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BLG 231E DIGITAL CIRCUITS MIDTERM SOLUTIONS

<u>Please show ALL work</u>. Answers with no supporting explanations or work will be given no partial credit. If we cannot read or follow your solution, no partial credit will be given. PLEASE BE NEAT!

QUESTION 1 (30 Points):

<u>Note</u> that Parts (a), (b), and (c) below are <u>not</u> related.

a) [10 points]

i. Since X is negative, its most significant bit is 1.

To obtain the **largest possible** <u>binary</u> X, we should fill in the blanks with 1s. X = 1111 1111

Since X is negative, to obtain its decimal value, we should take its 2's complement. X = 1111 1111
1's 0000 0000
+1 0000 0001 → Decimal X = -1

Binary X= 1111 1111
Decimal X= -1

ii. To obtain the **largest possible** <u>binary</u> X, we should fill in the blanks with 0s. X = 1000 1000

Since X is negative, to obtain its decimal value, we should take its 2's complement. X = 1000 1000
1's 0111 0111
+1 0111 1000

Decimal |X| = 64 + 32 + 16 + 8 = 120 → X = -120

Binary X= 1000 1000
Decimal X= -120

b) [10 Points]

You do not need to convert hexadecimal numbers to decimal!

4-bit binary, **<u>signed</u>** number A = \$A, is **<u>not</u> decimal 10!**

A = \$A = 1010 (Negative)
B = \$7C = 0111 1100 (Positive)

Since A is a 4-bit number and it is shorter than B, we must extend it from 4 bits to 8 bits.
Since A is a negative number (most significant bit is 1), its high-order part is filled with 1s.

8-bit A = 1111 1010

To perform the **binary** operation Z = A - B using the **2's complement** system, we take 2's complement of B and add it to A.

```
B = 0111 1100              A 1111 1010
1's 1000 0011          + -B 1000 0100
+1  1000 0100: -B           10111 1110
```

Result is positive.

We subtract a positive number (B) from a negative number (A), and the result is positive**. Overflow occurred**.
negative – positive → positive

c) [10 Points]

Since the result is **positive,** and **overflow occurs**, A is negative, and B is positive.
Remember: negative – positive → positive
The result should be negative but it cannot be represented using 8 bits.
The range of signed integers that can be represented with 8 bits: -128 to +127
The largest decimal result that yields this result is -129. A - B < -129
The largest possible value of B is +127.
A - 127 < -129 → **largest possible** <u>decimal</u> A = -2.

QUESTION 2 (35 Points):
Note that Parts (a), (b), and (c) below are not related.

a) **[10 points]** For each of the three expressions below, decide which term (if any) can be eliminated without adding a new term to the expression, and write it in the blank space provided next to the expression, and explain why:

i. $(\bar{A}+B+C)(A+D)(B+C+D)$: $(B+C+D)$ (consensus with respect to A)

ii. $A\,\bar{B}\,C+\bar{A}\,B\,D+B\,C\,\bar{D}$: No terms can be eliminated.

The consensus with respect to B is $A\,C\,\bar{D}$, consensus with respect to D is $\bar{A}\,B\,C$. However, these do not appear in the expression. Nothing can be eliminated.

b) **[10 points]** Prove the theorem given below, using Boolean algebra without using the absorption theorem (Do not use it!). **State which axiom/theorem** you used next to each step. Show all steps.

$$AB(AB+C\,\bar{D})=AB$$

$$\begin{aligned} AB(AB+C\,\bar{D}) &\overset{\text{Distributive}}{=} A\,B\cdot A\,B+A\,B\,C\,\bar{D} && \text{Associative+Commutative}\\ &= A\cdot A\cdot B\cdot B+A\,B\,C\,\bar{D} && \text{Idempotency}\\ &= A\,B+A\,B\,C\,\bar{D} && \text{Identity}\\ &= A\,B\cdot 1+A\,B\,C\,\bar{D} && \text{Distributive}\\ &= A\,B(1+C\,\bar{D}) && \text{Annihilator/Dominance}\\ &= A\,B(1) && \text{Identity}\\ &= A\,B \end{aligned}$$

c) **[15 points]** Minimize the Boolean expression given below, using Boolean algebra. Use as few steps as possible**. State which axiom/theorem** you used next to each step. Show all steps.

(**Note:** To show complements, put a bar over literals, such as $\bar{X}_1$.)

$$Z=f(X_1,X_2,X_3,X_4,X_5,X_6)=X_1\bar{X}_2+X_2X_3X_4+\bar{X}_1X_2X_3+X_5X_6+X_1X_3+X_5=?$$

We immediately observe that the X_5X_6 term is redundant:
$X_5X_6+X_5=X_5(1+X_6)$. Since $1+X_6=1$, the expression evaluates to X_5.

$$\begin{aligned} Z &= X_1\bar{X}_2+X_2X_3X_4+\bar{X}_1X_2X_3+X_1X_3+X_5 && \text{Consensus (w.r.t. } X_1)\\ &= X_1\bar{X}_2+X_2X_3X_4+\bar{X}_1X_2X_3+X_1X_3+X_2X_3+X_5 && \text{Absorption}\\ &= X_1\bar{X}_2+X_2X_3X_4+X_1X_3+X_2X_3+X_5 && \text{Absorption}\\ &= X_1\bar{X}_2+X_1X_3+X_2X_3+X_5 && \text{Commutative}\\ &= X_1\bar{X}_2+X_2X_3+X_1X_3+X_5 && \text{Consensus (w.r.t. } X_2)\\ &= X_1\bar{X}_2+X_2X_3+X_5 && \text{Absorption} \end{aligned}$$

We observe that X_1X_3 is the consensus of $X_1\bar{X}_2$ and X_2X_3 with respect to X_2, so can be eliminated.

QUESTION 3 (35 Points):

Note that Parts (a) and (b) below are not related.

a) [20 points]

i.

Num	a	b	c	d	
0	0	0	0	0	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
10	1	0	1	0	✓
12	1	1	0	0	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

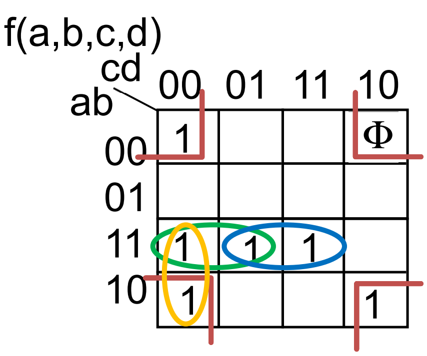
Num	a	b	c	d	
0,2	0	0	–	0	✓
0,8	–	0	0	0	✓
2,10	–	0	1	0	✓
8,10	1	0	–	0	✓
8,12	1	–	0	0	PI
12,13	1	1	0	–	PI
13,15	1	1	–	1	PI

Num	a	b	c	d	
0,2,8,10	–	0	–	0	PI
0,8,2,10	–	0	–	0	

The set of all prime implicants in SOP (Sum-of-Products) form:

$a\bar{c}\bar{d}$, $ab\bar{c}$, abd, $\bar{b}\cdot\bar{d}$

ii.



Essential prime implicants: $\bar{b}\cdot\bar{d}$, abd

The essential prime implicants cover all 1-generating input combinations except 1100.

To cover 1100, we can select either $a\bar{c}\bar{d}$ or $ab\bar{c}$.

Since the cost of $ab\bar{c}$ (7) is lower than cost of $a\bar{c}\bar{d}$ (8), we select $ab\bar{c}$.

The minimal covering sum with the lowest cost:

$$f(a,b,c,d) = \bar{b}\cdot\bar{d} + abd + ab\bar{c}$$

b) [15 points]

We insert NOT gates (blue circles) to convert AND and OR gates to NOR gates without changing the function of the given circuit.