



The Abdus Salam  
**International Centre**  
for Theoretical Physics

Kings College, London  
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# ANDERSON LOCALIZATION ON A BETHE LATTICE AND RELATED GRAPHS: THE PROBLEM OF EXISTENCE OF NON-ERGODIC EXTENDED PHASE

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Emilio Cuevas, Murcia

Andrea de Luca, Paris

Manuel Pino, Rutgers

Antonello Scardicchio, ICTP

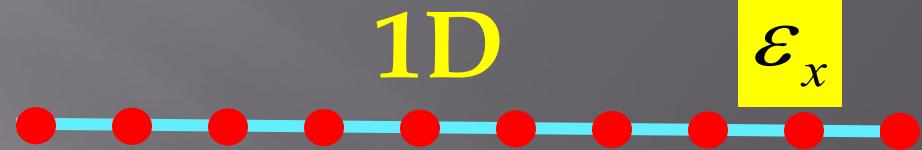
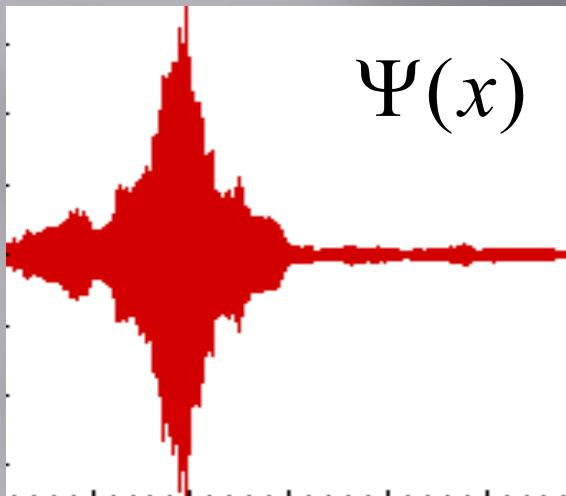
Mohsen Amini, Isfahan

V.E.Kravtsov ICTP, Trieste

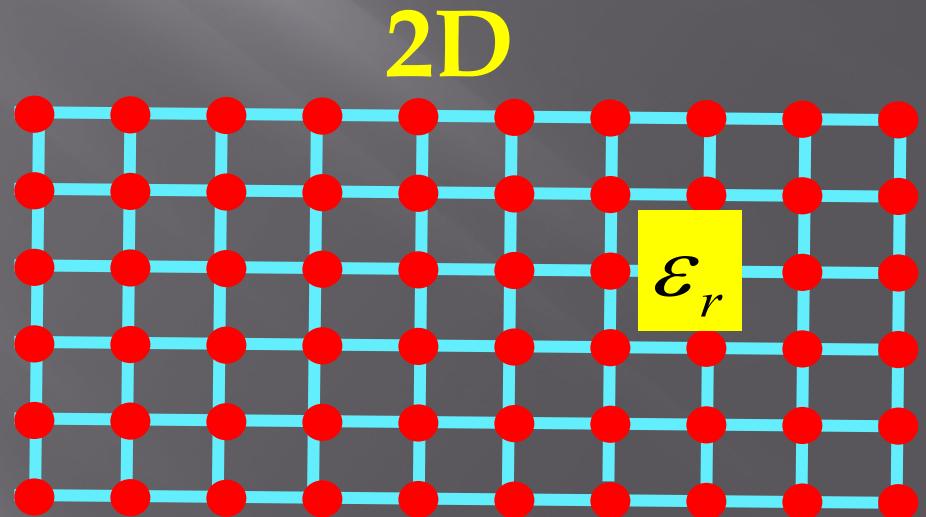
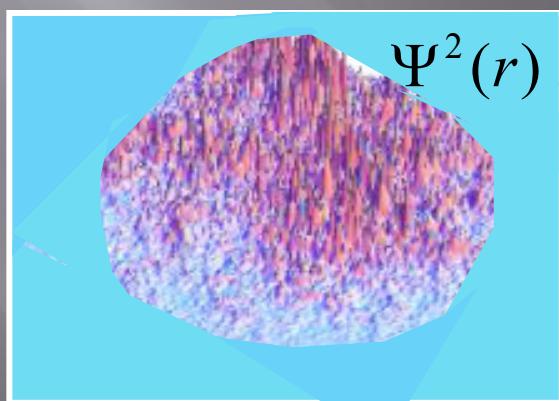


PRL v.113, 046806 (2014),  
New J. Phys. v.17, 122002 (2015)  
PRL v. 117, 156601 (2016),  
ArXiv: 1610.00758

# “Single particle” localization on one -and two-dimensional lattices



Simple regular networks with on-site energy disorder and  $N=L^D$  size of configurational space



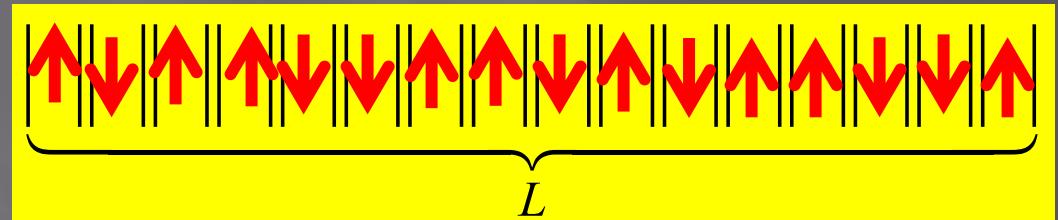
# Many-Body Hilbert space

basic state in Many-Body  
Hilbert/Fock Space

One-  
dimensional  
spin-chains

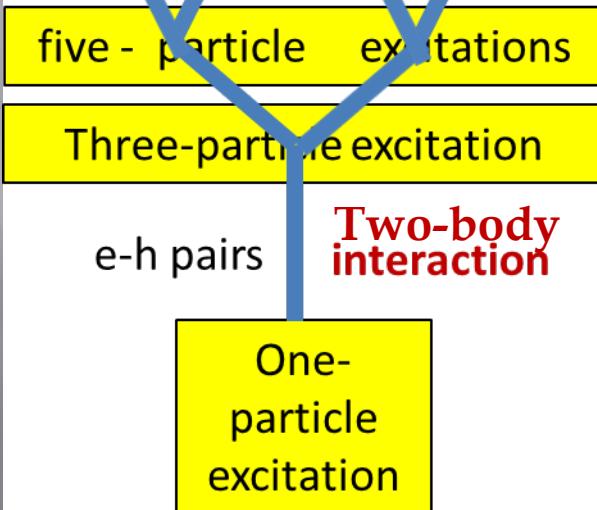
$$\Psi_b = | s_1^z, s_2^z, \dots s_L^z \rangle, \quad \text{Dim} = 2^L$$

Exponentially  
large  
dimension of  
Hilbert space

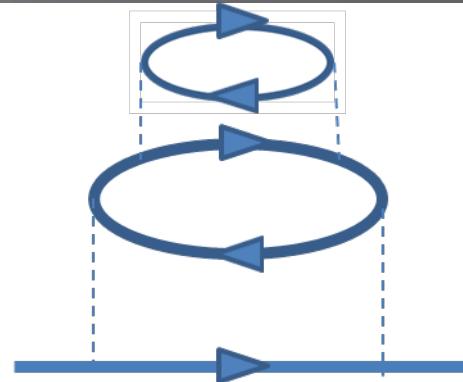


# Hierarchical structure of many-body Fock space

**Tree-like structure  
of many-body  
interaction**



Altshuler, Gefen, Kamenev,  
Levitov , 1997



Basko, Aleiner, Altshuler,  
2005



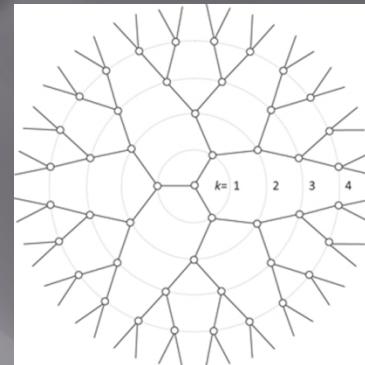
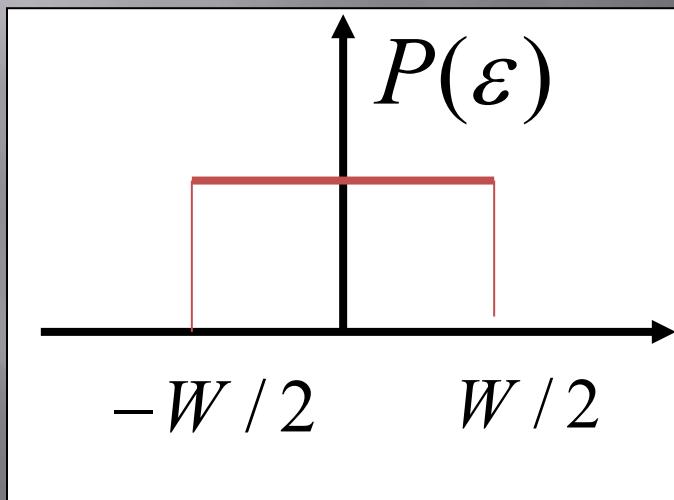
“Many-body localization” is  
a localization on graphs  
with local tree structure.

Much richer family of  
networks (depending on  
the interaction  
Hamiltonian) compared to  
lattices

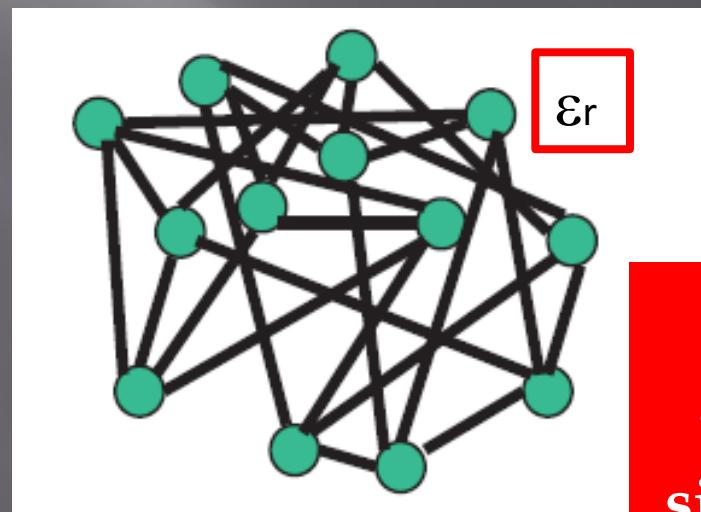
# Anderson model on random regular graphs (RRG) and on a Cayley tree

$$H = -I \sum_{\langle r, r' \rangle} c_r^+ c_{r'} + \sum_r \varepsilon_r n_r$$

Disorder strength  $W$

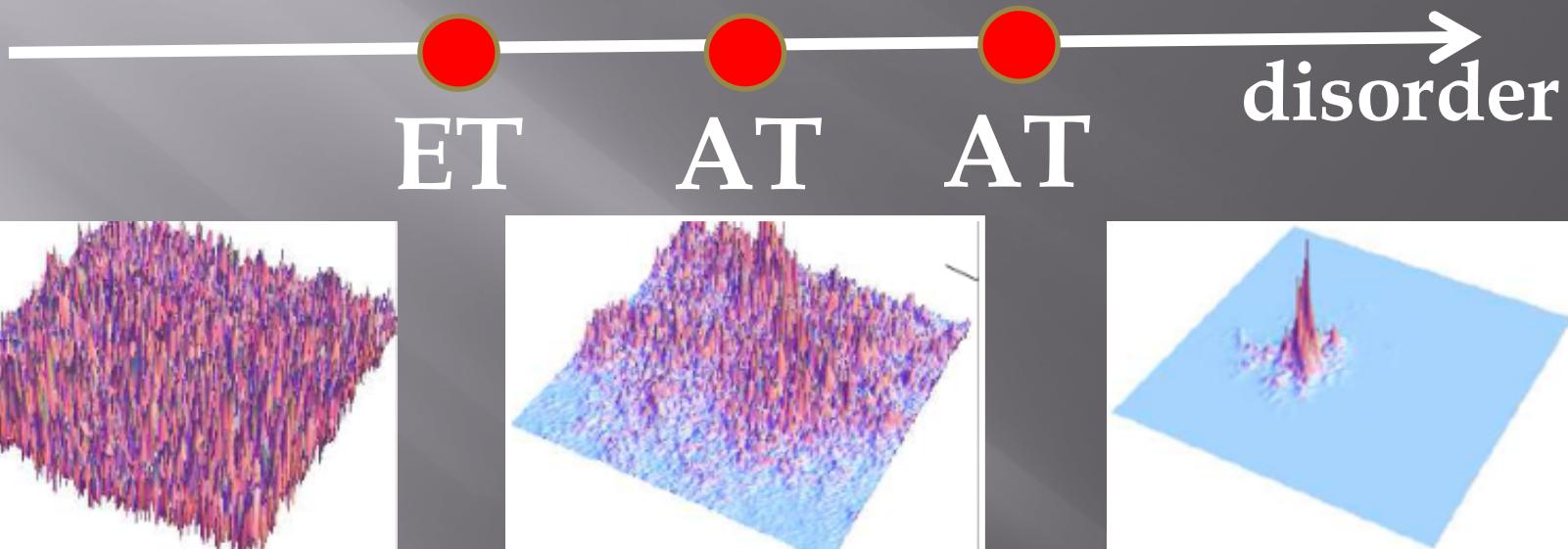


Finite fraction of states on the boundary



**RRG  
with on-  
site energy  
disorder**

# Localization and ergodic transitions



Search for NEE states

• • •  
3D Anderson model:  
NEE states only at  
the AT point.

G. Biroli,  
A. Ribeiro-Teixeira,  
and M. Tarzia,  
arXiv:1211.7334.

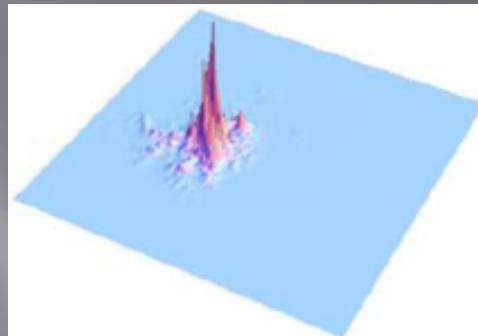
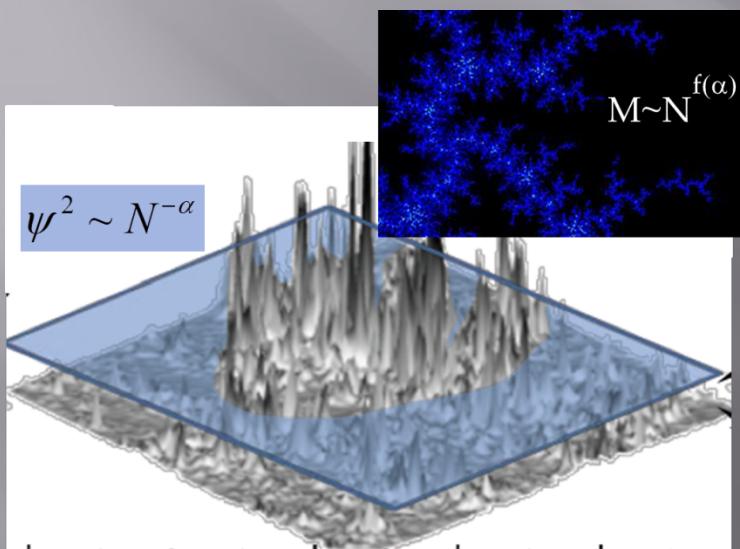
# Multifractal PDF and the moments

$$P(\ln |\psi|^2) = A N^{f\left(-\underbrace{\frac{\ln|\psi|^2}{\ln N}}_{\alpha}\right)-1}$$

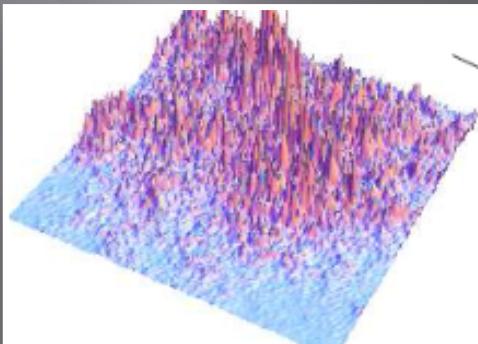
$$I_q = \sum_r \langle \psi(r)^{2q} \rangle = N^{-\tau(q)}$$

$$\tau(q) = D(q)(q-1)$$

# LOCALIZED, EXTENDED ERGODIC AND EXTENDED NON-ERGODIC PHASES

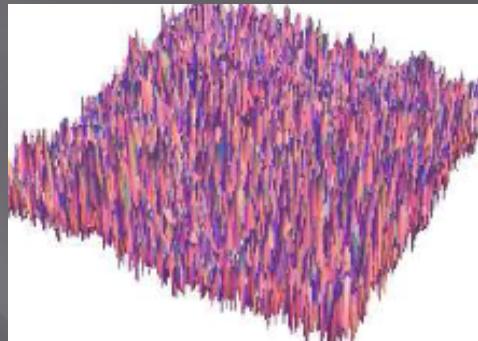


localized  $D_q = 0$



Non-ergodic  
extended  
(NEE)  $0 < D_q < 1$

$$P(\ln |\psi|^2) = A N^{f(\alpha)-1}$$



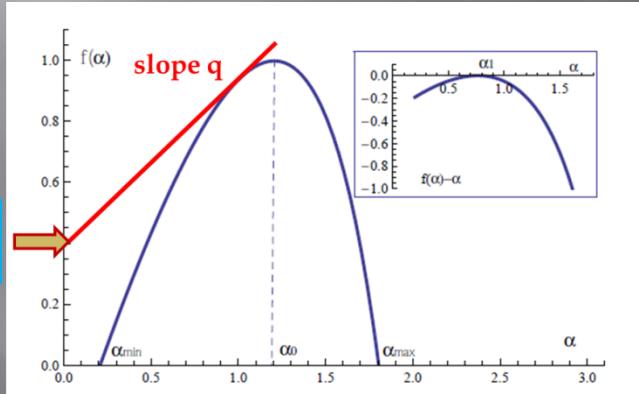
Ergodic  
extended (EE)  
 $D_q = 1$

# Legendre transform

$$I_q = \frac{\int_0^\infty d\alpha e^{\ln N [f(\alpha) - q\alpha]}}{\int_0^\infty d\alpha e^{\ln N [f(\alpha) - \alpha]}}$$

$$\tau(q) = D(q) (q-1)$$

Saddle point approximation:



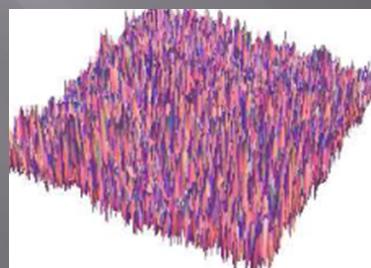
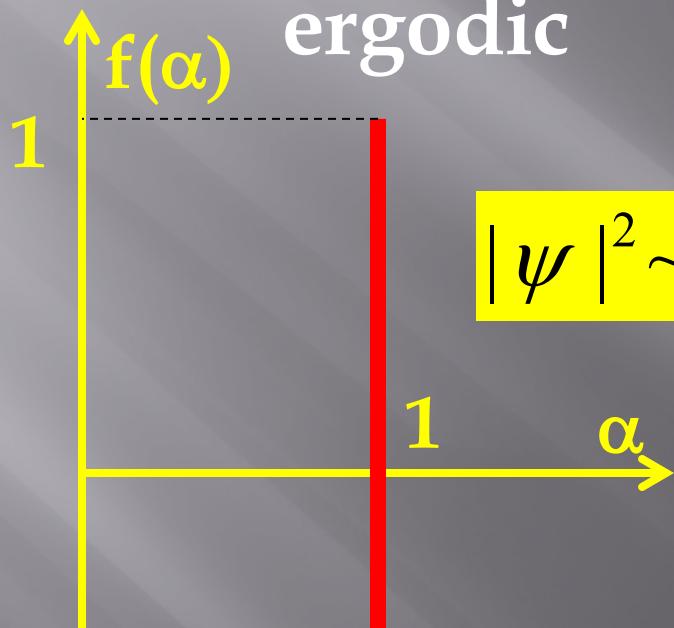
$$I_q = \sum_r \langle \psi(r)^{2q} \rangle = N^{-\tau(q)}$$

Set of critical exponents

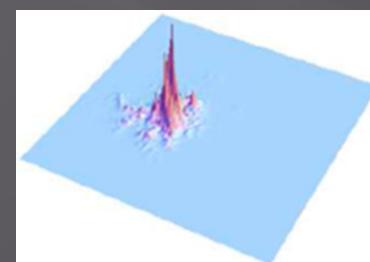
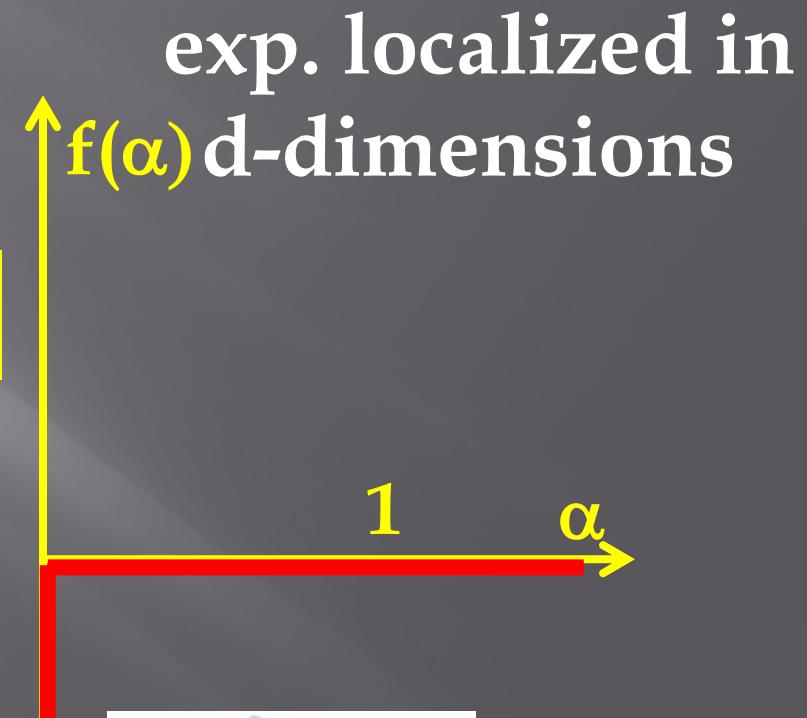
Legendre transform:

$$\tau_q = q\alpha_q - f(\alpha_q), \quad f'(\alpha_q) = q$$

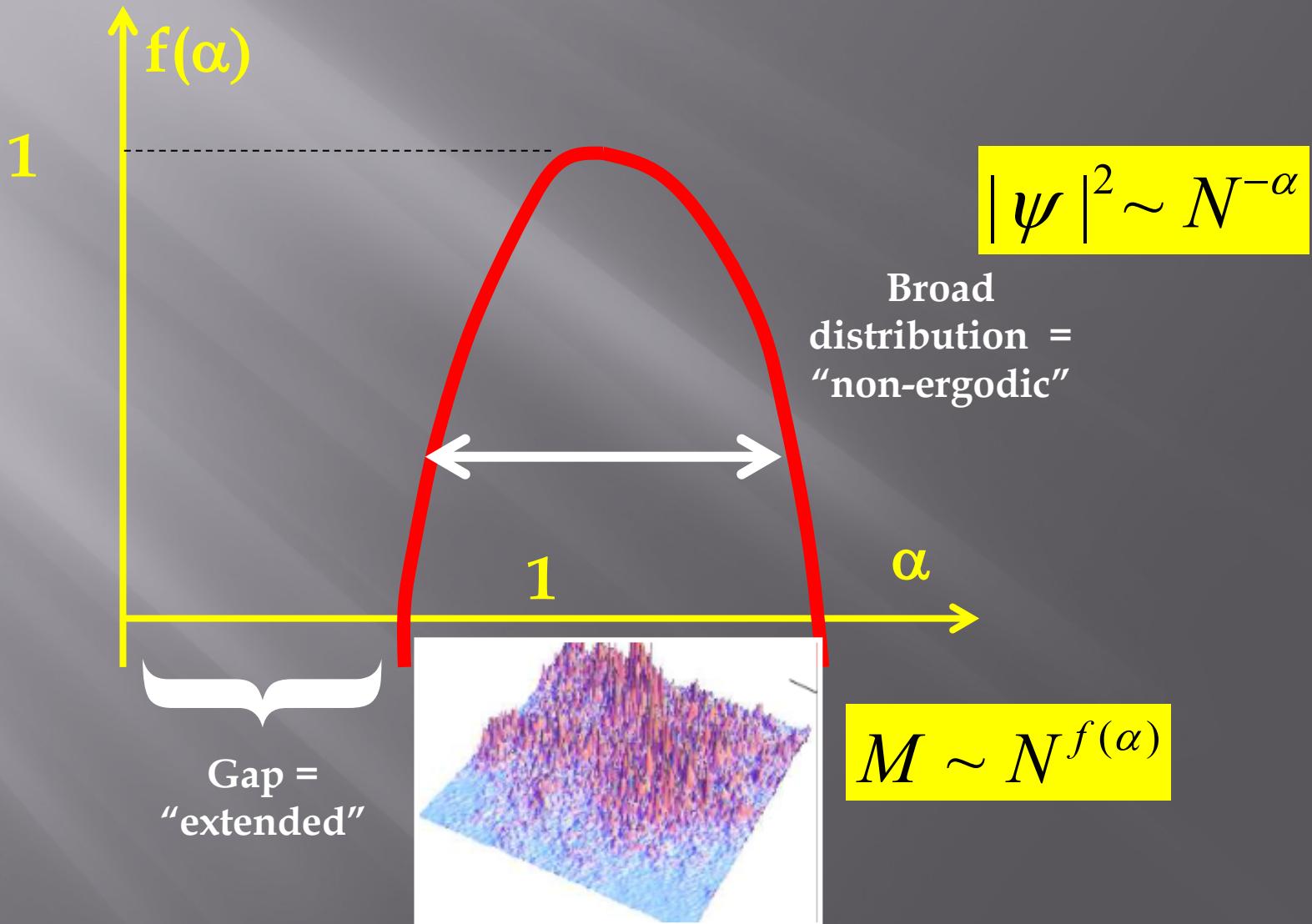
# Degenerate cases: ergodic and exponentially localized w.f.



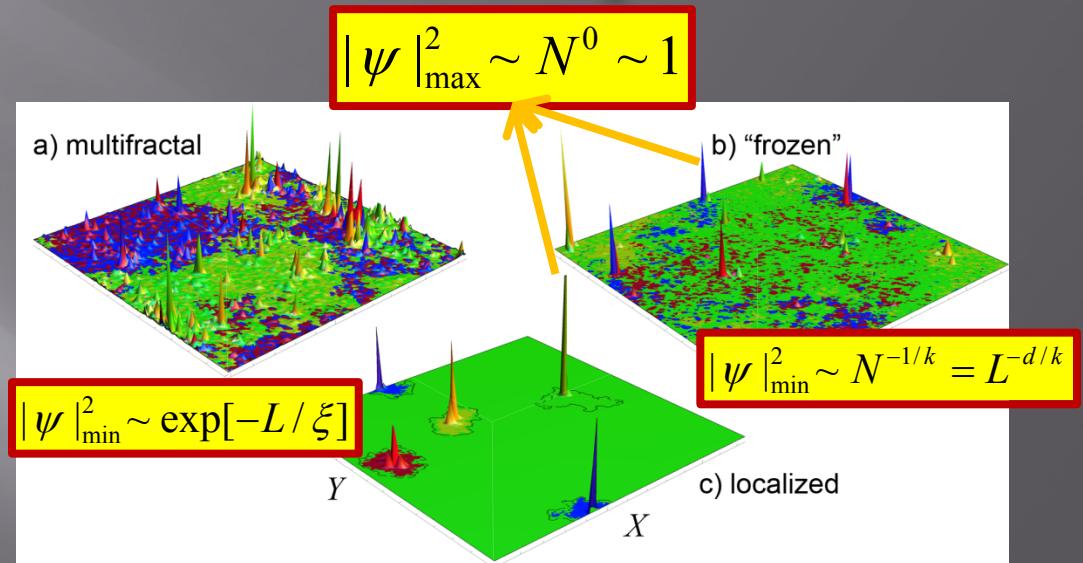
$$|\psi|^2 \sim N^{-\alpha}$$



# Extended non-ergodic phase

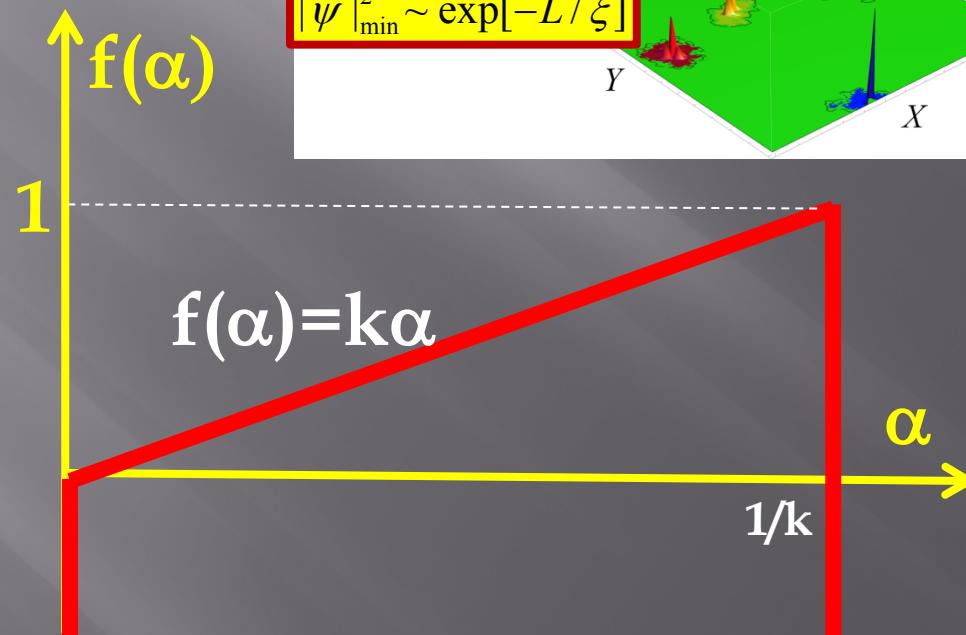


# Bad insulator: “frozen phase”.



For Bethe lattice:

$$\begin{aligned} |\psi|_{\min}^2 &\sim N^{-1/k} = K^{-L/k} \\ &\sim \exp[-L/\xi] \\ \xi &= \ln K / k \end{aligned}$$

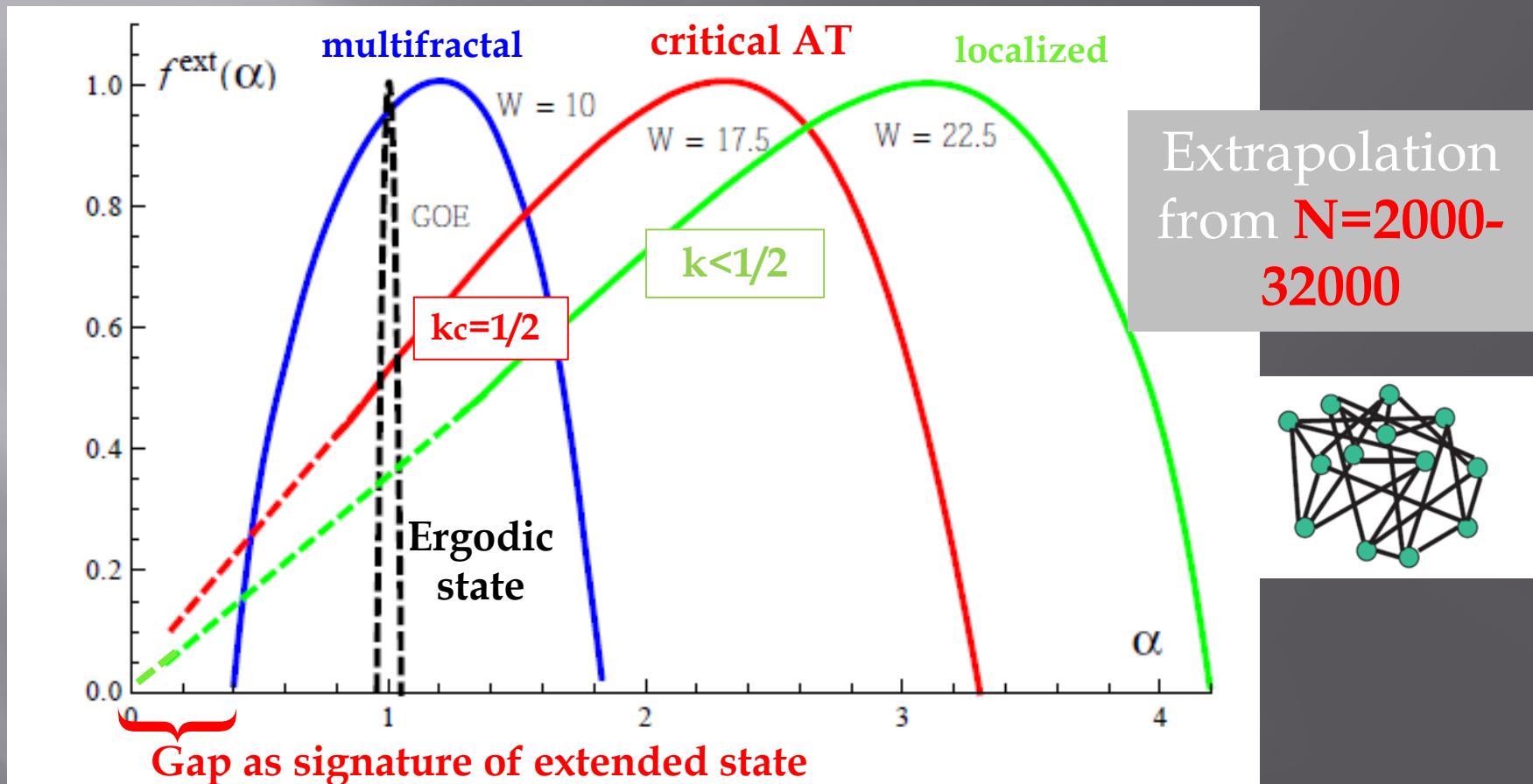


M.Foster et al.

$$\tau(q)=0, \text{ for all } q>k \quad \tau(q)=q/k-1, \text{ for } 0<q<k$$

# Search for ergodic transition on RRG

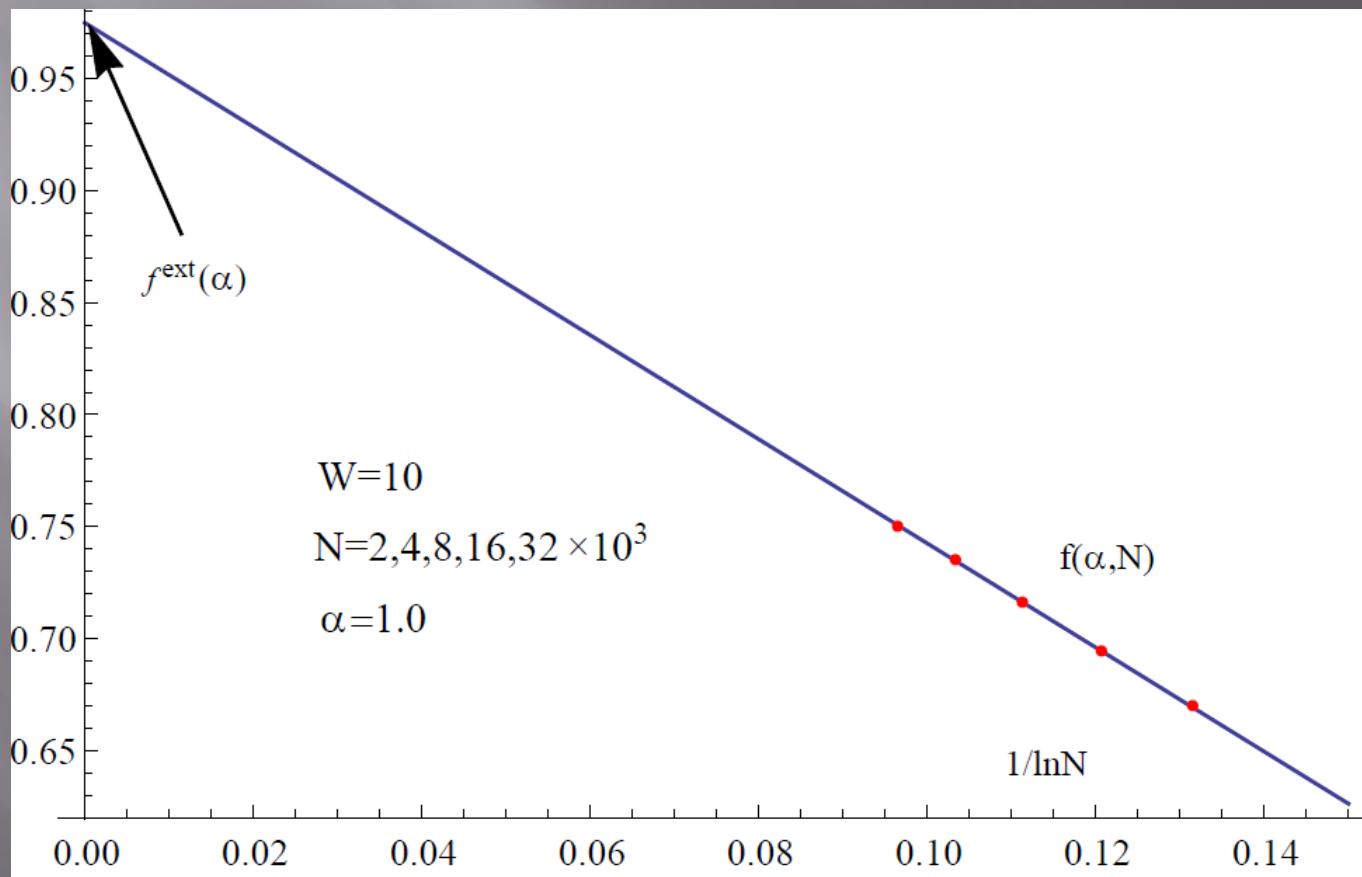
De Luca, Altshuler, V.E.K., Scardicchio al. PRL v.113, 046806 ( 2014)



Continuous deformation from localized to extended states  
A wide region of non-ergodic extended states

# Extrapolation

$$\frac{\ln P_{rec}(\ln |\psi_{env}|^2)}{\ln N} = (f(\alpha, N) - 1) + \frac{A}{\ln N}$$



# Rectification (de-convolution)

envelope    oscillating

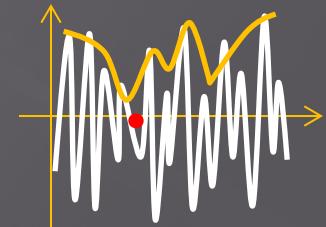
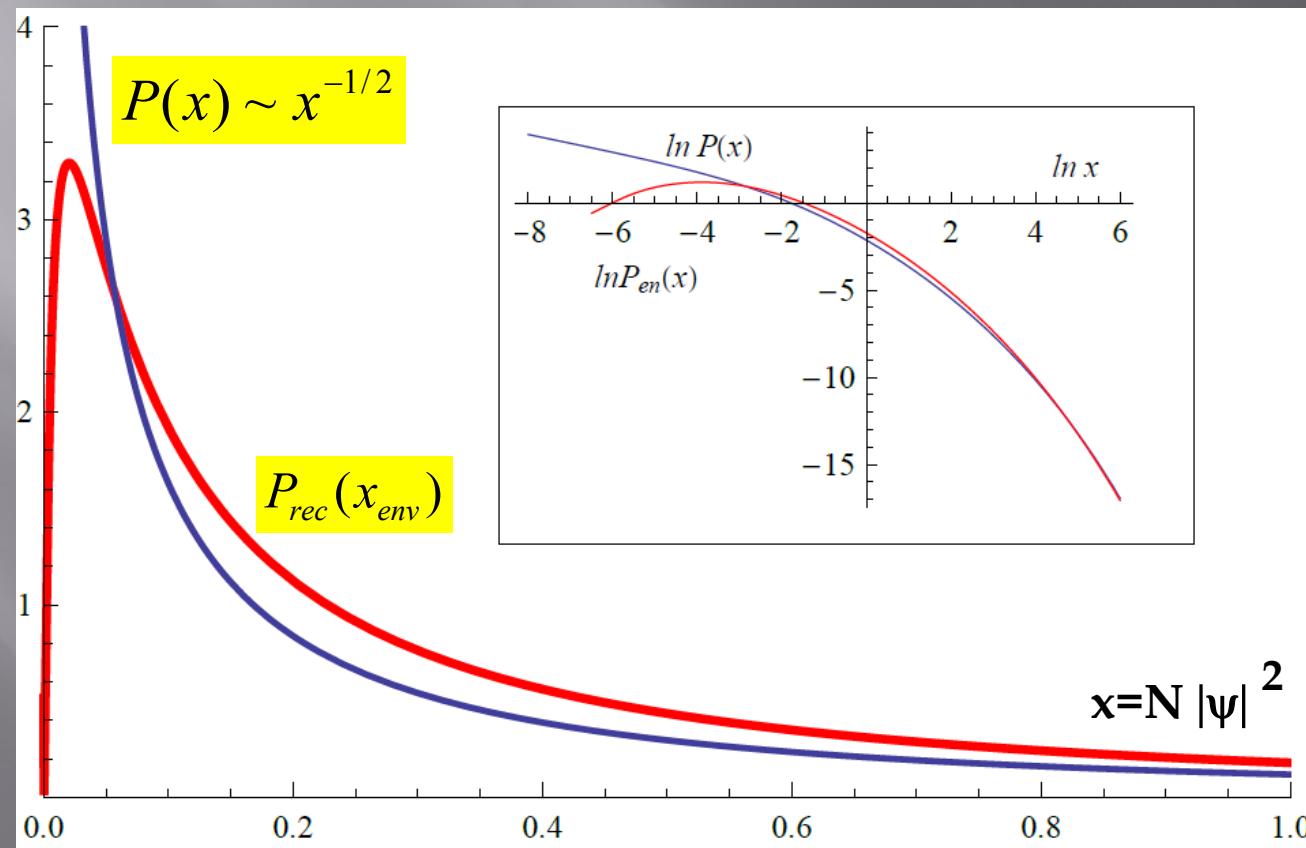
$$\psi = \psi_{env} \circ \psi_{GOE}$$

$$P(\ln |\psi|^2) = \text{convol}\{P_{rec}(\ln |\psi_{env}|^2) * P_{PT}(\ln |\psi_{GOE}|^2)\}$$

observed

wanted

known



# EXACT DIAGONALIZATION CALCULATIONS: IPR AND SHANNON ENTROPY OF EIGENSTATES

$$\ln I_2 = \overline{\ln \sum_r |\psi(r)|^4} = -\boxed{D_2} \ln N + c_2 + r_2(\ln N)$$

localized  $D_q = 0$

$$\ln S = \overline{\ln \sum_r |\psi(r)|^2 \ln(|\psi(r)|^{-2})} = \boxed{D_1} \ln N + c_1 + r_1(\ln N)$$

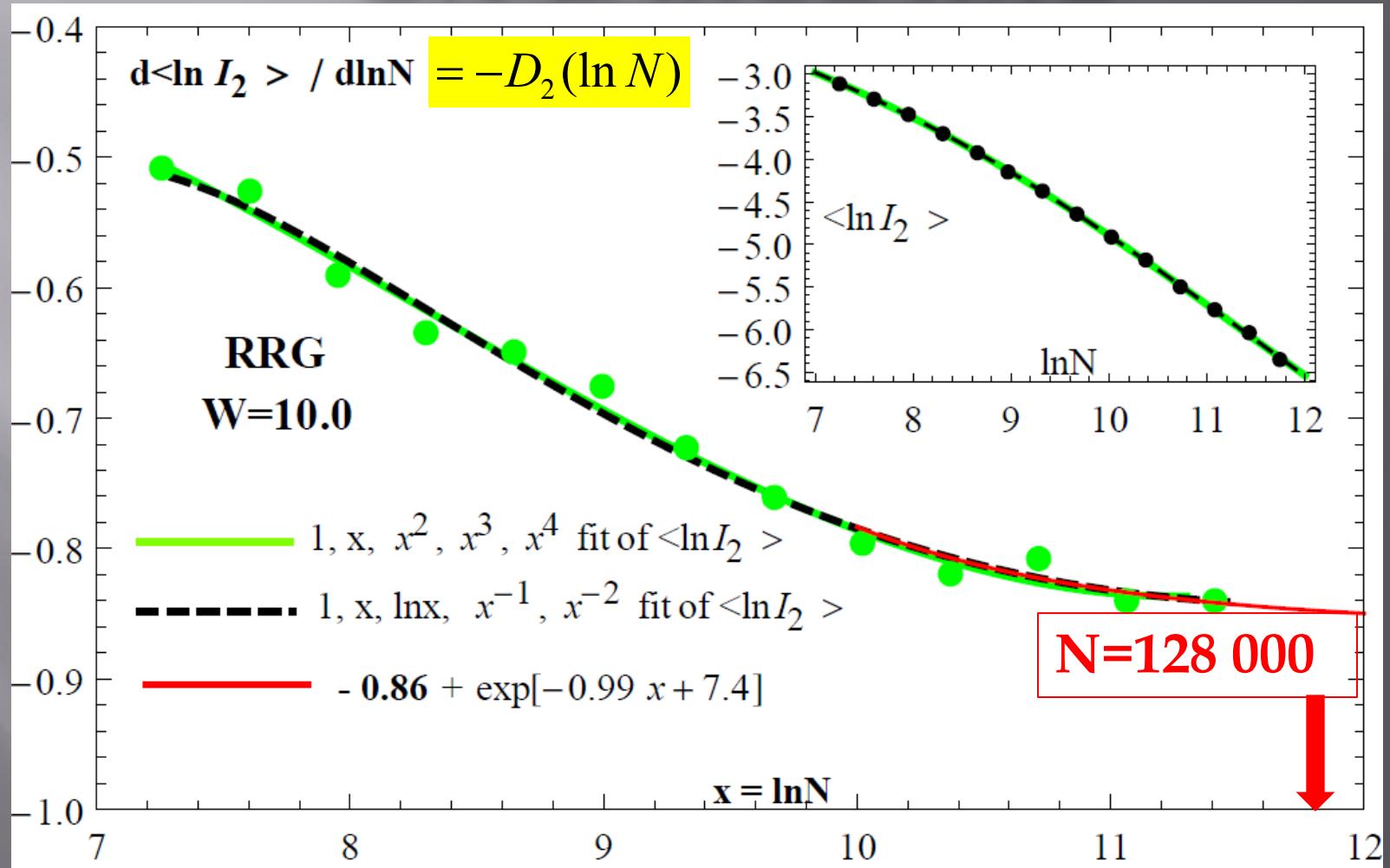
Ergodic  
extended (EE)  
 $D_q = 1$

$$D_2(\ln N) = -d \ln I_2 / d \ln N$$

Non-ergodic  
extended  
(NEE)  $0 < D_q < 1$

$$D_1(\ln N) = d \ln S / d \ln N$$

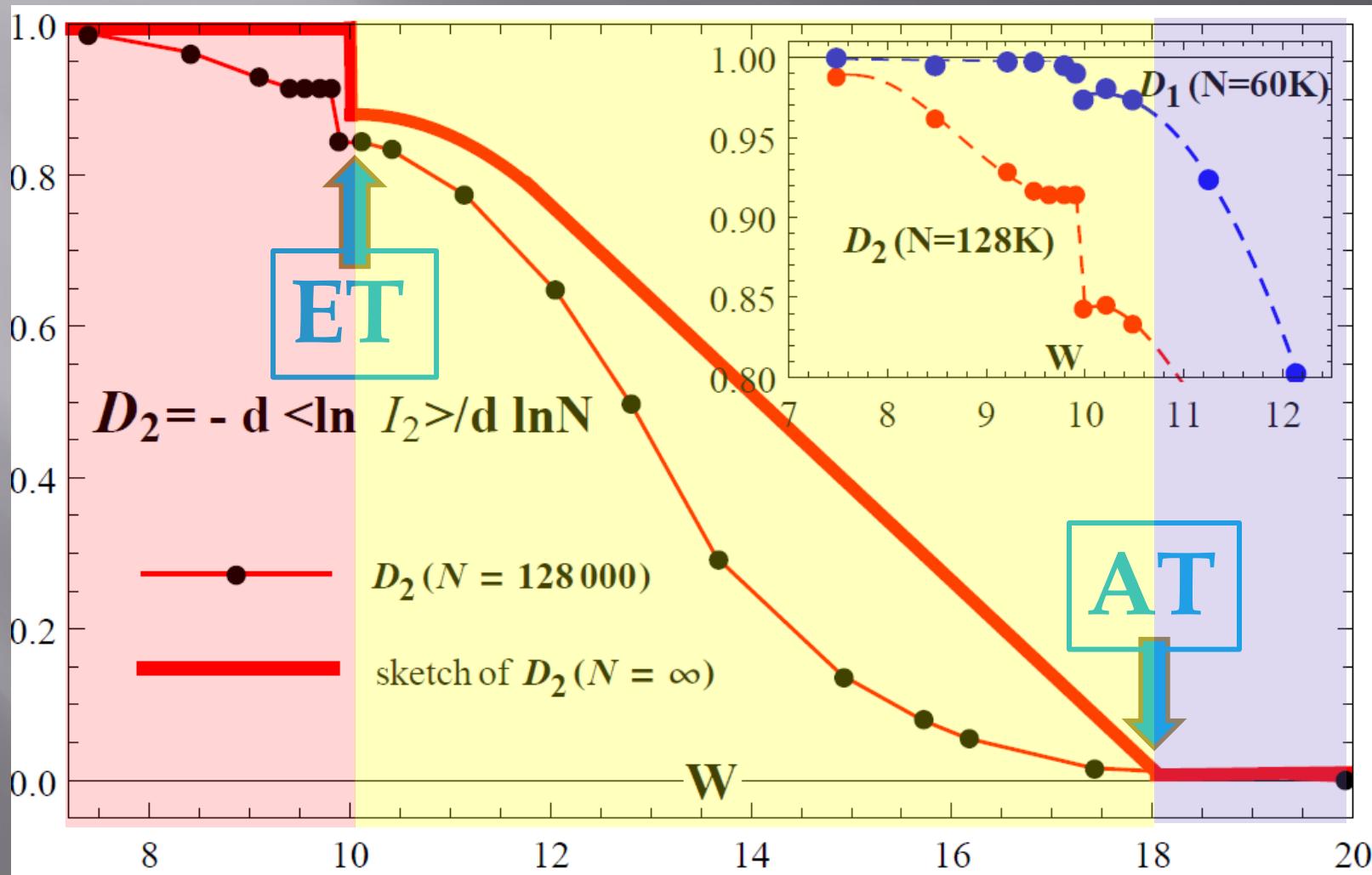
# EXACT DIAGONALIZATION ON LARGE RRG



# Running fractal dimensions vs disorder

Altshuler, Cuevas, Ioffe, V.E.K

ArXiv:1605.02295



# First order ergodic transition?

