İTÜ Computer Engineering Dep.
Assoc. Prof. Feza BUZLUCA
Asst. Prof. Sanem KABADAYI
Asst. Prof. Ayşe YILMAZER
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BLG 231E DIGITAL CIRCUITS FINAL EXAM SOLUTIONS

QUESTION 1 (35 Points):

You can answer Parts (a), (b), and (c) independently of each other.

a) [10 Points]

Overflow cases (V=1):

Addition:	pos. + pos. → neg.	neg. + neg. → pos.
X = 0:	$A_3 = 0, B_3 = 0, Z_3 = 1$	$A_3 = 1, B_3 = 1, Z_3 = 0$
Subtraction:	pos. – neg. → neg.	neg. – pos. → pos.
X = 1:	$A_3 = 0, B_3 = 1, Z_3 = 1$	$A_3 = 1, B_3 = 0, Z_3 = 0$

The logic expression for the output V in the **first canonical form**:

$$\mathbf{V = \overline{X} \cdot \overline{A_3} \cdot \overline{B_3} \cdot Z_3 + \overline{X} \cdot A_3 \cdot B_3 \cdot \overline{Z_3} + X \cdot \overline{A_3} \cdot B_3 \cdot Z_3 + X \cdot A_3 \cdot \overline{B_3} \cdot \overline{Z_3}}$$

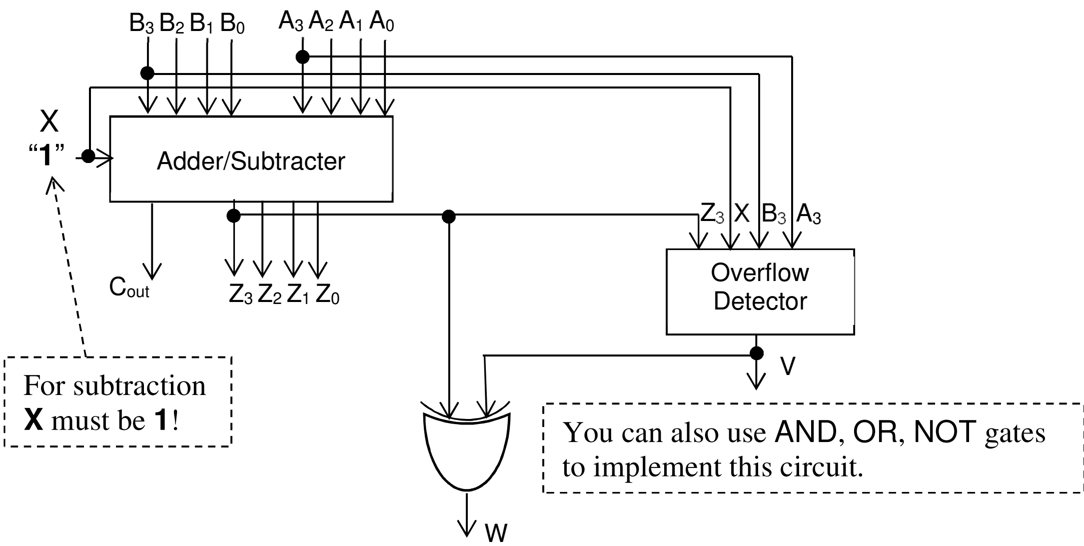
b) [10 Points]

After the subtraction operation (**X=1**):

If $Z_3 = 0$ AND $V = 0$ OR $Z_3 = 1$ AND $V = 1$, then $A \geq B \rightarrow W = 0$

If $Z_3 = 1$ AND $V = 0$ OR $Z_3 = 0$ AND $V = 1$, then $A < B \rightarrow W = 1$

$$\mathbf{W = Z_3 \cdot \overline{V} + \overline{Z_3} \cdot V = Z_3 \oplus V}$$

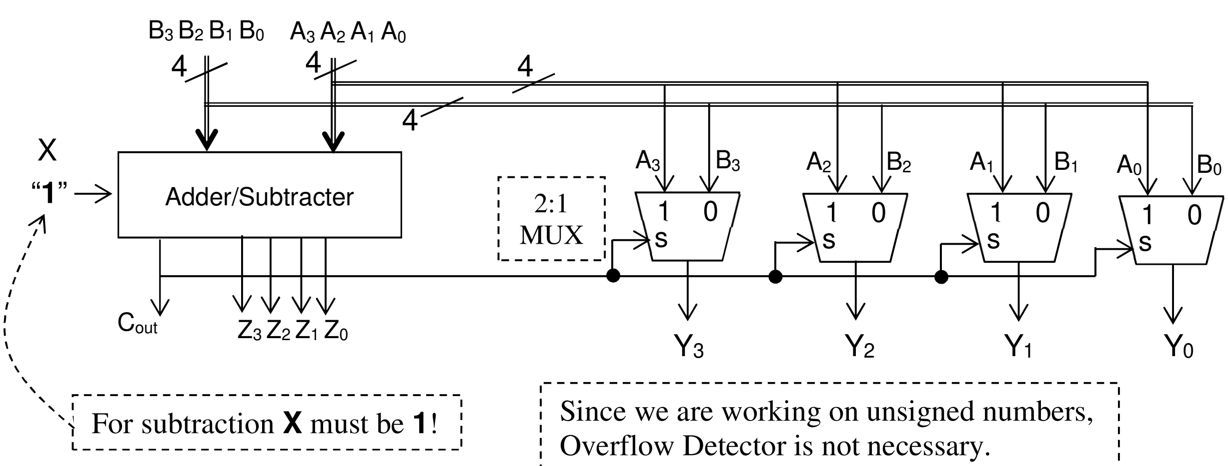


c) [15 Points]

After the subtraction operation (**X=1**):

If $C_{out} = 1$ (no borrow), then $A \geq B \rightarrow Y = A$

If $C_{out} = 0$ (borrow), then $A < B \rightarrow Y = B$



QUESTION 2 (35 Points):

<u>Note</u> that Parts (a), (b), and (c) below are <u>not</u> related.

a) [15 points]

i.

$$
\boxed{\begin{aligned} Q_1^+ &= \overline{Q_0}\cdot 0 + Q_0\cdot\overline{Q_1} = \overline{Q_1}\cdot Q_0 \\ Q_0^+ &= \overline{\overline{Q_1}}\cdot 0 + \overline{Q_1}\cdot\overline{Q_0} = \overline{Q_1}\cdot\overline{Q_0} \end{aligned}}
$$

ii.

Q_1Q_0	$Q_1^+Q_0^+$
00 →	01
01 →	10
10 →	00
00 →	01

b) [10 points]

i.

State transition, output table

S^+, Z (S \ X)	0	1	Z
A	A	B	0
B	A	B	1

State transition, output table with state codes

Q^+, Z (Q \ X)	0	1	Z
0	0	1	0
1	0	1	1

ii.

Q transitions

QQ^+ (Q \ X)	0	1
0	00	01
1	10	11

Transition table for T flip-flop :

QQ^+	T
00	0
01	1
10	1
11	0

T (Q \ X)	0	1
0	0	1
1	1	0

Flip-flop excitation: $T = \bar{Q}X + Q\bar{X} = Q \oplus T$

Output function: $Z = Q$

c) [10 points]

Z(t+1) (Z(t) \ AB)	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$\mathbf{Z(t+1) = A\,B + B\,Z(t) + A\,Z(t)}$$

A memory unit must have

1) two stable states: There are two stable states here (Z=0 and Z=1)
2) control input(s), which can be used to change or preserve the state of the unit: AB=11 sets the circuit, AB=00 resets it, AB=01 or AB=10 preserves the state of the unit.

∴ YES, this circuit can be used as a memory unit.

QUESTION 3 (30 Points):

Note that Parts (a) and (b) below are not related.

a) [15 points]

i.

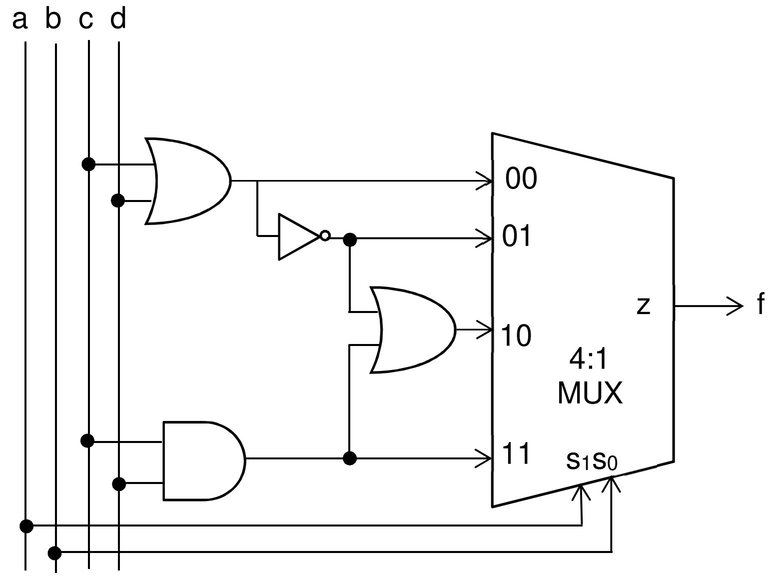
abcd	f
0000	0
0001	1
0010	1
0011	1
0100	1
0101	0
0110	0
0111	0
1000	1
1001	0
1010	0
1011	1
1100	0
1101	0
1110	0
1111	1

ab = 00:
$c + d$

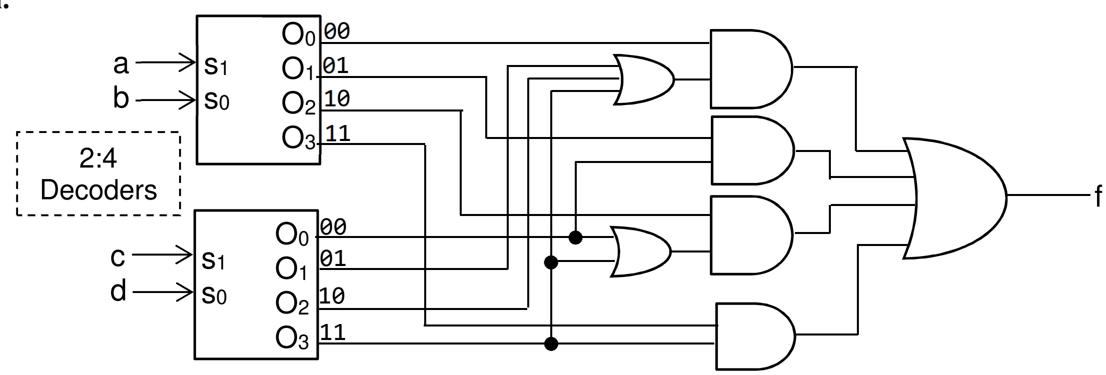
ab = 01:
$\overline{c + d}$

ab = 10:
$\bar{c} \cdot \bar{d} + c \cdot d$
$= \overline{c + d} + c \cdot d$

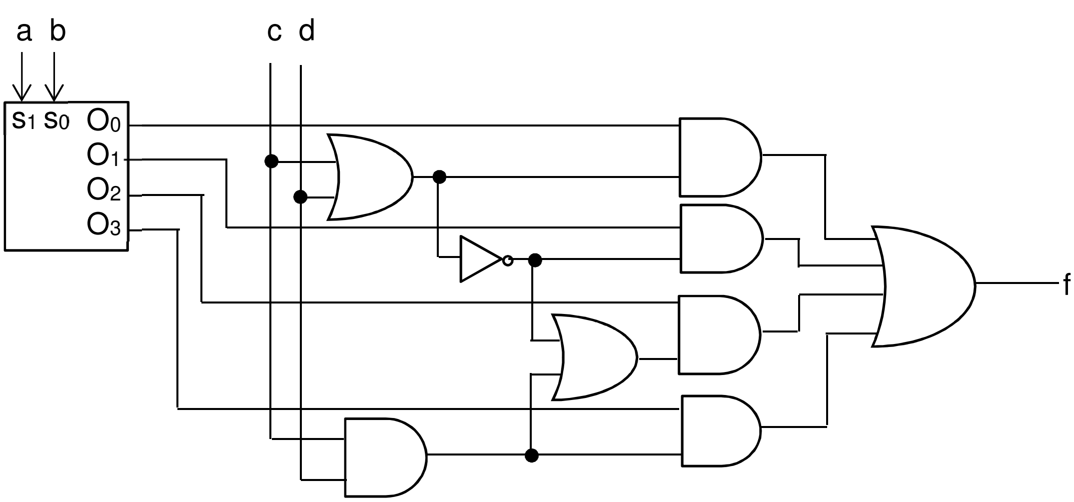
ab = 11:
$c \cdot d$



ii.



Another possible solution using one decoder. However, it uses two more logic gates.



b) [15 points]

Remember: NMOS: if VGS>0, ON; if VGS=0, OFF

PMOS: if VGS<0, ON; if VGS=0, OF

Gates (G) of transistors Q3 and Q4 are connected to the drains (D) of the Q1 and Q2.

A	B	Q1	Q2	Q3	Q4	Q5	Q6	Z
L	L	OFF	ON	ON	OFF	OFF	ON	L
L	H	ON	OFF	OFF	ON	OFF	ON	H
H	L	OFF	ON	ON	OFF	ON	OFF	L
H	H	ON	OFF	OFF	ON	ON	OFF	L