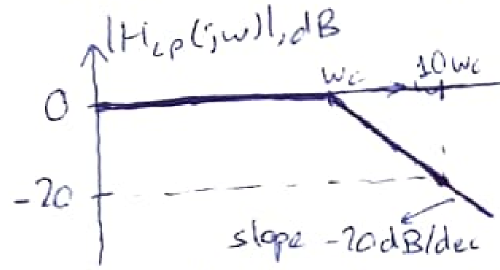
Example: Transfer function of a system is

$$H(s) = \frac{10.(s+0.01)(s+10)}{(s+0.1)^2(s+1)}$$ Find frequency response of this system.

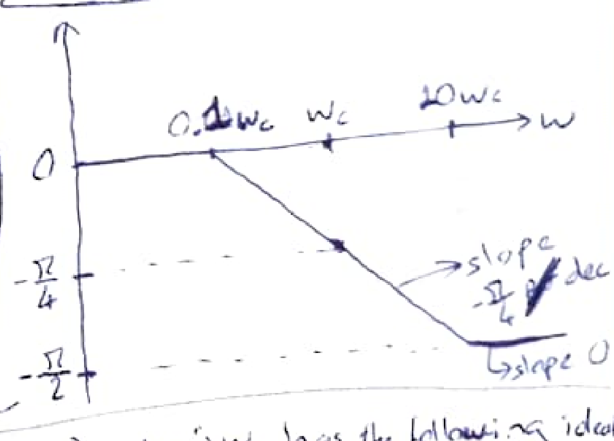
$$H(s) = \frac{10.\left(\frac{100s+1}{100}\right)\left(\frac{s}{10}+1\right)10}{(0.1)^2.\left(\frac{s}{0.1}+1\right)^2.(s+1)} = \frac{10.\left(\frac{s}{0.01}+1\right)0.01\left(\frac{s}{10}+1\right)10}{(0.1)^2.\left(\frac{s}{0.1}+1\right)^2.(s+1)}$$

$$H(s) = \frac{100\left(1+\frac{s}{0.01}\right)\left(1+\frac{s}{10}\right)}{\left(1+\frac{s}{0.1}\right)^2\left(1+\frac{s}{1}\right)} \Rightarrow H(jw) = \frac{100.\left(1+\frac{jw}{0.01}\right)\left(1+\frac{jw}{10}\right)}{\left(1+\frac{jw}{0.1}\right)^2\left(1+\frac{jw}{1}\right)}$$

We know a low-pass filter $H_{LP}(jw) = \frac{1}{1+\frac{jw}{w_c}}$ has following ideal characteristics

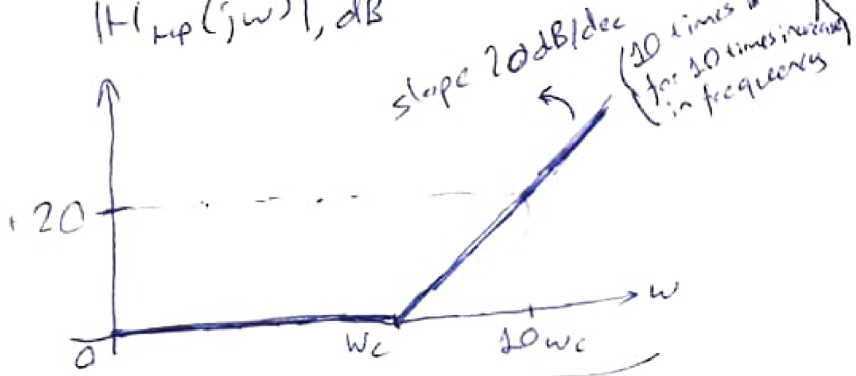


Magnitude response

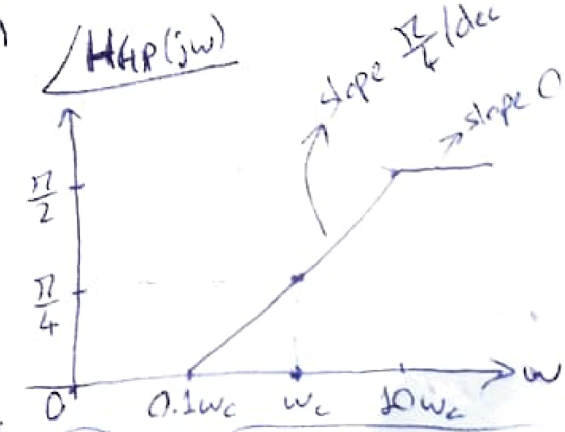


Phase response

We know a high-pass filter $H_{HP}(jw) = 1 + \frac{jw}{w_c}$ has the following ideal characteristics.



Magnitude response

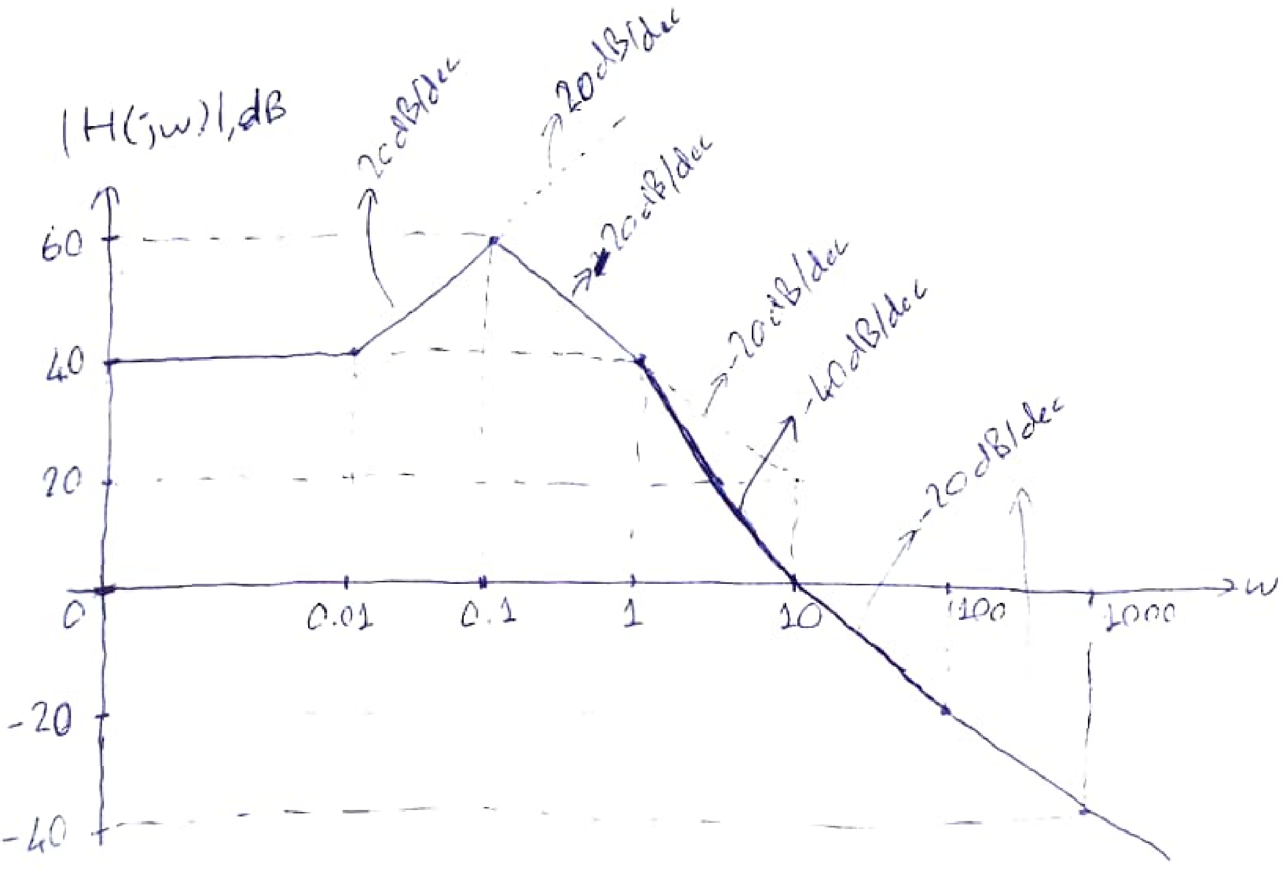


phase response

For $H(jw) = \dfrac{100 \cdot \left(1+\frac{jw}{0.01}\right)\left(1+\frac{jw}{10}\right)}{\left(1+\frac{jw}{0.1}\right)^2 \cdot \left(1+\frac{jw}{1}\right)}$, we have a gain of 100 (40 dB) at $w=0$, 2 zeros ($w=0.01$ & $w=10$), 1 single pole at $w=1$, 1 double pole at $w=0.1$. Each pole $\underset{\downarrow}{,} \, w_p$, brings a low-pass filter with form $\dfrac{1}{1+\frac{jw}{w_p}}$ whereas each zero, w_z, brings a high-pass filter with form $1+\dfrac{jw}{w_z}$.

In idealistic manner, each pole decreases the slope -20 dB/dec (if it is double pole, it decreases the slope -40 dB/dec and so on). On the other hand, each zero increases the slope +20 dB/dec. As we have a gain of 100, our magnitude response start with 40 dB. Then, we will consider all the poles and zeros.

$$H(jw) = 100 \cdot \left(1+\frac{jw}{0.01}\right) \cdot \frac{1}{\left(1+\frac{jw}{0.1}\right)} \cdot \frac{1}{\left(1+\frac{jw}{0.1}\right)} \cdot \frac{1}{\left(1+\frac{jw}{1}\right)} \cdot \left(1+\frac{jw}{10}\right)$$



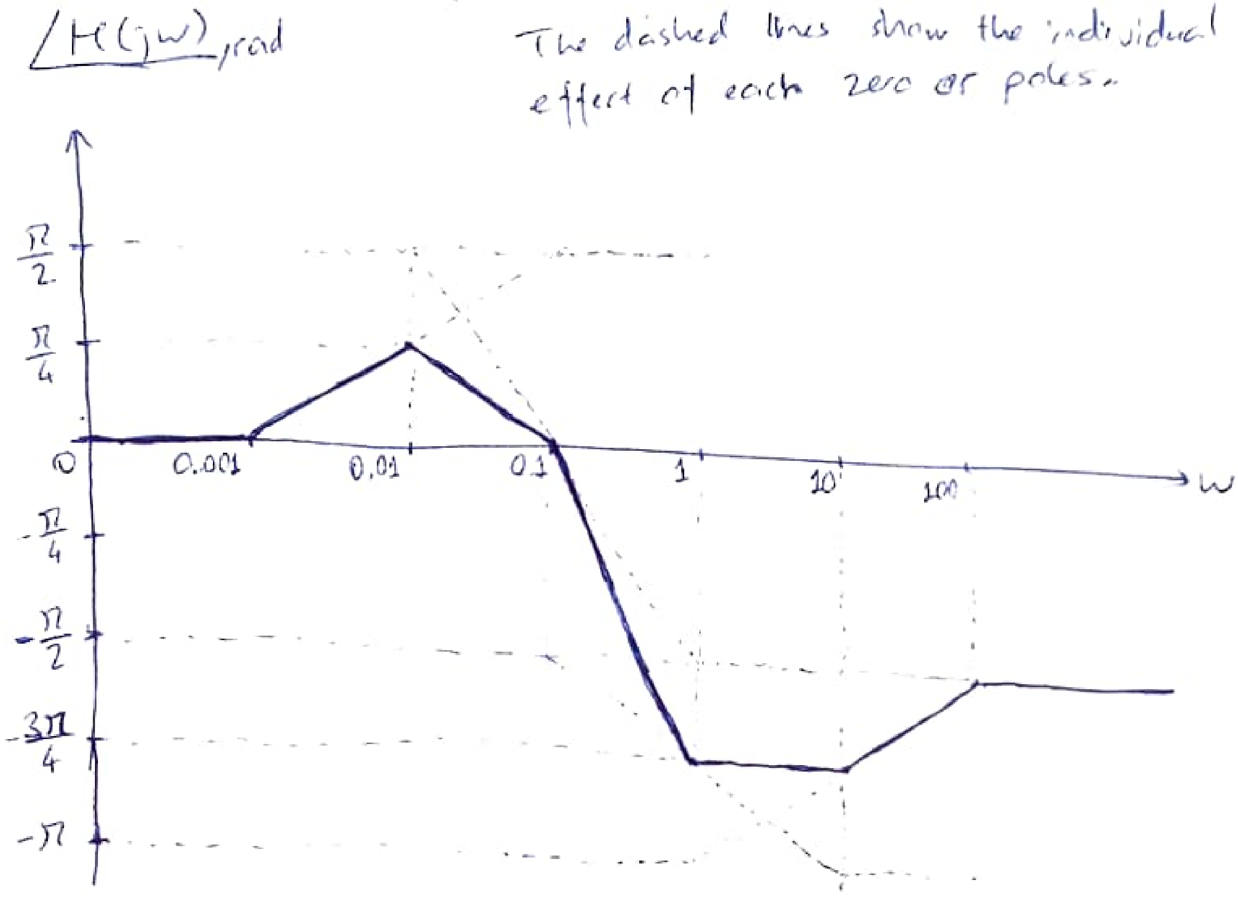
Magnitude response of this system

For phase response, each zero brings a phase increase of $\frac{\pi}{2}$ such that phase increase starts from $\frac{1}{10}$th of the zero and continues upto 10th times of the zero.

Similarly, each pole brings a phase decrease of $\frac{\pi}{2}$ such that phase decrease starts from $\frac{1}{10}$th of the pole and continues upto 10th times of the pole.

~~It has a positive~~ The system has a positive, real gain so the phase response starts with an angle of 0.

The dashed lines show the individual effect of each zero or poles.



Phase response of this system

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Note: Please note that this solution consider idealistic characteristics of the lowpass and highpass filters.
For example, for a lowpass filter, we consider a gain of 0dB at corner frequency w_c in ideal; however, it should be -3 dB in real.