Week 5 Examples

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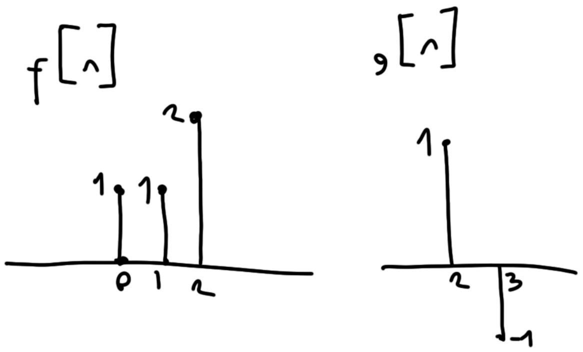
Q1: Derive the convolution $x[n]=f[n]*g[n]$ for the given signals.

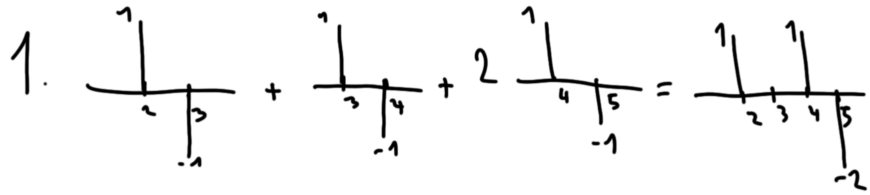
a) $f[n]=\delta[n]+\delta[n-1]+2\delta[n-2]$

$g[n]=\delta[n-2]-\delta[n-3]$



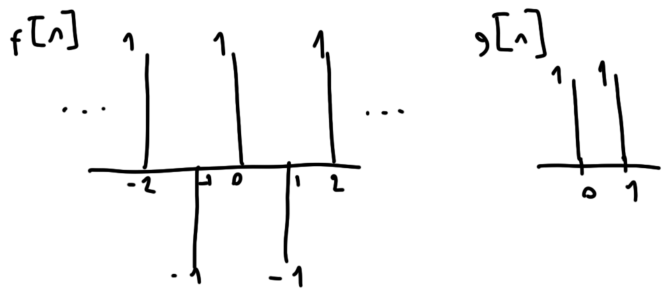
$y[n]=\sum_{k=-\infty}^{\infty} x[k].h[n-k]$ is nonzero for $k=0,1,2.$

$y[n]=f[0].g[n]+f[1].g[n-1]+f[2].g[n-2]$



b) $f[n]=(-1)^n$

$g[n]=\delta[n]+\delta[n-1]$



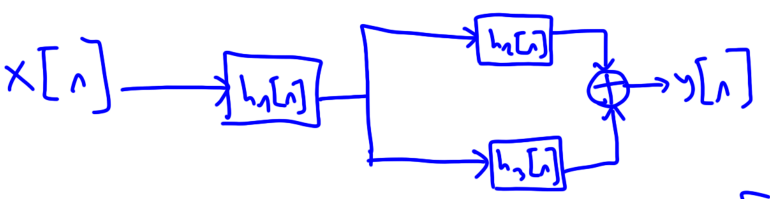
$y[n]=g[0].f[n]+g[n].f[n-1]$

$=$

Q2: For the interconnected system given below, the impulse responses are

$h_1[n]=\left(\frac{1}{2}\right)^n.u[n+2]$

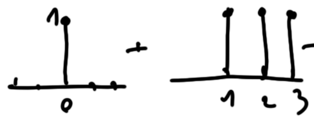
$h_2[n]=\delta[n]$ $\quad h_3[n]=u[n-1]$



Find the overall impulse response of the system.

$$y[n] = x[n] * h_1[n] * h_2[n] + x[n] * h_1[n] * h_2[n]$$

$$= x[n] * h_1[n] * \left(h_2[n] + h_3[n]\right)$$



$\longrightarrow u[n]$

Thus,

$$h[n] = \left(\left(\frac{1}{2}\right)^n . u[n+2]\right) * u[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k+2] . u[n+2] = \sum_{k=-2}^{n} \left(\frac{1}{2}\right)^k . \underbrace{u[n+2]}$$

Needed! To make the summation, n shall be greater than −2.

$$\boxed{\begin{array}{l}\underline{\text{Geometric Sum}} \\ \displaystyle\sum_{k=m}^{n} \alpha . r^k = \frac{\alpha\left(r^m - r^{n+1}\right)}{1-r}\end{array}}$$

$$= \frac{\left(\frac{1}{2}\right)^{-2} - \left(\frac{1}{2}\right)^{n+1}}{1 - \frac{1}{2}} = \left(8 - 2 . \left(\frac{1}{2}\right)^{n+1}\right) . u[n+2]$$

$$= \left(8 - \left(\frac{1}{2}\right)^n\right) . u[n+2]$$