Middle East Technical University
Electrical and Electronics Engineering Department

# EE430 Computer Assignment 4

*Given: May 21, 2018*
*Due: June 5, 2018 at 23:55*

In this assignment, there are questions about the generalized-linear phase systems, structures for discrete-time systems, amplitude and phase equalization of non-minimum phase systems, and the properties of minimum phase systems.

Before starting the homework, please read the notes at the end.

## Question 1

**a)** Consider the filter whose z-transform is given by

$$H_1(z) = 1 + 2z^{-1} - 4z^{-2} + 3z^{-3} + z^{-4} + 3z^{-5} - 4z^{-6} + 2z^{-7} + z^{-8}.$$

Plot the magnitude and phase response of this filter. Find its group delay analytically. Check your answer by plotting the group delay using "`grpdelay`" command in MATLAB. Is it a linear phase filter? If your answer is yes, specify its type. Obtain the pole-zero plot of this filter using "`zplane`" command. Comment on the placement of the zeros.

**b)** Repeat part (a) for the filter whose z-transform is given by

$$H_2(z) = 1 + 2z^{-1} - 4z^{-2} + 3z^{-3} - 3z^{-5} + 4z^{-6} - 2z^{-7} - z^{-8}.$$

**c)** Repeat part (a) for the filter whose z-transform is given by

$$H_3(z) = 1 + 2z^{-1} - 4z^{-2} + 3z^{-3} - 3z^{-4} + 4z^{-5} - 2z^{-6} - z^{-7}.$$

**d)** Repeat part (a) for the filter whose z-transform is given by

$$H_4(z) = 1 + 2z^{-1} - 4z^{-2} + 3z^{-3} + 3z^{-4} - 4z^{-5} + 2z^{-6} + z^{-7}.$$

.

## Question 2

Consider the input sequence

$$x[n] = \begin{cases} \cos\left(\frac{\pi}{5}n\right) + 2\cos\left(\frac{\pi}{2}n\right), & 0 \le n < 64 \\ 0, & otherwise \end{cases}.$$

Consider the system function

$$H(z) = \frac{(1 - 0.4z^{-1})(1 + 0.7z^{-1})}{(1 - 0.2z^{-1})(1 - 0.7z^{-1})}.$$

**a)** Find the first 64 point of the sequence which is the output of the above system for the input $x[n]$ using "`filter`" command in MATLAB. Plot the result.

**b)** Find the first 64 point of the output sequence using Direct Form 1 structure without any built-in functions of MATLAB. Plot the result. Verify that the result is the same as the one in part (a).

**c)** Find the first 64 point of the output sequence using Direct Form 2 structure without any built-in functions of MATLAB. Plot the result. Verify that the result is the same as the one in part (a).

**d)** Find the first 64 point of the output sequence using Cascade structure of two subsystems $\frac{(1-0.4z^{-1})}{(1-0.2z^{-1})}$ and $\frac{(1+0.7z^{-1})}{(1-0.7z^{-1})}$ with Direct Form 2 implementation without any built-in functions of MATLAB. Plot the result. Verify that the result is the same as the one in part (a).

**e)** Find the first 64 point of the output sequence using Parallel structure without any built-in functions of MATLAB. Plot the result. Verify that the result is the same as the one in part (a).

## Question 3

**a)** Generate a 100 Hz square wave, $x[n]$, with sample rate 1000 Hz in [0-0.05] sec range using the following MATLAB code fragment.

```
>> t=0:0.001:0.05;
>> x=square(2*pi*100*t);
```

Plot $x[n]$. Plot the magnitude and phase characteristics of $x[n]$ using "`freqz`" command.

**b)** Now, $x[n]$ will be distorted by an undesirable non-minimum phase system. The z-transform of this system is given by

$$H(z) = (1 - 0.8e^{j0.1\pi}z^{-1})(1 - 0.8e^{-j0.1\pi}z^{-1})(1 - 1.5e^{j0.3\pi}z^{-1})(1 - 1.5e^{-j0.3\pi}z^{-1})$$

Using "`filter`" command, obtain the output of this system when the input is $x[n]$. Denote the output sequence by $z[n]$. Note that the length of $z[n]$ should be the same as that of $x[n]$ due to "`filter`" command.

Plot $z[n]$. Comment on the distortion introduced by the above system. Plot the magnitude and phase characteristics of $z[n]$ using "`freqz`" command.

**c)** Find a minimum-phase compensating system, $G(z)$ such that the magnitude characteristics of $x[n]$ and the output of compensating system will be the same. Obtain the output of the compensating system when the input is $z[n]$ using "`filter`" command. Denote the result by $v[n]$.

Plot $v[n]$. Compare it with $x[n]$ and $z[n]$. Plot the magnitude and phase characteristics of $v[n]$ using "`freqz`" command. Compare them with those in part (a).

**d)** In part (c), we realized amplitude equalization. In this part, we will try to equalize the phase distorted by the overall system $H(z)G(z)$. First, obtain the 1000 length group delay function of $H(z)G(z)$ using the following code fragment. `B` and `A` are the vectors whose elements are the numerator and denominator coefficients of $H(z)G(z)$.

```
>> [Gd,W] = grpdelay(B,A,1000);
```

Plot `Gd` versus `W`.

In order to compensate phase distortion of the overall system, we will use "`iirgrpdelay`" command which designs the optimal allpass filter for the given group delay specifications. We will give the desired group delay as "`max(Gd)-Gd`" in order to compensate group delay distortion. Use the following code fragment to design phase equalizer filter.

```
>>[num,den]=iirgrpdelay(8,     W/pi,     [0     W(end)/pi],
max(Gd)-Gd);
```

Denote the designed filter by $T(z)$. Plot the group delay of the overall system function, $H(z)G(z)T(z)$ using "`grpdelay`" command.

Using "`filter`" command, obtain the output of phase equalizer. Denote the result by $y[n]$. Plot $y[n]$. Compare it with $x[n]$, $z[n]$, and $v[n]$ . Plot the magnitude and phase characteristics of $y[n]$ using "`freqz`" command. Compare them with those in part (a).

## Question 4

Consider the systems whose z-transforms are given by

$$H_1(z) = \left(1 - \frac{7}{10}e^{j0.4\pi}z^{-1}\right)\left(1 - \frac{7}{10}e^{-j0.4\pi}z^{-1}\right)\left(1 - \frac{3}{10}e^{j0.8\pi}z^{-1}\right)\left(1 - \frac{3}{10}e^{-j0.8\pi}z^{-1}\right)$$

$$H_2(z) = \left(1 - \frac{7}{10}e^{j0.4\pi}z^{-1}\right)\left(1 - \frac{7}{10}e^{-j0.4\pi}z^{-1}\right)\left(1 - \frac{10}{3}e^{j0.8\pi}z^{-1}\right)\left(1 - \frac{10}{3}e^{-j0.8\pi}z^{-1}\right)$$

$$H_3(z) = \left(1 - \frac{10}{7}e^{j0.4\pi}z^{-1}\right)\left(1 - \frac{10}{7}e^{-j0.4\pi}z^{-1}\right)\left(1 - \frac{3}{10}e^{j0.8\pi}z^{-1}\right)\left(1 - \frac{3}{10}e^{-j0.8\pi}z^{-1}\right)$$

$$H_4(z) = \left(1 - \frac{10}{7}e^{j0.4\pi}z^{-1}\right)\left(1 - \frac{10}{7}e^{-j0.4\pi}z^{-1}\right)\left(1 - \frac{10}{3}e^{j0.8\pi}z^{-1}\right)\left(1 - \frac{10}{3}e^{-j0.8\pi}z^{-1}\right)$$

**a)** Plot the pole-zero diagram of $H_1(z)$, $H_2(z)$, $H_3(z)$, and $H_4(z)$ using "`zplane`" command in MATLAB.

**b)** Find the scaling factors, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, such that the system-functions $G_1(z) = \alpha_1 H_1(z)$, $G_2(z) = \alpha_2 H_2(z)$, $G_3(z) = \alpha_3 H_3(z)$, and $G_4(z) = \alpha_4 H_4(z)$ have the same magnitude characteristics.

**c)** Denote the inverse z-transform of $G_1(z), G_2(z), G_3(z)$, and $G_4(z)$ by $g_1[n], g_2[n], g_3[n]$, and $g_4[n]$, respectively. Plot $g_1[n]$, $g_2[n]$, $g_3[n]$, and $g_4[n]$.

**d)** Obtain the 256-point DFT's of $g_1[n]$, $g_2[n]$, $g_3[n]$, and $g_4[n]$. Plot the magnitude of DFT's on the same figure with different colors. Comment on the results.

**e)** Plot the phase of DFT's on the same figure with different colors Can you differentiate the minimum-phase system by just looking at the phase spectrums? Explain.

**f)** Plot the group delays of the system functions $G_1(z), G_2(z), G_3(z)$, and $G_4(z)$ on the same figure with different colors. You can use "`grpdelay`" command in MATLAB. Comment on the results. Can you differentiate the minimum-phase system by just looking at the group delays? Explain.

**g)** Partial energy sequence of an impulse response $g[n]$ is defined as

$$PE_g[n] = \sum_{k=0}^{n} |g[k]|^2.$$

Plot the partial energy sequences of $g_1[n]$, $g_2[n]$, $g_3[n]$, and $g_4[n]$ on the same figure with different colors. Comment on the results. Can you differentiate the minimum-phase system by just looking at the graph? Explain.

## Notes

**1)** Returning <u>all</u> of the homework assignments and attempting each question is compulsory for this course.

**2)** This homework will be evaluated by Özlem Tuğfe Demir (deozlem@metu.edu.tr, E-102). Any specific question about the homework is to be addressed to her.

**3)** Submission of the homework will be through the ODTUClass system. Unexpected problems happen. Therefore, do not wait until the last minute to submit.

**4)** You will submit m-files and a word or a pdf document all zipped in a <u>single zip-file</u>. The m-files will contain working code. In addition to the m-files, provide a word or a pdf document (preferred format is pdf: you can easily convert a word file into a pdf) to give your comments, observations and other material.

**5)** Before submitting the homework, be sure that all the m-files work in a clear workspace.

**6)** Clarity and the structure of the code will also be graded. The evaluator must be able to easily read and understand what your code does. Place comments if you think they are necessary.

**7)** Format and appearance of your figures/(numeric outputs)/(text outputs) will also be graded. Do not forget figure titles, legends, labels, etc. Please take some time for the consideration of those issues. Do not just randomly give an unknown plot. Do not just randomly throw some unknown values to the command prompt.

**8)** Do not hesitate to contact Özlem Tuğfe Demir any time you need.