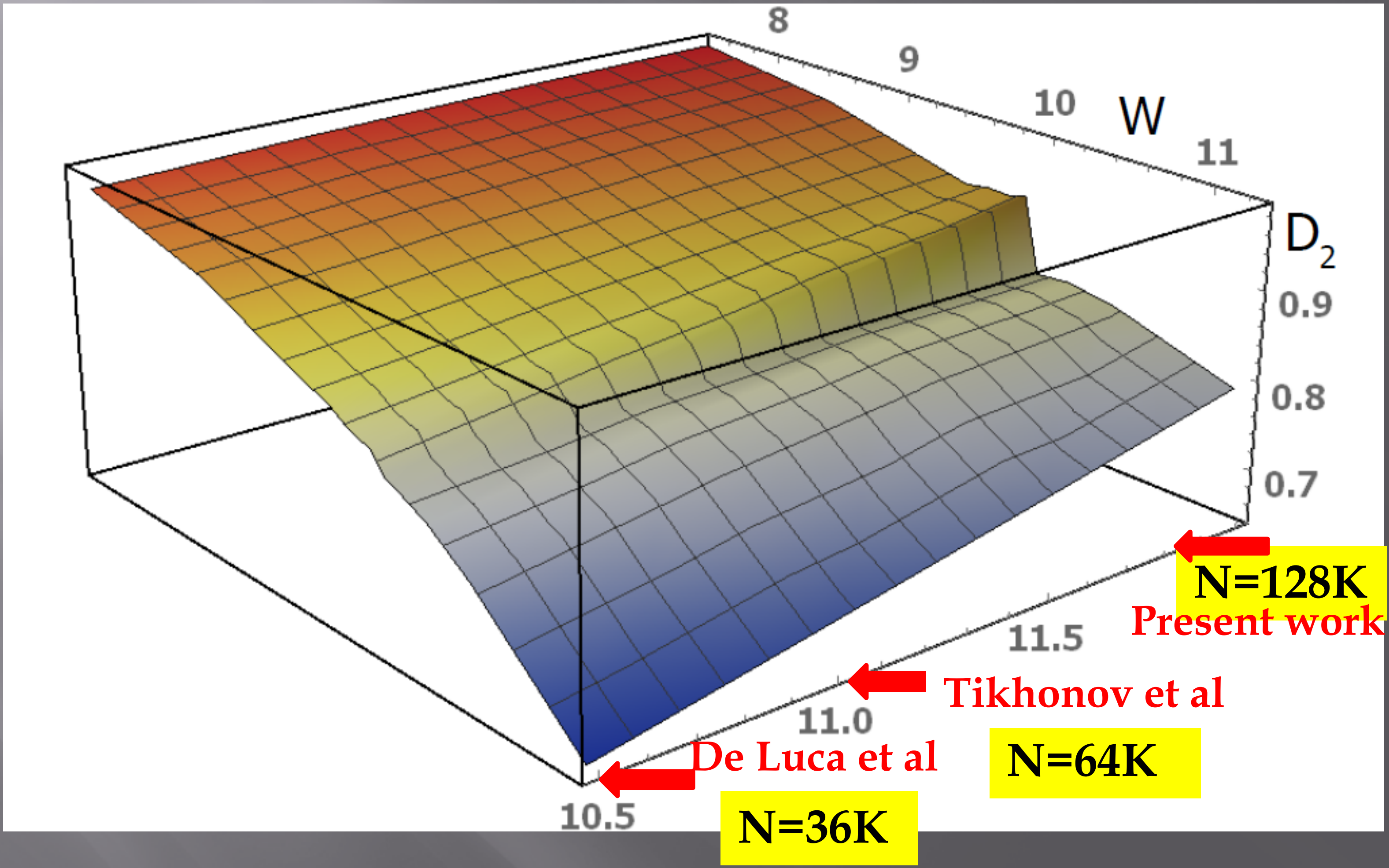
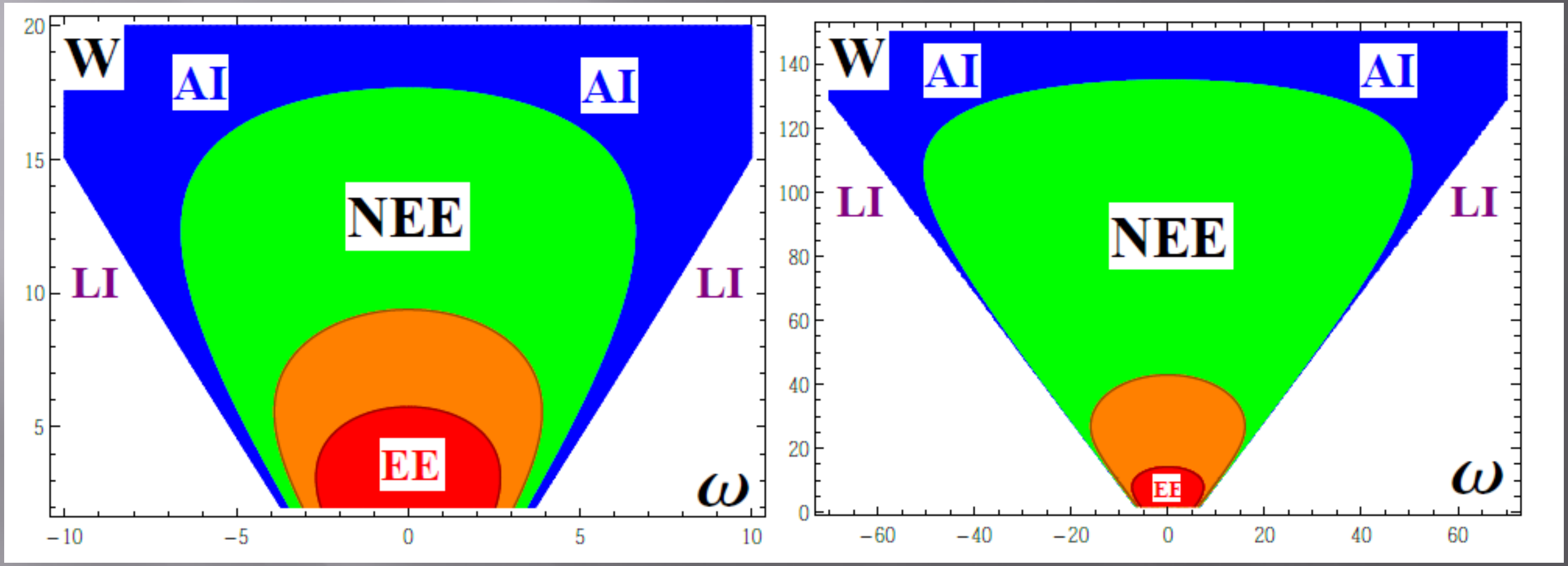
EVOLUTION OF JUMP WITH N



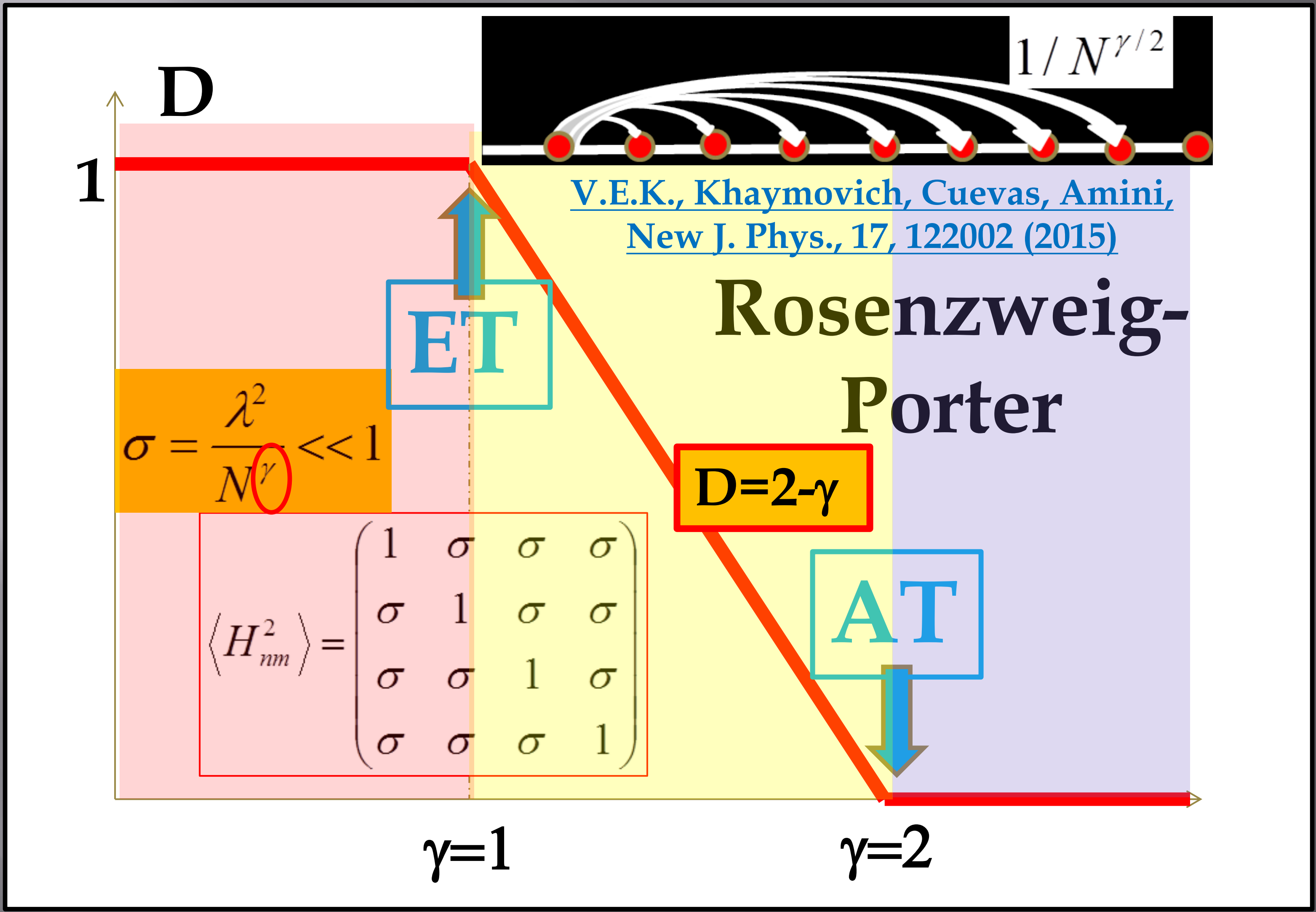
Phase diagram from the RSB solution on infinite Bethe lattice

K=2 K=8

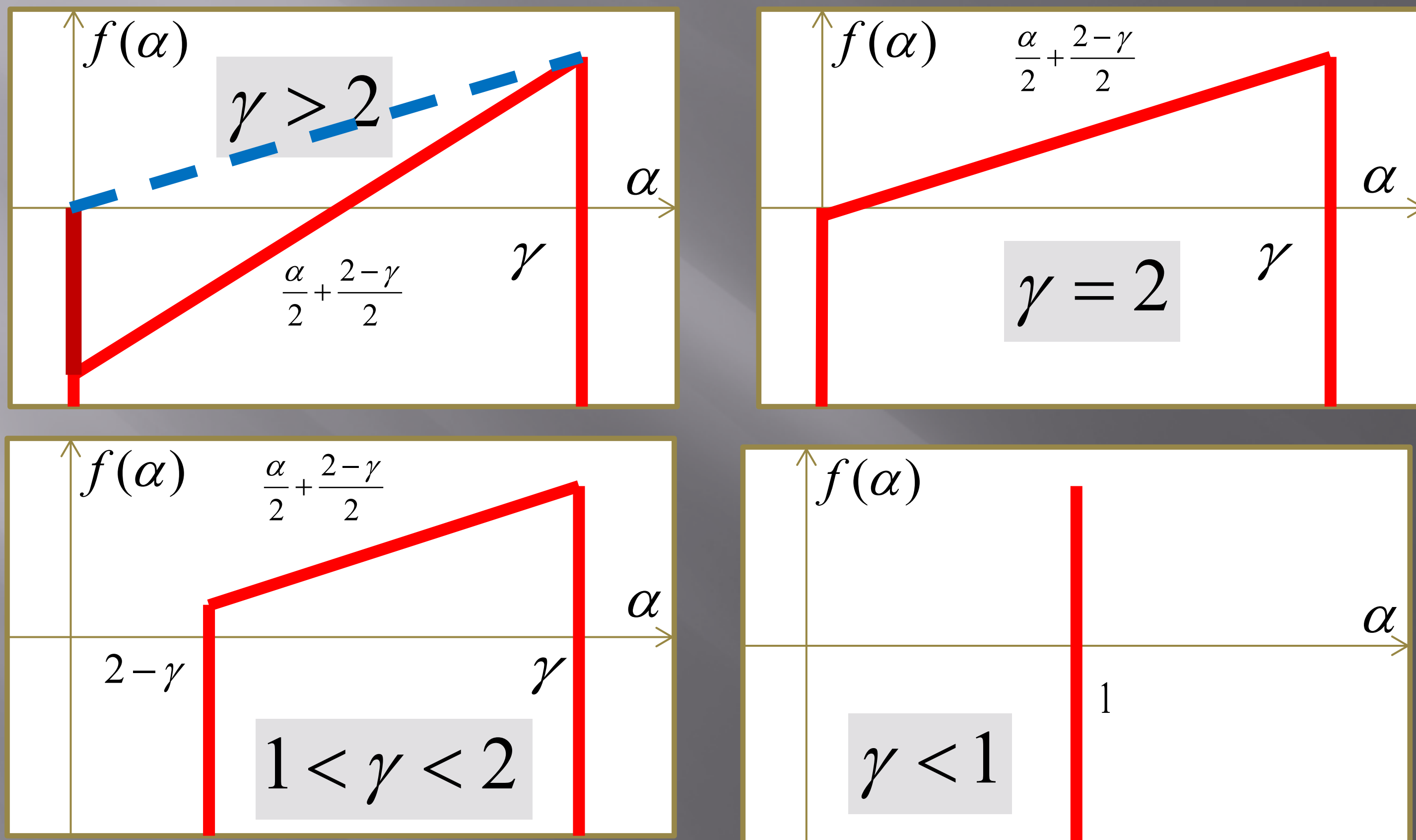


In the limit of branching K>>1 NEE phase takes the lion share of the entire extended phase

Rosenzweig-Porter RMT:

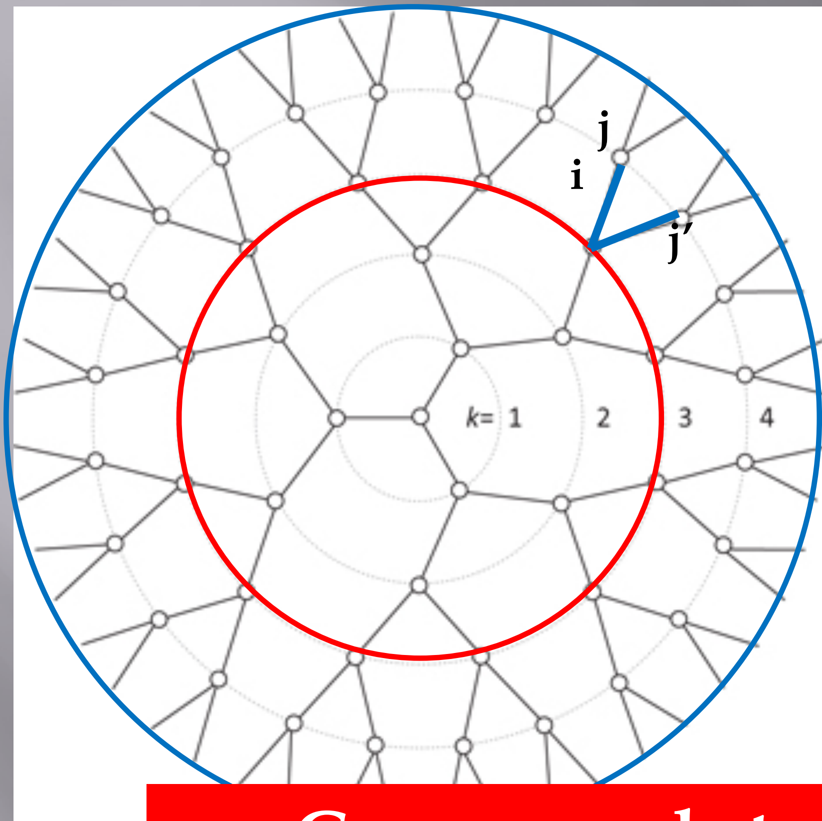


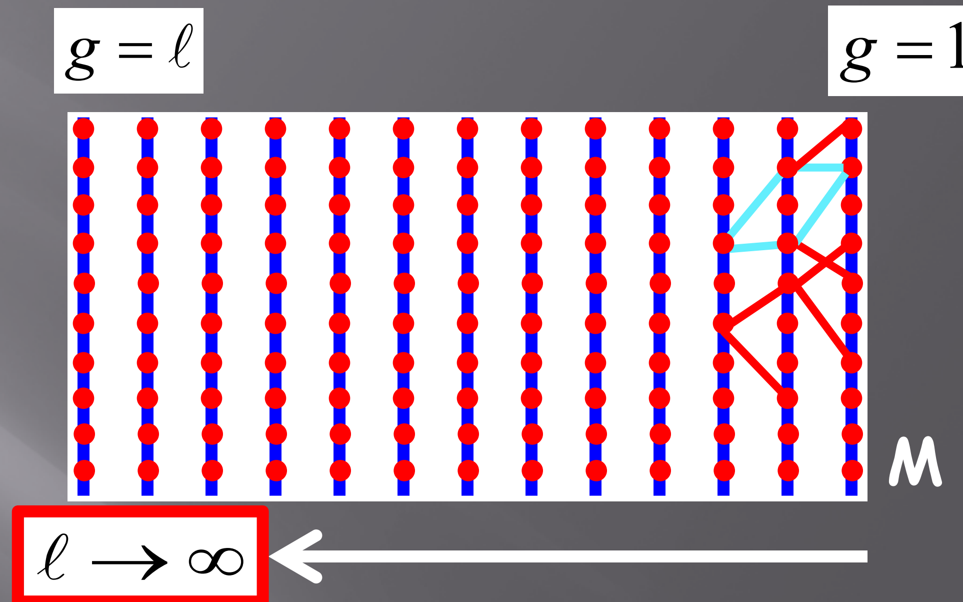
Multifractality spectrum f(α)



$$D_{q>1/2} = 2-\gamma$$

POPULATION DYNAMICS NETWORK





Corresponds to infinite BL

$$G_i^{(n+1)} = \frac{1}{E + i\eta - \varepsilon_n - \sum_{<j,i>}^{\boxed{K}} G_j^{(n)}}$$

Each site of the generation n+1 is connected at random to K parent sites of generation n

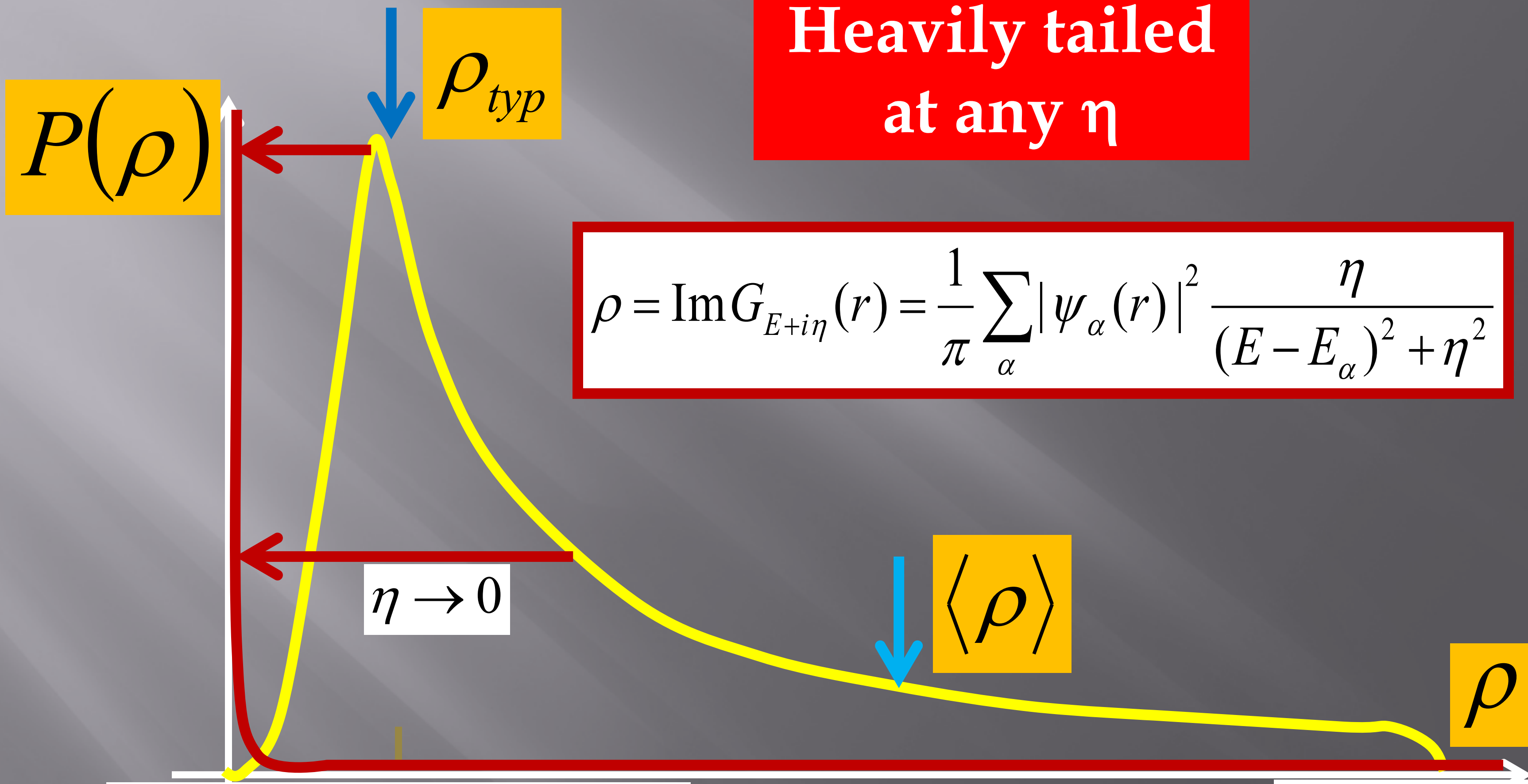
Local Density of States (LDoS): an instrument to distinguish between different phases

$$\rho = \operatorname{Im} G_{E+i\eta}(r) = \frac{1}{\pi}\sum_{\alpha} |\psi_{\alpha}(r)|^{2} \frac{\eta}{(E-E_{\alpha})^{2}+\eta^{2}}$$

Singular distribution in insulator

Heavily tailed at any η

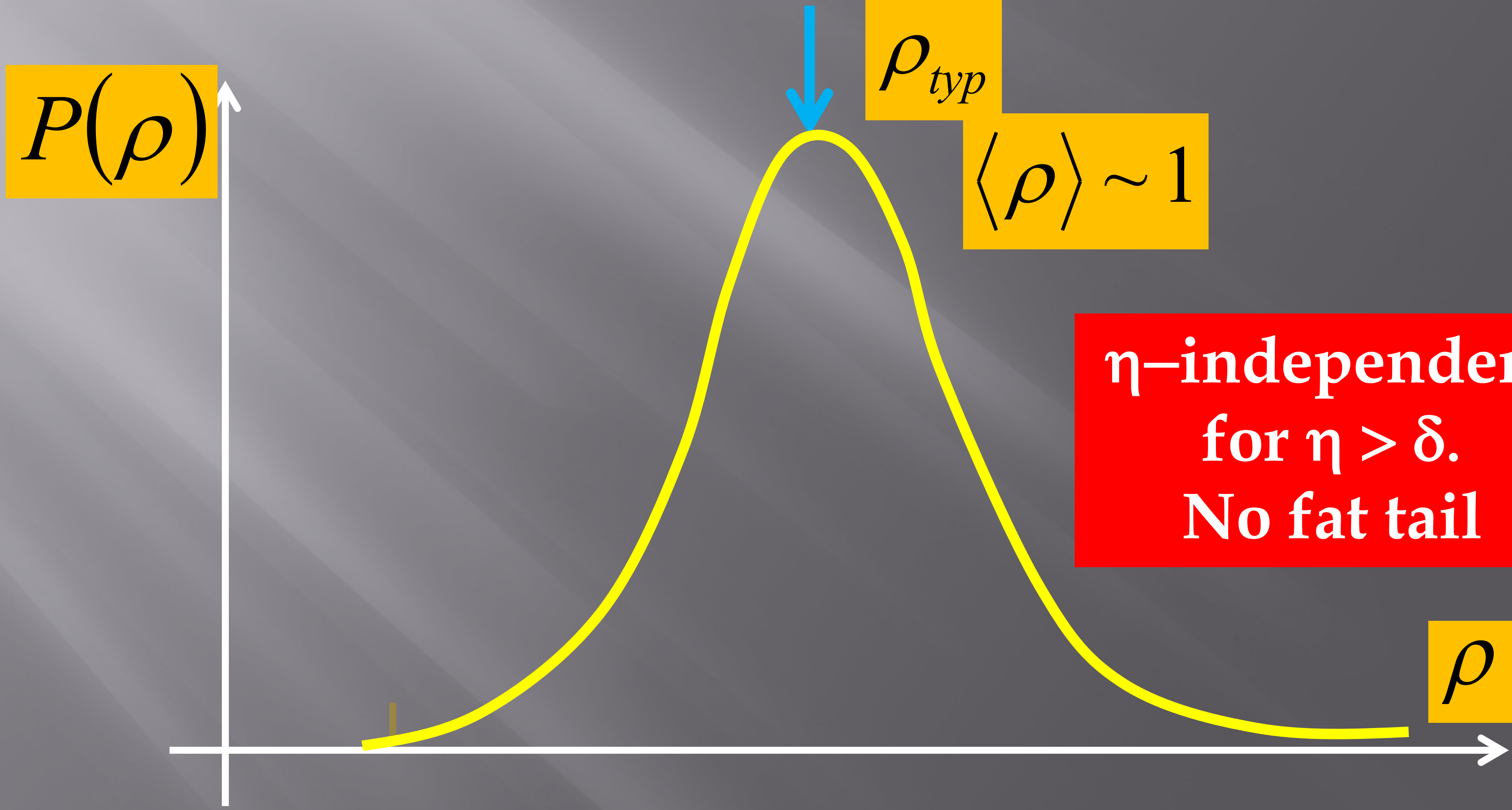
$$\rho = \operatorname{Im} G_{E+i\eta}(r) = \frac{1}{\pi}\sum_{\alpha} |\psi_{\alpha}(r)|^2 \frac{\eta}{(E-E_{\alpha})^2+\eta^2}$$



$$\exp\langle \ln \rho \rangle = \rho_{typ} \sim \eta$$

$$\rho_{\max} \sim \eta^{-1}$$

η- and N-independent distribution in ergodic phase



η–independent for η > δ. No fat tail

HYBRID DISTRIBUTION IN NON-ERGODIC EXTENDED PHASE

$$\rho_{typ} \sim \eta\, N^{D_1}, \quad (\eta < \delta \sim N^{-1})$$

$$\boxed{\rho_{typ} \sim N^{-1+D_1}}, \quad (\delta < \eta < \boxed{\eta_c} \sim N^{-z})$$

$$\rho_{typ} \sim \eta^{(1-D_1)/z}, \; (\eta > \boxed{\eta_c})$$

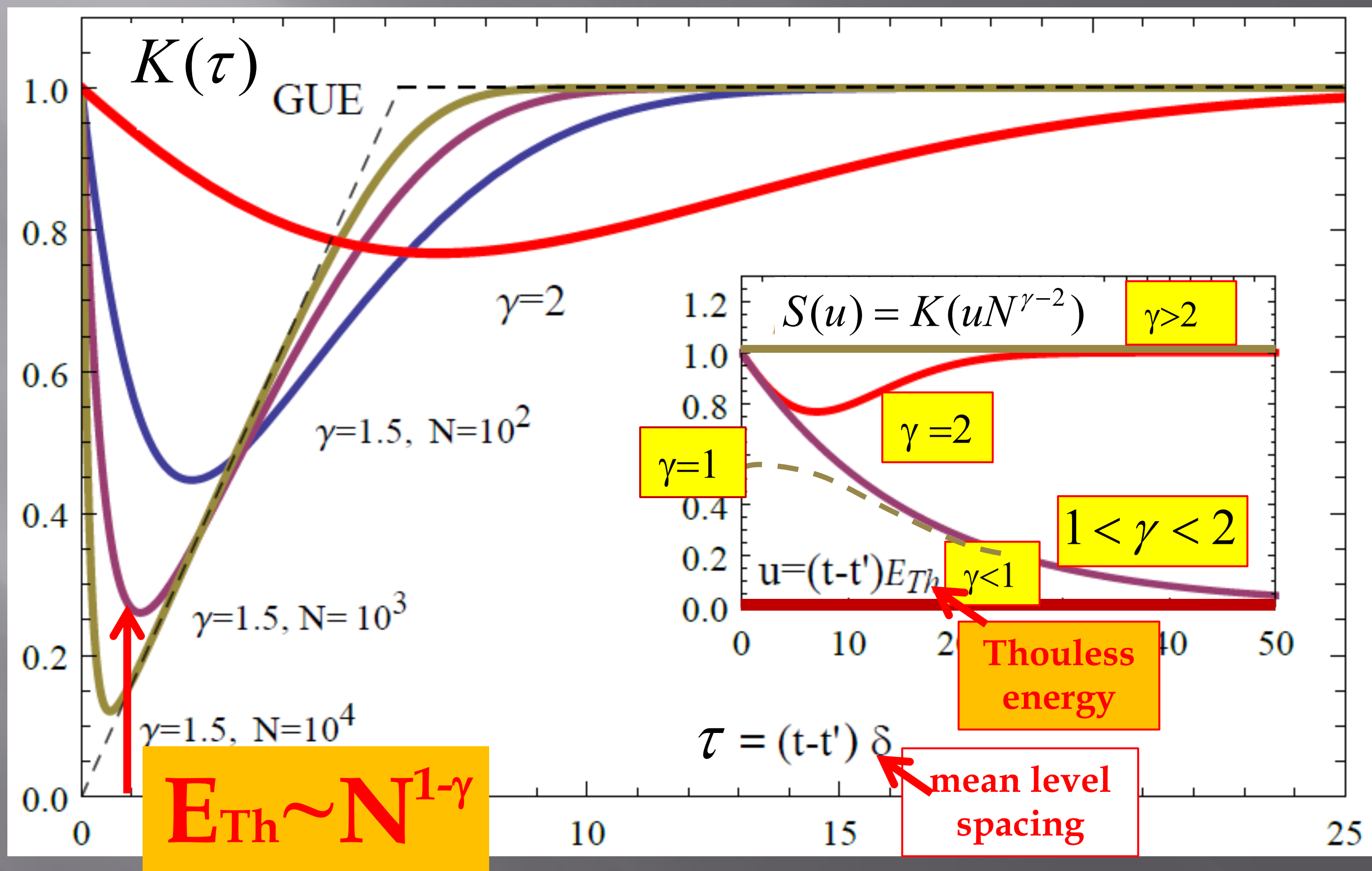
Tailed but η–independent for δ<η<ηc

Realized in Rosenzweig-Porter RMT with z=1-D

D. Facoetti, P. Vivo, G. Biroli, *EPL*, 2016

New energy scale and new intermediate regime

ANALYTICAL SOLUTION FOR SPECTRAL FORM-FACTOR



D from population dynamics

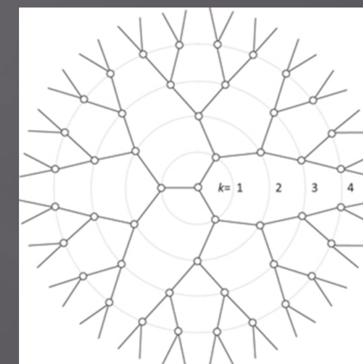
Non-stationry, "inflating" regime of PD

$$\langle \operatorname{Im} G \rangle_{typ} \big|_{\eta \to 0} = \eta \exp[\Lambda \ell]$$

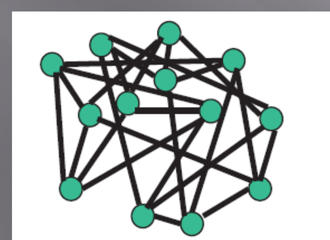
$$K^{\ell_t} = N$$

Termination at the size of the tree:

$$\ell = \ell_t = \frac{\ln N}{\ln K}$$



Where to terminate on RRG? Diameter of the graph =lnN/lnK ? Or larger (repetitions)?



$$\langle \operatorname{Im} G \rangle_{typ} \big|_{\eta \to 0} \overset{\text{def}}{\sim} \eta N^{D} \sim \eta\, N^{\frac{\Lambda}{\ln K}}$$

$$D = \frac{\Lambda}{\ln K}$$

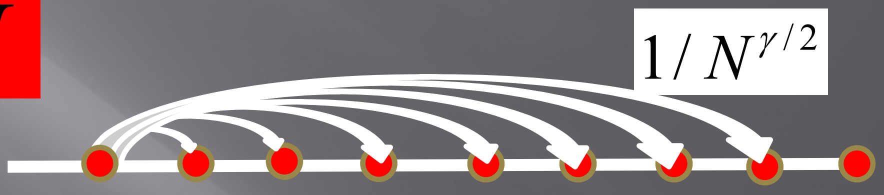
Can be found analytically by RSB on infinite BL, or by numerics in PD

Rosenzweig-Porter model

$$K = N$$

$$D = \frac{\Lambda}{\ln N}$$

$$\Lambda = 2\ln\left(\frac{W_c}{W}\right) = 2\ln\left(\frac{N^{2/2}}{N^{\gamma/2}}\right) = \boxed{(2-\gamma)}\ln N$$



$$\ell_t = \frac{\ln N}{\ln K} = d = 1$$

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

Termination at the diameter of the graph

Rosenzweig-Porter RMT, 1960

V.E.K., Khaymovich, Cuevas, Amini, New J. Phys., 17, 122002 (2015)

D(W) can be computed analiticaly using one-step replica symmetry breaking

B.L.Altshuler, L.B.Ioffe, V.E.K, E. Cuevas
PRL v. 117, 156601 (2016)

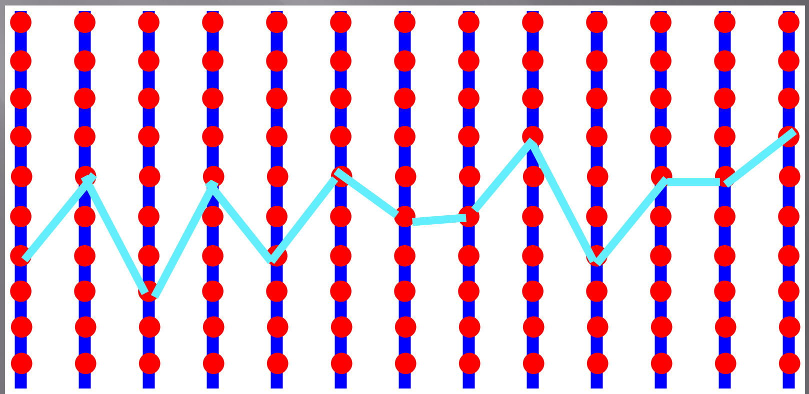
$$\left\langle \mathrm{Im} G \right\rangle_{typ} \Big|_{\eta \to 0} = \eta \exp[\Lambda \ell]$$

$$\mathrm{Im}\, G_i^{(g)}(\omega) = \frac{\sum_{j(i)} \mathrm{Im}\, G_j^{(g+1)}}{\left(\omega - \epsilon_i - \mathrm{Re}\, \Sigma_i^{(g+1)}\right)^2}$$

$$\Lambda(\omega) = \overline{\ln Z(\omega)}/\ell$$

$$Z(\omega) = \sum_P \prod_{k=1}^{\ell} \frac{1}{\left(\omega - \epsilon_P^{(k)}\right)^2}.$$

$g = \ell$ $\qquad$ $g = 1$



Mapping to directed polymer problem

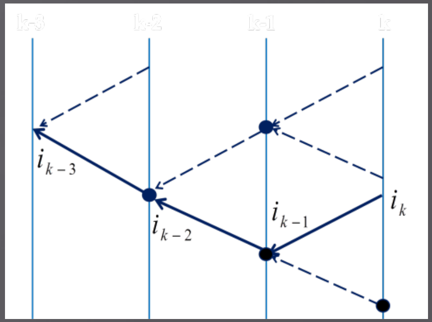
Replica trick for Λ

$$G_i = \frac{1}{\omega + i\eta - \varepsilon_n - \sum_{<j,i>}^{K} G_j}$$

$$Z = \sum_P \prod_{k=1}^{\ell} \left[\mathrm{Re}\, G_\omega(i_k^{(P)})\right]^2$$

$$\mathrm{Im}\, G = \eta \exp[\Lambda\, \ell]$$

$$\Lambda(\omega) = \overline{\ln Z(\omega)}/\ell$$



Replica trick:

$$\Lambda(\omega) = \lim_{n\to 0} \frac{1}{n\ell}\left(\overline{Z^n} - 1\right)$$

One-step RSB

$$\begin{aligned}\Lambda(\omega, m) &= \lim_{n\to 0} \frac{1}{n}\left[\left(K \int F(\epsilon)\, \frac{d\epsilon}{|\omega-\epsilon|^{2m}}\right)^{n/m} - 1\right] \\ &= \frac{1}{m} \ln\left(K \int F(\epsilon)\, \frac{d\epsilon}{|\omega-\epsilon|^{\boxed{2m}}}\right),\end{aligned}$$

RS solution: m=1
RSB: m ≠ 1

m m m m

n/m groups

Equivalent to SUSY approach (Fyodorov& Mirlin, ,Tikhonov & Mirlin) for computing Λ

at an assumption that

$$D = \frac{\Lambda}{\ln K}$$

(termination of exp growth of ImG~ηexp[Λ^{ℓ}], where ℓ = graph's diameter)

Minimal account for ReΣ

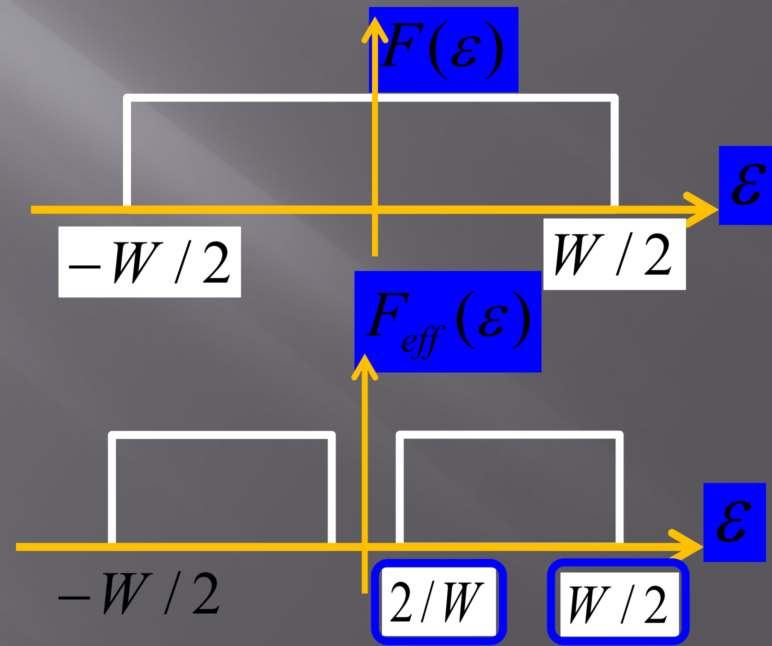
$$\Lambda(m,W)=\frac{1}{m}\ln\left(K\int \boxed{F_{eff}(\varepsilon)}\frac{d\varepsilon}{\varepsilon^{\boxed{2m}}}\right)\equiv\frac{1}{m}\ln\left(K\, I_m\right)$$

Conventional approximation Re Σ =0

With Re Σ : SUMMETRY

$$\boxed{F_{eff}(\varepsilon)=F_{eff}(1/\varepsilon)}$$

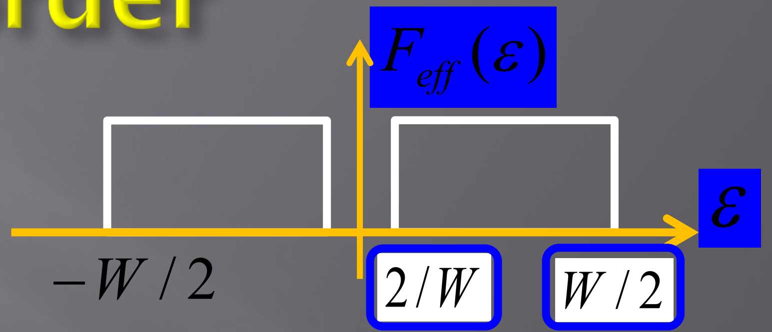
$$\boxed{I_m = I_{1-m}}$$



Approximation for critical disorder

With Re Σ : SUMMETRY

$$F_{eff}(\varepsilon) = F_{eff}(1/\varepsilon)$$



$$\Lambda(m,W) = \frac{1}{m}\ln\left(K\int F_{eff}(\varepsilon)\frac{d\varepsilon}{\varepsilon^{2m}}\right) \equiv \frac{1}{m}\ln\left(K\, I_m\right)$$

$$\Lambda(m_c, W_c) = 0,$$
$$\partial_m \Lambda(m_c, W_c) = 0$$

$$K\ln\left(W_c/2\right) = W_c/2 - 2/W_c$$

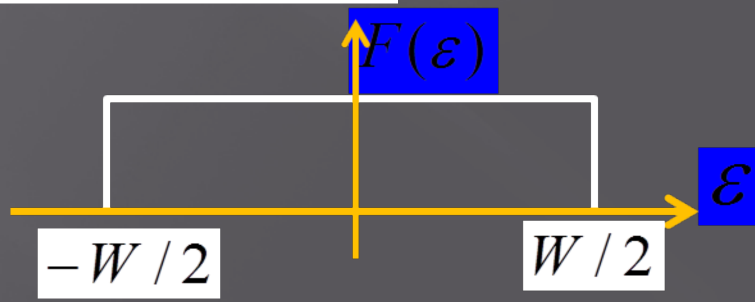
$$m_c = 1/2$$

K	2	3	4	5	6	7	8
Eq.(35)	17.65	34.18	52.30	71.62	91.91	113.0	134.8
Ref.[31]	17.4	33.2	50.1	67.7	87.3	105	125.2
Eq.(28)	29.1	53.6	80.3	108	138	169	200

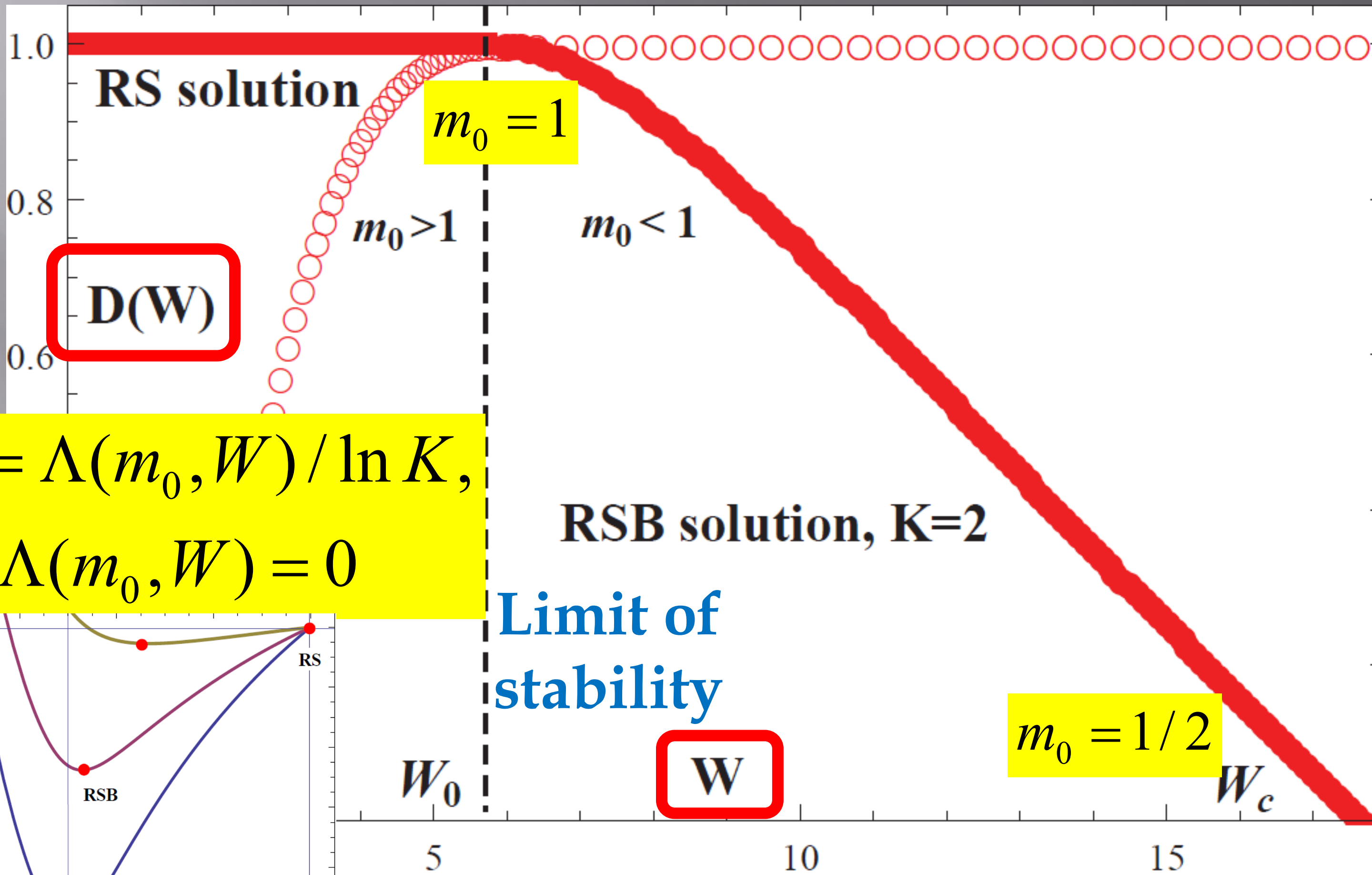
Our result

PD, Biroli

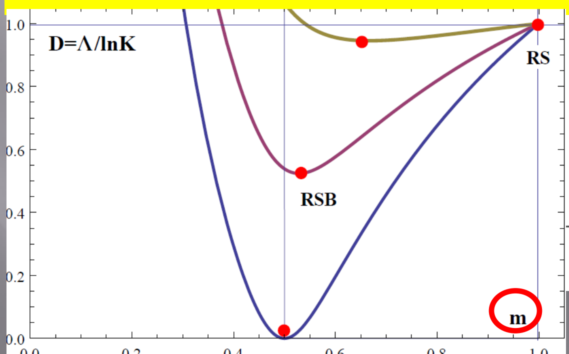
Abou-Chakra, Thouless, Anderson, 1973



RS and RSB solution



$$D(W) = \Lambda(m_0, W) / \ln K,$$
$$\partial_m \Lambda(m_0, W) = 0$$



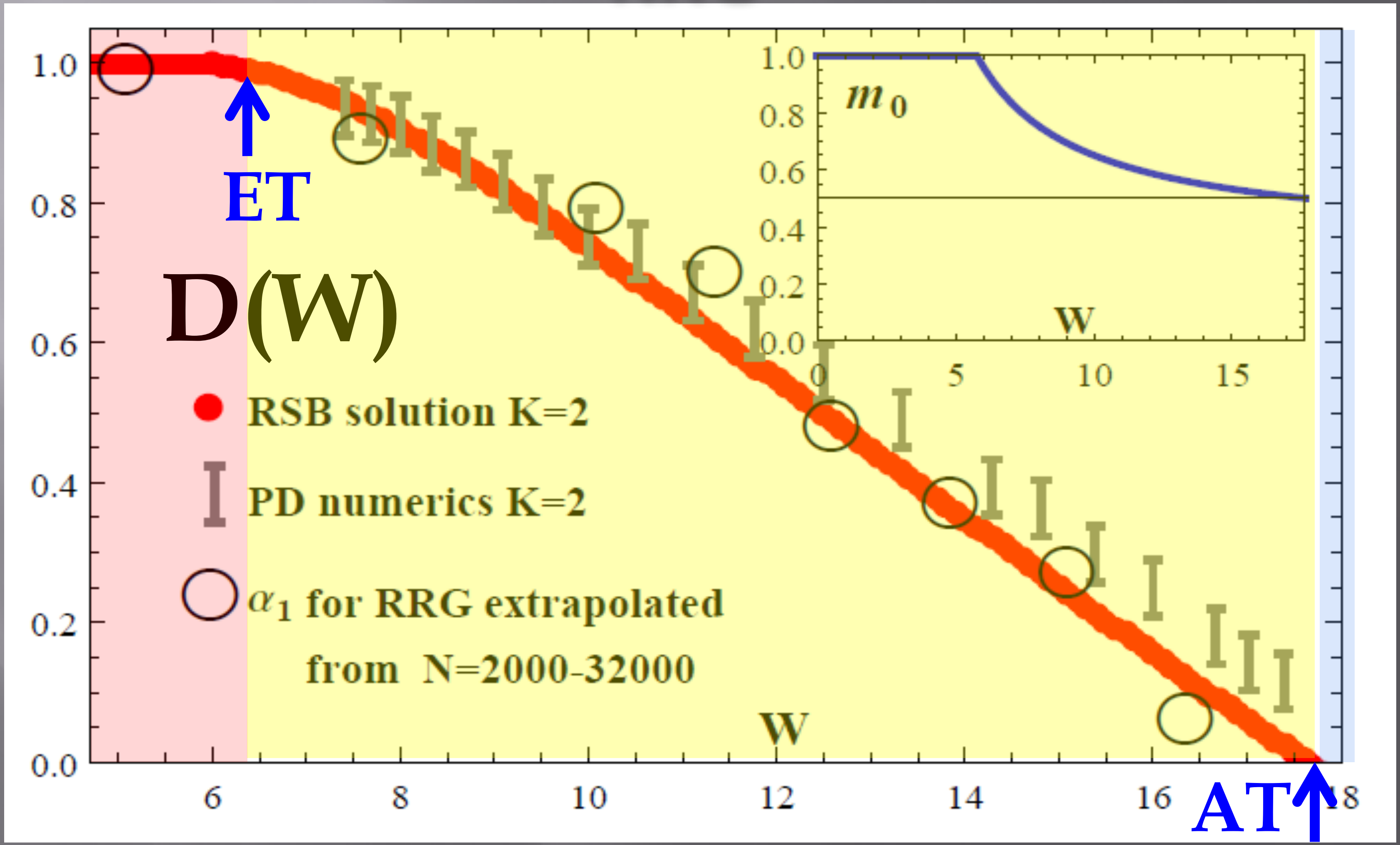
Solutions with minimal Λ is selected

Why minimal Λ?

Fast flow (strong instability, large Λ). Small basin of attraction (small probability to attain the solution)

Slow flow (weak instability, small Λ). Large basin of attraction (large probability to attain the solution)

D from analytical replica symmetry breaking approach on BL, from numerics in PD and on modestly large RRG



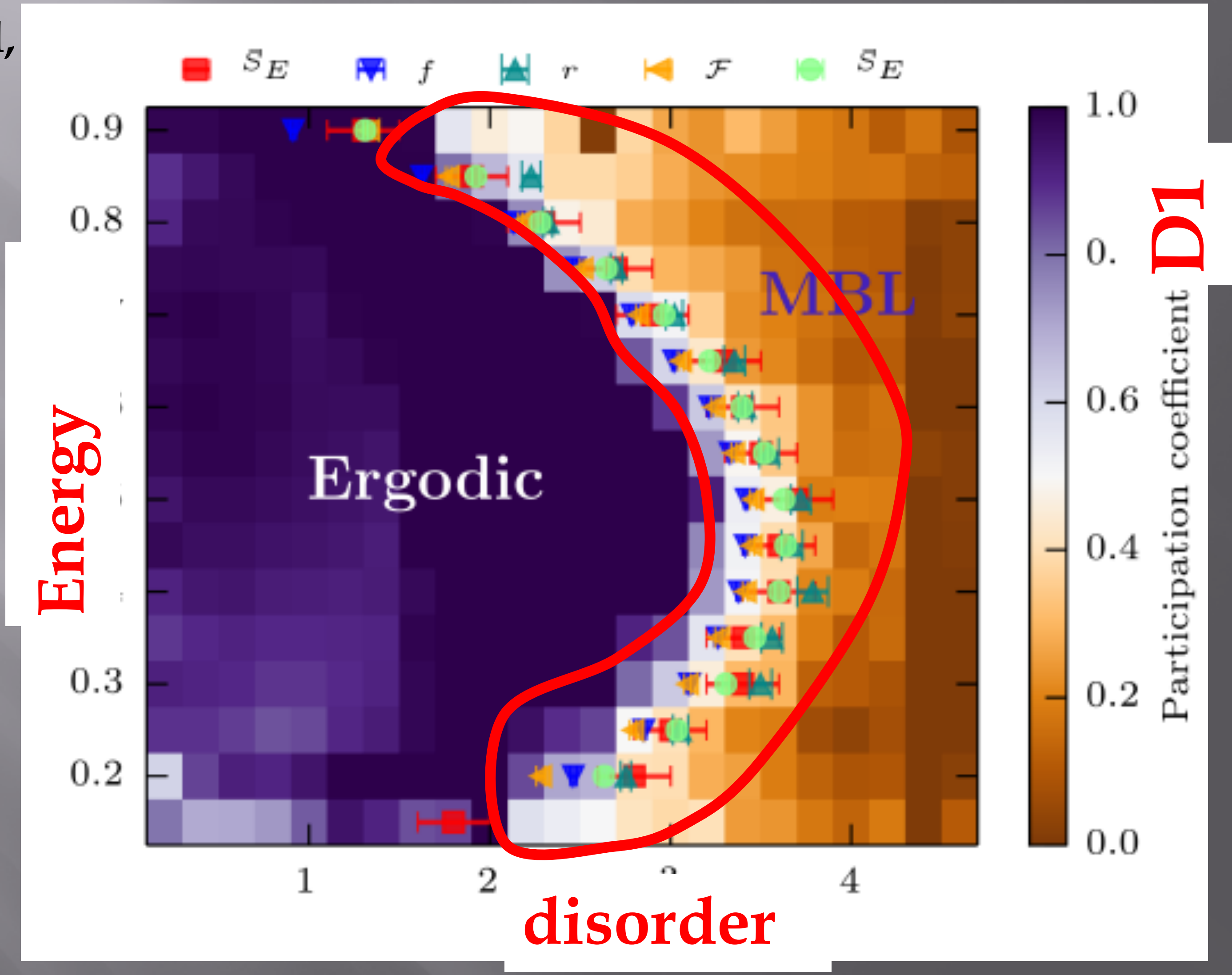
Main conclusion:

"Many-body localization" is a localization on **graphs with local tree structure.**

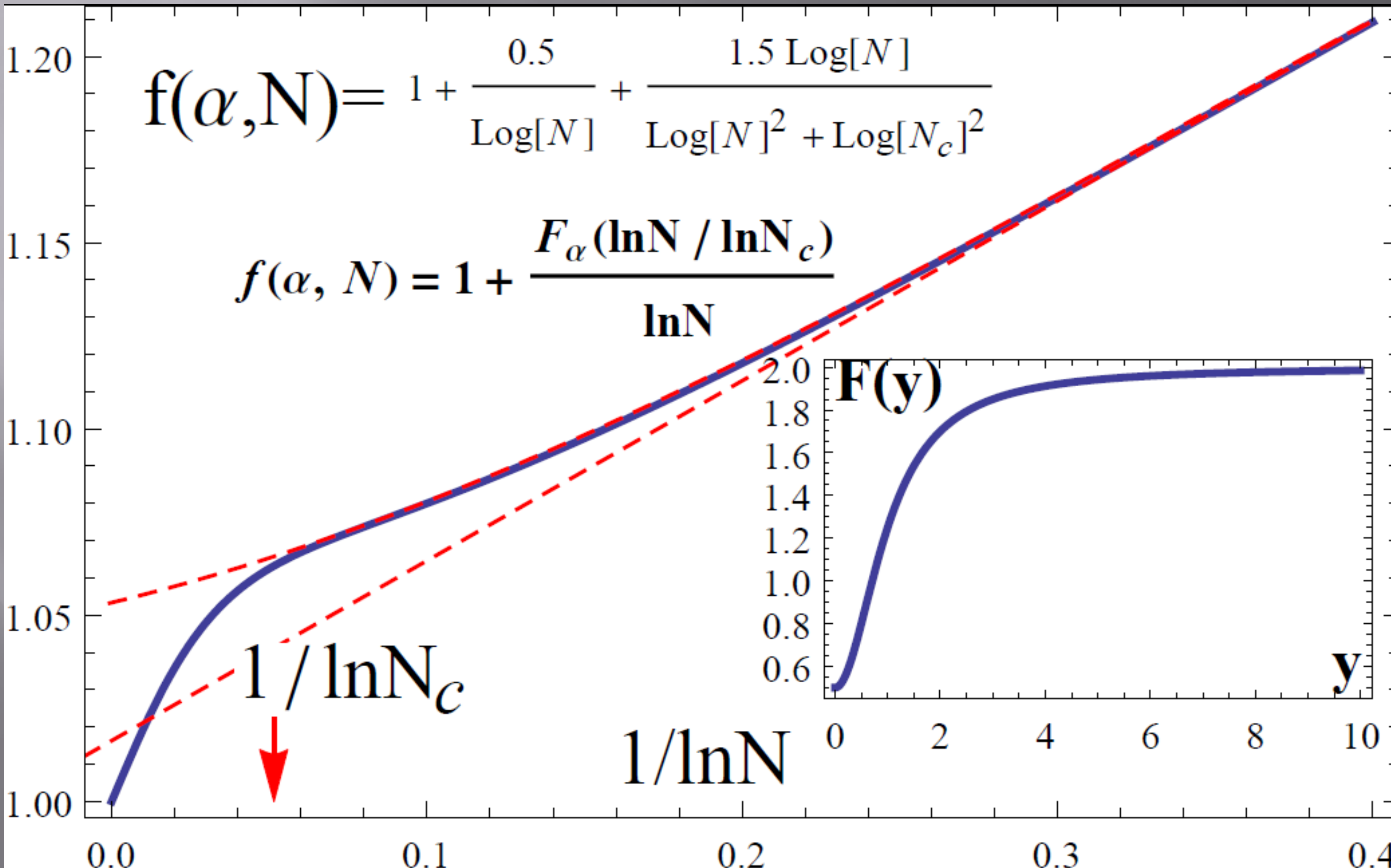
Much work is ahead!

Disordered Heisenberg chain

Luitz et al, 2014



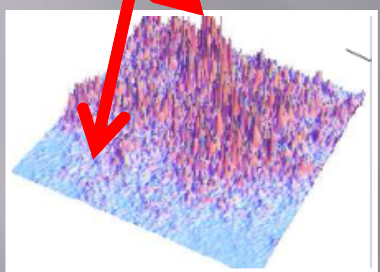
Extrapolation



Definition of D=D1

Mirlin-Fyodorov duality

$$f(\alpha) = f(2-\alpha) + \alpha - 1$$



$$\rho_{typ} \overset{\text{def}}{\sim} \eta\, N^{\boxed{D}}, \qquad \text{for } \eta << \delta \sim 1/N.$$

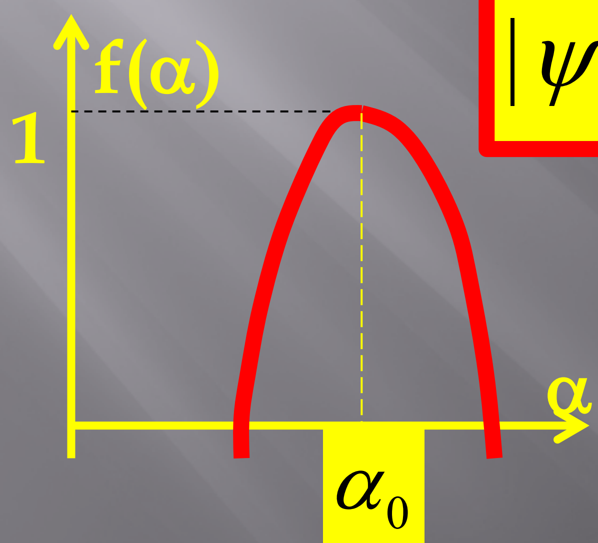
One state inside η

$$\rho = \mathrm{Im}\, G_{E+i\eta}(r) = \frac{1}{\pi}\sum_{\alpha} |\psi_{\alpha}(r)|^2 \frac{\eta}{(E-E_{\alpha})^2+\eta^2}$$

$$\rho_{typ} \sim \frac{\eta}{\delta^2} |\psi|^2_{typ} \sim \eta\, N^2\, |\psi|^2_{typ}$$

$$|\psi|^2_{typ} \sim N^{-2+D} \sim N^{-\alpha_0}$$

Fractal dimension of the w.f. support set



$$2 - \alpha_0 = \alpha_1 = \boxed{D_1 = D}$$

M-F duality