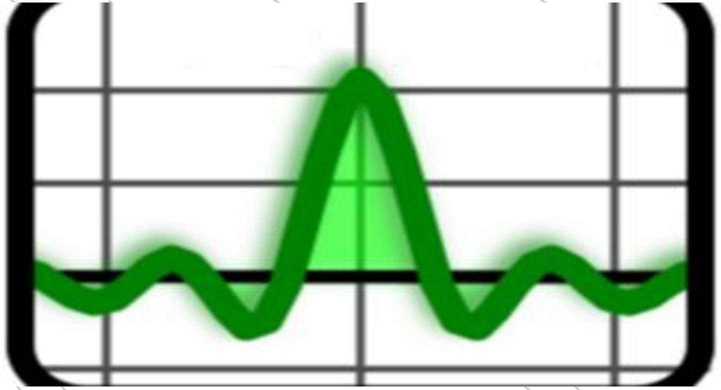

Signals & Systems for Computer Engineering

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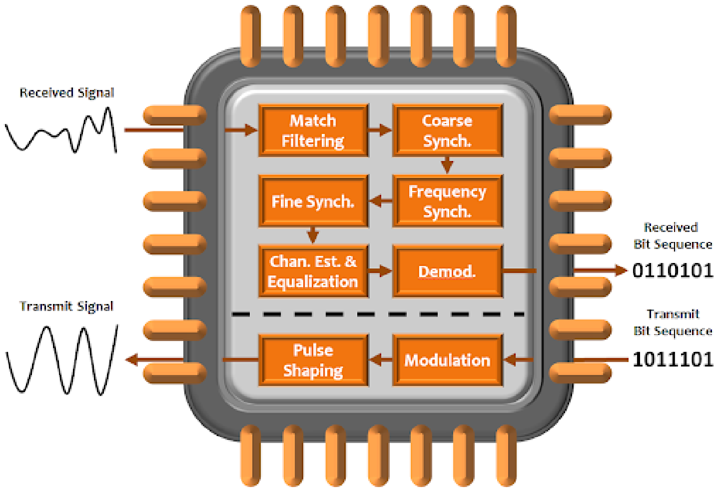
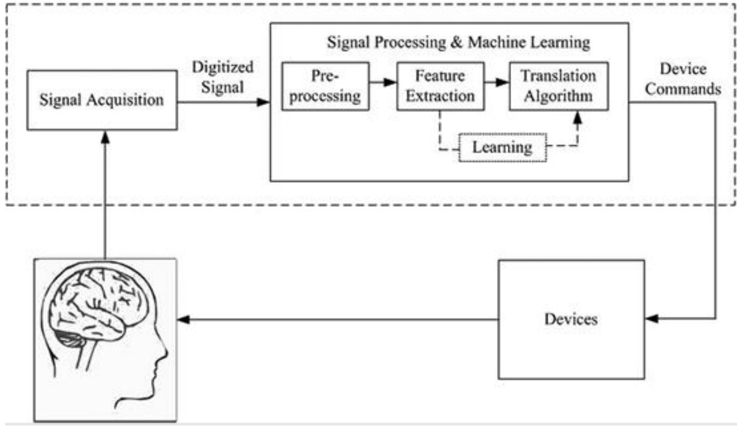
BLG354E / CRN: 21350
1st Week Lecture

Introduction:



signal processing for communications

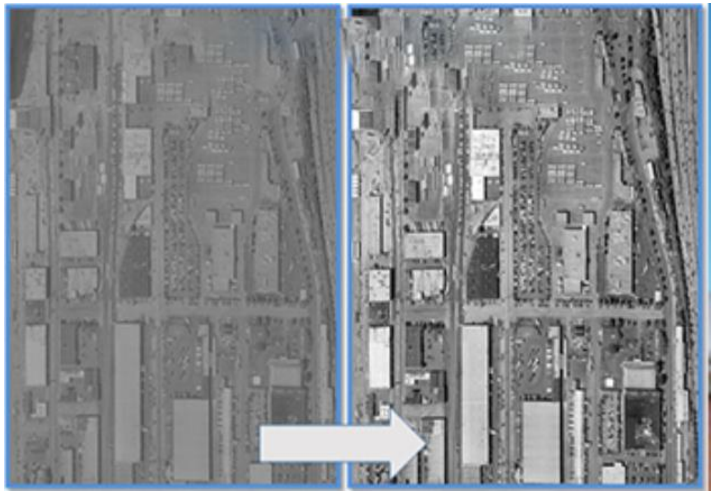


Neuro science and machine learning

Financial analysis and predictions

Autonomous Navigation

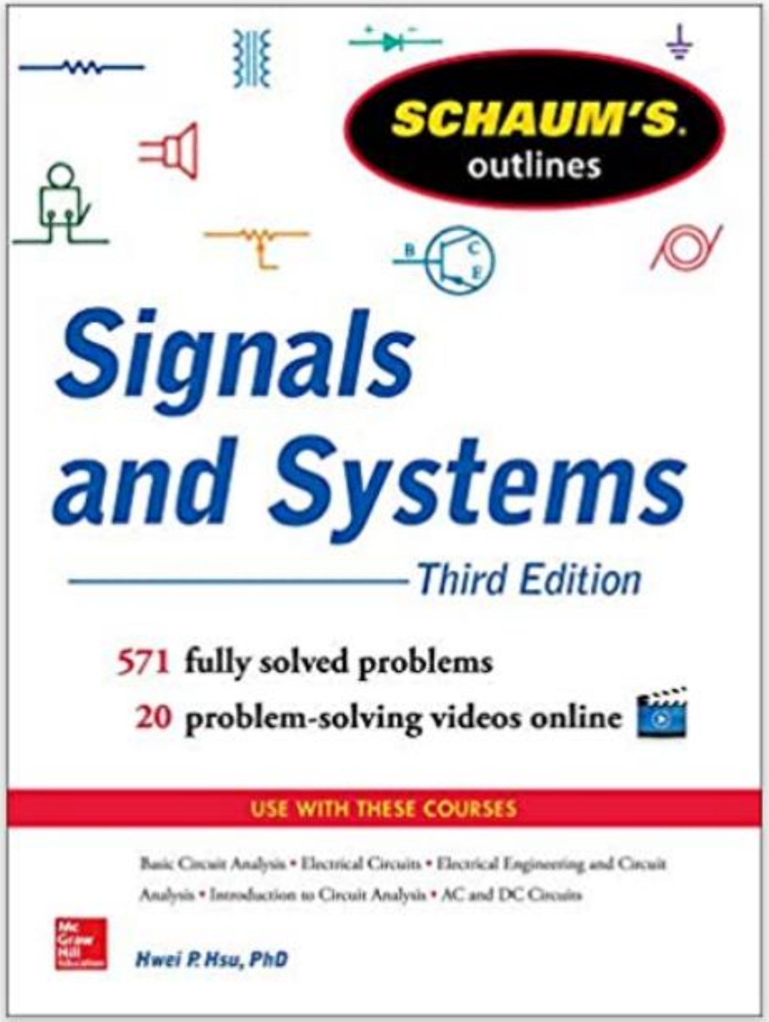
Image processing



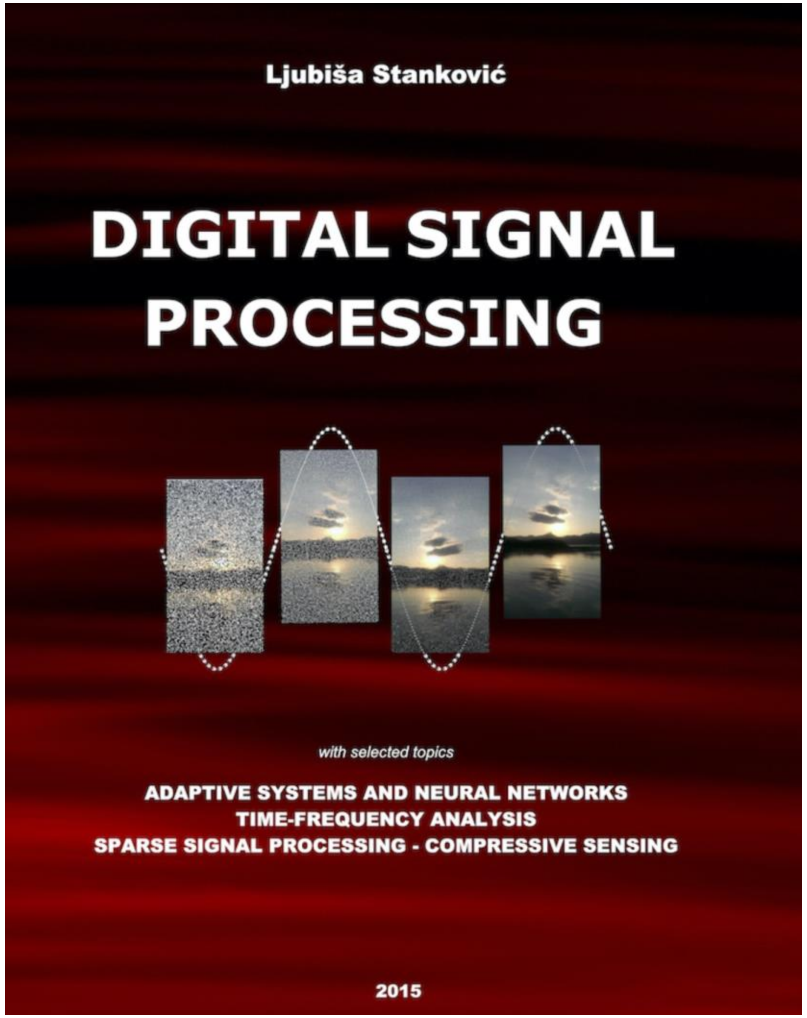
Sensor fusion and control systems

Text Books of the Course:

Main Text Book



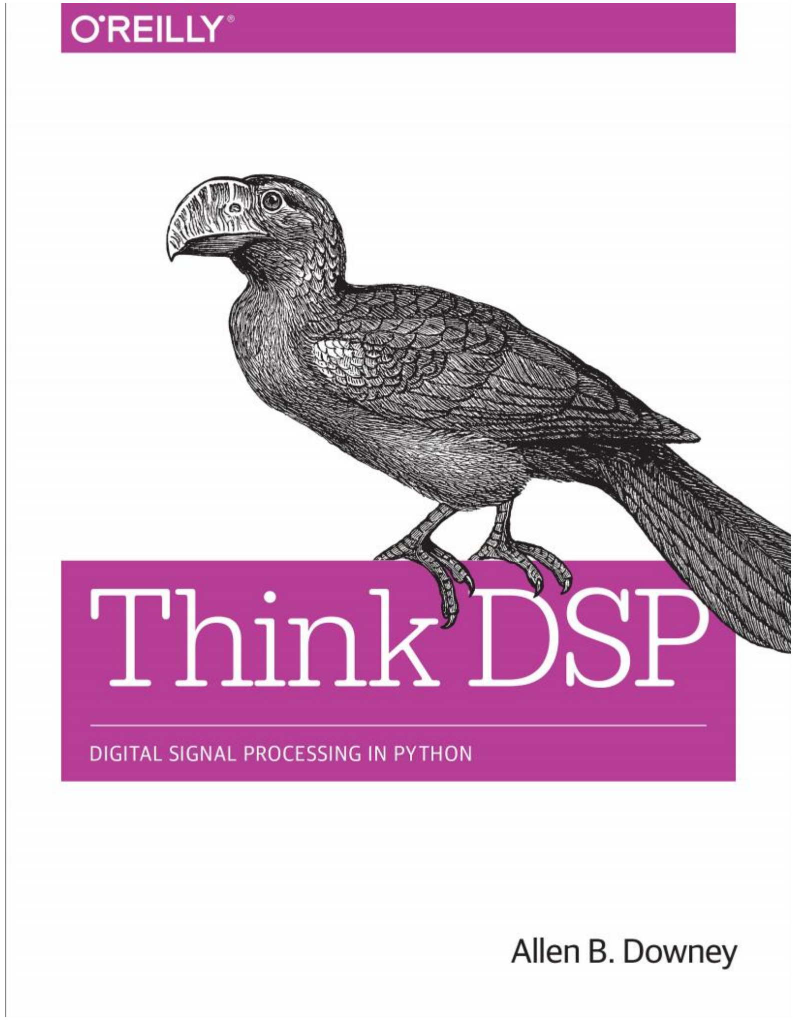
Hwei Hsu, 2013, McGraw-Hill



Ljubiša Stankovic, Revised edition 2020

ISBN-13: 978-1514179987
CreateSpace Independent Publishing Platform

Programming Examples with Python



Allen B. Downey, 2016, O'REILLY

https://github.com/AllenDowney/ThinkDSP

Course Plan (2023-2024 Spring):

1st week: Introduction, Course Outline, Signals and Classification of the Signals
2nd week: Properties of Continuous Time Signals and Discrete Time Signals, Classification of the Systems
3rd week: Convolution Integral, Introduction to Linear Time Invariant (LTI) Systems
4th week: Properties and application examples of CT LTI Systems
5th week: Properties and application examples of DT LTI systems, FIR Filters
6th week: Real Time Programming and Simulation of DT Systems
7th week: Signal Processing Domains, Transforms and Laplace Transform of LTI Systems
8th week: Frequency Response of the Systems
9th week: Fourier Transform and its Applications, Spectral Analysis with Fourier Transform, Modulation
10th week: Convolution in frequency domain, Z Transform of DT Systems and convolution
11th week: Fourier Analysis of Discrete-Time Signals and Systems
12th week: Fast Fourier Transform
13th week: Digital Filter Design and Software Implementation
14th week: Generalized System design, analysis and programming applications

Grading Policy:

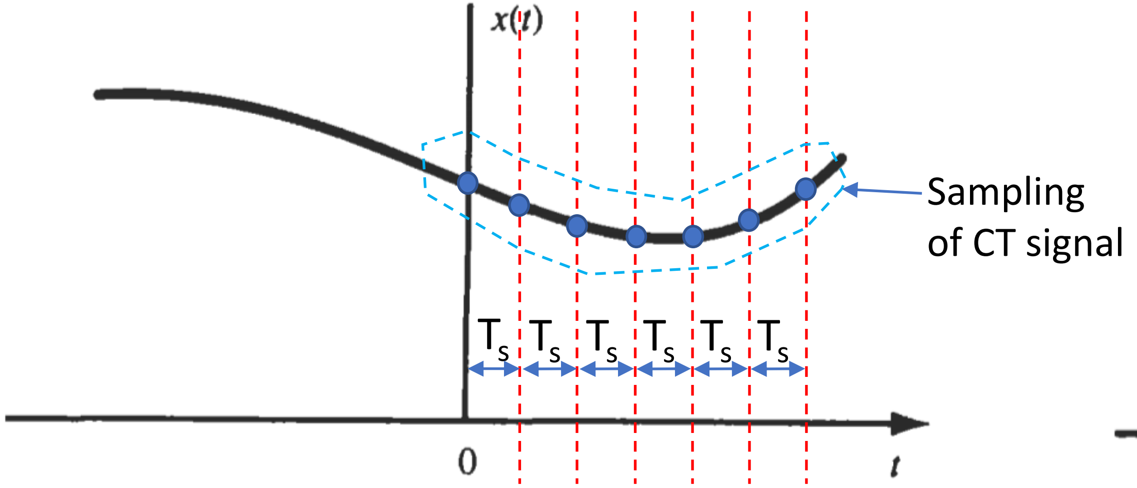
Quiz – 3 Weeks	15%
Homework Assignments (2)	15%
Midterm	30%
Final exam	40%

Visa / VF Condition:

- **At least 70% Attendance required**
- **Overall visa score must be at least 30** ($0.15 \times \text{Quiz} + 0.15 \times \text{Homework} + 0.3 \times \text{Midterm} \geq 18$)

TAs: Asel Menekşe menekse16@itu.edu.tr
Onur Can Koyun okoyun@itu.edu.tr

Continuous Time Signal

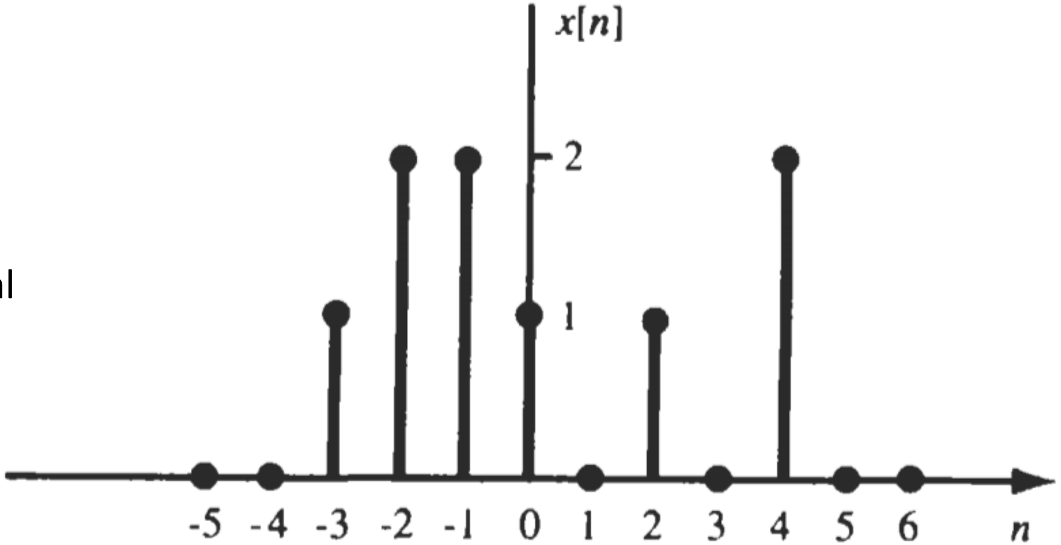


A signal x(t) is a continuous-time signal if t is a continuous variable.

Time interval between sampling instants T_s is called the sampling period. When the sampling intervals are equal (uniform sampling) then:

$$x_n = x[n] = x(t = nT_s)$$

Discrete Time Signal



If t is a discrete variable, that is, x(t) is defined at discrete times, then x(t) is a discrete-time signal.

$$\{x_n\} = \{\ldots, 0, 0, 1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2, 0, 0, \ldots\}$$

Notations for discrete time signal sequences:

a) Analytic expression:

$$x[n] = x_n = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases} \Rightarrow \{x_n\} = \left\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \left(\frac{1}{2}\right)^n, \ldots\right\}$$

b) Sequential expression:

$$\{x_n\} = \{\ldots, 0, 0, 1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2, 0, 0, \ldots\} \Rightarrow \{x_n\} = \{1, 2, 2, \underset{\uparrow}{1}, 0, 1, 0, 2\}$$

n=0, x_0=1

Sum and product of two sequences:

$$\{c_n\} = \{a_n\} + \{b_n\} \longrightarrow c_n = a_n + b_n$$

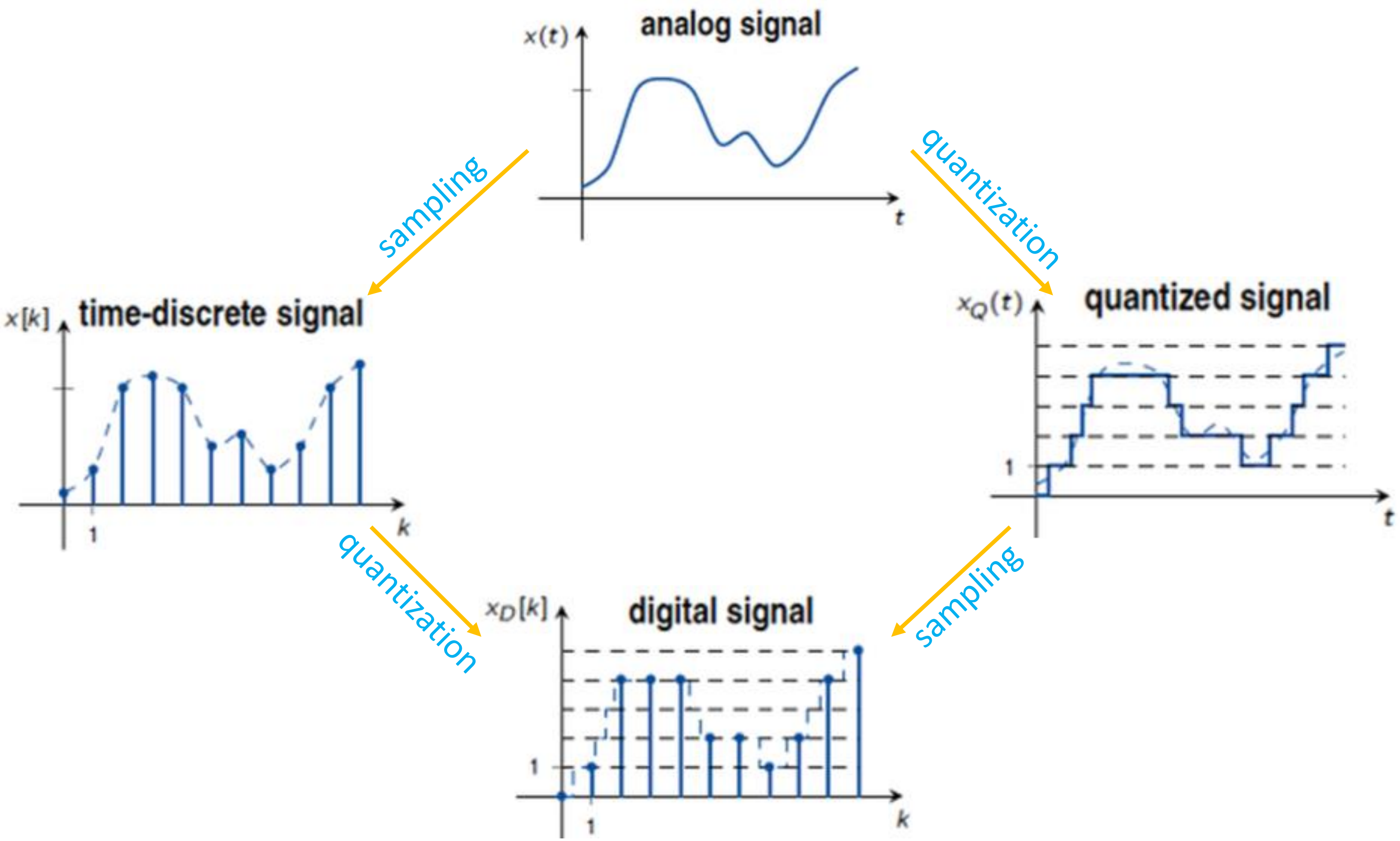
$$\{c_n\} = \{a_n\}\{b_n\} \longrightarrow c_n = a_n b_n$$

$$\{c_n\} = \alpha\{a_n\} \longrightarrow c_n = \alpha a_n$$

where $\alpha = \text{constant}$

Analog and Digital Signals:

If a continuous-time signal x(t) can take on any value in the continuous interval (a, b), where a may be -∞ and b may be + ∞, then the continuous-time signal x(t) is called an analog signal.



If a discrete-time signal x[n] can take on only a finite number of distinct values, then we call this signal a digital signal.

Real and Complex Signals:

A signal x(t) is a real signal if its value is a real number.

A signal x(t) is a complex signal if its value is a complex number such as $x(t)=x_1(t)+jx_2(t)$ where $j=\sqrt{-1}$

$$s \overset{\text{def}}{=} \sigma + j\omega$$

σ → Real part, $j\omega$ → Imaginary part

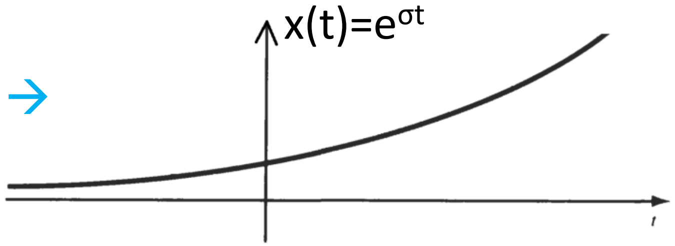
$$x(t) = e^{st} = e^{(\sigma+j\omega)t} = e^{\sigma t}(\cos \omega t + j \sin \omega t)$$

If $x(t)=A\cdot e^{j(\omega t+\Theta)}$ then $x(t)= A\cdot \mathrm{Re}\{e^{j(\omega t+\Theta)}\}+A\cdot \mathrm{Im}\{e^{j(\omega t+\Theta)}\}$

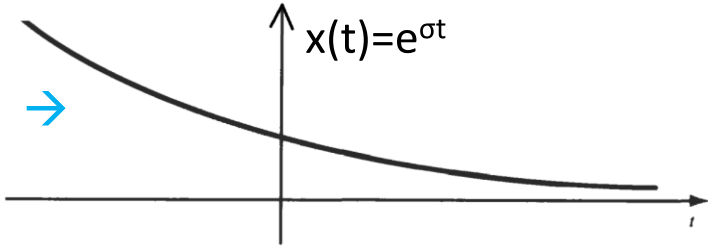
$$x(t)= A\cdot\cos(\omega t+\Theta)+J\cdot A\cdot\sin(\omega t+\Theta)$$

$$x_1(t) \; + \; jx_2(t)$$

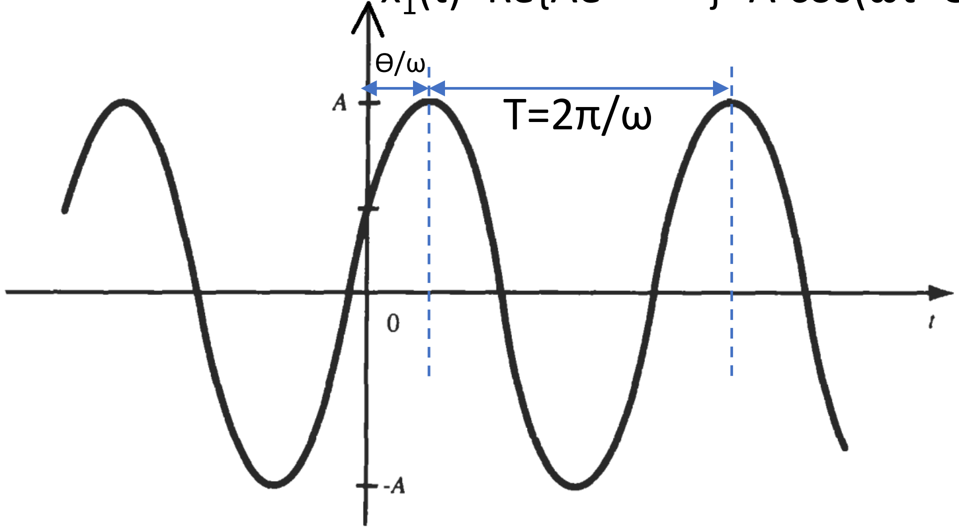
$\omega=0$, $\sigma>0$ →



$\omega=0$, $\sigma<0$ →



$x_1(t)=\mathrm{Re}\{Ae^{j(\omega t+\Theta)}\}=A\cdot\cos(\omega t+\Theta)$



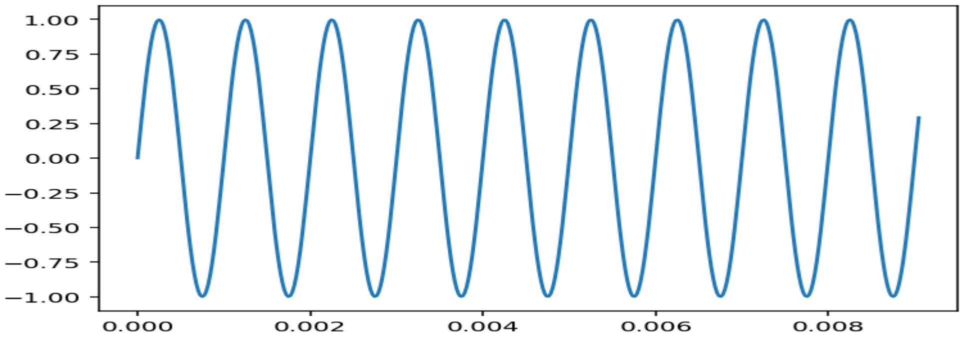
Discrete time exponential signal: $x[n] = e^{j\Omega_0 n} = \cos \Omega_0 n + j \sin \Omega_0 n$

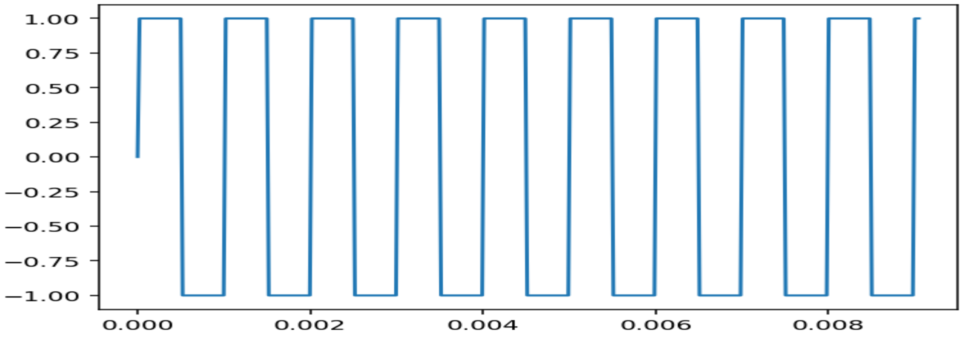
Waveform generation, draw and play in Python

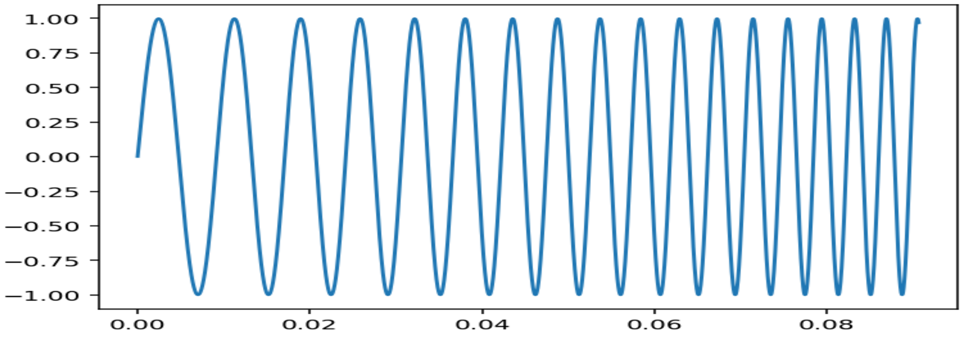
```
# Sinusoidal and Square waveform sound generator
import numpy as np
import sounddevice as sd
import matplotlib.pyplot as plt

fs = 44100
time = np.arange(0, 4, 1/fs)

# 1- Fixed frequency sinusoidal sound at 1kHz
sound=np.sin(2*np.pi*1000*time)
```

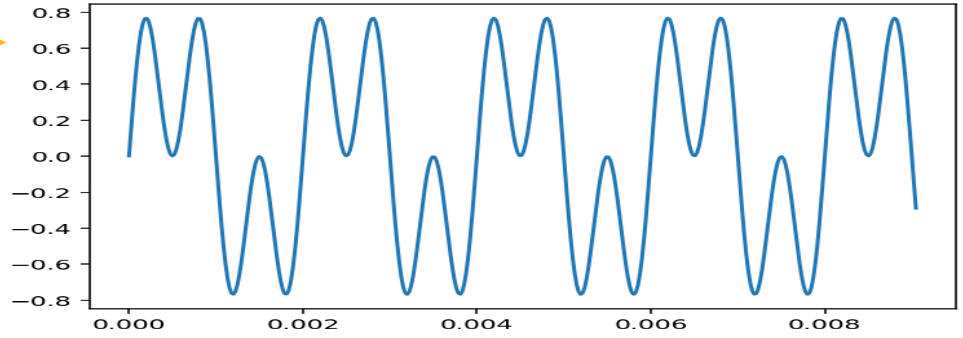
$\underbrace{2\pi \cdot 1000}_{\omega=2\pi f}$

```
# 2- square waveform:
#sound=np.sign(np.sin(2*np.pi*1000*time))

# 3- variable frequency sinusoidal waveform:
# sound=np.sin(2*np.pi*(100+1000*time)*time)

# 4- Multitone sinusoidal waveform:
#sound=0.5*(np.sin(2*np.pi*500*time)+np.sin(2*np.pi*1500*time))

plt.plot(time[0:400],sound[0:400])
sd.play(sound, fs)
```



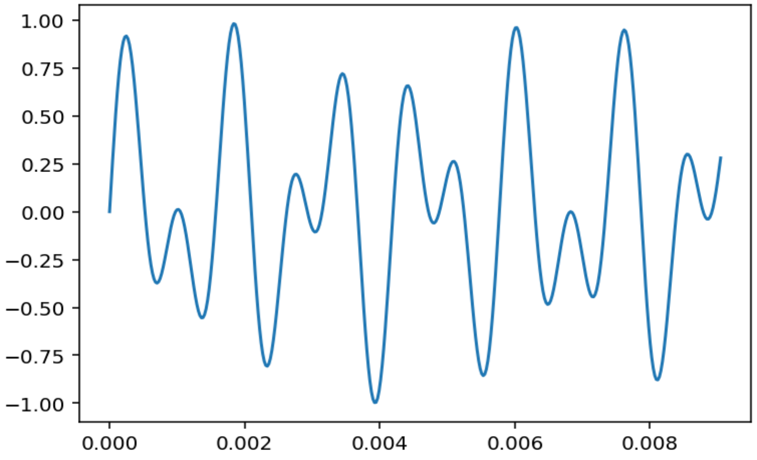






DTMF (Dual-Tone Multi-Frequency) :

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#



Tone code for “1” → 697Hz + 1209Hz
(sound1)

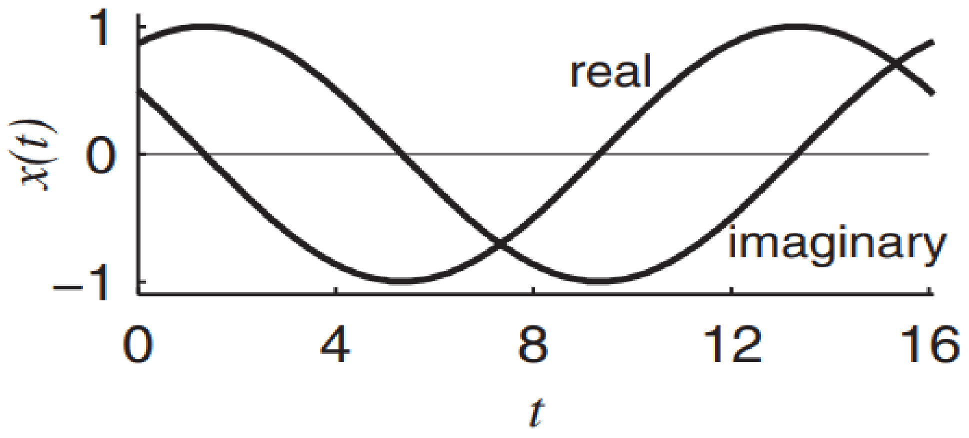
```
# DTMF Tone Code Sequence
import numpy as np
import sounddevice as sd
import matplotlib.pyplot as plt

fs = 44100
time = np.arange(0, 1, 1/fs)

# DTMF waveform for the key sequence "1“ , "5", "9", "4"
sound1=0.5*(np.sin(2*np.pi*697*time)+np.sin(2*np.pi*1209*time))
sound2=0.5*(np.sin(2*np.pi*770*time)+np.sin(2*np.pi*1336*time))
sound3=0.5*(np.sin(2*np.pi*852*time)+np.sin(2*np.pi*1477*time))
sound4=0.5*(np.sin(2*np.pi*770*time)+np.sin(2*np.pi*1209*time))

sound=np.concatenate((sound1,sound2,sound3,sound4))
plt.plot(time[0:400],sound1[0:400])
sd.play(sound, fs)
```

Example:

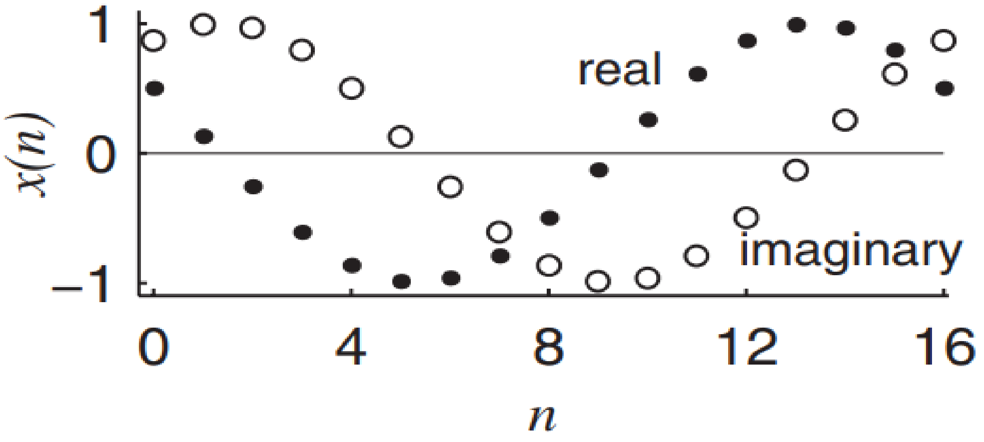
Illustrate the signal $x(t) = e^{j(\frac{2\pi}{16}t+\frac{\pi}{3})}$ as CT signal and for the case if it is discretized at f_s=1Hz

$$x(t) = e^{j(\frac{2\pi}{16}t+\frac{\pi}{3})} = \cos\left(\frac{2\pi}{16}t + \frac{\pi}{3}\right) + j\sin\left(\frac{2\pi}{16}t + \frac{\pi}{3}\right)$$

$$x(t) = e^{j(\frac{2\pi}{16}t+\frac{\pi}{3})}$$



$$x[n] = e^{j(\frac{2\pi}{16}n+\frac{\pi}{3})}$$



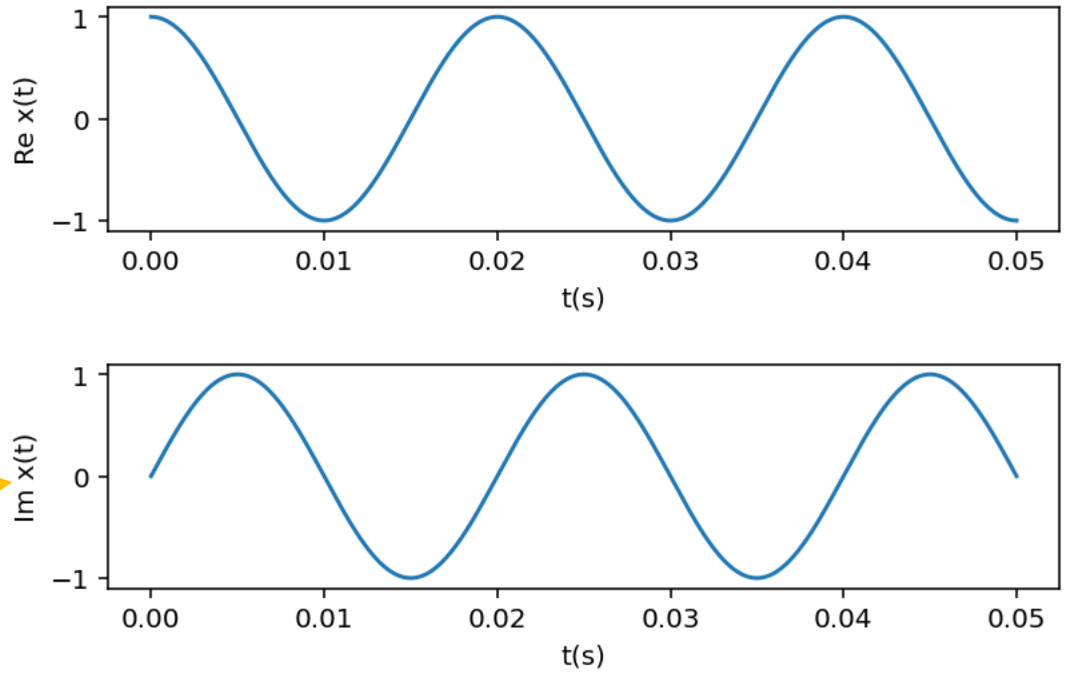
Plot of Complex CT Signal in Python

$$x(t)=e^{j\omega t}= e^{j2\pi ft}$$

```
# Continuous time Complex signal @50Hz
import numpy as np
import matplotlib.pyplot as plt

Duration=0.05   # 50ms time window
Resolution=1000
f=50  # Frequency=50Hz
t = np.linspace(0, Duration, Resolution)

plt.subplot(2,1,1);
plt.plot(t, np.exp(2j*np.pi*f*t).real );
plt.xlabel('t(s)');
plt.ylabel('Re x(t)');
plt.subplot(2,1,2);
plt.plot(t, np.exp(2j*np.pi*f*t).imag);
plt.xlabel('t(s)');
plt.ylabel('Im x(t)');
plt.tight_layout(pad=2.0)
plt.show()
```

Real(x(t))

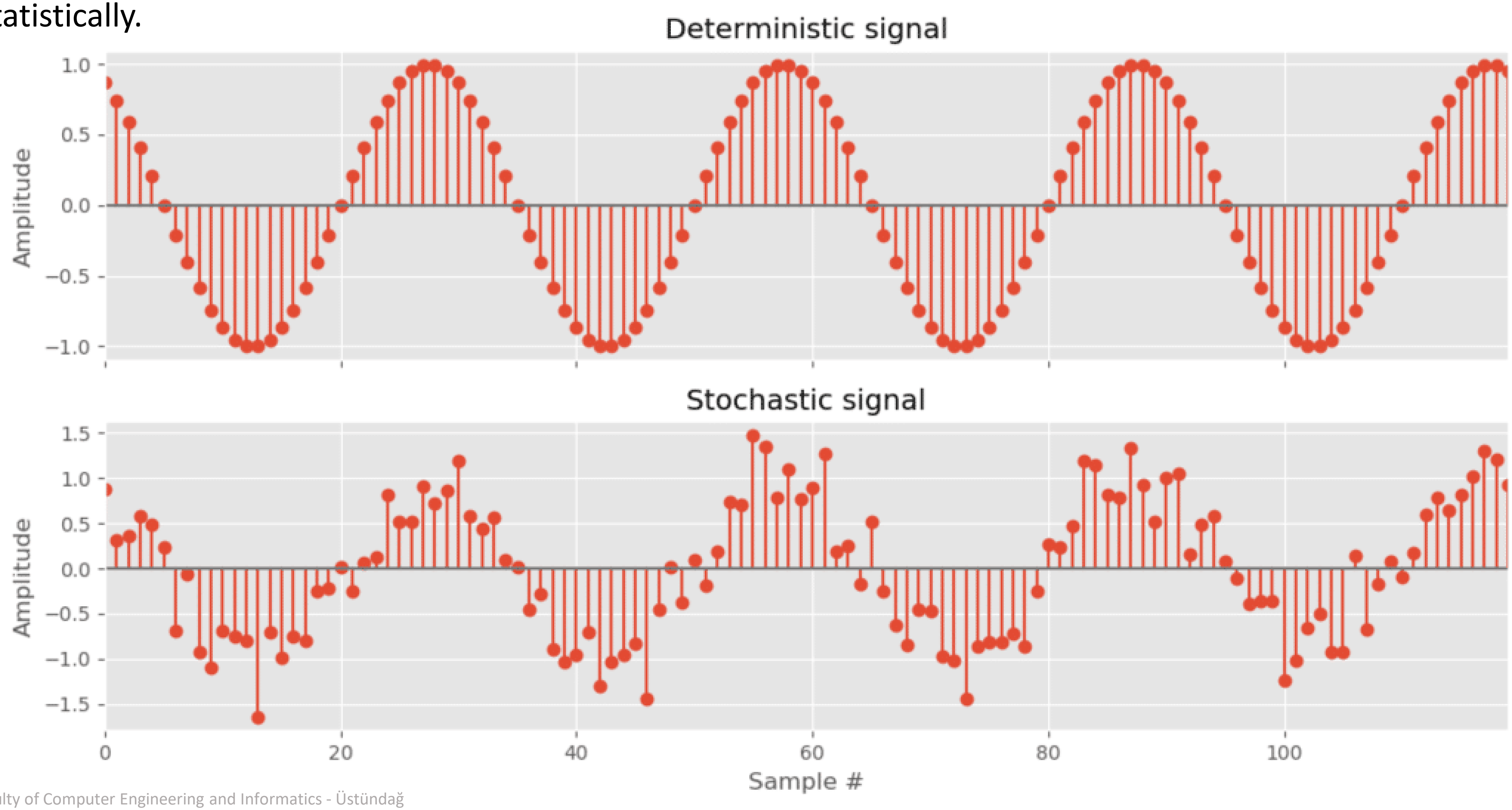
Imaginary(x(t))



Deterministic and Stochastic Signals:

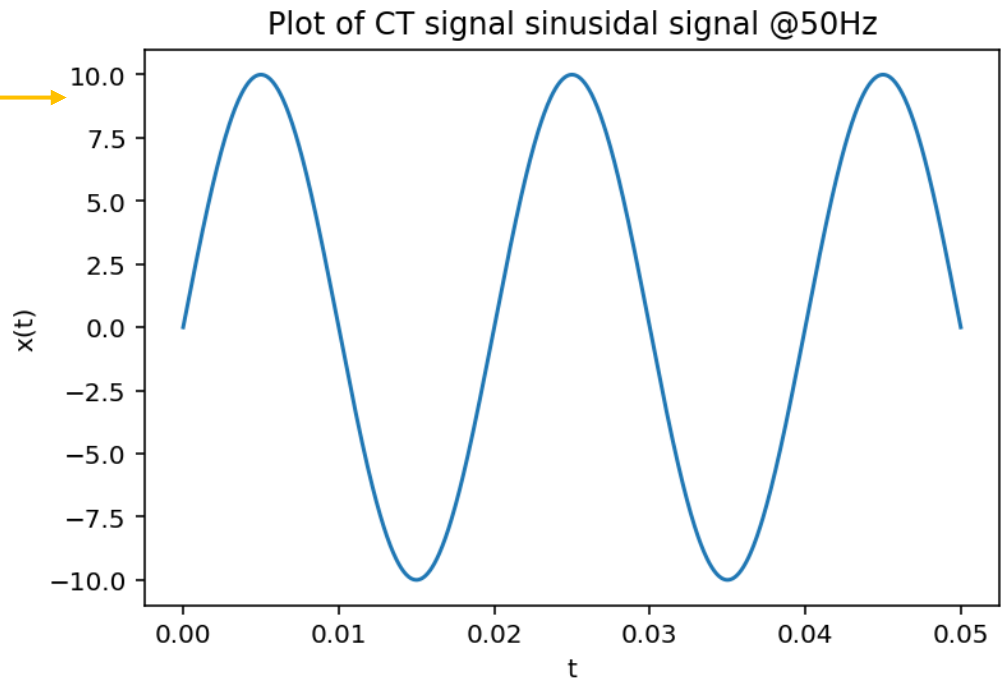
Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time

Random/Stochastic signals are those signals that take random values at any given time and must be characterized statistically.

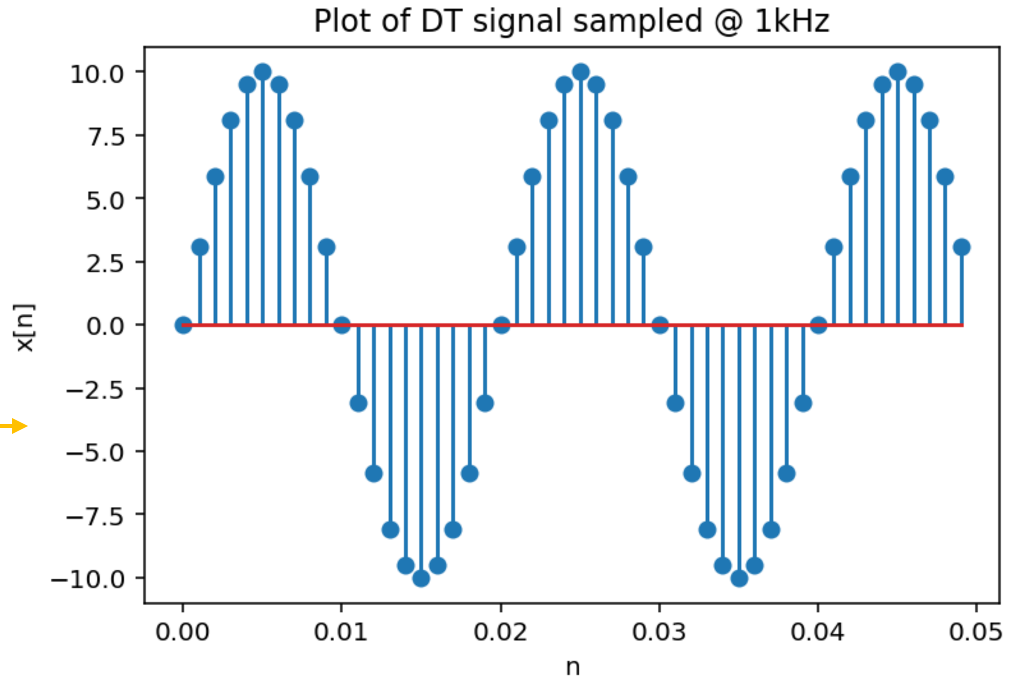


Plot of Discrete Time Signal in Python

```
#Continuous time sinusoidal signal @50Hz
# and its discrete form when it is sampled @1000kHz

import numpy as np
import matplotlib.pyplot as plt
Duration=0.05   # 50ms time window
Resolution=1000
t = np.linspace(0, Duration, Resolution)
plt.plot(t, 10 * np.sin(2*np.pi*50*t));
plt.xlabel('t');
plt.ylabel('x(t)');
plt.title('Plot of CT signal sinusidal signal @50Hz');
plt.show()
```

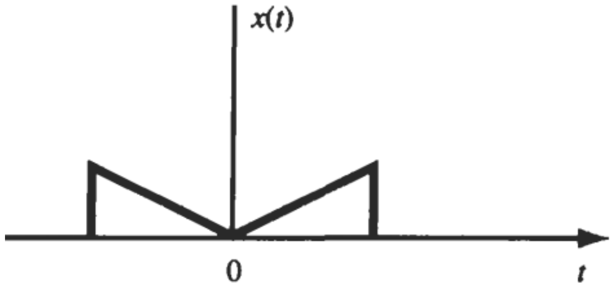



```
# Sample at 1ms and draw the DT signal:
Ts = 0.001  # Sampling period
n = np.arange(0,Duration,Ts);

x = 10*np.sin(2 * np.pi * 50 * n)
plt.xlabel('n');
plt.ylabel('x[n]');
plt.title('Plot of DT signal sampled @ 1kHz');
plt.stem(n, x);
```

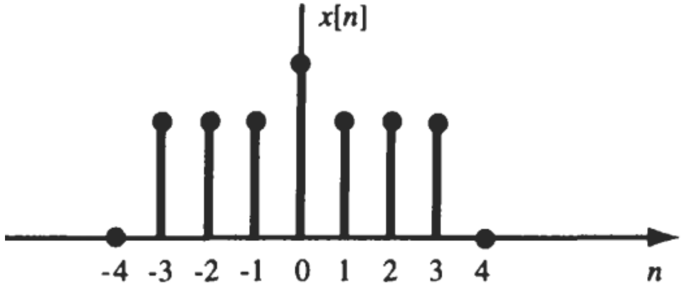


Even and Odd Signals:

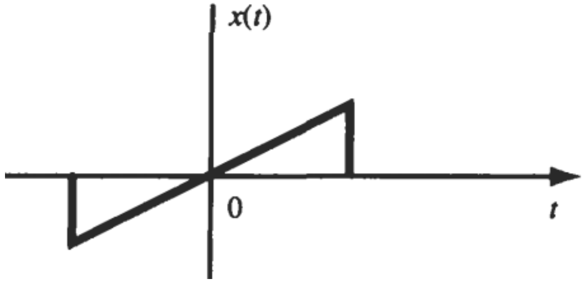
A CT signal x(t) is referred to as an even signal if **x(-t)=x(t)**



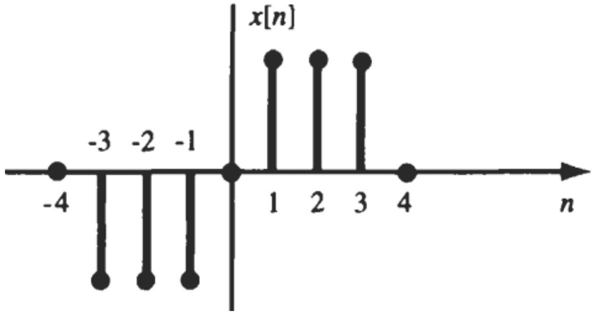
A DT signal x[n] is referred to as an even signal if **x[-n]=x[n]**



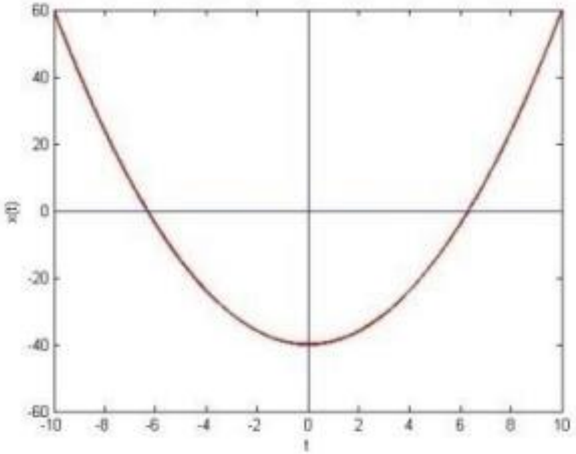
A CT signal x(t) is referred to as an odd signal if **x(-t)=-x(t)**



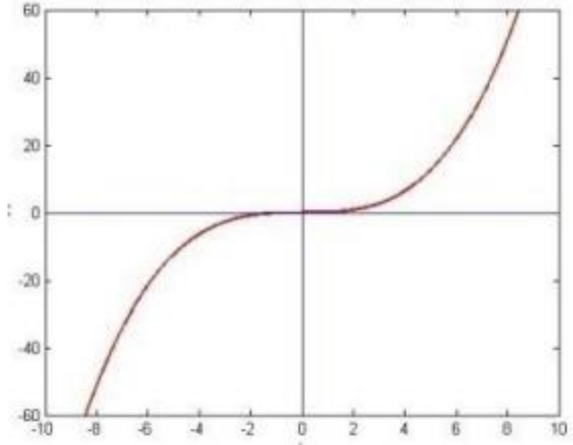
A DT signal x[n] is referred to as an odd signal if **x[-n]=-x[n]**



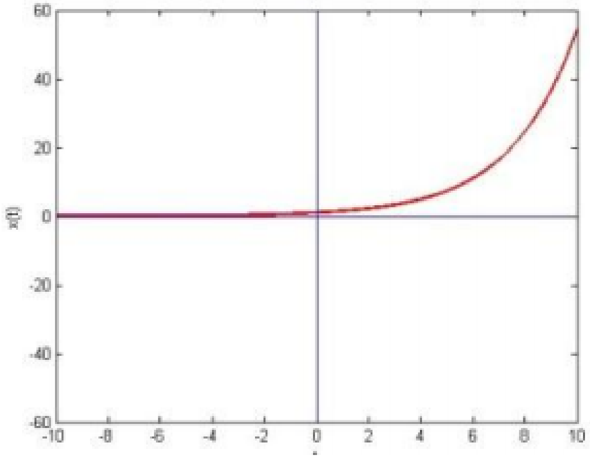
Example:



$x(t) = t^2 - 40$ is even.



$x(t) = 0.1t^3$ is odd.



$x(t) = e^{0.4t}$ is neither even nor odd.

Decomposition Theorem:

Any signal x(t) or x[n] can be expressed as a sum of two signals, one of which is even and one of which is odd.

$$x(t) = x_e(t) + x_o(t)$$
$$x[n] = x_e[n] + x_o[n]$$

where

$$\begin{cases} x_e(t) = \frac{1}{2}\{x(t) + x(-t)\} & \text{even part of } x(t) \\ x_e[n] = \frac{1}{2}\{x[n] + x[-n]\} & \text{even part of } x[n] \\ x_o(t) = \frac{1}{2}\{x(t) - x(-t)\} & \text{odd part of } x(t) \\ x_o[n] = \frac{1}{2}\{x[n] - x[-n]\} & \text{odd part of } x[n] \end{cases}$$

Example:

$$x(n) = \cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) \overset{?\,\rightarrow}{=} \frac{\cos\left(\frac{2\pi}{8}n\right)}{2} - \frac{\sqrt{3}}{2}\sin\left(\frac{2\pi}{8}n\right)$$

The even-symmetric component of x(n):

$$x_e(n) = \frac{x(n) + x(-n)}{2} = \frac{\cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) + \cos\left(\frac{2\pi}{8}(-n) + \frac{\pi}{3}\right)}{2} = \frac{2\cos\left(\frac{2\pi}{8}n\right)\cos\left(\frac{\pi}{3}\right)}{2} = \frac{\cos\left(\frac{2\pi}{8}n\right)}{2}$$

The odd-symmetric component of x(n):

$$x_o(n) = \frac{x(n) - x(-n)}{2} = \frac{\cos\left(\frac{2\pi}{8}n + \frac{\pi}{3}\right) - \cos\left(\frac{2\pi}{8}(-n) + \frac{\pi}{3}\right)}{2} = \frac{-2\sin\left(\frac{2\pi}{8}n\right)\sin\left(\frac{\pi}{3}\right)}{2} = -\frac{\sqrt{3}}{2}\sin\left(\frac{2\pi}{8}n\right)$$

Integral of decomposed signals:

Integral of an odd-symmetric signal $x_o(t)$ over symmetric limits is zero ➡ $$\int_{-t_0}^{t_0} x_o(t)\,dt = 0$$

$$\int_{-t_0}^{t_0} x(t)\,dt = \int_{-t_0}^{t_0} x_e(t)\,dt = 2\int_{0}^{t_0} x_e(t)\,dt$$

This property will be used for a simplification of Fourier Series

Example: Express the signal $x(t) = e^{j\left(\frac{2\pi}{16}t+\frac{\pi}{3}\right)}$ in terms of its symmetric components

$$x_e(t) = \frac{x(t)+x(-t)}{2} = e^{j\left(\frac{\pi}{3}\right)}\frac{e^{j\left(\frac{2\pi}{16}t\right)} + e^{j\left(\frac{2\pi}{16}(-t)\right)}}{2} = e^{j\left(\frac{\pi}{3}\right)}\cos\left(\frac{2\pi}{16}t\right)$$

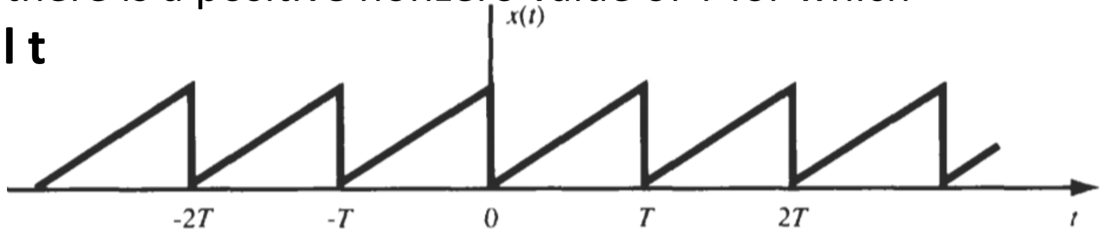
$$x_o(t) = \frac{x(t)-x(-t)}{2} = e^{j\left(\frac{\pi}{3}\right)}\frac{e^{j\left(\frac{2\pi}{16}t\right)} - e^{j\left(\frac{2\pi}{16}(-t)\right)}}{2} = je^{j\left(\frac{\pi}{3}\right)}\sin\left(\frac{2\pi}{16}t\right)$$

$$x(t) = x_e(t) + x_o(t) = e^{j\left(\frac{\pi}{3}\right)}\cos\left(\frac{2\pi}{16}t\right) + je^{j\left(\frac{\pi}{3}\right)}\sin\left(\frac{2\pi}{16}t\right)$$

Periodic and Nonperiodic Signals:

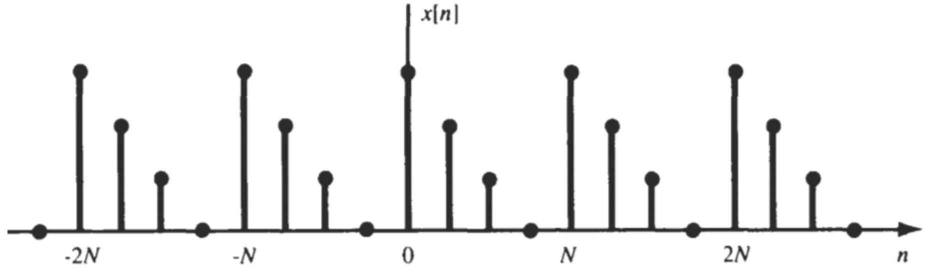
A continuous-time signal x(t) is said to be periodic with period T if there is a positive nonzero value of T for which

x(t+T)=x(t)   for all t



A discrete-time signal x[n] is said to be periodic with period N if there is a positive nonzero value of N for which

x[n+N]=x[n]   for all n



Smallest value of T or N that satisfies the above condition is called fundamental period

Signals do not satisfy periodicity conditions are called non-periodic signals

Example:

Determine the fundamental period of $x(t) = e^{j\frac{3\pi t}{5}}$ **x(t+T)=x(t)** →

$$\left\{\begin{array}{l} e^{j3\pi t/5} = e^{j3\pi(t+T)/5} \\ 1 = e^{j3\pi T/5} \\ e^{j2k\pi} = e^{j3\pi T/5} \\ T = \dfrac{10}{3} \qquad (k = 1) \end{array}\right.$$

Determine the fundamental period of $x[n] = e^{j\frac{3\pi n}{5}}$ **x[n+N]=x[n]** →

$$\left\{\begin{array}{l} e^{j3\pi n/5} = e^{j3\pi(n+N)/5} \\ 1 = e^{j3\pi N/5} \\ e^{j2k\pi} = e^{j3\pi N/5} \\ T = 10 \qquad (k = 3) \end{array}\right.$$

Proof: Prove that $x(t) = e^{j\omega_0 t}$ is periodic and find its fundamental period

If x(t) is periodic then $e^{j\omega_0(t+T)} = e^{j\omega_0 t}$

$$e^{j\omega_0(t+T)} = e^{j\omega_0 t} e^{j\omega_0 T} \rightarrow e^{j\omega_0 T} = 1 \begin{cases} \omega_0 = 0 \rightarrow x(t)=1 \text{ and it is periodic for all T} \\ \omega_0 \neq 0 \rightarrow \omega_0 T = m2\pi \end{cases}$$

$T = m\frac{2\pi}{\omega_0}$ and the fundamental period T_0 is the smallest positive T $\rightarrow T_0 = \frac{2\pi}{\omega_0}$

If we consider the periodicity criteria for the DT case:

$x[n] = e^{j\Omega_0 n}$ is periodic if $e^{j\Omega_0(n+N)} = e^{j\Omega_0 n} e^{j\Omega_0 N} = e^{j\Omega_0 n} \rightarrow e^{j\Omega_0 N} = 1$

$\Omega_0 N = m2\pi$ where m is a positive integer $\rightarrow \frac{\Omega_0}{2\pi} = \frac{m}{N}$

x[n] is periodic only if $\frac{\Omega_0}{2\pi}$ is a rational number

Example:

x(t) is a CT signal given as x(t)=cos(15t). Find the fundamental period of the DT signal x[n] if x[n] is discretized by sampling x(t) at the sampling frequency $f_s = \frac{10}{\pi}$ Hz

$x[n] = x(nT_s)$ $T_s=1/f_s=0.1\pi$ seconds $x(t)=\cos(\omega t)=\cos(15t) \rightarrow \omega=15$ rad/s

The fundamental period of x(t) : $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{15}$

x[n] is periodic if $\frac{T_s}{T_0} = \frac{T_s}{2\pi/15} = \frac{m}{N_0}$

Since m and N_0 are positive integers $\rightarrow$ $T_s = \frac{m}{N_0} T_0 = \frac{m}{N_0} \frac{2\pi}{15}$ $\rightarrow$ $\frac{T_s}{T_0} = \frac{\pi/10}{2\pi/15} = \frac{15}{20} = \frac{3}{4}$

If x[n] is periodic then $N_0 = m\frac{T_0}{T_s} = m\frac{4}{3}$

m=3 provides N_0 to be the smallest positive integer. Fundamental period of x[n] is $N_0=4$

Example: (Case study for the signal having multiple periodic components)

Find the fundamental frequency of the signal x[n] $= e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$

$$\frac{\omega_0}{2\pi} = \frac{k}{N}$$

$$\frac{\frac{2\pi}{3}}{2\pi} = \frac{1}{3} \qquad \frac{\frac{3\pi}{4}}{2\pi} = \frac{3}{8}$$

Fundamental period of the first exponential is 3

Fundamental period of the second exponential is 8

Since the least common multiple of the periods of the two signals is 24,
x[n] is periodic with N_0=24

Example:

Find the fundamental frequency of $x(t) = \cos t + \sin \sqrt{2}\, t$

$x(t)=\boxed{x_1(t)}+\boxed{x_2(t)}$

$\cos t = \cos \omega_1 t$ $\sin \sqrt{2}\, t = \sin \omega_2 t$

Periods of x_1 and x_2: $T_1 = 2\pi/\omega_1 = 2\pi$ $T_2 = 2\pi/\omega_2 = \sqrt{2}\,\pi$

$T_1/T_2 = \sqrt{2}$ is irrational

Therefore x(t) is not periodic and there is no fundamental frequency

Example: Find the fundamental period of the DT signal x[n] $= sin(\frac{5\pi}{6}n) + cos(\frac{3\pi}{4}n) + sin(\frac{\pi}{3}n)$

The least common multiple of the denominators is 12 → $x[n] = sin(\frac{10\pi}{12}n) + cos(\frac{9\pi}{12}n) + sin(\frac{4\pi}{12}n)$

Fundamental frequency is $\omega_0 = \pi/12$ → The fundamental period is $T = 2\pi/\omega_0 = 24$ and the three terms are the 4th, 9th and 10th harmonic of ω_0

Example: Find the fundamental frequency of the CT signal x(t) $= \underbrace{sin(\frac{5\pi}{6}t)}_{x_1(t)} + \underbrace{cos(\frac{3\pi}{4}t)}_{x_2(t)} + \underbrace{sin(\frac{\pi}{3}t)}_{x_3(t)}$

$x(t) = x_1(t) + x_2(t) + x_3(t)$

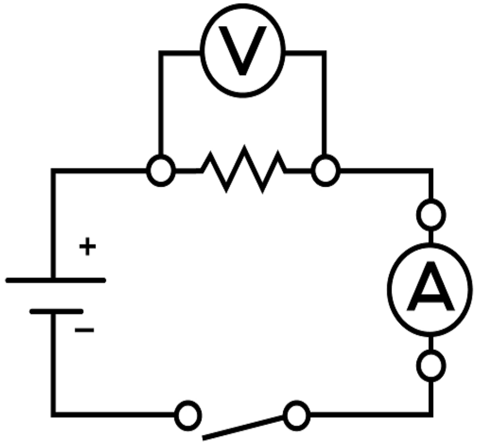
The frequencies and periods of $x_1(t)$, $x_2(t)$ and the $x_3(t)$ are:

$$\left.\begin{matrix} \omega_1 = \frac{5\pi}{6},\ f_1 = \frac{5}{12},\ T_1 = \frac{12}{5} \\ \omega_2 = \frac{3\pi}{4},\ f_2 = \frac{3}{8},\ T_2 = \frac{8}{3} \\ \omega_3 = \frac{\pi}{3},\ f_3 = \frac{1}{6},\ T_3 = 6 \end{matrix}\right\} f_0 = GCD(\frac{5}{12}, \frac{3}{8}, \frac{1}{6}) = GCD(\frac{10}{24}, \frac{9}{24}, \frac{4}{24}) = \frac{1}{24}$$

The fundamental angular frequency is $\omega_0 = \pi/12$
and the fundamental period is $T_0 = 2\pi/\omega = 24$
$f_0 = 1/24$

→ $x(t) = sin(\frac{10\pi}{12}t) + cos(\frac{9\pi}{12}t) + sin(\frac{4\pi}{12}t)$

Energy and Power Signals:



If v(t) is the voltage drop across the resistor R while the current is i(t) then the instantaneous power p(t) per ohm is defined as

$$p(t) = \frac{v(t)i(t)}{R} = i^2(t)$$

Then the total energy E and average power P on a per-ohm basis are

$$\begin{cases} E = \int_{-\infty}^{\infty} i^2(t)\,dt \quad \text{joules} \\ P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} i^2(t)\,dt \quad \text{watts} \end{cases}$$

If we make an analogy to 1 ohm basis Energy and Power definition, then normalized energy content E of an arbitrary continuous-time signal x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

and the normalized average power P of x(t) is $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$

Similarly the normalized energy content E of a discrete-time signal x[n] can be expressed as $$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Then the normalized average power P of x[n] is given as $$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Energy signal: x(t) is said to be an energy signal or x[n] is said to be an energy sequence if and only if $0<E<\infty$, and so the power P=0.

Power signal: x(t) is said to be a power signal or x[n] is said to be a power sequence if and only if $0<P<\infty$, and so the energy $E = \infty$

If a signal do not satisfy any of these properties then it is neither an energy signal nor a power signal.

Example: $x[n]=(-0.5)^n u[n]$ Determine if x[n] is an energy or power sequence

$$E = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=0}^{\infty} 0.25^n = \frac{1}{1-0.25} = \frac{4}{3} < \infty$$

$$P = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n] = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 0.25^n = \frac{1}{2\infty+1} \sum_{n=0}^{N} 0.25^n = 0$$

Hence it is an energy signal since we got finite energy and zero power