Week 4. Examples

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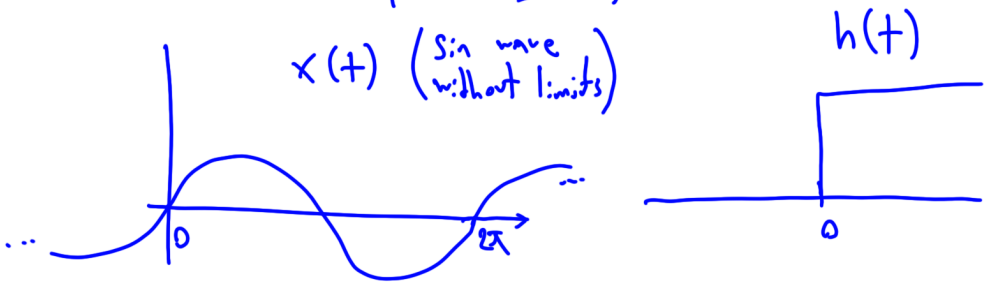
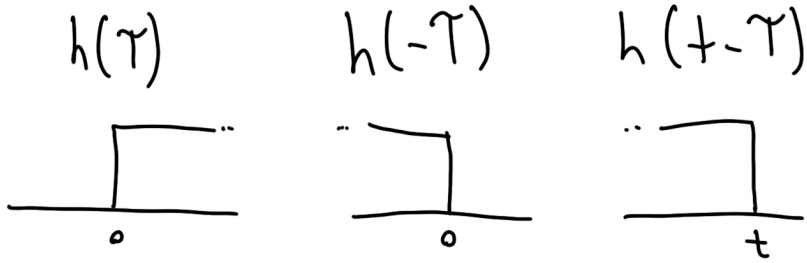
Q1: Convolve the following signals.



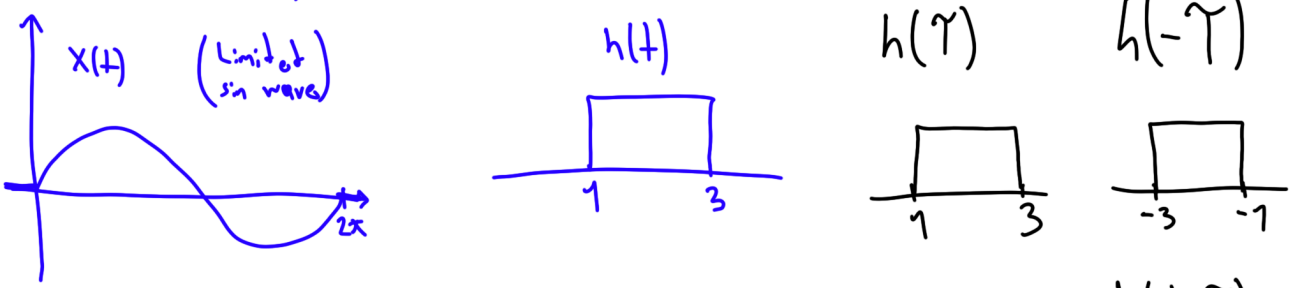
$$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

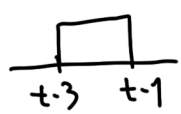


$$\int_{-\infty}^{\infty} \sin(\tau)\cdot u(t-\tau)\,d\tau = \int_{-\infty}^{t} \sin(\tau)\,d\tau$$

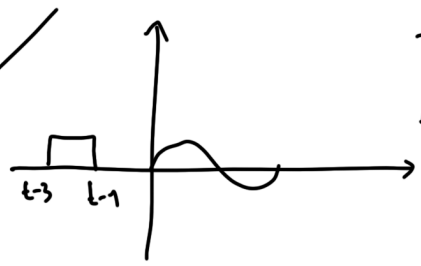
⇒ Step response could be obtained by integrating the impulse response.

Q2: Convolve the following signals





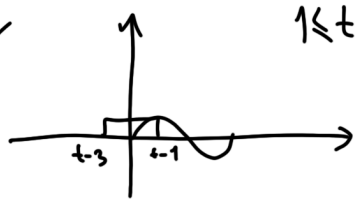
Position 1



$t<1$

No overlap. $y(t)=0$

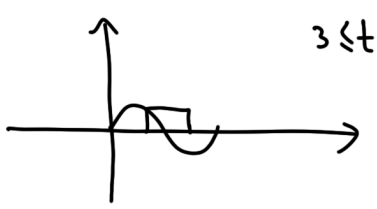
Position 2



$1\le t<3$

$$\int_0^{t-1} \sin(\tau)\,d\tau = -\cos(\tau)\Big|_{\tau=0}^{t-1} = 1-\cos(t-1)$$

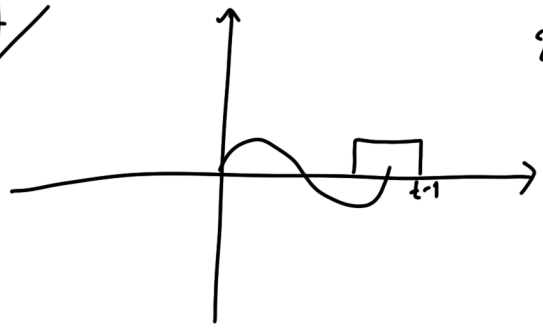
Position 3



$3\le t<2\pi+1$

$$\int_{t-3}^{t-1} \sin(\tau)\,d\tau = -\cos(\tau)\Big|_{\tau=t-3}^{t-1} = \cos(t-3)-\cos(t-1)$$

Position 4



$2\pi+1 \le t < 2\pi+3$

$$y(t) = \int_{t-3}^{2\pi} \sin(\tau)\,d\tau = -\cos(\tau)\Big|_{\tau=t-3}^{2\pi} = \cos(t-3) - 1$$

$$y(t) = \begin{cases} 1-\cos(t-1), & 1 \le t < 3 \\ \cos(t-3)-\cos(t-1), & 3 \le t < 2\pi+1 \\ \cos(t-3)-1, & 2\pi+1 \le t < 2\pi+3 \\ 0, & \text{otherwise} \end{cases}$$

Q3: For the following impulse responses of of CT LTI systems, determine whether each system is causal & stable.

a) $h(t) = e^{-4t}u(t-2)$ b) $h(t) = e^{-6t}u(3-t)$

Causal because $h(t)=0$ for $t<0$. Not causal because $h(t)\neq 0$ for $t<0$.

$$\int_{-\infty}^{\infty} \left|e^{-4t}u(t-2)\right| dt = \int_{2}^{\infty} e^{-4t}\,dt = \frac{-e^{-\infty}+e^{-8}}{4} = \frac{e^{-8}}{4} < \infty$$

Stable

$$\int_{-\infty}^{\infty} \left|e^{-6t}\cdot u(3-t)\right| dt = \int_{-\infty}^{3} e^{-6t}\,dt$$

$$= \frac{-e^{-18}+e^{\infty}}{6} = \infty$$

Unstable.