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MAT 281E – Linear Algebra and Applications

Final

1. (25p) Using the Gram-Schmidt method, find an **orthonormal** basis for the column space of the given matrice.

$$\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

2. (10p) Solve the following linear equations system by Cramer's rule.

$$\begin{aligned} 3x + 4y &= 9 \\ 2x - y &= -1 \end{aligned}$$

3. (20p) The linear equations system given below has no solution. Find the vector $\boldsymbol{a}^T = [x_0 \;\; y_0]$ that best approximates a solution.

$$\begin{aligned} 4x &= 2 \\ 2y &= 0 \\ x + y &= 11 \end{aligned}$$

4. (20p) Find an orthogonal matrice $\boldsymbol{P}$ such that $\boldsymbol{P^{-1}AP}$ is diagonal. Find $\boldsymbol{A^k}$.

$$\boldsymbol{A} = \begin{bmatrix} 3 & 0 & 7 \\ 0 & 5 & 0 \\ 7 & 0 & 3 \end{bmatrix}$$

5. (25p) Consider a system $\boldsymbol{x'(t)} = \boldsymbol{Ax(t)}$ where $\boldsymbol{x'(t)} = \frac{dx(t)}{dt}$, and $\boldsymbol{x}^T\boldsymbol{(t)} = [x_1(t) \quad x_2(t)]$ such that

$$\begin{aligned} x_1'(t) &= 2x_1(t) + x_2(t) \\ x_2'(t) &= 4x_1(t) + 5x_2(t) \end{aligned}$$

Find a solution satisfying the conditions $x_1(0) = 1$, $x_2(0) = 0$ to this system.

Explain all of your answers.

Exam Duration: ***110 minutes + 10 minutes*** *for scanning and uploading your answers.*