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> #Source term Q for the 2D Euler equations -
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> # Energy e
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$$\begin{aligned} > Q := & \frac{1}{2} \left( \rho_x \cos\left(\frac{a_{\rho x} \pi x}{L}\right) a_{\rho x} \pi \left( u_0 + u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \right. \right. \\ & + u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \left. \left. \right) \left( 2 p_x \cos\left(\frac{a_{px} \pi x}{L}\right) + 2 p_y \sin\left(\frac{a_{py} \pi y}{L}\right) + 2 p_0 \right. \right. \\ & - u_0^2 \rho_0 - v_0^2 \rho_0 - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_0 \\ & - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 + u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_0 \\ & + u_y^2 \cos^2\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_0 + u_0^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\ & + u_0^2 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\ & - u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\ & - u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\ & - u_y^2 \cos^2\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\ & - u_y^2 \cos^2\left(\frac{a_{uy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\ & - 2 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 - 2 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 \\ & + \gamma v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 + \gamma v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 \\ & - v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\ & - v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\ & - v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\ & - v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\ & + \gamma v_0^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) + \gamma v_0^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\ & + u_0^2 \gamma \rho_0 - u_x^2 \sin^2\left(\frac{a_{ux} \pi x}{L}\right) \rho_0 - u_y^2 \cos^2\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 \end{aligned}$$

$$\begin{aligned}
& -u_0^2 \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) - u_0^2 \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& -v_x^2 \cos\left(\frac{a_{v x} \pi x}{L}\right)^2 \rho_{00} - v_y^2 \sin\left(\frac{a_{v y} \pi y}{L}\right)^2 \rho_{00} + \gamma v_0^2 \rho_{00} \\
& -v_0^2 \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) - v_0^2 \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \gamma \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \gamma \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{v x} \pi x}{L}\right) v_y \sin\left(\frac{a_{v y} \pi y}{L}\right) \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{v x} \pi x}{L}\right) v_y \sin\left(\frac{a_{v y} \pi y}{L}\right) \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \gamma \rho_{00} \\
& + 2 u_0 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) \gamma \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) \gamma \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \gamma \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \gamma \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{u x} \pi x}{L}\right) u_y \cos\left(\frac{a_{u y} \pi y}{L}\right) \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{v x} \pi x}{L}\right) v_y \sin\left(\frac{a_{v y} \pi y}{L}\right) \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{v x} \pi x}{L}\right) v_y \sin\left(\frac{a_{v y} \pi y}{L}\right) \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{v x} \pi x}{L}\right) \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{v x} \pi x}{L}\right) \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{v y} \pi y}{L}\right) \rho_{0x} \sin\left(\frac{a_{\rho 0 x} \pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{v y} \pi y}{L}\right) \rho_{0y} \cos\left(\frac{a_{\rho 0 y} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_0
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-0} + 2 \gamma v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-0} \Big) \Big) / \\
& \left( L(\gamma - 1) \left( \rho_{-0} + \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) + \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \right) \right) \\
& + \frac{1}{2} \frac{1}{L(\gamma - 1)} \left( u_{-x} \cos\left(\frac{a_{-ux} \pi x}{L}\right) a_{-ux} \pi \left( 2 p_{-x} \cos\left(\frac{a_{-px} \pi x}{L}\right) \right. \right. \\
& + 2 p_{-y} \sin\left(\frac{a_{-py} \pi y}{L}\right) + 2 p_{-0} - u_{-0}^2 \rho_{-0} - v_{-0}^2 \rho_{-0} \\
& - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux} \pi x}{L}\right) \rho_{-0} - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy} \pi y}{L}\right) \rho_{-0} \\
& + u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \gamma \rho_{-0} + u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \gamma \rho_{-0} \\
& + u_{-0}^2 \gamma \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) + u_{-0}^2 \gamma \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - 2 v_{-0} v_{-x} \cos\left(\frac{a_{-vx} \pi x}{L}\right) \rho_{-0} - 2 v_{-0} v_{-y} \sin\left(\frac{a_{-vy} \pi y}{L}\right) \rho_{-0} \\
& + \gamma v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-0} + \gamma v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-0} \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx} \pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy} \pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& + \gamma v_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) + \gamma v_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right) \\
& + u_{-0}^2 \gamma \rho_{-0} - u_{-x}^2 \sin\left(\frac{a_{-ux} \pi x}{L}\right)^2 \rho_{-0} - u_{-y}^2 \cos\left(\frac{a_{-uy} \pi y}{L}\right)^2 \rho_{-0} \\
& - u_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-rhox} \pi x}{L}\right) - u_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-rho y} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_0 - v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_0 + \gamma v_x \rho_0^2 \rho_0 \\
& - v_x \rho_0^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) - v_y \rho_0^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0
\end{aligned}$$

$$\begin{aligned}
& -2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \Big) \Big)
\end{aligned}$$

$$\begin{aligned}
& + \left( \left( u_0 + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \right. \right. \\
& + \left. u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \right) \left( \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \gamma \rho_0^2 \right. \\
& - \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0^2 \\
& - \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0^2 \\
& + \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_0 \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& + \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_0 \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& + \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& - \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_0^2 \\
& - \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0^2 \\
& - \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_0^2 \\
& - \pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0^2 \\
& + \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_0 \rho_0^2 \\
& + \pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0^2 \\
& - p_x \sin\left(\frac{a_{px}\pi x}{L}\right) a_{px} \pi \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -p_x \sin\left(\frac{a_{px}\pi x}{L}\right) a_{px}\pi \rho_0 - \rho_x \cos\left(\frac{a_{\rho x}\pi x}{L}\right) a_{\rho x}\pi p_0 \\
& -2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0 \\
& -2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0 \\
& -2\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0 \\
& -2\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0 \\
& +2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0 \\
& +2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}v_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0 \\
& +2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0 \\
& +2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0 \\
& +\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_0 \gamma \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& +\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_0 \gamma \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& +\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& +\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& +\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0^2 \\
& -\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& -\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& +\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& +\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& -\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& -\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_0 \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2
\end{aligned}$$



$$\begin{aligned}
& -\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& -\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& -\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& + 2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \gamma \rho_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \gamma \rho_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos(1 \\
& / (L)(a_{\rho y}\pi y)) \\
& + 2\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2\pi u_x^2 \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \rho_x \sin(1/ \\
& (L)(a_{\rho x}\pi x)) \\
& - 2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \rho_y \cos(1/ \\
& (L)(a_{\rho y}\pi y)) \\
& + 2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos(1 \\
& / (L)(a_{\rho y}\pi y)) \\
& + 2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \rho_x \sin(1/
\end{aligned}$$

$$(L)(a_{rho x \pi x}))$$

$$+ 2 \pi v_x \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy \pi y}}{L}\right) rho_0 rho_y \cos(1/$$

$$(L)(a_{rho y \pi y}))$$

$$- 2 \pi v_x \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} \gamma v_0 rho_0 rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right)$$

$$- 2 \pi v_x \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} \gamma v_0 rho_0 rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right)$$

$$- 2 \pi v_x^2 \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx \pi x}}{L}\right) rho_0 rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right)$$

$$- 2 \pi v_x^2 \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} \gamma \cos\left(\frac{a_{vx \pi x}}{L}\right) rho_0 rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right)$$

$$+ 2 \pi u_x \cos\left(\frac{a_{ux \pi x}}{L}\right) a_{ux} u_0 \gamma rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) rho_y$$

$$\cos\left(\frac{1}{L}(a_{rho y \pi y})\right)$$

$$+ 2 \pi u_x^2 \cos\left(\frac{a_{ux \pi x}}{L}\right) a_{ux} \sin\left(\frac{a_{ux \pi x}}{L}\right) \gamma rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) rho_y$$

$$\cos\left(\frac{a_{rho y \pi y}}{L}\right)$$

$$+ 2 \pi u_x \cos\left(\frac{a_{ux \pi x}}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy \pi y}}{L}\right) \gamma rho_0 rho_x \sin(1$$

$$/(L)(a_{rho x \pi x}))$$

$$+ 2 \pi u_x \cos\left(\frac{a_{ux \pi x}}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy \pi y}}{L}\right) \gamma rho_0 rho_y \cos(1$$

$$/(L)(a_{rho y \pi y}))$$

$$- 2 \pi u_x \cos\left(\frac{a_{ux \pi x}}{L}\right) a_{ux} u_y \cos\left(\frac{a_{uy \pi y}}{L}\right) rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right)$$

$$rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right)$$

$$+ 2 \pi v_x \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} v_y \sin\left(\frac{a_{vy \pi y}}{L}\right) rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right)$$

$$rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right)$$

$$- 2 \pi v_x \sin\left(\frac{a_{vx \pi x}}{L}\right) a_{vx} \gamma v_0 rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) rho_y$$

$$\begin{aligned}
& \cos\left(\frac{1}{L}(a_{rho}y\pi y)\right) \\
& - 2\pi v_x^2 \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}\gamma \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) rho_y \\
& \cos\left(\frac{a_{rho}y\pi y}{L}\right) \\
& - 2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}\gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 rho_x \sin\left(\frac{1}{L}(a_{rho}x\pi x)\right) \\
& (L)(a_{rho}x\pi x)) \\
& - 2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}\gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 rho_y \cos\left(\frac{1}{L}(a_{rho}y\pi y)\right) \\
& (L)(a_{rho}y\pi y)) \\
& + \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x^2 \sin\left(\frac{a_{rho}x\pi x}{L}\right)^2 \\
& + 2\pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) \\
& rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) \\
& + \pi u_x \cos\left(\frac{a_{ux}\pi x}{L}\right) a_{ux}u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y^2 \cos\left(\frac{a_{rho}y\pi y}{L}\right)^2 \\
& - \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}\gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x^2 \sin\left(\frac{a_{rho}x\pi x}{L}\right)^2 \\
& - 2\pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}\gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) \\
& rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) \\
& - \pi v_x \sin\left(\frac{a_{vx}\pi x}{L}\right) a_{vx}\gamma v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y^2 \cos\left(\frac{a_{rho}y\pi y}{L}\right)^2 \\
& - p_x \sin\left(\frac{a_{px}\pi x}{L}\right) a_{px}\pi rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) \\
& - rho_x \cos\left(\frac{a_{rho}x\pi x}{L}\right) a_{rho}x\pi p_x \cos\left(\frac{a_{px}\pi x}{L}\right) \\
& - rho_x \cos\left(\frac{a_{rho}x\pi x}{L}\right) a_{rho}x\pi p_y \sin\left(\frac{a_{py}\pi y}{L}\right) \Big) \Big) \Big) / \Big( (\gamma \\
& - 1) L \Big( rho_0 + rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) + rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) \Big) \Big) \\
& - \frac{1}{2} \Big( rho_y \sin\left(\frac{a_{rho}y\pi y}{L}\right) a_{rho}y\pi \Big( v_0 + v_x \cos\left(\frac{a_{vx}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + v_{-y} \sin\left(\frac{a_{-vy}\pi y}{L}\right) \left( 2 p_{-x} \cos\left(\frac{a_{-px}\pi x}{L}\right) + 2 p_{-y} \sin\left(\frac{a_{-py}\pi y}{L}\right) + 2 p_{-0} \right. \\
& - u_{-0}^2 \rho_{-0} - v_{-0}^2 \rho_{-0} - 2 u_{-0} u_{-x} \sin\left(\frac{a_{-ux}\pi x}{L}\right) \rho_{-0} \\
& - 2 u_{-0} u_{-y} \cos\left(\frac{a_{-uy}\pi y}{L}\right) \rho_{-0} + u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \gamma \rho_{-0} \\
& + u_{-y}^2 \cos\left(\frac{a_{-uy}\pi y}{L}\right)^2 \gamma \rho_{-0} + u_{-0}^2 \gamma \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) \\
& + u_{-0}^2 \gamma \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) \\
& - u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy}\pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) \\
& - u_{-y}^2 \cos\left(\frac{a_{-uy}\pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& - 2 v_{-0} v_{-x} \cos\left(\frac{a_{-vx}\pi x}{L}\right) \rho_{-0} - 2 v_{-0} v_{-y} \sin\left(\frac{a_{-vy}\pi y}{L}\right) \rho_{-0} \\
& + \gamma v_{-x}^2 \cos\left(\frac{a_{-vx}\pi x}{L}\right)^2 \rho_{-0} + \gamma v_{-y}^2 \sin\left(\frac{a_{-vy}\pi y}{L}\right)^2 \rho_{-0} \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx}\pi x}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx}\pi x}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy}\pi y}{L}\right)^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) \\
& - v_{-y}^2 \sin\left(\frac{a_{-vy}\pi y}{L}\right)^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& + \gamma v_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) + \gamma v_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& + u_{-0}^2 \gamma \rho_{-0} - u_{-x}^2 \sin\left(\frac{a_{-ux}\pi x}{L}\right)^2 \rho_{-0} - u_{-y}^2 \cos\left(\frac{a_{-uy}\pi y}{L}\right)^2 \rho_{-0} \\
& - u_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) - u_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right) \\
& - v_{-x}^2 \cos\left(\frac{a_{-vx}\pi x}{L}\right)^2 \rho_{-0} - v_{-y}^2 \sin\left(\frac{a_{-vy}\pi y}{L}\right)^2 \rho_{-0} + \gamma v_{-0}^2 \rho_{-0} \\
& - v_{-0}^2 \rho_{-x} \sin\left(\frac{a_{-rhox}\pi x}{L}\right) - v_{-0}^2 \rho_{-y} \cos\left(\frac{a_{-rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0 \\
& -2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& -2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \Big) \Big) / \\
& \left( L(\gamma-1) \left( \rho_0 + \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) + \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \right) \right) \\
& + \frac{1}{2} \frac{1}{L(\gamma-1)} \left( v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy}\pi \left( 2 p_x \cos\left(\frac{a_{px}\pi x}{L}\right) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 2 p_y \sin\left(\frac{a_{py} \pi y}{L}\right) + 2 p_0 - u_0^2 \rho_0 - v_0^2 \rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \rho_0 - 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_0 + u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \gamma \rho_0 \\
& + u_0^2 \gamma \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) + u_0^2 \gamma \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& - u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) \\
& - u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& - u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) \\
& - u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 - 2 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_0 + \gamma v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_0 \\
& - v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) \\
& - v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& - v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) \\
& - v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& + \gamma v_0^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) + \gamma v_0^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& + u_0^2 \gamma \rho_0 - u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \rho_0 - u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \rho_0 \\
& - u_0^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) - u_0^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& - v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_0 - v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_0 + \gamma v_0^2 \rho_0 \\
& - v_0^2 \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right) - v_0^2 \rho_y \cos\left(\frac{a_{rho y} \pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{rhox} \pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0 \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right)
\end{aligned}$$



$$\begin{aligned}
& -2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux} \pi x}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) \gamma \rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \gamma \rho_0 \\
& + u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \gamma \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy} \pi y}{L}\right)^2 \gamma \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_{ux} \pi x}{L}\right) u_y \cos\left(\frac{a_{uy} \pi y}{L}\right) \rho_0 \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx} \pi x}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_x \sin\left(\frac{a_{\rho x} \pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_{vy} \pi y}{L}\right)^2 \rho_y \cos\left(\frac{a_{\rho y} \pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) \rho_0 + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \rho_0 \Big) \Big) \\
& + \left( \left( v_0 + v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) + v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) \right) \left( \right. \right. \\
& - \pi u_y \sin\left(\frac{a_{uy} \pi y}{L}\right) a_{uy} u_0 \gamma \rho_0^2 \\
& \left. \left. + \pi u_y \sin\left(\frac{a_{uy} \pi y}{L}\right) a_{uy} u_0 \rho_x^2 \sin\left(\frac{a_{\rho x} \pi x}{L}\right)^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_0^2 \\
& - \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_0^2 \\
& + \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& + \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0^2 \\
& - \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& + \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_0^2 \\
& + \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& + p_y \cos\left(\frac{a_{py}\pi y}{L}\right) a_{py} \pi \rho_0 + \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_0^2 \\
& + \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_0^2 \\
& - \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_0^2 \\
& - \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_0^2 \\
& - \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2 \\
& - \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_y^2 \cos\left(\frac{a_{\rho y}\pi y}{L}\right)^2 \\
& - \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma \rho_0^2 \\
& + \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{\rho x}\pi x}{L}\right)^2
\end{aligned}$$

$$\begin{aligned}
& + \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{rho_y}\pi y}{L}\right)^2 \\
& - \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_x^2 \sin\left(\frac{a_{rho_x}\pi x}{L}\right)^2 \\
& - \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma \rho_y^2 \cos\left(\frac{a_{rho_y}\pi y}{L}\right)^2 \\
& - \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x^2 \sin\left(\frac{a_{rho_x}\pi x}{L}\right)^2 \\
& - \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y^2 \cos\left(\frac{a_{rho_y}\pi y}{L}\right)^2 \\
& + \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_x^2 \sin\left(\frac{a_{rho_x}\pi x}{L}\right)^2 \\
& + \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_y^2 \cos\left(\frac{a_{rho_y}\pi y}{L}\right)^2 \\
& + \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0^2 \\
& + \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x^2 \sin\left(\frac{a_{rho_x}\pi x}{L}\right)^2 \\
& + \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y^2 \cos\left(\frac{a_{rho_y}\pi y}{L}\right)^2 \\
& + 2 \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& + 2 \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \rho_0 \\
& + 2 \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& + 2 \pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \rho_0 \\
& - 2 \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& - 2 \pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \rho_0 \\
& - 2 \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& - 2 \pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{rho_y}\pi y}{L}\right) \rho_0 \\
& - 2 \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_0 \\
& - 2 \pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma \rho_x \sin\left(\frac{a_{rho_x}\pi x}{L}\right) \rho_y
\end{aligned}$$

$$\begin{aligned}
& \cos\left(\frac{1}{L}(a_{rho}y\pi y)\right) \\
& - 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \gamma rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) rho_0 \\
& - 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_0 rho_x \sin\left(\frac{1}{L}(a_{rho}x\pi x)\right) \\
& - 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_0 rho_y \cos\left(\frac{1}{L}(a_{rho}y\pi y)\right) \\
& + 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) rho_0 \\
& + 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) \\
& + 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) rho_0 \\
& - 2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) rho_0 \\
& - 2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) \\
& - 2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_{rho}y\pi y}{L}\right) rho_0 \\
& - 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rho}x\pi x}{L}\right) rho_0
\end{aligned}$$

$$\begin{aligned}
& -2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
& \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& -2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& \rho_0 + 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0 \\
& + 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right) \\
& + 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_0 \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0 \\
& + 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 \rho_x \sin\left(\frac{1}{L}(a_{\rho x}\pi x)\right) \\
& + 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_0 \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right) \\
& + 2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_0 \\
& + 2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& + 2\pi v_y^2 \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} \gamma \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \rho_0 \\
& + 2\pi u_y \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} u_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right) \\
& + 2\pi u_y^2 \sin\left(\frac{a_{uy}\pi y}{L}\right) a_{uy} \cos\left(\frac{a_{uy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \\
& \cos\left(\frac{a_{\rho y}\pi y}{L}\right) \\
& - 2\pi v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} v_0 \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \rho_y \cos\left(\frac{1}{L}(a_{\rho y}\pi y)\right)
\end{aligned}$$

$$\begin{aligned}
& / (L) (a_{rho y \pi y})) \\
& - 2 \pi v_y^2 \cos\left(\frac{a_{vy \pi y}}{L}\right) a_{vy} \sin\left(\frac{a_{vy \pi y}}{L}\right) rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) rho_y \\
& \cos\left(\frac{a_{rho y \pi y}}{L}\right) \\
& - \pi u_y \sin\left(\frac{a_{uy \pi y}}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux \pi x}}{L}\right) \gamma rho_x^2 \sin\left(\frac{a_{rho x \pi x}}{L}\right)^2 \\
& - 2 \pi u_y \sin\left(\frac{a_{uy \pi y}}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux \pi x}}{L}\right) \gamma rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) \\
& rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right) \\
& - \pi u_y \sin\left(\frac{a_{uy \pi y}}{L}\right) a_{uy} u_x \sin\left(\frac{a_{ux \pi x}}{L}\right) \gamma rho_y^2 \cos\left(\frac{a_{rho y \pi y}}{L}\right)^2 \\
& + \pi v_y \cos\left(\frac{a_{vy \pi y}}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx \pi x}}{L}\right) rho_x^2 \sin\left(\frac{a_{rho x \pi x}}{L}\right)^2 \\
& + 2 \pi v_y \cos\left(\frac{a_{vy \pi y}}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx \pi x}}{L}\right) rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) \\
& rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right) \\
& + \pi v_y \cos\left(\frac{a_{vy \pi y}}{L}\right) a_{vy} \gamma v_x \cos\left(\frac{a_{vx \pi x}}{L}\right) rho_y^2 \cos\left(\frac{a_{rho y \pi y}}{L}\right)^2 \\
& + p_y \cos\left(\frac{a_{py \pi y}}{L}\right) a_{py} \pi rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) \\
& + p_y \cos\left(\frac{a_{py \pi y}}{L}\right) a_{py} \pi rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right) \\
& + rho_y \sin\left(\frac{a_{rho y \pi y}}{L}\right) a_{rho y} \pi p_x \cos\left(\frac{a_{px \pi x}}{L}\right) \\
& + rho_y \sin\left(\frac{a_{rho y \pi y}}{L}\right) a_{rho y} \pi p_y \sin\left(\frac{a_{py \pi y}}{L}\right) \\
& + rho_y \sin\left(\frac{a_{rho y \pi y}}{L}\right) a_{rho y} \pi p_0 \Big) \Big) / \Big( (\gamma - 1) L \Big( rho_0 \\
& + rho_x \sin\left(\frac{a_{rho x \pi x}}{L}\right) + rho_y \cos\left(\frac{a_{rho y \pi y}}{L}\right) \Big) \Big) + \frac{1}{L} \Big( \pi \Big( p_0 \\
& + p_x \cos\left(\frac{a_{px \pi x}}{L}\right) + p_y \sin\left(\frac{a_{py \pi y}}{L}\right) \Big) \Big( v_y \cos\left(\frac{a_{vy \pi y}}{L}\right) a_{vy} \\
& + u_x \cos\left(\frac{a_{ux \pi x}}{L}\right) a_{ux} \Big) \Big) \\
& + \frac{1}{L} \Big( p_y \cos\left(\frac{a_{py \pi y}}{L}\right) a_{py} \pi \Big( v_0 + v_x \cos\left(\frac{a_{vx \pi x}}{L}\right)
\end{aligned}$$

$$\left[ \begin{aligned}
 &+ v_y \sin\left(\frac{a_v y \pi y}{L}\right) \right) \right) \\
 &- \frac{1}{L} \left( p_x \sin\left(\frac{a_p x \pi x}{L}\right) a_p x \pi \left( u_0 + u_x \sin\left(\frac{a_u x \pi x}{L}\right) \right. \right. \\
 &\left. \left. + u_y \cos\left(\frac{a_u y \pi y}{L}\right) \right) \right) :
 \end{aligned} \right] >$$