8 March 2023

BLG311E – FORMAL LANGUAGES AND AUTOMATA
2022-2023 Spring
Homework 1
Araş. Gör. Elif Ak

1) Consider the following requirement set for a dumb turnstile machine:

- Turnstile machine is "locked" by default
- User can push one of the following coins at once to the machine: 25, 50, 100
- Machine checks the coins presented to the machine so far and
  - either waits for additional coins if the amount of credits is not enough,
  - or unlocks to let **the maximum amount of people possible** to pass one by one.
  - Once total credits exceed the amount necessary to let one-person pass, machine lets the maximum amount of people pass, returns change and goes back to its initial state
- The amount of credits required for a single person to pass is 30
- Machine will not accept any coins while it is unlocked.

Examples:

- If the user initially inputs 25, machine will stay locked and wait for additional credits
- If the user initially inputs 100, machine will let 3 people pass, return 10 with the last person (warning: it's not possible to pay 100 and let a single person pass in this machine)

Define and explain each element in the following sets and draw the diagram for this machine using Mealy Model.

Input alphabet of your design	Output alphabet of your design
State machine diagram	

2) Let $\sum$ be any alphabet and let $L_1 \subseteq \sum^*$ and $L_2 \subseteq \sum^*$

a) Let $\lambda \notin L_1$. Explain why $L_1 \sum^* \neq \sum^*$
b) Let $\lambda \in L_1$ and $\lambda \in L_2$.
Show using axioms and theorems of languages $(L_1 \sum^* L_2)^* = \sum^*$

3) Consider the following languages A, B and C defined over the alphabet $\sum = a, b$.

- $A = abb^+ba$
- $B = a(bb)^+ba$
- $C = a(bbb)^*a$

Answer each of the following questions considering the definition above.

a) Give an example string that is accepted by the all three languages.
b) Give an example string that is accepted by only A.
c) Give an example string that is accepted by only A and B.
d) Give an example string that is accepted by only A and C.
e) Give an example string that is accepted by only C.
f) Indicate if there is a subset/superset relation between any pair of the three languages.

4) Consider the inductive definition of the reverse operation on a string.
$|w| = 0 \Rightarrow w^R = w = \lambda$
$|w| = n + 1 \wedge n \in \mathbb{N} \Rightarrow |u| = n \wedge\ a \in \Sigma \wedge w = ua \Rightarrow w^R = au^R$
Using the definition above, show that $\left(w^i\right)^R = (w^R)^i$ where i is a natural number.

5) Let A and B denote different languages over the alphabet Σ. Prove that $A \subseteq B \Rightarrow A^* \subseteq B^*$.

6) Consider the following grammar,

$$
\begin{aligned}
n_0 &= A \mid n_0 A \\
A &= X \mid Y \mid Z \\
X &= ab \\
Y &= aab \\
Z &= abb
\end{aligned}
$$

a) Which type does this grammar correspond to in Chomsky hierarchy? Why?
b) Give an equivalent grammar which correspond to a more restrictive Chomsky type.
c) Give a regular expression that can be used to represent this language.

7) Find out the language produced by the following grammar G.

S → 1A

A → 1A | B

B → 0B | λ

8) Design context-free grammars for the following languages:

a) The $\{0^n 1^n \mid n \geq 1\}$, that is, the set of all strings of one or more 0's followed by an equal number of 1's.

b) The $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

9) Consider the following grammar

$$
\begin{aligned}
&< S >::= \ a < A > \mid b < B > \mid a \mid b \\
&< A >::= \ b < S > \\
&< B >::= a < S > \mid \ b < S >
\end{aligned}
$$

a) Write a Type-3 grammar that contains a single non-terminal to define the language that this grammar defines.

b) Provide the shortest string that may never occur as a sub-string in the words that these grammars produce.

10) Consider the grammar

$$S \rightarrow aS \mid aSbS \mid \lambda$$

This grammar is ambiguous. Show in particular that the string aab has two:

a) Parse trees

b) Leftmost derivations

c) Rightmost derivations

d) Find an unambiguous grammar for the language.