BLG 311E - Formal Languages and Automata
Assignment #3

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Q1. In the advanced city of Lexicoville, an AI-driven society relied on innovations in language theory and computational linguistics. A secret research institute named TuringLabs focused on developing AI models capable of deciphering complex languages to advance human understanding.

One day, Dr. Syntax, the head of TuringLabs, stumbled upon a mysterious coded message that seemed to follow a specific pattern. As a renowned linguistic prodigy, you were invited to join the team and help decipher the language.

For the language $L = \{a^i b^{(i+j)} c^j \mid i \geq 1, j \geq 0\}$:

1. Write the grammar production rules.
2. Design a PDA for this language.
3. Show how the strings abc and $aabcbc$ are accepted by the PDA you designed.

Q2. Cem and İdil, a tech-savvy couple, are planning their wedding and they want to create a custom hashtag for their special day. They've decided to create a hashtag generator that produces hashtags with a specific pattern: sequences of lowercase English letters (a, b, c, ..., z) separated by underscores (_). To ensure the hashtags follow this pattern, they design a context-free grammar to represent their ideal wedding hashtags:

$$S \to AB$$
$$A \to aAb \mid a$$
$$B \to \_bBc \mid \lambda$$

Construct a pushdown automaton (PDA) that accepts this language by empty stack, so that Cem and İdil can generate the perfect hashtag for their big day.

Q3. Elon Musk is working on a Mars colonization project and has devised a context-free grammar G for communicating with Martian civilizations. Convert the following context-free grammar G to Chomsky Normal Form (CNF):

$$S \to AB \mid AC$$
$$A \to aA \mid \lambda$$
$$B \to bB \mid b$$
$$C \to cC \mid c$$

Q4. In the futuristic city of Machina, TuringBots have become an essential part of daily life. They are Turing Machines with advanced capabilities, helping citizens with various tasks. One day, a team of Machina engineers was tasked with designing a TuringBot that could manage water distribution to the city's hydroponic gardens.

The engineers need to create a Turing Machine that recognizes the following language: $L = \{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } i + j = k\}$.

Design a Turing Machine for the given language, including the state diagram and the transition function.