BLG202E –Midterm Exam Part B
Spring 2023, Duration: 30 minutes exam + 10 minutes for uploading

Instructions:

- *Do NOT communicate with other people, including your friends, classmates, and family members!*
- *This is an open-book exam.*
- *Give your answers in English.*
- *Use an A4 paper for each question.*
- *Write the question number, your Name and İTÜ ID on the top of each page and __sign all pages__.*
- *Scan or take photo of your answers and upload them on Ninova within a pdf file __before the deadline__!*
- *There will be no extension for time without penalty. There will be a late submission option for 5 mins where you will lose 10 points.*

ANSWER ONLY ONE OPTION FROM THE FOLLOWIG QUESTIONS:

Question 3)

OPTION 1

a) Let $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 8 & 5 \\ 3 & 6 & 9 \end{bmatrix}$ by applying Gaussian elimination, the following $PA = LU$ factorization can be written:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 8 & 5 \\ 3 & 6 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 \\ 0 & -6 & -12 \\ 0 & 0 & -9 \end{bmatrix}$$

Compute the determinant of A using the factorization.

b) Consider the following linear system

$$2x + 2cy = 2c$$
$$x + y = 2$$

where c is a parameter that can take very large values

i) Apply simple Gaussian elimination without any pivoting. Find the solution.

ii) Apply Gaussian elimination with scaled pivoting. Find the solution.

iii) Compare the results

Let

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 7 \\ 3 & 5 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 8 \end{pmatrix}$$

(a) (15 pts) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 7 \\ 3 & 5 & 8 \end{pmatrix}$. Find a matrix P such that $PA = LU$ where L is a lower

triangular and U is an upper triangular matrix (only use Gaussian Elimination).

(b) (10 pts) By using the results of part a), explain how to find a solution for the above system.

QUESTION 4)

OPTION 1

An experiment was carried by varying the parameter x and the measurements, f(x), are given in the table below.

x	f(x)
7.0	2
8.3	3
9.4	4
11.3	5

i) Assume that the measurements are modeled as $\hat{f}(x) = k(x - 5)$. **Derive** the optimal value of the parameter k in the **the least squares sense** .

ii) Compute the least squares error corresponding to the value of k found in part i).

OPTION 2

Fit the least square line $y = ax + b$ for the data given below:

x	-3	0	1	7
y	10	46	2	1