EE496 : COMPUTATIONAL INTELLIGENCE
FS01: FUZZY SYSTEMS: INTRODUCTION

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Motivation

Every day humans use imprecise linguistic terms

e.g. big, fast, about 12 o'clock, old, etc.

All complex human actions are decisions based on such concepts:

- driving and parking a car,
- financial/business decisions,
- law and justice,
- giving a lecture,
- listening to the professor/tutor.

So, these terms and the way they are processed play crucial role.

Computers need a mathematical model to express and process such complex semantics.

Concepts in classical mathematics are inadequate for such models.

Imprecision

Any notion is said to be imprecise when its meaning is not fixed by sharp boundaries.

Can be applied fully/to certain degree/not at all.

Gradualness (“membership gradience”) also called fuzziness.

Proposition is imprecise if it contains gradual predicates.

Such propositions may be neither true nor false, but in-between.

They are true to a certain degree, i.e. partial truth.

Forms of such degrees can be found in natural language, e.g. very, rather, almost not, etc.

Example I – The Sorites Paradox

sorites: a series of propositions whereby each conclusion is taken as the subject of the next

1) If a sand dune is small, adding one grain of sand to it leaves it small.
2) A sand dune with a single grain is small.

___

⟹ Hence all sand dune are small.

Paradox comes from all-or-nothing treatment of small.

Degree of truth of “heap of sand is small” decreases by adding one grain after another.

Certain number of words refer to continuous numerical scales.

Example I – The Sorites Paradox

How many grains of sand has a sand dune at least?

Statement A(n): “n grains of sand are a sand dune.”

Let $d_n = T(A(n))$ denote “degree of acceptance” for A(n).

Then

$$0 = d_0 \leq d_1 \leq . . . \leq d_n \leq . . . \leq 1$$

can be seen as truth values of **a many valued logic**.

Why is there imprecision in all languages?

Why is there imprecision?

Any language is discrete and real world is continuous!

Gap between **discrete representation** and **continuous perception**, i.e. prevalence of ambiguity in languages.

Consider the word *young*, applied to humans.
The more fine-grained the scale of age,
e.g. going from years to months, weeks, days, etc.,
the more difficult is it to fix threshold
below which young fully applies,
above which young does not at all.

Conflict between linguistic and numerical representation:
finite term set {young, mature, old},
real-valued interval [0, 140] years for humans.

Imprecision

Is there a membership threshold for imprecisely defined classes?

Consider the notion *bald*:

A man without hair on his head is bald,

a hairy man is not bald.

Usually, bald is only partly applicable.

Where to set baldness/non baldness threshold?

Fuzzy set theory does not assume any threshold!

This has consequences for the logic behind fuzzy set theory.

To be discussed in this course later.

Applications of Fuzzy Systems

Control Engineering

Approximate Reasoning

Data Analysis

Image Analysis

Advantages:

Use of imprecise or uncertain information

Use of expert knowledge

Robust nonlinear control

Time to market

Marketing aspects

Fuzzy Sets

- Membership Functions
- Fuzzy Numbers
- Linguistic Variables and Linguistic Values
- Semantics

Membership Functions

Lotfi A. Zadeh (1965)

“A fuzzy set is a class with a continuum of membership grades.”

Fuzzy set M is characterized **by membership function μ_M**.

μ_M associates real number in [0, 1] with each element $x \in X$.

Value of μ_M at x represents **grade of membership of x in M.**

Fuzzy set M is thus defined as mapping

$$\mu_M : X \rightarrow [0, 1].$$

So, μ_M generalizes traditional characteristic function

$$\chi_M : X \rightarrow \{0, 1\}.$$

Note: in **crisp** sets either $x \in X$ or $x \notin X$.

Membership Functions

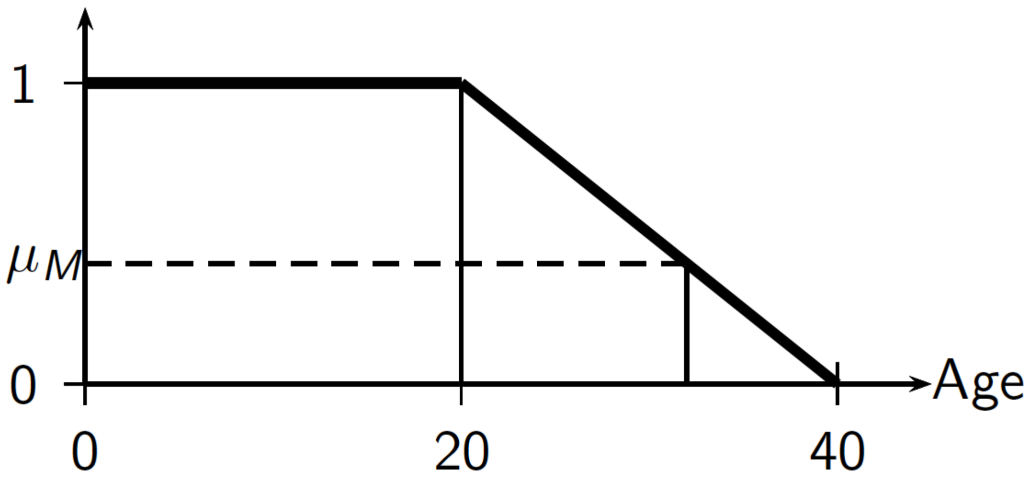
$\mu_M(u) = 1$ reflects **full membership** in M.

$\mu_M(u) = 0$ expresses **absolute non-membership** in M.

Sets can be viewed as special case of fuzzy sets where only full membership and absolute non-membership are allowed.

Such sets are called **crisp sets** or **Boolean sets**.

Membership degrees $0 < \mu_M < 1$ represent partial membership.



Representing young in “a young person”

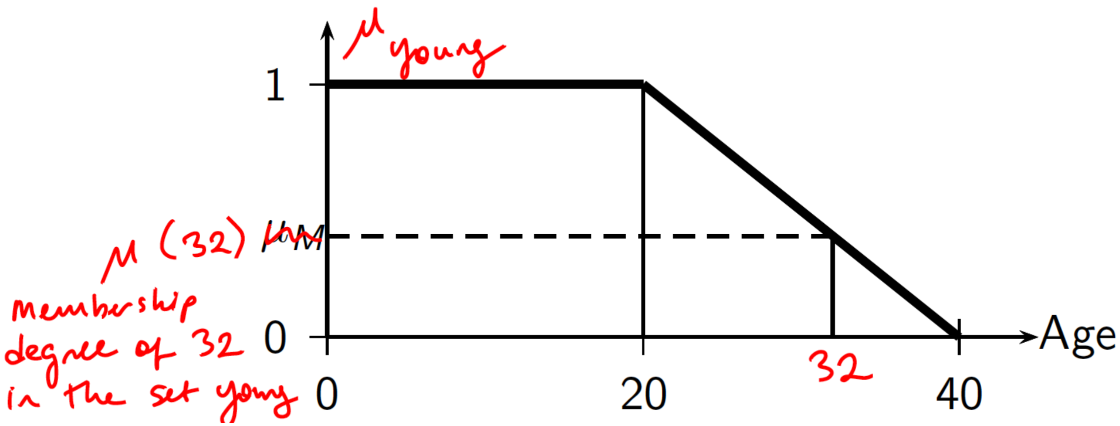
Membership Functions

Consider again representation for predicate young

There is no precise threshold between

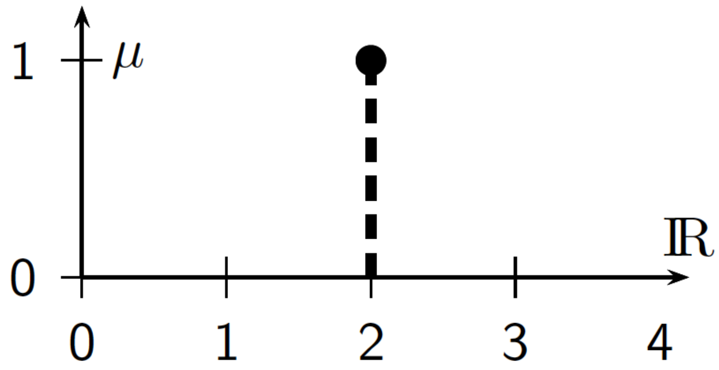
- prototypes of **young** and
- prototypes of **not young**.

Fuzzy sets offer **natural interface** between linguistic and numerical representations.

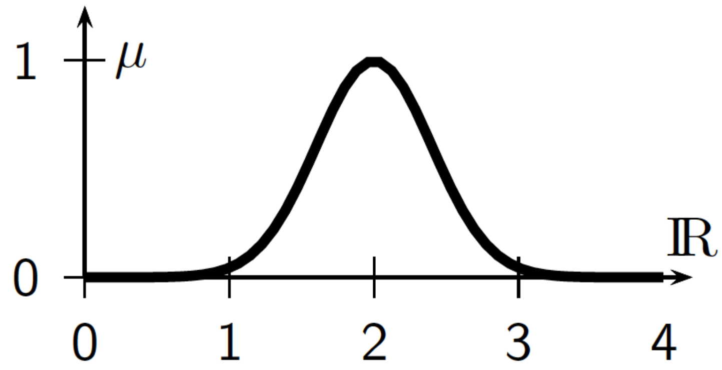


Representing young in “a young person”

Examples for Fuzzy Numbers



exactly two



around two

Exact numerical value has membership degree of 1.

Terms like **around** are modeled using triangular or Gaussian function.

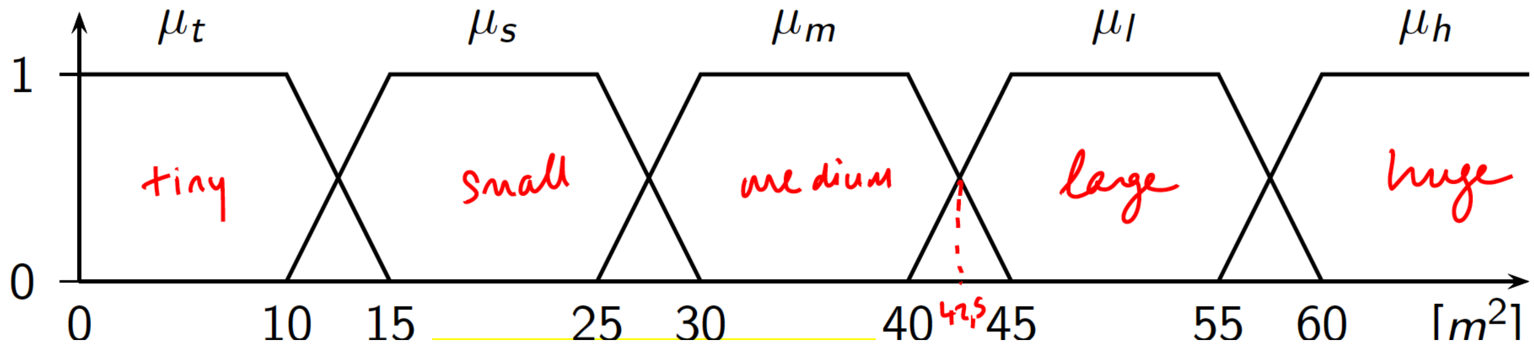
Linguistic Variables and Linguistic Values

Linguistic variables represent attributes in fuzzy systems.

They are **partitioned** into **linguistic values** (not numerical!).

Partition is usually chosen subjectively (based on human intuition).

All linguistic values have a meaning, not a precise numerical value.



Linguistic variable living area of a flat A stores linguistic values:

e.g. tiny, small, medium, large, huge

Every $x \in A$ has $\mu(x) \in [0, 1]$ to each value, e.g. let $a = 42.5m^2$.

So, $\mu_t(a) = \mu_s(a) = \mu_h(a) = 0$, $\mu_m(a) = \mu_l(a) = 0.5$.