

BLG202E –Midterm Exam Part A

Spring 2023, Duration: 30 minutes exam + 10 minutes for uploading

Instructions:

- Do NOT communicate with other people, including your friends, classmates, and family members!
- This is an open-book exam.
- Give your answers in English.
- Use an A4 paper for each question.
- Write the question number, your Name and İTÜ ID on the top of each page and **sign all pages**.
- Scan or take photo of your answers and upload them on Ninova within a pdf file **before the deadline!**
- There will be no extension for time without penalty. There will be a late submission option for 5 mins where you will lose 10 points.

ANSWER ONLY ONE OPTION FROM THE FOLLOWING QUESTIONS:

Question 1)

OPTION 1

a) The function $f(x) = x^{2^n}$ at a point x_0 can be computed as by carrying out the following sequences:

$$x_1 = x_0^2, x_2 = x_1^2, \dots, x_n = x_{n-1}^2$$

Let the floating point representation of x be $\text{fl}(x) = \hat{x}$ in floating point arithmetic, the following numbers are computed:

$$\hat{x}_1 = x_0^2(1 + \epsilon_1), \quad \hat{x}_2 = (\hat{x}_1)^2(1 + \epsilon_2), \dots, \hat{x}_n = (\hat{x}_{n-1})^2(1 + \epsilon_n)$$

With $|\epsilon_i| \leq \eta$ (rounding unit)

Find the \hat{x}_n in terms of η and x_0

b) Consider the following 10-bit hypothetical floating point representation:

1 bit for sign of the number (Positive 0, negative 1)	1 bit for sign of the exponent (Positive 0, negative 1)	4 bits for Mantissa	4 bits for exponent

i) Find the 10-bit representation of $(33.25)_{10}$

ii) Compute the absolute relative error.

iii) What are the largest and smallest numbers by magnitude that can be represented using this floating point representation?

OPTION 2

- (a) How do you write 121 and 113/1024 in binary ?
- (b) Represent 2/7 in a single precision floating point format.
- (c) Convert the following data which was stored by a computer using 32-bit representation of floating point numbers to a decimal.

1	0	0	0	1	1	1	0	1	0	1	1	1	0	all zero	0
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QUESTION 2)**OPTION 1**

i.) Assume that the following link matrix is constructed for 10 web pages to find the ranks using PageRank algorithm. The j th row of the matrix represents the inlinks from web page j and i th column represents the outlinks from web page i . Draw the link structure of the network as a graph.

$$\begin{bmatrix} 0 & 0 & 1/4 & 1 & 1/5 & 0 \\ 1/2 & 0 & 0 & 0 & 1/5 & 0 \\ 0 & 1/3 & 0 & 0 & 1/5 & 0 \\ 0 & 1/3 & 1/4 & 0 & 1/5 & 1/2 \\ 1/2 & 1/3 & 1/4 & 0 & 0 & 1/2 \\ 0 & 0 & 1/4 & 0 & 1/5 & 0 \end{bmatrix}$$

ii) Let $A = \begin{bmatrix} a & 1-a \\ 1-b & b \end{bmatrix}$ and $0 \leq a \leq 1$ and $0 \leq b \leq 1$ find the eigenvalues of the matrix A .

OPTION 2

- (a) Let $a = 0.1b_2b_3 \dots b_{53}b_{54} \dots \times 2^m$. If one is rounding a either $a_- = 0.1b_2b_3 \dots b_{53} \times 2^m$ or $a_+ = 0.1b_2b_3 \dots b_{53} + 2^{-53}) \times 2^m$ then determine absolute and relative errors.
- (b) Approximate $7^{\frac{1}{3}}$ via Bisection Method with error less than 10^{-2} .