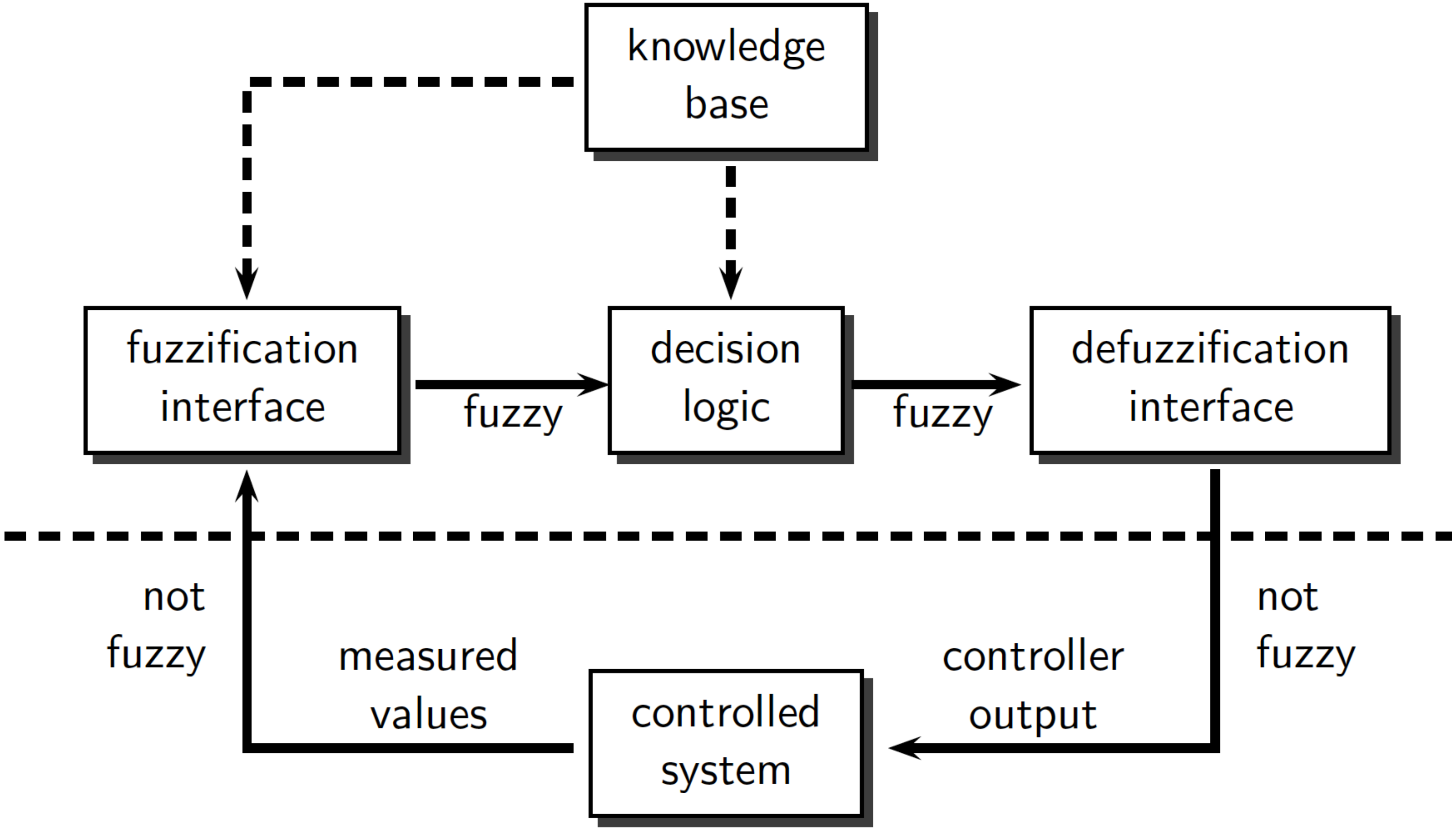
EE496 : COMPUTATIONAL INTELLIGENCE
FS06: FUZZY CONTROLLER: MAMDANI-ASSILIAN

UGUR HALICI
METU: Department of Electrical and Electronics Engineering (EEE)
METU-Hacettepe U: Neuroscience and Neurotechnology (NSNT)

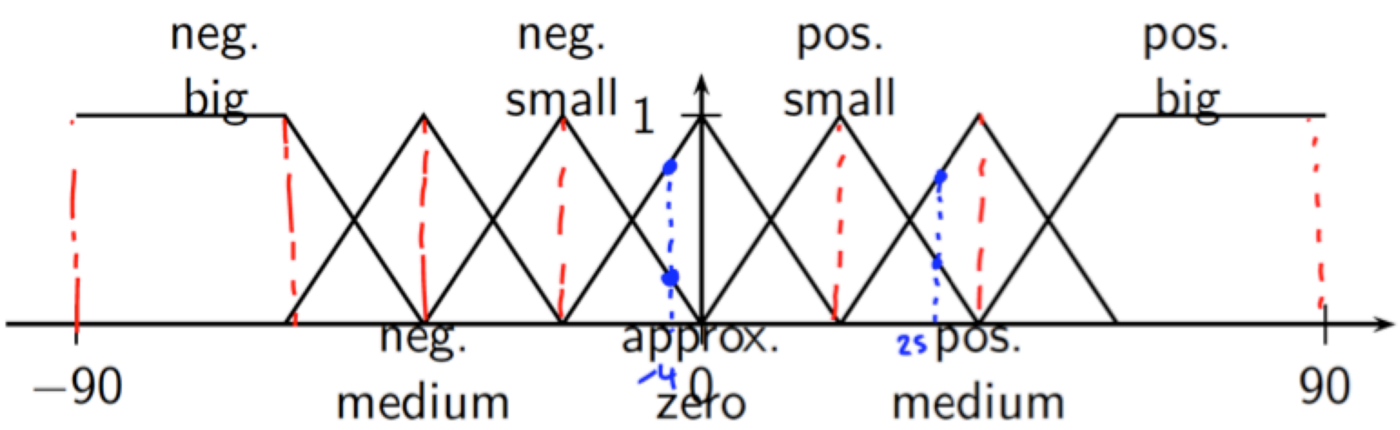
Architecture of a Fuzzy Controller

•



Example: Cartpole Problem (cont.)

X_1 is partitioned into 7 fuzzy sets.



Support of fuzzy sets: intervals with length 1/4 of whole range X_1.

Similar fuzzy partitions for X_2 and Y .

Example: Cartpole Problem (cont.)

Next step: specify rules

if $\xi 1$ is $A^{(1)}$ and . . . and ξ_n is $A^{(n)}$ then η is B,

$A^{(1)}, \ldots, A^{(n)}$ and B represent linguistic terms corresponding to $\mu^{(1)}, \ldots, \mu^{(n)}$ and μ according to $X_1, \ldots, X_n$ and Y .

Let the rule base consist of k rules.

Example: Cartpole Problem (cont.)

θ

$\dot{\theta}$	nb	nm	ns	az	ps	pm	pb
nb			ps	pb			
nm				pm			
ns	nm		ns	ps			
az	nb	nm	ns	az	ps	pm	pb
ps				ns	ps		pm
pm				nm			
pb				nb	ns		

FOR RULE EVALUATION AT PAGE 8

INPUT

$(\theta, \dot{\theta}) = (25, -4)$

$\mu_{ps}(25) = 0.3$

$\mu_{pm}(25) = 0.6$

others 0

$\mu_{ns}(-4) = \ldots$

$\mu_{az}(-4) = 0.5$

others 0

So indicated 4 cells should be considered. But 2 of them are defined as rule

19 rules for cartpole problem, *e.g.*

If θ is *approximately zero* and $\dot{\theta}$ is *negative medium* then F is *positive medium*.

Definition of Table-based Control Function

Measurement $(x_1, \ldots, x_n) \in X_1 \times \ldots \times X_n$ is forwarded to decision logic.

Consider rule

if ξ_1 is $A^{(1)}$ and . . . and ξ_n is $A^{(n)}$ then η is B.

Decision logic computes degree to $\xi_1, \ldots, \xi_n$ fulfills premise of rule.

For $1 \le v \le n$, the value $\mu^{(v)}(x_v)$ is calculated.

Combine values conjunctively by $\alpha = \min\{\mu^{(1)}, \ldots, \mu^{(n)}\}$.

For each rule R_r with $1 \le r \le k$, compute

$$\alpha_r = \min\{\mu^{(1)}_{i1,r}(x_1), \ldots, \mu^{(n)}_{in,r}(x_n)\} .$$

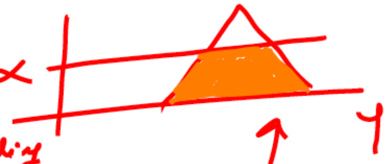
Definition of Table-based Control Function II

FOR A SINGLE RULE

Output of R_r = fuzzy set of output values.

Thus "cutting off" fuzzy set μ_{i_r} associated with conclusion of R_r at α_r.

So for input $(x_1, \ldots, x_n)$, R_r implies fuzzy set



For rule r the corresponding fuzzy set R_r

$$\mu_{x_1,\ldots,x_n}^{\text{output}(R_r)} : Y \to [0,1],$$

$$y \mapsto \min\left\{\mu_{i_{1,r}}^{(1)}(x_1), \ldots, \mu_{i_{n,r}}^{(n)}(x_n), \mu_{i_r}(y)\right\}.$$

If $\mu_{i_{1,r}}^{(1)}(x_1) = \ldots = \mu_{i_{n,r}}^{(n)}(x_n) = 1$, then $\mu_{x_1,\ldots,x_n}^{\text{output}(R_r)} = \mu_{i_r}$.

If for all $\nu \in \{1, \ldots, n\}$, $\mu_{i_{1,r}}^{(\nu)}(x_\nu) = 0$, then $\mu_{x_1,\ldots,x_n}^{\text{output}(R_r)} = 0$.

Combination of Rules

The decision logic combines the fuzzy sets from all rules.

The **maximum** leads to the output fuzzy set

$\mu^{output}_{x1,\dots,xn} : Y \rightarrow [0, 1],$

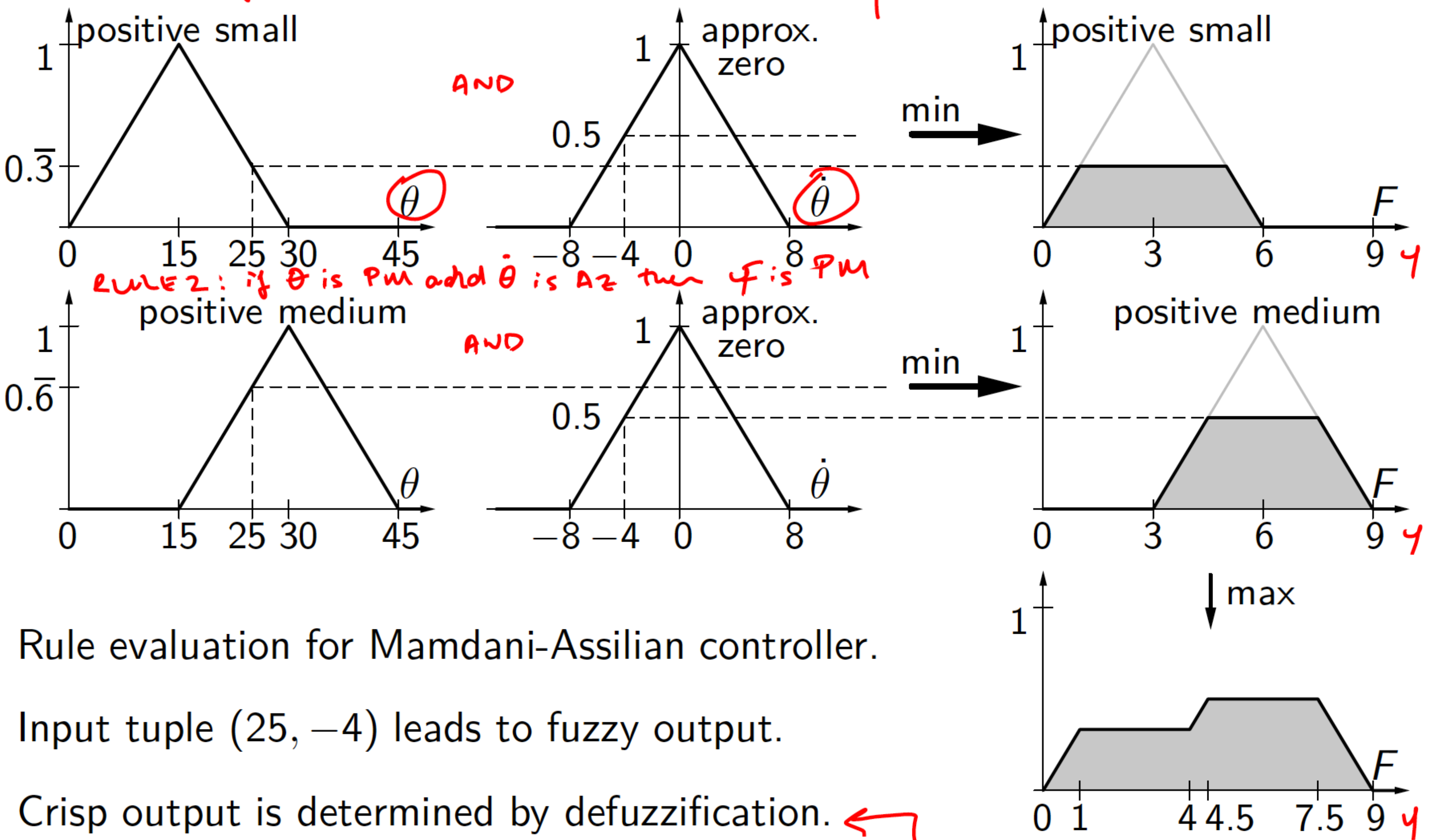
$y \rightarrow \max_{1\leq r\leq k} \{\min \{\mu^{(1)}_{i1,r}(x_1), . . . , \mu^{(n)}_{in,r}(xn), \mu_{ir}(y)\}\} .$

Then $\mu^{output}_{x1,\dots,xn}$ is passed to defuzzification interface.

Rule Evaluation

SEE PAGE 4
ONLY TWO RULES ARE RELATED

RULE 1: If θ is PS and $\dot{\theta}$ is AZ then F is PS



Rule evaluation for Mamdani-Assilian controller.

Input tuple (25, −4) leads to fuzzy output.

Crisp output is determined by defuzzification.

what should be y ?

Defuzzification

So far: mapping between each $(n_1, \ldots, n_n)$ and $\mu^{output}_{x1,\ldots,xn}$.

Output = description of output value as fuzzy set.

Defuzzification interface derives crisp value from $\mu^{output}_{x1,\ldots,xn}$.

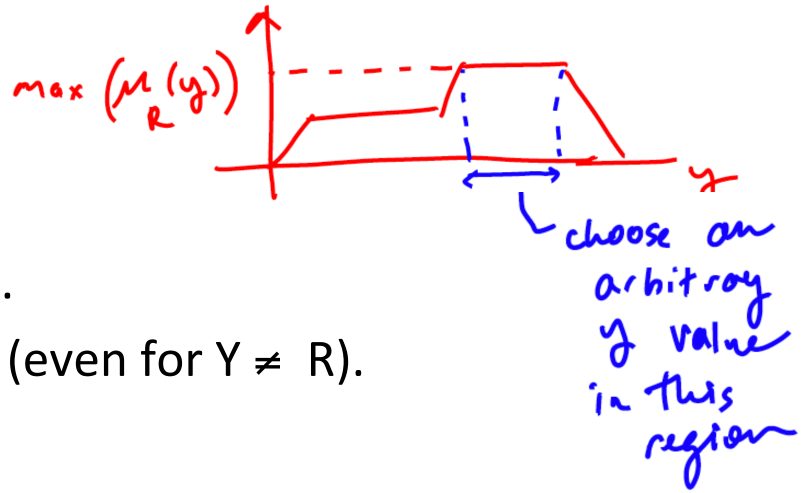
This step is called **defuzzification**.

Most common methods:

- max criterion,
- mean of maxima,
- center of gravity.

The Max Criterion Method

Choose an arbitrary $y \in Y$ for which $\mu^{output}_{x1,\ldots,xn}$ reaches the maximum membership value.



Advantages:

- Applicable for arbitrary fuzzy sets.
- Applicable for arbitrary domain Y (even for $Y \neq \mathbb{R}$).

Disadvantages:

- Rather class of defuzzification strategies than single method.
- Which value of maximum membership?
- Random values and thus non-deterministic controller.
- Leads to discontinuous control actions.

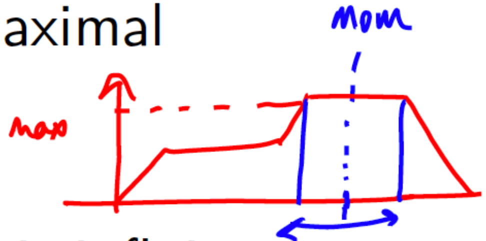
The Mean of Maxima (MOM) Method

Preconditions:

(i) Y is interval

(ii) $Y_{\text{Max}} = \{y \in Y \mid \forall y' \in Y : \mu^{\text{output}}_{x_1,\ldots,x_n}(y') \leq \mu^{\text{output}}_{x_1,\ldots,x_n}(y)\}$ is non-empty and measurable

(iii) Y_{Max} is set of all $y \in Y$ such that $\mu^{\text{output}}_{x_1,\ldots,x_n}$ is maximal



Crisp output value = mean value of Y_{Max}.

if Y_{Max} is finite:

$$\eta = \frac{1}{|Y_{\text{Max}}|} \sum_{y_i \in Y_{\text{Max}}} y_i$$

if Y_{Max} is infinite:

$$\eta = \frac{\int_{y \in Y_{\text{Max}}} y \, dy}{\int_{y \in Y_{\text{Max}}} dy}$$

find mean value of y in this region

MOM can lead to discontinuous control actions.

Center of Gravity (COG) Method

Same preconditions as MOM method.

η = center of gravity/area of $\mu^{\text{output}}_{x1,\ldots,xn}$

If Y is finite, then

$$\eta = \frac{\sum_{y_i \in Y} y_i \cdot \mu^{\text{output}}_{x_1,\ldots,x_n}(y_i)}{\sum_{y_i \in Y} \mu^{\text{output}}_{x_1,\ldots,x_n}(y_i)}.$$

If Y is infinite, then

$$\eta = \frac{\int_{y \in Y} y \cdot \mu^{\text{output}}_{x_1,\ldots,x_n}(y)\, dy}{\int_{y \in Y} \mu^{\text{output}}_{x_1,\ldots,x_n}(y)\, dy}.$$

Center of Gravity (COG) Method

Advantages:

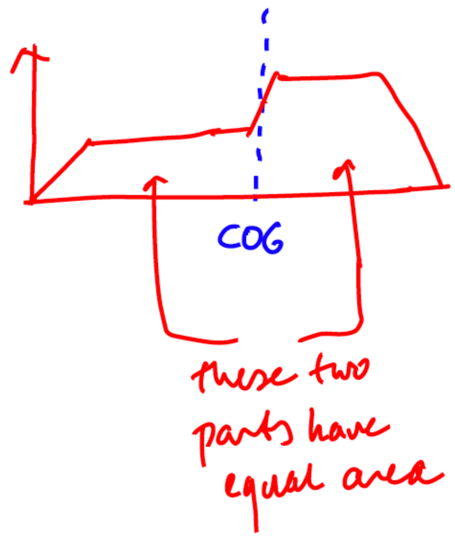
- Nearly always smooth behavior,
- A certain rule does not dominate.

Disadvantage:

- No semantic justification,
- Long computation,
- Counterintuitive results possible.

Also called center of area (COA) method:

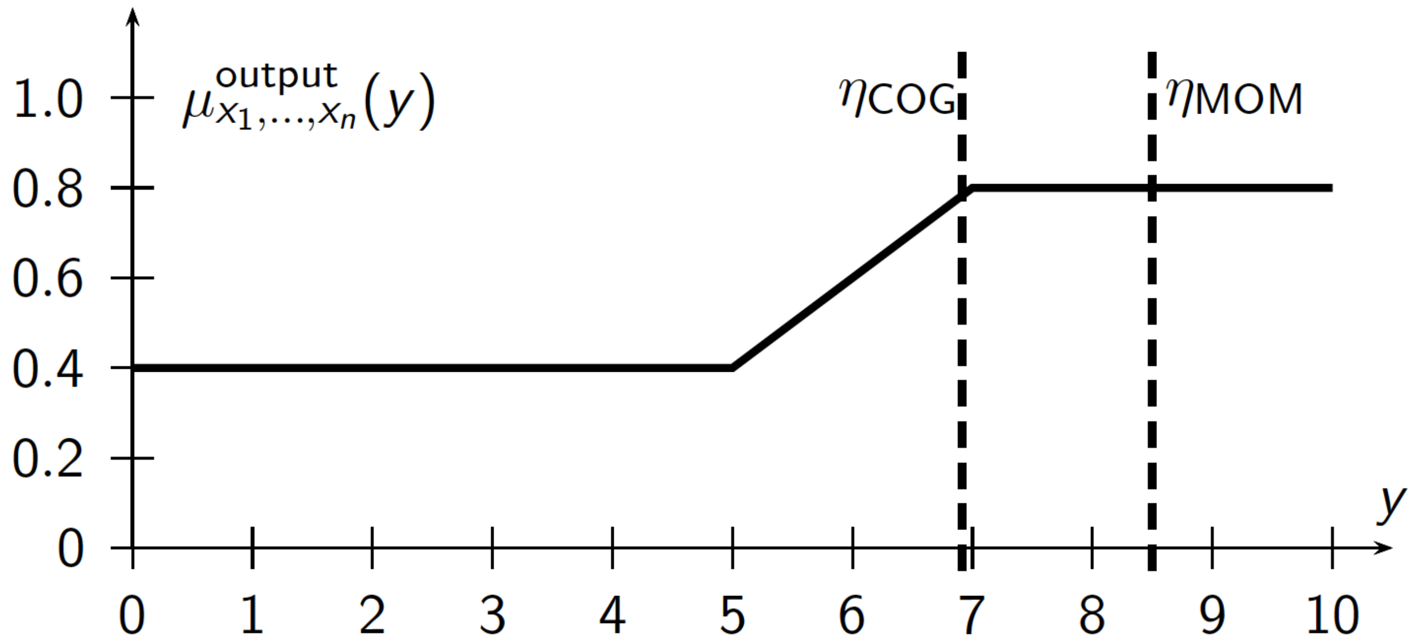
take value that splits $\mu^{output}_{x1,\ldots,xn}$ into 2 equal parts.



Example

Task: compute η_{COG} and η_{MOM} of fuzzy set shown below.

Based on finite set Y = 0, 1, . . . , 10 and infinite set Y = [0, 10].



Example for COG

Continuous and Discrete Output Space

$$\eta_{\text{COG}} = \frac{\int_0^{10} y \cdot \mu^{\text{output}}_{x_1,\ldots,x_n}(y)\,dy}{\int_0^{10} \mu^{\text{output}}_{x_1,\ldots,x_n}(y)\,dy}$$

$$= \frac{\int_0^5 0.4y\,dy + \int_5^7 (0.2y - 0.6)y\,dy + \int_7^{10} 0.8y\,dy}{5 \cdot 0.4 + 2 \cdot \frac{0.8+0.4}{2} + 3 \cdot 0.8}$$

$$\approx \frac{38.7333}{5.6} \approx 6.917$$

$$\eta_{\text{COG}} = \frac{0.4 \cdot (0+1+2+3+4+5) + 0.6 \cdot 6 + 0.8 \cdot (7+8+9+10)}{0.4 \cdot 6 + 0.6 \cdot 1 + 0.8 \cdot 4}$$

$$= \frac{36.8}{6.2} \approx 5.935$$

Example for COG

Continuous and Discrete Output Space

$$\eta_{\mathrm{MOM}} = \frac{\int_7^{10} y \, dy}{\int_7^{10} \, dy}$$

$$= \frac{50 - 24.5}{10 - 7} = \frac{25.5}{3}$$

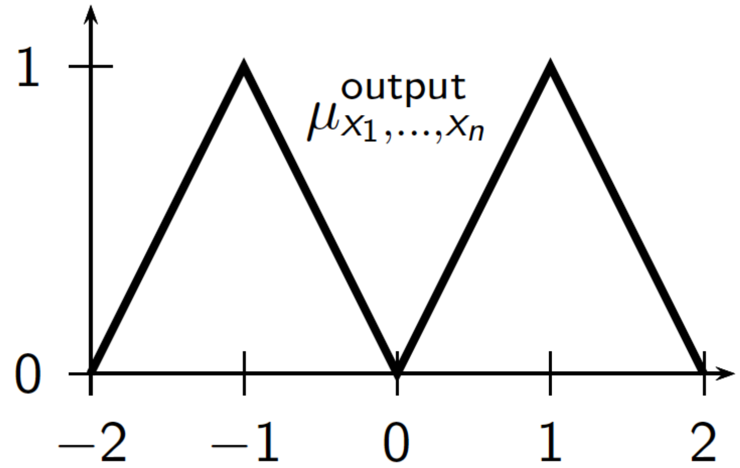
$$= 8.5$$

$$\eta_{\mathrm{MOM}} = \frac{7 + 8 + 9 + 10}{4}$$

$$= \frac{34}{4}$$

$$= 8.5$$

Problem Case for MOM and COG



- What would be the output of MOM or COG?
- Is this desirable or not?