Famous Problems in Computer Science

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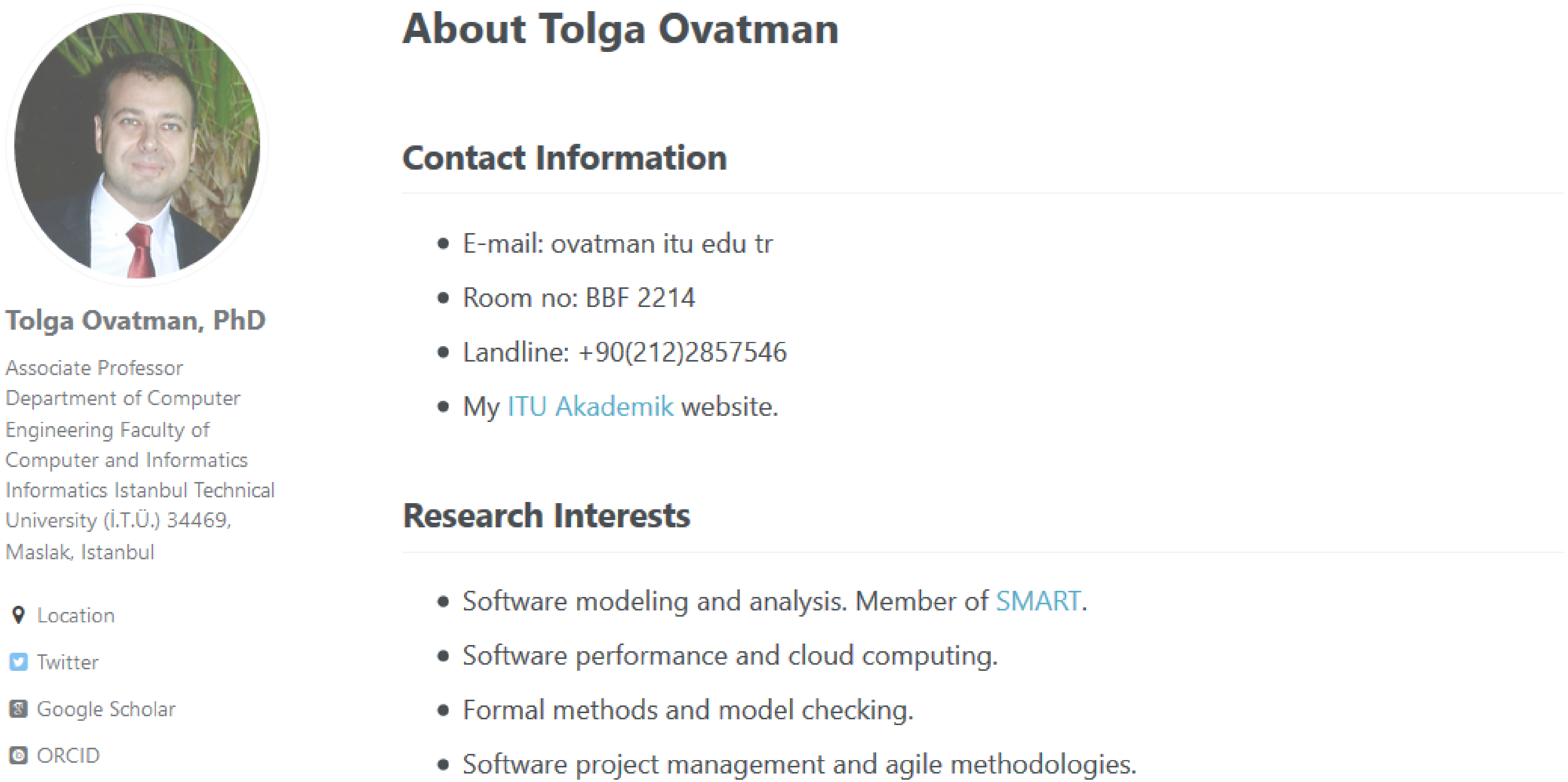

About Tolga Ovatman

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Research Interests

- Software modeling and analysis. Member of SMART.
- Software performance and cloud computing.
- Formal methods and model checking.
- Software project management and agile methodologies.

Searching

Predecessor/Successor

Problem ?

Challenge

Problems of practical importance usually contains too many possibilities.

15	7	3	25	20	18	13	11	6	17

Can we provide a strategy for guess the number game? [1-N]

1. Guess 1
2. *Increase until the right guess*

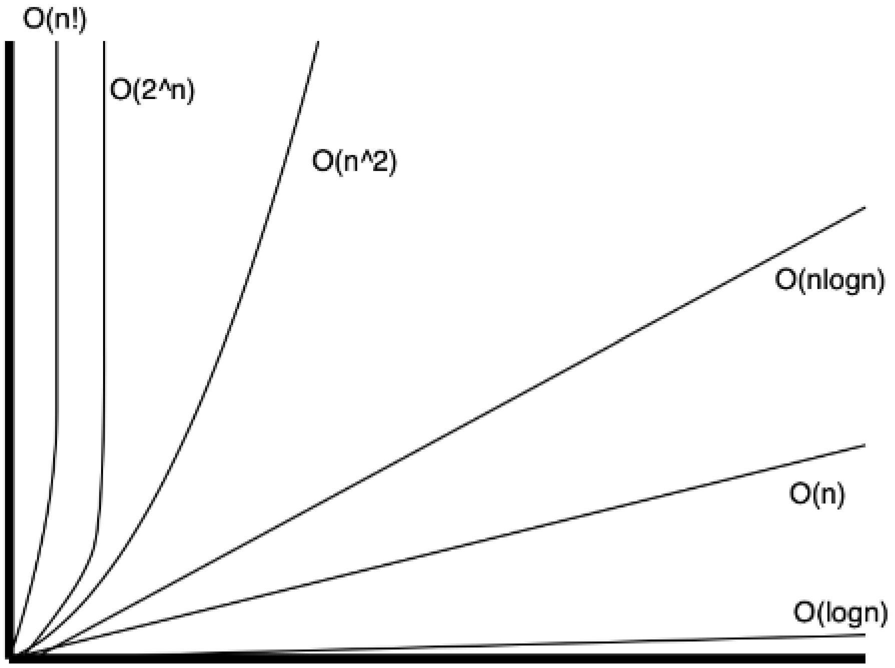
1. Make a random guess
2. *Get feedback*
3. *Make another random guess*

1. Guess N/2
2. *Get feedback*
    1. *If lower, guess [0 - N/2]*
    2. *If higher, guess [N/2 - N]*

Algorithm Performance

15	7	3	25	20	18	13	11	6	17

How many steps
do I need to take
for input of size N



3	6	7	11	13	15	17	18	20	25

Sorting

Why important?

It makes lookup or search efficient

It makes merging of sequences efficient.

It enables processing of data in a defined order.

Solution

- It is a multi-dimensional problem
- *For an n elements it takes around n logn steps.*

Sorting

BOGO SORT

```
while not is_sorted(list):
  shuffle(list)
```

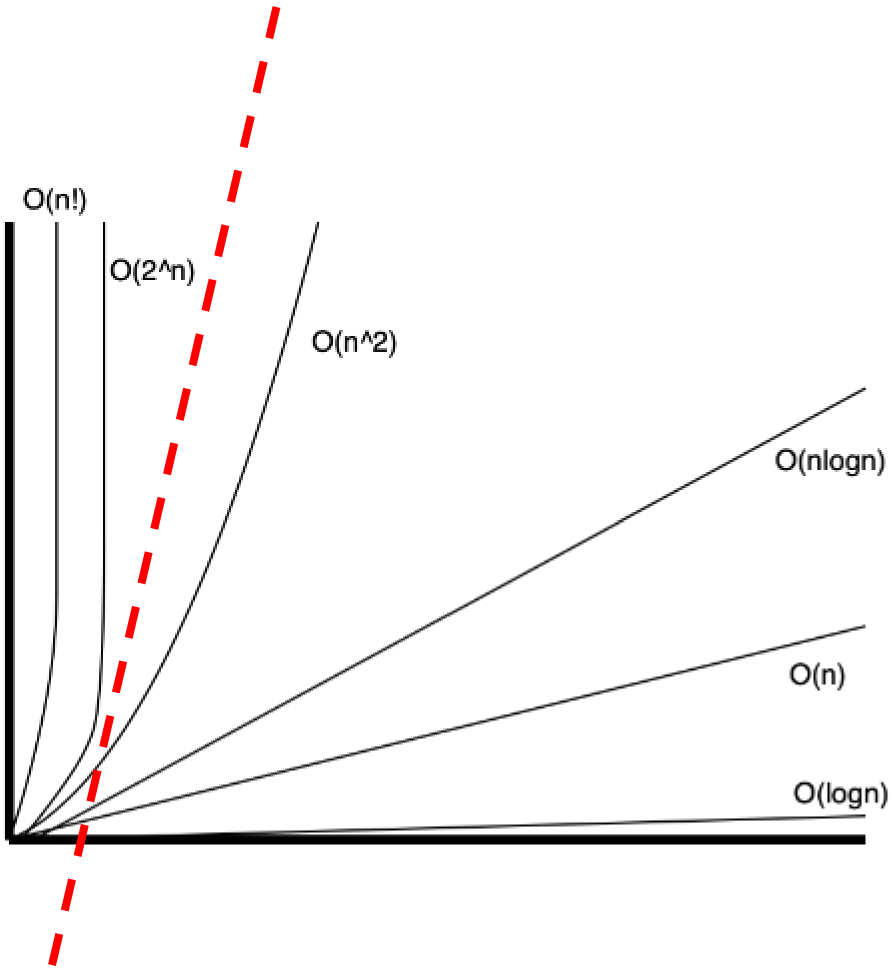
STALIN SORT

```
for a in list:
      if out_of_order(list, a):
            list.remove(a)
```

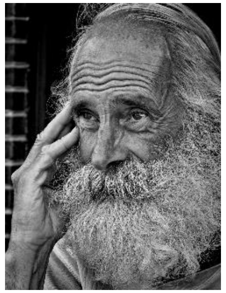
SLEEP SORT

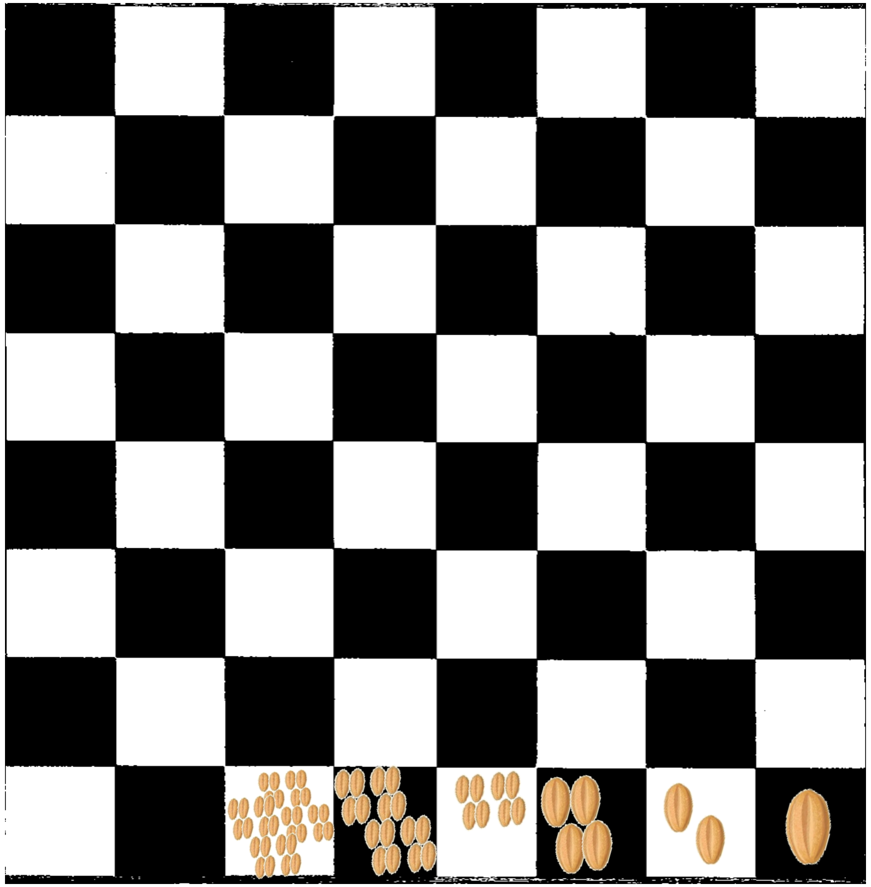
```
parallel for a in list:
      sleep(a)
      print(a)
```

Algorithm Performance



Exponential Growth – Story of chessman

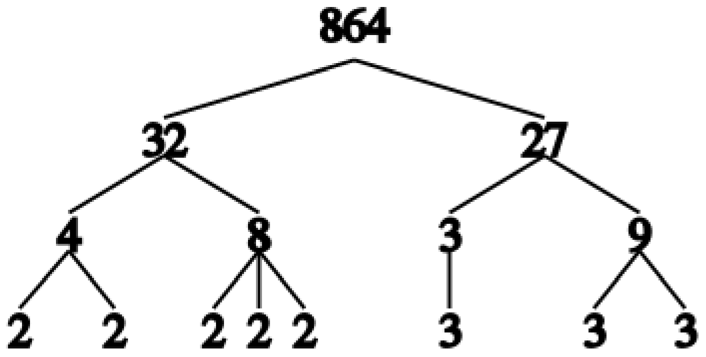




How much?

- *Approx. 2x10 wheat grains weigh a gram.*
- $\sum_{i=0}^{63} 2^i \cong 2^{64} \cong 2 \times 10^{19}$ *grains almost* 10^{12} *tons*
- *Earth produced much less than* 10^9 *tons a year for the last 20 years.*
- *Chess is less than* $2x10^3$ *years old.*

Prime Decomposition



35 = 7 x 5

437 = *19 x 23*

Why important?

It is the fundamental technique that lies behind modern cryptography

Primality Test? ✓

- *Thought to be as hard as Prime decomposition*
- *In 2002 AKS test has been developed which does the job in* ***log(n)^{12}*** *for an* ***n*** *bit number.*

Prime Decomposition

1459067680075833232301869393490706352924018723753571643995818710198734387990053589383695714026701498021218180862924674222828157022922076746906543401224889672472407926969987100581290103199317858753663710862357656510507883714297115637342788911463535102712032765166518411726859837988672111837205085526346618740053

=

12131072439211271897323671531612440428472427633701410925634549312301964373042085619324197365322416866541017057361365214171711713797974299334871062829803541

X

12027524255478748885956220793734512128733387803682075433653899983955179850988797899869146900809131611153346817050832096022160146366346391812470987105415233

Challenge

Not all numbers composed of multiplication of small primes

Solutions

- No known efficient solutions exists
- The most effecient general solution is $e^{\boldsymbol{f(n)}}$ steps for an ***n*** bit number.
- An n^3 algorithm exists (Shur’s) for quantum computers.

Byzantine Generals

Idea?

N generals in a siege communicate unreliably to reach a decision.

How many trusted generals at least may exists to still reach a correct decision?

Why important?

In a distributed system there always exist faulty peers, etiher willingly or unwillingly.

Consensus protocols designed to overcome this problem.

Byzantine Generals

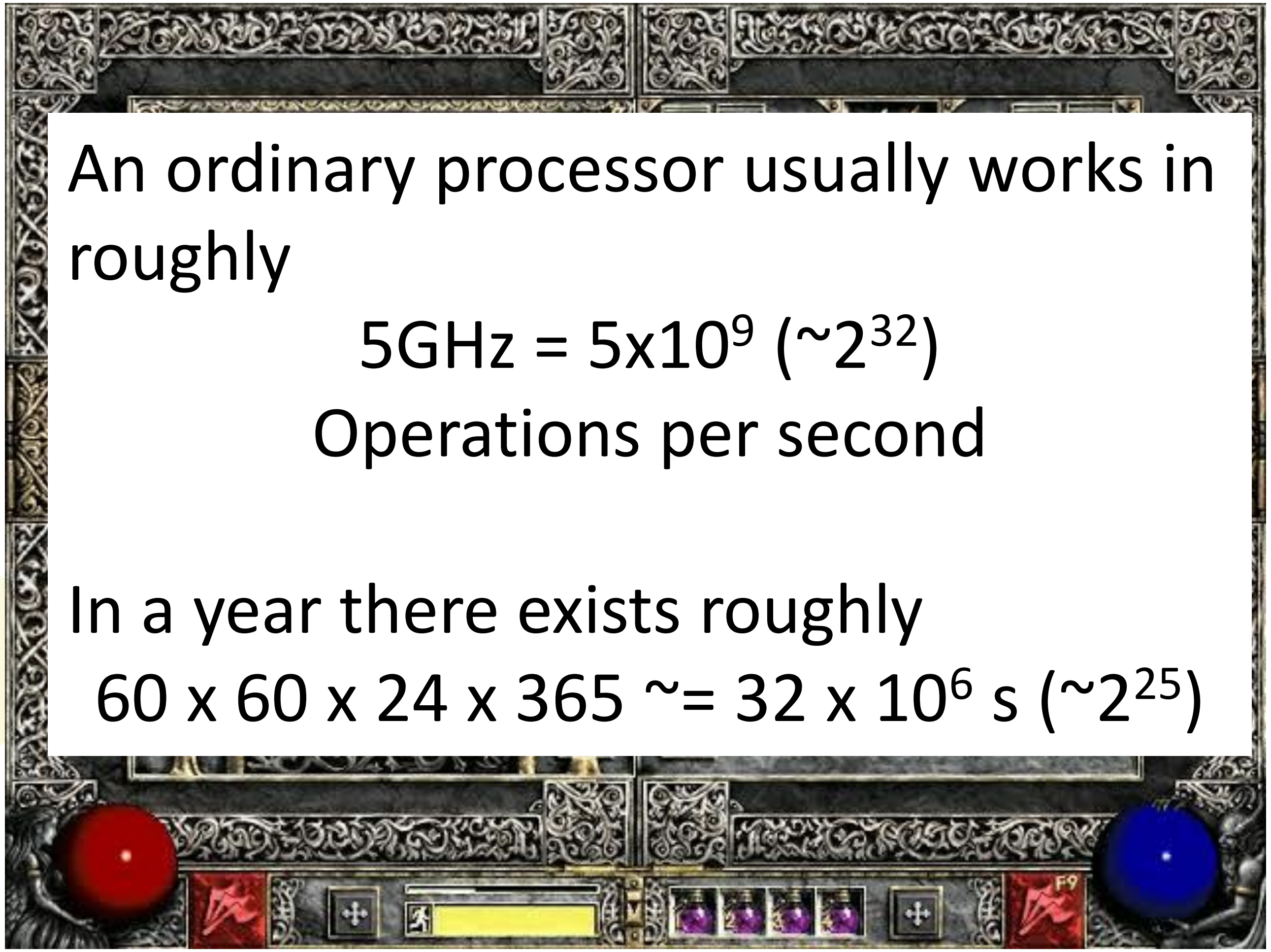
Challenge

Not always possible to know how many peers fail…

Solutions

- For N faulty peers at least 3N+1 peers needed
- PBFT, Paxos, Raft …

Knapsack Problem



An ordinary processor usually works in roughly

5GHz = 5×10^9 (~2^{32})

Operations per second

In a year there exists roughly

60 x 60 x 24 x 365 ~= 32 x 10^6 s (~2^{25})

Why important?

In real world optimization problems, you search for the most valuable combinations

How hard?

- *0-1 knapsack problem with integer wieghts for n items requires* ***2^n*** *steps*

That is ….. for 60 items ….

~8 years !!!

….. for 90 items ….

~8 billion years !!!

Knapsack Problem

Challenge

It takes very long to try all combinations…

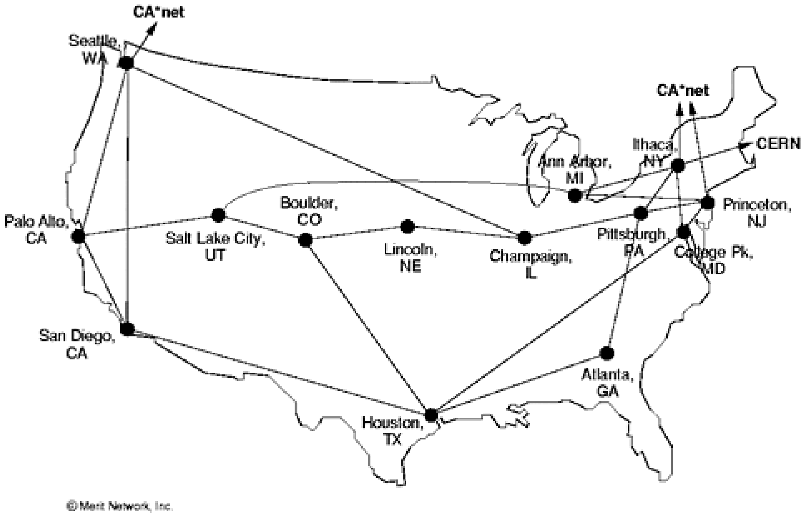
Solutions

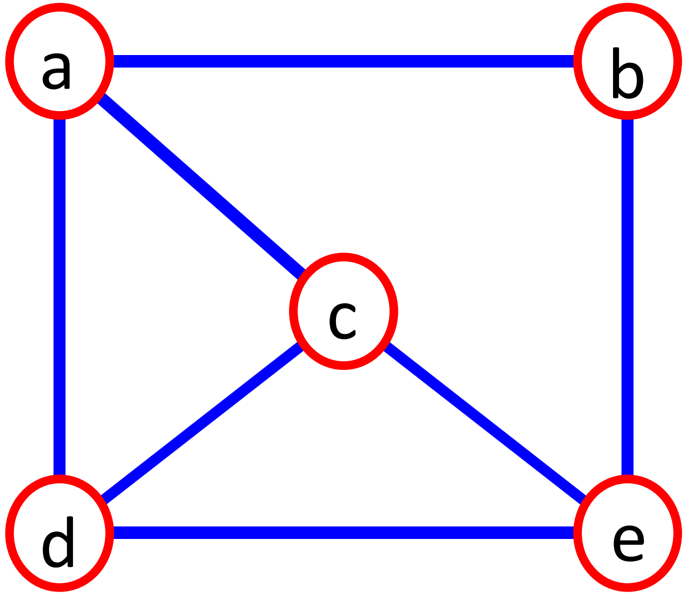
- Can a value of at least V be achieved without exceeding the weight W?
    - No known efficient solutions
- What is the maximum value V?
    - Harder than the decision.

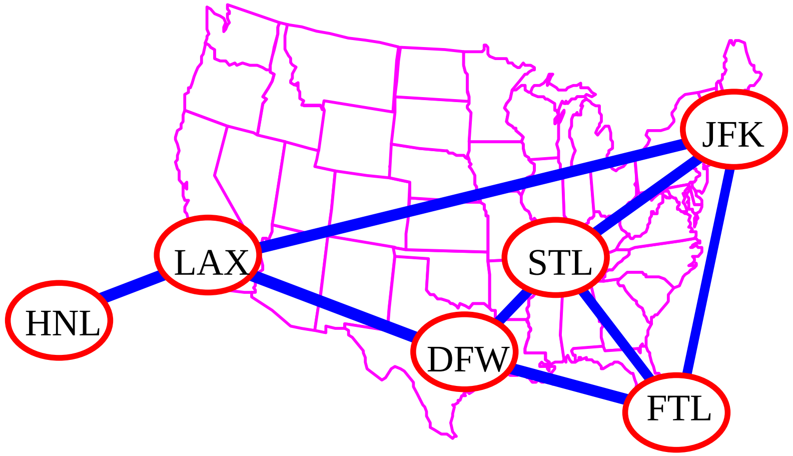
Graph primers

BULGARIA
Black Sea
GEORGIA
ARMENIA
AZER.
IRAN
IRAQ
SYRIA
GREECE
Mediterranean Sea
THRACE
ANKARA
ANATOLIA
İstanbul
Kocaeli
Sakarya
Ereğli
Samsun
Trabzon
Erzurum
Sivas
Bursa
Eskişehir
Balıkesir
Aliağa
Manisa
İzmir
Denizli
Konya
Kayseri
Malatya
Elazığ
Diyarbakır
Van
Kahramanmaraş
Adana
İçel
Toros
Gaziantep
Şanlıurfa
İskenderun
Antalya
E80
E87
E84
E90
E881
E89
E88
E95
E70
E97
E691
E96
E99
E98
E91
E92
CYP.

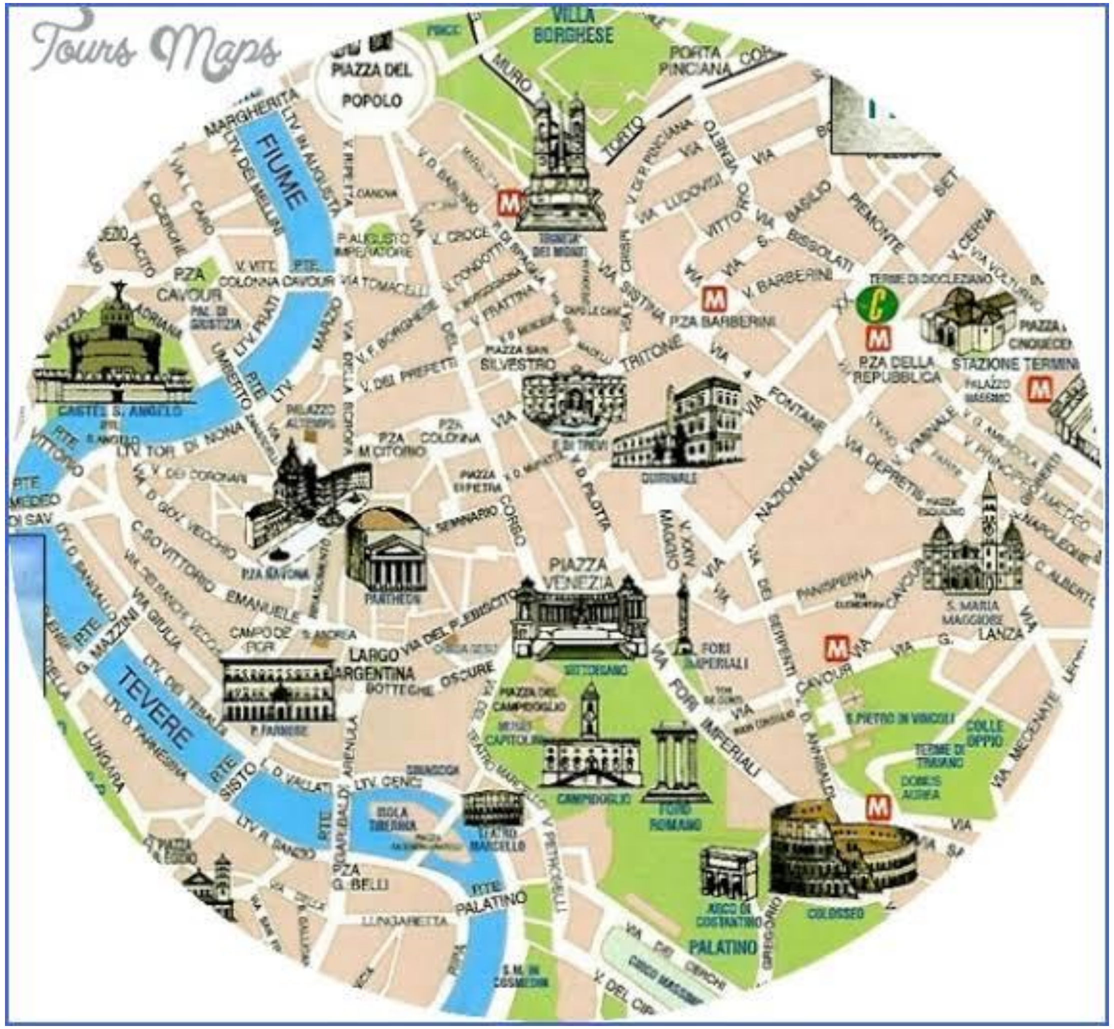
NSFNET T1 Network 1991







Hamiltonian Path



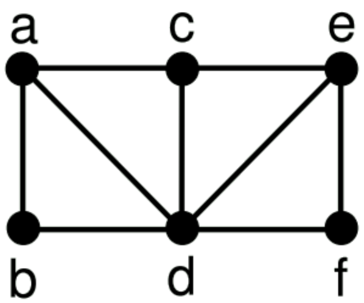
Idea?

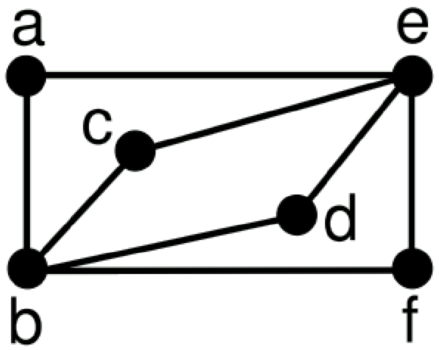
Can we stop by all the attractions without visiting an attraction twice?

Why important?

We want to convey information to a network of peers without repetition

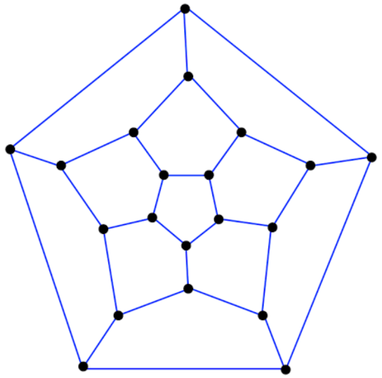
Hamiltonian Path

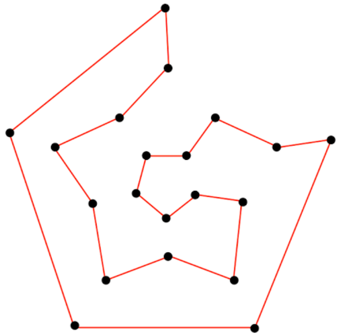




Challenge

Real World graphs are too complicated to try all possible variations…





Solutions

- No known efficient solutions exists
- The most effecient general solution is $1.657^{\boldsymbol{n}}$ steps for ***n*** cities
- Efficient solutions exist under special conditions

Traveling Salesman

WASHINGTON
MONTANA
NORTH DAKOTA
MINNESOTA
SOUTH DAKOTA
WISCONSIN
OREGON
IDAHO
WYOMING
MICHIGAN
Ottawa
Montreal
QUEBEC
ONTARIO
MAINE
VT
Toronto
NEW YORK
MA
CT RI
Chicago
IOWA
NEBRASKA
ILLINOIS
PENN
OHIO
NEVADA
UTAH
United States
COLORADO
KANSAS
MISSOURI
INDIANA
MD
DE NJ
Philadelphia
San Francisco
WEST VIRGINIA
VIRGINIA
KENTUCKY
CALIFORNIA
Las Vegas
Los Angeles
ARIZONA
San Diego
NEW MEXICO
OKLAHOMA
ARKANSAS
TENNESSEE
NORTH CAROLINA
SOUTH CAROLINA
Dallas
MISSISSIPPI
ALABAMA
GEORGIA
TEXAS
LOUISIANA
Houston
FLORIDA
Gulf of California
Gulf of Mexico
Google
Map data ©2018 Google, INEGI

Why important?

Frequently confronted in geo-optimization problems.

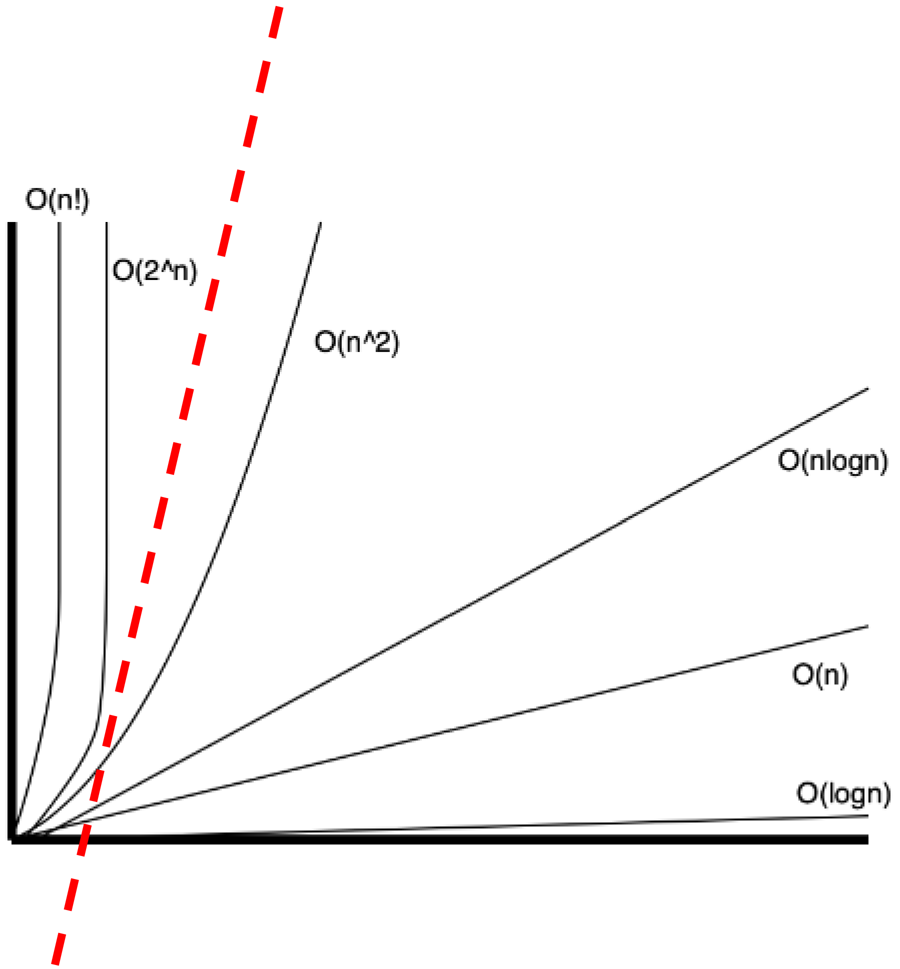
How hard?

- *A special case of Ham-Path where we try to obtain the **shortest** path where we end up at the point we start.*

(n-1)!/2

- *For 16 cities ~2^{40} tours to check*
- *There are ~2^{265} atoms in the universe.*
- *There are ~2^{90} nanosec. since the big bang.*
- *There are ~2^{355} tours to check for* ***~75 cities***

P vs NP



Class P: Easy to solve

- Search for a number in a non-sorted list
- Search for a number in a sorted list

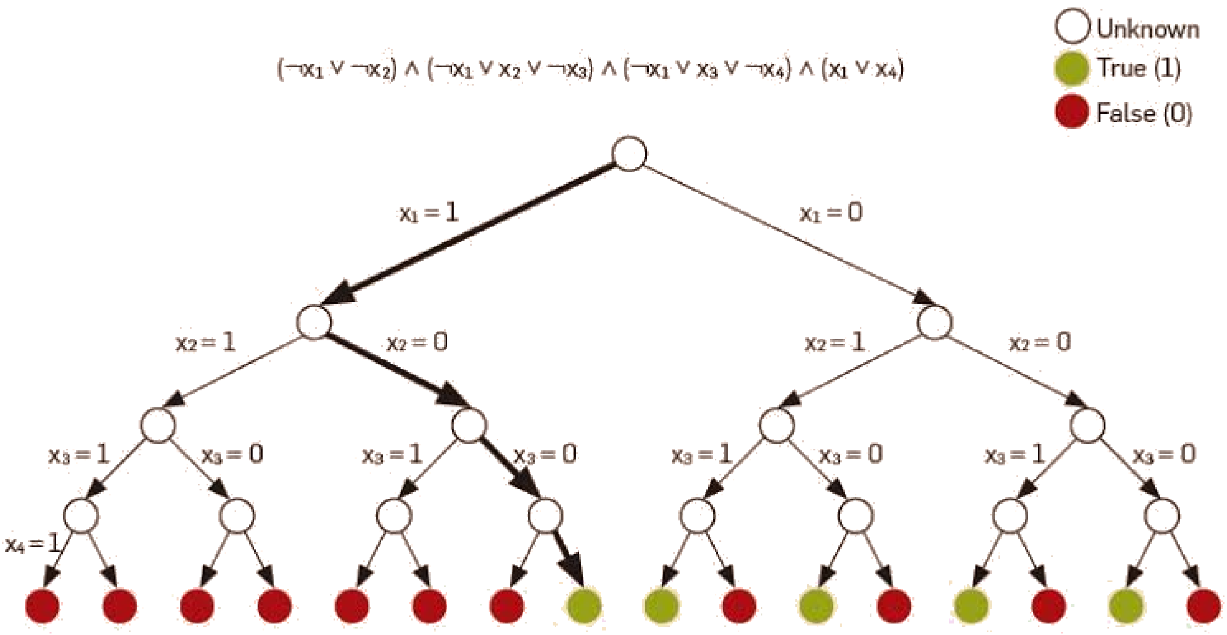
Class NP: Easy to check

- Hamiltonian Path
- Traveling salesman

Boolean Satisfiablity Problem

$$(p \wedge q) \vee r$$

T T F



Idea?

Can we find values that makes a boolean formula TRUE?

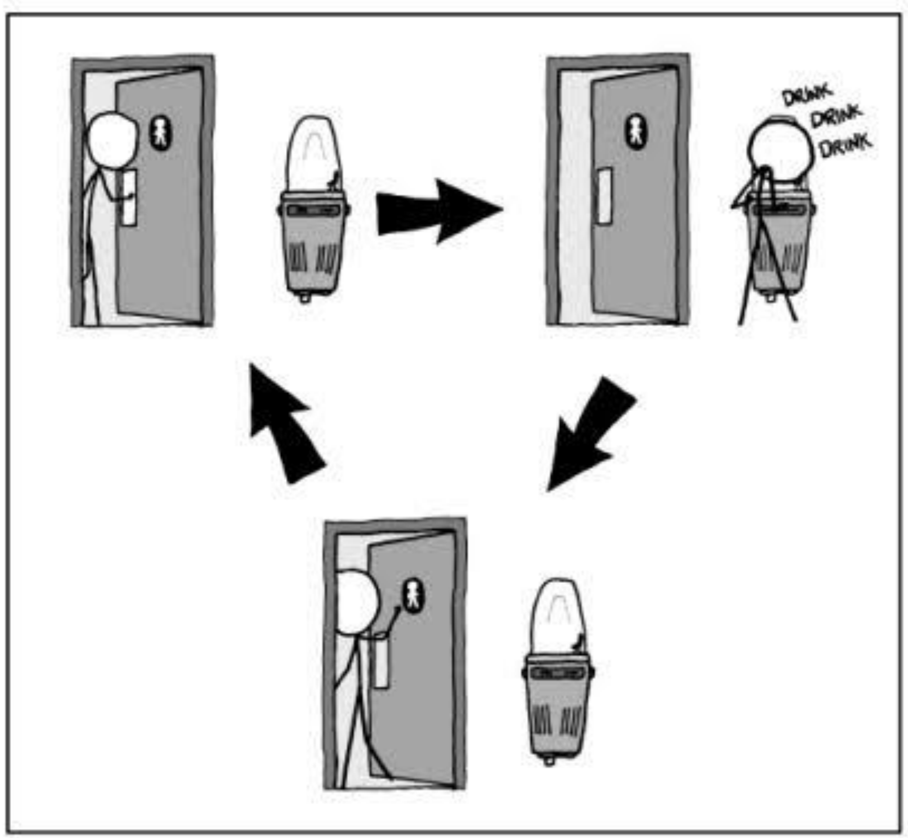
Why important?

It is the foundation of computer inference...

Solutions

- NP problem
- Most NP problems can be transformed to SAT

The Halting Problem



I AVOID DRINKING FOUNTAINS OUTSIDE BATHROOMS BECAUSE I'M AFRAID OF GETTING TRAPPED IN A LOOP.

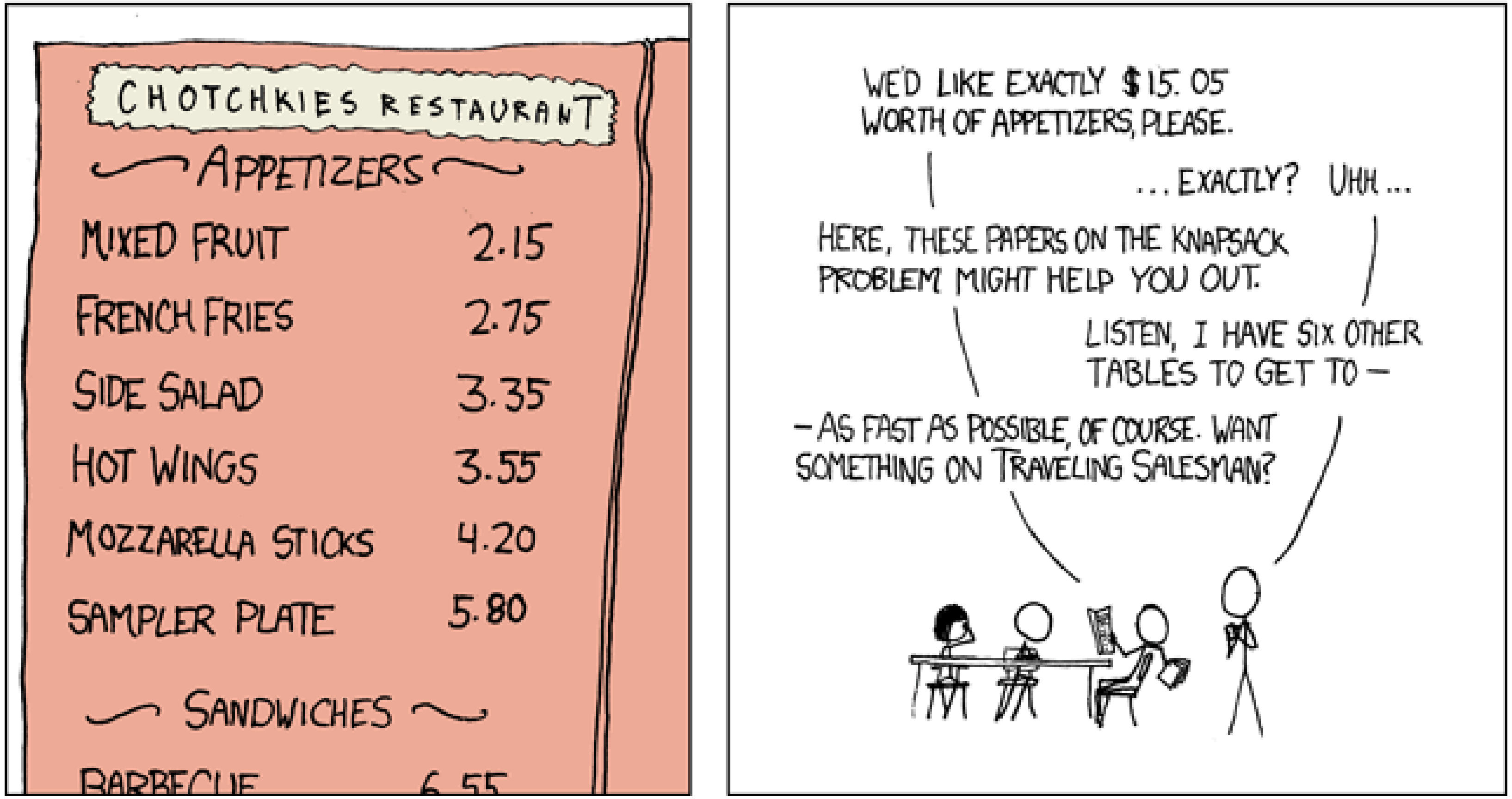
Idea?

Can we write a program that understands if a program would stop computing with a certain input, or not?

Why important?

It shows that it is impossible to solve some problems, in theory

Thank you very much…


CHOTCHKIES RESTAURANT
APPETIZERS
MIXED FRUIT 2.15
FRENCH FRIES 2.75
SIDE SALAD 3.35
HOT WINGS 3.55
MOZZARELLA STICKS 4.20
SAMPLER PLATE 5.80
SANDWICHES
BARBECUE 6.55
WE'D LIKE EXACTLY $15.05 WORTH OF APPETIZERS, PLEASE.
...EXACTLY? UHH...
HERE, THESE PAPERS ON THE KNAPSACK PROBLEM MIGHT HELP YOU OUT.
LISTEN, I HAVE SIX OTHER TABLES TO GET TO –
– AS FAST AS POSSIBLE, OF COURSE. WANT SOMETHING ON TRAVELING SALESMAN?