EE496 : COMPUTATIONAL INTELLIGENCE
FS04: FUZZY CONTROL

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Fuzzy Control

The most successful aplication area of Fuzzy Systems, currently used in industry.

Special kind of non-linear table-based control method.

Definition of non-linear transition function can be made without specifying each entry individually.

Examples: technical systems

- Electrical engine moving an elevator,
- Heating installation

Goal: define certain behavior

- Engine should maintain certain number of revolutions per minute.
- Heating should guarantee certain room temperature.

Table-based Control

Control systems all share a time-dependent **output variable**:

- Revolutions per minute,
- Room temperature.

Output is controlled by *control variable*:

- Adjustment of current,
- Thermostat.

Also, **disturbance variables** influence output:

- Load of elevator, . . . ,
- Outside temperature or sunshine through a window, . . .

Table-based Control

Computation of actual value incorporates both control variable measurements of current output variable ξ and

change of output variable $\Delta\xi = d\xi/dt$.

If ξ is given in finite time intervals, then set

$$\Delta\xi(t_n+1) = \xi(t_n+1) - \xi(t_n).$$

In this case measurement of $\Delta\xi$ not necessary.

Example: Cartpole Problem

Balance an upright standing pole by moving its foot.

Lower end of pole can be moved unrestrained along horizontal axis.

Mass m at foot and mass M at head.

Influence of mass of shaft itself is negligible.

Determine force F (control variable) that is necessary to balance pole standing upright.

That is measurement of following output variables:

- angle θ of pole in relation to vertical axis,
- change of angle, i.e. triangular velocity θ' = dθ/dt .

Both should converge to zero.

Notation

Input variables $\xi_1, \ldots, \xi_n$, control variable η

Measurements: used to determine actual value of η

η may specify change of η.

Assumption: ξ_i , $1 \leq i \leq n$ is value of X_i , $\eta \in Y$

Solution: control function φ

$$\begin{aligned} \varphi : X_1 \times \ldots \times X_n &\to Y \\ (x_1, \ldots, x_n) &\to y \end{aligned}$$

Example: Cartpole Problem (cont.)

Angle θ ∈ X1 = [−90°, 90°]

Theoretically, every angle velocity θ’ possible.

Extreme θ’ are artificially achievable.

Assume −45°/s ≤ θ’ ≤ 45°/s holds,

i.e. θ’ ∈ X2 = [−45°/s, 45°/s].

Absolute value of force |F| ≤ 10N.

Thus define F ∈ Y = [−10N, 10N].

Example: Cartpole Problem (cont.)

Differential equation of cartpole problem:

$$(M+m)\sin^2\theta\cdot l\cdot\ddot{\theta}+m\cdot l\cdot\sin\theta\cos\theta\cdot\dot{\theta}^2-(M+m)\cdot g\cdot\sin\theta=-F\cdot\cos\theta$$

Compute F(t) such that θ(t) and θ’(t) converge towards zero quickly.

Physical analysis demands knowledge about physical process

Problems of Classical Approach

Often very difficult or even impossible to specify accurate mathematical model.

Description with differential equations is very complex.

Profound physical knowledge from engineer.

Exact solution can be very difficult.

Should be possible: to control process without physical-mathematical model,

e.g. human being knows how to ride bike without knowing existence of differential equations.

Fuzzy Approach

Simulate behavior of human who knows how to control.

That is a **knowledge-based** analysis.

Directly ask expert to perform analysis.

Then expert specifies knowledge as **linguistic rules**, e.g. for cartpole problem:

“If θ is approximately zero and θ’ is also approximately zero,

then F has to be approximately zero, too.”

Fuzzy Approach: Fuzzy Partitioning

1. Formulate set of linguistic rules:

Determine linguistic terms (represented by fuzzy sets).

$X_1, \dots, X_n$ and Y is partitioned into fuzzy sets.

Define p1 distinct fuzzy sets $\mu^{(1)}_1, \dots, \mu^{(1)}_{p1} \in F(X_1)$ on set X_1.

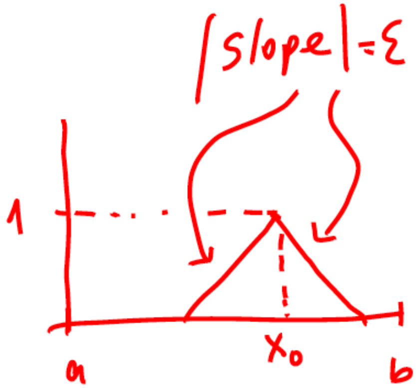
(similarly on X_i, $i=1..n$)

Associate linguistic term with each set.

Fuzzy Approach: Fuzzy Partitioning

Of set X_1 corresponds to interval [a, b] of real line,

then $\mu^{(1)}_1, \ldots, \mu^{(1)}_{p1} \in F(X_1)$ are triangular functions



$$\mu_{x0,\varepsilon} : [a, b] \to [0, 1]$$

$$x \;\&\!\!\to 1 - \min\{\varepsilon \cdot |x - x_0|, 1\}.$$

If $a < x_1 < \ldots < x_{p1} < b$, only $\mu^{(1)}_2, \ldots, \mu^{(1)}_{p1-1}$ are triangular.

Boundaries are treated differently.

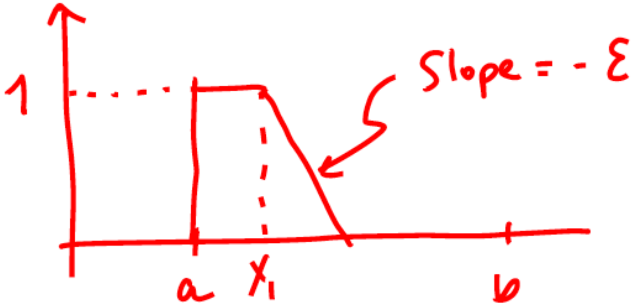
Fuzzy Approach: Fuzzy Partitioning

left fuzzy set:

$\mu^{(1)}_1 : [a, b] \rightarrow [0, 1]$

$x \rightarrow 1,$ if $x \leq x_1$

$1 - \min\{\varepsilon \cdot (x - x_1), 1\}$ otherwise

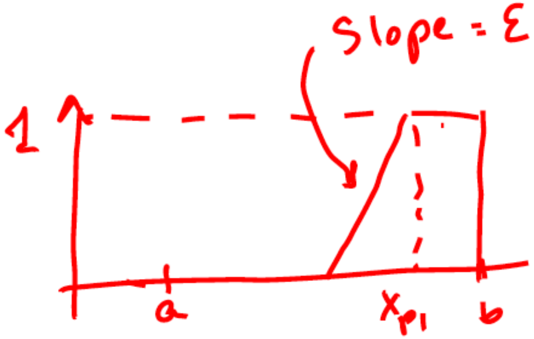


right fuzzy set:

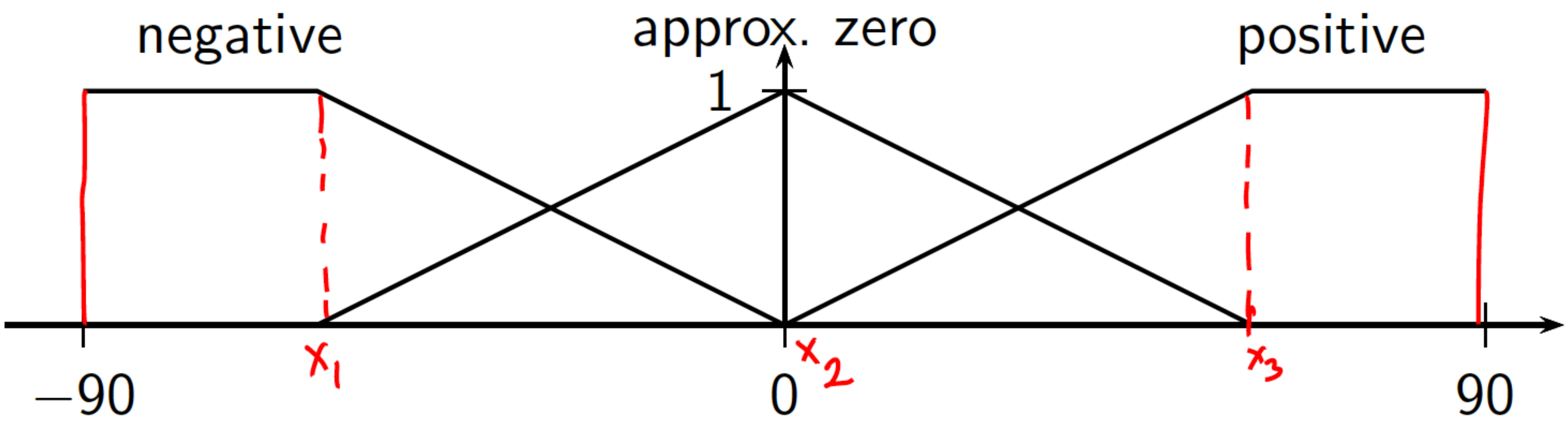
$\mu^{(1)}_{p1} : [a, b] \rightarrow [0, 1]$

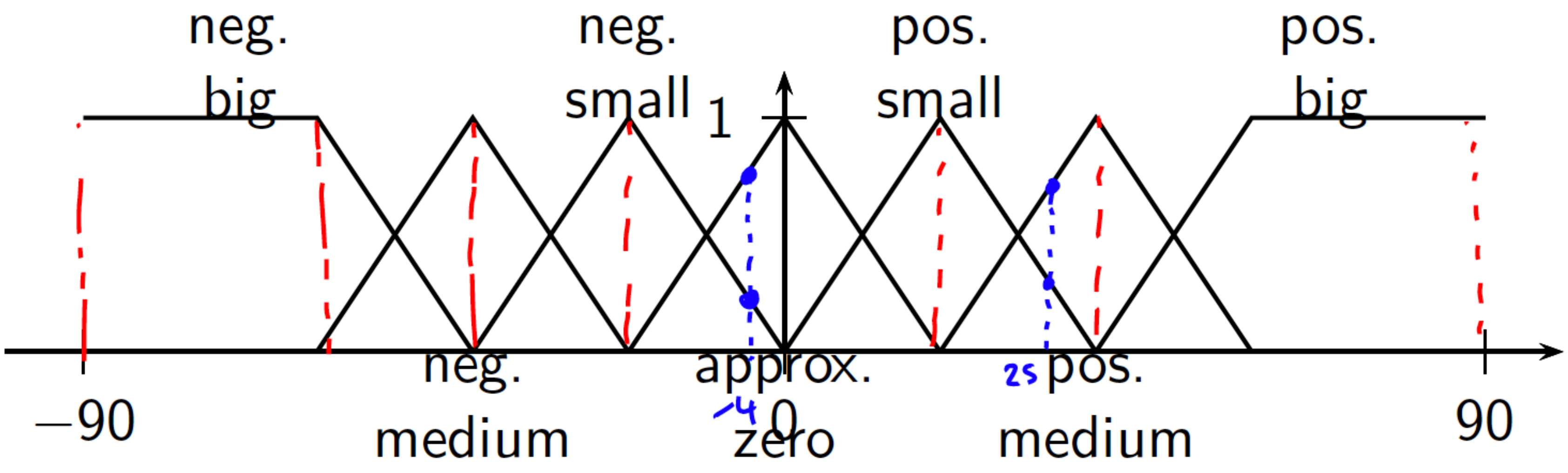
$x \rightarrow 1,$ if $x_{p1} \leq x$

$1 - \min\{\varepsilon \cdot (xp1 - x), 1\}$ otherwise



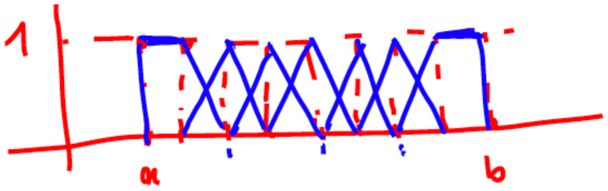
Coarse and Fine Fuzzy Partitions





Example: Cartpole Problem (cont.)

X_1 partitioned into 7 fuzzy sets.



Support of fuzzy sets: intervals with length 1/4 of whole range X_1.

Similar fuzzy partitions for X_2 and Y .

Next step: specify rules

if ξ_1 is $A^{(1)}$ and . . . and ξ_n is $A^{(n)}$ then η is B,

$A^{(1)}$, . . . ,$A^{(n)}$ and B represent linguistic terms corresponding to

$\mu^{(1)}$, . . . , $\mu^{(n)}$ and μ according to X_1, . . . ,X_n and Y .

Rule base consists of k rules.

Example: Cartpole Problem (cont.)

- 19 rules for cartpole problem,

		θ						
		nb	nm	ns	az	ps	pm	pb
$\dot{\theta}$	nb			ps	pb			
	nm				pm			
	ns	nm		ns	ps			
	az	nb	nm	ns	az	ps	pm	pb
	ps				ns	ps		pm
	pm				nm			
	pb				nb	ns		

e.g.

- If θ is approximately zero and θ’ is negative medium then F is positive medium.

Fuzzy Approach: Challenge

How to define function $\varphi : X \rightarrow Y$ that fits to rule set?

Idea:

Represent set of rules as fuzzy relation.

Specify desired table-based controller by this fuzzy relation.

Fuzzy Relation

Consider only crisp sets.

Then, solving control problem = specifying control function

$\varphi : X \to Y$.

φ corresponds to relation

$R_\varphi = \{(x,\varphi(x)) \mid x \in X\} \subseteq X \times Y$.

For measured input $x \in X$, control value

$\{\varphi(x)\} = \{x\} \circ R_\varphi$.

Fuzzy Control Rules

If temperature is very high
    and pressure is slightly low,
then heat change should be slightly negative.

If rate of descent = positive big
    and airspeed = negative big
    and glide slope = positive big,
then rpm change = positive big
    and elevator angle change = insignificant change.

Architecture of a Fuzzy Controller

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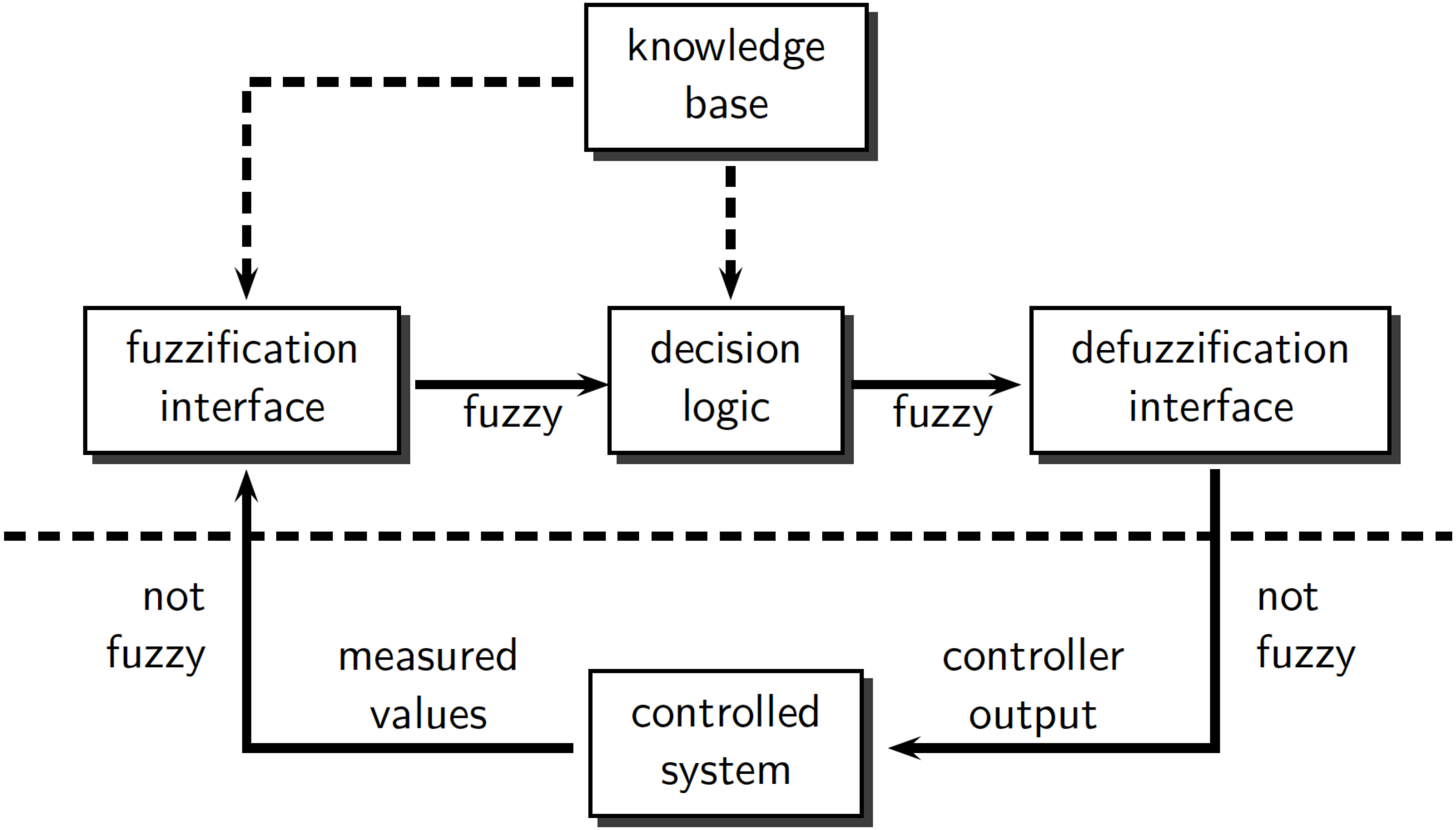

Architecture of a Fuzzy Controller

Fuzzification interface

- receives current input value (eventually maps it to suitable domain),
- converts input value into linguistic term or into fuzzy set.

Knowledge base (consists of **data base** and **rule base**)

- Data base contains information about boundaries, possible domain transformations, and fuzzy sets with corresponding linguistic terms.
- Rule base contains linguistic control rules.

Decision logic (represents processing unit)

- computes output from measured input accord. to knowledge base.

Defuzzification interface (represents processing unit)

- determines crisp output value
  (and eventually maps it back to appropriate domain).