MAT 175E

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Homework 2

Only one randomly chosen question (which is the same for all of you) will be graded!

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(1) Prove or disprove each of the following:

(a) There are sets A, B and C satisfying the conditions: $A \cap B \neq \emptyset$ and $A \cap C = \emptyset$ and $(A \cap B) - C = \emptyset$.

(b) $\mathcal{P}(A - B) \subseteq \mathcal{P}(A) - \mathcal{P}(B)$ for any sets A and B where $\mathcal{P}(\ )$ denotes the power set of its argument.

(c) For any sets A, B, C, if $A \cup C = B \cup C$ and $A - C = B - C$, then $A = B$.

(d) For any sets A and B, every subset of $A \times B$ is of the form $U \times V$ for some subset U of A and for some subset V of B.

(e) For any sets A, B, P, Q, if $A \times B \subseteq P \times Q$ then $A \subseteq P$.

(f) $A \cap \mathcal{P}(A) = \emptyset$ for any nonempty set A.

(2) Let A, B, C be nonempty sets. Prove that:

$$(A \times B) - (B \times C) \subseteq A \times (B - C) \text{ if and only if } A \subseteq B \text{ or } B \cap C = \emptyset$$

(3) (a) Let A be an infinite subset of $\mathbb{N}$. Show that:

For any natural number m, there is an element a of A such that $a > m$.

(b) Prove that

$$\underbrace{\sqrt{2 + \sqrt{2 + \ldots + \sqrt{2 + \sqrt{2}}}}}_{n \text{ radicals}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$$

for all positive natural numbers n. (Hint: You may need to use the half angle formula: $\cos 2\alpha = 2\cos^2\alpha - 1$)

(4) (a) Let a_n be a sequence of real numbers satisfying

$$a_n a_{n-2} = (a_{n-1} - 6)(a_{n-1} - 12) \quad \text{for any } n \in \mathbb{N} \text{ with } n \geq 3.$$

If $a_1 = a_2 = 4$, prove that $a_n = 4$ for any positive natural number n.

(b) Prove by induction that, for any natural number n with $n \geq 2$ and for any set A with n elements, the number of subsets of A containing exactly two elements is $\dfrac{(n-1)n}{2}$.