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(Q3)
opt 1:

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(a)$$

$$f'(x) = \frac{f(x+h) - f(x)}{h} \underbrace{- \frac{h}{2} f''(a)}_{\text{truncation error}} \left(O(h) \right)$$

→ Error decreases linearly with h

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(b)$$

$$f(x-h) = f(x) - h f'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(c)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \underbrace{- \frac{h^2}{6} f'''(d)}_{\text{truncation error}} \left(O(h^2) \right)$$

→ Error decreases quadratically with h

b is more accurate than a

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Q4)

opt 2;

a)

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix} \rightarrow (3-\lambda)^2 - 1 = 0$$

$$(3-\lambda)^2 = 1$$

$$\lambda^2 - 6\lambda + 8 = 0$$

$$\lambda_1 = 2 \quad \lambda_2 = 4$$

eigenvector of $\lambda = 2$:

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} v_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigenvector of $\lambda = 4$:

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} v_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

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Q4)

opt 2)

b)

$$x_1 = A x_0 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix} = -1 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$x_2 = A x_1 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 10 \\ -6 \end{bmatrix} = -6 \begin{bmatrix} -5/3 \\ 1 \end{bmatrix}$$

$$x_3 = A x_2 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ -6 \end{bmatrix} = \begin{bmatrix} 36 \\ -28 \end{bmatrix} = -28 \begin{bmatrix} -9/7 \\ 1 \end{bmatrix}$$

$$A x = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} -9/7 \\ 1 \end{bmatrix} = \begin{bmatrix} -102/7 \\ -44/7 \end{bmatrix}$$