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BLG 231E DIGITAL CIRCUITS MIDTERM SOLUTIONS

QUESTION 1 (30 Points):

Note that Parts (a) and (b) below are not related.

a) [15 points]

i.

Operation: Addition

Reason: A is negative because its most significant bit (MSB) is 1. Since B<A, B is also negative.
Overflow can occur **only after the addition** of two **negative** integers.
Overflow: negative + negative → positive

ii.

8th bit: 0

9th bit: 1

Reason: Because of overflow, result is incorrectly positive (negative + negative → positive). Therefore sign (8th) bit is 0.
Since the MSBs of both numbers A and B are 1, when we add them the 9th bit of the result will be 1.

Notes:

1. You will get no credit for simply stating "addition", "subtraction", 0, or 1. Your explanations must be correct.
2. 9th bit of the result does not represent the overflow. "9th bit =1 because there is an overflow" is NOT a correct explanation. The 9th bit of the result can be 0, even if there is an overflow, or it can be 1, even if there is no overflow. See the examples in the lecture notes (slides 1.24 and 1.25). Remember: this bit is ignored when working with signed numbers.
3. "9th bit =1 because the result cannot be represented using 8 bits" is NOT a correct explanation. The 9th bit of the result can be 0, even if the result cannot be represented using 8 bits, or it can be 1, even if the result can be represented.
4. "9th bit =1 because there is a carry" is NOT a sufficient explanation. The reason for a carry should be explained.
5. This is not a general solution if you give arbitrary integers A and B as examples and explain the reason based on them only. A solution that is valid for these exemplary numbers may not hold for other numbers.

b) [15 Points]:

i. A and B are **unsigned** integers.

A = \$7D= 0111 1101
B = \$C = 1100 unsigned extension → B = 0000 1100
2's complement of B: B'+1 = 1111 0011 + 1 = 1111 0100
A>B because there is NO borrow.

```
A -B:    0111 1101
       + 1111 0100
        10111 0001
```

Carry =1 → no borrow

ii. A and B are **signed** integers.

A = \$7D= 0111 1101
B = \$C = 1100 sign extension → B = 111 1100 (B is negative)
2's complement of B: B'+1 = 0000 0011 + 1 = 0000 0100
A>B because the result is negative and there is an overflow.

```
A -B:    0111 1101
       + 0000 0100
         1000 0001
```

Sign bit =1 → result is negative.
pos-neg=neg → overflow
Since there is an overflow, the result should be positive.

Notes:

1. To compare unsigned integers, you cannot use the sign of the result.
2. To compare signed integers, checking the sign of the result is not enough. You must also consider the overflow.

QUESTION 2 (35 Points):

<u>Note</u> that Parts (a) and (b) below are <u>not</u> related.

a) [15 points]

$$X \cdot Y = 0 \text{ and } X + Y = 1 \rightarrow X = \overline{Y} \text{ and } Y = \overline{X}$$

$$\begin{aligned} X \cdot Y &= 0 \\ &= X \cdot \overline{X} \\ X + Y &= 1 \\ &= X + \overline{X} \end{aligned}$$

So, $Y = \overline{X}$

OR

$$\begin{aligned} X \cdot Y &= 0 \\ &= \overline{Y} \cdot Y \\ X + Y &= 1 \\ &= \overline{Y} + Y \end{aligned}$$

So, $X = \overline{Y}$

$$\begin{aligned} X \cdot Z + \overline{X} \cdot Y + Y \cdot Z &= Z(X + Y) + \overline{X} \cdot Y \\ &= Z(X + \overline{X}) + \overline{X} \cdot Y \\ &= Z(1) + Y \cdot Y \\ &= Z + Y \end{aligned}$$

Solution 1

$$\begin{aligned} X \cdot Z + \overline{X} \cdot Y + Y \cdot Z &= (X + Y)(\overline{X} + Z) \\ &= (1)(\overline{X} + Z) \\ &= (\overline{X} + Z) + 0 \\ &= (\overline{X} + Z) + X \cdot Y \\ &= \overline{X} + X \cdot Y + Z \\ &= (\overline{X} + Y) + Z \\ &= (X + Y)(\overline{X} + Y) + Z \\ &= Y + Z \end{aligned}$$

$X + Y = 1$

$X \cdot Y = 0$

$X + Y = 1$

Solution 2

$$\begin{aligned} X \cdot Z + \overline{X} \cdot Y + Y \cdot Z &= (X + Y)(\overline{X} + Z) \\ &= (X + Y)(\overline{X} + Z)(Y + Z) \\ &= (1)(\overline{X} + Z)(Y + Z) \\ &= \overline{X} \cdot Y + \overline{X} \cdot Z + Y \cdot Z + Z \\ &= \overline{X} \cdot Y + Z \\ &= X \cdot Y + \overline{X} \cdot Y + Z \\ &= Y + Z \end{aligned}$$

$X + Y = 1$

$X \cdot Y = 0$

Solution 3

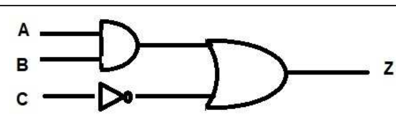
b) [20 points]

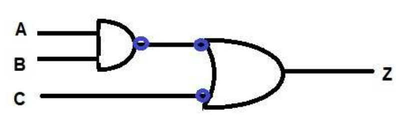
i.

$$\begin{aligned} (\overline{A} + \overline{B} + C) \oplus \overline{(AB)} &= (\overline{A} + \overline{B} + C)AB + \overline{(\overline{A} + \overline{B} + C)} \cdot \overline{(AB)} + \overline{C} \\ &= ABC + AB\overline{C}(\overline{A} + \overline{B}) + \overline{C} \\ &= ABC + \overline{C} \\ &= \overline{C} + AB \end{aligned}$$

ii.

$$\begin{aligned} Z = f(A, B, C) &= \overline{C} + AB \\ &= \overline{[\overline{(AB)} + C]} \end{aligned}$$





QUESTION 3 (35 Points):
Note that Parts (a) and (b) below are not related.

a) [20 points]

i.

Num	a b c d
0	0 0 0 0 ✓
1	0 0 0 1 ✓
2	0 0 1 0 ✓
8	1 0 0 0 ✓
10	1 0 1 0 ✓
12	1 1 0 0 ✓
14	1 1 1 0 ✓

Num	a b c d
$\boxed{0, 1}$	$\boxed{0\,0\,0\,-}$
0, 2	0 0 - 0 ✓
0, 8	- 0 0 0 ✓
2, 10	- 0 1 0 ✓
8, 10	1 0 - 0 ✓
8, 12	1 - 0 0 ✓
10, 14	1 - 1 0 ✓
12, 14	1 1 - 0 ✓

Num	a b c d
$\boxed{0,2,8,10}$	$\boxed{-\,0\,-\,0}$
0,8,2,10	- 0 - 0
$\boxed{8,10,12,14}$	$\boxed{1\,-\,-\,0}$
8,12,10,14	1 - - 0

No need to rewrite the same items

Set of all prime implicants (not checked off): $\bar{a}\,\bar{b}\,\bar{c}$, $\bar{b}\,\bar{d}$, $a\,\bar{d}$

ii.

	$\bar{C}.\bar{D}$	$\bar{C}.D$	$C.D$	$C.\bar{D}$
$\bar{A}.\bar{B}$	1	1	0	x
$\bar{A}.B$	0	0	0	0
$A.B$	x	0	0	x
$A.\bar{B}$	1	0	0	1

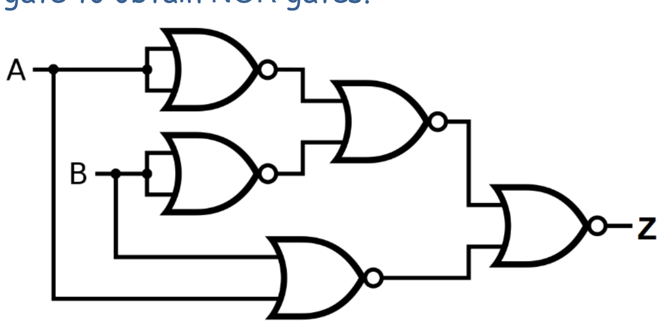
We need to cover true points 0, 1, 8, and 10.
$\bar{a}\,\bar{b}\,\bar{c}$ is an essential prime implicant. So, $\bar{a}\,\bar{b}\,\bar{c}$ has to be included in the minimal covering sum. Between $\bar{b}\,\bar{d}$ and $a\,\bar{d}$, we decide based on the cost criteria. Since $a\,\bar{d}$ has one less complementation,
Minimal covering sum: $\mathbf{f(a, b, c, d) = \bar{a}\,\bar{b}\,\bar{c} + a\,\bar{d}}$

b)
[15 points]

For NOR gates, starting with POS form is advantageous.
To implement the function using the fewest possible number of 2-input NOR gates, we must convert the given expression to POS form.

$\mathbf{Z = A\,\bar{B} + \bar{A}\,B = (A + B)(\bar{A} + \bar{B})}$ (POS form)
$\mathbf{Z = [\,(A + B)' + (\bar{A} + \bar{B})'\,]'}$ (De Morgan's Law)
$\mathbf{Z = [(A \downarrow B) \downarrow (\bar{A} \downarrow \bar{B})]}$ (NOR gates only)

Alternative: You can also first implement the expression in POS form using AND, OR, and NOT gates.
Then, you can add NOT gates to the outputs of OR gates, and to the inputs of the AND gate to obtain NOR gates.



Note:
If you implement the circuit using the given expression in SOP form, you will obtain a circuit that contains more gates.