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[> #Source term Q for the 2D Navier-Stokes equations -
[> # Energy e

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[>

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$$\begin{aligned}
> Q := & \frac{1}{2} \left(\rho_x \cos\left(\frac{a_{\rho x} \pi x}{L}\right) a_{\rho x} \pi \left(u_0 + u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \right. \right. \\
& + u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \left. \right) \left(2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) + 2 p_y \sin\left(\frac{a_{py} \pi y}{L}\right) + 2 p_0 \right. \\
& + 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - u_0^2 \rho_0 - v_0^2 \rho_0 + u_0^2 \gamma \rho_0 - u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \rho_0 \\
& - u_y^2 \cos^2\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 - u_0^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - u_0^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) - v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 \\
& - v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 + \gamma v_0^2 \rho_0 - v_0^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - v_0^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0 - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma \rho_0 + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_0 \\
& + u_0^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) + u_0^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_0^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) + \gamma v_0^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) rho_{-0} - 2 v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) rho_{-0} \\
& + \gamma v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 rho_{-0} + \gamma v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 rho_{-0} \Big) \Big/ \Big(L (\gamma \\
& - 1) \Big(rho_{-0} + rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) + rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \Big) \Big) \\
& + \frac{1}{2} \frac{1}{L (\gamma - 1)} \Big(u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} \pi \Big(2 p_{-x} \cos\left(\frac{a_{-px} \pi x}{L}\right) \\
& + 2 p_{-y} \sin\left(\frac{a_{-py} \pi y}{L}\right) + 2 p_{-0} \\
& + 2 u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) \gamma rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& + 2 u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) \gamma rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& + 2 \gamma v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& + 2 \gamma v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - u_{-0}^2 rho_{-0} - v_{-0}^2 rho_{-0} + u_{-0}^2 \gamma rho_{-0} - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 rho_{-0} \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 rho_{-0} - u_{-0}^2 rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - u_{-0}^2 rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 rho_{-0} \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 rho_{-0} + \gamma v_{-0}^2 rho_{-0} - v_{-0}^2 rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - v_{-0}^2 rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \gamma rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \gamma rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0 - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma \rho_0 + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_0 \\
& + u_0^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) + u_0^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_0^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) + \gamma v_0^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) rho_{-0} - 2 v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) rho_{-0} \\
& + \gamma v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 rho_{-0} + \gamma v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 rho_{-0} \Big) + \Big(\Big(u_{-0} \\
& + u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) + u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) \Big) \\
& \Big(\pi u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} u_{-0} \gamma rho_{-0}^2 \\
& - \pi u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} u_{-0} rho_{-x}^2 \sin\left(\frac{a_{-rhox} \pi x}{L}\right)^2 \\
& - \pi u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} u_{-0} rho_{-y}^2 \cos\left(\frac{a_{-rho y} \pi y}{L}\right)^2 \\
& + \pi u_{-x}^2 \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} \sin\left(\frac{a_{-ux} \pi x}{L}\right) \gamma rho_{-0}^2 \\
& - \pi u_{-x}^2 \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} \sin\left(\frac{a_{-ux} \pi x}{L}\right) rho_{-x}^2 \sin\left(\frac{a_{-rhox} \pi x}{L}\right)^2 \\
& - \pi u_{-x}^2 \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} \sin\left(\frac{a_{-ux} \pi x}{L}\right) rho_{-y}^2 \cos\left(\frac{a_{-rho y} \pi y}{L}\right)^2 \\
& - \pi u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) rho_{-0}^2 \\
& + \pi v_{-x} \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} v_{-0} rho_{-x}^2 \sin\left(\frac{a_{-rhox} \pi x}{L}\right)^2 \\
& + \pi v_{-x} \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} v_{-0} rho_{-y}^2 \cos\left(\frac{a_{-rho y} \pi y}{L}\right)^2 \\
& + \pi v_{-x}^2 \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} \cos\left(\frac{a_{-vx} \pi x}{L}\right) rho_{-x}^2 \sin\left(\frac{a_{-rhox} \pi x}{L}\right)^2 \\
& + \pi v_{-x}^2 \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} \cos\left(\frac{a_{-vx} \pi x}{L}\right) rho_{-y}^2 \cos\left(\frac{a_{-rho y} \pi y}{L}\right)^2 \\
& + \pi v_{-x} \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) rho_{-0}^2 \\
& - \pi v_{-x} \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} \gamma v_{-0} rho_{-0}^2 \\
& - \pi v_{-x}^2 \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} \gamma \cos\left(\frac{a_{-vx} \pi x}{L}\right) rho_{-0}^2 \\
& - p_{-x} \sin\left(\frac{a_{-px} \pi x}{L}\right) a_{-px} \pi rho_{-0} - \pi u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} u_{-0} rho_{-0}^2 \\
& - \pi u_{-x}^2 \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} \sin\left(\frac{a_{-ux} \pi x}{L}\right) rho_{-0}^2 \\
& + \pi v_{-x} \sin\left(\frac{a_{-vx} \pi x}{L}\right) a_{-vx} v_{-0} rho_{-0}^2
\end{aligned}$$

$$\begin{aligned}
& + \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0^2 \\
& - p_x \sin\left(\frac{a_{px}\pi x}{L}\right) a_{px} \pi \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \gamma \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& + \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \gamma \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& + \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0^2 \\
& - \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& + \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& - 2 \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 \pi v_x \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} v_0 \rho_0 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + 2 \pi v_x \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} v_0 \rho_0 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 \pi v_x^2 \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + 2 \pi v_x^2 \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_0 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_0 \\
& + 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_0 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_y \\
& \cos\left(\frac{1}{L} (a_{\rho y} \pi y)\right) \\
& + 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_0 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \rho_0 \\
& + 2 \pi u_x^2 \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_0 \\
& + 2 \pi u_x^2 \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 \pi u_x^2 \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \rho_0 \\
& + 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_0 \rho_x \sin\left(\frac{1}{L} (a_{\rho x} \pi x)\right) \\
& + 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_0 \rho_y \cos\left(\frac{1}{L} (a_{\rho y} \pi y)\right) \\
& - 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_0 \\
& - 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \\
& \rho_0 \\
& +2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \\
& \rho_0 \\
& +2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \\
& \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \\
& +2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \\
& \rho_0 -2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& -2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_y \\
& \cos\left(1/\left(\frac{L}{a_{rho_y}\pi y}\right)\right) \\
& -2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \rho_0 \\
& -2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& -2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \\
& -2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \rho_0 \\
& -2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \rho_x \sin(1/ \\
& (L)(a_{rho_x}\pi x)) \\
& -2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \rho_y \cos(1/ \\
& (L)(a_{rho_y}\pi y)) \\
& -2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_y \cos(1
\end{aligned}$$

$$\begin{aligned}
& / (L) (a_{rho y} \pi y)) \\
& - 2 \pi u_x^2 \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux} \pi x}{L}\right) rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) rho_y \\
& \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& + 2 \pi v_x \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} v_0 rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) rho_y \cos(1 \\
& / (L) (a_{rho y} \pi y)) \\
& + 2 \pi v_x^2 \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx} \pi x}{L}\right) rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) rho_y \\
& \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& + \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma rho_x^2 \sin\left(\frac{a_{rho x} \pi x}{L}\right)^2 \\
& + 2 \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) \\
& rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& + \pi u_x \cos\left(\frac{a_{ux} \pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma rho_y^2 \cos\left(\frac{a_{rho y} \pi y}{L}\right)^2 \\
& - \pi v_x \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) rho_x^2 \sin\left(\frac{a_{rho x} \pi x}{L}\right)^2 \\
& - 2 \pi v_x \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) \\
& rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& - \pi v_x \sin\left(\frac{a_{vx} \pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) rho_y^2 \cos\left(\frac{a_{rho y} \pi y}{L}\right)^2 \\
& - p_x \sin\left(\frac{a_{px} \pi x}{L}\right) a_{px} \pi rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) \\
& - rho_x \cos\left(\frac{a_{rho x} \pi x}{L}\right) a_{rho x} \pi p_0 \\
& - rho_x \cos\left(\frac{a_{rho x} \pi x}{L}\right) a_{rho x} \pi p_x \cos\left(\frac{a_{px} \pi x}{L}\right) \\
& - rho_x \cos\left(\frac{a_{rho x} \pi x}{L}\right) a_{rho x} \pi p_y \sin\left(\frac{a_{py} \pi y}{L}\right) \Big) \Big) / \Big(\Big(\gamma \\
& - 1 \Big) L \Big(rho_0 + rho_x \sin\left(\frac{a_{rho x} \pi x}{L}\right) + rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \Big) \Big)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \left(\rho_y \sin\left(\frac{a_{\rho y} \pi y}{L}\right) a_{\rho y} \pi \left(v_0 + v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \right. \right. \\
& + v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \left. \left. \right) \left(2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) + 2 p_y \sin\left(\frac{a_{py} \pi y}{L}\right) + 2 p_0 \right. \right. \\
& + 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - u_0^2 \rho_0 - v_0^2 \rho_0 + u_0^2 \gamma \rho_0 - u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \rho_0 \\
& - u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \rho_0 - u_0^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - u_0^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) - v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_0 \\
& - v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_0 + \gamma v_0^2 \rho_0 - v_0^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - v_0^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_0 \\
& - 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& + 2 \gamma v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& + 2 \gamma v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& + 2 \gamma v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& + 2 \gamma v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-0} \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) \rho_{-0} - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) \rho_{-0} \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \gamma \rho_{-0} + u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \gamma \rho_{-0} \\
& + u_{-0}^2 \gamma \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) + u_{-0}^2 \gamma \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& + \gamma v_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) + \gamma v_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - 2 v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-0} - 2 v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-0} \\
& + \gamma v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-0} + \gamma v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-0} \Big) \Big/ \Big(L (\gamma \\
& - 1) \Big(\rho_{-0} + \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) + \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \Big) \Big)
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{1}{L(\gamma-1)} \left(v_{-y} \cos\left(\frac{a_{-vy}\pi y}{L}\right) a_{-vy} \pi \left(2 p_{-x} \cos\left(\frac{a_{-px}\pi x}{L}\right) \right. \right. \\
& + 2 p_{-y} \sin\left(\frac{a_{-py}\pi y}{L}\right) + 2 p_{-0} \\
& + 2 u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \gamma \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& + 2 u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \gamma \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_{-x} \cos\left(\frac{a_{-vx}\pi x}{L}\right) v_{-y} \sin\left(\frac{a_{-vy}\pi y}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_{-x} \cos\left(\frac{a_{-vx}\pi x}{L}\right) v_{-y} \sin\left(\frac{a_{-vy}\pi y}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) \\
& - u_{-0}^2 \rho_{-0} - v_{-0}^2 \rho_{-0} + u_{-0}^2 \gamma \rho_{-0} - u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \rho_{-0} \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy}\pi y}{L}\right)^2 \rho_{-0} - u_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& - u_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) - v_{-x}^2 \cos\left(\frac{a_{-vx}\pi x}{L}\right)^2 \rho_{-0} \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy}\pi y}{L}\right)^2 \rho_{-0} + \gamma v_{-0}^2 \rho_{-0} - v_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& - v_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) \\
& - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \gamma \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right) \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \gamma \rho_{-y} \cos\left(\frac{a_{-\rho y}\pi y}{L}\right) \\
& + 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) \gamma \rho_{-0} + 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \gamma \rho_{-0} \\
& - 2 u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \rho_{-0} \\
& + u_{-y}^2 \cos\left(\frac{a_{-uy}\pi y}{L}\right)^2 \gamma \rho_{-x} \sin\left(\frac{a_{-\rho x}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& + 2 \gamma v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& + 2 \gamma v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& + 2 \gamma v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& + 2 \gamma v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-0} \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) \rho_{-0} - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) \rho_{-0} \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \gamma \rho_{-0} + u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \gamma \rho_{-0} \\
& + u_{-0}^2 \gamma \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) + u_{-0}^2 \gamma \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& + \gamma v_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-\rho x} \pi x}{L}\right) + \gamma v_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-\rho y} \pi y}{L}\right) \\
& - 2 v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-0} - 2 v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-0} \\
& + \gamma v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-0} + \gamma v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-0} \Big) + \Big(\Big(v_{-0} \\
& + v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) + v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \Big) \Big(
\end{aligned}$$

$$\begin{aligned}
& -\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_0^2 \\
& +\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_{0x}^2 \sin\left(\frac{a_{\rho 0x}\pi x}{L}\right)^2 \\
& +\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_0^2 \\
& +\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0^2 \\
& -\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_0^2 \\
& -\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& +\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_{0y}^2 \cos\left(\frac{a_{\rho 0y}\pi y}{L}\right)^2 \\
& +\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0^2 \\
& -\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0^2 \\
& +\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_{0x}^2 \sin\left(\frac{a_{\rho 0x}\pi x}{L}\right)^2 \\
& +\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_{0y}^2 \cos\left(\frac{a_{\rho 0y}\pi y}{L}\right)^2 \\
& -\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_{0x}^2 \sin\left(\frac{a_{\rho 0x}\pi x}{L}\right)^2 \\
& -\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_{0y}^2 \cos\left(\frac{a_{\rho 0y}\pi y}{L}\right)^2 \\
& -\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0^2 \\
& -\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_{0x}^2 \sin\left(\frac{a_{\rho 0x}\pi x}{L}\right)^2 \\
& -\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_{0y}^2 \cos\left(\frac{a_{\rho 0y}\pi y}{L}\right)^2 \\
& +\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_0^2 \\
& +\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& +p_y \cos\left(\frac{a_{py}\pi y}{L}\right) a_{py} \pi \rho_0 \\
& -\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_{0x}^2 \sin\left(\frac{a_{\rho 0x}\pi x}{L}\right)^2
\end{aligned}$$

$$\begin{aligned}
& -\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_{-y}^2 \cos\left(\frac{a_{rho y}\pi y}{L}\right)^2 \\
& -\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_{-0}^2 \\
& +\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_{-x}^2 \sin\left(\frac{a_{rho x}\pi x}{L}\right)^2 \\
& +\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_{-y}^2 \cos\left(\frac{a_{rho y}\pi y}{L}\right)^2 \\
& -\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_{-x}^2 \sin\left(\frac{a_{rho x}\pi x}{L}\right)^2 \\
& -\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_{-y}^2 \cos\left(\frac{a_{rho y}\pi y}{L}\right)^2 \\
& -\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_{-x}^2 \sin\left(\frac{a_{rho x}\pi x}{L}\right)^2 \\
& -\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_{-y}^2 \cos\left(\frac{a_{rho y}\pi y}{L}\right)^2 \\
& +\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_{-x}^2 \sin\left(\frac{a_{rho x}\pi x}{L}\right)^2 \\
& +\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_{-y}^2 \cos\left(\frac{a_{rho y}\pi y}{L}\right)^2 \\
& +\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_{-0}^2 \\
& +\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_{-x}^2 \sin\left(\frac{a_{rho x}\pi x}{L}\right)^2 \\
& +\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_{-y}^2 \cos\left(\frac{a_{rho y}\pi y}{L}\right)^2 \\
& +2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_{-0} \rho_{-x} \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& +2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_{-0} \rho_{-y} \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& +2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_{-0} \rho_{-x} \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& +2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_{-0} \rho_{-y} \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& -2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_{-0} \rho_{-x} \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& -2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_{-0} \rho_{-y} \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& -2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_{-0} \rho_{-x} \sin\left(\frac{a_{rho x}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& -2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \rho_0 \\
& -2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{1}{L}(a_{rho y}\pi y)\right) \\
& -2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \rho_0 \\
& -2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 \rho_x \sin\left(\frac{1}{L}(a_{rho x}\pi x)\right) \\
& -2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 \rho_y \cos\left(\frac{1}{L}(a_{rho y}\pi y)\right) \\
& +2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \rho_0 \\
& +2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& +2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \rho_0 \\
& -2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \rho_0 \\
& -2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& -2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \rho_0
\end{aligned}$$

ρ_0

$$-2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right)$$

ρ_0

$$-2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right)$$

$$\rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)$$

$$-2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)$$

$$\rho_0 + 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0$$

$$+ 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y$$

$$\cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)$$

$$+ 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0$$

$$+ 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 \rho_x \sin\left(\frac{1}{L}(a_{\rho x}\pi x)\right)$$

$$+ 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)$$

$$+ 2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0$$

$$+ 2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y$$

$$\cos\left(\frac{a_{\rho y}\pi y}{L}\right)$$

$$+ 2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0$$

$$+ 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)$$

$$+ 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)$$

$$+ 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)$$

$$+ 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)$$

$$\begin{aligned}
& + 2 \pi u_y^2 \sin\left(\frac{a_{uy} \pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 \pi v_y \cos\left(\frac{a_{vy} \pi y}{L}\right) a_{vy} v_0 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_y \cos(1 \\
& / (L) (a_{\rho y} \pi y)) \\
& - 2 \pi v_y^2 \cos\left(\frac{a_{vy} \pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - \pi u_y \sin\left(\frac{a_{uy} \pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_x^2 \sin\left(\frac{a_{\rho x} \pi x}{L}\right)^2 \\
& - 2 \pi u_y \sin\left(\frac{a_{uy} \pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - \pi u_y \sin\left(\frac{a_{uy} \pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_y^2 \cos\left(\frac{a_{\rho y} \pi y}{L}\right)^2 \\
& + \pi v_y \cos\left(\frac{a_{vy} \pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x} \pi x}{L}\right)^2 \\
& + 2 \pi v_y \cos\left(\frac{a_{vy} \pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + \pi v_y \cos\left(\frac{a_{vy} \pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y} \pi y}{L}\right)^2 \\
& + p_y \cos\left(\frac{a_{py} \pi y}{L}\right) a_{py} \pi \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + p_y \cos\left(\frac{a_{py} \pi y}{L}\right) a_{py} \pi \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + \rho_y \sin\left(\frac{a_{\rho y} \pi y}{L}\right) a_{\rho y} \pi p_0 \\
& + \rho_y \sin\left(\frac{a_{\rho y} \pi y}{L}\right) a_{\rho y} \pi p_x \cos\left(\frac{a_{px} \pi x}{L}\right) \\
& + \rho_y \sin\left(\frac{a_{\rho y} \pi y}{L}\right) a_{\rho y} \pi p_y \sin\left(\frac{a_{py} \pi y}{L}\right) \Big) \Big) / \Big((\gamma - 1) L \Big(\rho_0 \\
& + \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) + \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \Big) \Big) + \Big(k \pi^2 \Big(
\end{aligned}$$

$$\begin{aligned}
& -2 p_x \sin\left(\frac{a_{px} \pi x}{L}\right) a_{px} \rho_x \cos\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox} \rho_{00} \\
& -2 p_x \sin\left(\frac{a_{px} \pi x}{L}\right) a_{px} \rho_x^2 \cos\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox} \sin\left(\frac{a_{rhox} \pi x}{L}\right) \\
& -2 p_x \sin\left(\frac{a_{px} \pi x}{L}\right) a_{px} \rho_x \cos\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox} \rho_y \\
& \cos\left(\frac{1}{L} (a_{rhox} \pi y)\right) \\
& +2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) a_{px}^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{rhox} \pi y}{L}\right) \\
& -\rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox}^2 \rho_{00} p_0 \\
& -\rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox}^2 \rho_{00} p_x \cos\left(\frac{a_{px} \pi x}{L}\right) \\
& -\rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox}^2 \rho_{00} p_y \sin\left(\frac{a_{py} \pi y}{L}\right) \\
& +2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) a_{px}^2 \rho_{00} \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) \\
& -2 \rho_x^2 \cos\left(\frac{a_{rhox} \pi x}{L}\right)^2 a_{rhox}^2 p_0 \\
& -2 \rho_x^2 \cos\left(\frac{a_{rhox} \pi x}{L}\right)^2 a_{rhox}^2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) \\
& -2 \rho_x^2 \cos\left(\frac{a_{rhox} \pi x}{L}\right)^2 a_{rhox}^2 p_y \sin\left(\frac{a_{py} \pi y}{L}\right) \\
& -\rho_x^2 \sin\left(\frac{a_{rhox} \pi x}{L}\right)^2 a_{rhox}^2 p_0 \\
& -\rho_x^2 \sin\left(\frac{a_{rhox} \pi x}{L}\right)^2 a_{rhox}^2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) \\
& -\rho_x^2 \sin\left(\frac{a_{rhox} \pi x}{L}\right)^2 a_{rhox}^2 p_y \sin\left(\frac{a_{py} \pi y}{L}\right) \\
& +p_x \cos\left(\frac{a_{px} \pi x}{L}\right) a_{px}^2 \rho_{00}^2 \\
& +p_x \cos\left(\frac{a_{px} \pi x}{L}\right) a_{px}^2 \rho_y^2 \cos\left(\frac{a_{rhox} \pi y}{L}\right)^2 \\
& -\rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox}^2 \rho_y \cos\left(\frac{a_{rhox} \pi y}{L}\right) p_0 \\
& -\rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox}^2 \rho_y \cos\left(\frac{a_{rhox} \pi y}{L}\right) p_x \cos\left(\frac{a_{px} \pi x}{L}\right) \\
& -\rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) a_{rhox}^2 \rho_y \cos\left(\frac{a_{rhox} \pi y}{L}\right) p_y \sin\left(\frac{a_{py} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + p_x \cos\left(\frac{a_{px}\pi x}{L}\right) a_{px}^2 \rho_{0x}^2 \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 p_x \cos\left(\frac{a_{px}\pi x}{L}\right) a_{px}^2 \rho_{0x} \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) \Big) \Big) / \\
& \left(R \left(\rho_{0x} + \rho_{0x} \sin\left(\frac{a_{rhox}\pi x}{L}\right) + \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) \right)^3 L^2 \right) \\
& + \left(k^2 \left(-2 p_y \cos\left(\frac{a_{py}\pi y}{L}\right) a_{py} \rho_{0y} \sin\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy} \rho_{0x} - 2 p_y \cos\left(\frac{a_{py}\pi y}{L}\right) \right. \right. \\
& \left. \left. - 2 p_y \cos\left(\frac{a_{py}\pi y}{L}\right) a_{py} \rho_{0y}^2 \sin\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) \right. \right. \\
& \left. \left. (L) (a_{rhoxy}\pi y) \right) \right. \\
& + 2 p_y \sin\left(\frac{a_{py}\pi y}{L}\right) a_{py}^2 \rho_{0x} \sin\left(\frac{a_{rhox}\pi x}{L}\right) \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) \\
& - \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy}^2 \rho_{0x} p_{0x} \\
& - \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy}^2 \rho_{0x} p_x \cos\left(\frac{a_{px}\pi x}{L}\right) \\
& - \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy}^2 \rho_{0x} p_y \sin\left(\frac{a_{py}\pi y}{L}\right) \\
& + 2 p_y \sin\left(\frac{a_{py}\pi y}{L}\right) a_{py}^2 \rho_{0x} \rho_{0x} \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 \rho_{0y}^2 \sin\left(\frac{a_{rhoxy}\pi y}{L}\right)^2 a_{rhoxy}^2 p_{0x} \\
& - 2 \rho_{0y}^2 \sin\left(\frac{a_{rhoxy}\pi y}{L}\right)^2 a_{rhoxy}^2 p_x \cos\left(\frac{a_{px}\pi x}{L}\right) \\
& - 2 \rho_{0y}^2 \sin\left(\frac{a_{rhoxy}\pi y}{L}\right)^2 a_{rhoxy}^2 p_y \sin\left(\frac{a_{py}\pi y}{L}\right) \\
& - \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy}^2 \rho_{0x} \sin\left(\frac{a_{rhox}\pi x}{L}\right) p_{0y} \\
& - \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy}^2 \rho_{0x} \sin\left(\frac{a_{rhox}\pi x}{L}\right) p_x \cos\left(\frac{a_{px}\pi x}{L}\right) \\
& - \rho_{0y} \cos\left(\frac{a_{rhoxy}\pi y}{L}\right) a_{rhoxy}^2 \rho_{0x} \sin\left(\frac{a_{rhox}\pi x}{L}\right) p_y \sin\left(\frac{a_{py}\pi y}{L}\right) \\
& + p_y \sin\left(\frac{a_{py}\pi y}{L}\right) a_{py}^2 \rho_{0x}^2 \\
& + p_y \sin\left(\frac{a_{py}\pi y}{L}\right) a_{py}^2 \rho_{0y}^2 \cos\left(\frac{a_{rhoxy}\pi y}{L}\right)^2 \\
& - \rho_{0y}^2 \cos\left(\frac{a_{rhoxy}\pi y}{L}\right)^2 a_{rhoxy}^2 p_{0x}
\end{aligned}$$

$$\begin{aligned}
& -\rho_y^2 \cos\left(\frac{a_{rho_y \pi y}}{L}\right)^2 a_{rho_y^2} p_x \cos\left(\frac{a_{p_x \pi x}}{L}\right) \\
& -\rho_y^2 \cos\left(\frac{a_{rho_y \pi y}}{L}\right)^2 a_{rho_y^2} p_y \sin\left(\frac{a_{p_y \pi y}}{L}\right) \\
& + p_y \sin\left(\frac{a_{p_y \pi y}}{L}\right) a_{p_y^2} \rho_x^2 \sin\left(\frac{a_{rho_x \pi x}}{L}\right)^2 \\
& + 2 p_y \sin\left(\frac{a_{p_y \pi y}}{L}\right) a_{p_y^2} \rho_0 \rho_y \cos\left(\frac{a_{rho_y \pi y}}{L}\right) \Big) \Big) / \\
& \left(R \left(\rho_0 + \rho_x \sin\left(\frac{a_{rho_x \pi x}}{L}\right) + \rho_y \cos\left(\frac{a_{rho_y \pi y}}{L}\right) \right)^3 L^2 \right) \\
& + \frac{1}{L} \left(\pi \left(p_0 + p_x \cos\left(\frac{a_{p_x \pi x}}{L}\right) \right. \right. \\
& + p_y \sin\left(\frac{a_{p_y \pi y}}{L}\right) \Big) \left(v_y \cos\left(\frac{a_{v_y \pi y}}{L}\right) a_{v_y} \right. \\
& + u_x \cos\left(\frac{a_{u_x \pi x}}{L}\right) a_{u_x} \Big) \Big) + \frac{1}{3} \frac{1}{L^2} \left(\left(v_0 + v_x \cos\left(\frac{a_{v_x \pi x}}{L}\right) \right. \right. \\
& + v_y \sin\left(\frac{a_{v_y \pi y}}{L}\right) \Big) \left(3 p_y \cos\left(\frac{a_{p_y \pi y}}{L}\right) a_{p_y} \pi L \right. \\
& + 3 \mu v_x \cos\left(\frac{a_{v_x \pi x}}{L}\right) a_{v_x^2} \pi^2 + 4 \mu v_y \sin\left(\frac{a_{v_y \pi y}}{L}\right) a_{v_y^2} \pi^2 \Big) \Big) \\
& + \frac{1}{3} \frac{1}{L^2} \left(\left(u_0 + u_x \sin\left(\frac{a_{u_x \pi x}}{L}\right) \right. \right. \\
& + u_y \cos\left(\frac{a_{u_y \pi y}}{L}\right) \Big) \left(4 \mu u_x \sin\left(\frac{a_{u_x \pi x}}{L}\right) a_{u_x^2} \pi^2 \right. \\
& - 3 p_x \sin\left(\frac{a_{p_x \pi x}}{L}\right) a_{p_x} \pi L + 3 \mu u_y \cos\left(\frac{a_{u_y \pi y}}{L}\right) a_{u_y^2} \pi^2 \Big) \Big) \\
& - \frac{2}{3} \frac{1}{L} \left(u_x \cos\left(\frac{a_{u_x \pi x}}{L}\right) a_{u_x} \pi \mu \left(\frac{2 u_x \cos\left(\frac{a_{u_x \pi x}}{L}\right) a_{u_x} \pi}{L} \right. \right. \\
& - \frac{v_y \cos\left(\frac{a_{v_y \pi y}}{L}\right) a_{v_y} \pi}{L} \Big) \Big) + \frac{1}{L} \left(v_x \sin\left(\frac{a_{v_x \pi x}}{L}\right) a_{v_x} \pi \mu \left(\right. \right. \\
& - \frac{u_y \sin\left(\frac{a_{u_y \pi y}}{L}\right) a_{u_y} \pi}{L} - \frac{v_x \sin\left(\frac{a_{v_x \pi x}}{L}\right) a_{v_x} \pi}{L} \Big) \Big) \\
& + \frac{1}{L} \left(u_y \sin\left(\frac{a_{u_y \pi y}}{L}\right) a_{u_y} \pi \mu \left(- \frac{u_y \sin\left(\frac{a_{u_y \pi y}}{L}\right) a_{u_y} \pi}{L} \right. \right. \\
& - \frac{v_x \sin\left(\frac{a_{v_x \pi x}}{L}\right) a_{v_x} \pi}{L} \Big) \Big)
\end{aligned}$$

$$\left[\begin{array}{l} -\frac{2}{3} \frac{1}{L} \left(v_y \cos \left(\frac{a_{vy} \pi y}{L} \right) a_{vy} \pi \mu \left(\frac{2 v_y \cos \left(\frac{a_{vy} \pi y}{L} \right) a_{vy} \pi}{L} \right. \right. \\ \left. \left. - \frac{u_x \cos \left(\frac{a_{ux} \pi x}{L} \right) a_{ux} \pi}{L} \right) \right) \end{array} \right]$$