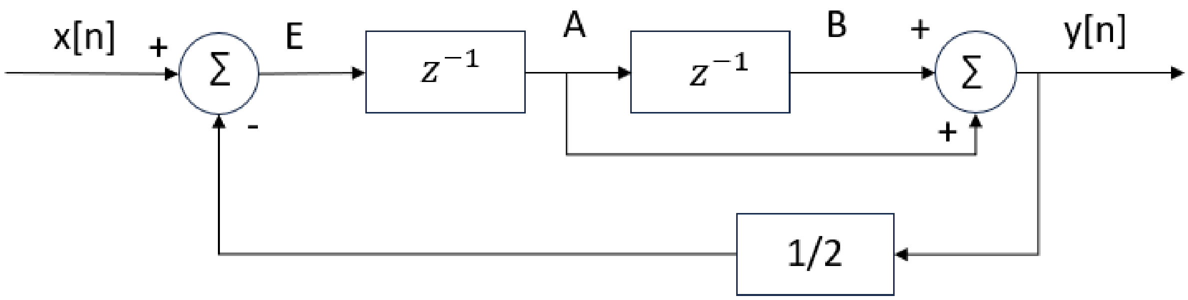
BLG354E – Quiz 2

Q1) A system diagram is given below where x[n] is a discrete-time signal obtained by sampling a continuous-time signal x(t) with sampling frequency of 100 Hz. (20p+10p+20p)



a) **Find the impulse response of the system via simulation (Present the result in a tabular form by also providing a pseudocode).**
Psuedo Code:
@T_S=1/(100 Hz)=0.01 sec, t=kT_S
Input X
E=X-0.5Y
B=A ; z^{-1}
A=E ; z^{-1}
Output Y=A+B
Return

n	X	A	B	E	Y
0	1	0.000	0.000	1.000	0.000
1	0	1.000	0.000	-0.500	1.000
2	0	-0.500	1.000	-0.250	0.500
3	0	-0.250	-0.500	0.375	-0.750
4	0	0.375	-0.250	-0.063	0.126
5	0	-0.063	0.375	-0.156	0.312
6	0	-0.156	-0.063	0.109	-0.218
7	0	0.109	-0.156	0.023	-0.046
8	0	0.023	0.109	-0.066	0.132
9	0	-0.066	0.023	0.021	-0.042
10	0	0.021	-0.066	0.022	-0.044
11	0	0.022	0.021	-0.022	0.044
12	0	-0.022	0.022	0.000	0.000
13	0	0.000	-0.022	0.011	-0.022
14	0	0.011	0.000	-0.005	0.010

b) Find the system response for x[n]=n(u[n]-u[n-2]) by simulation.

x[n]=n(u[n]-u[n-2])= n($\delta[n]+\delta[n-1]$)={0, 1}   x[1]=1 and x[n]=0 if $n \neq 1$

n	X	A	B	E	Y
0	0	0.000	0.000	0.000	0.000
1	1	0.000	0.000	1.000	0.000
2	0	1.000	0.000	-0.500	1.000
3	0	-0.500	1.000	-0.250	0.500
4	0	-0.250	-0.500	0.375	-0.750
5	0	0.375	-0.250	-0.063	0.126
6	0	-0.063	0.375	-0.156	0.312
7	0	-0.156	-0.063	0.109	-0.218
8	0	0.109	-0.156	0.023	-0.046
9	0	0.023	0.109	-0.066	0.132
10	0	-0.066	0.023	0.021	-0.042
11	0	0.021	-0.066	0.022	-0.044
12	0	0.022	0.021	-0.022	0.044
13	0	-0.022	0.022	0.000	0.000
14	0	0.000	-0.022	0.011	-0.022
15	0	0.011	0.000	-0.005	0.010

c) Find the system response for x[n]=n(u[n-1]-u[n-3]) by convolution for the first 5 values where u[n] is the unit step function.

x[n]=n(u[n-1]-u[n-3])=n($\delta[n-1]+\delta[n-2]$)={1, 2}
As we find impulse response in part (a), h[n] is equal to the output in part (a), i.e., h[n]={0.000, 1.000, 0.500, -0.750, 0.126, 0.312} for n=0,1,2,3,4,5.
y[n]= x[n]* h[n]

n							**x1**	**x2**							
							1	**2**							
8									0.312	0.126	-0.75	0.5	1.0	0	
7								0.312	0.126	-0.75	0.5	1.0	0		
6							0.312	0.126	-0.75	0.5	1.0	0			
5						0.312	0.126	-0.75	0.5	1.0	0				
4					0.312	0.126	-0.75	0.5	1.0	0					
3				0.312	0.126	-0.75	0.5	1.0	0						
2			0.312	0.126	-0.75	0.5	1.0	0							
1		0.312	0.126	-0.75	0.5	1.0	0								
0	0.312	0.126	-0.75	0.5	1.0	0									
	0	**0**	**1.0**	**2.5**	**0.25**	**-1.375**	**0.562**	**0.625**	**0**						
	y0	**y1**	**y2**	**y3**	**y4**	**y5**	**y6**	**y7**	**y8**						

Q2) For the system function $H(z) = \frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}$,

a) Draw the block diagram of the system as two cascaded sub-systems.
b) Is the system BIBO stable? Explain briefly.

$$H(z) = \frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = \underbrace{\left(\frac{1}{1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}}\right)}_{H_1(z)} \cdot \underbrace{\left(\frac{1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}}{1}\right)}_{H_2(z)}$$

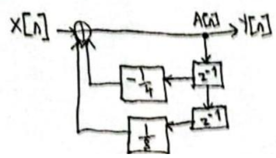
a-)

$H_1(z)$

$$H_1(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = 1.A(z)$$

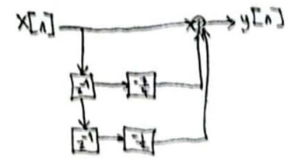
$$A(z) = X(z) - \left(\frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right)A(z)$$



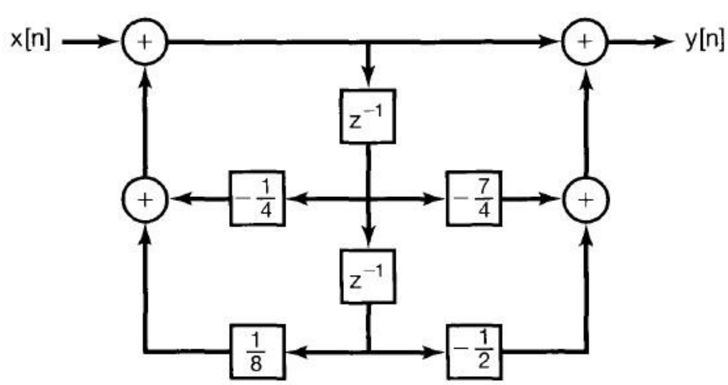
$H_2(z)$

$$Y(z) = 1.\left(1-\frac{7}{4}z^{-1}-\frac{1}{2}z^{-2}\right)A(z)$$

$$A(z) = X(z) - 0.A(z) \rightarrow X(z) = A(z)$$

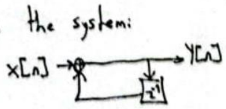


Combining these two systems:

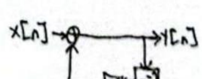


b)

Consider the system:



The system accumulates Y[n] and thus there is no limits. However, if a constant multiplier lesser than 1 was applied, the result will be different.



$$Y[n] = x[n] + K.Y[n-1] + K^2\,Y[n-2] + \ldots = x[n] + \underbrace{\sum_{k=1}^{\infty} K^k . Y[n-k]}_{\text{Decreasing Geometrical series}}$$

Thus, having multipliers absolutely lesser than 1, the first subsystem is BIBO stable. The second system which could be written as $Y[n] = X[n] - \frac{7}{4}X[n-1] - \frac{1}{2}X[n-2]$ is also BIBO stable. Thus the overall system is BIBO stable