İTÜ Computer Engineering Dep.
Assoc. Prof. Feza BUZLUCA
Asst. Prof. Sanem KABADAYI
Prof. Mustafa KAMAŞAK
November 24, 2023

BLG 231E DIGITAL CIRCUITS MIDTERM SOLUTIONS

QUESTION 1 (30 Points):

Note that Parts (a) and (b) below are not related.

a) [15 points]

i. After an addition operation, overflow can occur in the following two cases:
1. negative + negative = positive
2. positive + positive = negative

Based on the given information, only the second case is possible.

r_3: 1 | **a_3: 0** | **b_3: 0**

Case 1: both −, result +	Case 2: both +, result -
1 1 a_1 a_0	0 0 a_1 a_0
1 1 b_1 b_0	0 0 b_1 b_0
0 r_3 r_1 r_0 <u>not possible</u>	1 r_3 r_1 r_0 possible

ii.

Answer: No
Reason: Overflow occurring in their addition means they have the same sign, so overflow cannot occur in their subtraction.

b) [15 Points]

i.

```
29/2 = 2x14 + 1 (least significant bit)
14/2 = 2x7  + 0
7/2  = 2x3  + 1
3/2  = 2x1  + 1
1/2  = 2x0  + 1
```

```
29(decimal)      = 11101 (5-bit binary)
                 = 0001 1101 (8-bit binary)
1's complement   = 1110 0010
+1                 0000 0001
2's complement   = 1110 0011 = -29
```

ii.

```
Y          = 1110 0011        Y     =     1110 0011
X = $9A    = 1001 1010        X'+1  =   + 0110 0110
X'         = 0110 0101                   10100 1001
X'+1       = 0110 0110         ignore 9th bit
```

Since negative – negative = positive, there is **<u>no overflow</u>**.
Since the result is **<u>positive</u>** <u>and there is</u> **<u>no overflow</u>**, **Y>X**.

Notes:
1. Only checking the sign of the result to compare signed integers is not sufficient. You must also **consider the overflow**. The sign is valid only if there is **no overflow**.
2. As the integers provided are **<u>signed</u>** numbers, comparison between them using **borrow is not possible**.
3. $9A is **<u>not</u>** 154 (decimal). You don't need to convert a hexadecimal number to decimal before converting to binary. The hexadecimal representation is only used for documentation (to write and read easily). Converting hexadecimal to binary, and vice versa, can be done easily. (Slide 1.21)

QUESTION 2 (35 Points):
<u>Note</u> that Parts (a), (b), and (c) below are <u>not</u> related.

a) [15 points]

$(a+f(0,b))(\bar{a}+f(1,b))$

$= a\bar{a} + af(1,b) + \bar{a}f(0,b) + f(0,b)f(1,b)$	(multiply out (distributive law))
$= 0 + af(1,b) + \bar{a}f(0,b) + f(0,b)f(1,b)$	(since $a\bar{a}=0$ (inverse law))
$= af(1,b) + \bar{a}f(0,b)$	(consensus theorem)

Note: $f(0,b)f(1,b)$ is the consensus term of $af(1,b) + \bar{a}f(0,b)$ with respect to a.

b) [10 points]

Yes.

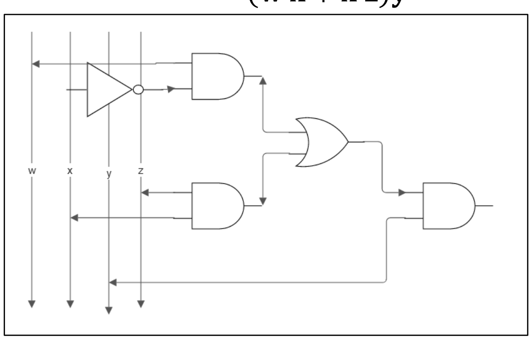
- NOT: $f(0,x) = 0 + \bar{x} = \bar{x}$.
- OR : $f(x,f(0,y)) = f(x,\bar{y}) = x + \overline{(\bar{y})} = x + y$.

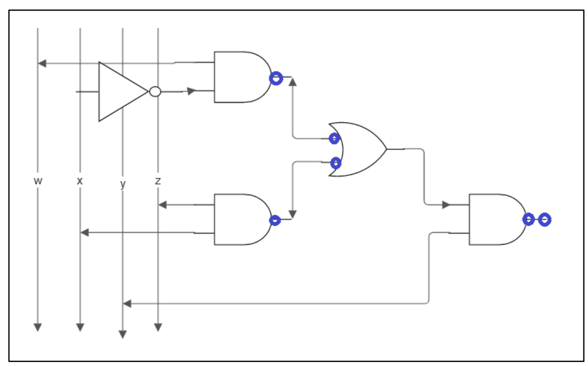
Since {OR, NOT} is a complete set, AND can also be implemented. Specifically,

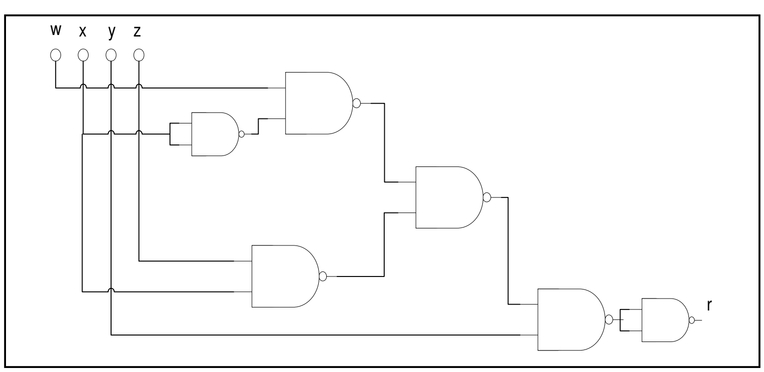
- AND: $x\,y = (\bar{x}+\bar{y})' = f(0, f(f(0,x), y))$
  $= f(0, f(\bar{x}, y))$
  $= f(0, \bar{x}+\bar{y})$
  $= (\bar{x}+\bar{y})'$

c) [10 points]

$$\mathbf{r = f(w,x,y,z) = w\,\bar{x}\,y + x\,y\,z}$$
$$\mathbf{= (w\,\bar{x} + x\,z)y}$$







Six NAND gates are sufficient.

QUESTION 3 (35 Points):

Note that Parts (a), (b), and (c) below are not related.

a) [10 points]

	abc	
(0)	000	✓
(1)	001	✓
(2)	010	✓
(4)	100	✓
(6)	110	✓

	abc	
(0,1)	00-	
(0,2)	0-0	✓
(0,4)	-00	✓
(2,6)	-10	✓
(4,6)	1-0	✓

	abc
(0,2,4,6)	--0

The set of all prime implicants= { $\bar{c}$, $\bar{a}\,\bar{b}$ }

b) [10 points]

	$\bar{C}.\bar{D}$	$\bar{C}.D$	$C.D$	$C.\bar{D}$
$\bar{A}.\bar{B}$	1	0	1	1
$\bar{A}.B$	1	0	x	1
$A.B$	0	0	0	x
$A.\bar{B}$	1	1	0	1

$\bar{A}\ C$

$\bar{A}.\bar{D}$

$\bar{B}.\bar{D}$

$C.\bar{D}$

$A.\bar{B}\ \bar{C}$

When we are finding the set of all prime implicants of a function in SOP form, we assign 1 to Φ's.

The set of all prime implicants = {$\bar{a}\,c$, $\bar{a}\,\bar{d}$, $\bar{b}\,\bar{d}$, $c\,\bar{d}$, $a\,\bar{b}\,\bar{c}$}

c) [15 points]

	i	*j*	*k*	*l*	*Cost*
A		X		X	4
B	X	X			3
C	X	X	X		5
D		X	X		2
E			X	X	2

Since column j covers i, we remove column j.

	i	*k*	*l*	*Cost*
~~**A**~~			~~X~~	~~4~~
B	X			3
C	X	X		5
~~**D**~~		~~X~~		~~2~~
E		X	X	2

Prime implicant E covers more points than A, and its cost is lower.
Prime implicant E covers more points than D, and their cost is the same.
Therefore, we remove rows A and D.

		i	*k*	*l*	*Cost*
	B	X			3
	C	X	X		5
√	~~**E**~~		~~X~~	~~X~~	~~2~~

To cover *l*, we must take the prime implicant E.
We mark E and remove it from the table.

		i	*Cost*
√	**B**	X	3
	~~**C**~~	~~X~~	~~5~~

B is cheaper than C.
To cover *i*, we take the prime implicant B. We mark B.
We covered all the true points of the function.

The minimal covering sum: f = B + E

Note: Since the given prime implicant chart is simple, the solution is obvious (readily apparent). However, you are expected to simplify the chart in a **systematic way** and provide **all the steps** of the process.