Differential Amplifiers

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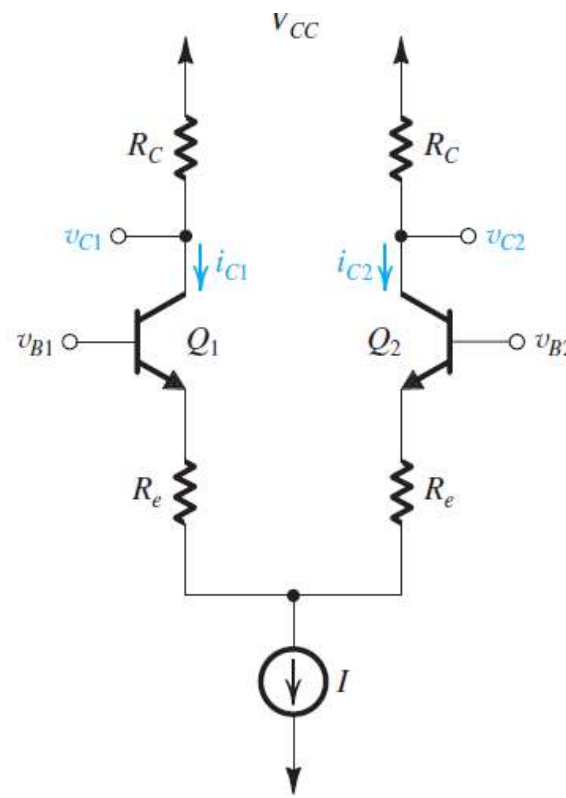
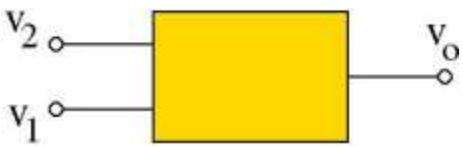


Common-Mode and Differential-Mode Signals & Gain

Differential and Common-Mode Signals/Gain

Consider a <u>linear</u> circuit with TWO inputs



By superposition:

$$\boxed{v_o = A_1 \cdot v_1 + A_2 \cdot v_2}$$

Define: $\boxed{v_d = v_2 - v_1}$ Difference (or differential) Mode

$\boxed{v_c = \dfrac{v_1 + v_2}{2}}$ Common Mode

$$v_1 = v_c - \frac{v_d}{2}$$

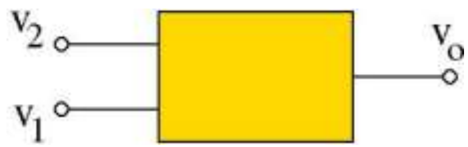
$$v_2 = v_c + \frac{v_d}{2}$$

Substituting for $v_1 = v_c - \dfrac{v_d}{2}$ and $v_2 = v_c + \dfrac{v_d}{2}$ in the expression for v_o:

$$v_o = A_1 \cdot \left(v_c - \frac{v_d}{2} \right) + A_2 \cdot \left(v_c + \frac{v_d}{2} \right) = (A_1 + A_2) \cdot v_c + \left(\frac{A_2 - A_1}{2} \right) \cdot v_d$$

$$\boxed{v_o = A_c \cdot v_c + A_d \cdot v_d}$$

Differential and common-mode signal/gain is an alternative way of finding the system response



$$v_o = A_1 \cdot v_1 + A_2 \cdot v_2$$

$$v_d = v_2 - v_1 \qquad A_c = A_1 + A_2$$

$$v_c = \frac{v_1 + v_2}{2} \qquad A_d = \frac{A_2 - A_1}{2}$$

$$v_o = A_c \cdot v_c + A_d \cdot v_d$$

$$v_1 = v_c - \frac{v_d}{2} \qquad A_1 = \frac{A_c}{2} - A_d$$

$$v_2 = v_c + \frac{v_d}{2} \qquad A_2 = \frac{A_c}{2} + A_d$$

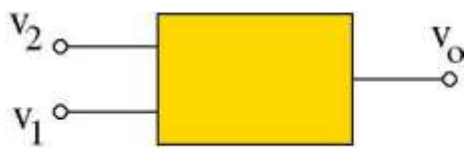
Differential Gain: A_d

Common Mode Gain: A_c

Common Mode Rejection Ratio (CMRR)*: $|A_d| / |A_c|$

* CMRR is usually given in dB: CMRR(dB) = 20 log $(|A_d| / |A_c|)$

To find v_o, we can calculate/measure either A_1, A_2 pair or A_c, A_d pair



Superposition (finding A_1 and A_2):

1. Set $v_2 = 0$, compute A_1 from $v_o = A_1 v_1$
2. Set $v_1 = 0$, compute A_2 from $v_o = A_2 v_2$
3. For any v_1 and v_2: $v_o = A_1 v_1 + A_2 v_2$

Difference Method (finding A_d and A_c):

1. Set $v_c = 0$ (or set $v_1 = -0.5\ v_d$ & $v_2 = +0.5\ v_d$) compute A_d from $v_o = A_d v_d$
2. Set $v_d = 0$ (or set $v_1 = +v_c$ & $v_2 = +v_c$) compute A_c from $v_o = A_c v_c$
3. For any v_1 and v_2: $v_o = A_d v_d + A_c v_c$

$$v_d = v_2 - v_1 \qquad v_c = 0.5(v_1 + v_2)$$

➢ **Both methods give the same answer for v_o (or A_v).**

$$v_1 = v_c - \frac{v_d}{2}$$

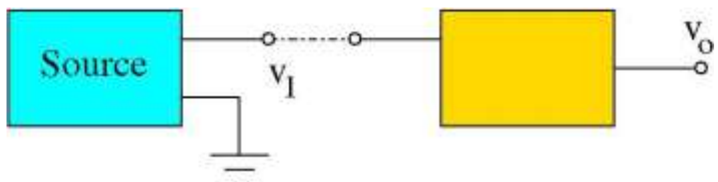
$$v_2 = v_c + \frac{v_d}{2}$$

Why are Differential Amplifiers popular?

- They are much less sensitive to noise (CMRR >>1).

Why is a large CMRR useful?

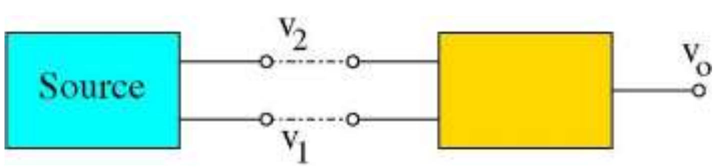
- A major goal in circuit design is to **minimize the noise level** (or improve signal-to-noise ratio). Noise comes from many sources (thermal, EM, …)
- A regular amplifier “amplifies” both signal and noise.



$$\boxed{v_1 = v_{sig} + v_{noise}}$$

$$\boxed{v_o = A \cdot v_1 = A \cdot v_{sig} + A \cdot v_{noise}}$$

- However, if the signal is applied between two inputs and <u>we use a differential amplifier with a large CMRR</u>, the signal is amplified a lot more than the noise which improves the signal to noise ratio.*



$$v_1 = -0.5 v_{sig} + v_{noise} \quad \& \quad v_2 = +0.5 v_{sig} + v_{noise}$$
$$v_d = v_2 - v_1 = v_{sig} \quad \& \quad v_c = v_{noise}$$

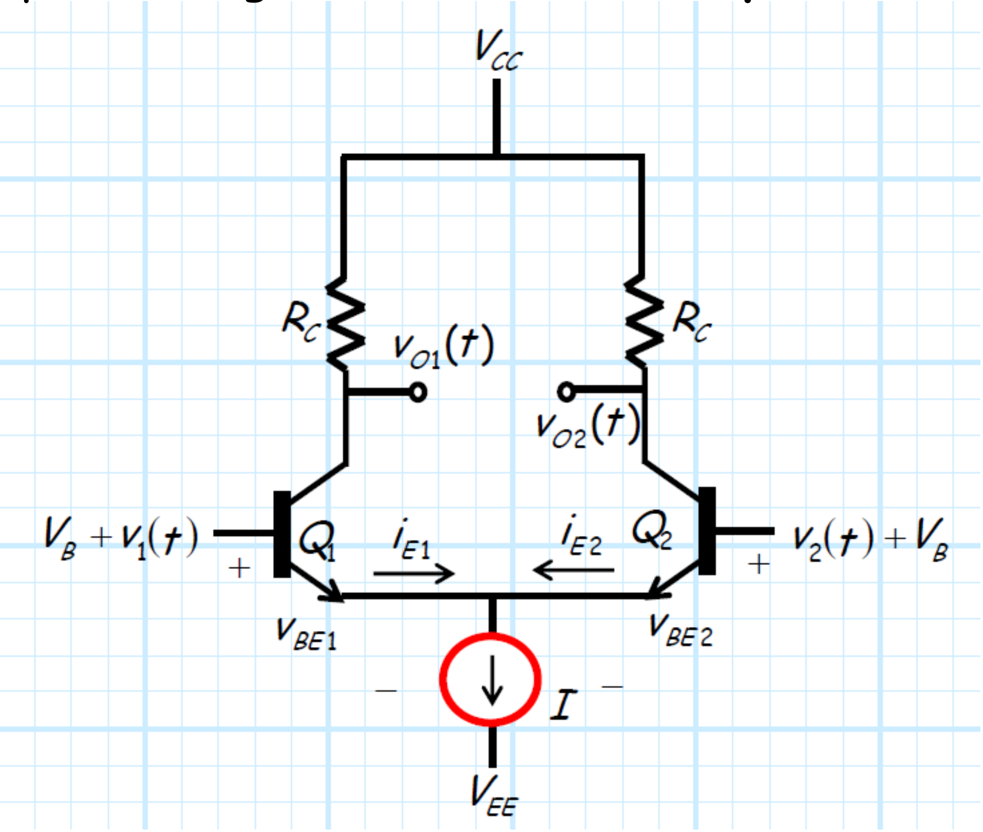
CMRR=$(|A_d|/|A_c|)$

$$\boxed{v_o = A_d \cdot v_d + A_c \cdot v_c = A_d \cdot v_{sig} + \frac{A_d}{CMRR} \cdot v_{noise}}$$

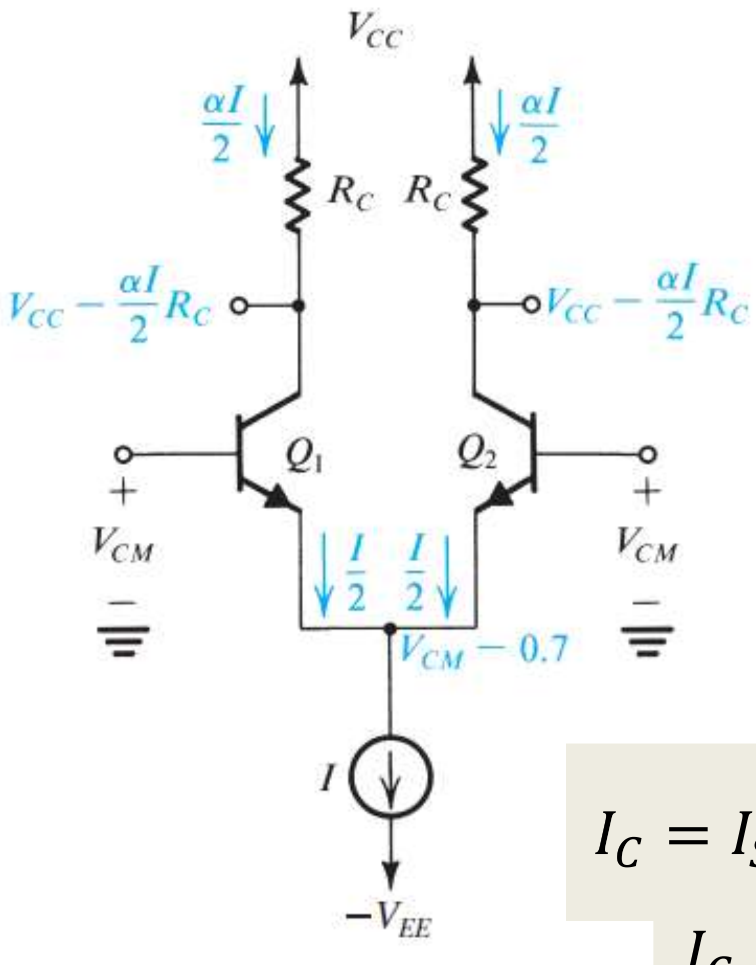
* Assuming that noise levels are similar to both inputs.

BJT Differential Amplifier

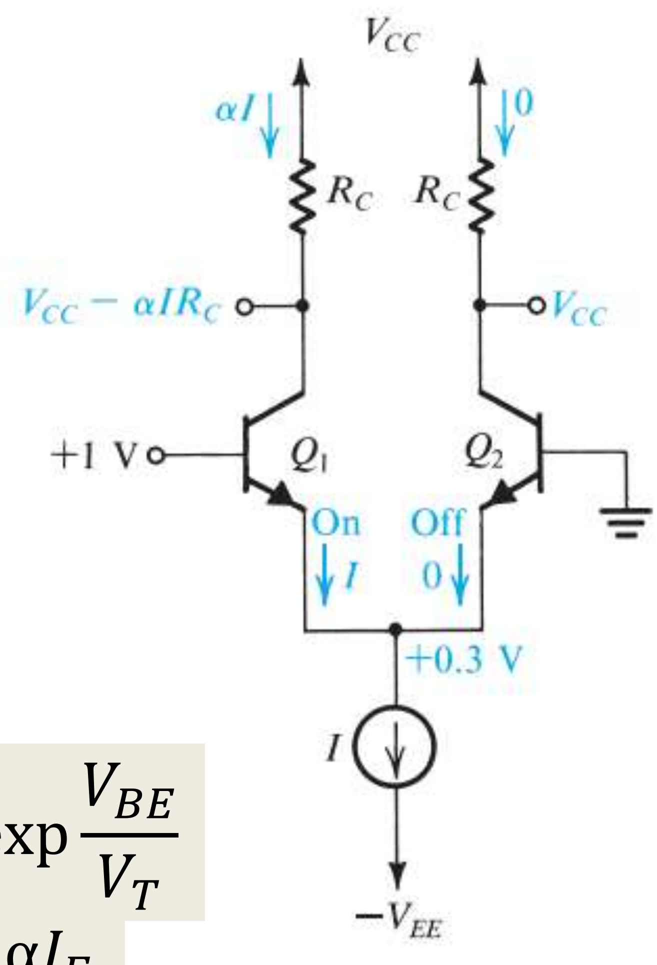
In addition to common emitter, common-collector (i.e., the emitter follower), and common-base amplifiers, a fourth important and “classic” **BJT** amplifier stage is the **differential pair**.



Differential Amplifiers



(a)

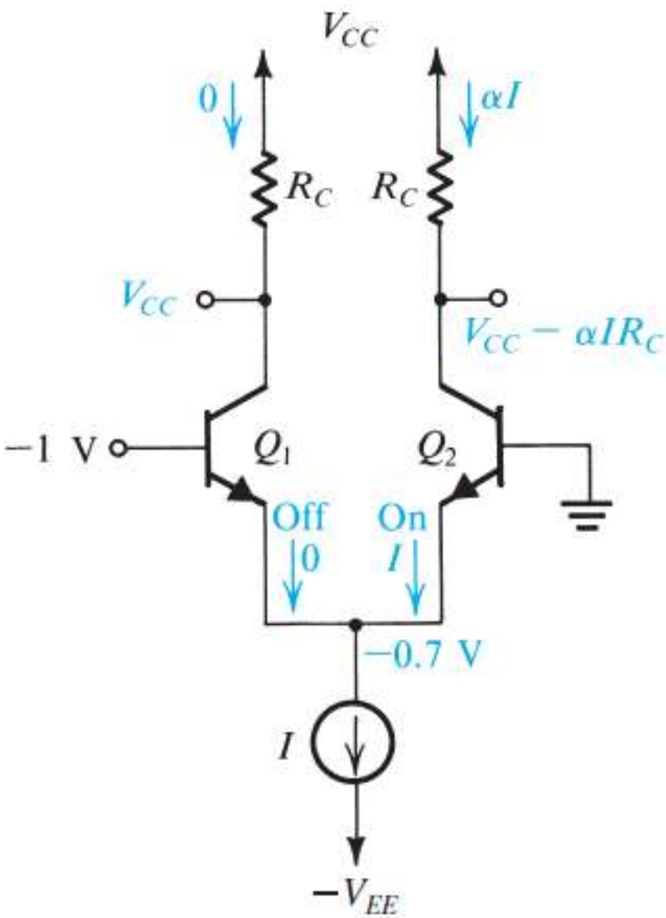


(b)

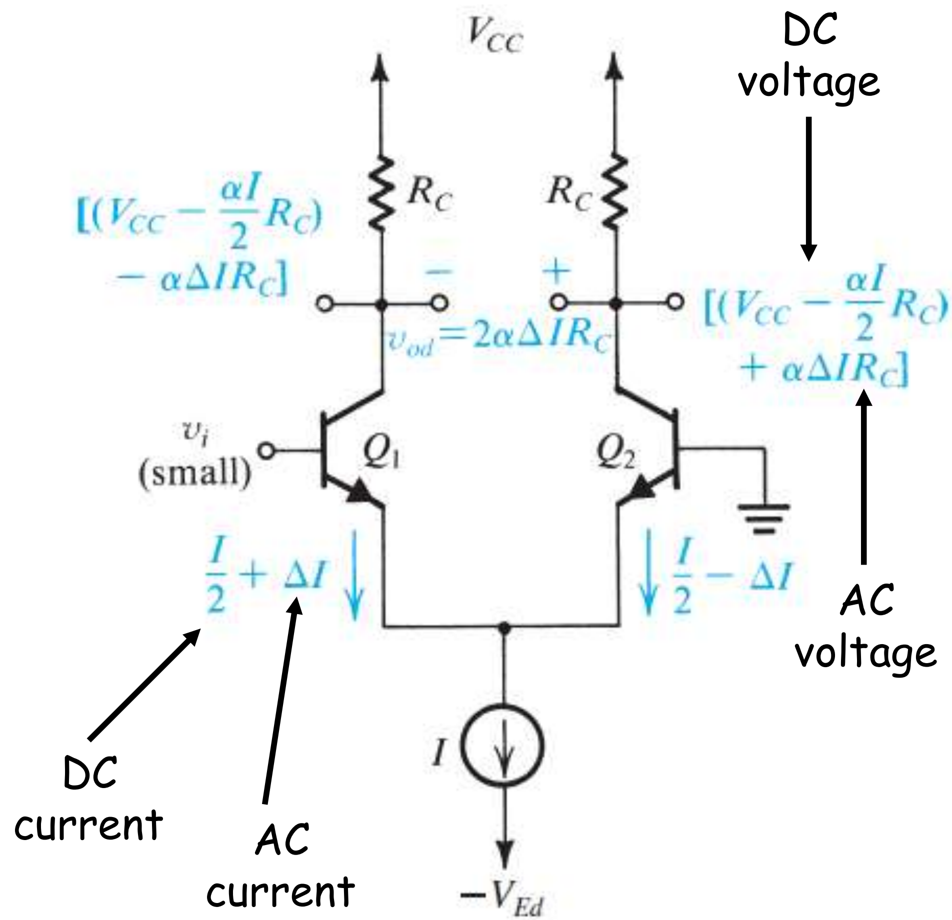
$$I_C = I_S \exp\frac{V_{BE}}{V_T}$$

$$I_C = \alpha I_E$$

Differential Amplifiers

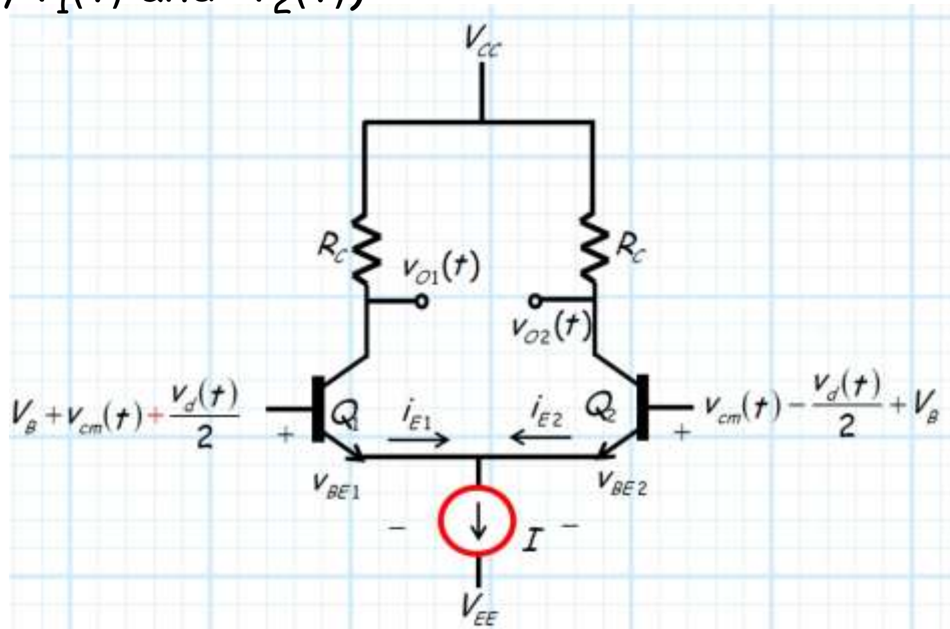


(c)



(d)

Now lets consider the case where each input of the differential pair consists of an **identical DC bias** term V_B, and also **an AC small-signal** component (i.e., $v_1(t)$ and $v_2(t)$)

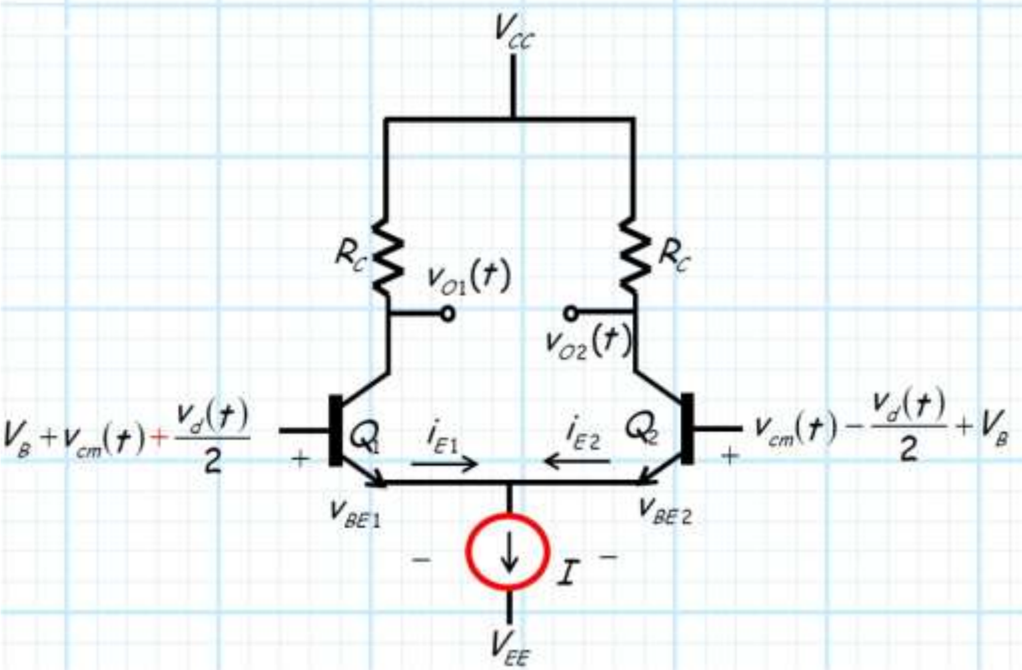


$$v_{cm}(t) \doteq \frac{v_1(t) + v_2(t)}{2}$$

$$v_1(t) = v_{cm}(t) + \frac{v_d(t)}{2}$$

$$v_d(t) \doteq v_1(t) - v_2(t)$$

$$v_2(t) = v_{cm}(t) - \frac{v_d(t)}{2}$$



$$v_{cm}(t) \doteq \frac{v_1(t) + v_2(t)}{2}$$

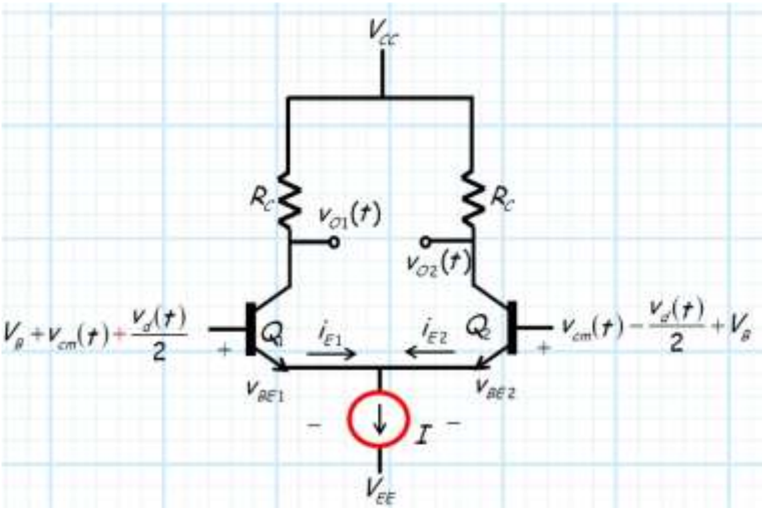
$$v_d(t) \doteq v_1(t) - v_2(t)$$

This is a **differential** amplifier, so we typically define gain in terms of its **common-mode** (A_{cm}) and **differential** (A_d) gains:

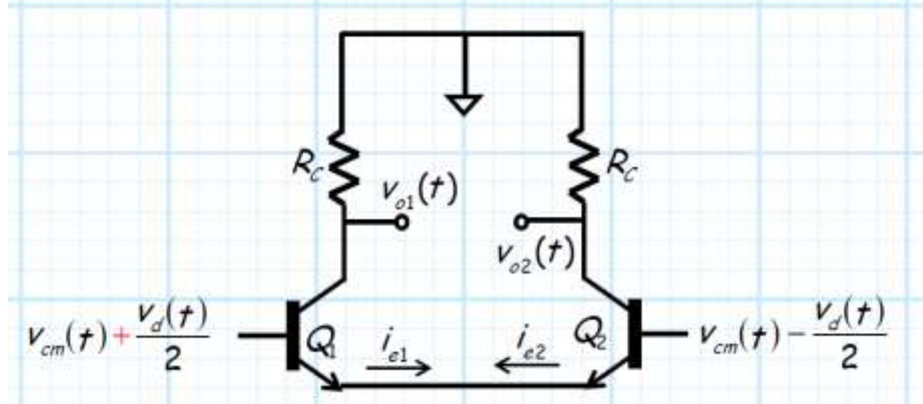
$$A_{cm} \doteq \frac{v_{o1}}{v_{cm}} = \frac{v_{o2}}{v_{cm}} \qquad \text{and} \qquad A_d \doteq \frac{v_{o1}}{v_d} = -\frac{v_{o2}}{v_d}$$

$$v_{o1}(t) = A_{cm}\, v_{cm}(t) + A_d\, v_d(t)$$

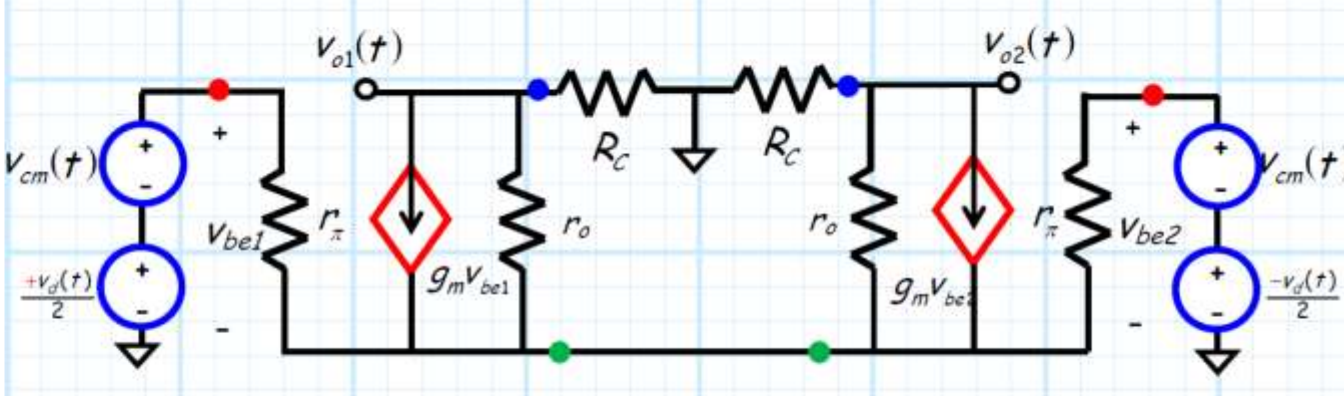
$$v_{o2}(t) = A_{cm}\, v_{cm}(t) - A_d\, v_d(t)$$



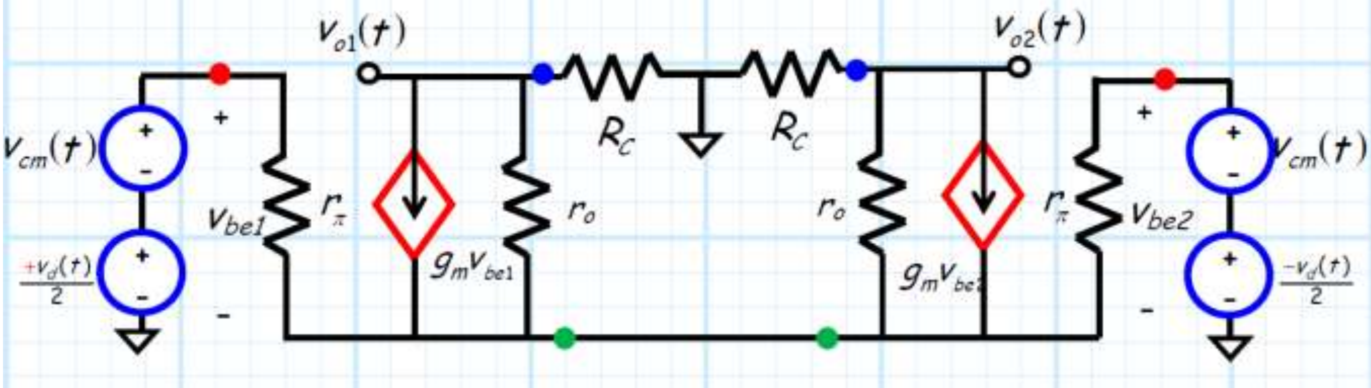
Turning **off** the DC sources:

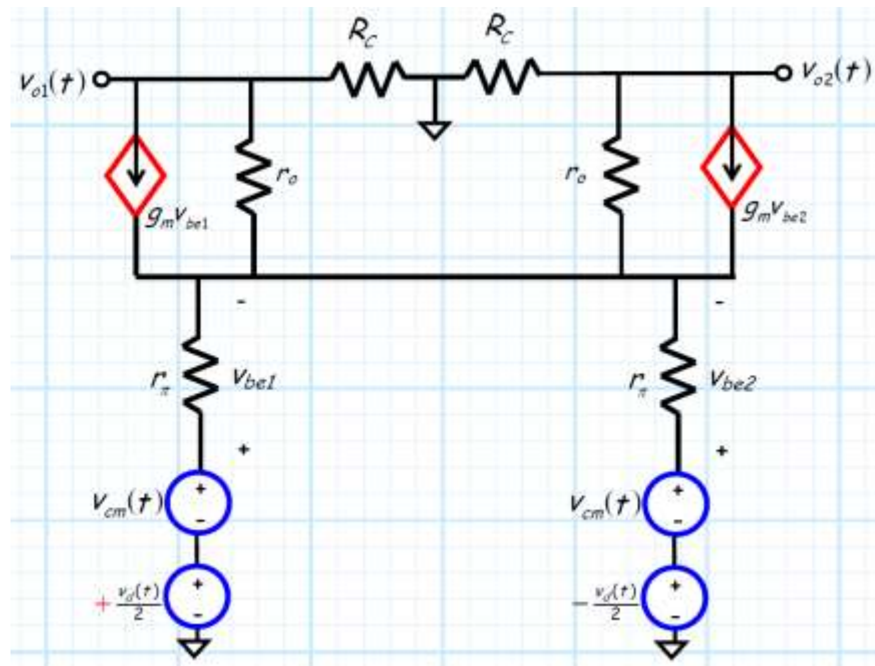


And now inserting the **hybrid-pi** BJT model:



And now inserting the **hybrid-pi** BJT model:

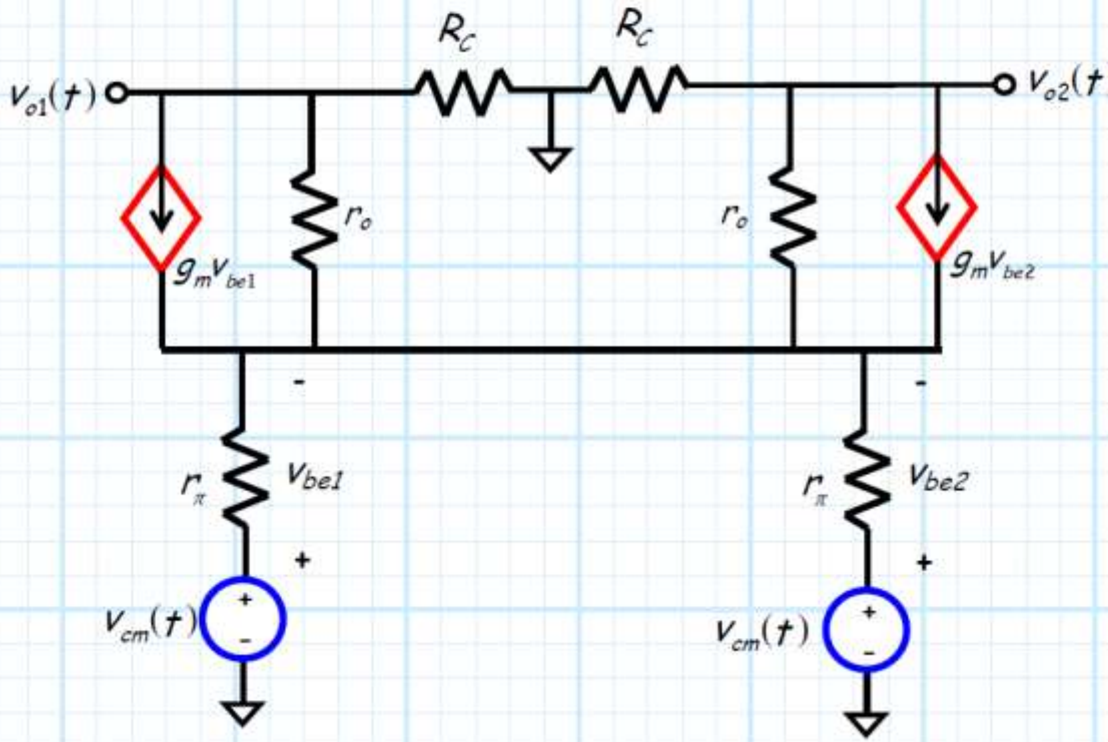




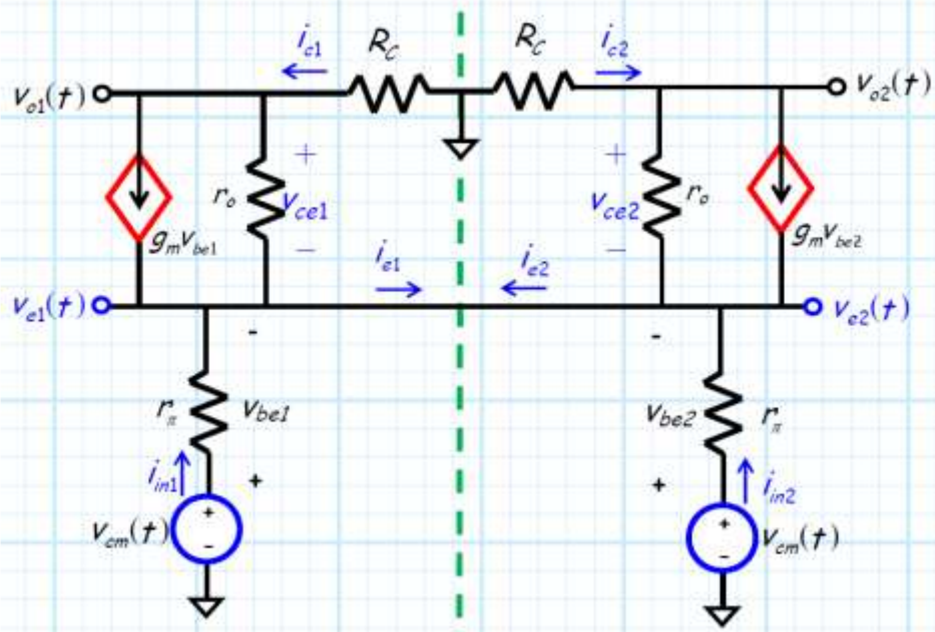
How do we analyze this?

A: In a word, **superposition**!

We first turn off the **two** differential-mode sources, and analyze the circuit with only the two **remaining** (equal valued) **common-mode** sources.



We notice that the common-mode circuit has a perfect plane of **reflection** (i.e., bilateral) **symmetry**:

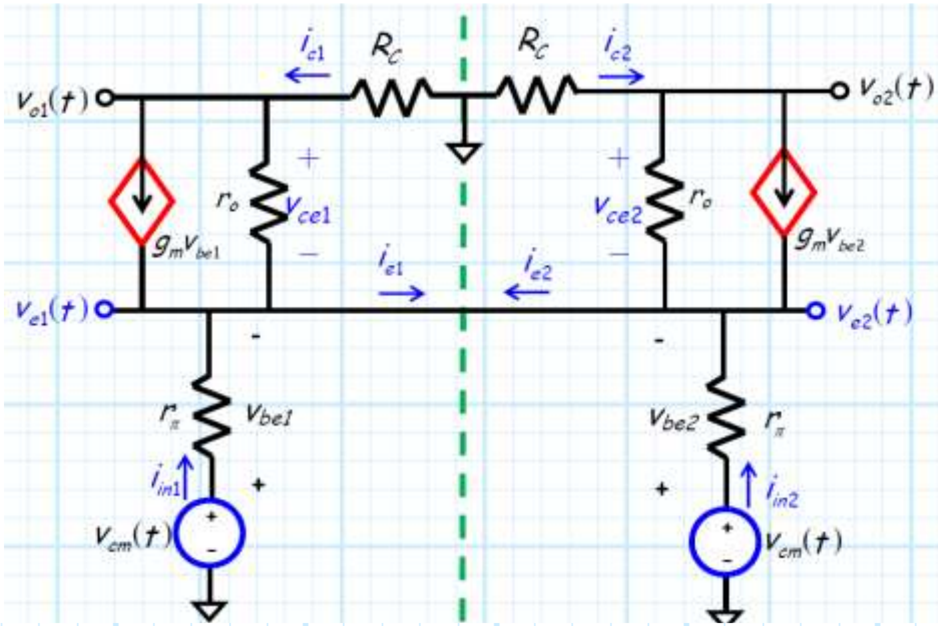


For example:

$$
\begin{aligned}
v_{be1} &= v_{be2} \\
v_{o1} &= v_{o2} \\
v_{ce1} &= v_{ce2} \\
v_{e1} &= v_{e2}
\end{aligned}
\qquad \text{and} \qquad
\begin{aligned}
i_{in1} &= i_{in2} \\
g_m v_{be1} &= g_m v_{be2} \\
&\boxed{\begin{aligned} i_{c1} &= i_{c2} \\ i_{e1} &= i_{e2} \end{aligned}}
\end{aligned}
$$

The left and right side of the circuit above are **mirror images** of each other (including the sources with **equal** value v_{cm}).

The two sides of the circuit a perfectly and precisely **equivalent**, and so the currents and voltages on each side of the circuit must likewise be perfectly and precisely **equal**!



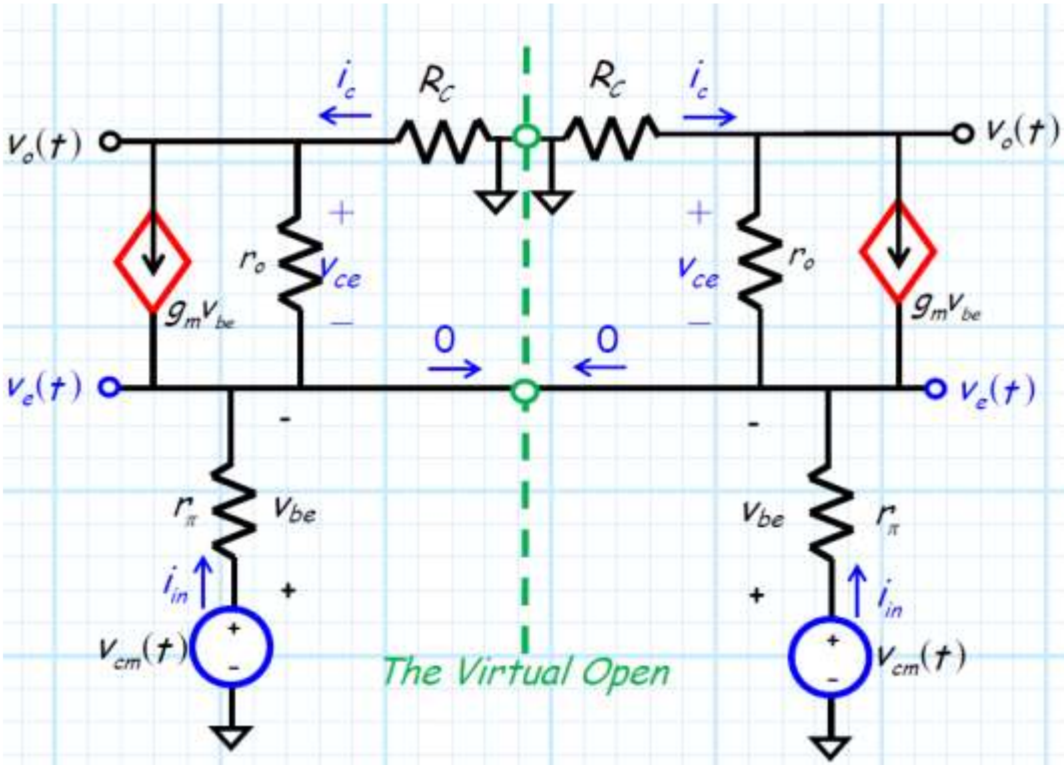
$$i_{e1} = i_{e2}.$$

*But, just **look** at the circuit; from **KCL** it is evident that:*

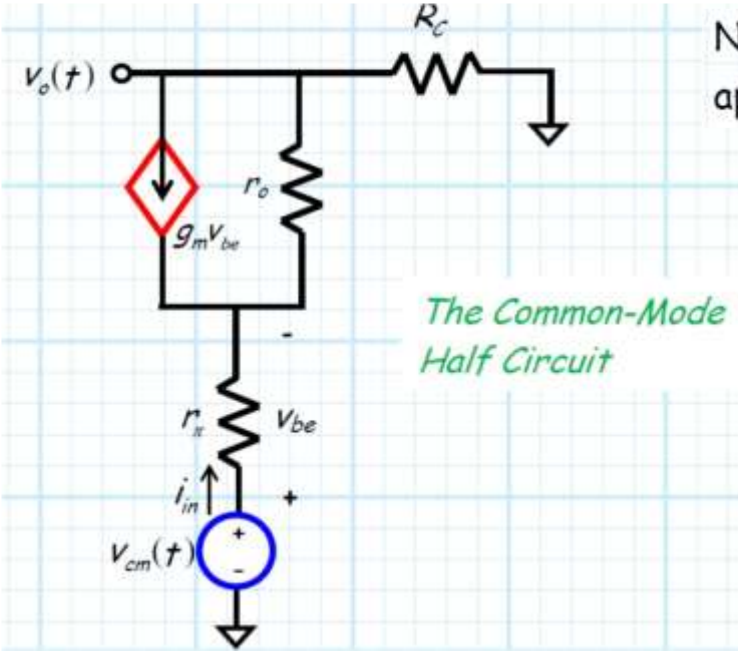
$$i_{e1} = -i_{e2}$$

There is **only one possible solution** that satisfies the two equations—the common-mode, small-signal emitter currents must be equal to **zero**!

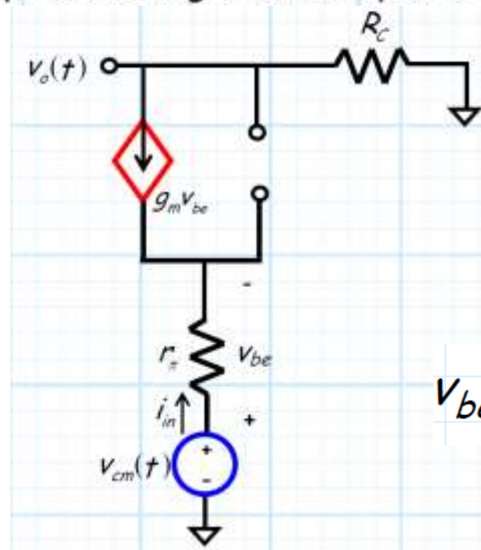
$$i_{e1} = i_{e2} = -i_{e2} = 0$$



Thus, we can take pair of scissors and cut this circuit into **two identical half-circuits**, without affecting **any** of the currents or voltages—the two circuits on either side of the virtual open are **completely independent**!



Now, since $r_o \gg r_\pi$ and $r_o \gg R_C$, we can simplify the circuit by approximating it as an open circuit:



$$v_{be} = r_\pi i_{in}$$

$$i_{in} = -g_m v_{be}$$

$$v_{be} = -(g_m r_\pi) v_{be} = -\beta v_{be}$$

$$v_{be} = -\beta v_{be}$$

Q: *No way! If $v_{be} = 0$, then $g_m v_{be} = 0$. **No current** is flowing, and so the **output voltage** v_o must likewise be equal to **zero**!*

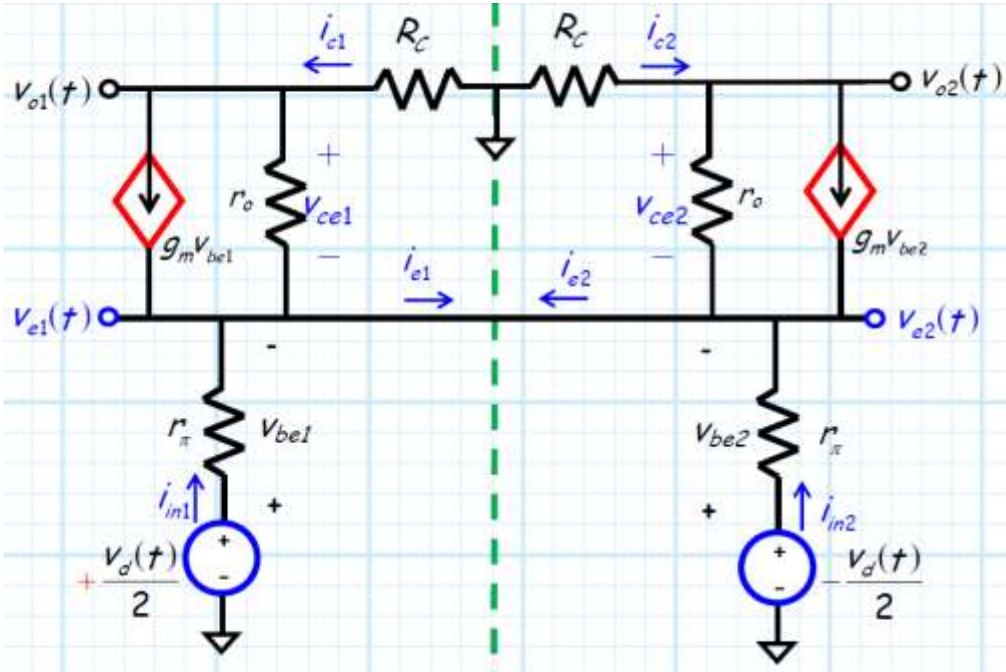
A: It means that the **common-mode gain** of a BJT differential pair is very small (**almost zero**!).

If we do not neglect r_o, common mode gain will be almost zero.

$$A_{cm} = \frac{v_o}{v_{cm}} \cong 0$$

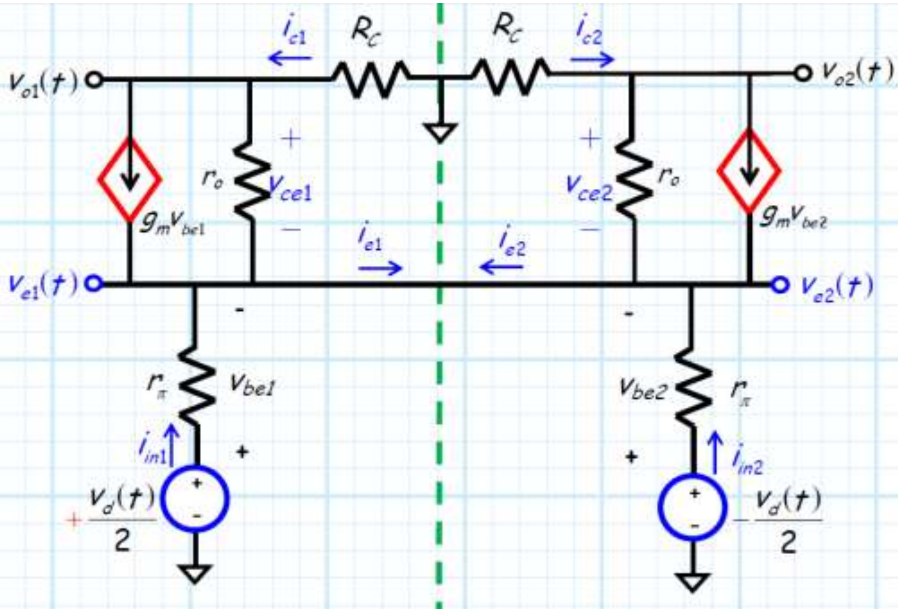
$$output\ resistance:\ r_0 = \frac{V_A}{I_{CQ}}$$

We then turn **off** the two common-mode sources, and analyze the circuit with only the two (equal but opposite valued) **differential-mode** sources.



Look at the two-small signal sources—they are "equal but **opposite**". The fact that the two sources have **opposite "sign"** changes the symmetry of the circuit.

$$\begin{aligned} v_{be1} &= -v_{be2} \\ v_{o1} &= -v_{o2} \\ v_{ce1} &= -v_{ce2} \\ v_{e1} &= -v_{e2} \end{aligned} \quad \text{and} \quad \begin{aligned} i_{in1} &= -i_{in2} \\ g_m v_{be1} &= -g_m v_{be2} \\ i_{c1} &= -i_{c2} \\ i_{e1} &= -i_{e2} \end{aligned}$$



$$
\begin{aligned}
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\end{aligned}
\qquad \text{and} \qquad
\begin{aligned}
i_{in1} &= -i_{in2} \\
g_m v_{be1} &= -g_m v_{be2} \\
i_{c1} &= -i_{c2} \\
i_{e1} &= -i_{e2}
\end{aligned}
$$

Q: *Wait! You say that—because of "circuit symmetry"—that:*

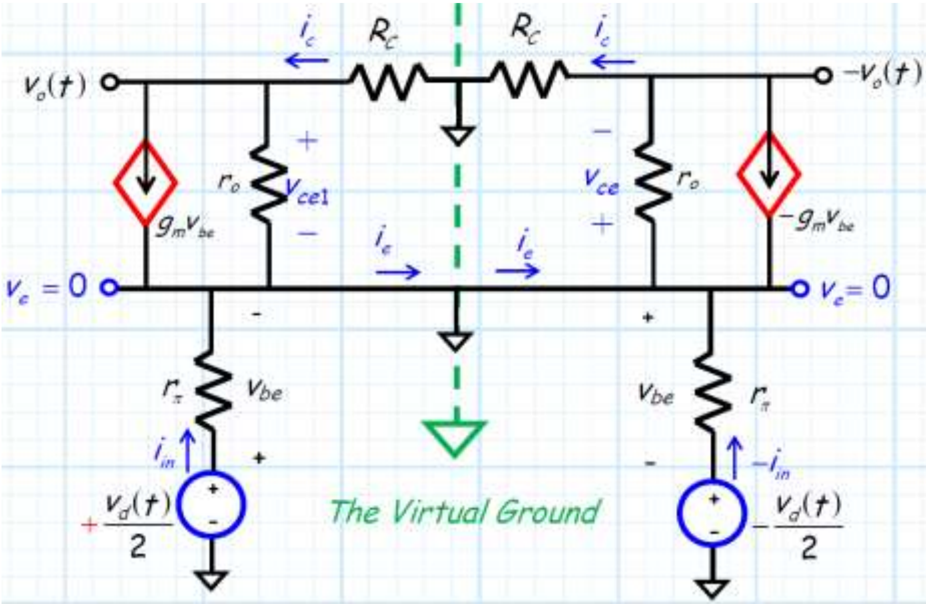
$$v_{e1} = -v_{e2}.$$

*But, just **look** at the circuit; from **KVL** it is evident that:*

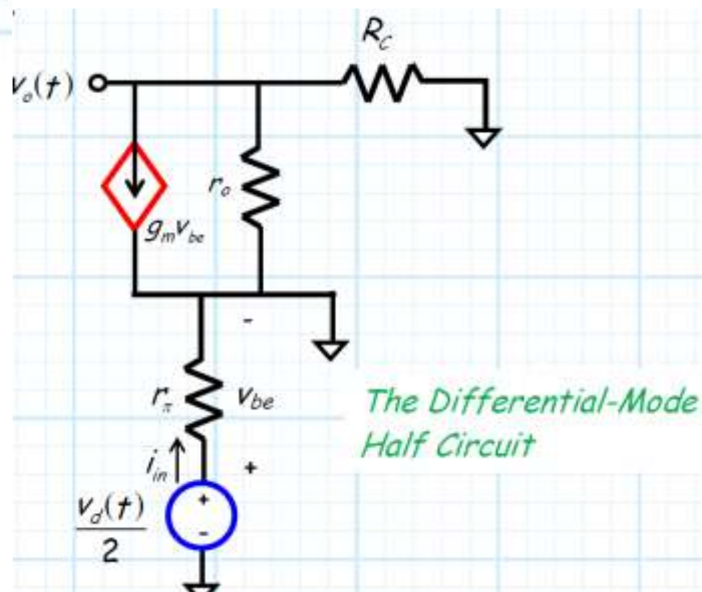
$$v_{e1} = v_{e2}$$

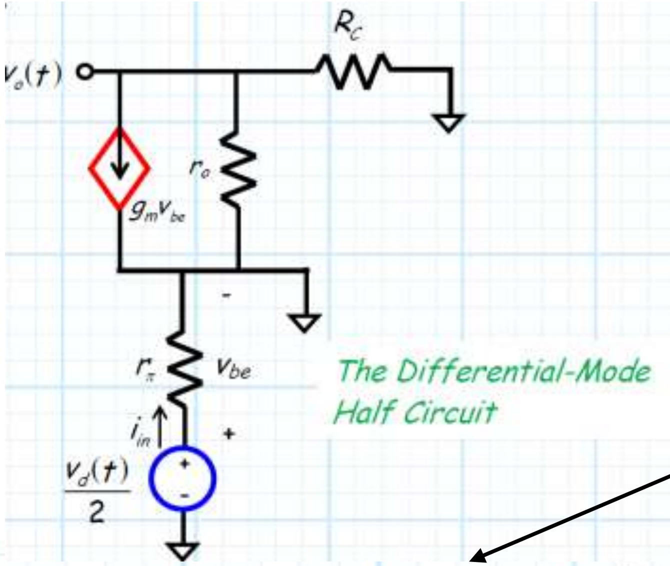
There is **only one possible solution** that satisfies the two equations—the differential-mode, small-signal emitter voltages must be equal to **zero**!

$$v_{e1} = -v_{e2} = v_{e2} = 0$$

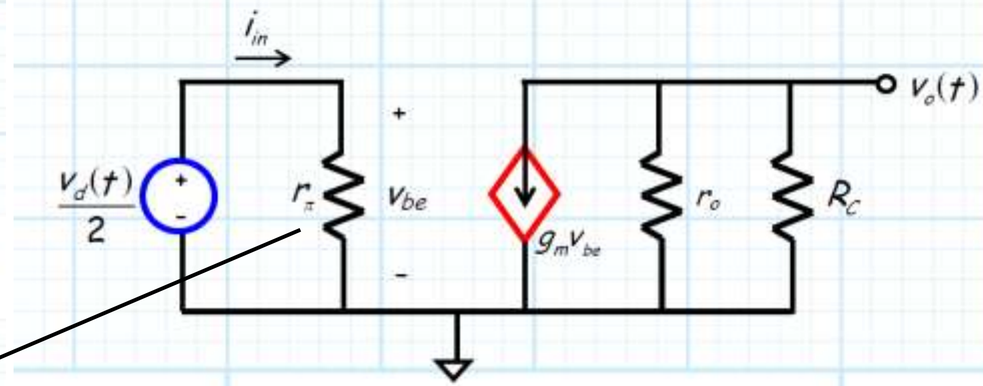


Again, the circuit has two **isolated and independent** halves. We can take our scissors and cut it into two separate "**half-circuits**":





Let's **redraw** this half-circuit and see if **you** recognize it:



$$v_o(t) = -g_m\left(r_o \| R_C\right)\frac{v_d(t)}{2} \cong -\frac{g_m R_C}{2} v_d(t)$$

$$i_{in}(t) = \frac{1}{r_\pi}\frac{v_d(t)}{2}$$

From this we can conclude that the **differential-mode small-signal gain** is:

$$A_d \doteq \frac{v_o(t)}{v_d(t)} = -\tfrac{1}{2} g_m R_C$$

And the **differential mode-input resistance** is:

$$R_{in}^d \doteq \frac{v_d(t)}{i_{in}(t)} = 2r_\pi$$

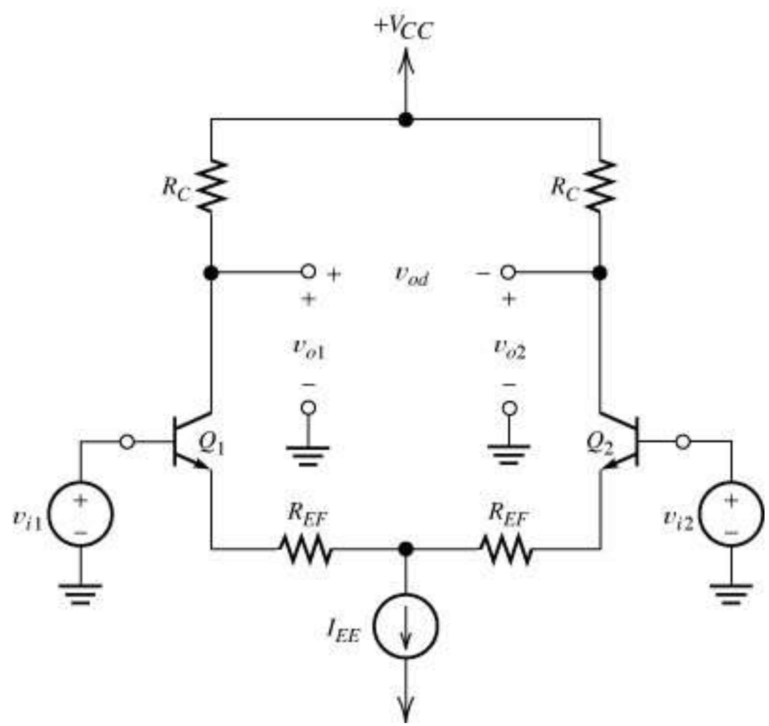
In addition, it is evident (from past analysis) that the **output resistance** is:

$$R_{out}^d = r_o \| R_C \cong R_C$$

Emitter degeneration

Sometimes it is advantageous to **add emitter generation resistor R_{EF}** to the circuit, as shown in the Figure.

There resistors have the disadvantage of reducing the differential voltage gain of the circuit. However, two reasons for this is to **increase input impedance** and to **reduce distortion** due to the nonlinearity of the BJTs.



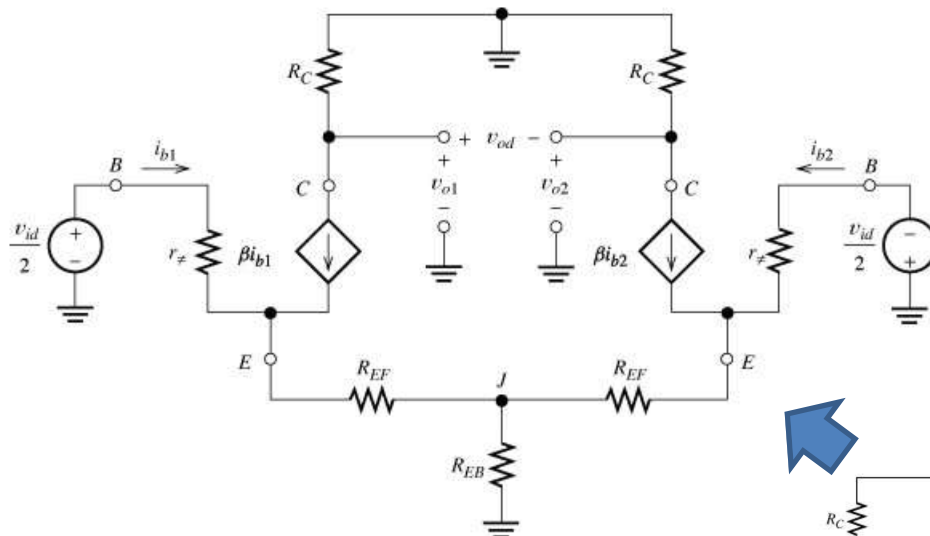
Small-signal analysis of the differential pair

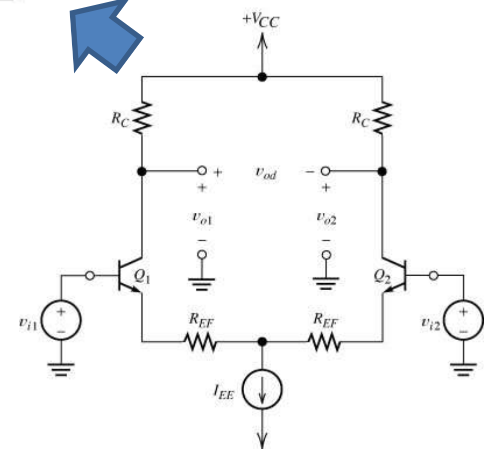
Using small-signal analysis, we can derive expressions for voltage gain, input impedance and output impedance of the emitter-coupled differential pair.

The small-signal equivalent circuit for the differential pair is shown below by replacing the transistors by their small-signal models.

Note that DC power supply has been shorted to GND in small-signal circuit.

Also note that **the I_{EE} current source is replaced by a resistance R_{EB}** in the small-signal circuit, as practical current sources has a finite output impedance.





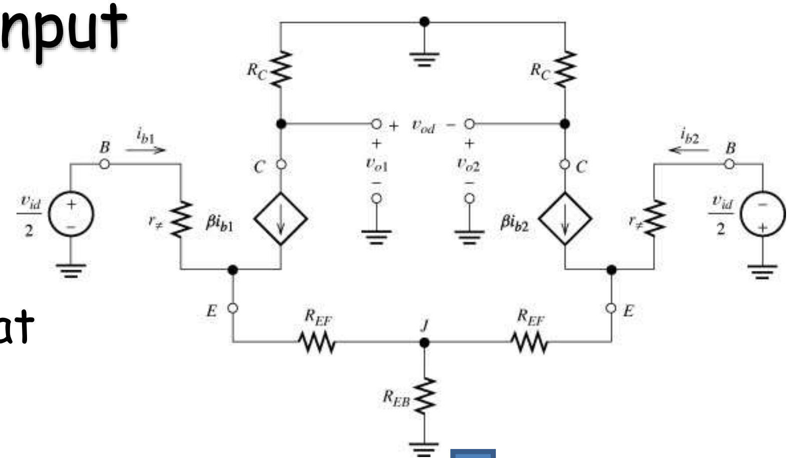
Small-signal analysis: differential input

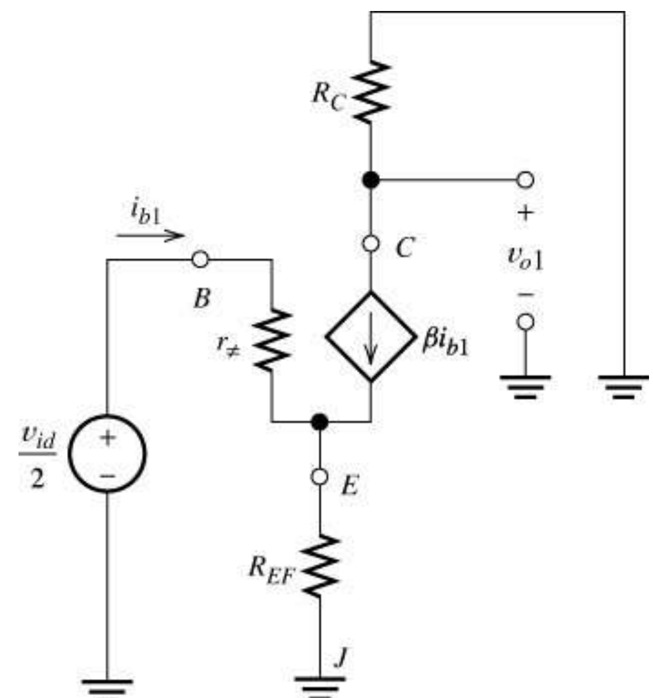
First, we analyze the circuit for a **pure differential input signal**. Therefore the input voltage are $v_{i1}=-v_{i2}=v_{id}/2$.

The analysis can be simplified by observing that the equivalent circuit is symmetrical.

Due to this symmetry and opposite polarity of the independent sources, the **voltage at point J is zero.** The circuit behavior would not change by shorting point J to Ground.

We can then consider only the **left-hand side circuit as shown** in the Figure. We need to analyze only this half circuit as the right half is the same except different polarity.





Half-circuit for a differential input signal.

Small-signal analysis: differential input II

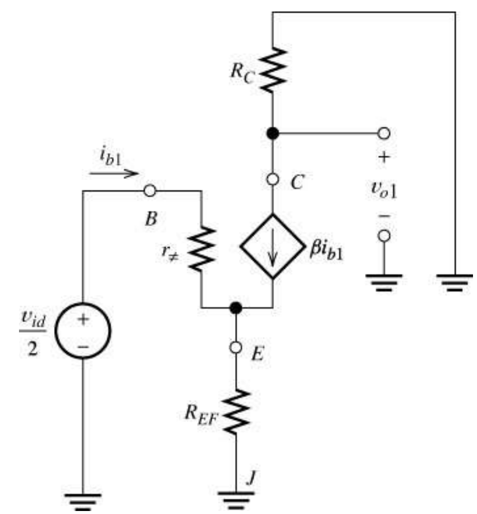
The voltage gain is:

$$v_{O1} = -R_C \beta i_{b1}$$
$$v_{id}/2 = [r_\pi + (\beta + 1) R_{EF}]\, i_{b1}$$

$$A_{vds} = \frac{v_{O1}}{v_{id}} = \frac{-R_C \beta}{2[r_\pi + (\beta + 1) R_{EF}]}$$

$$A_{vdb} = \frac{v_{od}}{v_{id}} = \frac{v_{o2} - v_{o1}}{v_{id}} = 2A_{vds}$$

subscript v for voltage gain,
d for differential input,
s for single−ended output



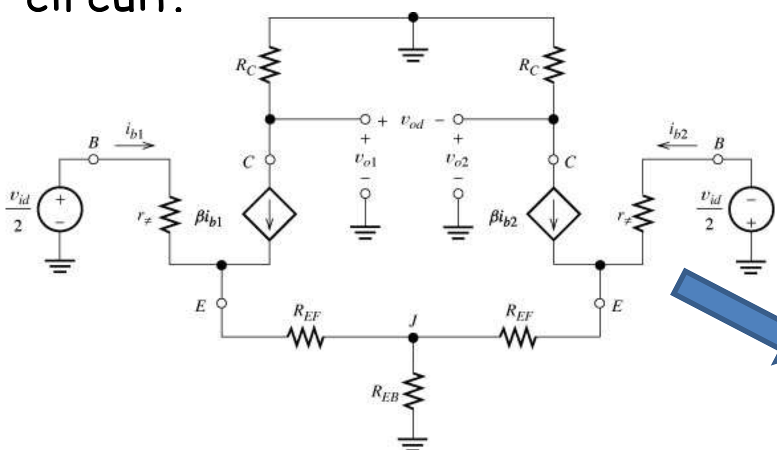
Half-circuit for a differential input signal.

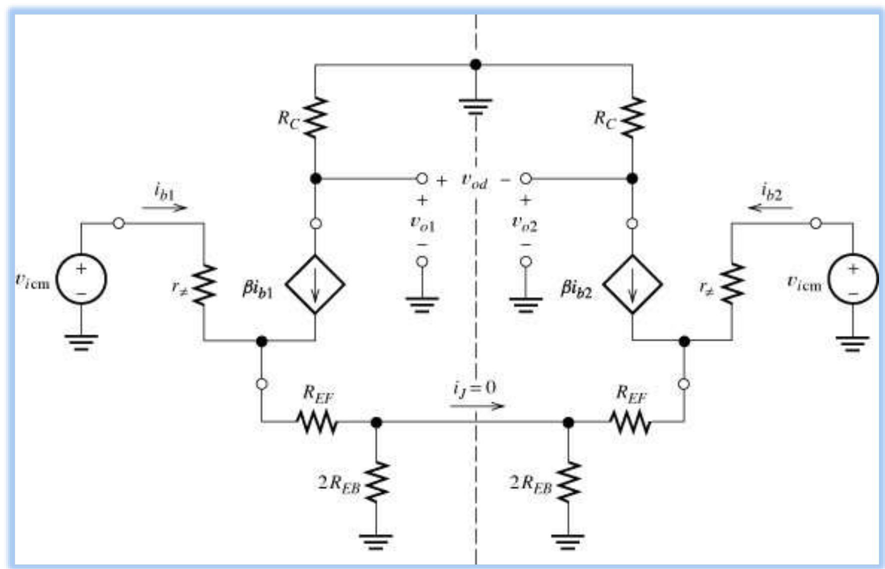
Small-signal analysis: common-mode input,

When the input voltage are $v_{i1}=v_{i2}=v_{icm}$, the equivalent circuit is depicted in the figure. **We have shown the output impedance of the current source as the parallel combination of two resistors.**

The equivalent circuit is symmetrical with respect to the dashed line including the polarities of the signal sources. Therefore, we conclude that **current *i_J* must be zero.**

As such, we can open the connection and consider only left or right hand half circuit.





Small-signal equivalent circuit with a pure common-mode input signal.

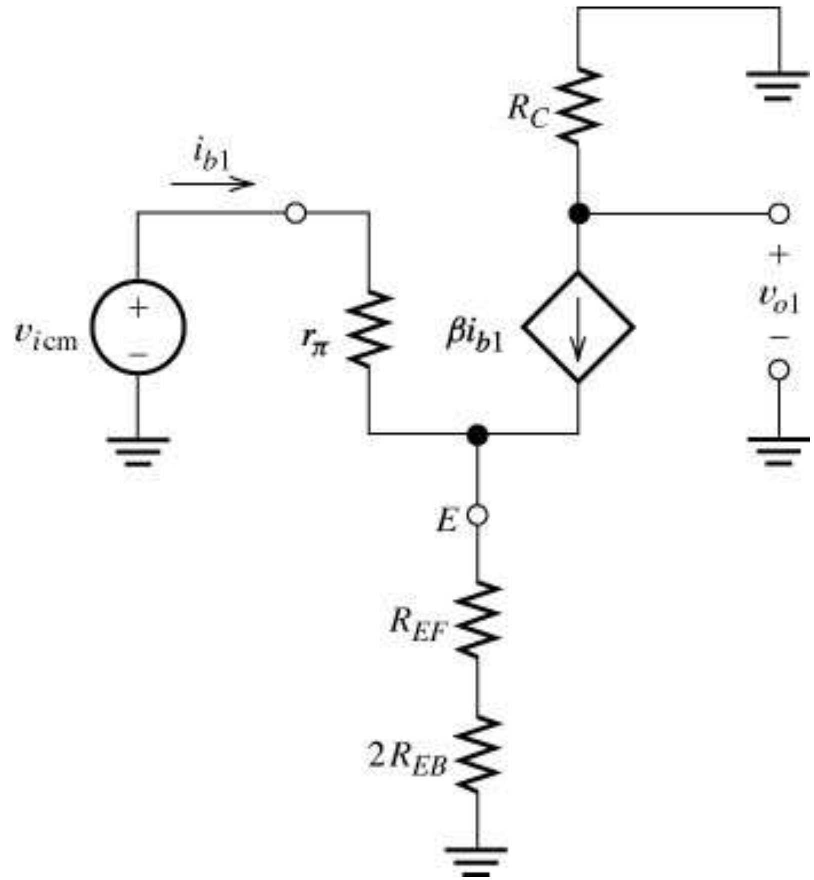
Small-signal analysis: common-mode input

Note that we have defined the common-mode input impedance to be the voltage divided by the total current the source must deliver to both terminals.

The gain from a single-ended load to common-mode input is:

As $v_{o1}=v_{o2}=v_{ocm}$

$$A_{vcm}=\frac{v_{O1}}{v_{icm}}=\frac{-R_C\beta}{r_\pi+(\beta+1)(R_{EF}+2R_{EB})}=\frac{v_{Ocm}}{v_{icm}}$$



Half-circuit for a pure common-mode input signal.