Middle East Technical University
Electrical and Electronics Engineering Department

EE430 Computer Assignment 2

Given: April 4, 2018
Due: April 19, 2018 at 23:55

In this assignment, there are questions about the overlap-add & overlap-save methods, relation of decimation in time with DFT, sampling of continuous-time signals, downsamling and upsampling.

Before starting the homework, please read the notes at the end.

Question 1

This question is about overlap-add and overlap-save methods. You will implement the methods as described in the lectures. Consider the sequence

$$x[n] = \begin{cases} 1 + (0.97)^n \cos(2\pi n/7), & 0 \leq n < 96 \\ 0, & \text{otherwise} \end{cases}$$

together with a discrete-time LTI system represented by the impulse response

$$h[n] = \begin{cases} (-1)^n (n+2), & 0 \leq n < 4 \\ 0, & \text{otherwise} \end{cases}.$$

a) (Overlap-add) Partition this sequence into non-overlapping blocks of length $L = 12$. (If you have access to MATLAB's Signal Processing Toolbox, you may find the `buffer` function useful.) On a single figure, plot (using MATLAB's `stem`) the graph of each non-zero block using a different color.

b) Again on a single figure, plot the graph of the block outputs using a different color. Note that some samples from different colored graphs will overlap.

c) Construct the total system output using the previously computed block outputs. Plot its graph.

d) Compute the system output directly using "`conv`" and check your result in the previous part. Theoretically the two must be exactly the same. However, due to floating point arithmetic, there will be small numerical errors.

e) (Overlap-save) Now, for the overlap-save method, determine the proper overlap size and separate the sequence into overlapping blocks of length $L = 12$. (The `buffer` function is useful also here.) On a single figure, plot the graph of each non-zero block using a different color. Note that some samples from different colored graphs will overlap. In order to see the overlaps, you may, for example, use a different marker for every odd indexed block.

f) Calculate the block outputs using once circular convolution and then once DFT. Check that your results coincide. On a single figure, plot the block outputs before time aliased parts are removed.

g) Plot the graph of the block outputs after time aliased parts are removed. Note that, now, the graphs will not overlap.

h) Construct the total system output using the previously computed block outputs of overlap-save method and check your result. Plot its graph.

Question 2

This question is about the relation between decimation in time and DFT.

Let $x[n]$ be an N-point sequence which is zero outside the region $0 \leq n \leq N-1$ and define the subsequences $\{x_m[n]\}_{m=0}^{M-1}$ as

$$x_m[n] \triangleq x[Mn+m], \qquad n = 0,1,\dots,\frac{N}{M}-1, \qquad m = 0,1,\dots,M-1$$

where N is an integer multiple of M. Let $X[k]$ denote the N-point DFT coefficients of $x[n]$. Let $X_m[k]$ denote the (N/M)-point DFT coefficients of $x_m[n]$.

a) Show that $X[k]$ is given in terms of $\{X_m[k]\}_{m=0}^{M-1}$ as

$$X[k] = \sum_{m=0}^{M-1} X_m\big[((k))_{N/M}\big]e^{-\frac{j2\pi km}{N}}, \qquad k = 0,\dots,N-1.$$

where $((k))_{N/M}$ is "k *modulo* N/M".

b) Using the relation in Part (a), compute the 12-point DFT of the vector

$$\boldsymbol{x} = [7 \quad 5 \quad 2 \quad 9 \quad 1 \quad 5 \quad -4 \quad 7 \quad 2 \quad -3 \quad -1 \quad -6]^{\mathrm{T}}$$

using two 6-point DFT's.

c) Check your result in Part (b) by directly computing the 12-point DFT of the vector $\boldsymbol{x}$.

d) Now, compute the 12-point DFT of the vector in Part (b) using 4-point DFT's.

e) Check your result in Part (d) using the result of Part (c).

Question 3

Consider the following continuous-time chirp signal.

$x_c(t) = \cos\left(2\pi \frac{f_1 - f_0}{2T} t^2 + 2\pi f_0 t\right), \quad 0 \leq t \leq T$

Whose instantaneous frequency, $2\pi \frac{f_1 - f_0}{T} t + 2\pi f_0$, changes linearly from $2\pi f_0$ to $2\pi f_1$ in $0 \leq t \leq T$. Take $f_0 = 500$ Hz, $f_1 = 1000$ Hz and $T = 3$ sec.

a) Sample $x_c(t)$ with sampling period $\mathrm{T_s} = \frac{1}{4000}$ sec in $0 \text{ sec} \leq t \leq 3 \text{ sec}$ range and obtain $x_1[n] = x_c(nT_s)$. Let $X_1[k]$ denote the 12001-point DFT of $x_1[n]$. 12001 is the length of the sequence $x_1[n]$. Plot $|X_1[k]|$ vs. $k = [0{:}\,1{:}\,12000]$. Comment on the result.

b) Using "`sound`" command in MATLAB, convert $x_1[n]$ into continuous-time audio signal with sample frequency 4000 Hz and listen to it. Comment on it.

c) Using "`sound`", convert $x_1[n]$ into continuous-time audio signal with sample frequency 2000 Hz and listen to it. Compare it with part (b) and comment on it.

d) Generate the sequence $x_2[n]$ as

$$x_2[n] = \begin{cases} x_1\left[\frac{n}{4}\right], & n = 0, \pm 4, \pm 8 \ldots \\ 0 \quad , & otherwise \end{cases}$$

Using "`sound`", convert $x_2[n]$ into continuous-time audio signal with sample frequency 4000 Hz and listen to it. Compare it with part (b) and comment on it.

e) Now, design a lowpass filter with cutoff frequency $\frac{\pi}{4}$ using the following code fragment in MATLAB. Note that $0 - \pi$ range is normalized to $0 - 1$ and 0.25 corresponds to $\frac{\pi}{4}$.

```
f = [0 0.25 0.25 1];      % Frequency breakpoints
m = [4 4 0 0];            % Magnitude breakpoints
b = fir2(30,f,m);         % FIR filter design
```

Using "`conv`" command, convolve $x_2[n]$ and $b[n]$ filter sequence obtained above. Let $x_3[n]$ denote the result. Using "`sound`", convert $x_3[n]$ into continuous-time audio signal with sample frequency 4000 Hz and listen to it. Compare it with part (b) and (d). Comment on the result.

f) Using "`sound`", convert $x_3[n]$ into continuous-time audio signal with sample frequency 16000 Hz and listen to it. Compare it with part (b). Comment on the result.

g) Generate $x_4[n] \triangleq x_3[3n]$. Using "`sound`", convert $x_4[n]$ into continuous-time audio signal with sample frequency 16000 Hz and listen to it. Compare it with part (b) and (f). Comment on the result.

h) Now, using "`sound`", convert $x_4[n]$ into continuous-time audio signal with sample frequency $\frac{16000}{3}$ Hz and listen to it. Compare it with part (b) and (g). Comment on the result.

Question 4

This question is about sampling a continuous-time (CT) signal. Consider the CT signal

$$x_c(t) = e^{-\alpha t^2}\cos(\Omega_c t)$$

This signal has a Fourier transform

$$X_c(j\Omega) = \sqrt{\frac{\pi}{4\alpha}}\left(e^{-\frac{(\Omega-\Omega_c)^2}{4\alpha}} + e^{-\frac{(\Omega+\Omega_c)^2}{4\alpha}}\right).$$

a) Set $\alpha = 2$ and $\Omega_c = 20\pi$ rad/s. Sample $x_c(t-0.75)$ in $0 \le t \le 1.5$ sec range with sampling period $\mathrm{T} = \frac{1}{50}$ sec and obtain $x_1[n] = x_c(nT - 0.75)$. Note that the first ($n = 0$) and last ($n = 75$) samples of $x_1[n]$ correspond to $x_c(-0.75)$ and $x_c(0.75)$, respectively. In addition, obtain $x_2[n] = x_c(nT - 2)$ by sampling $x_c(t-2)$ in $0 \le t \le 4$ sec range.

b) Plot $X_c(j\Omega)$ vs. $\Omega = [-50\pi{:}\frac{\pi}{1000}{:}50\pi]$. Here the spacing between consecutive frequencies is small enough such that we can easily visualize what the actual continuous function $X_c(j\Omega)$ looks like. Let $X_1[k]$ and $X_2[k]$ denote the 5001-point DFT of $x_1[n]$ and $x_2[n]$, respectively. Plot $T|X_1[k]|$ and $T|X_2[k]|$ vs. $\Omega = \left[-50\pi{:}\frac{\pi}{50}{:}50\pi\right]$ on the same figure you plot $X_c(j\Omega)$. Compare them and comment on the results. (Note that 5001 is the length of $\left[-50\pi{:}\frac{\pi}{50}{:}50\pi\right]$. Hint: You may need "`fftshift`" command in MATLAB.)

c) Now, sample $x_c(t-2)$ with sampling period $\mathrm{T} = \frac{1}{150}$ sec in $0 \le t \le 4$ sec range and obtain $x_3[n] = x_c(nT-2)$. Let $\hat{X}_2[k]$ and $\hat{X}_3[k]$ denote the 1024-point DFT of $x_2[n]$

and $x_3[n]$. Plot $|\hat{X}_2[k]|$ and $|\hat{X}_3[k]|$ vs. $k = [0:1:1023]$. Compare them and comment on your results.

d) Generate the sequence $x_4[n]$ as

$$x_4[n] = \begin{cases} x_2\left[\frac{n}{3}\right], & n = 0, \pm 3, \pm 6, \ldots \\ 0 \quad , & otherwise \end{cases}$$

Let $\hat{X}_4[k]$ denote the 1024-point DFT of $x_4[n]$. Plot $|\hat{X}_4[k]|$ vs. $k = [0:1:1023]$. Compare it with the previous graph in part (c). Comment on the result.

e) Let us define $x_5[n] \triangleq x_2[5n]$. Let $\hat{X}_5[k]$ denote the 1024-point DFT of $x_5[n]$. Plot $|\hat{X}_5[k]|$ vs. $k = [0:1:1023]$. . Compare it with the previous graph in part (c). Comment on the result. Can you see two pulses?

f) Sample $x_c(t)$ with sampling period $\mathrm{T} = \frac{1}{30}$sec in $-2 \text{ sec} \leq t \leq 2 \text{ sec}$ range and obtain $x_6[n] = x_c(nT)$. Use the following approximation to sinc interpolation, i.e.,

$$x_r(t) = \sum_{n=-60}^{60} x_6[n] \operatorname{sinc}\left(\frac{t}{T} - n\right).$$

Plot $x_R(t)$ and $x_c(t)$ vs. $t = \left[-2:\frac{1}{1000}:2\right]$. Compare them and comment on your results.

g) Repeat part (f) for $x_c(t) = e^{-2t^2} \cos(40\pi t)$. Compare the results with part (f).

Notes

1) Returning all of the homework assignments and attempting each question is compulsory for this course.
2) This homework will be evaluated by Özlem Tuğfe Demir (deozlem@metu.edu.tr, E-102). Any specific question about the homework is to be addressed to her.
3) Submission of the homework will be through the ODTUClass system. Unexpected problems happen. Therefore, do not wait until the last minute to submit.
4) You will submit m-files and a word or a pdf document all zipped in a single zip-file. The m-files will contain working code. In addition to the m-files, provide a word or a pdf document (preferred format is pdf: you can easily convert a word file into a pdf) to give your comments, observations and other material.
5) Before submitting the homework, be sure that all the m-files work in a clear workspace.

6) Clarity and the structure of the code will also be graded. The evaluator must be able to easily read and understand what your code does. Place comments if you think they are necessary.

7) Format and appearance of your figures/(numeric outputs)/(text outputs) will also be graded. Do not forget figure titles, legends, labels, etc. Please take some time for the consideration of those issues. Do not just randomly give an unknown plot. Do not just randomly throw some unknown values to the command prompt.

8) Do not hesitate to contact Özlem Tuğfe Demir any time you need.