```
> # This program calculates the source term Q for the 2D Navier-
  Stokes equations -
> # Energy e

> restart;
> with(CodeGeneration) :
> #with(Student[VectorCalculus]) :
> #SetCoordinates( 'cartesian'[x, y, z] ) :
> alias( rho = rho(x, y, t) ) : alias( ρ_an = rho_an(x, y, t) ) :
> alias( u = u(x, y, t) ) : alias( u_an = u_an(x, y, t) ) :
> alias( v = v(x, y, t) ) : alias( v_an = v_an(x, y, t) ) :
> alias( w = w(x, y, t) ) : alias( w_an = w_an(x, y, t) ) :
> alias( p = p(x, y, t) ) : alias( p_an = p_an(x, y, t) ) :
> alias( e = e(x, y, t) ) : alias( e_t = e_t(x, y, t) ) :
> alias( Q = Q(x, y, t) ) :

> #2D Navier-Stokes equation - ENERGY - Prior to substitutions of
  auxiliary relations:
> Diff(rho·e_t, t) + Diff(rho·u·e_t + p·u, x) + Diff(rho·v·e_t + p·v, y) ;
```

$$\frac{\partial}{\partial t}\left(\rho\, e_t\right) + \frac{\partial}{\partial x}\left(\rho\, u\, e_t + p\, u\right) + \frac{\partial}{\partial y}\left(\rho\, v\, e_t + p\, v\right) \tag{1}$$

```
> #Auxiliary relations for energy:
> #p=rho·R·T;  e := 1/(gamma - 1)·R·T; e_t := e + (u·u + v·v)/2;
> #e_t := p/((γ - 1) ρ) + (u^2 + v^2)/2;

> #2D Navier-Stokes equation - ENERGY:
> Diff(rho·e_t, t) + Diff(rho·u·e_t + p·u - u·τ_xx - v·τ_xy + q_x, x) + Diff(rho·v·e_t + p·v
      - u·τ_xy - v·τ_yy + q_y, y) ;
```

$$\frac{\partial}{\partial t}\left(\rho\, e_t\right) + \frac{\partial}{\partial x}\left(\rho\, u\, e_t + p\, u - u\,\tau_{xx} - v\,\tau_{xy} + q_x\right) + \frac{\partial}{\partial y}\left(\rho\, v\, e_t + p\, v - u\,\tau_{xy} - v\,\tau_{yy} + q_y\right) \tag{2}$$

```
> # Energy equation, writen as differential operator:
> L1 := diff(rho·e_t, t) + diff(rho·u·e_t + p·u - u·τ_xx - v·τ_xy + q_x, x) + diff(rho·v·e_t
      + p·v - u·τ_xy - v·τ_yy + q_y, y) :
> τ_xx := 2/3·mu·(2 diff(u, x) - diff(v, y)) :
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> τ_xy := mu·( diff(u, y) + diff(v, x) ) :
> τ_yy := 2/3·mu·(2 diff(v, y) − diff(u, x)) :
>
> L2 := algsubs( τ_xx = 2/3·mu· (2 diff(u, x) − diff(v, y)), L1 ) :
> L2 := algsubs( τ_yy = 2/3·mu·(2 diff(v, y) − diff(u, x)), L2 ) :
> L2 := algsubs( τ_xy = mu·( diff(u, y) + diff(v, x)), L2)
```

$$L2 := \left(-\left(\frac{\partial}{\partial y}u\right)^2 - 2\left(\frac{\partial}{\partial y}u\right)\left(\frac{\partial}{\partial x}v\right) - \left(\frac{\partial}{\partial x}v\right)^2\right)\mu + \left(-\frac{4}{3}\left(\frac{\partial}{\partial x}u\right)^2 + \frac{4}{3}\left(\frac{\partial}{\partial x}u\right)\left(\frac{\partial}{\partial y}v\right) - \frac{4}{3}\left(\frac{\partial}{\partial y}v\right)^2\right)\mu + (\rho e_t + p)\left(\frac{\partial}{\partial x}u\right) + (\rho e_t + p)\left(\frac{\partial}{\partial y}v\right) + \left(\frac{\partial}{\partial t}\rho\right)e_t + \rho\left(\frac{\partial}{\partial t}e_t\right) + \left(\frac{\partial}{\partial x}\rho\right)u e_t + \rho u\left(\frac{\partial}{\partial x}e_t\right) + \left(\frac{\partial}{\partial x}p\right)u + \left(\frac{\partial}{\partial y}\rho\right)v e_t + \rho v\left(\frac{\partial}{\partial y}e_t\right) + \left(\frac{\partial}{\partial y}p\right)v \quad (3)$$

```
>
> #2D Analitycal solutions proposed by Roy, Smith & Ober, AIAA, 2002:
> ρ_an := ρ_0 + ρ_x·sin( a_rho,x·pi·x / L ) + ρ_y·cos( a_rho,y·pi·y / L ) :
> u_an := u_0 + u_x·sin( a_u,x·pi·x / L ) + u_y·cos( a_u,y·pi·y / L ) :
> v_an := v_0 + v_x·cos( a_v,x·pi·x / L ) + v_y·sin( a_v,y·pi·y / L ) :
> p_an := p_0 + p_x·cos( a_p,x·pi·x / L ) + p_y·sin( a_p,y·pi·y / L ) :

> # Applying operator L1 on u,v,rho and p, in order to obtain source term Q:
> L1 := algsubs( e_t = p/((γ−1) ρ) + (u^2 + v^2)/two, L1 );
      #using variable "two" instead of number 2 for easier future manipulations.
```

$$L1 := -\frac{1}{3}\frac{1}{(\gamma-1)\,two}\left(-6\rho u\gamma\left(\frac{\partial}{\partial t}u\right) - 6\rho v\gamma\left(\frac{\partial}{\partial t}v\right) - 3\rho\left(\frac{\partial}{\partial y}v\right)u^2\gamma - 9\rho\left(\frac{\partial}{\partial y}v\right)v^2\gamma - 9\rho\left(\frac{\partial}{\partial x}u\right)u^2\gamma - 3\rho\left(\frac{\partial}{\partial x}u\right)v^2\gamma + 6u\rho v\left(\frac{\partial}{\partial x}v\right) + 6v\rho u\left(\frac{\partial}{\partial y}u\right) + 4\left(\frac{\partial}{\partial x}u\right)^2\mu\,two\gamma + 4\left(\frac{\partial}{\partial x}u\right)\mu\,two\left(\frac{\partial}{\partial y}v\right) \quad (4)$$

$$-3\left(\frac{\partial}{\partial x}\rho\right)u v^2\gamma-3u\left(\frac{\partial}{\partial x}p\right)two\gamma-3\left(\frac{\partial}{\partial x}u\right)p\,two\gamma-3\left(\frac{\partial}{\partial y}p\right)v\,two\gamma$$
$$-6\left(\frac{\partial}{\partial x}v\right)\mu\,two\left(\frac{\partial}{\partial y}u\right)+3\left(\frac{\partial}{\partial x}v\right)^2\mu\,two\gamma-3\left(\frac{\partial}{\partial y}v\right)p\,two\gamma$$
$$-3\left(\frac{\partial}{\partial y}\rho\right)v u^2\gamma+3\left(\frac{\partial}{\partial y}u\right)^2\mu\,two\gamma+4\left(\frac{\partial}{\partial y}v\right)^2\mu\,two\gamma-3\left(\frac{\partial}{\partial t}\rho\right)u^2\gamma$$
$$+6\rho u\left(\frac{\partial}{\partial t}u\right)-3\left(\frac{\partial}{\partial t}\rho\right)v^2\gamma+6\rho v\left(\frac{\partial}{\partial t}v\right)-3\left(\frac{\partial}{\partial t}p\right)two+3\left(\frac{\partial}{\partial t}\rho\right)u^2$$
$$+3\left(\frac{\partial}{\partial t}\rho\right)v^2-4\left(\frac{\partial}{\partial x}u\right)\mu\,two\left(\frac{\partial}{\partial y}v\right)\gamma+6\left(\frac{\partial}{\partial x}v\right)\mu\,two\left(\frac{\partial}{\partial y}u\right)\gamma$$
$$-6u\rho v\gamma\left(\frac{\partial}{\partial x}v\right)-6v\rho u\gamma\left(\frac{\partial}{\partial y}u\right)-4\left(\frac{\partial}{\partial x}u\right)^2\mu\,two-3\left(\frac{\partial}{\partial x}\rho\right)u^3\gamma$$
$$+3\left(\frac{\partial}{\partial x}\rho\right)u v^2-3\left(\frac{\partial}{\partial x}v\right)^2\mu\,two+3\left(\frac{\partial}{\partial y}\rho\right)v u^2-3\left(\frac{\partial}{\partial y}\rho\right)v^3\gamma$$
$$-3\left(\frac{\partial}{\partial y}u\right)^2\mu\,two-4\left(\frac{\partial}{\partial y}v\right)^2\mu\,two+3\rho\left(\frac{\partial}{\partial y}v\right)u^2+9\rho\left(\frac{\partial}{\partial y}v\right)v^2$$
$$+9\rho\left(\frac{\partial}{\partial x}u\right)u^2+3\rho\left(\frac{\partial}{\partial x}u\right)v^2+3\left(\frac{\partial}{\partial x}\rho\right)u^3+3\left(\frac{\partial}{\partial y}\rho\right)v^3\Big)$$

```
> Q := algsubs(u = u_an, L1):
> Q := algsubs(v = v_an, Q):
> Q := algsubs(rho = rho_an, Q):
> Q := algsubs(p = p_an, Q):
> #Q:=simplify(Q, trig):
    #not a good idea because it expandes the expression a lot!
> Q := collect(Q, [pi, a_p,x, a_p,y, a_ρ,x, a_ρ,y, a_u,x, a_u,y, a_v,x, a_v,y, gamma],
    distributed):
    #it did not work well as in the previous cases for u,v, and rho. So I will split
    the equations in 8 terms (a_p,x, a_p,y, a_ρ,x,...) and factor them.
> Q := sort(Q);
```

$$Q := -\frac{1}{3}\,\frac{\left(-4\mu\,two\,u_x^2\cos\left(\frac{\pi x a_{u,x}}{L}\right)^2+4\gamma\mu\,two\,u_x^2\cos\left(\frac{\pi x a_{u,x}}{L}\right)^2\right)\pi^2 a_{u,x}^2}{(\gamma-1)L^2\,two} \tag{5}$$
$$-\frac{1}{3}\,\frac{1}{(\gamma-1)L^2\,two}\left(\left(4\mu\,two\,u_x v_y\cos\left(\frac{\pi x a_{u,x}}{L}\right)\cos\left(\frac{\pi y a_{v,y}}{L}\right)\right.\right.$$
$$\left.\left.-4\gamma\mu\,two\,u_x v_y\cos\left(\frac{\pi x a_{u,x}}{L}\right)\cos\left(\frac{\pi y a_{v,y}}{L}\right)\right)\pi^2 a_{u,x}a_{v,y}\right)$$

$$-\frac{1}{3}\,\frac{\left(3\,\gamma\mu\,two\,u_y^2\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2-3\,\mu\,two\,u_y^2\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\right)\pi^2\,a_{u,\,y}^2}{(\gamma-1)\,L^2\,two}$$

$$-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\left(\left(6\,\gamma\mu\,two\,u_y\,v_x\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\right.\right.$$

$$\left.\left.-6\,\mu\,two\,u_y\,v_x\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\right)\pi^2\,a_{u,\,y}\,a_{v,\,x}\right)$$

$$-\frac{1}{3}\,\frac{\left(-3\,\mu\,two\,v_x^2\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2+3\,\gamma\mu\,two\,v_x^2\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2\right)\pi^2\,a_{v,\,x}^2}{(\gamma-1)\,L^2\,two}$$

$$-\frac{1}{3}\,\frac{\left(4\,\gamma\mu\,two\,v_y^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2-4\,\mu\,two\,v_y^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2\right)\pi^2\,a_{v,\,y}^2}{(\gamma-1)\,L^2\,two}$$

$$-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\left(\left(3\,L\,two\,p_x\,u_x\sin\left(\frac{\pi\,x\,a_{p,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\right.\right.$$

$$\left.\left.+3\,L\,two\,p_x\,u_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{p,\,x}}{L}\right)+3\,L\,two\,p_x\,u_0\sin\left(\frac{\pi\,x\,a_{p,\,x}}{L}\right)\right)\right.$$

$$\left.\gamma\pi\,a_{p,\,x}\right)-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\left(\left(-3\,L\,two\,p_y\,v_x\cos\left(\frac{\pi\,y\,a_{p,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\right.\right.$$

$$\left.\left.-3\,L\,two\,p_y\,v_y\cos\left(\frac{\pi\,y\,a_{p,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-3\,L\,two\,p_y\,v_0\cos\left(\frac{\pi\,y\,a_{p,\,y}}{L}\right)\right)\right.$$

$$\left.\gamma\pi\,a_{p,\,y}\right)-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\left(\left(3\,L\,\rho_x\,u_x^3\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^3\right.\right.$$

$$+9\,L\,\rho_x\,u_x^2\,u_y\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^2+9\,L\,\rho_x\,u_x$$

$$u_y^2\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)+3\,L\,\rho_x\,u_x$$

$$v_x^2\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$+6\,L\,\rho_x\,u_x\,v_x\,v_y\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$+3\,L\,\rho_x\,u_x\,v_y^2\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2+3\,L\,\rho_x$$

$$u_y^3 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^3 + 3 L \rho_x u_y$$
$$v_x^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$
$$+ 6 L \rho_x u_y v_x v_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$
$$+ 3 L \rho_x u_y v_y^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 + 9 L \rho_x u_0$$
$$u_x^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$
$$+ 18 L \rho_x u_0 u_x u_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) + 9 L \rho_x u_0$$
$$u_y^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 + 3 L \rho_x u_0 v_x^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$
$$+ 6 L \rho_x u_0 v_x v_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 3 L \rho_x u_0$$
$$v_y^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2$$
$$+ 6 L \rho_x u_x v_0 v_x \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$
$$+ 6 L \rho_x u_x v_0 v_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$
$$+ 6 L \rho_x u_y v_0 v_x \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)$$
$$+ 6 L \rho_x u_y v_0 v_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 9 L \rho_x$$
$$u_0^2 u_x \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) + 9 L \rho_x u_0^2 u_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)$$
$$+ 6 L \rho_x u_0 v_0 v_x \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)$$
$$+ 6 L \rho_x u_0 v_0 v_y \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 3 L \rho_x u_x$$
$$v_0^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) + 3 L \rho_x u_y v_0^2 \cos\left(\frac{\pi x a_{\rho,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)$$

$$+3\,L\rho_x\,u_0^3\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)+3\,L\rho_x\,u_0\,v_0^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)+\left(-3\,L\rho_x\right.$$

$$u_x^3\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)^3-9\,L\rho_x$$

$$u_x^2\,u_y\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)^2-9\,L\rho_x\,u_x$$

$$u_y^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)^2\sin\left(\frac{\pi x a_{u,x}}{L}\right)-3\,L\rho_x\,u_x$$

$$v_x^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)^2\sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$-6\,L\rho_x\,u_x\,v_x\,v_y\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)\sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$-3\,L\rho_x\,u_x\,v_y^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)\sin\left(\frac{\pi y a_{v,y}}{L}\right)^2-3\,L\rho_x$$

$$u_y^3\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)^3-3\,L\rho_x\,u_y$$

$$v_x^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$

$$-6\,L\rho_x\,u_y\,v_x\,v_y\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)\sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$-3\,L\rho_x\,u_y\,v_y^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)\sin\left(\frac{\pi y a_{v,y}}{L}\right)^2-9\,L\rho_x\,u_0$$

$$u_x^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$

$$-18\,L\rho_x\,u_0\,u_x\,u_y\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)-9\,L\rho_x\,u_0$$

$$u_y^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi y a_{u,y}}{L}\right)^2-3\,L\rho_x\,u_0\,v_x^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$

$$-6\,L\rho_x\,u_0\,v_x\,v_y\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)\sin\left(\frac{\pi y a_{v,y}}{L}\right)-3\,L\rho_x\,u_0$$

$$v_y^2\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\sin\left(\frac{\pi y a_{v,y}}{L}\right)^2$$

$$-6\,L\rho_x\,u_x\,v_0\,v_x\cos\left(\frac{\pi x a_{\rho,x}}{L}\right)\cos\left(\frac{\pi x a_{v,x}}{L}\right)\sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$
\begin{aligned}
&-6\,L\rho_x\,u_x\,v_0\,v_y\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&-6\,L\rho_x\,u_y\,v_0\,v_x\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&-6\,L\rho_x\,u_y\,v_0\,v_y\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-9\,L\rho_x\\
&u_0^2\,u_x\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)-9\,L\rho_x\,u_0^2\,u_y\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\rho_x\,u_0\,v_0\,v_x\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&-6\,L\rho_x\,u_0\,v_0\,v_y\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-3\,L\rho_x\,u_x\\
&v_0^2\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)-3\,L\rho_x\,u_y\,v_0^2\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-3\,L\rho_x\,u_0^3\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)-3\,L\rho_x\,u_0\,v_0^2\cos\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\Bigg)\gamma\Bigg)\pi\,a_{\rho,\,x}\Bigg)\\
&-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\Bigg(\Bigg(-3\,L\rho_y\\
&u_x^2\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^2-3\,L\rho_y\\
&u_x^2\,v_y\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^2\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&-6\,L\rho_y\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\\
&-6\,L\rho_y\,u_x\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&-3\,L\rho_y\,u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)-3\,L\rho_y\\
&u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-3\,L\rho_y\\
&v_x^3\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^3\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)-9\,L\rho_y\\
&v_x^2\,v_y\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2\sin\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-9\,L\rho_y\,v_x
\end{aligned}
$$

$$v_y^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 3 L \rho_y$$

$$v_y^3 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^3$$

$$- 6 L \rho_y u_0 u_x v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 6 L \rho_y u_0 u_x v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$- 6 L \rho_y u_0 u_y v_x \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)$$

$$- 6 L \rho_y u_0 u_y v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 3 L \rho_y$$

$$u_x^2 v_0 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$

$$- 6 L \rho_y u_x u_y v_0 \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) - 3 L \rho_y$$

$$u_y^2 v_0 \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) - 9 L \rho_y v_0 v_x^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)$$

$$- 18 L \rho_y v_0 v_x v_y \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 9 L \rho_y v_0$$

$$v_y^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 3 L \rho_y u_0^2 v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)$$

$$- 3 L \rho_y u_0^2 v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$- 6 L \rho_y u_0 u_x v_0 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 6 L \rho_y u_0 u_y v_0 \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) - 9 L \rho_y$$

$$v_0^2 v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) - 9 L \rho_y v_0^2 v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$- 3 L \rho_y u_0^2 v_0 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) - 3 L \rho_y v_0^3 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + \left(3 L \rho_y\right.$$

$$\left. u_x^2 v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2 + 3 L \rho_y\right.$$

$$u_x^2 v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2 \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$+ 6 L \rho_y u_x u_y v_x \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$+ 6 L \rho_y u_x u_y v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$+ 3 L \rho_y u_y^2 v_x \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + 3 L \rho_y$$

$$u_y^2 v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 3 L \rho_y$$

$$v_x^3 \cos\left(\frac{\pi x a_{v,x}}{L}\right)^3 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + 9 L \rho_y$$

$$v_x^2 v_y \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 9 L \rho_y v_x$$

$$v_y^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 + 3 L \rho_y$$

$$v_y^3 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^3$$

$$+ 6 L \rho_y u_0 u_x v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$+ 6 L \rho_y u_0 u_x v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$+ 6 L \rho_y u_0 u_y v_x \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)$$

$$+ 6 L \rho_y u_0 u_y v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 3 L \rho_y$$

$$u_x^2 v_0 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$

$$+ 6 L \rho_y u_x u_y v_0 \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) + 3 L \rho_y$$

$$u_y^2 v_0 \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + 9 L \rho_y v_0 v_x^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)$$

$$+ 18 L \rho_y v_0 v_x v_y \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 9 L \rho_y v_0$$

$$v_y^2 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 + 3 L \rho_y u_0^2 v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)$$
$$+ 3 L \rho_y u_0^2 v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$
$$+ 6 L \rho_y u_0 u_x v_0 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$
$$+ 6 L \rho_y u_0 u_y v_0 \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + 9 L \rho_y$$
$$v_0^2 v_x \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + 9 L \rho_y v_0^2 v_y \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$
$$+ 3 L \rho_y u_0^2 v_0 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right) + 3 L \rho_y v_0^3 \sin\left(\frac{\pi y a_{\rho,y}}{L}\right)\Bigg) \gamma\Bigg) \pi a_{\rho,y}\Bigg)$$
$$- \frac{1}{3} \frac{1}{(\gamma - 1) L^2 \, two} \Bigg(\Bigg(9 L \rho_x u_x^3 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$
$$+ 18 L \rho_x u_x^2 u_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$
$$+ 9 L \rho_x u_x u_y^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) + 3 L \rho_x u_x$$
$$v_x^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$
$$+ 6 L \rho_x u_x v_x v_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$
$$+ 3 L \rho_x u_x v_y^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 + 9 L \rho_y$$
$$u_x^3 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2 + 18 L \rho_y$$
$$u_x^2 u_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) + 9 L \rho_y u_x$$
$$u_y^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 + 3 L \rho_y u_x$$
$$v_x^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$
$$+ 6 L \rho_y u_x v_x v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$+3\,L\rho_y\,u_x\,v_y^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2+9\,L\rho_0$$

$$u_x^3\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^2+18\,L\rho_0$$

$$u_x^2\,u_y\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)+9\,L\rho_0\,u_x$$

$$u_y^2\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2+3\,L\rho_0\,u_x\,v_x^2\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2$$

$$+6\,L\rho_0\,u_x\,v_x\,v_y\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+3\,L\rho_0\,u_x$$

$$v_y^2\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2+18\,L\rho_x\,u_0$$

$$u_x^2\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$+18\,L\rho_x\,u_0\,u_x\,u_y\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)$$

$$+6\,L\rho_x\,u_x\,v_0\,v_x\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)$$

$$+6\,L\rho_x\,u_x\,v_0\,v_y\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+18\,L\rho_y\,u_0$$

$$u_x^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$+18\,L\rho_y\,u_0\,u_x\,u_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$+6\,L\rho_y\,u_x\,v_0\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$+6\,L\rho_y\,u_x\,v_0\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+18\,L\rho_0\,u_0$$

$$u_x^2\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)+18\,L\rho_0\,u_0\,u_x\,u_y\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$+6\,L\rho_0\,u_x\,v_0\,v_x\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$+6\,L\rho_0\,u_x\,v_0\,v_y\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+9\,L\rho_x$$

$$u_0^2\, u_x \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) + 3\, L \rho_x\, u_x\, v_0^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$

$$+ 9\, L \rho_y\, u_0^2\, u_x \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) + 3\, L \rho_y\, u_x$$

$$v_0^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) + 9\, L \rho_0\, u_0^2\, u_x \cos\left(\frac{\pi x a_{u,x}}{L}\right) + 3\, L \rho_0\, u_x$$

$$v_0^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) + \left(-9\, L \rho_x\, u_x^3 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2\right.$$

$$- 18\, L \rho_x\, u_x^2\, u_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 9\, L \rho_x\, u_x\, u_y^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) - 3\, L \rho_x\, u_x$$

$$v_x^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$

$$- 6\, L \rho_x\, u_x\, v_x\, v_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$- 3\, L \rho_x\, u_x\, v_y^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 9\, L \rho_y$$

$$u_x^3 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2 - 18\, L \rho_y$$

$$u_x^2\, u_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) - 9\, L \rho_y\, u_x$$

$$u_y^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 - 3\, L \rho_y\, u_x$$

$$v_x^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$

$$- 6\, L \rho_y\, u_x\, v_x\, v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$- 3\, L \rho_y\, u_x\, v_y^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 9\, L \rho_0$$

$$u_x^3 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2 - 18\, L \rho_0$$

$$u_x^2\, u_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) - 9\, L \rho_0\, u_x$$

$$u_y^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 - 3 L \rho_0 u_x v_x^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2$$

$$- 6 L \rho_0 u_x v_x v_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 3 L \rho_0 u_x$$

$$v_y^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 18 L \rho_x u_0$$

$$u_x^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 18 L \rho_x u_0 u_x u_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$

$$- 6 L \rho_x u_x v_0 v_x \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$

$$- 6 L \rho_x u_x v_0 v_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 18 L \rho_y u_0$$

$$u_x^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 18 L \rho_y u_0 u_x u_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)$$

$$- 6 L \rho_y u_x v_0 v_x \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)$$

$$- 6 L \rho_y u_x v_0 v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 18 L \rho_0 u_0$$

$$u_x^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) - 18 L \rho_0 u_0 u_x u_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)$$

$$- 6 L \rho_0 u_x v_0 v_x \cos\left(\frac{\pi x a_{u,x}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)$$

$$- 6 L \rho_0 u_x v_0 v_y \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 9 L \rho_x$$

$$u_0^2 u_x \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) - 3 L \rho_x u_x v_0^2 \cos\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$

$$- 9 L \rho_y u_0^2 u_x \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) - 3 L \rho_y u_x$$

$$v_0^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right) - 3 L two p_x u_x \cos\left(\frac{\pi x a_{p,x}}{L}\right) \cos\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$-3\,L\,two\,p_y\,u_x\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{p,\,y}}{L}\right)-9\,L\,\rho_0\,u_0^2\,u_x\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$-3\,L\,\rho_0\,u_x\,v_0^2\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)-3\,L\,two\,p_0\,u_x\cos\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\Bigg)\gamma\Bigg)\pi\,a_{u,\,x}\Bigg)$$

$$-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\Bigg(\Bigg($$

$$-6\,L\,\rho_x\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$-6\,L\,\rho_x\,u_x\,u_y\,v_y\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_x\,u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$-6\,L\,\rho_x\,u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_y\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$-6\,L\,\rho_y\,u_x\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_y\,u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$-6\,L\,\rho_y\,u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_0\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$-6\,L\,\rho_0\,u_x\,u_y\,v_y\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-6\,L\,\rho_0$$

$$u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)-6\,L\,\rho_0$$

$$u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_x\,u_0\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$-6\,L\,\rho_x\,u_0\,u_y\,v_y\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$
\begin{aligned}
&-6\,L\,\rho_x\,u_x\,u_y\,v_0\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)-6\,L\,\rho_x\\
&u_y^2\,v_0\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\,\rho_y\,u_0\,u_y\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\,\rho_y\,u_0\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&-6\,L\,\rho_y\,u_x\,u_y\,v_0\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)-6\,L\,\rho_y\\
&u_y^2\,v_0\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\,\rho_0\,u_0\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\,\rho_0\,u_0\,u_y\,v_y\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&-6\,L\,\rho_0\,u_x\,u_y\,v_0\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)-6\,L\,\rho_0\\
&u_y^2\,v_0\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\,\rho_x\,u_0\,u_y\,v_0\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&-6\,L\,\rho_y\,u_0\,u_y\,v_0\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)-6\,L\,\rho_0\,u_0\,u_y\,v_0\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+\left(6\,L\,\rho_x\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\right.\\
&+6\,L\,\rho_x\,u_x\,u_y\,v_y\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\,\rho_x\,u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\,\rho_x\,u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\,\rho_y\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+6\,L\rho_y\,u_x\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_0\,u_x\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_0\,u_x\,u_y\,v_y\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+6\,L\rho_0\\
&u_y^2\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)+6\,L\rho_0\\
&u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_x\,u_0\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_x\,u_0\,u_y\,v_y\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_x\,u_x\,u_y\,v_0\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)+6\,L\rho_x\\
&u_y^2\,v_0\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_0\,u_y\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_0\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_x\,u_y\,v_0\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)+6\,L\rho_y\\
&u_y^2\,v_0\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_0\,u_0\,u_y\,v_x\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\\
&+6\,L\rho_0\,u_0\,u_y\,v_y\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)
\end{aligned}
$$

$$+6\,L\,\rho_0\,u_x\,u_y\,v_0\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)+6\,L\,\rho_0$$

$$u_y^2\,v_0\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$+6\,L\,\rho_x\,u_0\,u_y\,v_0\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)$$

$$+6\,L\,\rho_y\,u_0\,u_y\,v_0\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)+6\,L\,\rho_0\,u_0\,u_y\,v_0\sin\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\Bigg)$$

$$\gamma\Bigg)\pi\,a_{u,\,y}\Bigg)-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\Bigg(\Bigg(-6\,L\,\rho_x\,u_x$$

$$v_x^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$-6\,L\,\rho_x\,u_x\,v_x\,v_y\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_x\,u_y\,v_x^2\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$-6\,L\,\rho_x\,u_y\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_y\,u_x\,v_x^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$-6\,L\,\rho_y\,u_x\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_y\,u_y\,v_x^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$-6\,L\,\rho_y\,u_y\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\,\rho_0\,u_x\,v_x^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$-6\,L\,\rho_0\,u_x\,v_x\,v_y\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-6\,L\,\rho_0\,u_y$$

$$v_x^2\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$-6\,L\,\rho_0\,u_y\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-6\,L\,\rho_x\,u_0$$

$$
\begin{aligned}
&v_x^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_x u_0 v_x v_y \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) \\
&-6 L \rho_x u_x v_0 v_x \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_x u_y v_0 v_x \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) - 6 L \rho_y u_0 \\
&v_x^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_y u_0 v_x v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) \\
&-6 L \rho_y u_x v_0 v_x \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_y u_y v_0 v_x \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) - 6 L \rho_0 u_0 \\
&v_x^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) - 6 L \rho_0 u_0 v_x v_y \sin\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) \\
&-6 L \rho_0 u_x v_0 v_x \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_0 u_y v_0 v_x \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_x u_0 v_0 v_x \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&-6 L \rho_y u_0 v_0 v_x \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) - 6 L \rho_0 u_0 v_0 v_x \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&+ \left(6 L \rho_x u_x v_x^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right)\right. \\
&+6 L \rho_x u_x v_x v_y \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) \\
&+6 L \rho_x u_y v_x^2 \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \\
&+6 L \rho_x u_y v_x v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{v,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+6\,L\rho_y\,u_x\,v_x^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_y\,u_x\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_y\,v_x^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_y\,u_y\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_0\,u_x\,v_x^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_0\,u_x\,v_x\,v_y\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+6\,L\rho_0\,u_y\\
&v_x^2\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_0\,u_y\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+6\,L\rho_x\,u_0\\
&v_x^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_x\,u_0\,v_x\,v_y\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_x\,u_x\,v_0\,v_x\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_x\,u_y\,v_0\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)+6\,L\rho_y\,u_0\\
&v_x^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_y\,u_0\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\\
&+6\,L\rho_y\,u_x\,v_0\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\\
&+6\,L\rho_y\,u_y\,v_0\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)+6\,L\rho_0\,u_0\\
&v_x^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)+6\,L\rho_0\,u_0\,v_x\,v_y\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)
\end{aligned}
$$

$$+6\,L\rho_0\,u_x\,v_0\,v_x\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$+6\,L\rho_0\,u_y\,v_0\,v_x\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$+6\,L\rho_x\,u_0\,v_0\,v_x\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)$$

$$+6\,L\rho_y\,u_0\,v_0\,v_x\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)+6\,L\rho_0\,u_0\,v_0\,v_x\sin\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\Bigg)$$

$$\gamma\Bigg)\pi\,a_{v,\,x}\Bigg)-\frac{1}{3}\,\frac{1}{(\gamma-1)\,L^2\,two}\Bigg(\Bigg(3\,L\rho_x$$

$$u_x^2\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^2$$

$$+6\,L\rho_x\,u_x\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$+3\,L\rho_x\,u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)+9\,L\rho_x$$

$$v_x^2\,v_y\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)+18\,L\rho_x\,v_x$$

$$v_y^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+9\,L\rho_x$$

$$v_y^3\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2+3\,L\rho_y$$

$$u_x^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)^2$$

$$+6\,L\rho_y\,u_x\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$+3\,L\rho_y\,u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+9\,L\rho_y$$

$$v_x^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+18\,L\rho_y\,v_x$$

$$v_y^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)+9\,L\rho_y$$

$$v_y^3\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2+3\,L\rho_0$$

$$u_x^2\, v_y \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{u,\,x}}{L}\right)^2$$

$$+ 6\, L \rho_0\, u_x\, u_y\, v_y \cos\left(\frac{\pi\, y\, a_{u,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{u,\,x}}{L}\right) + 3\, L \rho_0$$

$$u_y^2\, v_y \cos\left(\frac{\pi\, y\, a_{u,\,y}}{L}\right)^2 \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) + 9\, L \rho_0$$

$$v_x^2\, v_y \cos\left(\frac{\pi\, x\, a_{v,\,x}}{L}\right)^2 \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) + 18\, L \rho_0\, v_x$$

$$v_y^2 \cos\left(\frac{\pi\, x\, a_{v,\,x}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) + 9\, L \rho_0$$

$$v_y^3 \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right)^2$$

$$+ 6\, L \rho_x\, u_0\, u_x\, v_y \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{\rho,\,x}}{L}\right) \sin\left(\frac{\pi\, x\, a_{u,\,x}}{L}\right)$$

$$+ 6\, L \rho_x\, u_0\, u_y\, v_y \cos\left(\frac{\pi\, y\, a_{u,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{\rho,\,x}}{L}\right)$$

$$+ 18\, L \rho_x\, v_0\, v_x\, v_y \cos\left(\frac{\pi\, x\, a_{v,\,x}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{\rho,\,x}}{L}\right) + 18\, L \rho_x\, v_0$$

$$v_y^2 \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{\rho,\,x}}{L}\right) \sin\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right)$$

$$+ 6\, L \rho_y\, u_0\, u_x\, v_y \cos\left(\frac{\pi\, y\, a_{\rho,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{u,\,x}}{L}\right)$$

$$+ 6\, L \rho_y\, u_0\, u_y\, v_y \cos\left(\frac{\pi\, y\, a_{\rho,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{u,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right)$$

$$+ 18\, L \rho_y\, v_0\, v_x\, v_y \cos\left(\frac{\pi\, y\, a_{\rho,\,y}}{L}\right) \cos\left(\frac{\pi\, x\, a_{v,\,x}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) + 18\, L \rho_y\, v_0$$

$$v_y^2 \cos\left(\frac{\pi\, y\, a_{\rho,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right)$$

$$+ 6\, L \rho_0\, u_0\, u_x\, v_y \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) \sin\left(\frac{\pi\, x\, a_{u,\,x}}{L}\right)$$

$$+ 6\, L \rho_0\, u_0\, u_y\, v_y \cos\left(\frac{\pi\, y\, a_{u,\,y}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right)$$

$$+ 18\, L \rho_0\, v_0\, v_x\, v_y \cos\left(\frac{\pi\, x\, a_{v,\,x}}{L}\right) \cos\left(\frac{\pi\, y\, a_{v,\,y}}{L}\right) + 18\, L \rho_0\, v_0$$

$$v_y^2 \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) + 3 L \rho_x u_0^2 v_y \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right)$$

$$+ 9 L \rho_x v_0^2 v_y \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) + 3 L \rho_y$$

$$u_0^2 v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) + 9 L \rho_y v_0^2 v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right)$$

$$+ 3 L \rho_0 u_0^2 v_y \cos\left(\frac{\pi y a_{v,y}}{L}\right) + 9 L \rho_0 v_0^2 v_y \cos\left(\frac{\pi y a_{v,y}}{L}\right) + \left(-3 L \rho_x\right.$$

$$u_x^2 v_y \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$

$$- 6 L \rho_x u_x u_y v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 3 L \rho_x u_y^2 v_y \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) - 9 L \rho_x$$

$$v_x^2 v_y \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) - 18 L \rho_x v_x$$

$$v_y^2 \cos\left(\frac{\pi x a_{v,x}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 9 L \rho_x$$

$$v_y^3 \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{\rho,x}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 3 L \rho_y$$

$$u_x^2 v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$

$$- 6 L \rho_y u_x u_y v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)$$

$$- 3 L \rho_y u_y^2 v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{u,y}}{L}\right)^2 \cos\left(\frac{\pi y a_{v,y}}{L}\right) - 9 L \rho_y$$

$$v_x^2 v_y \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right)^2 \cos\left(\frac{\pi y a_{v,y}}{L}\right) - 18 L \rho_y v_x$$

$$v_y^2 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi x a_{v,x}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right) - 9 L \rho_y$$

$$v_y^3 \cos\left(\frac{\pi y a_{\rho,y}}{L}\right) \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi y a_{v,y}}{L}\right)^2 - 3 L \rho_0$$

$$u_x^2 v_y \cos\left(\frac{\pi y a_{v,y}}{L}\right) \sin\left(\frac{\pi x a_{u,x}}{L}\right)^2$$

$$-6\,L\rho_0\,u_x\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)-3\,L\rho_0$$

$$u_y^2\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-9\,L\rho_0$$

$$v_x^2\,v_y\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-18\,L\rho_0\,v_x$$

$$v_y^2\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-9\,L\rho_0$$

$$v_y^3\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)^2$$

$$-6\,L\rho_x\,u_0\,u_x\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$-6\,L\rho_x\,u_0\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)$$

$$-18\,L\rho_x\,v_0\,v_x\,v_y\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)-18\,L\rho_x\,v_0$$

$$v_y^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\rho_y\,u_0\,u_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$-6\,L\rho_y\,u_0\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-18\,L\rho_y\,v_0\,v_x\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-18\,L\rho_y\,v_0$$

$$v_y^2\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-6\,L\rho_0\,u_0\,u_x\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{u,\,x}}{L}\right)$$

$$-6\,L\rho_0\,u_0\,u_y\,v_y\cos\left(\frac{\pi\,y\,a_{u,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$

$$-18\,L\rho_0\,v_0\,v_x\,v_y\cos\left(\frac{\pi\,x\,a_{v,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-18\,L\rho_0\,v_0$$

$$v_y^2\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-3\,L\rho_x\,u_0^2\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)$$

$$-9\,L\,\rho_x\,v_0^2\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,x\,a_{\rho,\,x}}{L}\right)-3\,L\,\rho_y$$
$$u_0^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-9\,L\,\rho_y\,v_0^2\,v_y\cos\left(\frac{\pi\,y\,a_{\rho,\,y}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$
$$-3\,L\,two\,p_x\,v_y\cos\left(\frac{\pi\,x\,a_{p,\,x}}{L}\right)\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$
$$-3\,L\,two\,p_y\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\sin\left(\frac{\pi\,y\,a_{p,\,y}}{L}\right)-3\,L\,\rho_0\,u_0^2\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)$$
$$-9\,L\,\rho_0\,v_0^2\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)-3\,L\,two\,p_0\,v_y\cos\left(\frac{\pi\,y\,a_{v,\,y}}{L}\right)\Bigg)\gamma\Bigg)\pi\,a_{v,\,y}\Bigg)$$

```
> #Q=T1+T2+T3+T4+T5+T6+T7+T8.
> #N:=simplify(Q, size)

> #Q=T1+T2+...+T15.
> #-----------------------------------------------------------------
  ------------------------------
> # After the factorization conducted on files
  Euler_equation_2d_e_part1.mw to Euler_equation_2d_e_part5.mw
  and Euler_equation_2d_e_check.mw, it yelds that Q is reduced
  to:
```