

BLG 231E
Homework 3 Solutions
Mustafa Can Çalışkan
150200097

1)

a)

Don't care (Φ) values are assumed to be 1, when we are finding the set of all prime implicants of a function in SOP form.

The Karnaugh map is a 4x4 grid with the following layout:

- Columns (labeled 'cd' at the top left):** 00, 01, 11, 10
- Rows (labeled 'ab' on the left):** 00, 01, 11, 10
- Variables:** 'd' is at the top center, 'c' is at the bottom right, 'b' is on the right side, and 'a' is on the left side.
- Cell Contents:**
 - (00, 00): 1
 - (01, 00): 1
 - (11, 00): 1
 - (10, 00): 1
 - (00, 01): 1
 - (01, 01): Φ
 - (11, 01): 1
 - (10, 01): 1
 - (00, 11): 1
 - (01, 11): 1
 - (11, 11): Φ
 - (10, 11): Φ
 - (00, 10): Φ
 - (01, 10): Φ
 - (11, 10): Φ
 - (10, 10): Φ
- Groupings (indicated by colored rectangles):**
 - Yellow:** Groups (00, 00), (01, 00), (10, 00), (11, 00); (00, 01), (01, 01), (10, 01), (11, 01); (00, 10), (01, 10), (10, 10), (11, 10).
 - Blue:** Groups (00, 00), (01, 00), (00, 01), (01, 01).
 - Red:** Groups (10, 00), (11, 00), (10, 01), (11, 01).
 - Green:** Groups (00, 00), (01, 00), (10, 00), (11, 00).
 - Purple:** Groups (01, 00), (01, 01), (01, 10), (01, 11).
 - Brown:** Groups (00, 00), (00, 01), (00, 10), (00, 11).

Prime implicants are: $a'c$, $a'b'$, $a'd'$, $b'd'$, $bc'd$, cd'

b)

(Don't cares are assumed to be 1, and marked with *)

1-generating (true) input combinations:

Nums.	a	b	c	d	
0	0	0	0	0	+
1	0	0	0	1	+
2	0	0	1	0	+
4	0	1	0	0	+
8*	1	0	0	0	+
3	0	0	1	1	+
5*	0	1	0	1	+
6	0	1	1	0	+
10*	1	0	1	0	+
13	1	1	0	1	+
14*	1	1	1	0	+

Groups with 2 points:

Nums.	a	b	c	d	
0, 1	0	0	0	-	+
0,2	0	0	-	0	+
0,4	0	-	0	0	+
0,8*	-	0	0	0	+
1,3	0	0	-	1	+
1,5*	0	-	0	1	+
2,3	0	0	1	-	+
2,6	0	-	1	0	+
2,10*	-	0	1	0	+
4,5*	0	1	0	-	+
4,6	0	1	-	0	+
8*,10*	1	0	-	0	+
5*,13	-	1	0	1	
6,14*	-	1	1	0	+
10*,14*	1	-	1	0	+

Groups with 4 points:

Nums.	a	b	c	d	
0, 1,2,3	0	0	-	-	
0,1,4,5*	0	-	0	-	
0,2,1,3	0	0	-	-	
0,2,4,6	0	-	-	0	
0,2,8*,10*	-	0	-	0	
0,4,1,5*	0	-	0	-	
0,4,2,6	0	-	-	0	
0,8*,2,10*	-	0	-	0	
2,6,10*,14*	-	-	1	0	
2,10*,6,14*	-	-	1	0	

Prime implicants are (same colors are correspond to the same implicants):

cd' , $b'd'$, $a'd'$, $a'c'$, $a'b'$, $bc'd$

2)

Properties of the prime implicants:

$A = cd'$ (cost = 5) (Points covered 2, 6, 10*, 14*)

$B = b'd'$ (cost = 6) (Points covered 0, 2, 8*, 10*)

$C = a'd'$ (cost = 6) (Points covered 0, 2, 4, 6)

$D = a'c'$ (cost = 6) (Points covered 0, 1, 4, 5*)

$E = a'b'$ (cost = 6) (Points covered 0, 1, 2, 3)

$F = bc'd$ (cost = 7) (Points covered 5*, 13)

Prime implicant chart:

Don't care values are assumed to be 0, when we are forming the prime implicant chart (values with * are not included in the table).

a)

		0	1	2	3	4	6	13	Cost
	A			X			X		5
	B	X		X					6
	C	X		X		X	X		6
	D	X	X			X			6
	E	X	X	X	X				6
+	F							X	7

As F is essential prime implicant, the rows and columns that it covers should be crossed out (removed from the chart)

b)

		0	1	2	3	4	6	Cost
	A			X			X	5
	B	X		X				6
	C	X		X		X	X	6
	D	X	X			X		6
+	E	X	X	X	X			6

As E is essential prime implicant, the rows and columns that it covers should be crossed out (removed from the chart)

c)

		4	6	Cost
	A		X	5
	B			6
+	C	X	X	6
	D	X		6

Since C covers both points (4, 6) and the cost of this process (6) is lower than cost of A + B (11), C is added to final set.

d)

Selected prime implicants: C, E, F.

Total cost: $7 + 6 + 6 = 19$.

Expression for the function with the lowest cost:

$$F(a, b, c, d) = a'd' + a'b' + bc'd.$$