Quiz 1 Solution

1) To determine whether a signal is periodic and find its fundamental period, we need to check if the signal repeats itself after a certain interval.

a. $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$

To check periodicity, we need to find if there exists a positive integer N such that:

$x[n] = x[n + N]$

Let's compute the period for $x[n]$:

$\sin\left(\frac{6\pi}{7}n + 1\right) = \sin\left(\frac{6\pi}{7}(n + N) + 1\right)$

For $x[n]$ to be periodic, the coefficient of n must be a multiple of 2π, and N should be an integer. However, in this case, $\frac{6\pi}{7}$ is an irrational multiple of π, so $x[n]$ is not periodic.

b. $x(t) = \cos(2t) + \sin(3t)$

Similar to part a, to be periodic, $x(t)$ should satisfy:

$x(t) = x(t + T)$

where T is the period. We need to find T for $x(t)$. Since the period of $\cos(2t)$ is π and the period of $\sin(3t)$ is $\frac{2\pi}{3}$, the least common multiple of these two periods is 2π. So, $x(t)$ is periodic with a fundamental period of 2π.

c. $x[n] = \cos\left(\frac{\pi}{8}n^2\right)$

To check periodicity, we again need to find if there exists a positive integer N such that:

$x[n] = x[n + N]$

$\cos\left(\frac{\pi}{8}n^2\right) = \cos\left(\frac{\pi}{8}(n + N)^2\right)$

This function is not periodic because the square of n inside the cosine function prevents it from repeating at integer multiples of N.

d. $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$

To check periodicity, we once again need to find if there exists a positive integer N such that:

$x[n] = x[n+N]$

$2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{4}(n+N)\right) + \sin\left(\frac{\pi}{8}(n+N)\right) - 2\cos\left(\frac{\pi}{2}(n+N) + \frac{\pi}{6}\right)$

This function is not periodic because the coefficients and frequencies of the terms prevent it from having a common period.

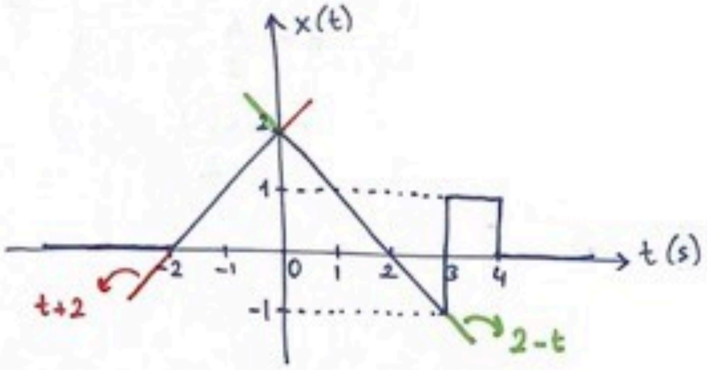
So, summarizing:

a. Not periodic.
b. Periodic with a fundamental period of 2π.
c. Not periodic.
d. Not periodic.

2) A continuous time signal $x(t)$ is given in the figure below.

a) Express $x(t)$ in terms of unit step functions.

b) Sketch $y(t) = 2\,x(2t+1)$



a) $x(t) = (t+2).\left[u(t+2) - u(t)\right] + (2-t).\left[u(t) - u(t-3)\right] + \left[u(t-3) - u(t-4)\right]$

$= (t+2).\,u(t+2) - 2.t.\,u(t) + (t-1).\,u(t-3) - u(t-4)$

b)