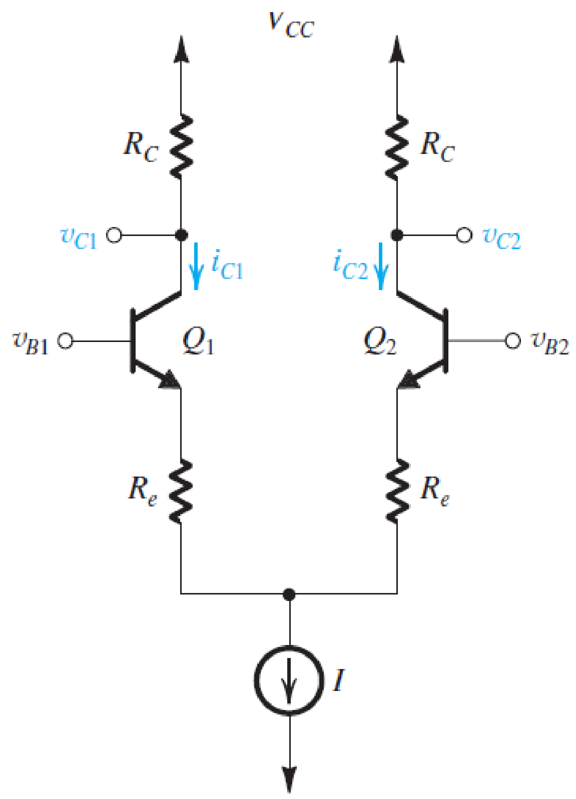
MOSFET Differential Amplifiers



Common-Mode and Differential-Mode Signals & Gain

Differential and Common-Mode Signals/Gain

Consider a <u>linear</u> circuit with TWO inputs

By superposition:

$$\boxed{v_o = A_1 \cdot v_1 + A_2 \cdot v_2}$$

Define:

$$\boxed{v_d = v_2 - v_1}$$

Difference (or differential) Mode

$$\boxed{v_c = \frac{v_1 + v_2}{2}}$$

Common Mode

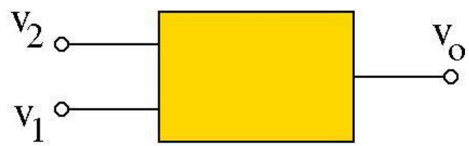
$$\boxed{\begin{aligned} v_1 &= v_c - \frac{v_d}{2} \\ v_2 &= v_c + \frac{v_d}{2} \end{aligned}}$$

Substituting for $v_1 = v_c - \dfrac{v_d}{2}$ and $v_2 = v_c + \dfrac{v_d}{2}$ in the expression for v_o:

$$v_o = A_1 \cdot \left(v_c - \frac{v_d}{2} \right) + A_2 \cdot \left(v_c + \frac{v_d}{2} \right) = (A_1 + A_2) \cdot v_c + \left(\frac{A_2 - A_1}{2} \right) \cdot v_d$$

$$\boxed{v_o = A_c \cdot v_c + A_d \cdot v_d}$$

Differential and common-mode signal/gain is an alternative way of finding the system response



$$v_o = A_1 \cdot v_1 + A_2 \cdot v_2$$

$$v_d = v_2 - v_1 \qquad A_c = A_1 + A_2$$

$$v_c = \frac{v_1 + v_2}{2} \qquad A_d = \frac{A_2 - A_1}{2}$$

$$v_o = A_c \cdot v_c + A_d \cdot v_d$$

$$v_1 = v_c - \frac{v_d}{2} \qquad A_1 = \frac{A_c}{2} - A_d$$

$$v_2 = v_c + \frac{v_d}{2} \qquad A_2 = \frac{A_c}{2} + A_d$$

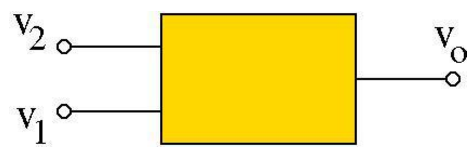
Differential Gain: A_d

Common Mode Gain: A_c

Common Mode Rejection Ratio (CMRR)*: $|A_d| / |A_c|$

* CMRR is usually given in dB: CMRR(dB) = 20 log $(|A_d| / |A_c|)$

To find v_o, we can calculate/measure either A_1 A_2 pair or A_c A_d pair



Superposition (finding A_1 and A_2):

1. Set $v_2 = 0$, compute A_1 from $v_o = A_1 v_1$
2. Set $v_1 = 0$, compute A_2 from $v_o = A_2 v_2$
3. For any v_1 and v_2: $v_o = A_1 v_1 + A_2 v_2$

Difference Method (finding A_d and A_c):

1. Set $v_c = 0$ (or set $v_1 = -0.5\, v_d$ & $v_2 = +0.5\, v_d$) compute A_d from $v_o = A_d v_d$
2. Set $v_d = 0$ (or set $v_1 = +v_c$ & $v_2 = +v_c$) compute A_c from $v_o = A_c v_c$
3. For any v_1 and v_2: $v_o = A_d v_d + A_c v_c$

   $v_d = v_2 - v_1 \quad v_c = 0.5(v_1 + v_2)$

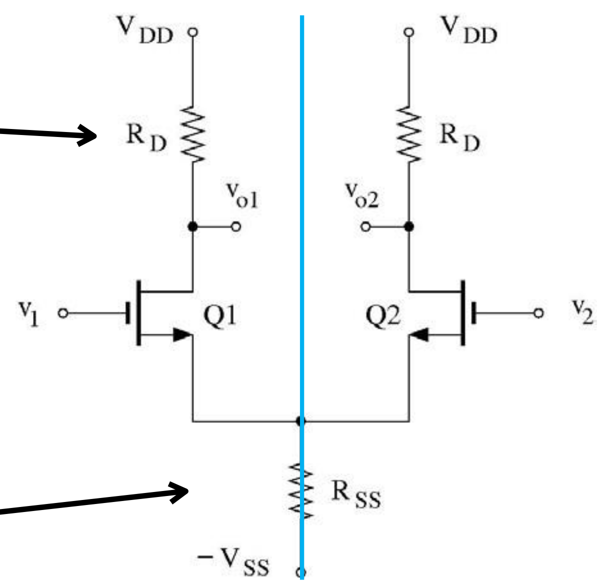
- **Both methods give the same answer for v_o (or A_v).**
- **The choice of the method is driven by application:**
  - **Easier solution**
  - **More relevant parameters**

MOSFET Differential Amplifiers:

Differential Amplifier

- Identical transistors.
- Circuit elements are symmetric about the mid-plane.
- Identical bias voltages at Q1 & Q2 gates ($V_{G1} = V_{G2}$).
- Signal voltages & currents are different because $v_1 \neq v_2$.

Load R_D: resistor, current-mirror, active load, …



R_{SS}: Bias resistor, current source (current-mirror)

Q1 & Q2 are in CS-like configuration (input at the gate, output at the drain) but with sources connected to each other.

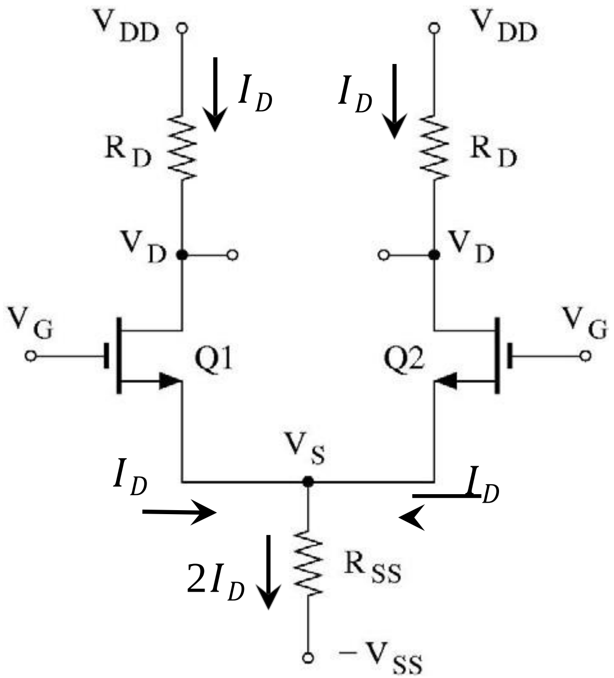
- For now, we keep track of "two" output, v_{o1} and v_{o2} , because there are several ways to configure "one" output from this circuit.

Differential Amplifier – DC Bias

Since $V_{G1} = V_{G2} = V_G$

and $V_{S1} = V_{S2} = V_S$

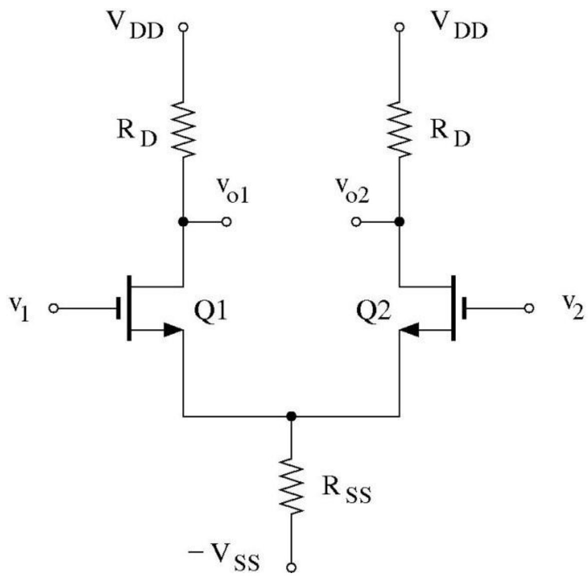
$$\boxed{\begin{aligned} V_{GS1} &= V_{GS2} = V_{GS} \\ V_{D1} &= V_{D2} = V_D \\ I_{D1} &= I_{D2} = I_D \\ V_{DS1} &= V_{DS2} = V_{DS} \end{aligned}}$$

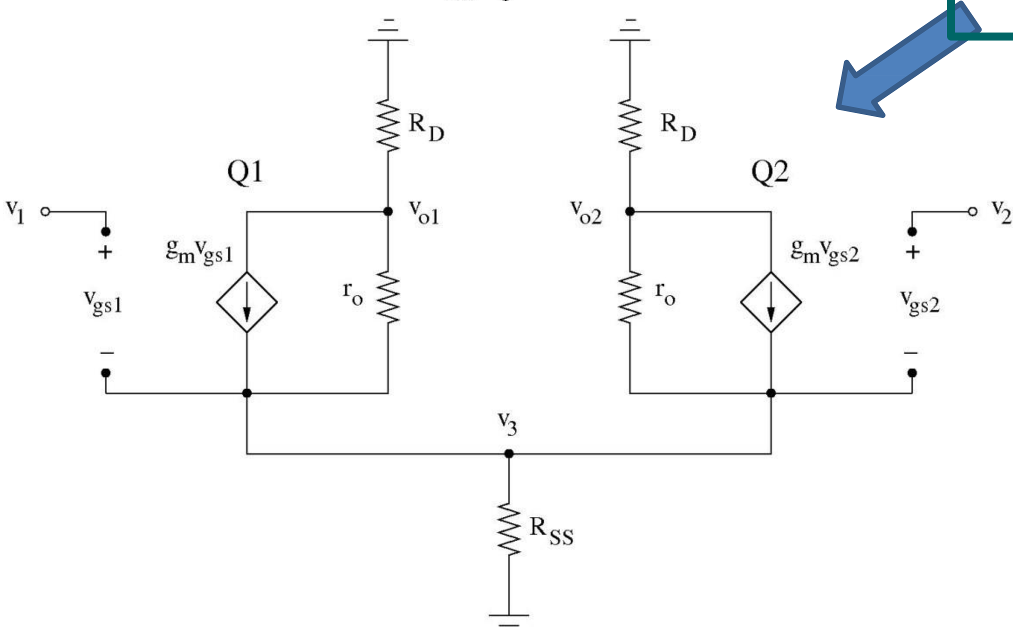


Also:

$$\boxed{\begin{aligned} g_{m1} &= g_{m2} = g_m \\ r_{o1} &= r_{o2} = r_o \end{aligned}}$$

Differential Amplifier – Gain

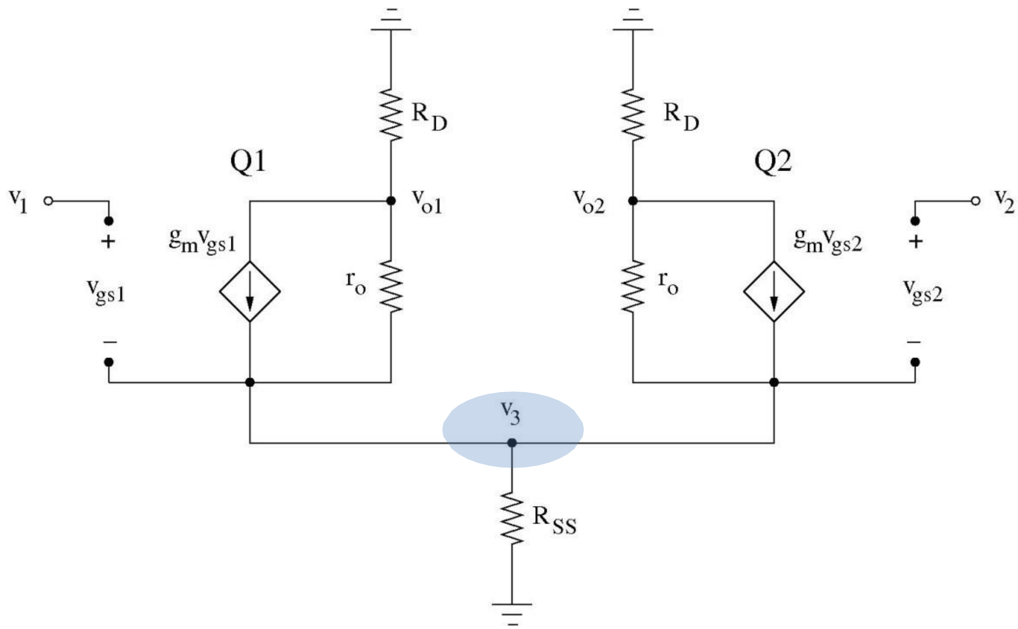




- Signal voltages & currents are different because $v_1 \neq v_2$
- We cannot use fundamental amplifier configuration for arbitrary values of v_1 and v_2.
- We have to replace each NMOS with its small-signal model.

Differential Amplifier – Gain

$$\boxed{\begin{aligned} v_{gs1} &= v_1 - v_3 \\ v_{gs2} &= v_2 - v_3 \end{aligned}}$$



Node Voltage Method:

Node v_{o1}: $$\frac{v_{o1}}{R_D} + \frac{v_{o1} - v_3}{r_o} + g_m(v_1 - v_3) = 0$$

Node v_{o2}: $$\frac{v_{o2}}{R_D} + \frac{v_{o2} - v_3}{r_o} + g_m(v_2 - v_3) = 0$$

Node v_3: $$\frac{v_3}{R_{SS}} + \frac{v_3 - v_{o2}}{r_o} + \frac{v_3 - v_{o1}}{r_o} - g_m(v_1 - v_3) - g_m(v_2 - v_3) = 0$$

Above three equations should be solved to find v_{o1} , v_{o2} and v_3 (lengthy calculations)

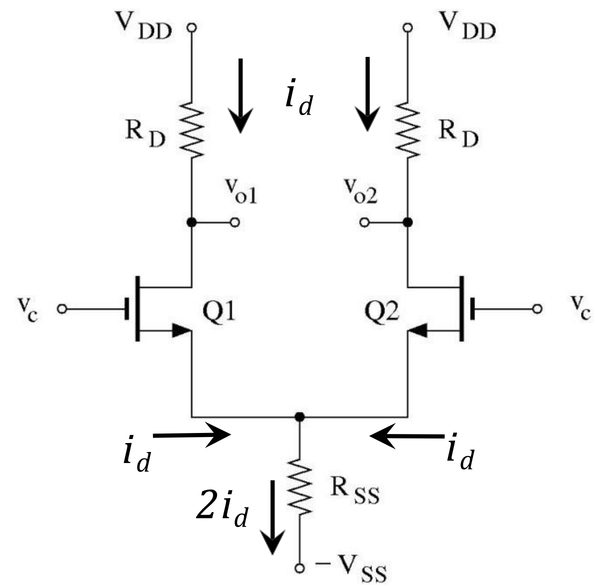
- Because the circuit is symmetric, **differential/common-mode method** is the preferred method to solve this circuit (and we can use fundamental configuration formulas).

Differential Amplifier – Common Mode (1)

Common Mode: Set $v_d = 0$ (or set $v_1 = +v_c$ and $v_2 = +v_c$)

$$v_1 = v_c - \frac{v_d}{2}$$

$$v_2 = v_c + \frac{v_d}{2}$$



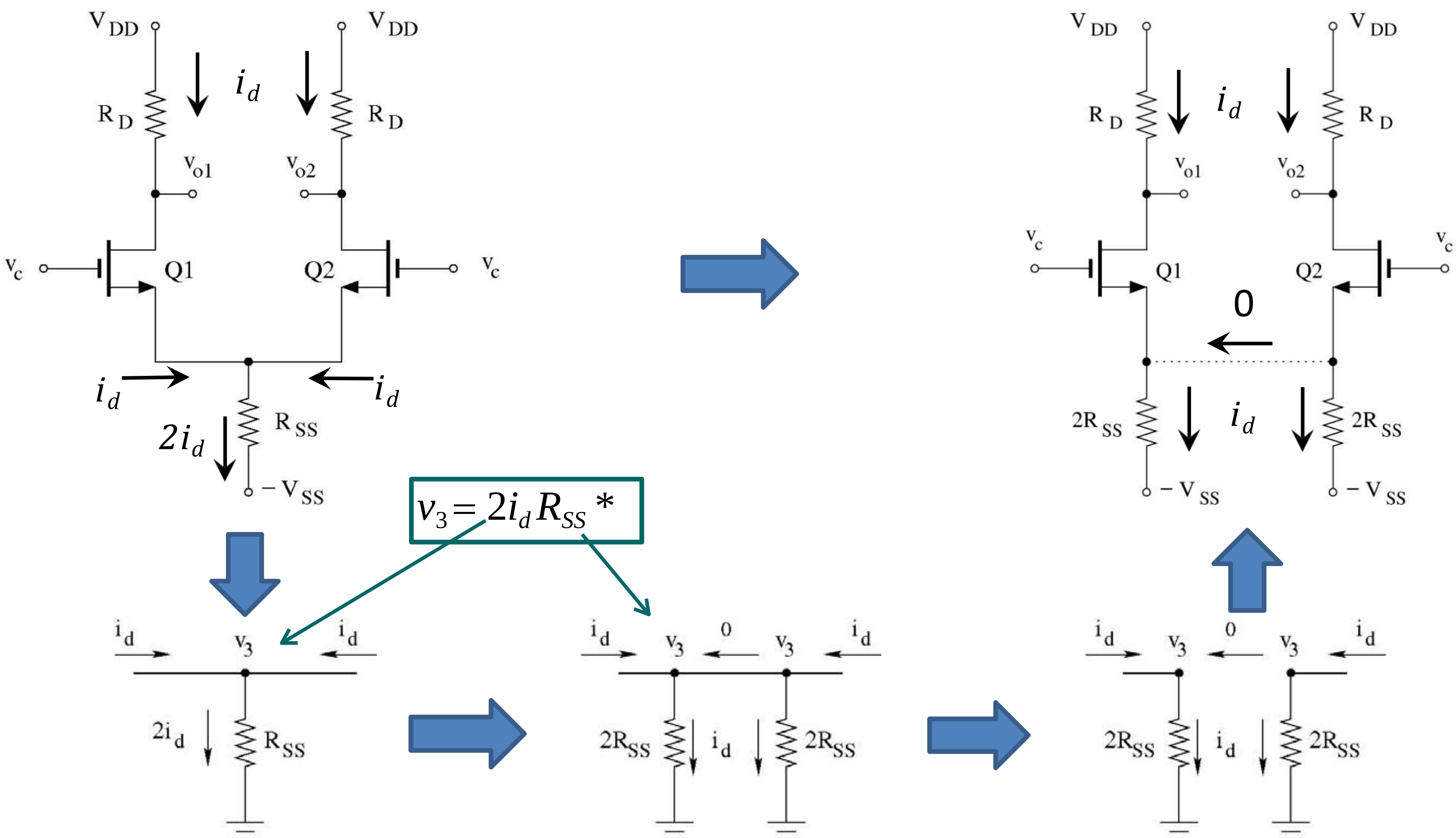
Because of summery of
the circuit and input signals*:

$$v_{o1} = v_{o2} \quad \text{and} \quad i_{d1} = i_{d2} = i_d$$

We can solve for v_{o1} by node voltage method but there is a simpler and more elegant way.

* If you do not see this, set $v_1 = v_2 = v_c$ in node equations of the previous slide, subtract the first two equations to get $v_{o1} = v_{o2}$. Ohm's law on R_D then gives $i_{d1} = i_{d2} = i_d$

Differential Amplifier – Common Mode (2)

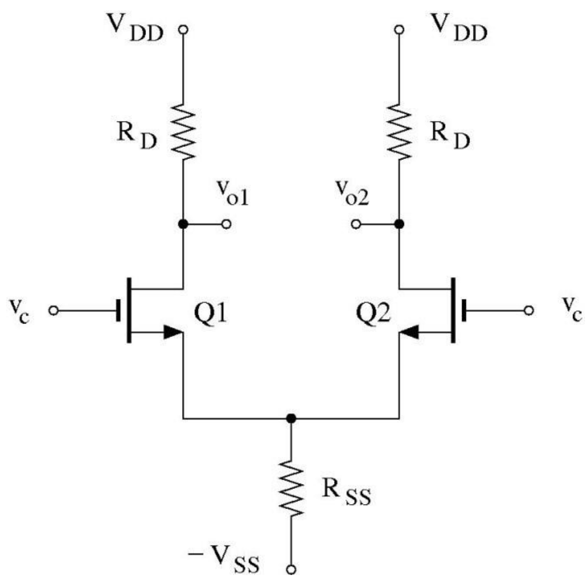


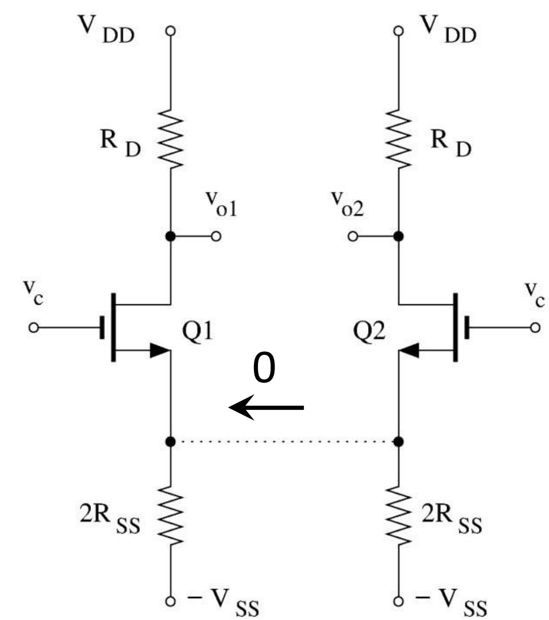
- **Because of the symmetry, the common-mode circuit breaks into two identical “half-circuits”.**

* V_{ss} is grounded for signal

Differential Amplifier – Common Mode (3)

➢ **The common-mode circuit breaks into two identical half-circuits.**





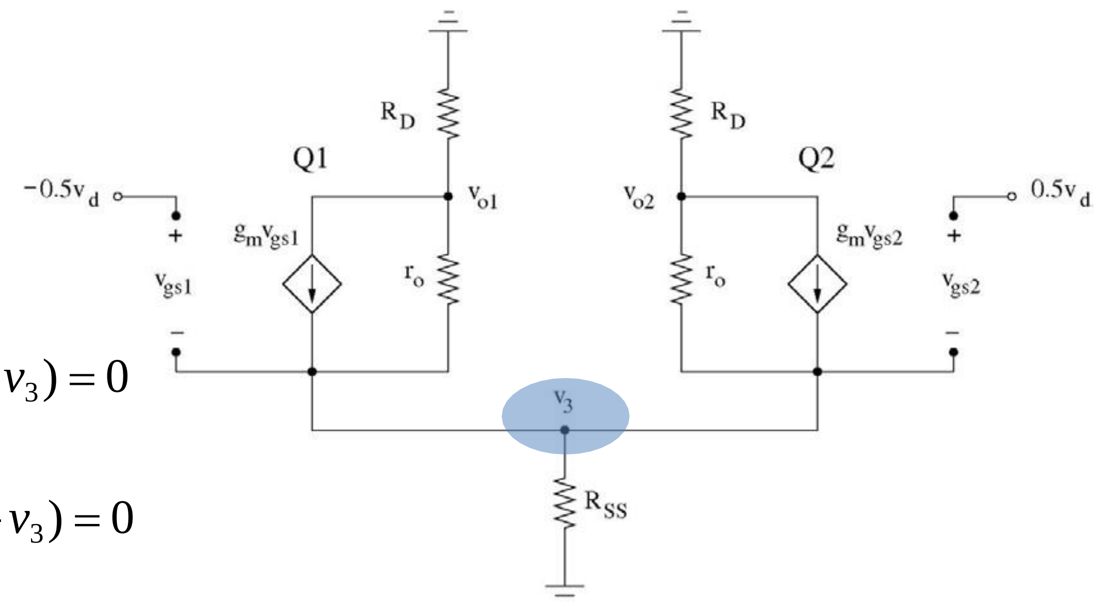
$$\frac{v_{o1}}{v_c} = \frac{v_{o2}}{v_c} = -\frac{g_m R_D}{1 + 2g_m R_{SS} + R_D / r_o}$$

Differential Amplifier – Differential Mode (1)

Differential Mode: Set $v_c = 0$ (or set $v_1 = -v_d/2$ and $v_2 = +v_d/2$)

$$v_{gs1} = -0.5v_d - v_3$$

$$v_{gs2} = +0.5v_d - v_3$$



Node Voltage Method:

Node v_{o1}: $\dfrac{v_{o1}}{R_D} + \dfrac{v_{o1} - v_3}{r_o} + g_m(-0.5v_d - v_3) = 0$

Node v_{o2}: $\dfrac{v_{o2}}{R_D} + \dfrac{v_{o2} - v_3}{r_o} + g_m(+0.5v_d - v_3) = 0$

Node v_3: $\dfrac{v_3}{R_{SS}} + \dfrac{v_3 - v_{o2}}{r_o} + \dfrac{v_3 - v_{o1}}{r_o} - g_m(-0.5v_d - v_3) - g_m(+0.5v_d - v_3) = 0$

Node v_{o1} + Node v_{o2} : $\left(\dfrac{1}{R_D} + \dfrac{1}{r_o}\right)(v_{o1} + v_{o2}) - \left(\dfrac{2}{r_o} + 2g_m\right)v_3 = 0$

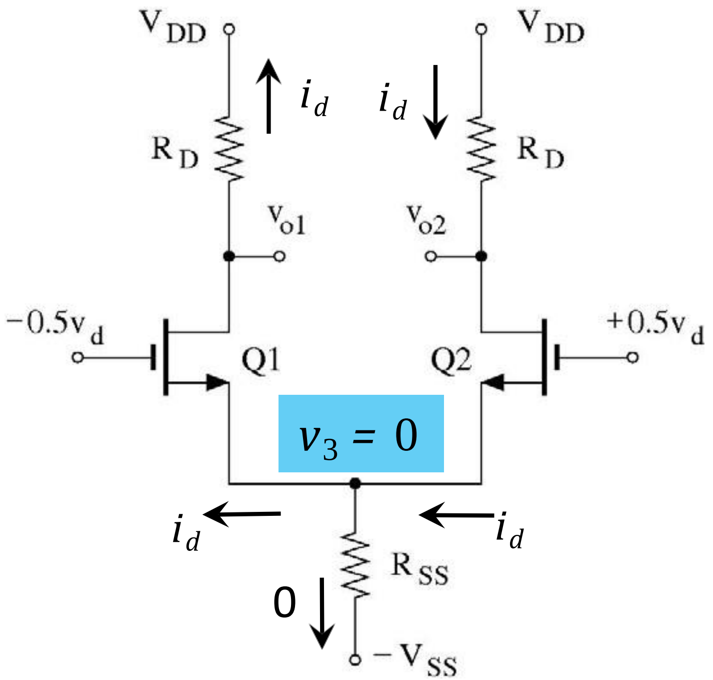
Node v_3: $-\dfrac{1}{r_o}(v_{o1} + v_{o2}) + \left(\dfrac{1}{R_{SS}} + \dfrac{2}{r_o} - 2g_m\right)v_3 = 0$

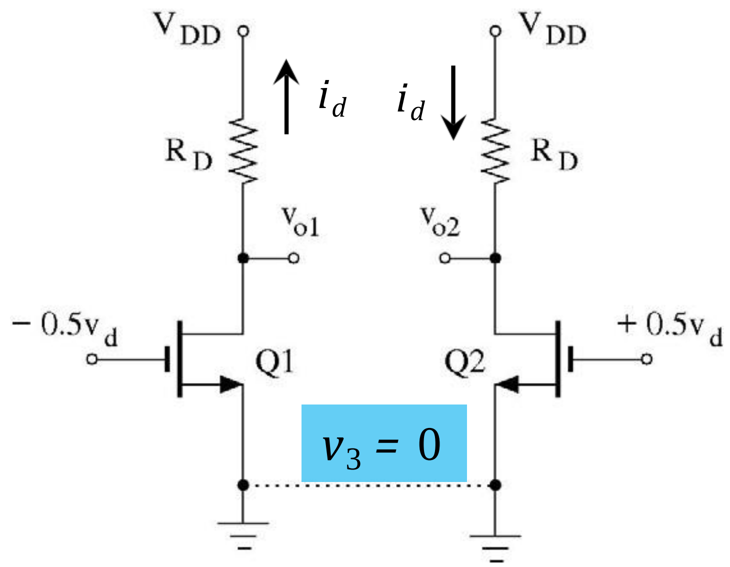
Only possible solution:

$$\boxed{\begin{array}{l} v_{o1} + v_{o2} = 0 \quad \Rightarrow \quad v_{o1} = -v_{o2} \\ v_3 = 0 \end{array}}$$

Differential Amplifier – Differential Mode (2)

$$\boxed{v_3 = 0 \quad \text{and} \quad v_{o1} = -v_{o2} \;\Rightarrow\; i_{d1} = -i_{d2}}$$





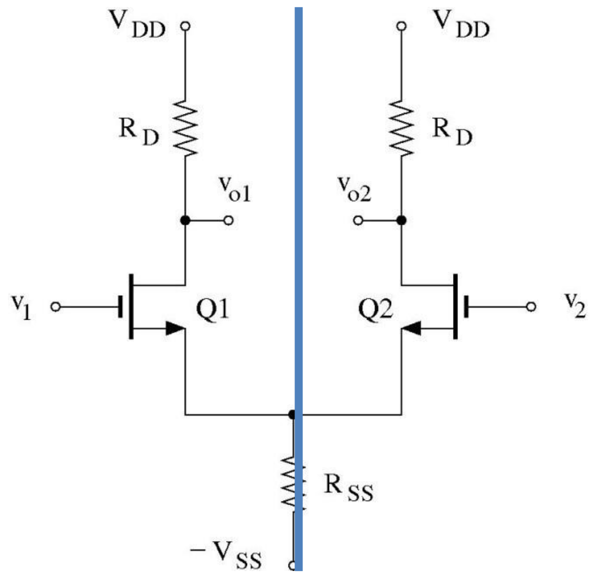
CS Amplifier

$$\frac{v_{o1}}{-0.5v_d} = -g_m\,(r_o \| R_D)\,, \quad \frac{v_{o2}}{+0.5v_d} = -g_m\,(r_o \| R_D)$$

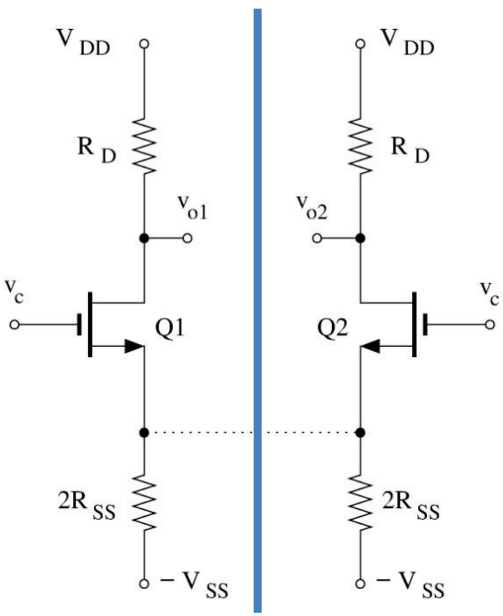
- Because of the symmetry, the differential-mode circuit also breaks into two identical half-circuits.

Concept of "Half Circuit"

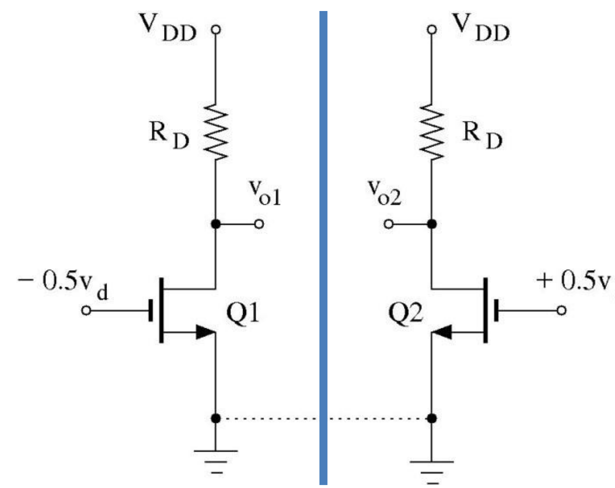
- For a symmetric circuit, differential- and common-mode analysis can be performed using "half-circuits."



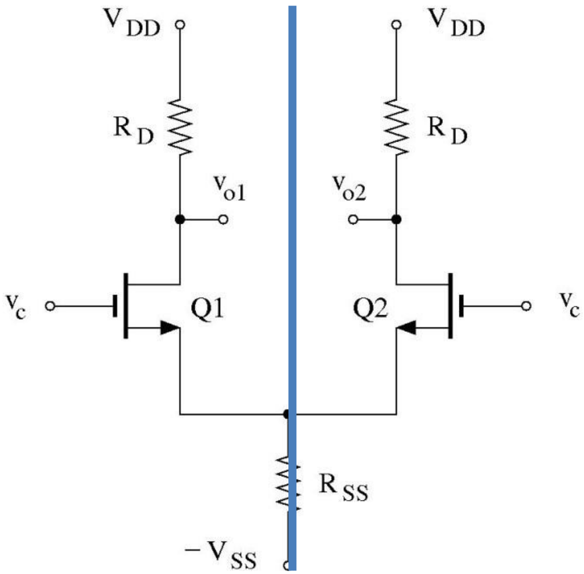
Common Mode



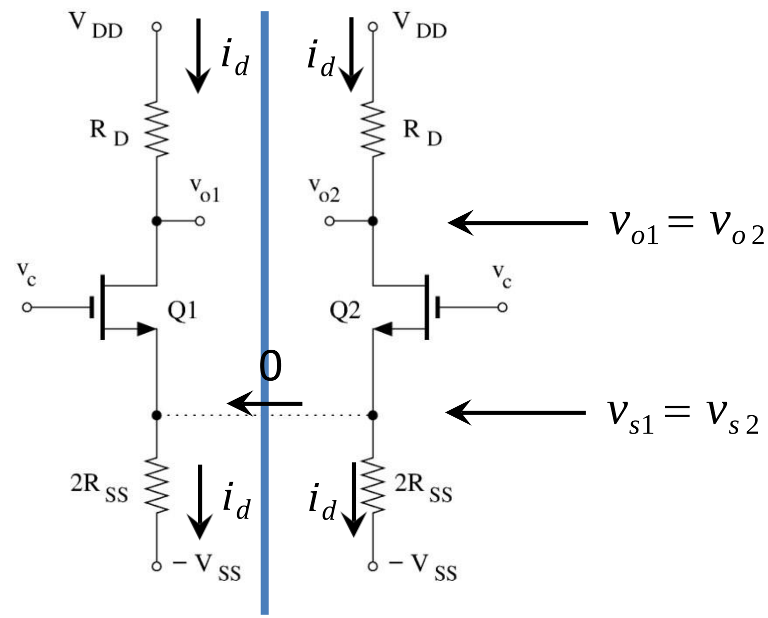
Differential Mode



Common-Mode "Half Circuit"



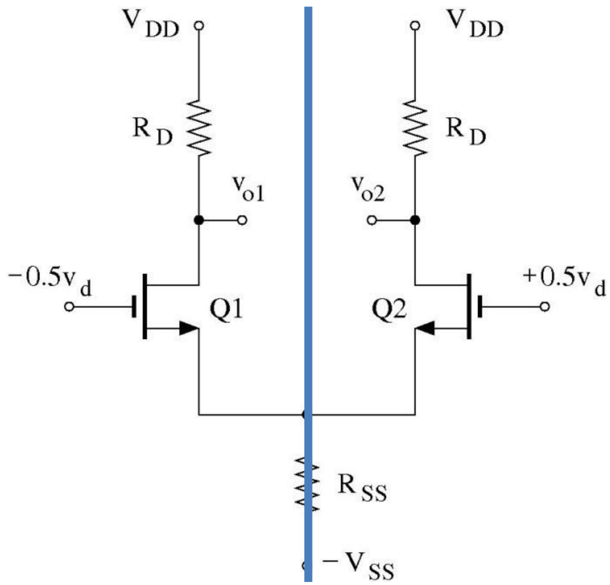
Common Mode circuit



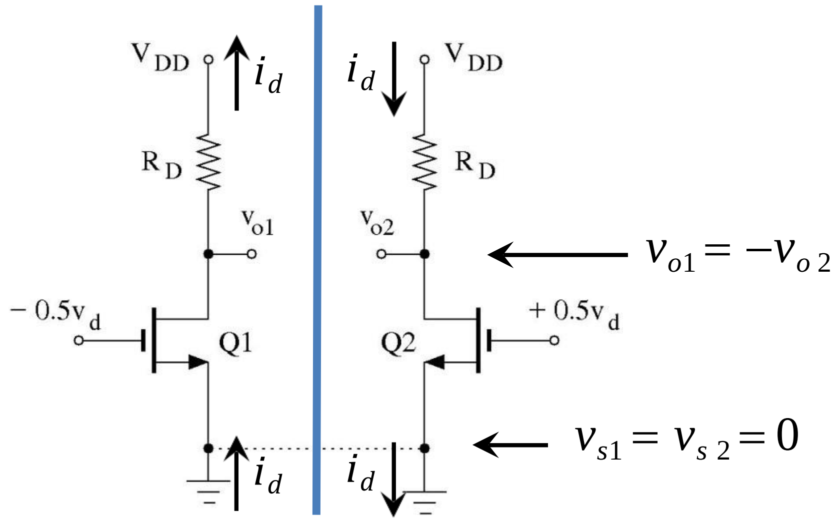
Common Mode Half-circuit

1. Currents about symmetry line are equal.
2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$)
3. No current crosses the symmetry line.

Differential-Mode “Half Circuit”



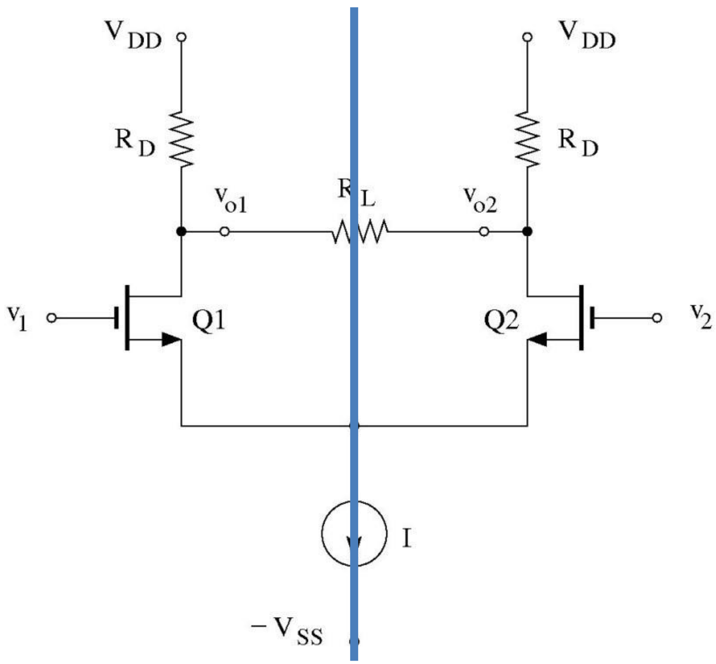
Differential Mode circuit

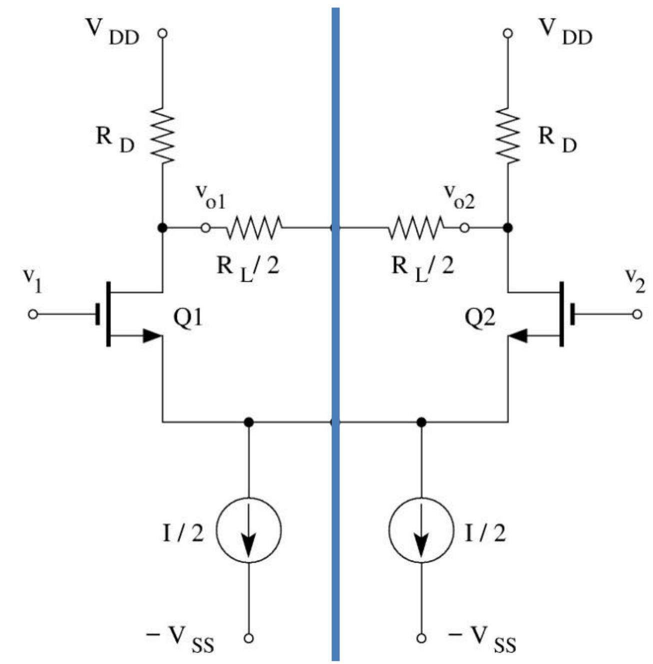


Differential Mode Half-circuit

1. Currents about the symmetry line are equal in value and opposite in sign.
2. Voltages about the symmetry line are equal in value and opposite in sign.
3. Voltage at the summery line is zero

Constructing “Half Circuits”





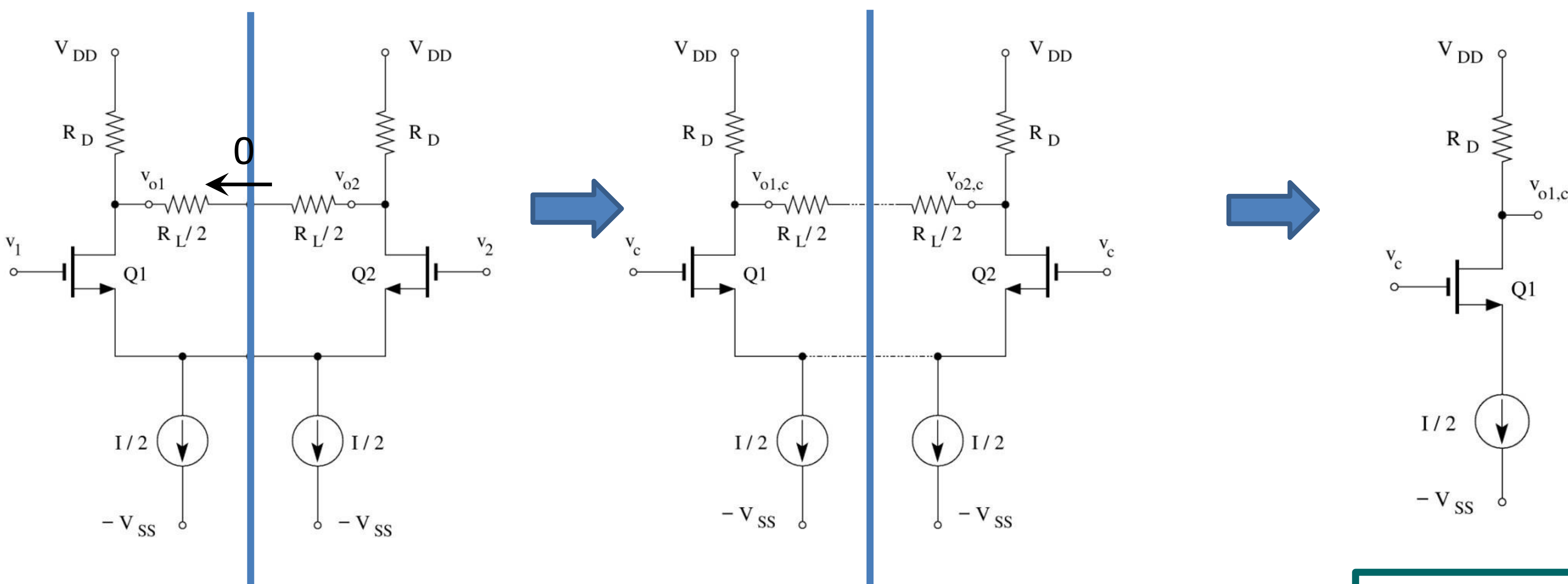
Step 1:

Divide **ALL elements** that <u>cross</u> the symmetry line (e.g., R_L) and/or <u>are located on</u> the symmetry line (current source) such that we have a symmetric circuit (only wires should cross the symmetry line, nothing should be located on the symmetry line!)

Constructing "Half Circuit"– Common Mode

Step 2: Common Mode Half-circuit

1. Currents about symmetry line are equal (e.g., $i_{d1} = i_{d2}$).
2. Voltages about the symmetry line are equal (e.g., $v_{o1} = v_{o2}$).
3. **No current crosses the symmetry line.**

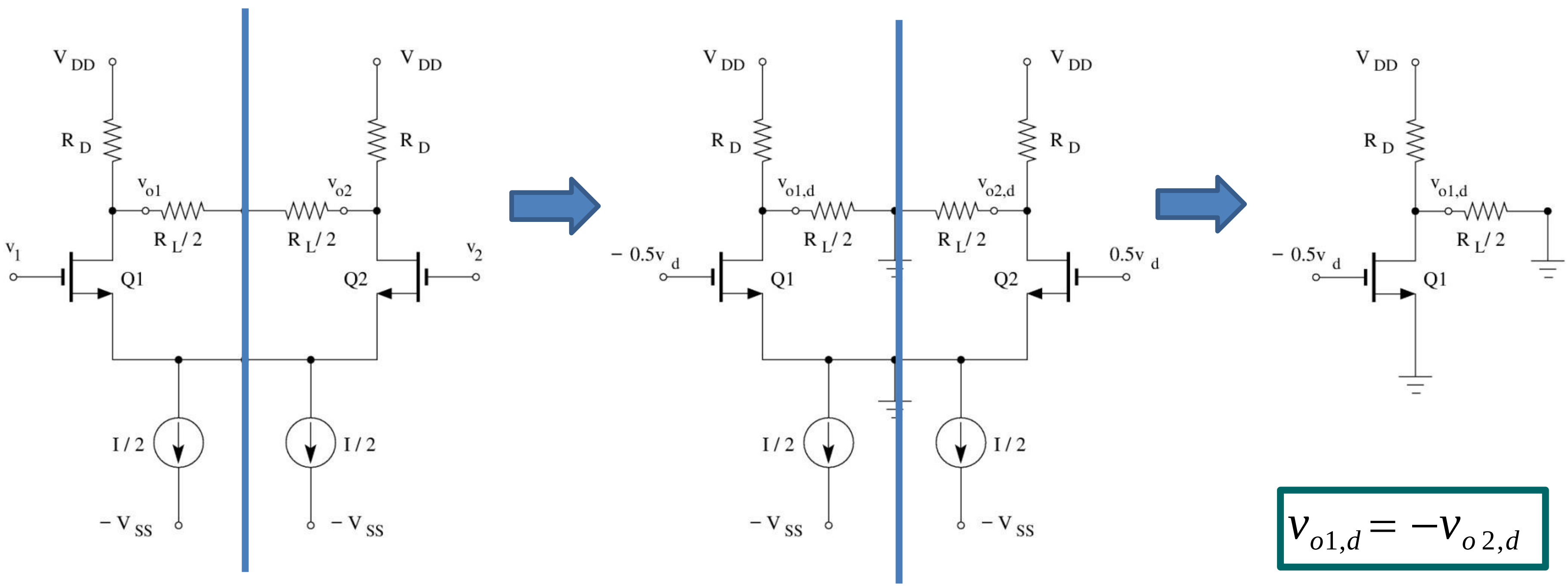


$$v_{o1,c} = v_{o2,c}$$

Constructing “Half Circuit”– Differential Mode

Step 3: Differential Mode Half-Circuit

1. Currents about symmetry line are equal but opposite sign (e.g., $i_{d1} = -i_{d2}$)
2. Voltages about the symmetry line are equal but opposite sign (e.g., $v_{o1} = -v_{o2}$)
3. **Voltage on the symmetry line is zero.**



$$v_{o1,d} = -v_{o2,d}$$