×10^{7} nerve impulses per second are being relayed back and forth between the brain and other parts of the body. This rate is increased significantly when awake [Fischer 1987].

CHAPTER I : *From Biological to Artificial Neuron Model*

1.1. Biological Neuron Neuron

- A neuron has a roughly spherical cell body called soma (Figure 1.1).
- The signals generated in soma are transmitted to other neurons through an extension on the cell body called ***axon*** or ***nerve fibres.***
- Another kind of extensions around the cell body like bushy tree is the ***dendrites***, which are responsible from receiving the incoming signals generated by other neurons. [Noakes 92]

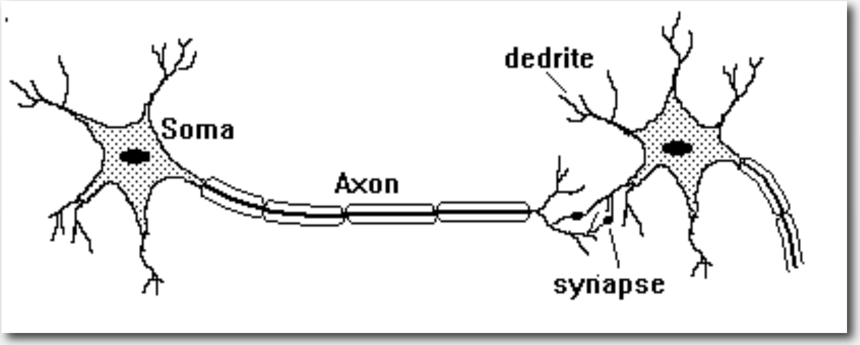


Figure 1.1. Typical Neuron

CHAPTER I : *From Biological to Artificial Neuron Model*

1.1. Biological Neuron Axon

- An axon (Figure 1.2), having a length varying from a fraction of a millimeter to a meter in human body, prolongs from the cell body at the point called ***axon hillock***.

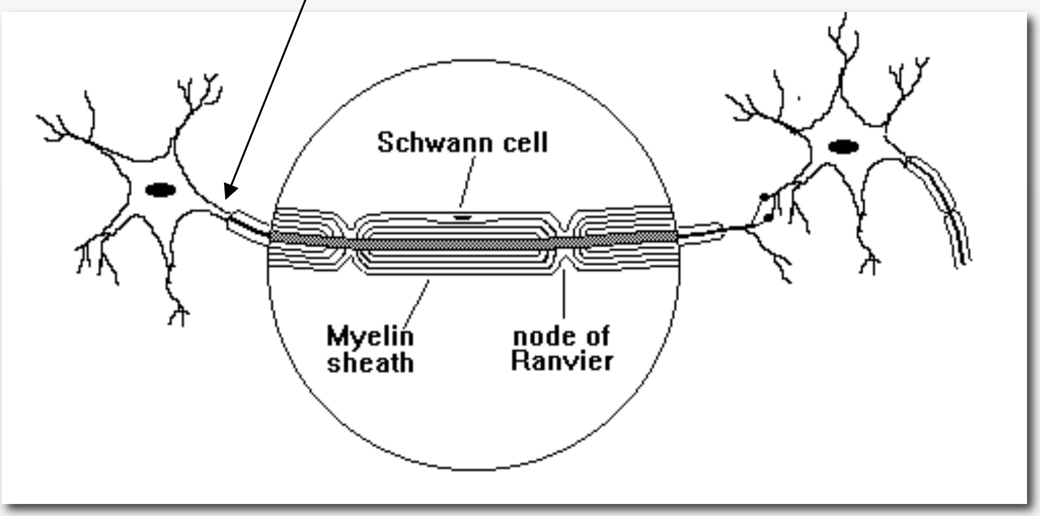


Figure 1.2. Axon

1.1. Biological Neuron Synapses

- At the other end, the axon is separated into several branches, at the very end of which the axon enlarges and forms ***terminal buttons***.
- Terminal buttons are placed in special structures called the ***synapses*** which are the junctions transmitting signals from one neuron to another (Figure 1.3).
- A neuron typically drive 10^3 to 10^4 synaptic junctions

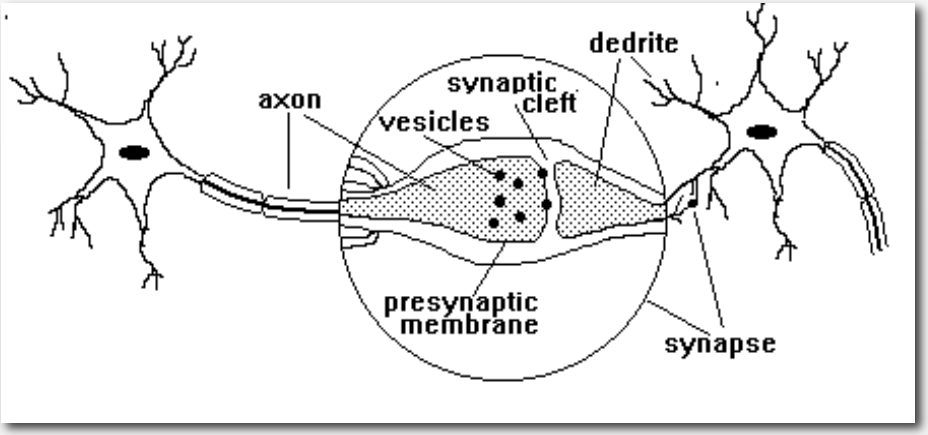
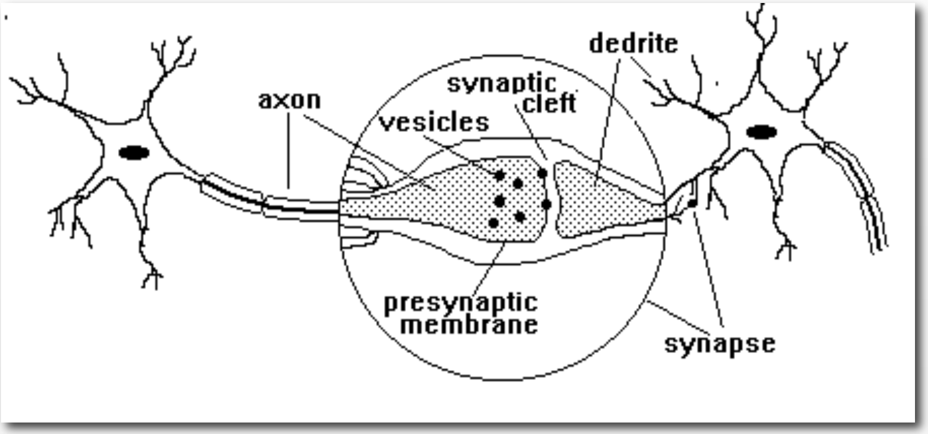


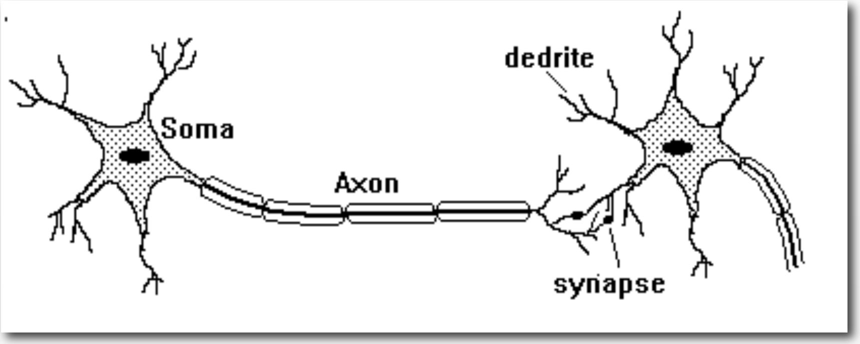
Figure 1.3. The synapse

1.1. Biological Neuron Synapses



- The ***synaptic vesicles*** holding several thousands of molecules of chemical transmitters, take place in terminal buttons.
- When a nerve impulse arrives at the synapse, some of these chemical transmitters are discharged into ***synaptic cleft***, which is the narrow gap between the terminal button of the neuron transmitting the signal and the membrane of the neuron receiving it.

1.1. Biological Neuron Synapses



- In general the synapses take place between an axon branch of a neuron and the dendrite of another one. Although it is not very common, synapses may also take place between two axons or two dendrites of different cells or between an axon and a cell body.

1.1. Biological Neuron Ion pumps

- Neurons are covered with a semi-permeable membrane, with only 5 nanometer thickness.
- The membrane is able to selectively absorb and reject ions in the intracellular fluid.
- The membrane basically acts as an ion pump to maintain a different ion concentration between the intracellular (internal) fluid and extracellular (external) fluid.
- While the sodium ions are continually removed from the internal fluid to external fluid, the potassium ions are absorbed from the external fluid in order to maintain an equilibrium condition.

1.1. Biological Neuron Resting Potential

- Due to the difference in the ion concentrations inside and outside, the cell membrane become polarized.
- In equilibrium the interior of the cell is observed to be 70 milivolts negative with respect to the outside of the cell. The mentioned potential is called the ***resting potential***.

1.1. Biological Neuron graded potential

- A neuron receives inputs from a large number of neurons via its synaptic connections.
- Nerve signals arriving at the presynaptic cell membrane cause chemical transmitters to be released in to the synaptic cleft.
- These chemical transmitters ***diffuse*** across the gap and join to the postsynaptic membrane of the receptor site. The membrane of the post-synaptic cell gathers the chemical transmitters.
- This causes either a ***decrease*** or an ***increase*** in the efficiency of the local sodium and potassium pumps depending on the type of the chemicals released in to the synaptic cleft.
- In turn, the soma potential, which is called ***graded potential***, changes.

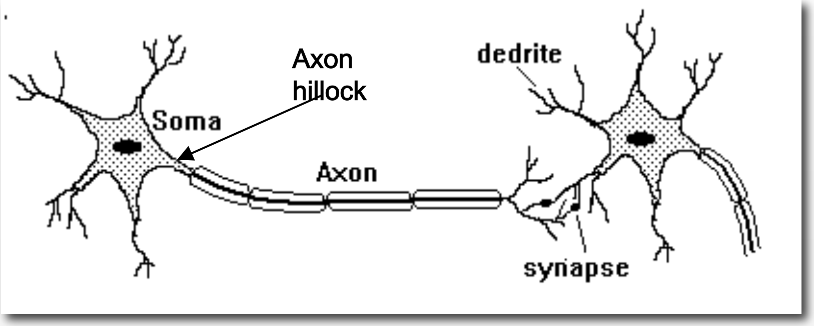
1.1. Biological Neuron Excitatory, Inhibitory synapses

- The synapses whose activation **decreasing** the efficiency of the pumps cause **depolarization** of the graded potential.
  - These are called **excitatory** synapses

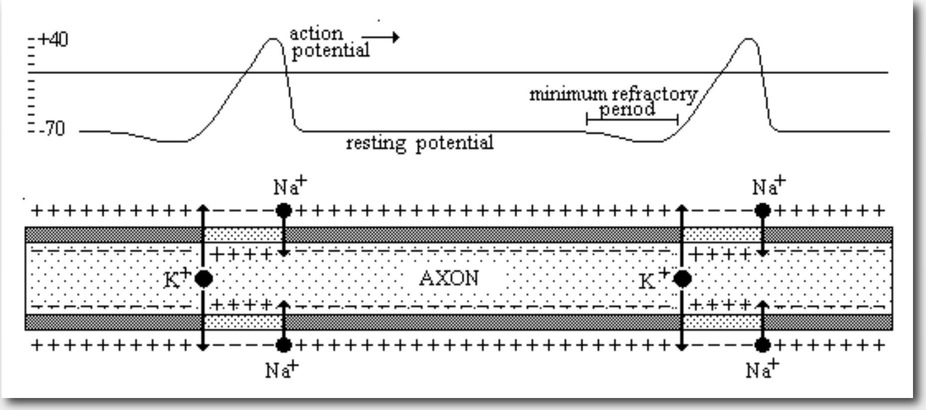
- The synapses whose activation **increasing** the efficiency of the pumps cause **depolarization** of the graded potential.
  - These are called **inhibitory** synapses

CHAPTER I : *From Biological to Artificial Neuron Model*

1.1. Biological Neuron Firing



- If the decrease in the neuron polarization is adequate to exceed a threshold at axon hillock then the neuron *fires*, i.e. generates pulses which are transmitted through axon.
- Once a pulse is created at the axon hillock, it is transmitted through the axon to other neurons.



1.1. Biological Neuron Firing

- In general, although the depolarization due to a single synapse is not enough to fire the neuron
- If some other areas of the membrane are depolarized at the same time by the arrival of nerve impulses through other synapses, it may be adequate to exceed the threshold at axon hillock and to fire.

1.1. Biological Neuron Action Potential

- At the axon hillock, the excitatory effects result in the interruption the regular ion transportation through the cell membrane,
- so that the ionic concentrations immediately begin to equalize as ions diffuse through the membrane.

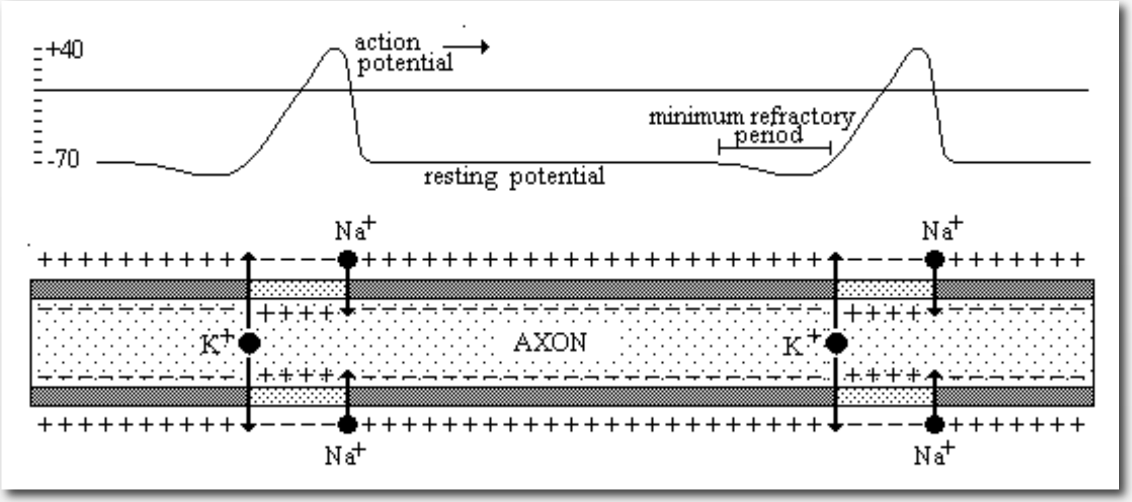
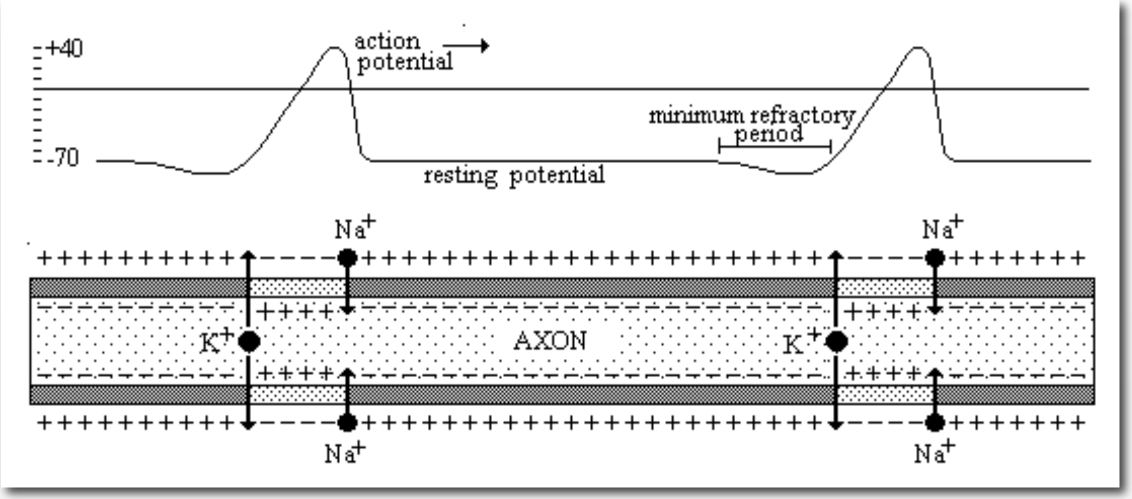


Figure 1.4. The action potential on axon

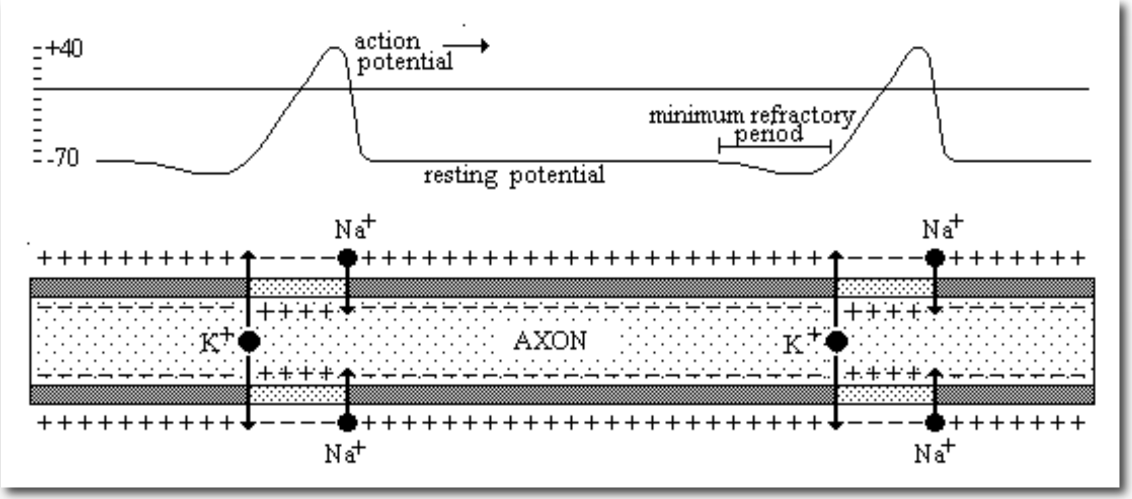
CHAPTER I : *From Biological to Artificial Neuron Model*

1.1. Biological Neuron Action Potential



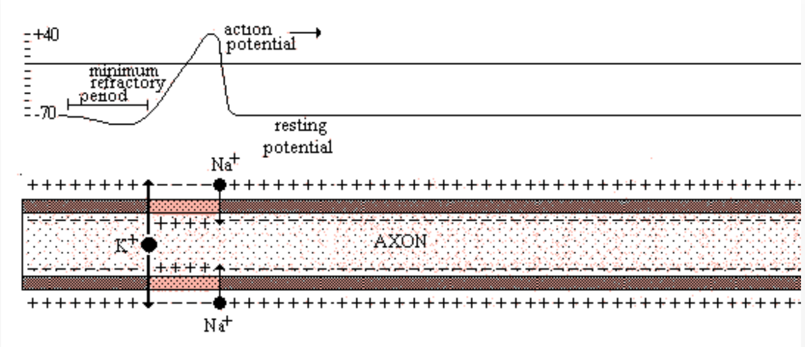
- If the depolarization is large enough, the membrane potential at axon hillock eventually collapses, and for a short period of time the internal potential becomes positive.
- The ***action potential*** is the name of this brief reversal in the potential, which results in an electric current flowing from the region at action potential to an adjacent axon region with a resting potential.

1.1. Biological Neuron Refractory Period



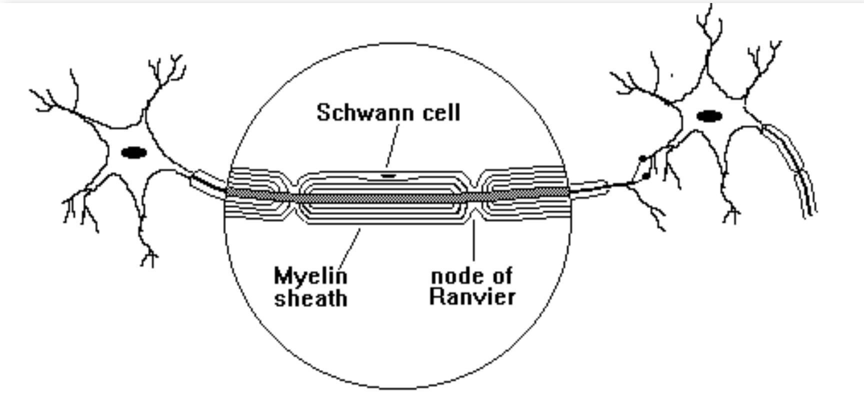
- This current causes the potential of the next resting region to change, so the effect propagates in this manner along the axon membrane wall.
- Once an action potential has passed a given point, it is incapable of being reexcited for a while called *refractory period.*

1.1. Biological Neuron Refractory Period



- Because the depolarized parts of the neuron are in a state of recovery and can not immediately become active again, the pulse of electrical activity always propagates in only forward direction.
- The previously triggered region then rapidly recovers to the polarized resting state due to the action of the sodium potassium pumps.
- The refractory period is about 1 milliseconds, and this limits the nerve pulse transmission so that a neuron can typically fire and generate nerve pulses at a rate up to 1000 pulses per second.

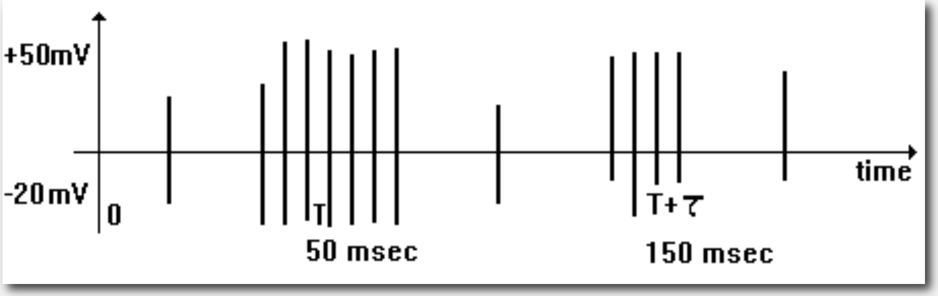
1.1. Biological Neuron Myelin sheath and nodes of Ranvier



- Most of the axons are covered by **myelin sheath** which is a **poor conductor**.
- The myelin sheath is **thin** at the points called node of **Ranvier**.
- At axons with myelin sheath, the action potential is transmitted as depolarizations occur at the nodes of Ranvier.
- This happens in a sequential manner so that the depolarization of a node triggers the depolarization of the next one.
- The nerve impulse effectively jumps from a node to the next one along the axon.

1.1. Biological Neuron The nature of signal transmission in the ns

- It is mostly tempted to conclude the signal transmission in the nervous system as having a digital nature in which a neuron is assumed to be either firing or not.
- However this conclusion is not that correct, because the intensity of a neuron signal is coded in the frequency of pulses.
- A better conclusion would be to interpret the biological neural systems as if using a form of pulse frequency modulation to transmit information.



1.2. Artificial Neuron Model

- As it is mentioned in the previous section, the **transmission of a signal** from one neuron to another through synapses is a **complex chemical process** in which specific transmitter substances are released from the sending side of the junction.
- The effect is to **raise or lower** the electrical potential inside the body of the receiving cell. If this potential **reaches a threshold**, the neuron **fires**.
- It is this characteristic that the artificial neuron model proposed by McCulloch and Pitts, [McCulloch and Pitts 1943] attempt to reproduce.

1.2. Artificial Neuron Model

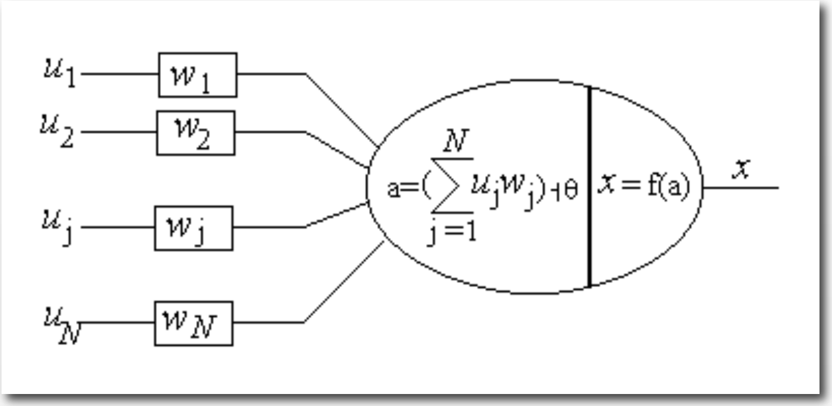


Figure 1.6. Artificial Neuron

- The neuron model shown in Figure 1.6 is the one that widely used in artificial neural networks with some minor modifications on it.
  - It has *N* **input**, denoted as $u_1, u_2, ...u_N$.
  - Each line connecting these inputs to the neuron is assigned a **weight**, which are denoted as $w_1, w_2, .., w_N$ respectively. Weights in the artificial model correspond to the synaptic connections in biological neurons.
  - θ represent ***threshold*** in artificial neuron.
  - The inputs and the weights are real values.

1.2. Artificial Neuron Model Neuron Activation

- The activation is given by the formula:

$$a = (\sum_{j=1}^{N} w_j u_j) + \theta \qquad (1.2.1)$$

- A **negative** value for a weight indicates an **inhibitory** connection while a **positive** value indicates an **excitatory** one.
- Although in biological neurons, θ has a negative value, it may be assigned a positive value in artificial neuron models. If θ is positive, it is usually referred as ***bias***. For its mathematical convenience we will use (+) sign in the activation formula.

1.2. Artificial Neuron Model Neuron Activation

- Sometimes, the threshold is combined for simplicity into the summation part by assuming an imaginary input u_0 =+1 and a connection weight $w_0 = \theta$. Hence the activation formula becomes:

$$a = \sum_{j=0}^{N} w_j u_j \qquad (1.2.2)$$

1.2. Artificial Neuron Model Neuron Activation

- The vector notation

$$a = \mathbf{w}^{\mathrm{T}}\mathbf{u} + \theta \qquad (1.2.3)$$

is useful for expressing the activation for a neuron.

- Here, the j^{th} element of the input vector $\mathbf{u}$ is u_j and the j^{th} element of the weight vector of $\mathbf{w}$ is w_j. Both of these vectors are of size N.
- Notice that, $\mathbf{w}^{\mathrm{T}}\mathbf{u}$ is the **inner product** of the **vectors** $\mathbf{w}$ and $\mathbf{u}$, resulting in a **scalar** value. The inner product is an operation defined on equal sized vectors. In the case these vectors have unit length, the inner product is a measure of similarity of these vectors.

1.2. Artificial Neuron Model Neuron output

- The **output value** of the neuron is **a function of its activation** in an analogy to the firing frequency of the biological neurons:

$$x = f(a) \tag{1.2.4}$$

- Originally the neuron output function $f(a)$ in McCulloch Pitts model proposed as threshold function, however linear, ramp and sigmoid functions (Figure 1.7.) are also widely used output functions.

1.2. Artificial Neuron Model Neuron output function

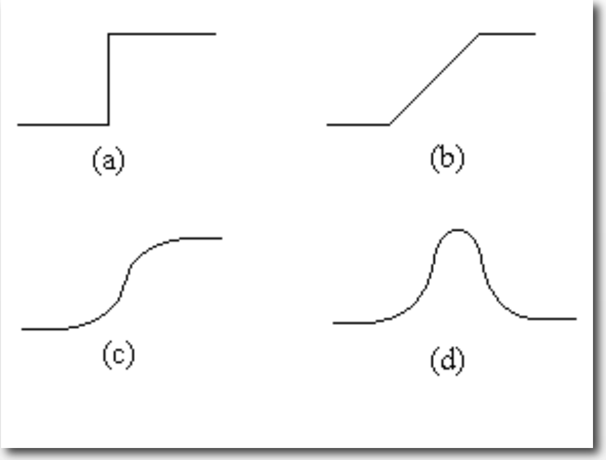


Figure 1.7. Some neuron output functions;

a) threshold function
b)ramp function,
c) sigmoid function,
d)Gaussian function

1.2. Artificial Neuron Model Neuron output

Linear: $f(a) = \kappa a$ (1.2.5)

Threshold: (1.2.6)

$$f(a) = \begin{cases} 0 & a \le 0 \\ 1 & 0 < a \end{cases}$$

Note: sign(a)=2*threshold(a)-1

Ramp: (1.2.7)

$$f(a) = \begin{cases} 0 & a \le 0 \\ a/\kappa & 0 < a \le \kappa \\ 1 & \kappa < a \end{cases}$$

Sigmoid: (1.2.8)

$$f(a) = \frac{1}{1 + e^{-\kappa a}}$$

Note: tanh(a)=2*sigmoid(a)-1

1.2. Artificial Neuron Model Implementation of lf by an

- Though its simple structure, McCulloch-Pitts neuron is a powerful computational device.
- McCulloch and Pitts proved that a synchronous **assembly of such neurons** is capable in principle to perform any computation that an ordinary digital computer can, though not necessarily so rapidly or conveniently [Hertz et al 91].

1.2. Artificial Neuron Model Example 1.1

When the threshold function is used as the neuron output function, and binary input values 0 and 1 are assumed, the basic Boolean functions AND, OR and NOT of two variables can be implemented by choosing appropriate weights and threshold values, as shown in Figure 1.8.

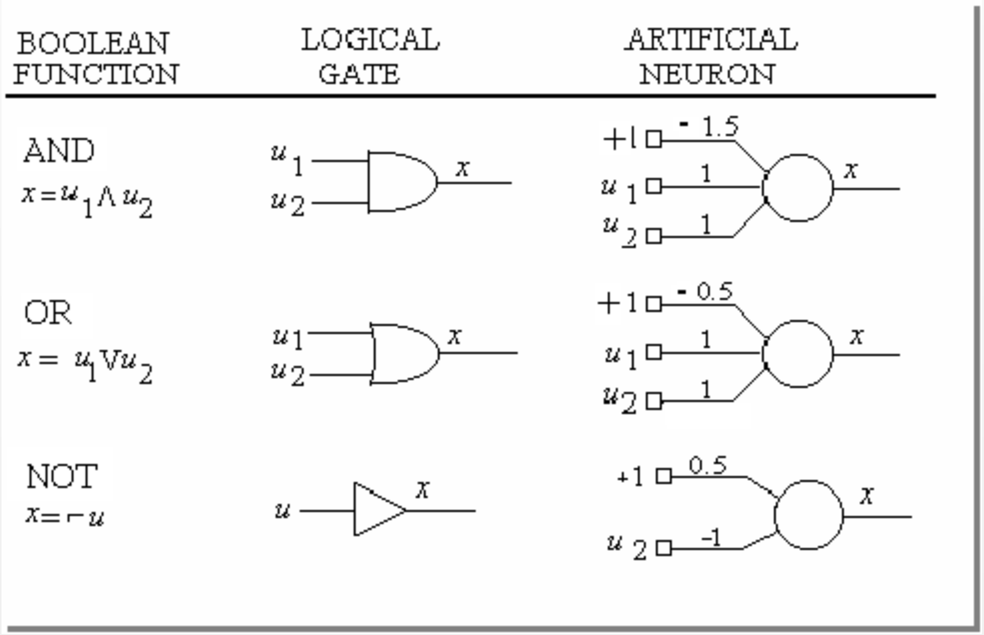


Figure 1.8. Implementation of Boolean functions by artificial neuron

1.3. Network of Neurons

- While a single artificial neuron is not able to implement some boolean functions, the problem is overcome by connecting the outputs of some neurons as input to the others, so constituting a neural network.
- Suppose that we have connected many artificial neurons that we introduced in Section 1.2 to form a network. In such a case, there are several neurons in the system, so we assign indices to the neurons to discriminate between them.

1.3. Network of Neurons

- Then to express the activation or output of i^{th} neuron, the formulas are modified as follows:

$$a_i = (\sum_{j=1}^{N} w_{ji} x_j) + \theta_i \tag{1.3.1}$$

where x_j may be either the output of a neuron determined as

$$x_j = f_j(a_j) \tag{1.3.2}$$

or an external input determined as:

$$x_j = u_j \tag{1.3.3}$$

1.3. Network of Neurons

Remember:

$$a_i = (\sum_{j=1}^{N} w_{ji} x_j) + \theta_i \qquad (1.3.1)$$

- In some applications the threshold value θ_i is determined by the external inputs.
- Due to the equation (1.3.1) sometimes it may be convenient to think all the inputs are connected to the network only through the threshold of some special neurons called the input neurons.
- They are just conveying the input value connected to their threshold as $\theta_j = u_j$ to their output x_j with a linear output transfer function $f_j(a)=a$.

1.3. Network of Neurons

- For a neural network we can define a state vector $\mathbf{x}$ in which the i^{th} component is the output of i^{th} neuron, that is x_i.

- Furthermore we define a weight matrix $\mathbf{W}$, in which the component w_{ji} is the weight of the connection from neuron j to neuron i.

- Therefore we can represent the system as:

$$\mathbf{x} = \mathbf{f}(\mathbf{W}^{\mathsf{T}}\mathbf{x} + \theta) \tag{1.3.4}$$

Here θ is the vector whose i^{th} component is θ_i and $\mathbf{f}$ is used to denote the vector function such that the function f_i is applied at the i^{th} component of the vector.

1.3. Network of Neurons

Example 1.2: A simple example often given to illustrate the behavior of a neural networks is the one used to implement the XOR (exclusive OR) function. Notice that it is not possible to obtain exclusive-or or equivalence function, by using a single neuron. However this function can be obtained when outputs of some neurons are connected as inputs to some other neurons. Such a function can be obtained in several ways, only two of them being shown in Figure 1.9.

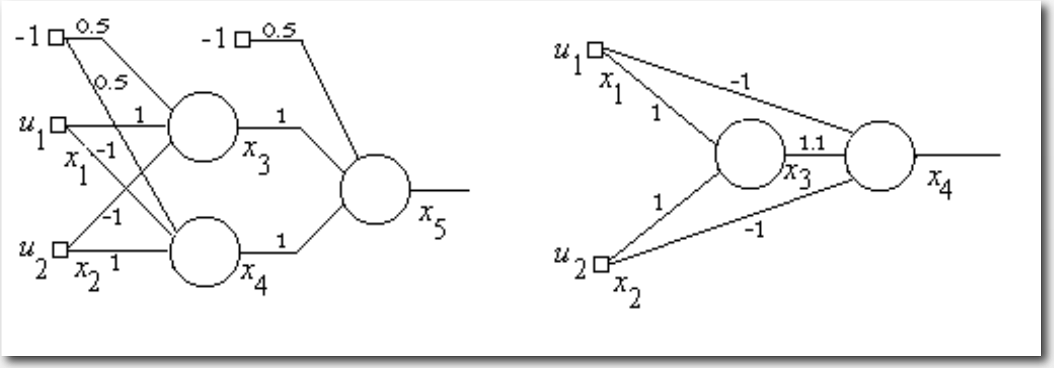
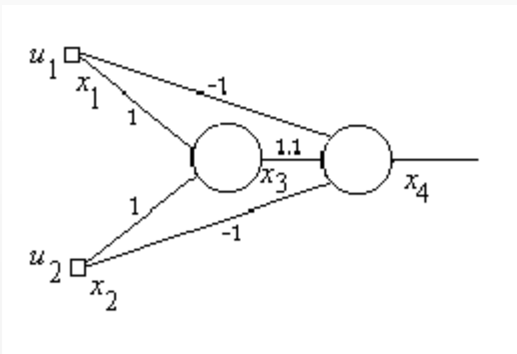


Figure 1.9. Various implementations of the exclusive-or function by using artificial neuron

1.3. Network of Neurons

The second neural network of Figure 1.9 can be represented as:



$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \mathbf{f}\left(\begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1.1 \\ 0 & 0 & 0 & 0 \end{bmatrix}^{T} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{bmatrix} \right)$$

where f_1 and f_2 being linear identity function, f_3 and f_4 being threshold functions.

In case of binary input, i.e. $u_i \in \{0,1\}$ or bipolar input, i.e. $u_i \in \{-1,1\}$, all of f_i may be chosen as threshold function.

The diagonal entries of the weight matrix are zero, since the neurons do not have self-feedback in this example.

The weight matrix is upper triangular, since the network is feedforward.

1.4. Network Architectures

- Neural computing is an alternative to programmed computing, which is a mathematical model inspired by biological models.
- This computing system is made up of a number of artificial neurons and a huge number of interconnections between them.
- According to the structure of the connections, we identify different classes of network architectures (Figure 1.10).

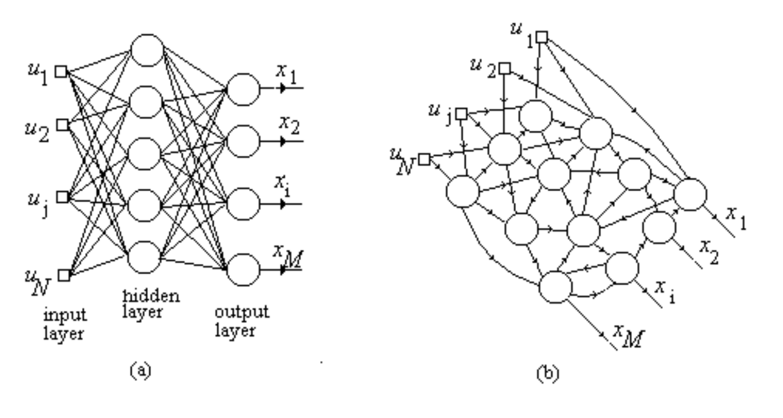
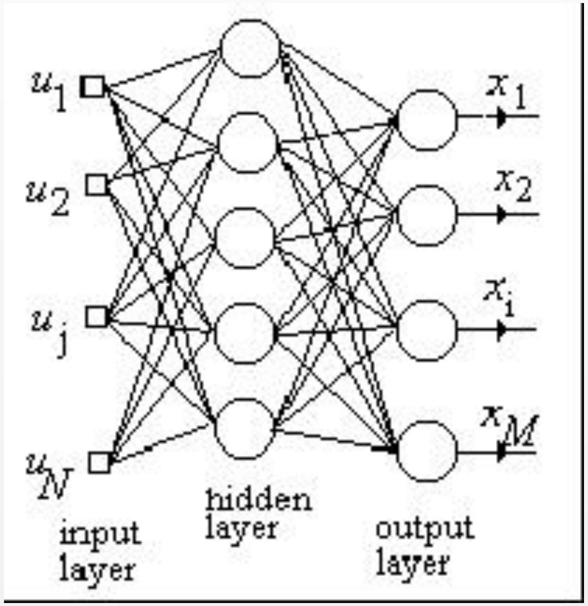


Figure 1.10 a) layered feedforward neural network
b) nonlayered recurrent neural network

CHAPTER I : *From Biological to Artificial Neuron Model*

1.4. Network Architectures *Feedforward Neural networks*

- In ***feedforward*** neural networks, the neurons are organized in the form of layers.
- The neurons in a layer get input from the previous layer and feed their output to the next layer. In this kind of networks connections to the neurons in the same or previous layers are not permitted.
- The last layer of neurons is called the ***output layer*** and the layers between the input and output layers are called the ***hidden layers***.
- The ***input layer*** is made up of special input neurons, transmitting only the applied external input to their outputs.



1.4. Network Architectures Feedforward Neural networks

- In a network, if there is only the layer of input nodes and a single layer of neurons constituting the output layer then they are called ***single layer*** network.
- If there are one or more hidden layers, such networks are called ***multilayer networks***.
- For a feed-forward network always exists an assignment of indices to neurons resulting in a triangular weight matrix.
- Furthermore the diagonal entries are zero indicating that there is no self feedback on the neurons.

1.4. Network Architectures *Recurrent Neural Networks*

- The structures, in which connections to the neurons of the same layer or to the previous layers are allowed, are called ***recurrent*** networks.

- In recurrent networks, due to feedback, it is not possible to obtain a triangular weight matrix with any assignment of the indices.