MUSTAFA CAN ÇALIŞKAN

150200097

MAT271E

HOMEWORK 2

1)

a)

* Events are independent
* Two possible outcomes (occupied or not) } Binomial distribution.

Random variable X: The situation that lines are occupied. $(p=0.4)$

$$P(X=3)=\binom{10}{3}\cdot\left(\frac{4}{10}\right)^3\cdot\left(1-\frac{4}{10}\right)^{10-3}$$

$$=120 \cdot 0.064 \cdot 0.028$$

$$\cong 0.215$$

b) At least 1 not occupied $\Leftrightarrow$ 1 − P(all occupied)

$$1-\left(\binom{10}{10}\cdot\left(\frac{4}{10}\right)^{10}\cdot\left(1-\frac{4}{10}\right)^{10-10}\right)$$

$$=1-(0.0001\cdot 1)$$

$$\cong 0.9999$$

c) For binomial distibution, $\mu_X = E[X] = n.p$

$$\mu_X = 10 \cdot \frac{4}{10}$$

$$= 4$$

2)

a) Two possible outcomes ✓
Independent trials ✓
Fixed num. of trials ✓

} Binomial distribution

Random variable X : Number of successes in a fixed number of independent trials. (Morning commutes) (0.2)

$$P(X=1) = \binom{5}{1} \cdot \left(\frac{2}{10}\right)^1 \left(1-\frac{2}{10}\right)^{5-1}$$

$$= 5 \cdot 0.2 \cdot 0.41$$

$$\cong \underline{0.41}$$

b)

$$P(X=4) = \binom{20}{4} \cdot \left(\frac{2}{10}\right)^4 \cdot \left(1-\frac{2}{10}\right)^{20-4}$$

$$= 4845 \cdot 0.0016 \cdot 0.028$$

$$\cong \underline{0.218}$$

c) $$P(X>4) = 1 - \sum_{i=0}^{4} P(X=i)$$

$$P(X=3) = \binom{20}{3} \cdot \left(\frac{2}{10}\right)^3 \cdot \left(1-\frac{2}{10}\right)^{17} = 0.205$$

$$P(X=1) = \binom{20}{1} \cdot \left(\frac{2}{10}\right)^1 \cdot \left(1-\frac{2}{10}\right)^{19} = 0.058$$

$$P(X=2) = \binom{20}{2} \cdot \left(\frac{2}{10}\right)^2 \cdot \left(1-\frac{2}{10}\right)^{18} = 0.137$$

2)

c)

$$P(x=0) = \binom{20}{0} \cdot \left(\frac{2}{10}\right)^0 \cdot \left(1-\frac{2}{10}\right)^{20} = 0.012$$

$$1 - \sum_{i=0}^{4} P(x=i) = 1 - 0.63$$

$$\cong \underline{0.37}$$

3)

a)

For Poisson : $P(X=k) = \dfrac{e^{-\lambda} \cdot \lambda^k}{k!}$, $\lambda = 0.5$

$$P(x<2) = P(x=1) + P(x=0)$$

$$P(x=1) = \frac{e^{-0.5} \cdot 0.5^1}{1!} \cong 0.303$$

$$P(x=0) = \frac{e^{-0.5} \cdot 0.5^0}{0!} \cong 0.606$$

$$\left.\right\} P(x<2) \cong \underline{0.909}$$

b) $P(x>2) = 1 - \sum_{i=0}^{2} P(x=i)$

$$P(x=2) = \frac{e^{-0.5} \cdot 0.5^2}{2!} \cong 0.076$$

$$1 - \sum_{i=0}^{2} P(x=i) = 1 - 0.986 \cong \underline{0.014}$$

3)

c) $\lambda_{(3\text{ weeks})} = 3 \cdot \lambda(1\text{ week}) = 1.5$

$$P(x=0) = \frac{e^{-1.5} \cdot (1.5)^0}{0!} \cong \underline{0.223}$$

4)

a) $\lambda = 1.2$

$$P(x=2) = \frac{e^{-1.2} \cdot 1.2^2}{2!} \cong \underline{0.217}$$

b) $P(x<3) = \sum_{i=0}^{2} P(x=i)$

$$P(x=0) = \frac{e^{-1.2} \cdot 1.2^0}{0} \cong 0.3$$

$$P(x=1) = \frac{e^{-1.2} \cdot 1.2^1}{1!} \cong 0.361$$

$$P(x=2) \cong 0.217$$

$$\sum_{i=0}^{2} P(x=i) \cong \underline{0.878}$$

4)

c) $\lambda_{(10 \text{ pages})} = \lambda_{(1 \text{ page})} \cdot 10 = \underline{12}$

$$P(X=5) = \frac{e^{-12} \cdot 12^5}{5!} \cong \underline{0.013}$$

d) $\lambda_{(40 \text{ pages})} = \lambda_{(1 \text{ page})} \cdot 40 = 48$

$$P(X \geq 3) = 1 - \sum_{i=0}^{2} P(X=i)$$

$$\left.\begin{aligned} P(x=0) &= \frac{e^{-48} \cdot 48^0}{0!} \cong 0 \\ P(x=1) &= \frac{e^{-48} \cdot 48^1}{1!} \cong 0 \\ P(x=2) &= \frac{e^{-48} \cdot 48^2}{2!} \cong 0 \end{aligned}\right\} 1 - \sum_{i=0}^{2} P(x=i) \cong \underline{1}$$