MAT 175E

Ergün Yaraneri
yaraneri@itu.edu.tr

Homework 4

Only one randomly chosen question (which is the same for all of you) will be graded!

---

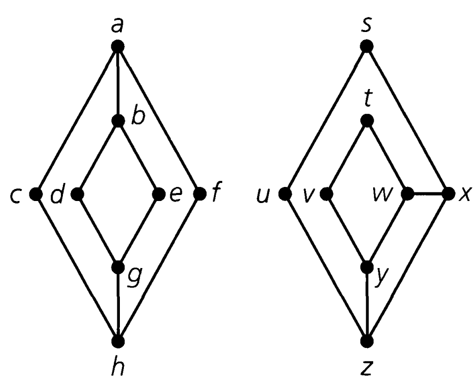
(1) Let G be a simple undirected graph with the vertex set

$$V = \{5, 6, 7, 8, 9, 10, 12, 14, 15\}$$

such that two distinct vertices i and j are adjacent if and only if i and j are not coprime (i.e., i and j have a common divisor > 1).

- (i) Find the degree of each vertex.
- (ii) Is there a circuit containing the vertex 7?
- (iii) Find a path from 7 to 9 of length 3.
- (iv) Find a trail from 7 to 9 which is not a path.
- (v) Find a path from 7 to 9 of length 8 (i.e., a path 7 to 9 passing through every vertex).
- (vi) Is G connected?
- (vii) Find a cycle of length 3.
- (viii) Is G bipartite?

(2) Find some cycles of length 4 in each of the following graphs. Are the following graphs isomorphic?



(3) Consider the sets A, B and A_k, where $k \in \mathbb{N}^+$, defined as follows

$$A = \{X \subseteq \mathbb{N}^+ \mid X \text{ is finite}\}, \quad B = \{X \subseteq \mathbb{N}^+ \mid X \text{ is infinite}\}, \quad A_k = \{X \subseteq \mathbb{N}^+ \mid |X| = k\}.$$

- (i) For any $k \in \mathbb{N}^+$ and for any $X \in A_k$, letting $X = \{x_1, x_2, ..., x_k\}$ where $x_1 < x_2 < \cdots < x_k$, show that the map

$$A_k \to \mathbb{N}^+ \times \mathbb{N}^+ \times \cdots \times \mathbb{N}^+ \ (k \text{ times}), \quad \text{defined by} \quad X \mapsto (x_1, x_2, ..., x_k),$$

  is injective.
- (ii) Show that A_k is countable for all $k \in \mathbb{N}^+$.
- (iii) Show that A is countable.
- (iv) Using the fact that $\mathcal{P}(\mathbb{N}^+) \sim \mathbb{R}$, show that B is uncountable.

(4) (a) Let A, B and X be sets such that $A \subseteq X \subseteq B$ and $A \sim B$. Prove that $A \sim X$ and $B \sim X$.

(b) Find a bijective map $\mathbb{R} \to \mathbb{R} - \{0\}$.