Signal&Systems for Comp.Eng.

BLG 354E

Project 2 Report

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1. Solutions

1.1. Question 1

1.1.1. Signal 1: $x_1[n] = \sin(\pi n/4)$

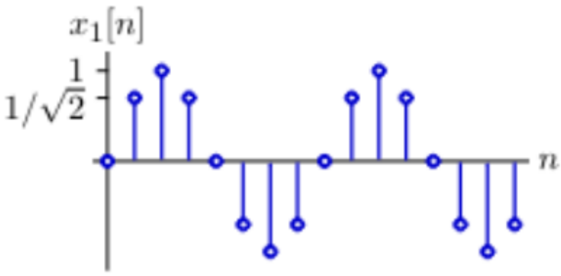

Figure 1.1: Graph of $x_1[n] = \sin(\pi n/4)$

Fourier Series Coefficients Calculation:

The Fourier series coefficients a_k for a periodic signal $x[n]$ with period N can be calculated using the formula:

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

For the given signal $x_1[n] = \sin(\pi n/4)$, we can express it using its exponential representation:

$$\sin\left(\frac{\pi n}{4}\right) = \frac{e^{j\frac{\pi n}{4}} - e^{-j\frac{\pi n}{4}}}{2j}$$

By plugging this expression into the formula, we find that the Fourier coefficients are:

$$a_k = \begin{cases} \frac{1}{j2} & k = 1 \\ -\frac{1}{j2} & k = -1 \\ 0 & \text{otherwise} \end{cases}$$

1.1.2. Signal 2

Fourier Series Coefficients Calculation:

For the signal $x_2[n]$, the Fourier series coefficients b_k are computed as:

$$b_k = \frac{1}{8} \sum_{n=0}^{7} x_2[n] e^{-j\frac{2\pi}{8}kn}$$

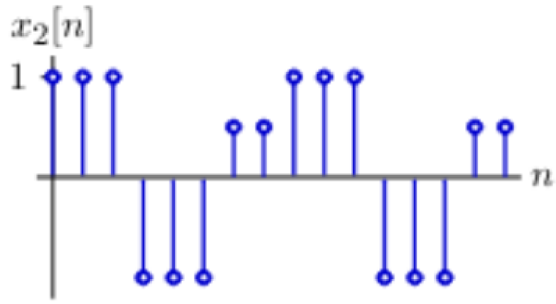

Figure 1.2: Graph of $x_2[n]$

Based on the given signal graph, we can find the value of b_k by substituting the n values. The reason for taking n from 0 to 7 is the periodic repetition of the signal.

$$b_k = \frac{1}{8}\left(1 + e^{-j\frac{k\pi}{4}} + e^{-j\frac{k\pi}{2}} - e^{-j\frac{3k\pi}{4}} - e^{-j\pi k} - e^{-j\frac{5k\pi}{4}} + \frac{1}{2}e^{-j\frac{3k\pi}{2}} + \frac{1}{2}e^{-j\frac{7k\pi}{4}}\right)$$

1.1.3. Signal 3

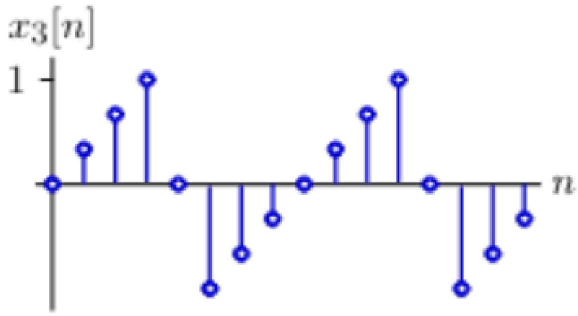

Figure 1.3: Graph of $x_3[n]$

Fourier Series Coefficients Calculation:
The Fourier series coefficients c_k for the signal $x_3[n]$ are given by:

$$c_k = \frac{1}{8}\sum_{n=0}^{7} x_3[n]e^{-j\frac{2\pi}{8}kn}$$

There will be no "cos" component in this signal. In addition, since signal is symmetrical about n axis, there will be no mean value. We can find the solution using the same method as for signal 2:

$$c_k = \frac{e^{-j\frac{k\pi}{4}} - e^{j\frac{k\pi}{4}}}{24} + \frac{e^{-j\frac{k\pi}{2}} - e^{j\frac{k\pi}{2}}}{12} + \frac{e^{-j\frac{3k\pi}{4}} - e^{j\frac{3k\pi}{4}}}{8} = -\frac{j}{12}\sin\frac{k\pi}{4} - \frac{j}{6}\sin\frac{k\pi}{2} - \frac{j}{4}\sin\frac{3k\pi}{4}$$

1.1.4. Signal 4: $x_4[n] = x_1[n-1]$

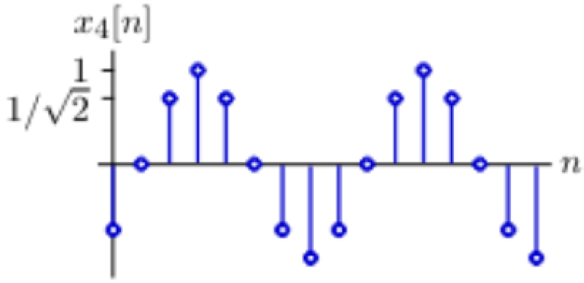

Figure 1.4: Graph of $x_4[n] = x_1[n-1]$

Fourier Series Coefficients Calculation:

For the signal $x_4[n] = x_1[n-1]$, the Fourier series coefficients d_k are derived by shifting the original signal $x_1[n]$:

$$d_k = a_k e^{-j\frac{2\pi}{8}1}$$

Using the previously calculated coefficients a_k of $x_1[n]$:

$$d_k = \begin{cases} \frac{1}{j2}e^{-j\frac{\pi}{4}} & k = 1 \\ -\frac{1}{j2}e^{-j\frac{\pi}{4}} & k = -1 \\ 0 & \text{otherwise} \end{cases}$$

This shift in the time domain translates to a phase shift in the frequency domain.

1.2. Question 2

1.2.1. Signal with Fourier coefficients For Part a: $a_k = \cos(\pi k/4)$

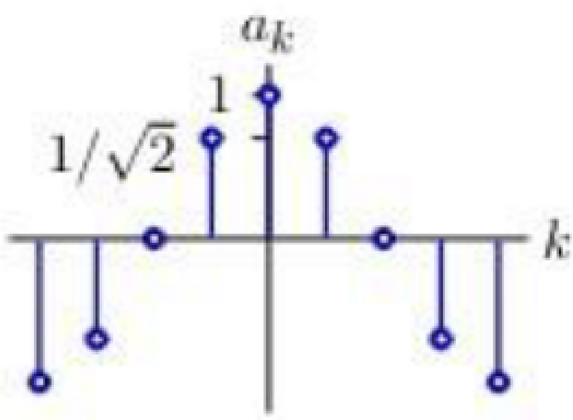

Figure 1.5: Fourier coefficients $a_k = \cos(\pi k/4)$

Finding the Signal $x_1[n]$ from a_k:

To determine the signal $x_1[n]$ from its Fourier series coefficients a_k, we use the inverse Fourier series formula:

$$x_1[n] = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$$

Given the coefficients $a_k = \cos(\pi k/4)$, we express it as:

$$a_k = \frac{e^{j\frac{\pi k}{4}} + e^{-j\frac{\pi k}{4}}}{2}$$

Now applying the inverse Fourier series:

$$x_1[n] = \frac{1}{8}\sum_{k=0}^{7}\left(\frac{e^{j\frac{\pi k}{4}} + e^{-j\frac{\pi k}{4}}}{2}\right)e^{j\frac{2\pi}{8}kn}$$

This simplifies to:

$$x_1[n] = \frac{1}{16}\sum_{k=0}^{7}\left(e^{j\left(\frac{\pi k}{4}+\frac{\pi kn}{4}\right)} + e^{-j\left(\frac{\pi k}{4}-\frac{\pi kn}{4}\right)}\right)$$

Evaluating this summation for $N = 8$:

$$x_1[n] = 4\delta[n-1] + 4\delta[n+1]$$

Thus, the resulting signal is:

$$x_1[n] = 4\delta[n-1+8k] + 4\delta[n+1+8k]$$

Due to the periodic nature of the signal, a +8k can be added.

1.2.2. Signal with Fourier coefficients For Part b

Finding the Signal

Since $b_{-k} \neq b_k^*$, we cannot calculate here as before. The left side correction and right side plane must be calculated separately. In the specified period, b_k takes values when $k = 1$ on the positive side and $k = -2$ on the negative side. If we substitute these values into the summation formula, we get the answer.

$$x_2[n] = \sum_{k=0}^{N-1} b_k e^{j\frac{2\pi}{N}kn}$$

For $N = 8$:

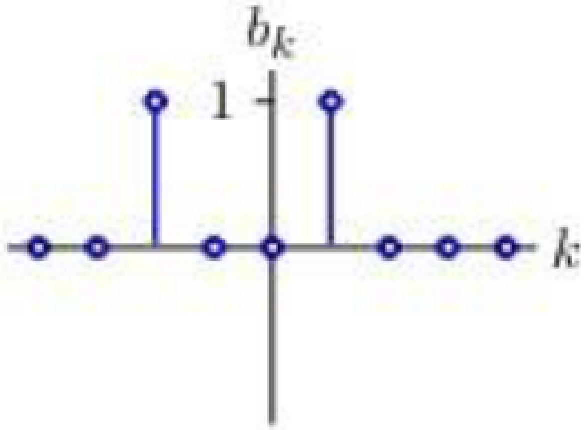

Figure 1.6: Fourier coefficients $b_k = e^{j2\pi k/8} + e^{-j2\pi k/8}$

$$b_k = \begin{cases} 1 & \text{for } k = 1 \\ 1 & \text{for } k = -2 \\ 0 & \text{otherwise} \end{cases}$$

We obtain the answer as follows by substituting the b_k values:

$$x_2[n] = e^{j\frac{2\pi}{8}n} + e^{-j\frac{2\pi}{8}2n}$$

1.3. Question 3

1.3.1. $\frac{d}{dt}x(t)$

The Fourier transform of the derivative of $x(t)$ is given by:

$$\mathcal{F}\left\{\frac{d}{dt}x(t)\right\} = j\omega X(j\omega)$$

This implies:

- The magnitude will be scaled by $|\omega|$.
- The phase will be shifted by $\frac{\pi}{2}$ for positive ω and $-\frac{\pi}{2}$ for negative ω.

Therefore:

- The magnitude plot $M5$ is a linear function of ω, which matches $|j\omega X(j\omega)|$ because the magnitude of $j\omega$ scales linearly with ω.
- The phase plot $A4$ shifts by $\frac{\pi}{2}$ at $\omega = 0$, consistent with the derivative's phase shift of $\frac{\pi}{2}$ for positive frequencies and $-\frac{\pi}{2}$ for negative frequencies.

1.3.2. $(x * x)(t)$

The Fourier transform of the convolution of $x(t)$ with itself is:

$$\mathcal{F}\left\{(x * x)(t)\right\} = X(j\omega) \cdot X(j\omega) = |X(j\omega)|^2 e^{j(2\angle X(j\omega))}$$

This means:

- The magnitude will be the square of the original magnitude.
- The phase will be doubled.

Therefore:

- The magnitude plot $M3$ represents $|X(j\omega)|^2$ since it shows the squared response, indicating a convolution in the time domain.
- The phase plot $A2$ is double the original phase, which is consistent with the convolution property where the phases add up.

1.3.3. $x\left(t - \frac{\pi}{2}\right)$

The Fourier transform of a time-shifted signal is:

$$\mathcal{F}\left\{x\left(t - \frac{\pi}{2}\right)\right\} = X(j\omega)e^{-j\omega\frac{\pi}{2}}$$

This implies:

- The magnitude remains unchanged.
- The phase is linearly shifted by $-\frac{\pi}{2}\omega$.

Therefore:

- The magnitude plot $M1$ is unchanged, consistent with time-shifting which does not affect the magnitude.
- The phase plot $A2$ shows a linear shift, indicating a time shift of $\frac{\pi}{2}$. This linear phase shift matches the property of a time shift in the time domain.

1.3.4. $x(2t)$

The Fourier transform of a time-scaled signal is:

$$\mathcal{F}\left\{x(2t)\right\} = \frac{1}{2}X\left(\frac{j\omega}{2}\right)$$

This implies:

- The magnitude is scaled and the frequency axis is compressed.

Therefore:

- The magnitude plot $M4$ shows a compression in frequency, consistent with time scaling by 2, where the frequency components are spread out by a factor of 2.
- The phase plot $A3$ corresponds to the compressed frequency axis, indicating a frequency scaling effect.

1.3.5. $x^2(t)$

The Fourier transform of a squared signal can be found using the convolution theorem. However, since $x(t)$ is real, the resulting Fourier transform will be:

$$\mathcal{F}\left\{x^2(t)\right\} = \int_{-\infty}^{\infty} X(j\omega_1)X(j(\omega - \omega_1))d\omega_1$$

The specific plots corresponding to this are:

- The magnitude plot $M6$ shows the convolution effect, consistent with the broadening and shifting of the frequency components due to squaring the signal.
- The phase plot $A1$ remains consistent with the real signal properties, where the phase is zero because $x^2(t)$ is an even function.

Matchings

By analyzing the Fourier transforms, I matched the given magnitude and phase plots to the derived signals. Here are the associations:

- $\frac{d}{dt}x(t)$: Magnitude - M5, Phase - A4
- $(x * x)(t)$: Magnitude - M3, Phase - A2
- $x\left(t - \frac{\pi}{2}\right)$: Magnitude - M1, Phase - A2
- $x(2t)$: Magnitude - M4, Phase - A3
- $x^2(t)$: Magnitude - M6, Phase - A1