MAT 175E

Ergün Yaraneri
yaraneri@itu.edu.tr

Homework 1

Only one randomly chosen question (which is the same for all of you) will be graded!

---

(1) (a) Find an assertion that is logically equivalent to $(P \vee Q) \wedge R$ and that contains only occurrences of the logical connectives:

(i) $\neg$ and $\wedge$ (ii) $\neg$ and $\vee$ (iii) $\neg$ and $\rightarrow$

(b) Determine whether each of the following assertions is a tautology, contradiction or contingent.

(i) $\big(P \rightarrow (Q \rightarrow R)\big) \rightarrow \big(Q \rightarrow (P \rightarrow R)\big)$
(ii) $\big((P \leftrightarrow \neg Q) \wedge P\big) \wedge Q$
(iii) $\big(P \rightarrow (Q \vee R)\big) \leftrightarrow (P \wedge \neg Q)$

(c) Suppose that we have two valid deductions having the same hypotheses. Is it possible that the conclusions of the deductions are P and $\neg P$ for some assertion P? Explain your answer.

(d) Write a proof of the following theorem in plain English.

"Every house in this town that is nice and not large is sold. Every expensive house in this town is nice. There is an expensive house in this town that is not sold. Therefore, there is a large house in this town."

(2) (a) Justify that $K \rightarrow (L \rightarrow M) \equiv (K \wedge L) \rightarrow M$

(b) Write a two-columns proof for the following theorem

$$G \rightarrow (H \vee C),\ \neg(A \wedge B \wedge C),\ \neg E \wedge B,\ \therefore\ A \rightarrow (G \rightarrow H)$$

(c) Find a counter example to the following invalid deduction

$$B \rightarrow (C \rightarrow D),\ E \rightarrow C,\ \therefore\ A \rightarrow (E \rightarrow D)$$

(d) Write a two column proof by contradiction for the following theorem

$$\begin{array}{l} \forall x\big(B(x) \vee C(x)\big) \\ \exists x\big(\neg E(x) \wedge D(x)\big) \\ \forall x\Big(\neg\big(D(x) \wedge C(x)\big) \vee A(x)\Big) \\ \hline \exists x\big(\neg A(x) \rightarrow B(x)\big) \end{array}$$

(3) (a) Suppose that the universes for the variables x and y are the set of all cars. Let a denote my car. Translate the following quantified assertions into sentences in plain English where the open statements $L(x,y), M(x,y)$, and $N(x,y)$ are given as follows:

$L(x,y) = x$ is faster than y; $M(x,y) = x$ is more expensive than y; $N(x,y) = x$ is older than y.

(i) $(\exists x)(\forall y)L(x,y)$
(ii) $(\forall x)(\exists y)M(x,y)$
(iii) $(\exists y)(\forall x)\big(L(x,y) \vee N(x,y)\big)$
(iv) $(\forall y)(\exists x)\big(\neg M(x,y) \rightarrow L(x,y)\big)$
(v) $\forall x L(a,x)$
(vi) $\forall x\Big(\big(M(x,a) \wedge N(x,a)\big) \rightarrow \neg L(x,a)\Big)$

(b) Write a negation in plain English for each of the following quantified assertions.

(i) There is a place in the world such that everyone who enter the place goes blind.
(ii) At least one person in Istanbul owns every newspapers published yesterday.
(iii) Everyone has a secret that is not known by more than two people.
(iv) Every integer is either odd or even, but not both.
(v) There are real numbers a and b such that $a < s < b$ for any element s of the set S.