EHB 211E
Basics of Electrical Circuits

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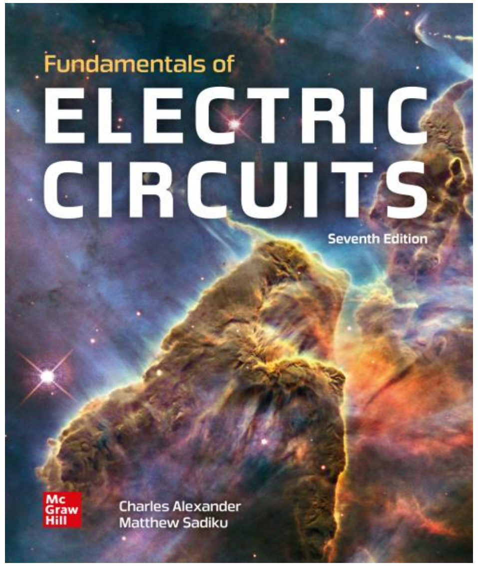
Circuit Theorems



Fundamentals of Electric Circuits by C. Alexander and M. Sadiku, Seventh Edition (McGraw-Hill Education)

Linearity

- Property of an element describing a linear relationship between cause and effect
- Two properties:
  - Homogeneity (aka scaling): If the input (or excitation) is multiplied by a constant, then the output (or response) is multiplied by the same constant.

$$v = iR \Rightarrow kiR = kv$$

  - Additivity: the response to a sum of inputs is the sum of the responses to each input applied separately.

$$v_1 = i_1 R \quad \& \quad v_2 = i_2 R$$

$$v = (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

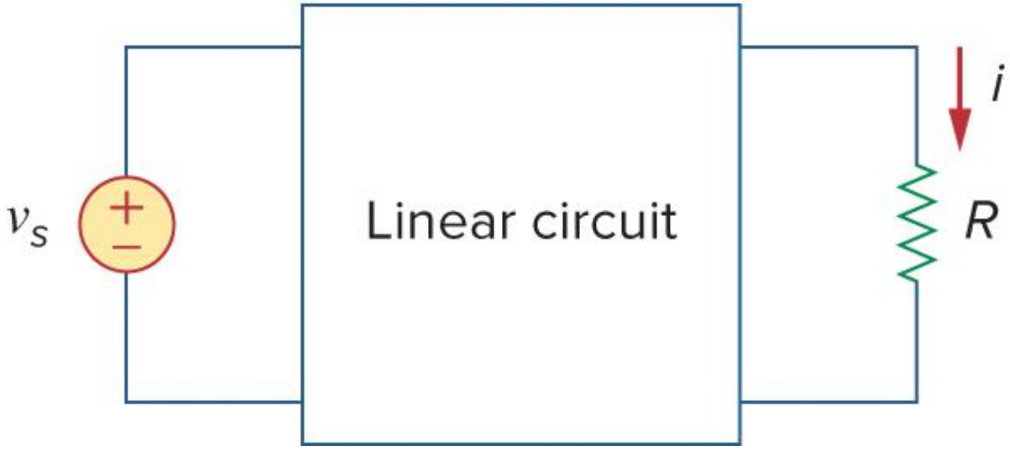
- In general, a circuit is linear if it is both additive and homogeneous
- Linear circuit: output is linearly related (or directly proportional) to its input

Linearity

- The relationship between power and voltage (or current) is nonlinear.

$$\mathrm{p} = i^2 R = \frac{v^2}{R} \Rightarrow$$

Quadratic equation
(not linear)



$p_1 = Ri_1^2 \quad \& \quad p_2 = Ri_2^2$

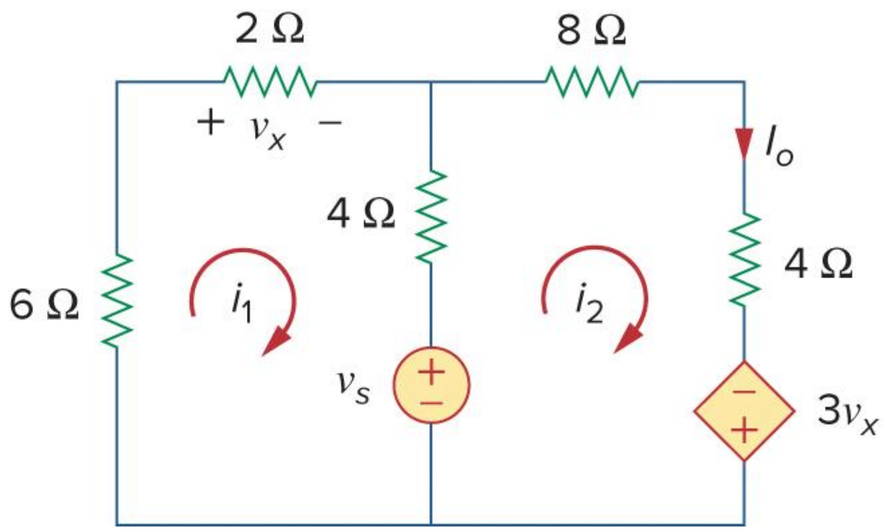
If $i_1 + i_2$ flows through R,

$$p = (i_1 + i_2)^2 R = Ri_1^2 + Ri_2^2 + 2Ri_1 i_2 \neq p_1 + p_2$$

Example 1

For the circuit shown below, find I_0 when $v_s = 12\ V$ and $v_s = 24\ V$.

Solution:



Applying KVL to the two loops, we obtain

$$12i_1 - 4i_2 + v_s = 0$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \qquad v_x = 2i_1$$

$$-10i_1 + 16i_2 - v_s = 0$$

$$2i_1 + 12i_2 = 0 \quad \Rightarrow \quad i_1 = -6i_2$$

$$-76i_2 + v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$$

When $v_s = 12$ V,

$$I_o = i_2 = \frac{12}{76}\text{A}$$

When $v_s = 24$ V,

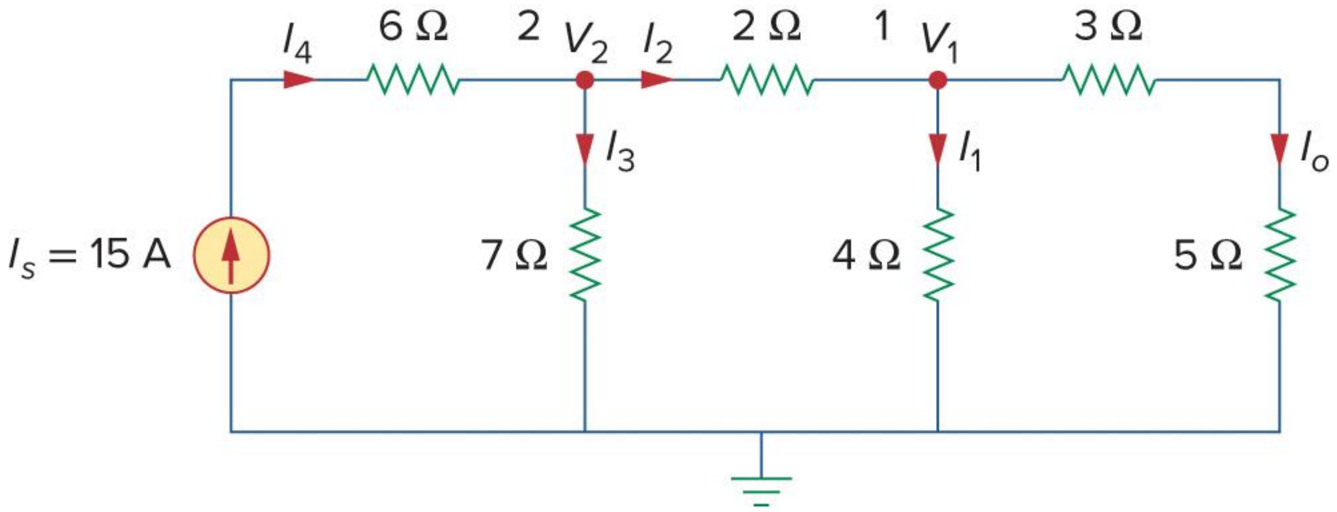
$$I_o = i_2 = \frac{24}{76}\text{A}$$

when the source value is doubled, I_o doubles.

Example 2

Assume $I_0 = 1\ A$ and use linearity to find the actual value of I_0 in the circuit given below.

Solution:



If $I_o = 1$ A, then $V_1 = (3 + 5)I_o = 8$ V and $I_1 = V_1/4 = 2$ A. Applying KCL at node 1 gives

$$I_2 = I_1 + I_o = 3 \text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14 \text{ V}, \quad I_3 = \frac{V_2}{7} = 2 \text{ A}$$

Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5 \text{ A}$$

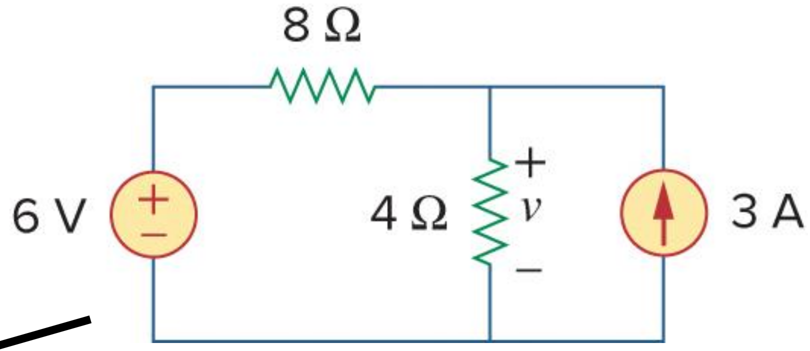
Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

Superposition

- Another way of determining voltage across or current through an element in a circuit
- Voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or current through) that element due to each independent source acting alone.
- **Steps to apply superposition principle:**
  - Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques we have covered so far.
    - Replace every other voltage sources by 0 V (short circuit)
    - Replace every other current source by 0 A (open circuit)
  - Repeat step 1 for each of the other independent sources.
  - Find the total contribution by adding algebraically all the contributions due to the independent sources.
- Note that dependent sources are left intact (Do not turn off dependent sources)
- Disadvantage: very likely to involve more work.
- Advantage: reduce a complex circuit to simpler circuits.
- Superposition is based on linearity. Superposition cannot be used for power calculation.

Example 3

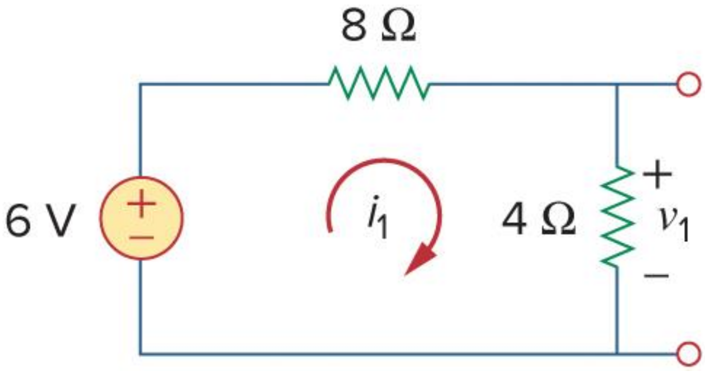
Use superposition theorem to find v in the circuit shown below.



Solution:

Since there are two sources, let $v = v_1 + v_2$ due to each independent sources

1st: Turn off current source (open circuit)

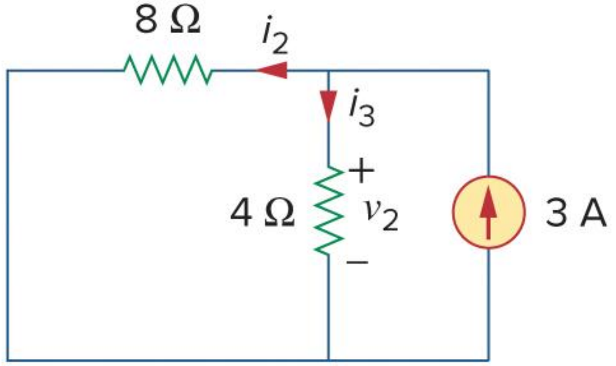


Applying KVL to the loop:

$-6 + 8i_1 + v_1 = 0 \qquad v_1 = 4i_1$

$-6 + 8i_1 + 4i_1 = 0 \Rightarrow 12i_1 = 6$
$\Rightarrow i_1 = 0.5\ A$

$v_1 = 4i_1 \Rightarrow v_1 = 4(0.5) \Rightarrow v_1 = 2\ V$



2nd: Turn off voltage sources (short circuit)

Current division: $i_3 = \dfrac{8}{12} \times 3 \Rightarrow i_3 = 2\ A$

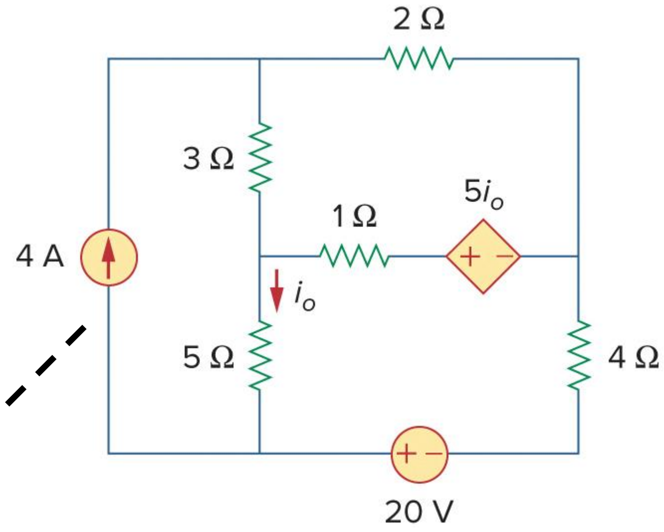
$v_2 = 4i_3 \Rightarrow v_2 = 4(2) \Rightarrow v_2 = 8\ V$

$v = v_1 + v_2 \Rightarrow v = 2 + 8 \Rightarrow v = 10\ V$

Example 4

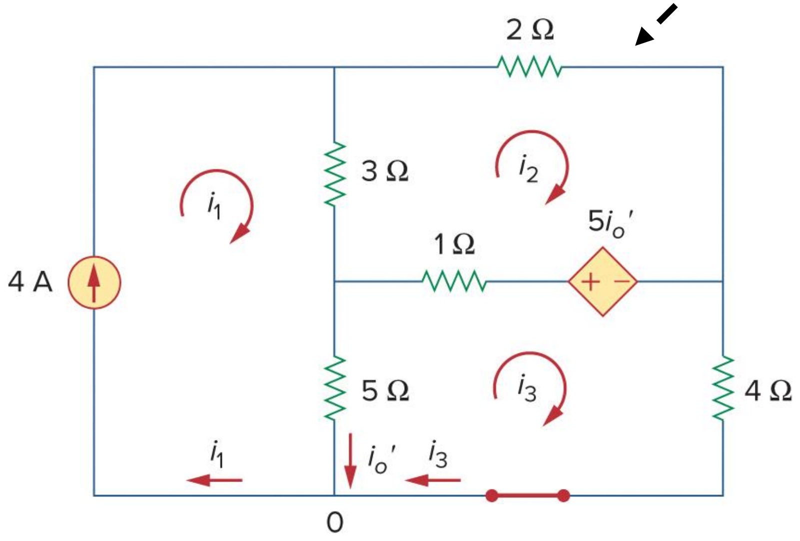
Find i_0 in the circuit shown below using superposition.

Solution: For superposition, the following two cases:



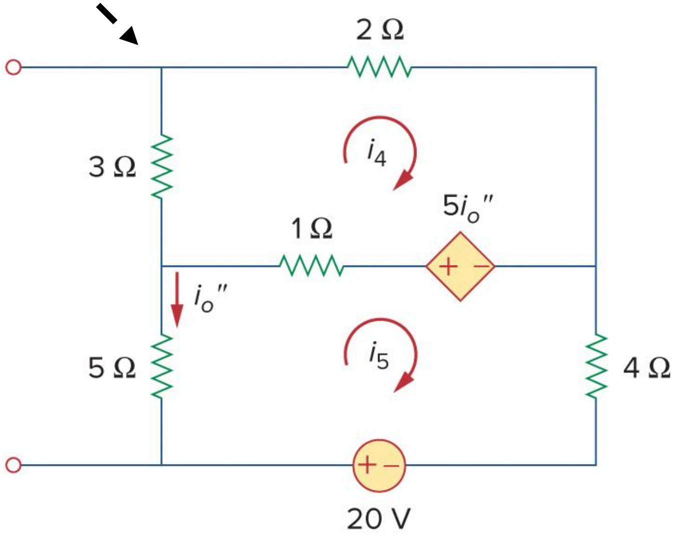
1st: Turn off independent voltage source (short circuit)

2nd: Turn off independent current source (open circuit)



Let $i_0 = i_0' + i_0''$ due to each independent source

Do not turn of dependent source



Solution

1st: Turn off 20 V independent voltage source (short circuit)

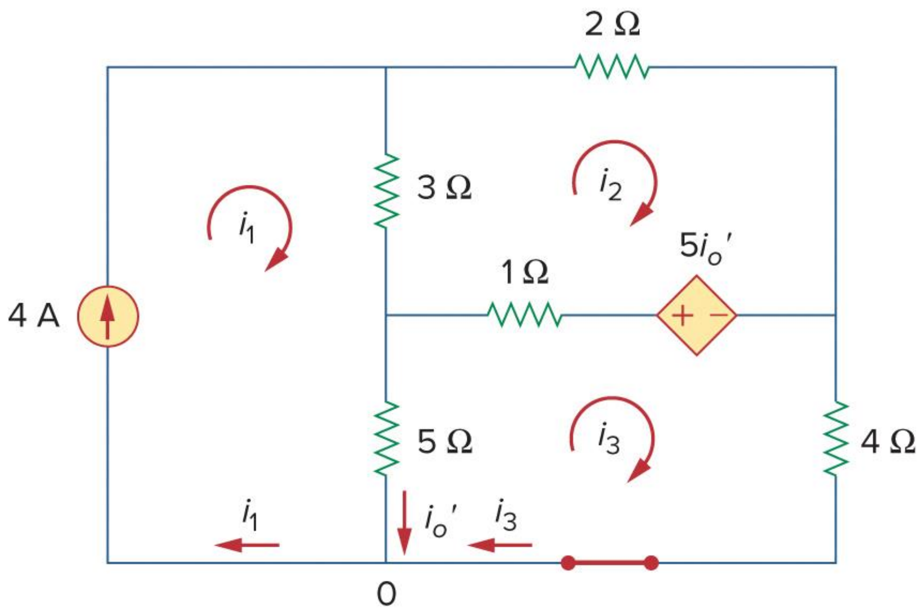
Apply mesh analysis to obtain i_0'

For loop 1: $i_1 = 4\ A$

For loop 2: $-3i_1 + 6i_2 - 1i_3 - 5i_o' = 0$

For loop 3: $-5i_1 - 1i_2 + 10i_3 + 5i_o' = 0$

At node 0: $i_3 = i_1 - i_o' = 4 - i_o'$



$$3i_2 - 2i_o' = 8$$

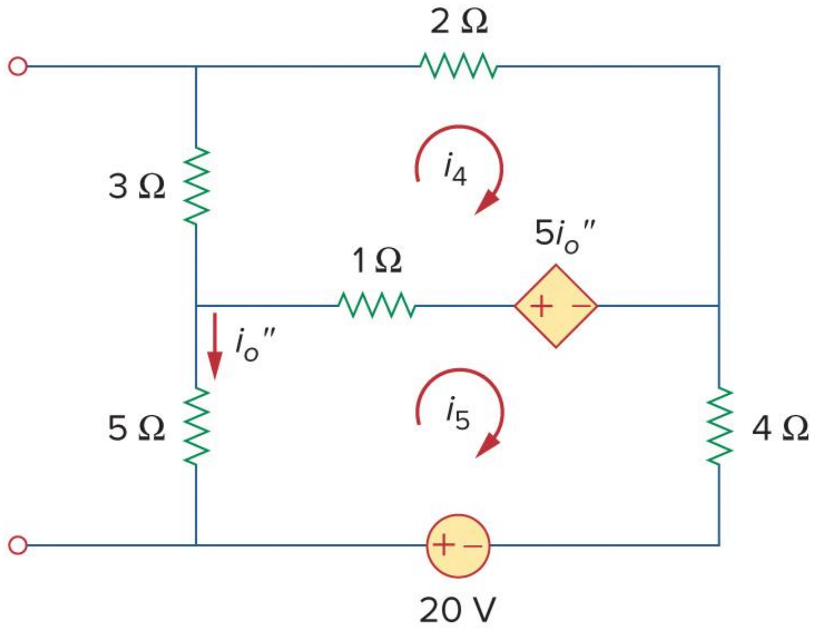
$$i_2 + 5i_o' = 20$$

$$i_o' = \frac{52}{17}\ \text{A}$$

Solution

2nd: Turn off 4 A independent current source (open circuit)

Apply mesh analysis to obtain i_0''



For loop 4: $6i_4 - i_5 - 5i_o'' = 0$

For loop 5: $-i_4 + 10i_5 - 20 + 5i_o'' = 0$

$$i_5 = -i_o''$$

$$6i_4 - 4i_o'' = 0$$

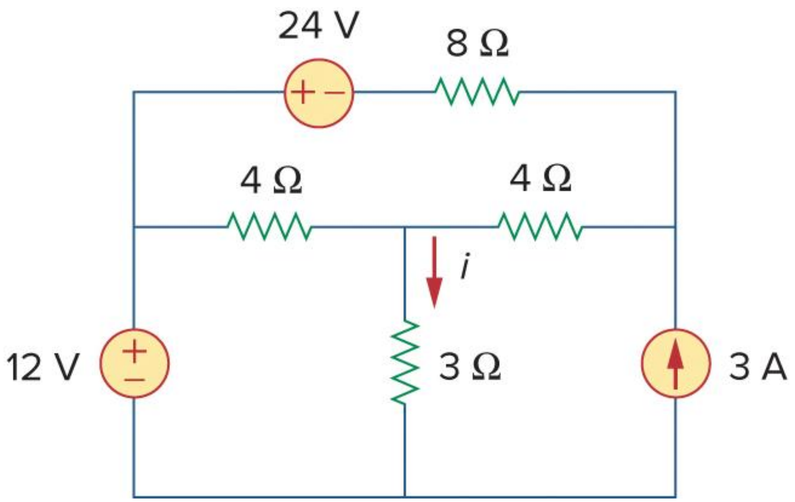
$$i_4 + 5i_o'' = -20$$

$$i_o'' = -\frac{60}{17}\text{ A}$$

$$\left.\begin{aligned} i_o' &= \frac{52}{17}\text{ A} \\ i_o'' &= -\frac{60}{17}\text{ A} \end{aligned}\right\} \quad i_o = -\frac{8}{17} = -0.4706\text{ A}$$

Example 5

For the circuit shown below, use superposition theorem to find i.

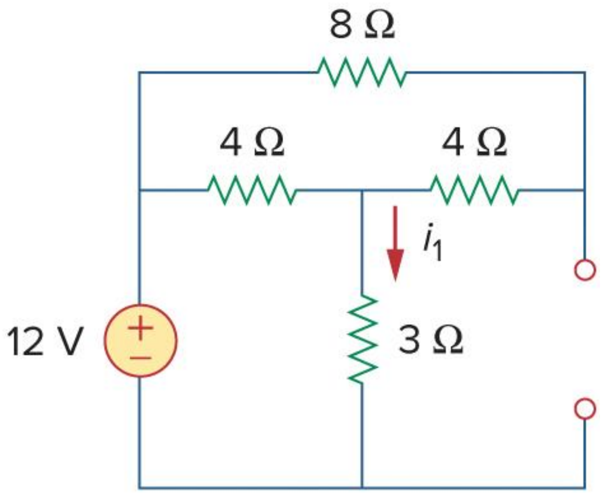


Solution: In this case, there are three sources

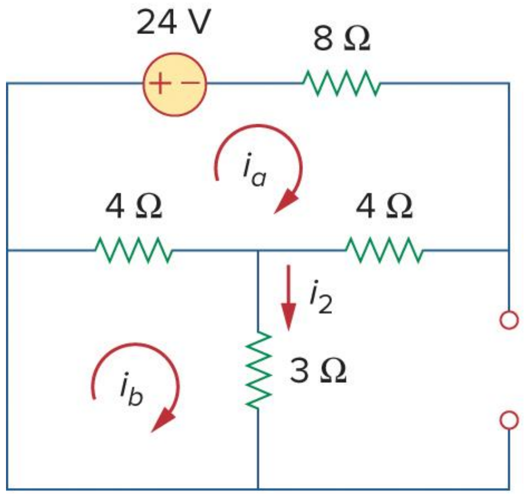
Let $i = i_1 + i_2 + i_3$ due to each independent source

Turn off two sources and keep only one of them when calculating current in each case

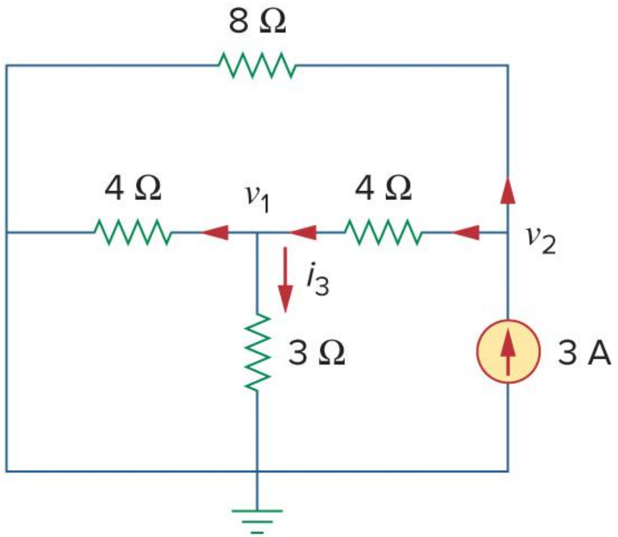
Case I



Case II

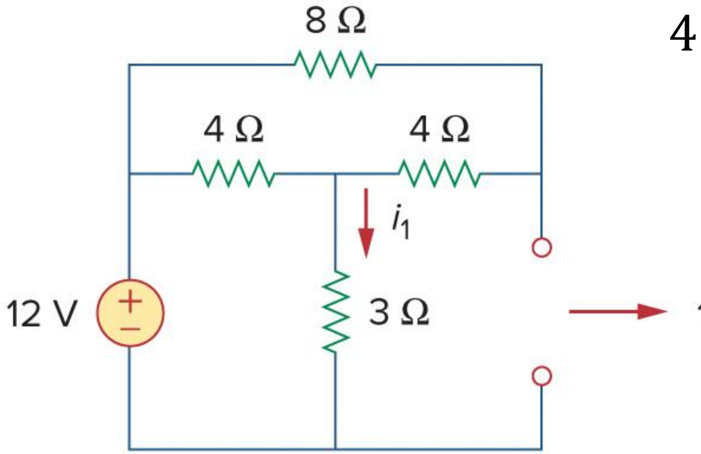


Case III

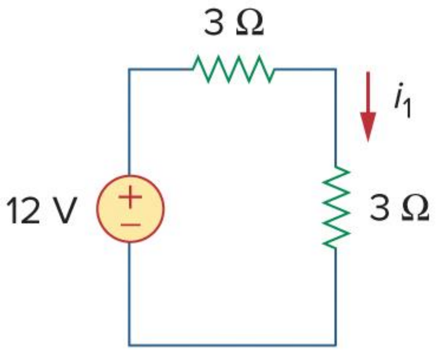


Solution

Case I:

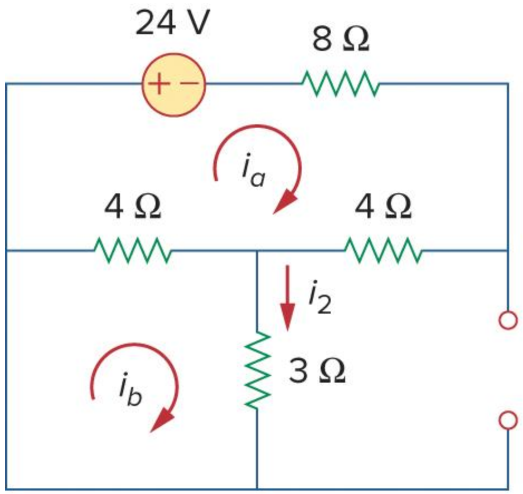


$$4\ \Omega || (8\ \Omega + 4\ \Omega) = 3\ \Omega$$



$$i_1 = \frac{12}{6} = 2\ \text{A}$$

Case II:



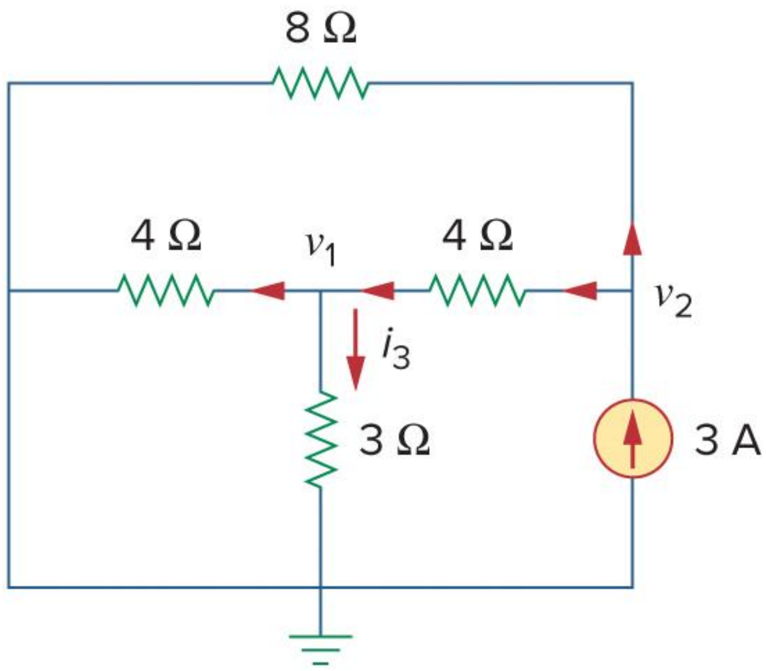
$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b$$

$$i_2 = i_b = -1$$

Solution

Case III:



$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1$$

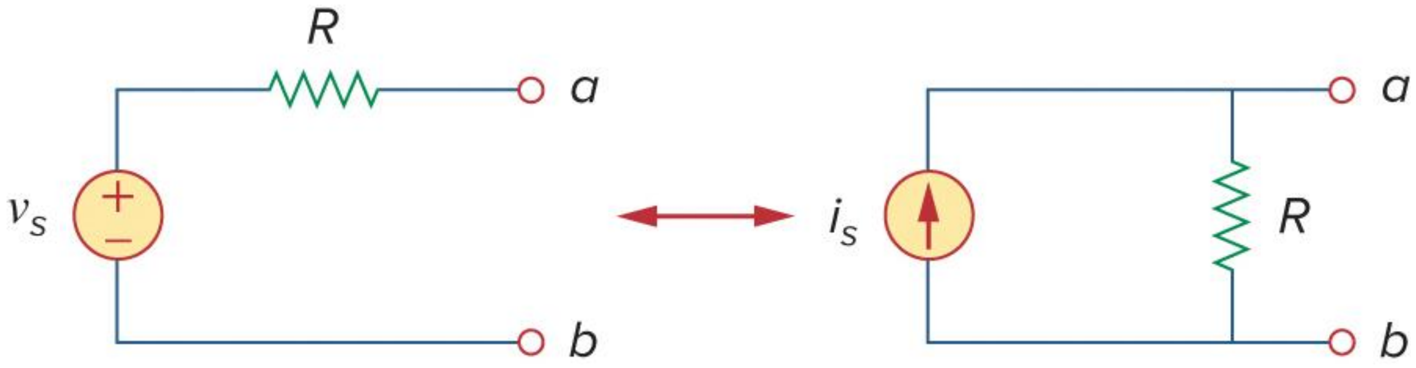
$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3} v_1$$

$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

Source Transformation

- Another tool for simplifying circuits.
- Process of replacing a voltage source v_s in series with a resistor R, by a current source i_s in parallel with a resistor R, or vice versa.
- Two circuits shown below are equivalent since they have the same voltage-current relationship at terminals *a*-*b*.
  - If sources are turned off, equivalent resistance at terminals *a*-*b* in both circuits is equal to R
  - When terminals *a*-*b* are short circuits, the short circuit current flowing from *a* to *b* is:



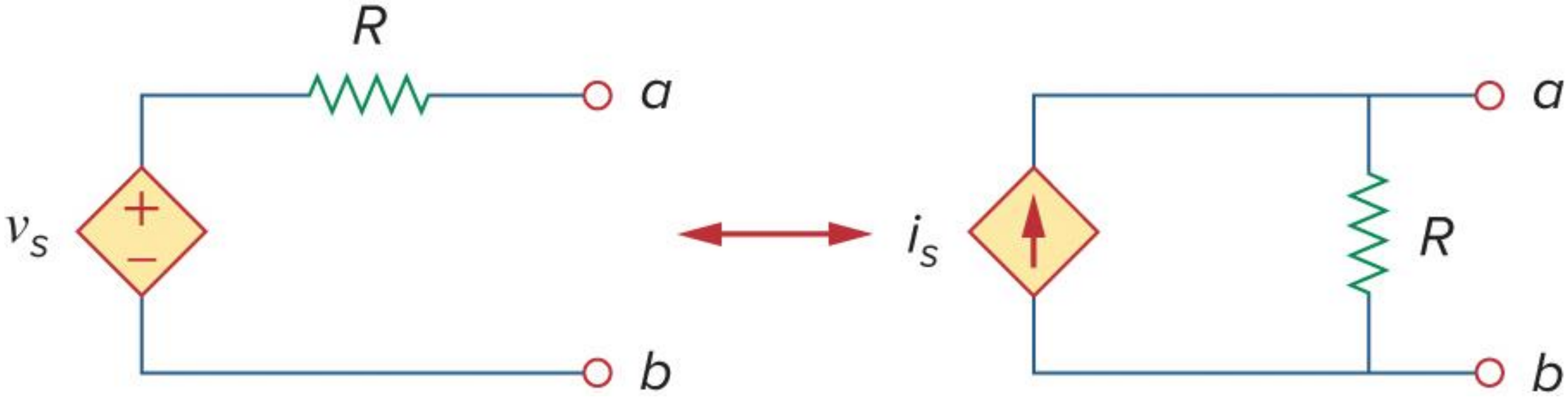
$$i_{sc} = \frac{v_s}{R}$$

$$i_{sc} = i_s$$

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Source Transformation

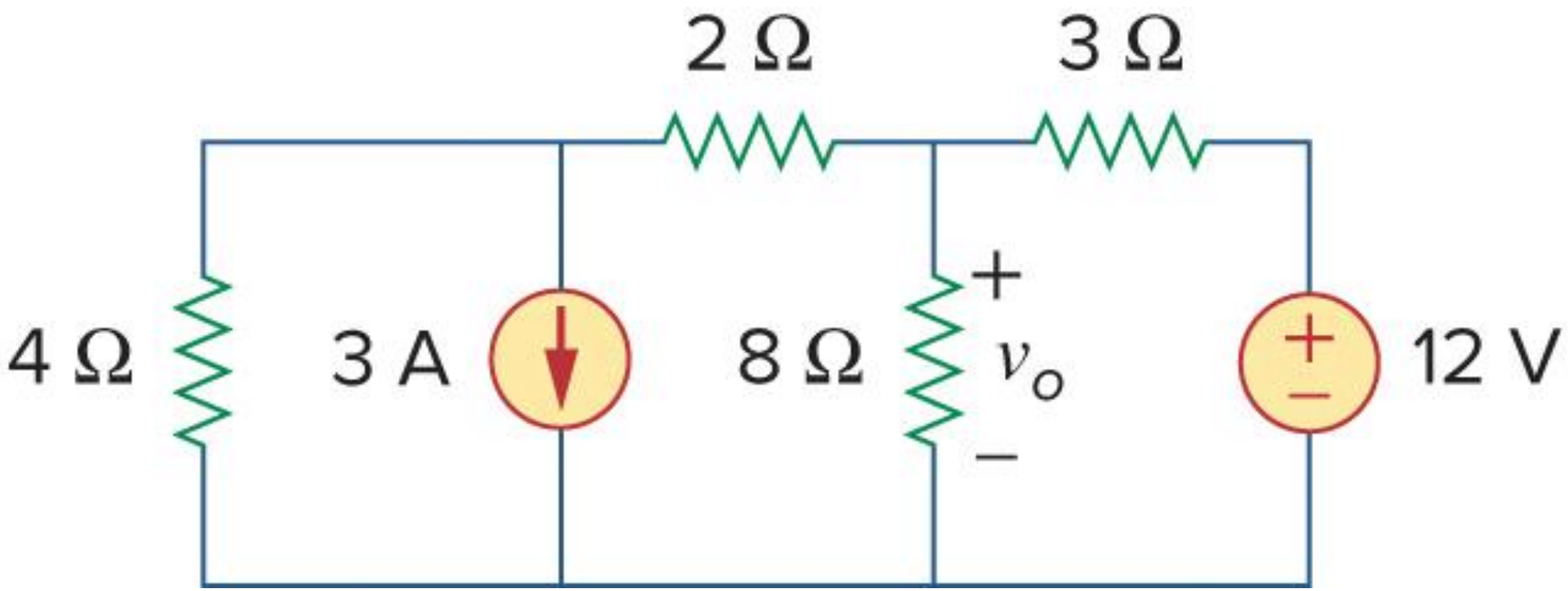
- Source transformation also applies to dependent sources
- Dependent voltage source in series with a resistor can be transform to a dependent current source in parallel with the resistor or vice versa.



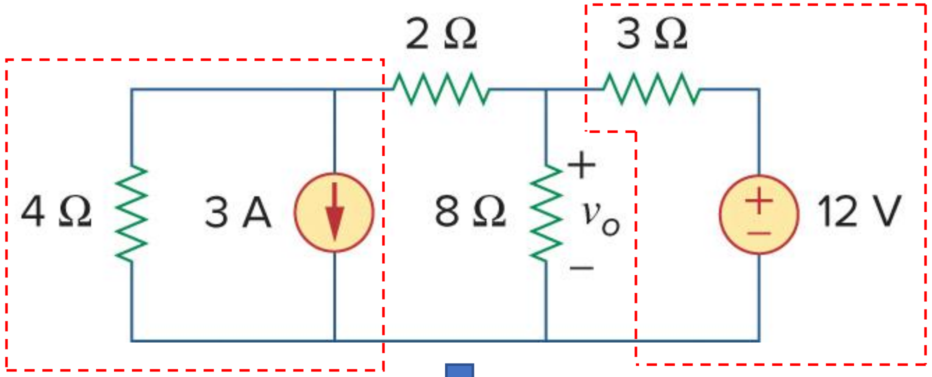
- Following important point should be kept in mind when dealing source transformation:
  - Arrow of the current source is directed toward the positive terminal of the voltage source
  - $R \neq 0$
  - $R \neq \infty$

Example 6

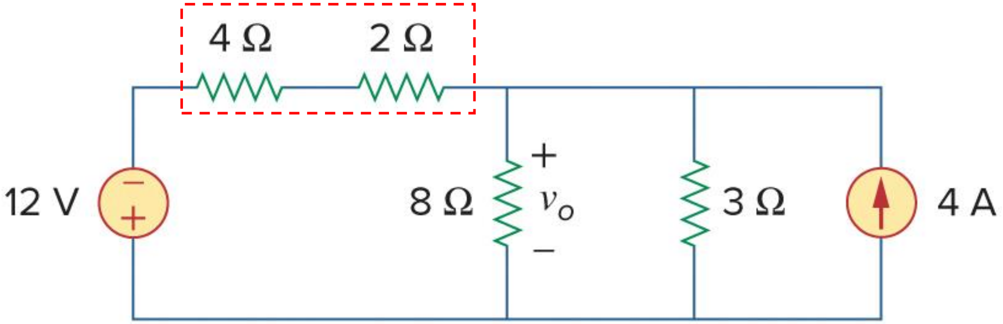
Use source transformation to find v_0 in the circuit shown below.

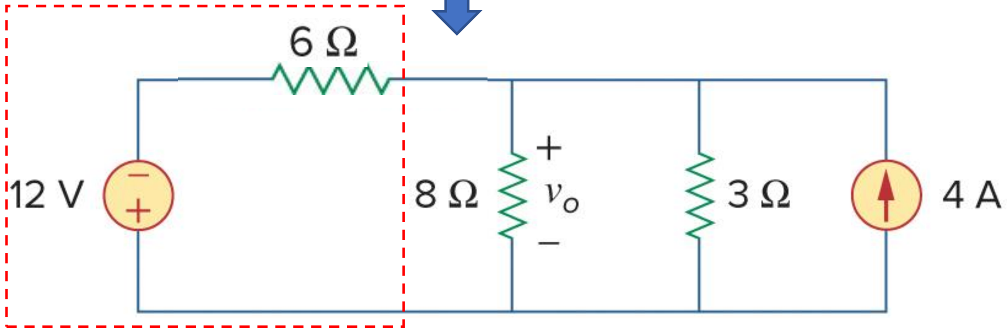


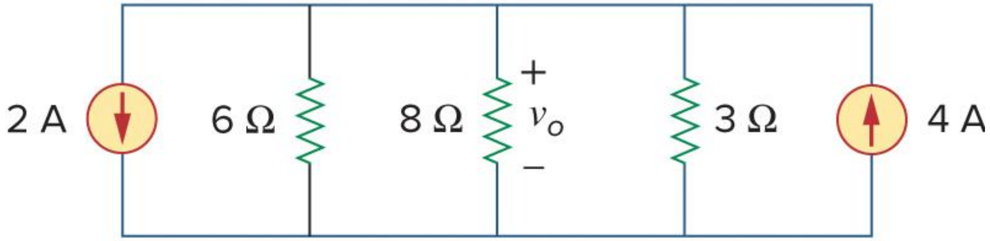
Solution



$v = iR$

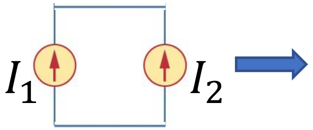




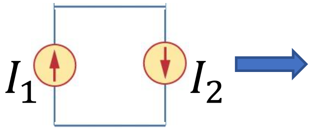


$6\ \Omega || 3\ \Omega = 2\ \Omega$

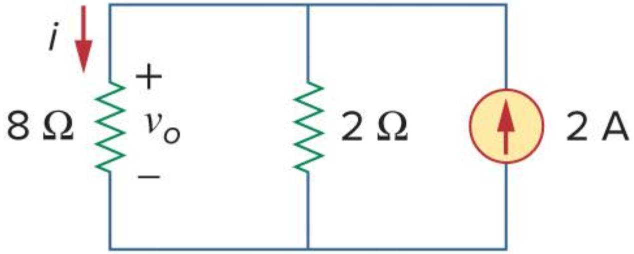
Recall:



$I = I_1 + I_2$



$I = I_1 - I_2$



Current division

$$i = \frac{2}{10} 2 \Rightarrow i = 0.4\ A$$

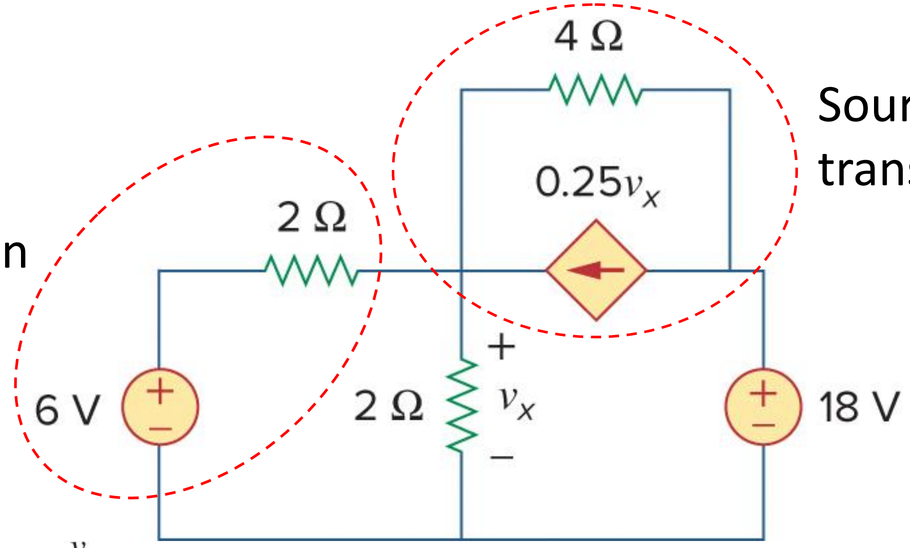
$$v_0 = 8i \Rightarrow v_0 = 8(0.4) = 3.2\ V$$

Example 7

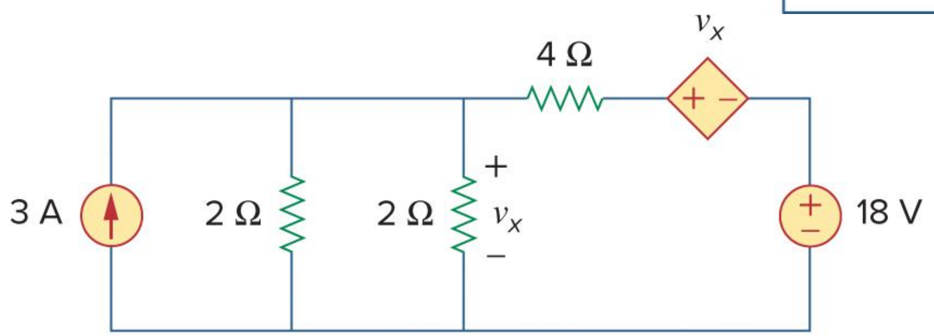
Find v_x using source transformation in the circuit shown below.

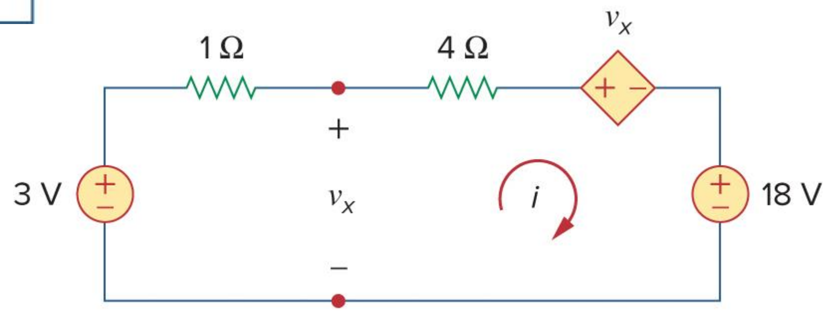
Solution:

Source transformation



Source transformation





Applying KVL to the loop in the circuit:

$$-3 + 5i + v_x + 18 = 0$$

Applying KVL to the loop containing 3 V, 1Ω, and v_x :

$$-3 + 1i + v_x = 0$$

$$v_x = 3 - i$$

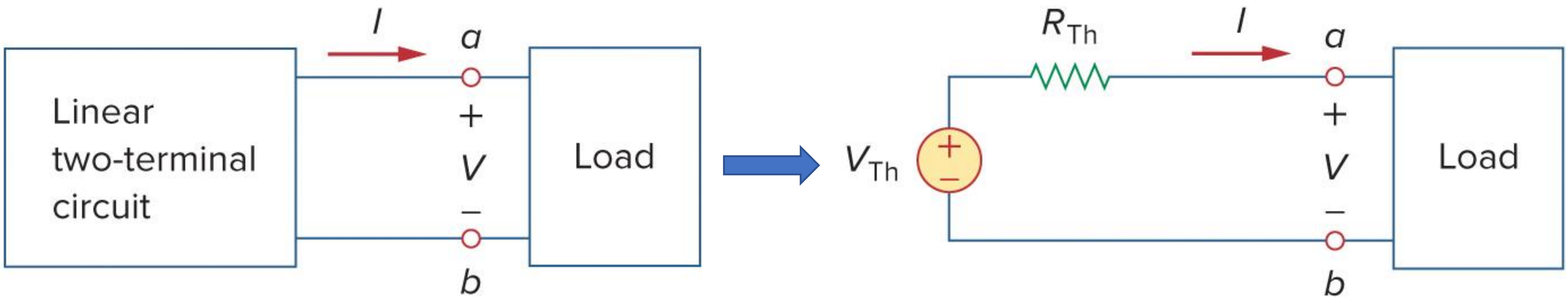
$$15 + 5i + 3 - i = 0$$

$$i = -4.5 \text{ A}$$

$$v_x = 3 - i = 7.5 \text{ V.}$$

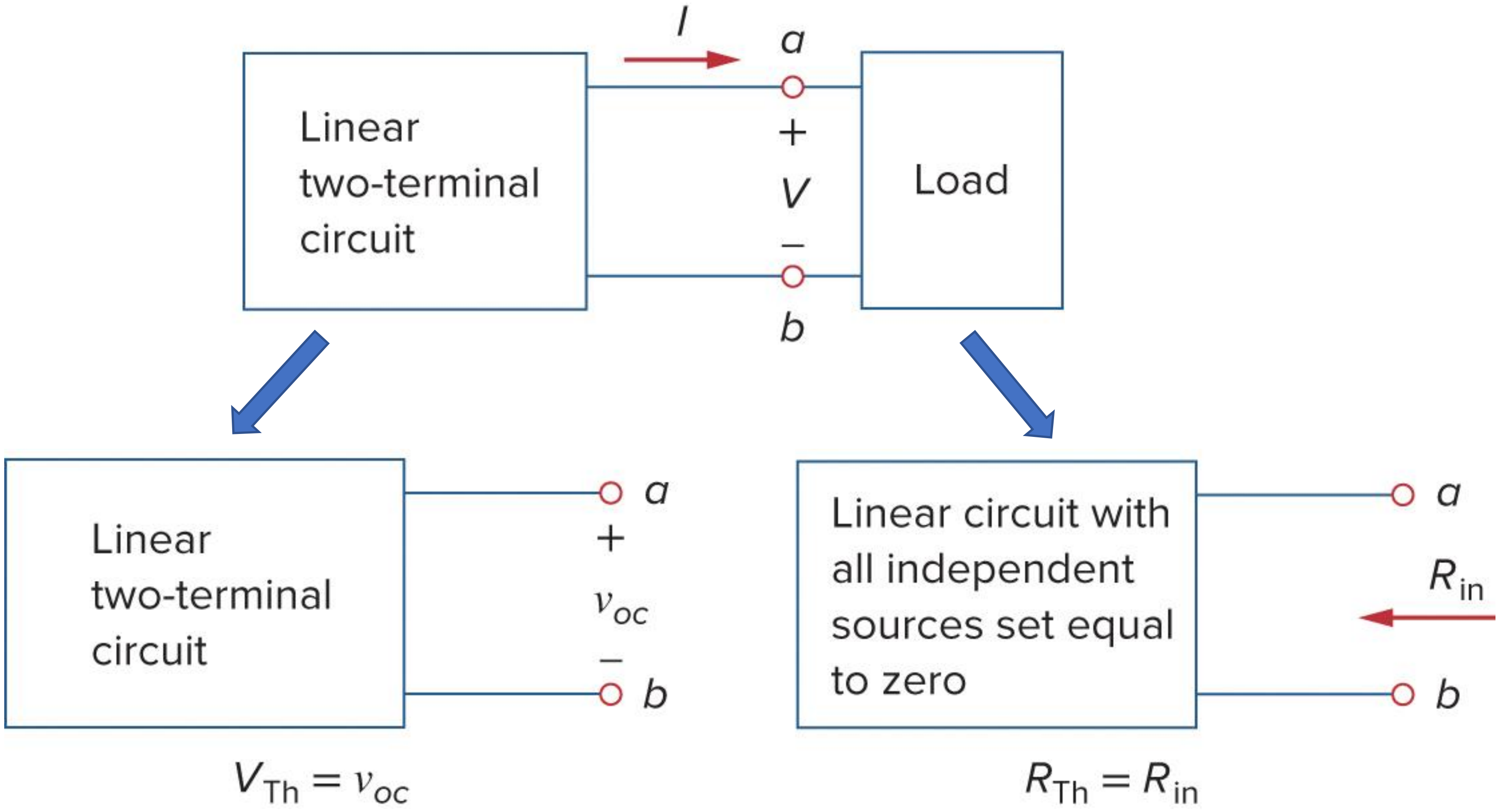
Thevenin's Theorem

- Occurs when a particular element in a circuit is variable (usually called the load) and other elements are fixed.
- Typical example: household outlet terminal may be connected to different appliances constituting a variable load.
- Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.
- **Thevenin Theorem:** Linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



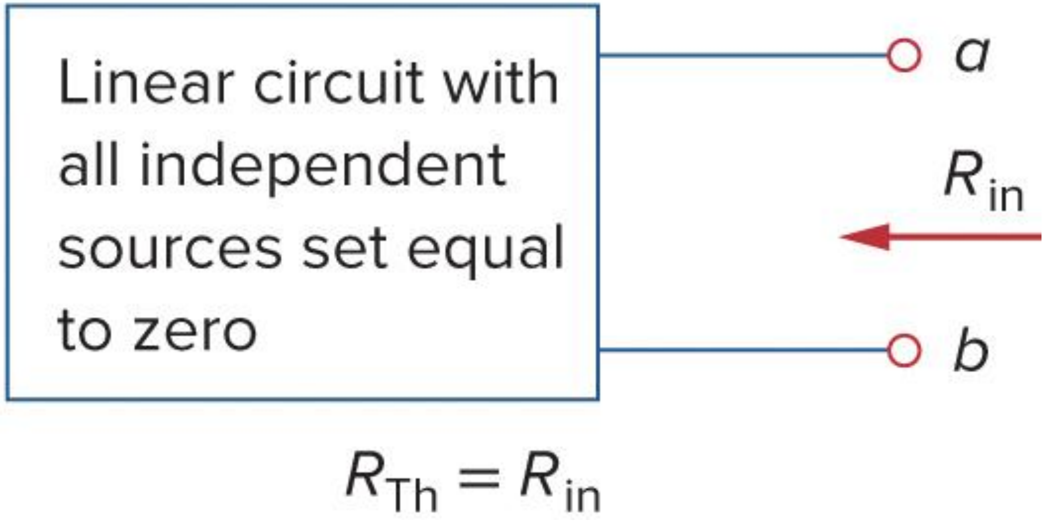
Thevenin's Theorem

- To find V_{Th}, the terminals *a-b* are made open-circuit by removing the load.
- To find R_{Th}, load is disconnected, and terminal *a-b* open circuited. And then turn off all independent sources.



Thevenin's Theorem

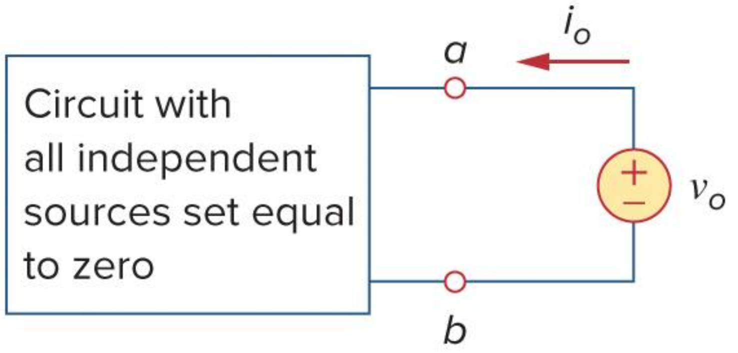
- To find R_{Th} (Thevenin resistance), two cases:
- Case I: If the network has no dependent sources, turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals *a* and *b*.



Thevenin's Theorem

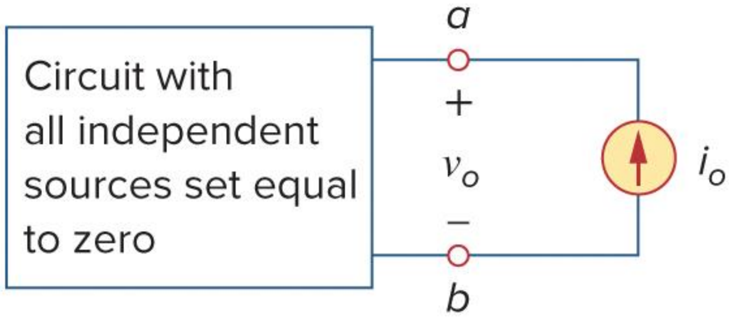
- Case II: If the network has dependent sources, turn off only all independent sources. As with superposition, do not turn off dependent sources. Apply a voltage source v_0 at terminals *a*-*b* and determine the resulting current i_0. Then R_{Th} can be calculated as

$$R_{Th} = \frac{v_0}{i_0}$$



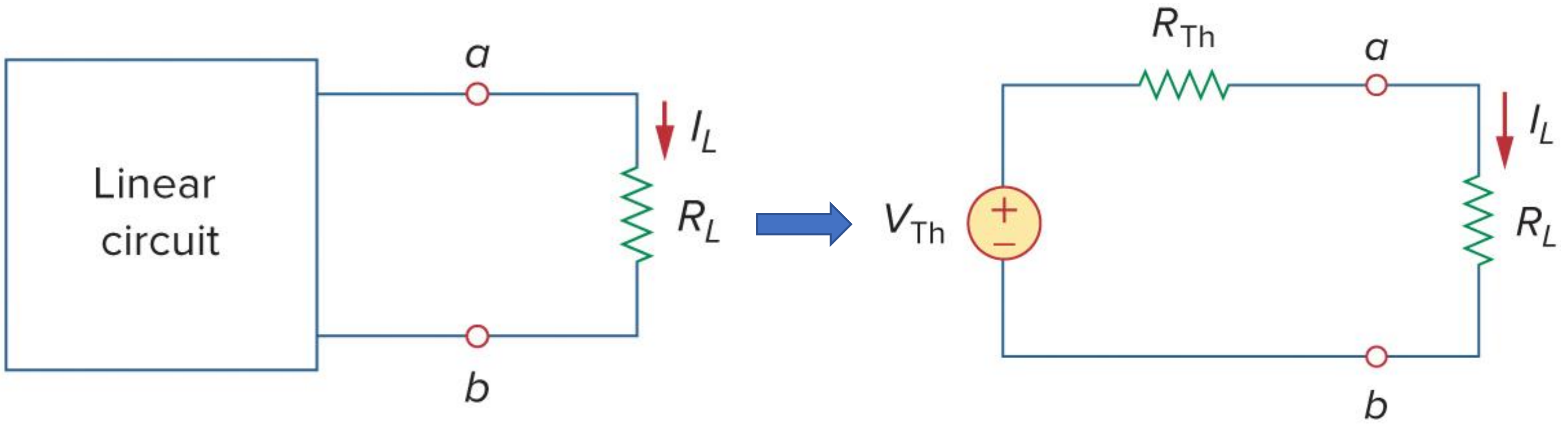
- Alternatively, insert a current source i_0 at terminal *a*-*b* and find terminal voltage v_0. Again, R_{Th} is obtained by

$$R_{Th} = \frac{v_0}{i_0}$$



Thevenin's Theorem

- Very important in circuit analysis because it helps simplify a circuit.
- A large circuit maybe replaced by a single independent voltage source and a single resistor as shown in the figure below.

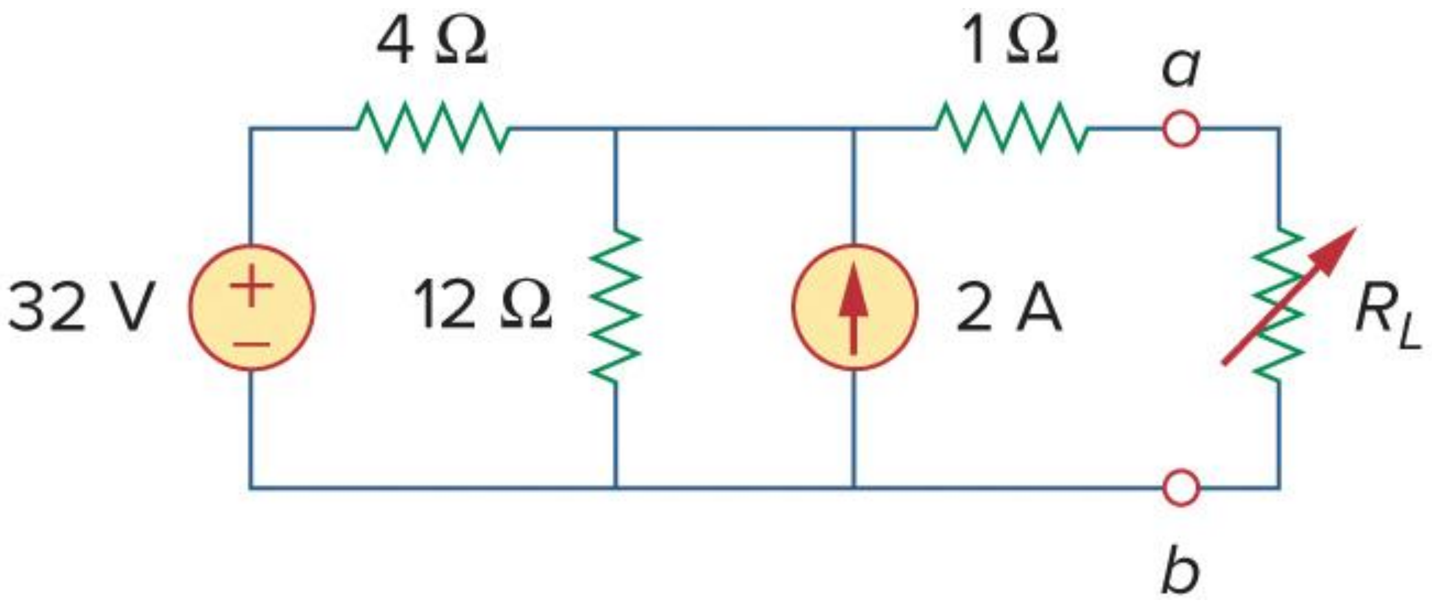


$$\underbrace{I_L = \frac{V_{Th}}{R_{Th} + R_L}}_{\text{Ohm's law}}$$

$$V_L = R_L I_L = \underbrace{\frac{R_L}{R_{Th} + R_L} V_{Th}}_{\text{Voltage division}}$$

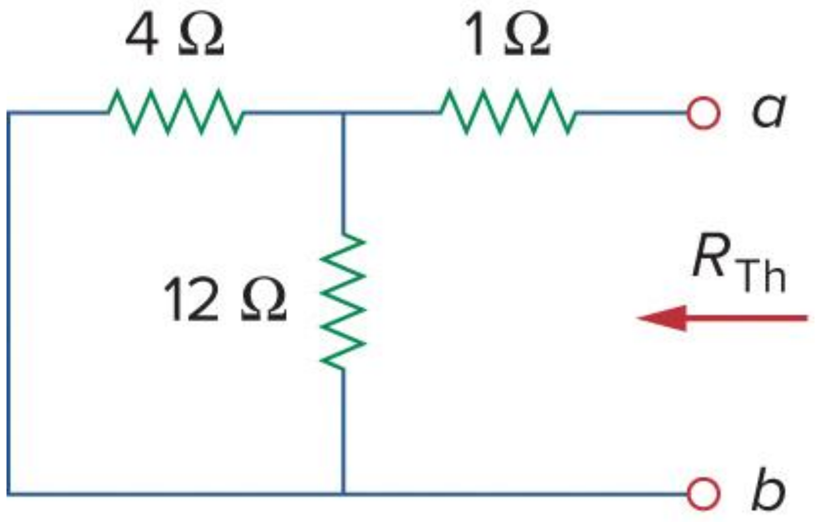
Example 8

Find the Thevenin equivalent circuit to the left of the terminals *a*-*b* in the circuit shown below. Then find the current through $R_L = 6, 16$, and $36\ \Omega$.



Solution

First, let's find Thevenin equivalent resistance R_{Th} by removing the load resistance and turning of the 32 V voltage source (short circuit) and 2 A current source (open circuit). Thus, the circuit becomes as follows:

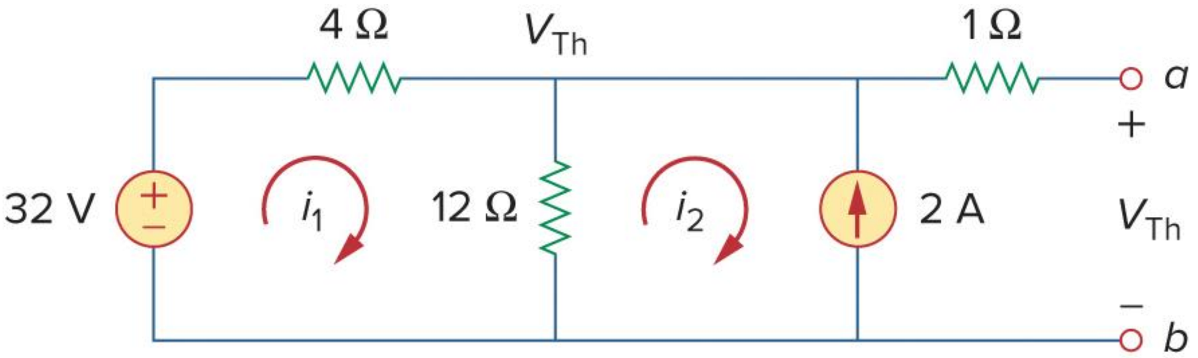


$$R_{Th} = (4\ \Omega || 12\ \Omega) + 1\Omega \qquad (4\ \Omega || 12\ \Omega) = \frac{4 \times 12}{4 + 12} = 3\Omega$$

$$R_{Th} = 3\Omega + 1\Omega = 4\ \Omega$$

Solution

To find Thevenin voltage V_{Th}, remove the load resistance and apply mesh analysis to two loops for the following circuit:



$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2 \text{ A}$$

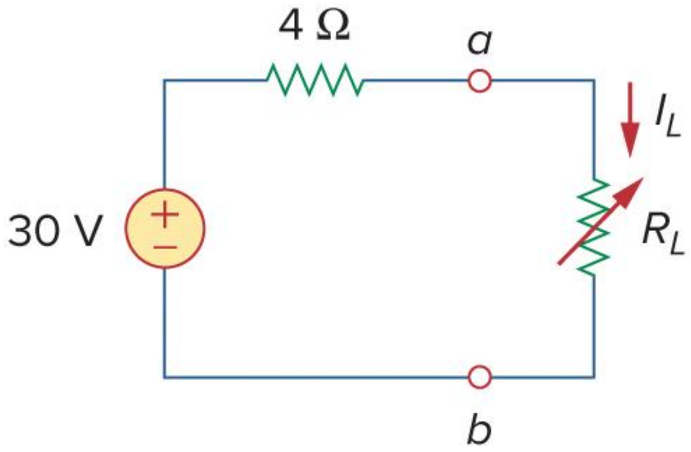
$$V_{\text{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30 \text{ V}$$

Alternatively, use nodal analysis to find V_{Th}.
Ignore 1 Ω resistance since no current flows there

$$\frac{32 - V_{\text{Th}}}{4} + 2 = \frac{V_{\text{Th}}}{12}$$

$$96 - 3V_{\text{Th}} + 24 = V_{\text{Th}} \quad \Rightarrow \quad V_{\text{Th}} = 30 \text{ V}$$

Thevenin equivalent circuit

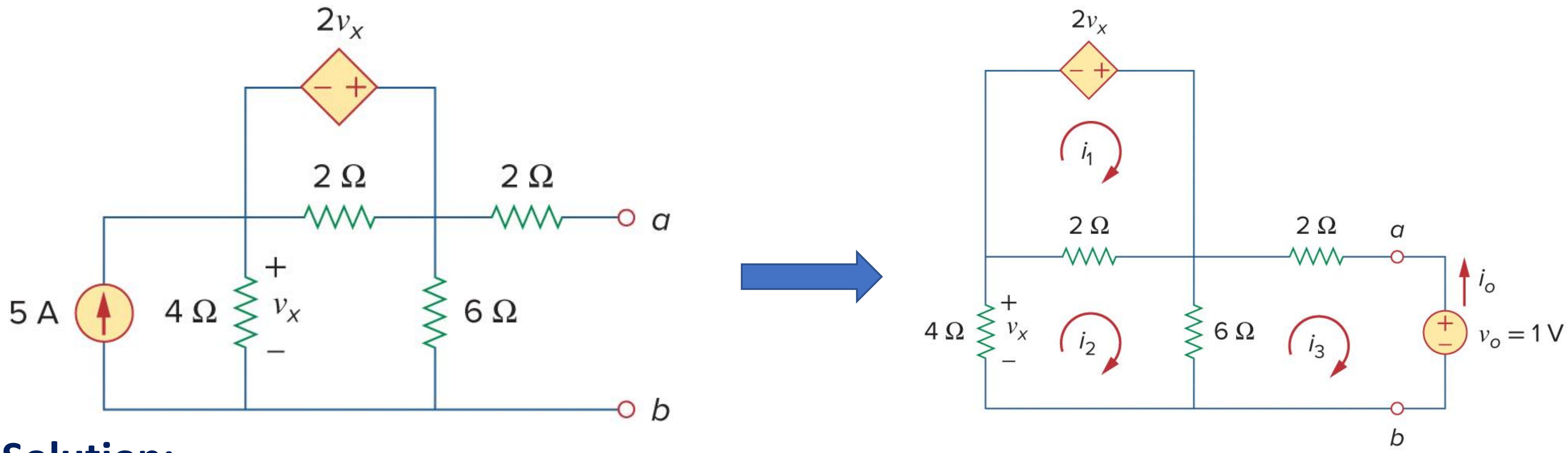


$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L} \qquad I_L = \frac{30}{20} = 1.5 \text{ A}$$

$$I_L = \frac{30}{10} = 3 \text{ A} \qquad I_L = \frac{30}{40} = 0.75 \text{ A}$$

Example 9

Find the Thevenin equivalent of the circuit shown below at terminals *a*-*b*.



Solution:

- 1st: find R_{Th} by turning off all independent sources but leave the dependent source as it is.
- Due to the presence of dependent source, we excite the circuit with a voltage source v_0 connected to the terminal *a*-*b*.
- Set $v_0 = 1$ V for ease of calculation. Our goal is to find i_0 in the following circuit in order to find Thevenin resistance R_{Th}. (Alternatively, we may insert 1 A current source as well)

$$R_{Th} = \frac{1}{i_0}$$

Solution

Applying mesh analysis to loop 1 in the circuit

$$-2v_x + 2(i_1 - i_2) = 0 \quad \Rightarrow \quad v_x = i_1 - i_2$$

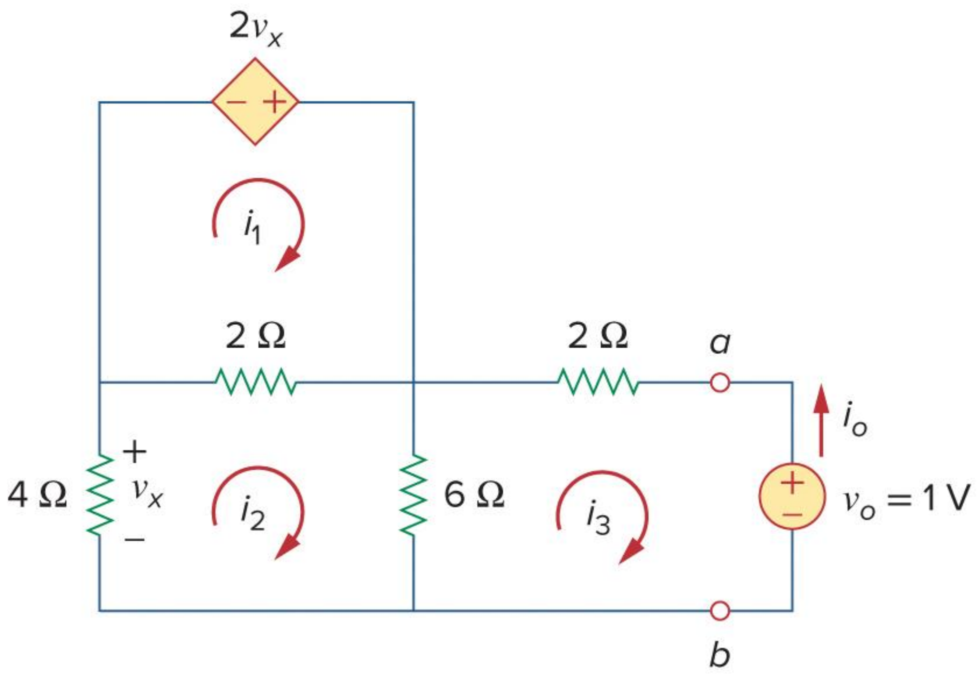
$$-4i_2 = v_x = i_1 - i_2 \quad \Rightarrow \quad i_1 = -3i_2$$

For loops 2 and 3, applying KVL produces

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

$$i_3 = -\frac{1}{6}\text{ A} \qquad i_o = -i_3 = 1/6 \text{ A.}$$



$$R_{\text{Th}} = \frac{1\text{ V}}{i_o} = 6\ \Omega$$

Solution

To get V_{Th}, we find v_{oc}

$$i_1 = 5$$

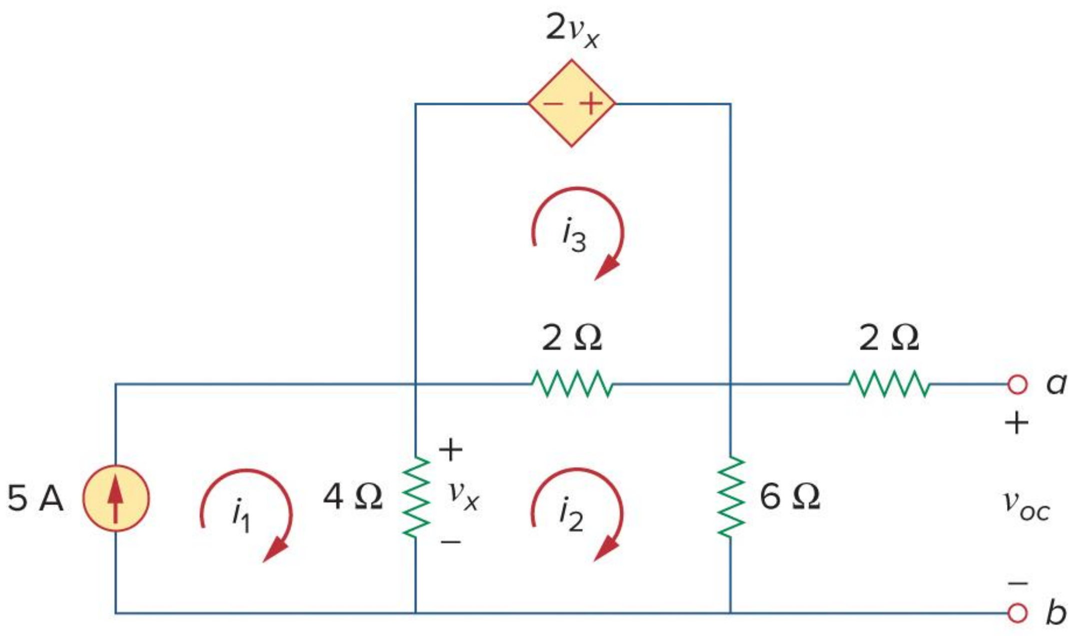
For loops 2 and 3, applying KVL produces

$$-2v_x + 2(i_3 - i_2) = 0 \quad \Rightarrow \quad v_x = i_3 - i_2$$
$$4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$$

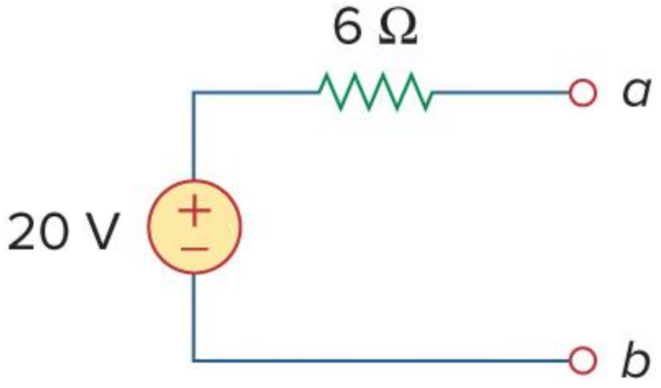
$$12i_2 - 4i_1 - 2i_3 = 0$$

$$4(i_1 - i_2) = v_x.$$

$$V_{\text{Th}} = v_{oc} = 6i_2 = 20 \text{ V}$$

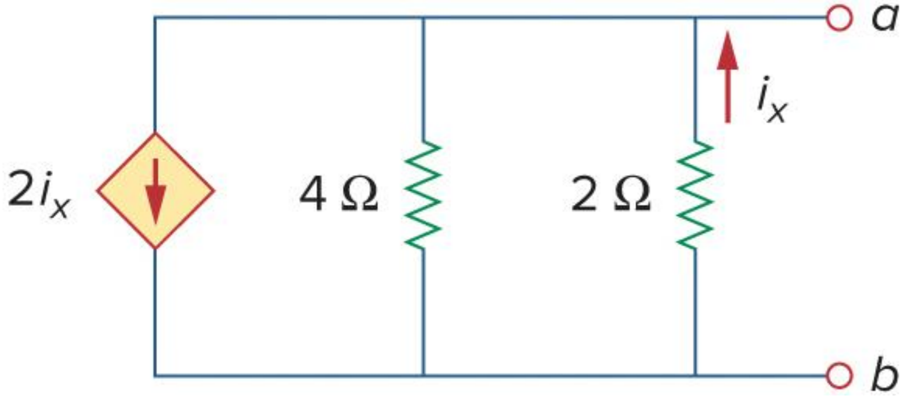


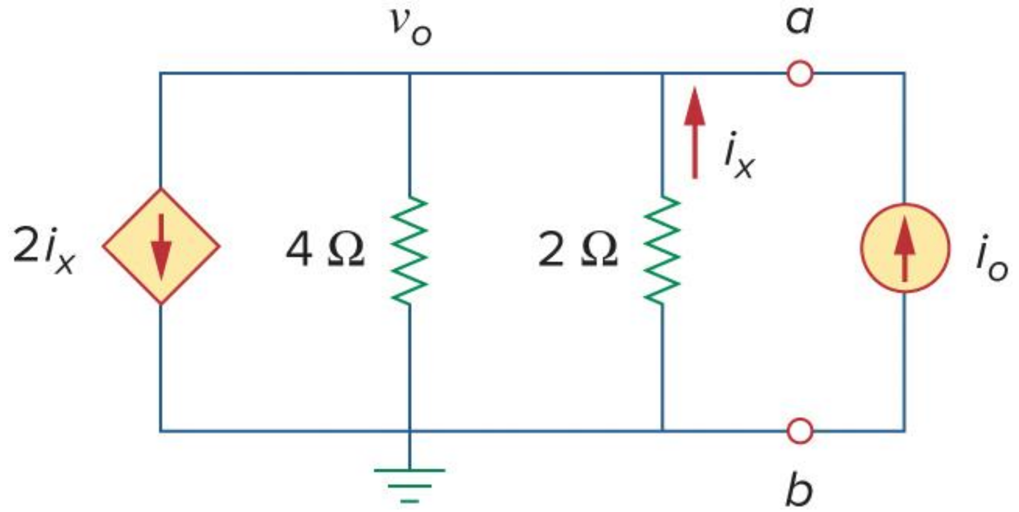
Thevenin equivalent circuit



Example 10

Determine the Thevenin equivalent of the circuit given below at terminals *a*-*b*.





Solution:

- First, let's find R_{Th}:
- No independent sources.
- Due to the presence of dependent source, we excite the circuit with either a voltage source v_0 or current source i_0 connected to the terminal *a*-*b*.
- Set $i_0 = 1\ \text{A}$ for sake of simplicity. Our goal is to find v_0 at the terminals *a*-*b* in the following circuit in order to find Thevenin resistance R_{Th}.

$$R_{Th} = \frac{v_0}{1}$$

Solution

$i_o = 1 \text{ A}.$

KCL at node a:

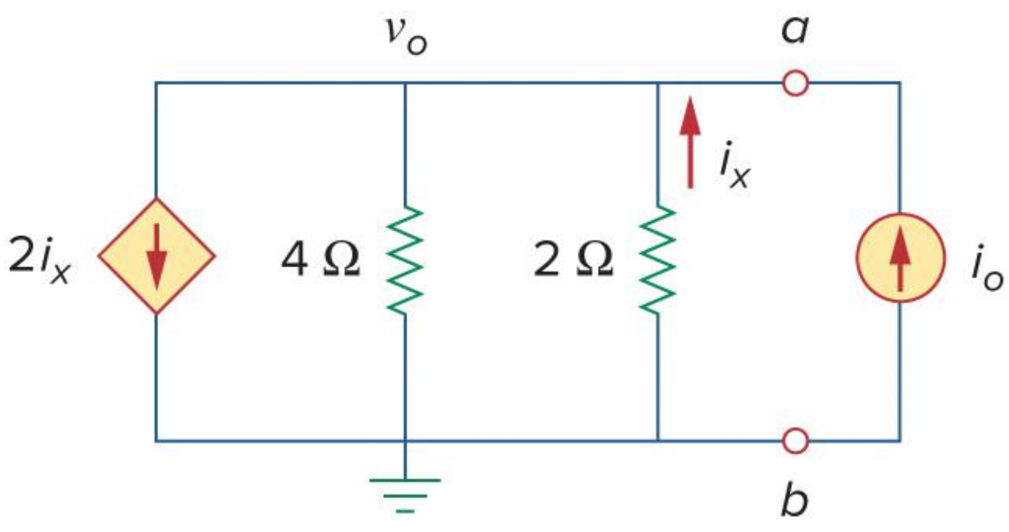
$$2i_x + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0$$

$$i_x = (0 - v_o)/2 = -v_o/2$$

$$2(-v_o/2) + (v_o - 0)/4 + (v_o - 0)/2 + (-1) = 0$$

$$= (-1 + \tfrac{1}{4} + \tfrac{1}{2})v_o \; - 1 \qquad \text{or} \qquad v_o = -4 \text{ V}$$

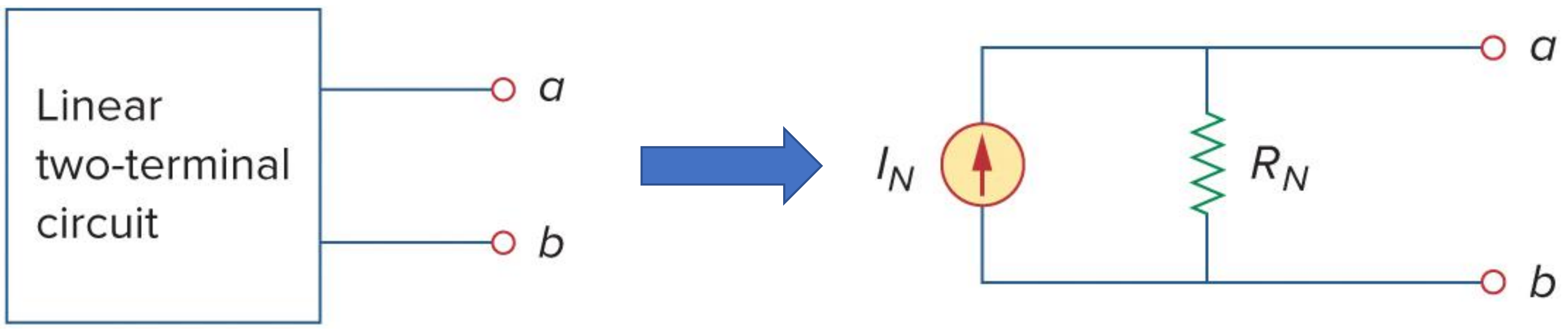
$$R_{\text{Th}} = v_o/1 = \mathbf{-4\ \Omega}.$$



- Negative sign of the resistance indicates that the circuit is supplying power. Since resistors cannot supply power (they absorb power), it is the dependent source that supplies power. This is an example of how a dependent source and resistors could be used to stimulate negative resistance.
- $V_{Th} = 0$ because there is no independent source in the circuit.

Norton's Theorem

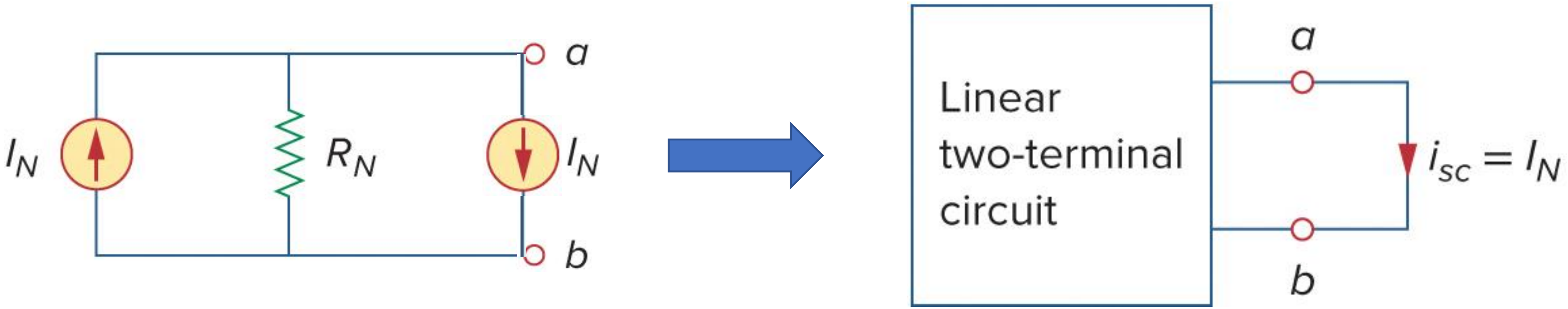
- What is Norton's theorem?
  - A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



- We find the R_N in the same way we find R_{Th}.
- From source transformation, the Thevenin and Norton resistance are equal, i.e., $R_N = R_{Th}$
- Dependent and independent sources are treated the same way as in Thevenin's theorem

Norton’s Theorem

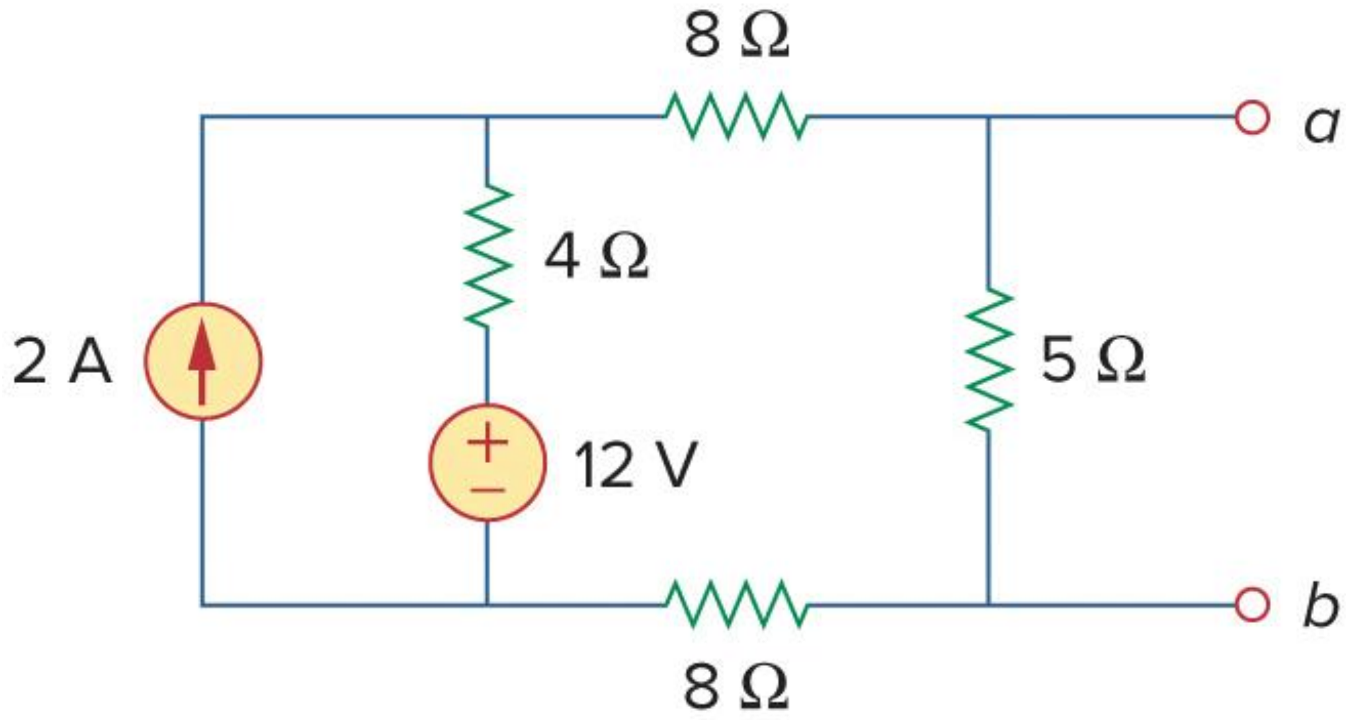
- To find the Norton current I_N, determine the short-circuit current flowing from terminal *a* to *b* as shown below.



$$R_N = R_{Th} \qquad I_N = \frac{V_{Th}}{R_{Th}}$$

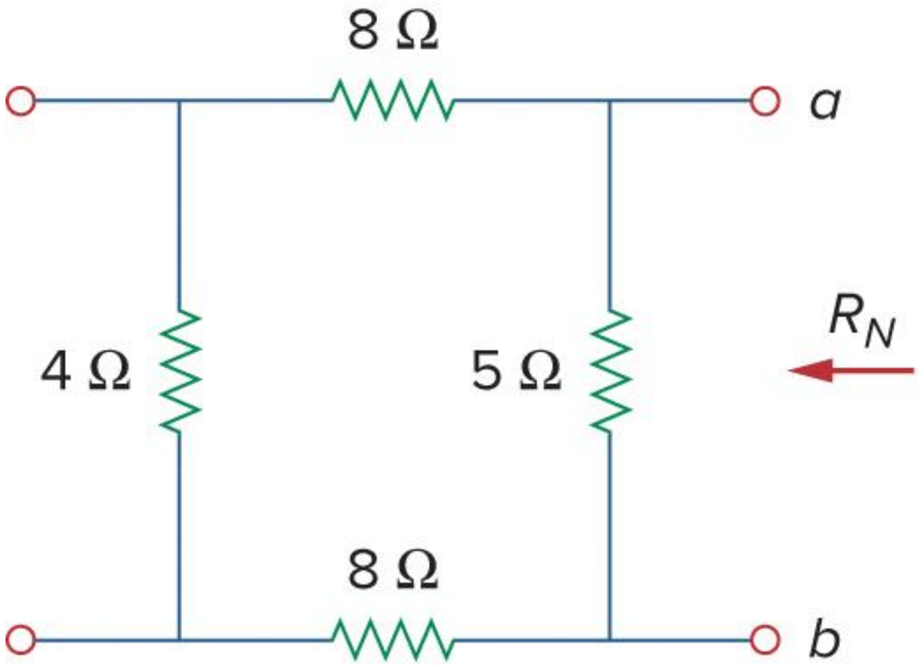
Example 11

Find the Norton equivalent circuit of the circuit shown below at terminals *a*-*b*.



Solution

- Find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit.
- Turn of all independent sources (set them equal to zero)
- Voltage source is short circuit while current source is open circuit.



$$(8\,\Omega + 4\,\Omega + 8\,\Omega)||5\,\Omega$$

$$R_N = \frac{20 \times 5}{20 + 5} \Rightarrow R_N = 4\,\Omega$$

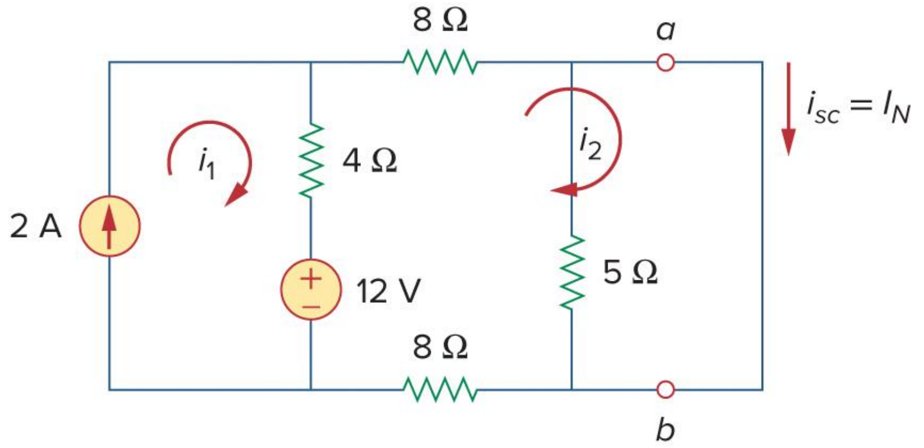
Solution

To find I_N, short circuit terminals *a*-*b*.

Apply KVL:

$$i_1 = 2 \text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1 \text{ A} = i_{sc} = I_N$$



Alternatively, we may determine I_N as follows:

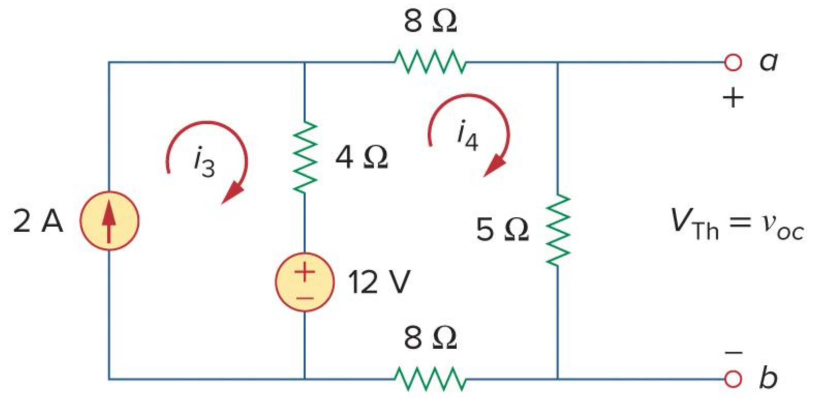
$$I_N = \frac{V_{Th}}{R_{Th}} \qquad R_N = R_{Th} = 4\,\Omega$$

$$i_3 = 2 \text{ A}$$

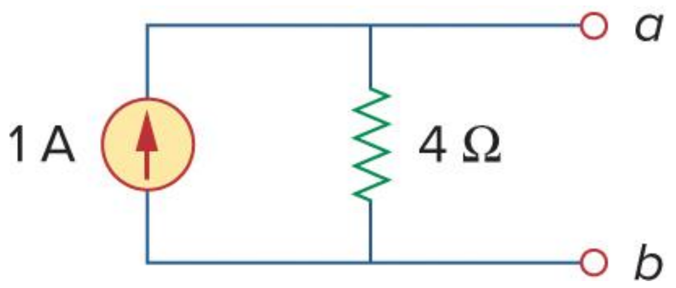
$$25i_4 - 4i_3 - 12 = 0 \Rightarrow i_4 = 0.8 \text{ A}$$

$$v_{oc} = V_{\text{Th}} = 5i_4 = 4 \text{ V}$$

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}} = \frac{4}{4} = 1 \text{ A}$$

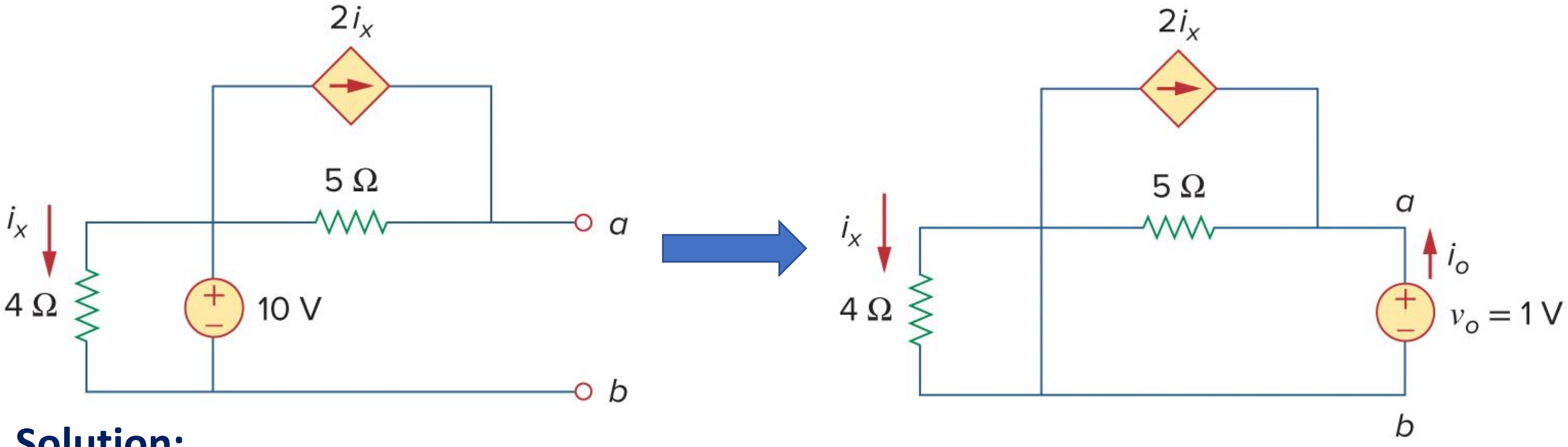


Norton equivalent circuit



Example 12

Using the Norton's theorem, find R_N and I_N of the circuit shown below at terminals *a*-*b*.



Solution:

To find R_N, turn off independent sources and leave dependent source as it is and also connect a voltage source of $v_0 = 1\ V$ to terminals *a*-*b*.

$$v = iR \Rightarrow i_0 = \frac{v_0}{R} = \frac{1}{5} \Rightarrow i_0 = 0.2\ A \qquad R_N = \frac{v_0}{i_0} = \frac{1}{0.2} \Rightarrow R_N = 5\ \Omega$$

Solution

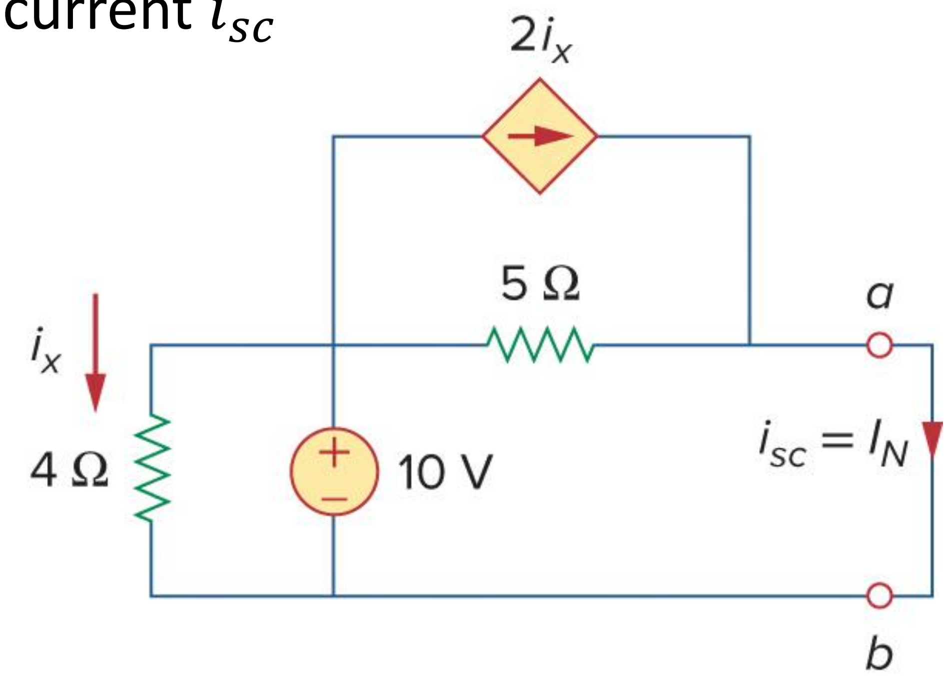
To find I_N, short-circuit terminals *a*-*b* and find current i_{sc}

$$i_x = \frac{10}{4} = 2.5\ \text{A}$$

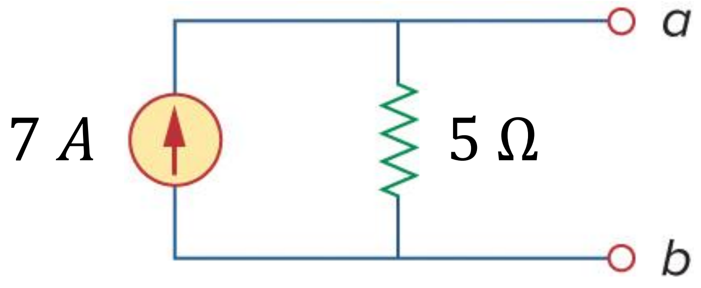
At node a, KCL gives

$$i_{sc} = \frac{10}{5} + 2i_x = 2 + 2(2.5) = 7\ \text{A}$$

$$I_N = 7\ \text{A}$$



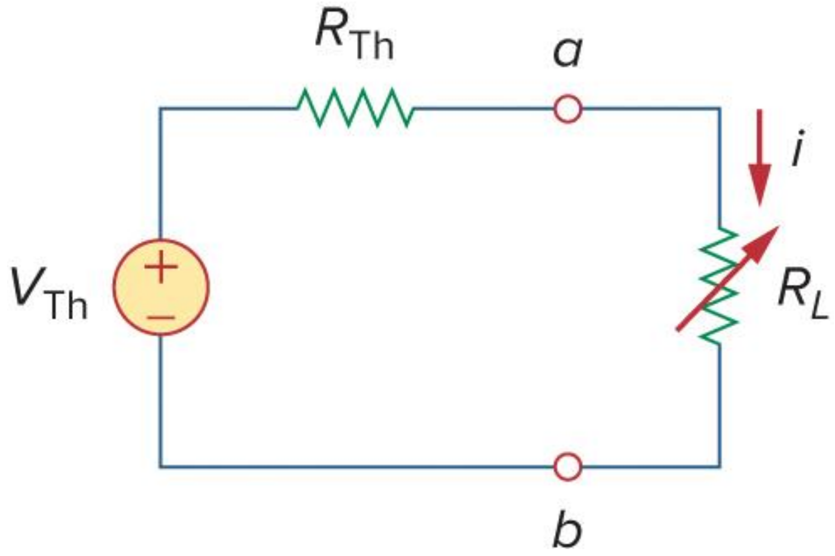
Norton equivalent circuit



Maximum Power Transfer

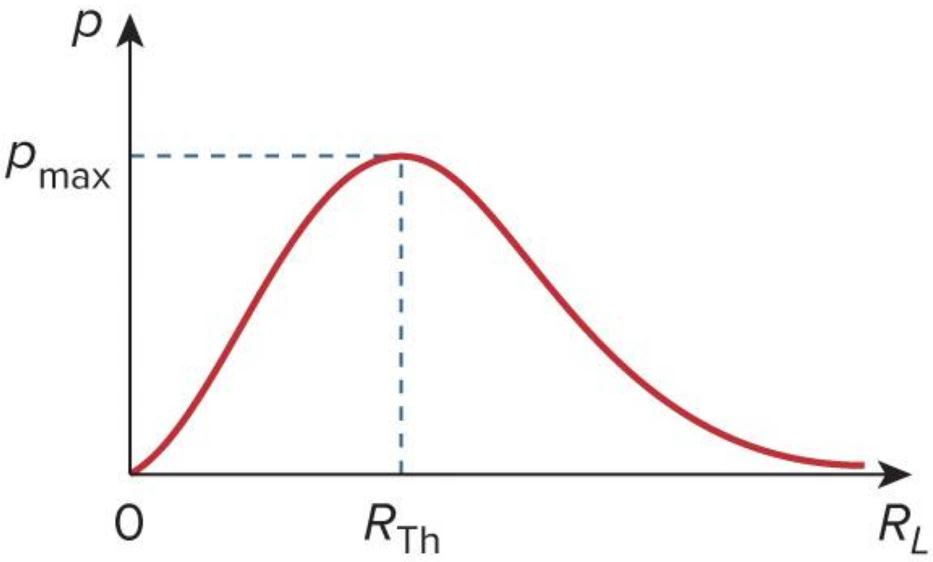
- In many practical application, a circuit is designed to provide power to a load.
- There are applications in areas such as communication where it is desirable to maximize the power delivered to a load.
- Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load.
- Power delivered to a load is given by

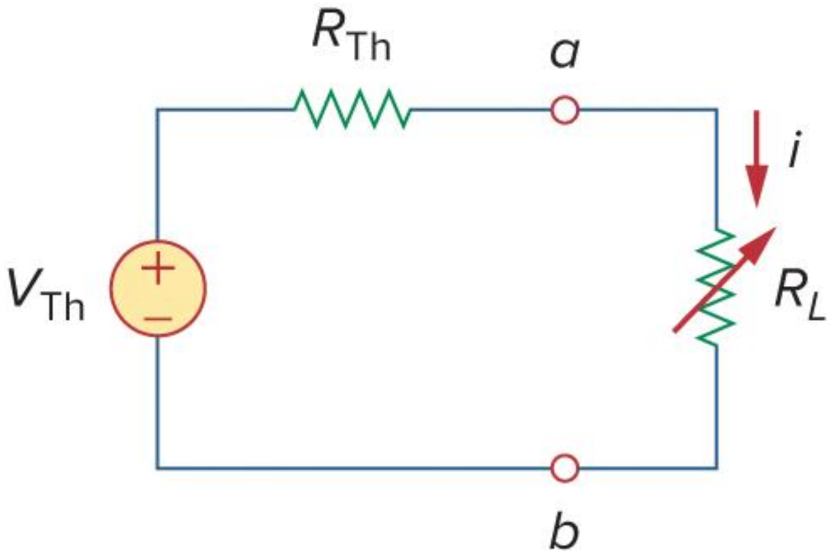
$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$



Maximum Power Transfer

- For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as follows:





$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

- Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load

$R_L = R_{Th}$ ⟶ Source and load are matched

Maximum Power Transfer: Proof

- To proof the maximum power theorem, differentiate p with respect to R_L and set the result equal to zero.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \quad \Longrightarrow \quad \boxed{p_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4}\right]$$

$$= V_{\text{Th}}^2 \left[\frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3}\right] = 0$$

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L)$$

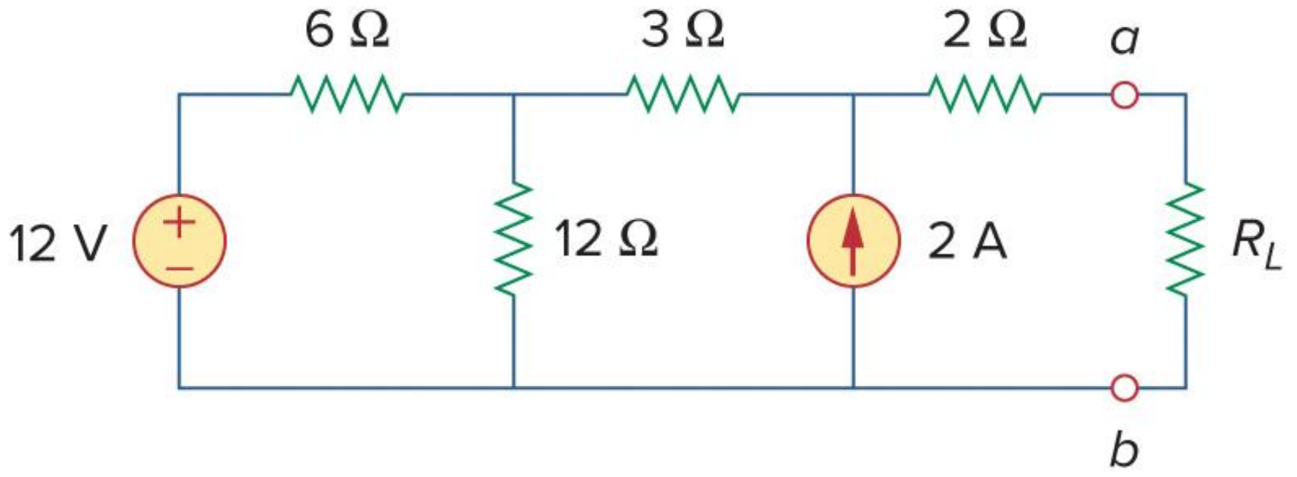
$$\boxed{R_L = R_{Th}}$$

Recall: Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).f'(x) - f(x).g'(x)}{[g(x)]^2}$$

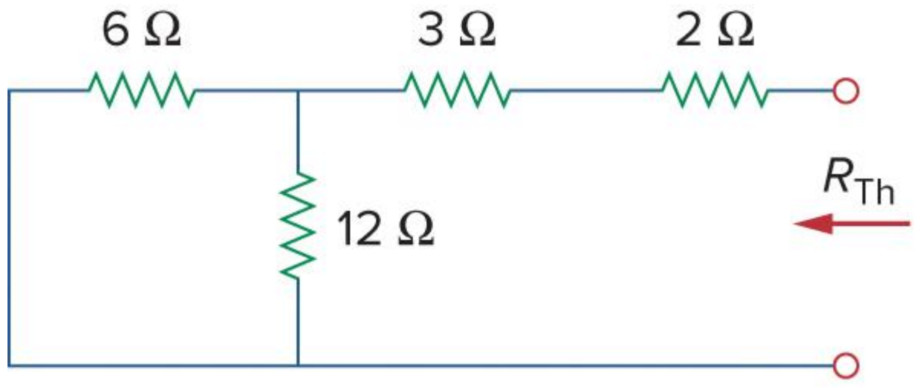
Example 13

Find the value of R_L for maximum power transfer in the circuit shown below. Find the maximum power.



Solution:

- To find R_L for maximum power transfer, find R_{Th} since $R_L = R_{Th}$ for maximum power transfer.
- To find R_{Th}, turn off all independent sources and disconnect the load resistance.



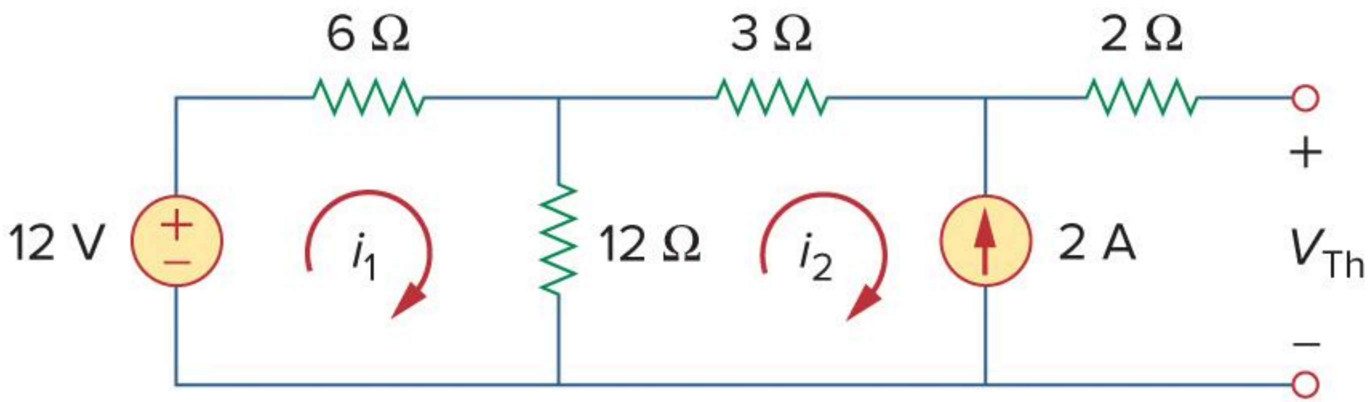
$$R_{Th} = (6\ \Omega || 12\ \Omega) + 3\ \Omega + 2\ \Omega$$

$$R_{Th} = R_L = 9\ \Omega$$

Solution

- To find maximum power transfer, determine V_{Th}:

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$



- Apply KVL to Loop 1: $-12 + 18i_1 - 12i_2 = 0, \; i_2 = -2 \text{ A} \;\; \Rightarrow \;\; i_1 = -2/3$
- Apply KVL to outer Loop : $-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \;\; \Rightarrow V_{\text{Th}} = 22 \text{ V}$

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$