You should hand in your homework on Thursday, November 3, at 14:30 in the class!
Late submissions will not be accepted!

Abstract Mathematics Homework Fall 2022

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1. Is the argument below valid? If it is valid give a proof, if not provide a counter-example.

$$\begin{array}{l} \neg R \to S \\ \neg\left(\neg P \wedge S\right) \\ \hline R \to Q \\ \hline P \vee Q \end{array}$$

2. Is the argument below valid? If it is valid give a proof, if not provide a counter-example.

$$\begin{array}{l} \forall x\big(P(x) \wedge \neg S(x)\big) \\ \forall x\Big(\big(P(x) \to Q(x)\big) \vee R(x)\Big) \\ \hline \exists x\big(R(x) \to S(x)\big) \\ \hline \exists x Q(x) \end{array}$$

3. Is the argument below valid? If it is valid give a proof, if not provide a counter-example.

$$\begin{array}{l} \forall x\big(A(x) \to B(x)\big) \\ \hline \exists x\big(A(x) \wedge B(x)\big) \end{array}$$

4. Let a, b, c, $d \in \mathbb{Z}$ and $d|a$ and $d|b$ but d does not divide c. Show that the equaiton $ax + by = c$ has no integer solutions x, y.

5. Let X be a set and $A, B, C \subset X$. If $A \cap B = A \cap C$ and $(X - A) \cap B = (X - A) \cap C$, prove that $B = C$.