BLG 231E

HW1

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1)

a)

Representation of $(102)_{10}$ in binary (base-2):

102 / 2 = 51, remainder is 0. (Least Significant Bit)

51 / 2 = 25, remainder is 1.

25 / 2 = 12, remainder is 1.

12 / 2 = 6, remainder is 0.

6 / 2 = 3, remainder is 0.

3 / 2 = 1, remainder is 1.

1 / 2 = 0, remainder is 1. (Most Significant Bit)

Thus, the representation of $(102)_{10}$ in 7-bits binary is $(1100110)_2$.

To expand 7-bits to 8-bits, it can be filled with 0: $(01100110)_2$.

In order to convert to $(102)_{10}$ to $(-102)_{10}$, 2's complement of $(102)_{10}$ must be used:

2's complement of (A) = A' + 1

A = $(01100110)_2$

A' = $(10011001)_2$

A' + 1 = $(10011010)_2 = (-102)_{10}$

$(27)_{10}$ can be represented in a similar way:

27 / 2 = 13, remainder is 1. (Least Significant Bit)

13 / 2 = 6, remainder is 1.

6 / 2 = 3, remainder is 0.

3 / 2 = 1, remainder is 1.

1 / 2 = 0, remainder is 1. (Most Significant Bit)

Thus, the representation of $(27)_{10}$ in 5-bit binary is $(11011)_2$.

To expand 5-bits binary to 8-bits, it can be filled with 0: $(00011011)_2$.

In order to convert to $(27)_{10}$ to $(-27)_{10}$, 2's complement of $(27)_{10}$ must be used:

2's complement of (A) = A' + 1

A = $(00011011)_2$

A' = $(11100100)_2$

A' + 1 = $(11100101)_2$ = $(-27)_{10}$

b)

i)

$(-27)_{10}$ = $(11100101)_2$
$(-102)_{10}$ = $(10011010)_2$
$(-27)_{10}$ + $(-102)_{10}$:

```
  11100101
  10011010
+_________
 101111111
```

The leftmost 1 is ignored. Since the sign digit (2nd leftmost digit) is 0, there is an overflow. (negative + negative cannot be positive). Thus, this summation cannot be represented with 8-bits digits. This is the summation of two signed numbers, so there is no borrow or carry.

ii)

$(-27)_{10}$ = $(11100101)_2$
$(-102)_{10}$ = $(10011010)_2$
$(27)_{10}$ = $(00011011)_2$
$(-102)_{10}$ - $(-27)_{10}$ = $(-102)_{10}$ + 2's complement of $(-27)_{10}$ = $(-102)_{10}$ + $(27)_{10}$:

```
  10011010
  00011011
+_________
  10110101
```

There is no overflow (sign bit is consistent), no borrow or no carry (This is the summation of two signed numbers).

iii)

$(102)_{10}$ = $(01100110)_2$
$(27)_{10}$ = $(00011011)_2$
$(27)_{10}$ - $(102)_{10}$ = $(27)_{10}$ + 2's complement of $(102)_{10}$ = $(27)_{10}$ + $(-102)_{10}$:

```
  00011011
  10011010
+_________
  10110101
```

There is no overflow (sign bit is consistent), yet there is a borrow (there is no carry). The first operand is smaller than the second.

2)

a)

1. (Y+X'Z')(X+Y'+Z')(X'+Y)
2. [(Y+X'Z')(X+Y'+Z')](X'+Y)
3. [YX+YY'+YZ'+X'Z'X+X'Z'Y'+X'Z'Z'](X'+Y) (Distributivity)
4. [YX+0+YZ'+X'Z'X+X'Z'Y'+X'Z'Z'](X'+Y) (Inverse) (YY' = 0)
5. [YX+YZ'+X'Z'X+X'Z'Y'+X'Z'Z'](X'+Y) (Identity) (YX+0 = YX)
6. [YX+YZ'+Z'0+X'Z'Y'+X'Z'Z'](X'+Y) (Inverse) (X'X = 0)
7. [YX+YZ'+0+X'Z'Y'+X'Z'Z'](X'+Y) (Null Law) (Z'0 = 0)
8. [YX+YZ'+X'Z'Y'+X'Z'Z'](X'+Y) (Identity) (YZ'+0 = YZ')
9. [YX+YZ'+X'Z'Y'+X'Z'](X'+Y) (Idempotency) (Z'Z' = Z')
10. (YXX'+YXY+YZ'X'+YZ'Y+X'Z'Y'X'+X'Z'Y'Y+Z'Z'X'+X'Z'Y) (Distributivity)
11. (Y0+YXY+YZ'X'+YZ'Y+X'Z'Y'X'+X'Z'0+0X'+X'Z'Y) (Inverse)
12. (0+YXY+YZ'X'+YZ'Y+X'Z'Y'X'+0+0+X'Z'Y) (Null Law)
13. (YXY+YZ'X'+YZ'Y+X'Z'Y'X'+X'Z'Y) (Identity)
14. (XY+YZ'X'+YZ'+X'Z'Y'+X'Z'Y) (Idempotency)
15. (X'Z')(Y+Y'+Y)+XY+YZ' (Distributivity)
16. (X'Z')(1)+XY+YZ' (Idempotency and Inverse)
17. X'Z'+XY+YZ' (Identity)

b)

1. X'Y'Z'T'+XY'T+X'Y'Z+XZT'+XY'ZT+X'Y'Z'
2. X'Y'Z'+XY'T+X'Y'Z+XZT'+XY'ZT (Absorption) (X'Y'Z'+X'Y'Z'T' = X'Y'Z')
3. X'Y'Z'+XY'T+X'Y'Z+XZT' (Absorption) (XY'T+XY'TZ = X'Y'T)
4. X'Y'(Z'+Z)+XY'T+XZT' (Distributivity)
5. X'Y'1+XY'T+XZT' (Inverse) (Z'+Z = 1)
6. X'Y'+XY'T+XZT' (Identity) (X'Y'1 = X'Y')
7. Y'(X'+XT)+XZT' (Distributivity)
8. Y'(X'T+X'+XT)+XZT' (Absorption) (X'T+X'=X')
9. Y'(T(X'+X)+X')+XZT' (Distributivity)
10. Y'(T1+X')+XZT' (Inverse) (X'+X = 1)
11. Y'(T+X')+XZT' (Identity) (T1 = T)
12. Y'T+Y'X'+XZT' (Distributivity)