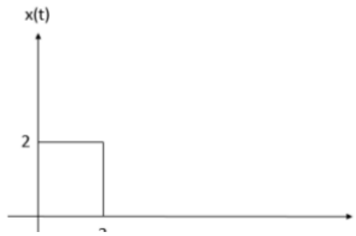
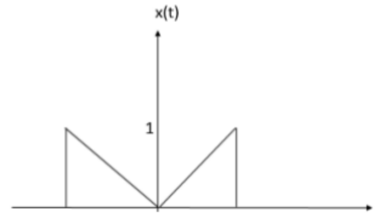
BLG354E – Final Exam - Example Questions (27.05.2024)

Q1) Find the Fourier transform for each of the following signals.

a-)



b-)



1-)

a-) $X(\omega) = \int_0^2 2 . e^{-j\omega t} dt = \frac{2}{-j\omega} e^{-j\omega t}\Big|_{t=0}^{2} = \frac{2}{j\omega}\left(e^{-2j\omega} - 1\right) = \frac{2-2e^{-2j\omega}}{j\omega} = \frac{2-2\cos 2\omega + 2j\sin 2\omega}{j\omega}$

$= 2 . \frac{\sin 2\omega}{\omega} + j . \frac{2\cos 2\omega - 2}{\omega}$

b-) $\int_0^T t/T . e^{-j\omega t} dt + \int_{-T}^{0} -t/T \, e^{-j\omega t} dt$ $\qquad u = t \quad dv = e^{-j\omega t} dt$

$du = dt \quad v = \frac{-1}{j\omega} . e^{-j\omega t}$

$= \frac{1}{T}\left[\int_0^T t . e^{-j\omega t} dt - \int_{-T}^{0} t . e^{-j\omega t} dt\right]$

$= \frac{1}{T}\left[t . \left(\frac{-1}{j\omega} e^{-j\omega t}\right)\Big|_{t=0}^{T} - \int_0^T \frac{-1}{j\omega} . e^{-j\omega t} dt - t\left(\frac{-1}{j\omega} . e^{-j\omega t}\right)\Big|_{t=-T}^{0} + \int_{-T}^{0} \frac{-1}{j\omega} e^{-j\omega t} dt\right]$

$= \frac{1}{T}\left[\frac{1}{j\omega} . \frac{-1}{j\omega} . e^{-j\omega t}\Big|_0^T + \left(\frac{-1}{j\omega}\right) . \left(\frac{-1}{j\omega}\right) . e^{-j\omega t}\Big|_{t=-T}^{0}\right]$

$- \frac{1}{T}\left[\frac{-1}{j^2\omega^2}\left(e^{-j\omega T} - 1\right) + \frac{1}{j^2\omega^2}\left(1 - e^{j\omega T}\right)\right]$

$= \frac{1}{T}\left[\frac{e^{-j\omega T} + 2 + e^{j\omega T}}{\omega^2}\right] = \frac{1}{T} . \left(\frac{2\cos\omega T - 2}{\omega^2}\right)$

Q2) A continuous-time signal x(t) is defined as $x(t) = 1 + 2\sin(12\pi t) + 4\cos(18\pi t)$. What should be the sampling frequency f_s to discretize x(t) with 4 samples in one period?

$x(t) = 1 + 2\sin(12\pi t) + 4\cos(18\pi t)$

Calculate frequencies: $1 \to$ constant, $\frac{12\pi}{2\pi} = 6$, $\frac{18\pi}{2\pi} = 9$

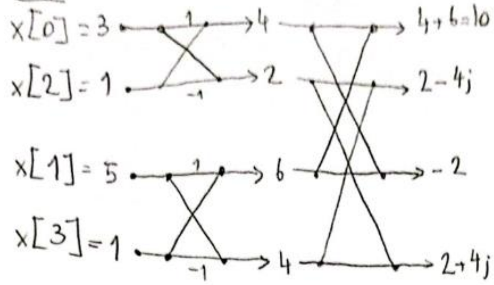
Largest common divisor of 6 and 9: $LCD(6,9) = 3$

$f_t = 3Hz.$

To have 4 samples in each period $\longrightarrow f_s = 12\ Hz$

Q3) Using Decimation-In-Time and Decimation-In-Frequency Radix-2 algorithms find the DFT of the following signal : [3,5,1,1].

DIT

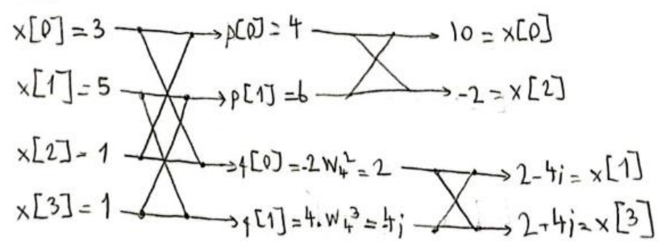


$W_4^0 = e^{-j(2\pi/N)0} = e^{-j\left(\frac{2\pi}{4}\right)\cdot 0} = 1$

$W_4^1 = e^{-j(2\pi/4)} = e^{-\frac{j\pi}{2}} = \frac{1}{e^{+\frac{j\pi}{2}}} = \frac{1}{\sqrt{-1}} = -j$

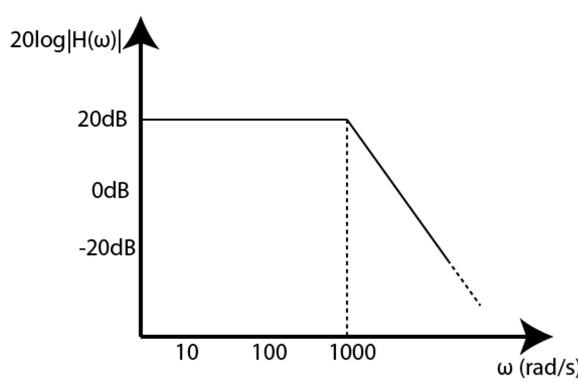
$$X = \begin{bmatrix} 10 \\ 2-4j \\ -2 \\ 2+4j \end{bmatrix}$$

DIF



(Please also take a look at the examples from the following link: https://www.rcet.org.in/uploads/academics/rohini_17557159296.pdf)

Q4) A Bode diagram for a second-order lowpass filter is given below.



a) Find the transfer function of the LPF in s-domain.
b) Find the DT implementation of the filter with a sampling frequency of 5kHz

a) $H(s) = \dfrac{K}{(s+1000)^2}$

$H(0) = \dfrac{K}{1000^2} = 20\,dB = 10 \Rightarrow K = 10^7$

$H(s) = \dfrac{10^7}{(s+1000)^2}$

b) $f_s = 5kHz \Rightarrow 2.10^{-4}$ sampling period T $\quad s = \dfrac{\cancel{2}}{\cancel{2}.10^{-4}} \cdot \dfrac{1-z^{-1}}{1+z^{-1}}$

$$H(z) = \frac{10^7}{\left(\frac{1-z^{-1}}{1+z^{-1}} \cdot 10^4 + 1000\right)^2} = \frac{10^7 . (1+z^{-1})^2}{(10^4 + 1000 - 10^4 . z^{-1} + 1000 . z^{-1})}$$

$$= \frac{10^7 . (1+z^{-1})^2}{(11000 - 9000 z^{-1})^2}$$

$$= \frac{10^7 + 2.10^7 . z^{-1} + 10^7 . z^{-2}}{121.10^6 - 198.10^6 + 81.10^6 . z^{-2}} = \frac{10 + 20 z^{-1} + 10 z^{-2}}{121 - 198 z^{-1} + 81 z^{-2}} = \frac{\frac{10}{121} + \frac{20}{121} z^{-1} + \frac{10}{121} . z^{-2}}{1 - \frac{198}{121} z^{-1} + \frac{81}{121} z^{-2}}$$

To make $a_0 = 1$

Q5) Find the z transforms of the following signals. Also define the ROCs.

a) $x[k] = a^{k-1} u[k-1]$

b) $x[k] = \cos(\Omega_0 k)\, u[k]$

a) $$\sum_{k=-\infty}^{\infty} a^{k-1} . u[k-1] . z^{-k} = \sum_{k=1}^{\infty} \left(\frac{a}{z}\right)^k . \frac{1}{a} = \frac{1}{a} \sum_{k=1}^{\infty} \left(\frac{a}{z}\right)^k = \frac{1}{a}\left(\sum_{k=0}^{\infty} \left(\frac{a}{z}\right)^k - 1\right)$$

$$= \frac{1}{a}\left(\frac{1}{1-\frac{a}{z}} - 1\right) = \frac{1}{a}\left(\frac{z}{z-a} - 1\right) = \frac{1}{z-a}$$

ROC: $|z| > |a|$

b) $$\sum_{k=-\infty}^{\infty} \frac{1}{2} . \left(e^{j\Omega_0 k} + e^{-j\Omega_0 k}\right) . u[k] . z^{-k} = \frac{1}{2} \sum_{k=0}^{\infty} \left(\frac{e^{j\Omega_0}}{z}\right)^k + \frac{1}{2} \sum_{k=0}^{\infty} \left(\frac{e^{-j\Omega_0}}{z}\right)^k$$

$$= \frac{1}{2} \cdot \frac{1}{1 - \frac{e^{j\Omega_0}}{z}} + \frac{1}{2} \cdot \frac{1}{1 - \frac{e^{-j\Omega_0}}{z}} = \frac{1}{2} \cdot \left(\frac{z}{z - e^{j\Omega_0}} + \frac{z}{z - e^{-j\Omega_0}}\right)$$

$$= \frac{1}{2} \cdot \frac{z^2 - z . e^{-j\Omega_0} + z^2 - z . e^{j\Omega_0}}{z^2 - z\, e^{j\Omega_0} - z\, e^{-j\Omega_0} + 1} = \frac{z[z - \cos\Omega_0]}{z^2 - 2z\cos\Omega_0 + 1}$$

ROC: $|z| > 1$

Q6)) Determine the poles and zeros of LTID system which is defined by $H(z) = \frac{z}{z^2 - 3z + 2}$. Also find the inverse z transform of the system. Assume that the system is right sided.

$$X(z) = \frac{z}{z^2-3z+2}$$

$$\frac{X(z)}{z} = \frac{1}{z^2-3z+2} = \frac{k_1}{z-1} + \frac{k_2}{z-2}$$

$$k_1 = \left[(z-1)\cdot\frac{1}{(z-1)\cdot(z-2)}\right]_{z=1} = -1 \qquad k_2 = \left[(z-2)\cdot\frac{1}{(z-1)\cdot(z-2)}\right]_{z=2} = 1$$

$$X(z) = \underbrace{\frac{-z}{(z-1)}}_{\to ROC: |z|>1} + \underbrace{\frac{z}{(z-2)}}_{\to ROC: |z|>2} = \frac{-1}{(1-z^{-1})} + \frac{1}{(1-2z^{-1})}$$

Poles: 1, 2

Zeros: 0