BLG202E –Final Exam Part A

Spring 2023, Duration: 30 minutes exam + 10 minutes for uploading

Instructions:

- *Do NOT communicate with other people, including your friends, classmates, and family members! Do NOT use any online tool such as https://chat.openai.com/.*
- *This is an open-book exam.*
- *Give your answers in English.*
- *Use an A4 paper for each question.*
- *Write the question number, your Name and İTÜ ID on the top of each page and **sign all pages**.*
- *Scan or take photo of your answers and upload them on Ninova within a pdf file **before the deadline**!*
- *There will be no extension for time without penalty. There will be a late submission option for 5 mins where you will lose 10 points.*

ANSWER ONLY ONE OPTION FROM THE FOLLOWIG QUESTIONS:

QUESTION 3)

OPTION 1

(25 pts) Compare errors while computing $f'(x)$ via

(a) $\dfrac{f(x+h)-f(x)}{h}$

(b) $\dfrac{f(x+h)-f(x-h)}{2h}$

OPTION 2

You are asked to derive a formula for the third derivative of function f around x_o using the Taylor's series expansion at $(x_o \mp h)$ and $(x_o \mp 2h)$.

a) (15 points) What is the formula for the third derivative?
b) (10 points) What is the truncation error for this formula?

QUESTION 4)

OPTION 1

Let

$$\begin{pmatrix} 2 & -2 & 3 \\ 24 & -27 & 31 \\ 6 & 0 & 23 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 7 \\ 63 \\ 75 \end{pmatrix}$$

(a) (15 pts) Let $A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 7 \\ 3 & 5 & 8 \end{pmatrix}$. Find matrices L and U such that $A = LU$ where L is a lower triangular and U is an upper triangular matrix

(b) (10 pts) By using the results in part a) find a solution for the above system.

OPTION 2

Let $A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

a) (15 points) Find the eigenvalues and eigenvectors of A.

b) (10 points) Apply Power method to find the eigenvalue and eigenvector of A. start with $v_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ initial guess and iterate three iterations(You should find v_3 and λ_3).

Algorithm: Power Method.
Input: matrix A and initial guess $\mathbf{v}_0$.

for $k = 1, 2, \ldots$ until termination
$\tilde{\mathbf{v}} = A\mathbf{v}_{k-1}$
$\mathbf{v}_k = \tilde{\mathbf{v}} / \|\tilde{\mathbf{v}}\|$
$\lambda_1^{(k)} = \mathbf{v}_k^T A \mathbf{v}_k$
end