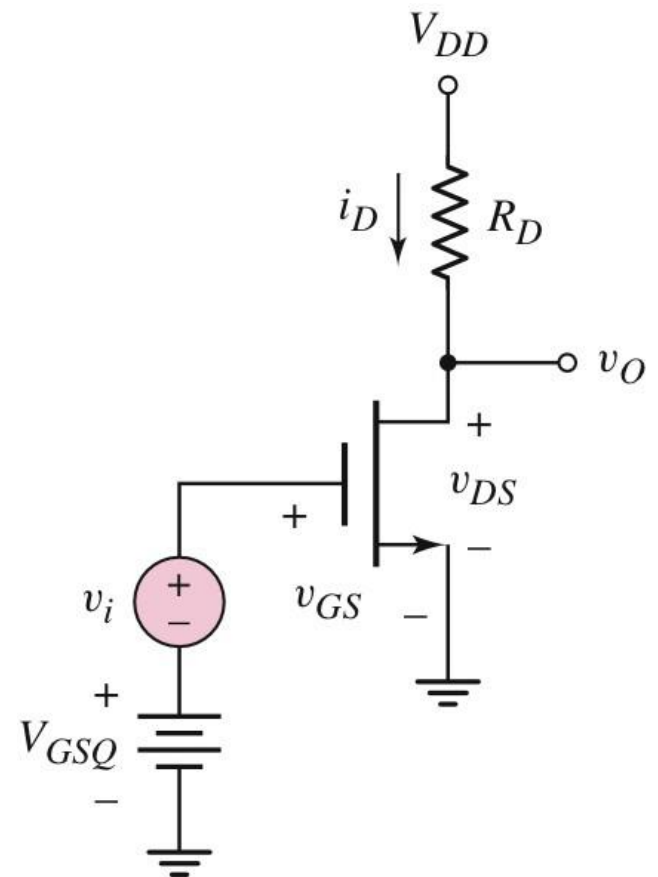


The MOSFET Amplifier

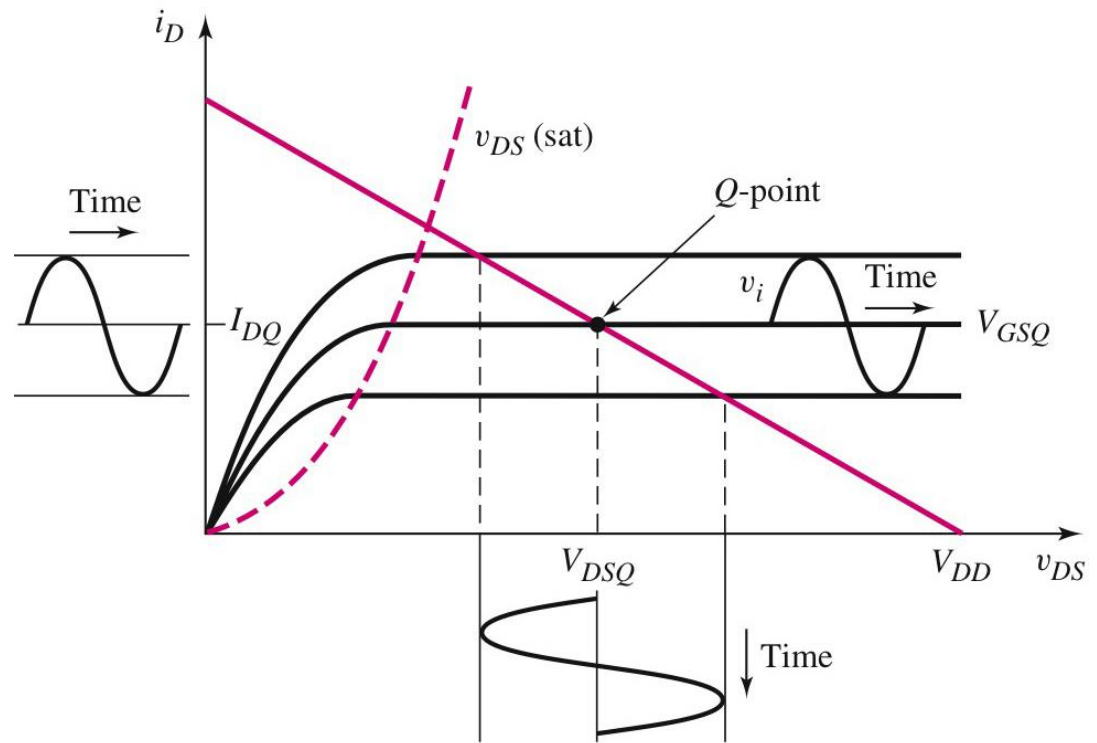
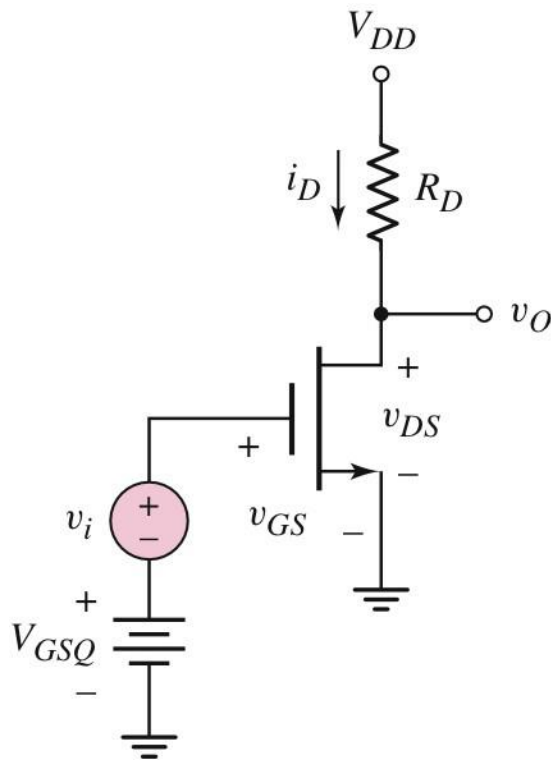
The figure shows a basic MOSFET amplifier where the dc source V_{GSQ} is used to provide the bias voltage between gate and source.

The ac voltage source v_i which represents the signal source is connected in series with the bias voltage V_{GSQ} .



The MOSFET Amplifier

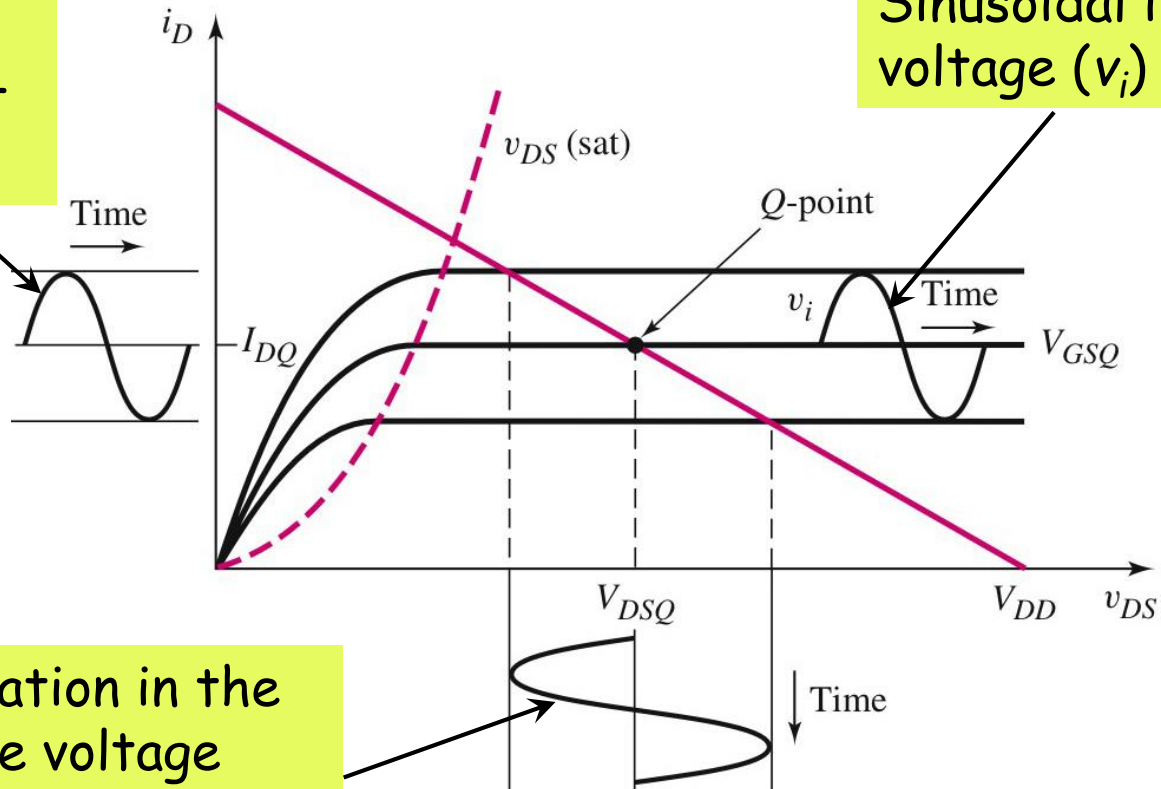
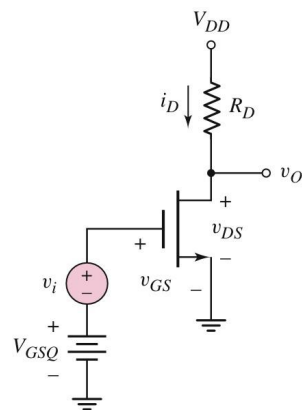
The Q-point is on dc load line and Q-point are functions of v_{GS} , V_{DD} and R_D and the transistor parameters.



The MOSFET Amplifier

The sinusoidal input voltage (v_i) causes variations in the gate-to-source voltage (v_{GS}), drain current (i_D) and drain-to-source voltage (v_{DS}).

Sinusoidal variation in the drain current (i_D)



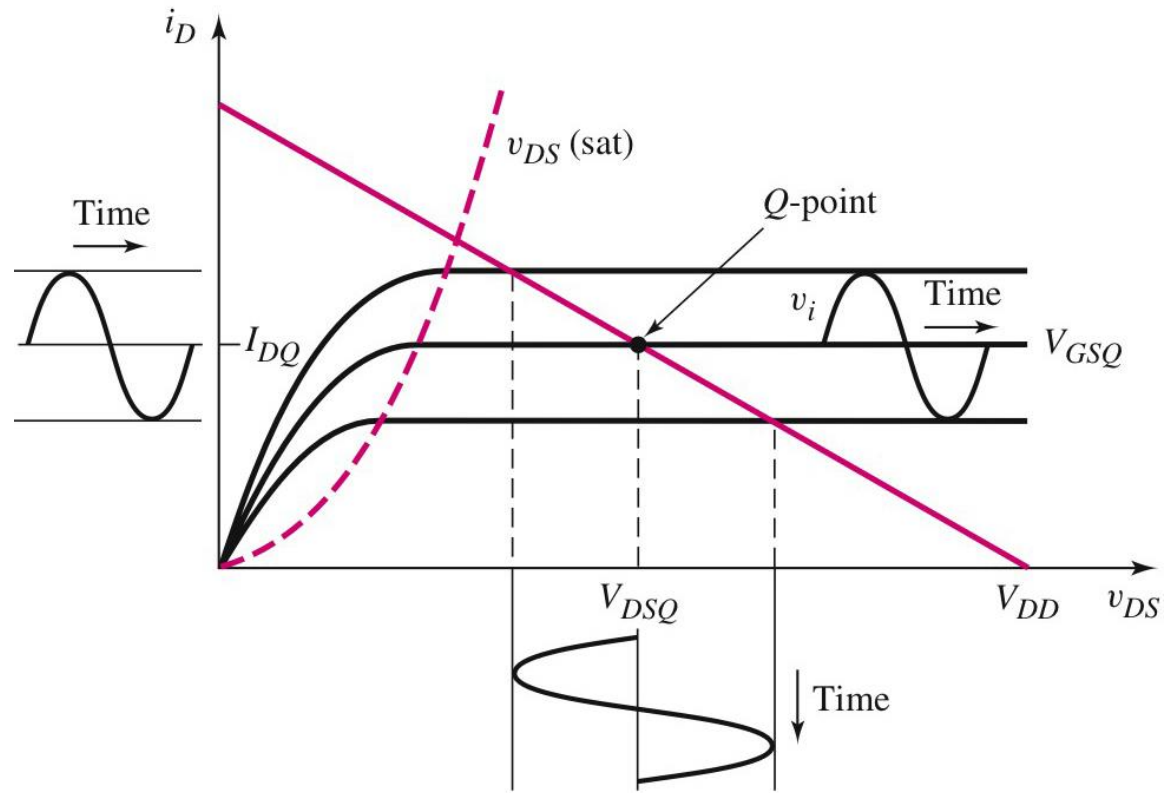
Sinusoidal variation in the drain-to-source voltage (v_{DS})

The MOSFET Amplifier

The total v_{GS} is the sum of V_{GSQ} and v_i i.e;

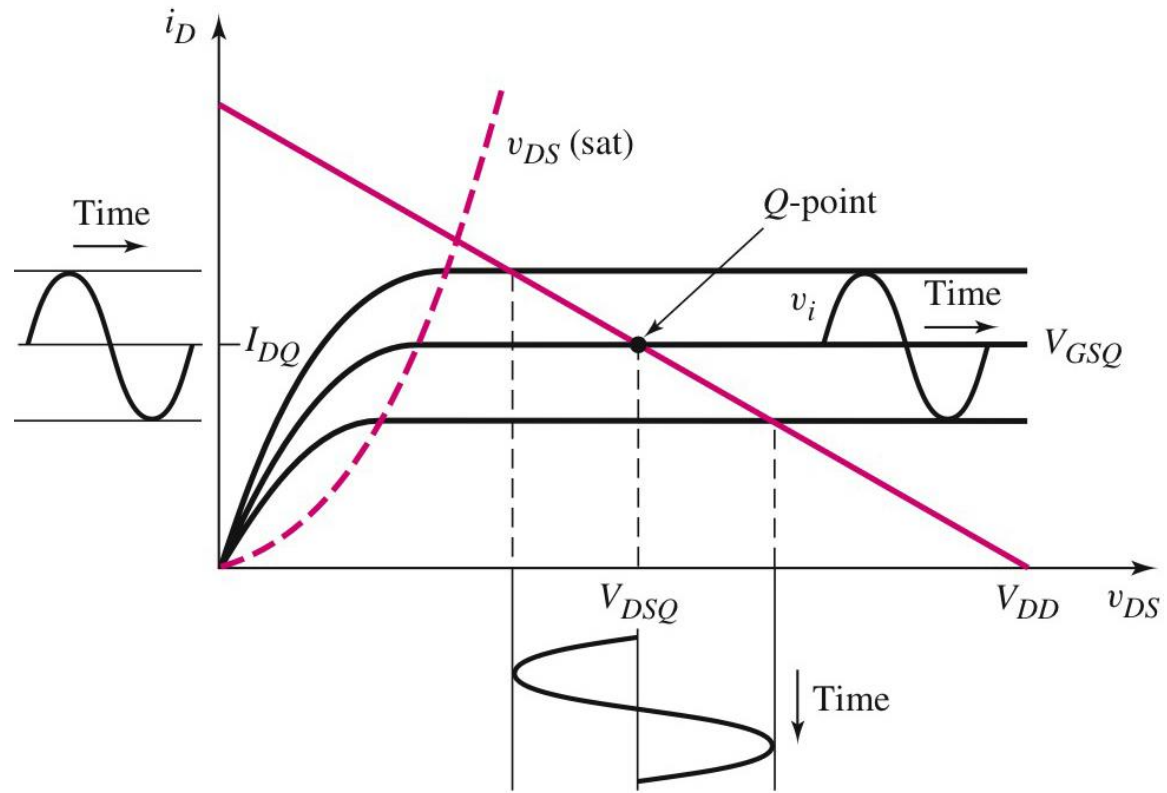
$$v_{GS} = V_{GSQ} + v_i$$

As v_i increases, the instantaneous value of v_{GS} increases, and the **bias point moves up on the load line.**



The MOSFET Amplifier

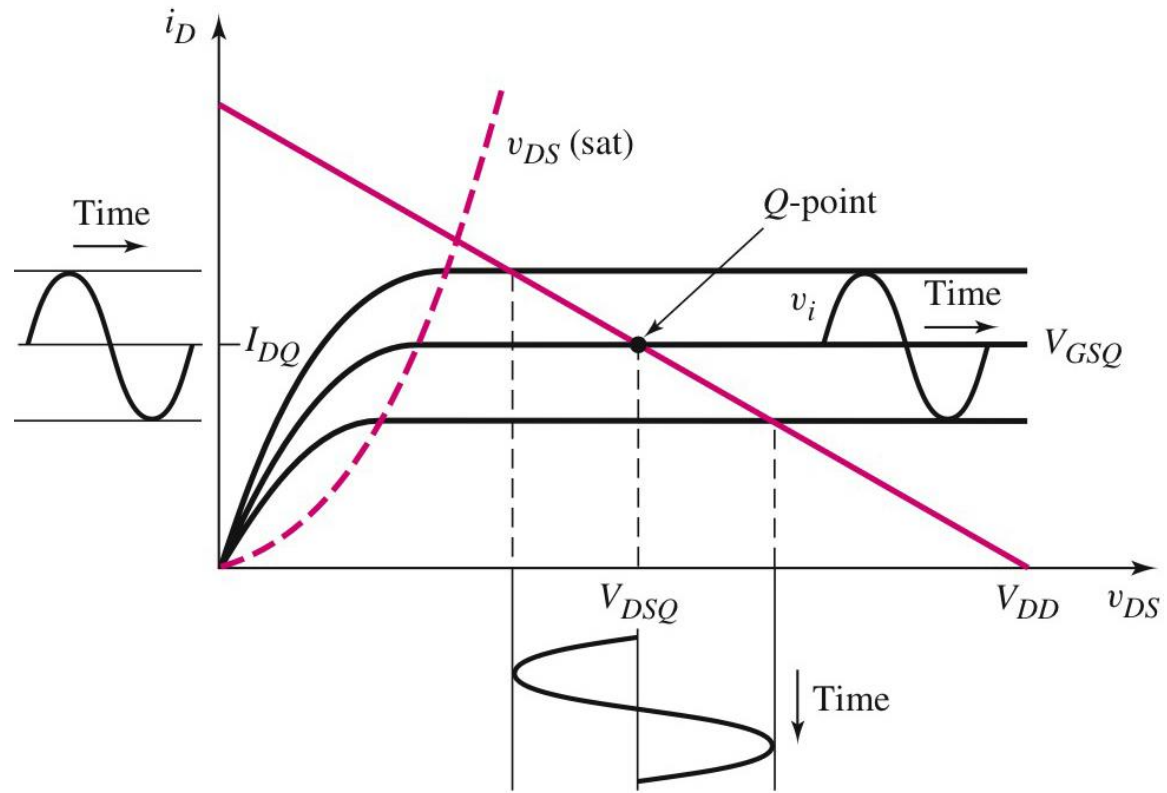
Larger value of v_{GS} means larger drain current i_D and smaller value of v_{DS} .



The MOSFET Amplifier

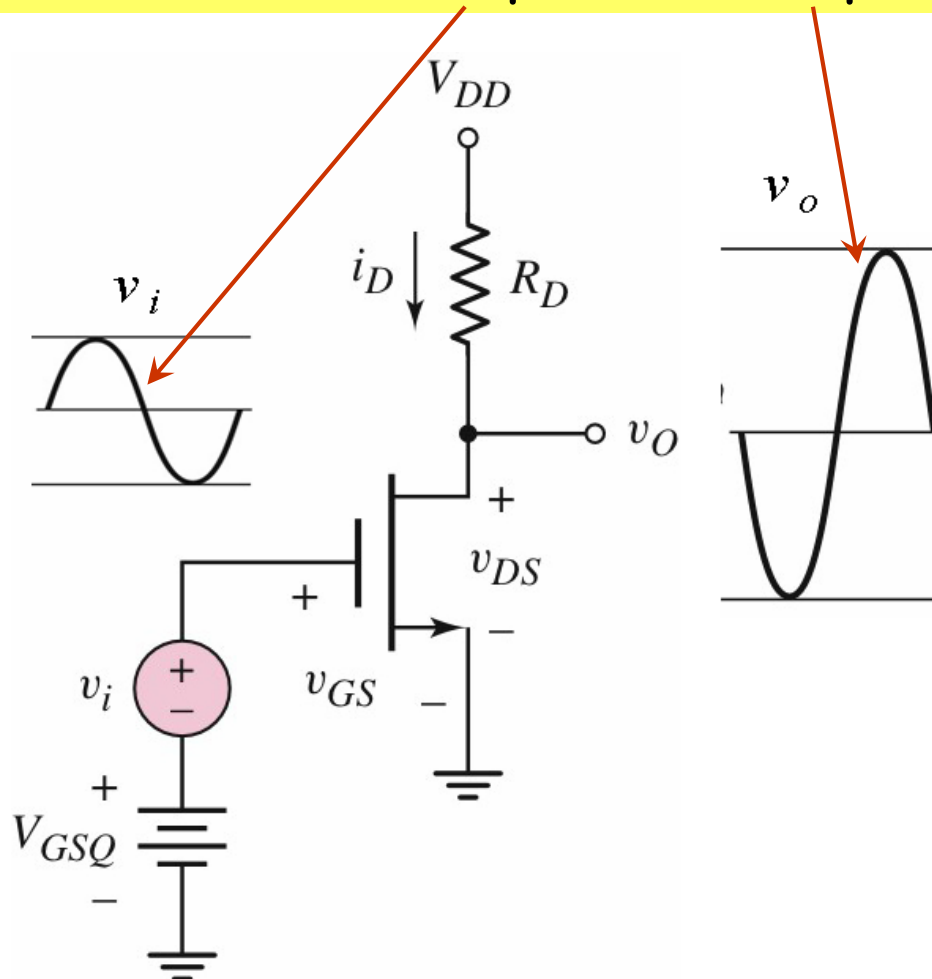
A smaller value of v_{GS} means smaller drain current i_D and larger value of v_{DS} .

Thus this configuration causes **phase inversion**.



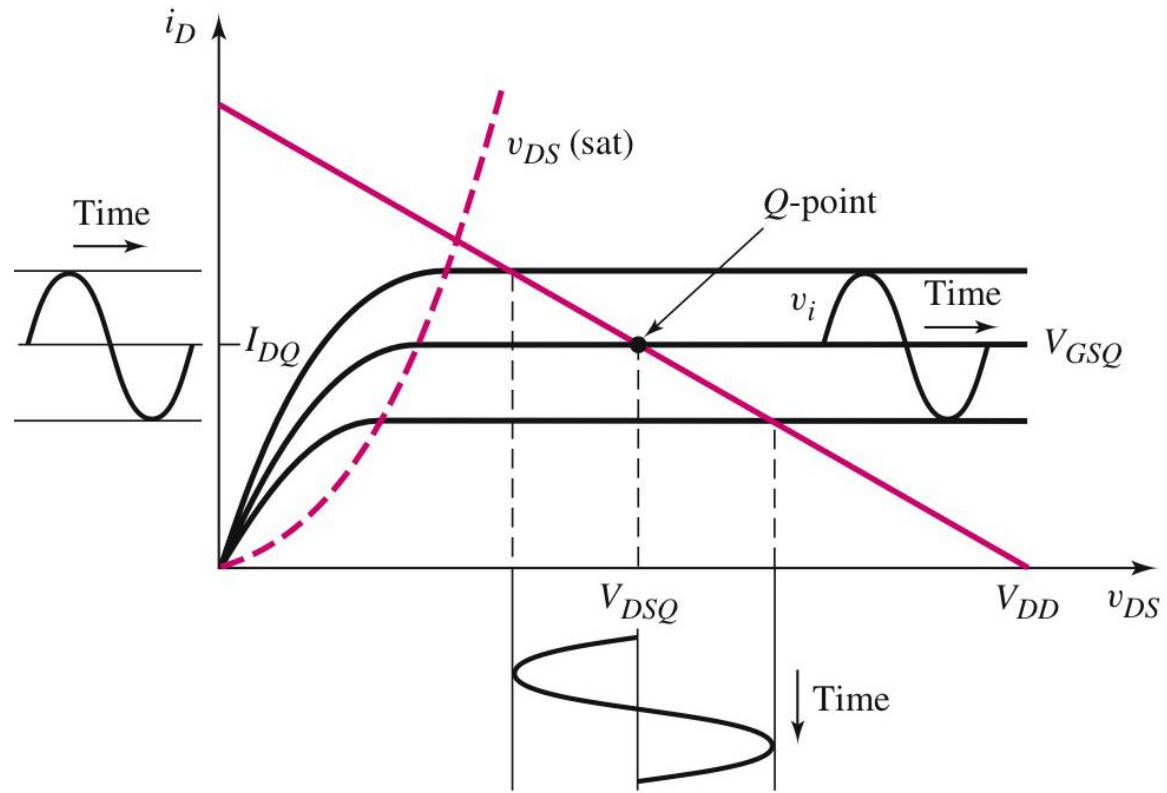
The MOSFET Amplifier

Phase inversion between input and output voltages



The MOSFET Amplifier

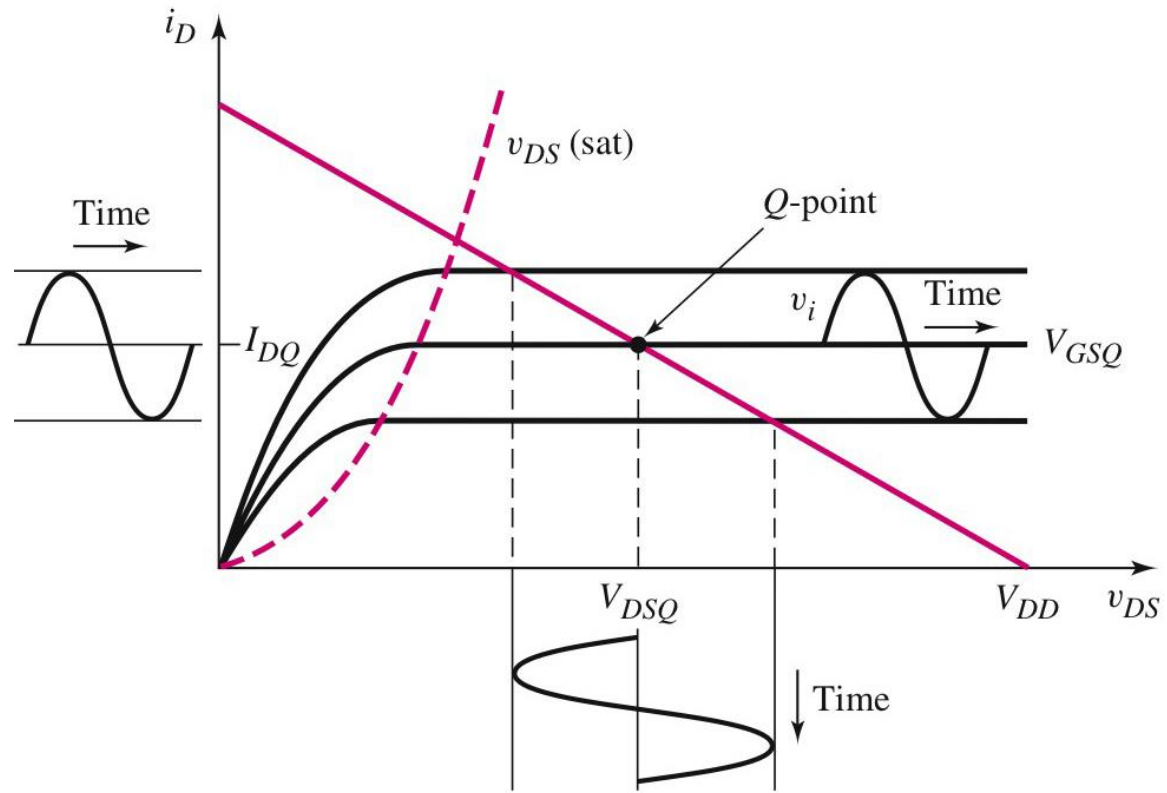
For the MOSFET to operate as a **linear amplifier**, it must be biased in the saturation region and the instantaneous drain current and drain-to-source voltage must also be confined to this region.



The MOSFET Amplifier

As long as the amplifier operates in the linear region, symmetrical input signal will produce symmetrical output.

If the limit is exceeded, the output signal will be clipped and distortion will occur.



The MOSFET Amplifier

The total instantaneous value of the gate-to-source voltage is given by;

$$v_{GS} = V_{GSQ} + v_i$$

Considering only ac component;

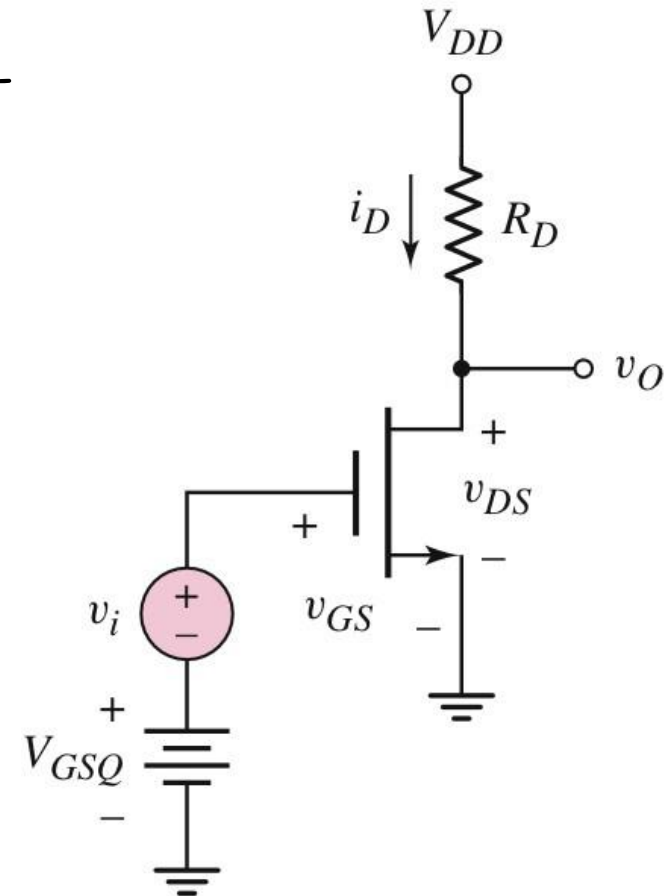
$$v_{gs} = v_i$$

Hence;

$$v_{GS} = V_{GSQ} + v_{gs}$$

dc component

ac component



The MOSFET Amplifier

The total instantaneous drain current is;

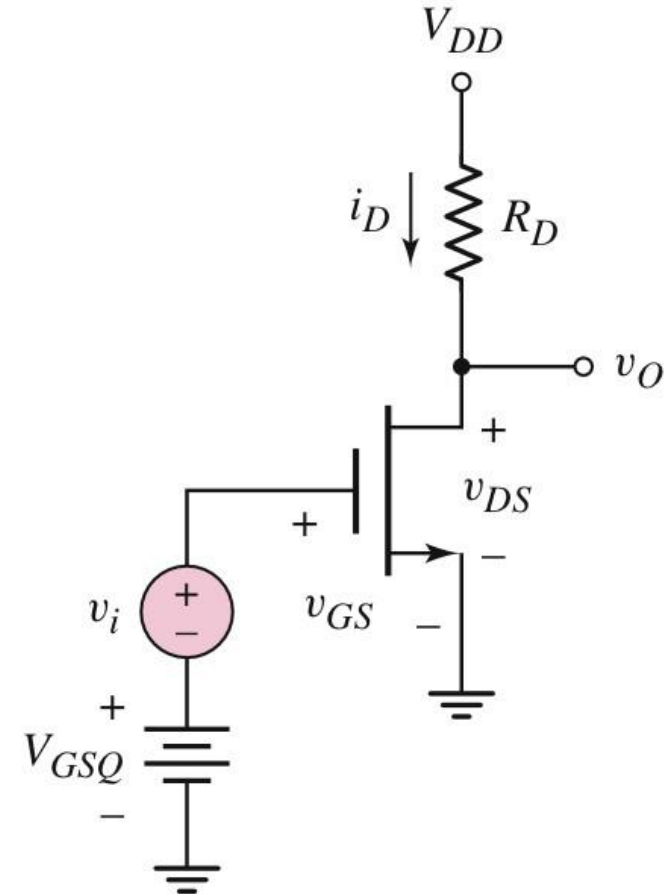
$$i_D = K_n (v_{GS} - V_{TN})^2$$

Substituting for v_{GS} :

$$\begin{aligned} i_D &= K_n (V_{GSQ} + v_{gs} - V_{TN})^2 \\ &= K_n [(V_{GSQ} - V_{TN}) + v_{gs}]^2 \end{aligned}$$

or

$$i_D = K_n (V_{GSQ} - V_{TN})^2 + 2K_n (V_{GSQ} - V_{TN}) v_{gs} + K_n v_{gs}^2$$

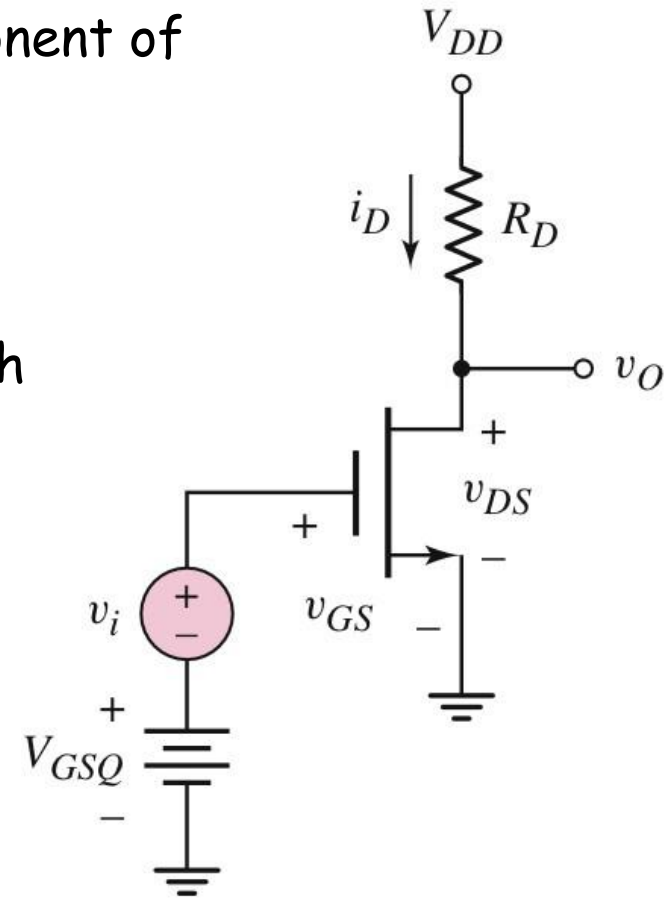


The MOSFET Amplifier

The term $K_n (V_{GSQ} - V_{TN})^2$ is the dc component of the drain current

The term $2K_n (V_{GSQ} - V_{TN}) v_{gs}$ is the time-varying component of the drain current which is linearly related to the input signal v_{gs} .

The term $K_n v_{gs}^2$ is the component of the drain current which is proportional to the square of the input signal v_{gs} .



$$i_D = K_n (V_{GSQ} - V_{TN})^2 + 2K_n (V_{GSQ} - V_{TN}) v_{gs} + K_n v_{gs}^2$$

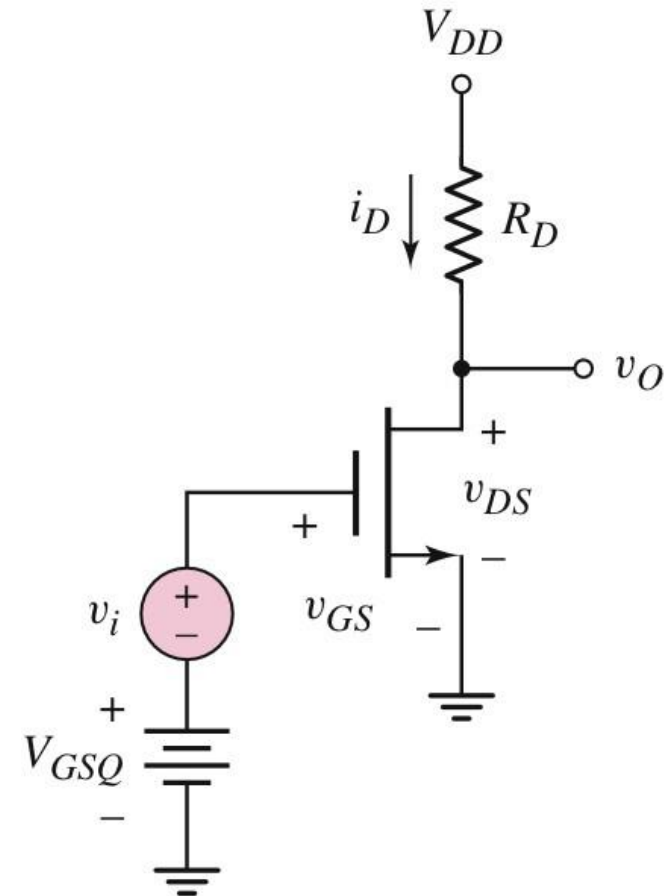
The MOSFET Amplifier

The term $K_n v_{gs}^2$ produces undesirable harmonics or non-linear distortion in the output voltage.

To minimize these harmonics, we must have;

$$v_{gs} \ll 2(V_{GSQ} - V_{TN})$$

This is the small-signal condition that must be satisfied for **linear amplifier**.



$$i_D = K_n (V_{GSQ} - V_{TN})^2 + 2K_n (V_{GSQ} - V_{TN}) v_{gs} + K_n v_{gs}^2$$

The MOSFET Amplifier

If the small-signal condition is satisfied, the term containing v_{gs}^2 may be neglected and the equation for i_D becomes;

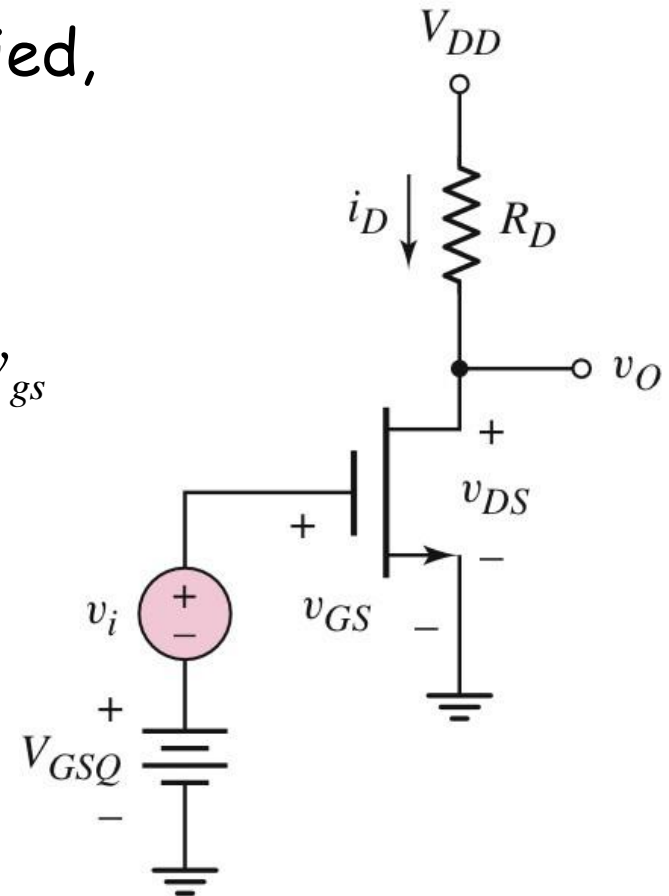
$$i_D = K_n (V_{GSQ} - V_{TN})^2 + 2K_n (V_{GSQ} - V_{TN}) v_{gs}$$

The equation may be written as;

$$i_D = I_D + i_d$$

where;

$$I_D = K_n (V_{GSQ} - V_{TN})^2 \quad \text{-- the dc component}$$



The MOSFET Amplifier

and;

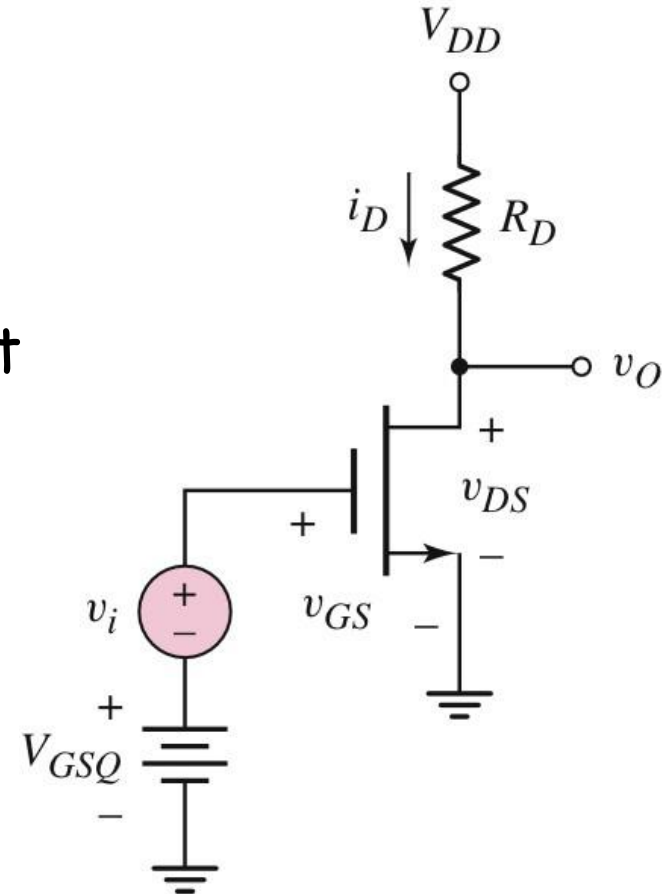
$$i_d = 2K_n (V_{GSQ} - V_{TN}) v_{gs}$$

– the ac or signal component

From the above equation, we can write;

$$g_m = \frac{i_d}{v_{gs}} = 2K_n (V_{GSQ} - V_{TN})$$

– the transconductance relating the output current to the input voltage



The MOSFET Amplifier

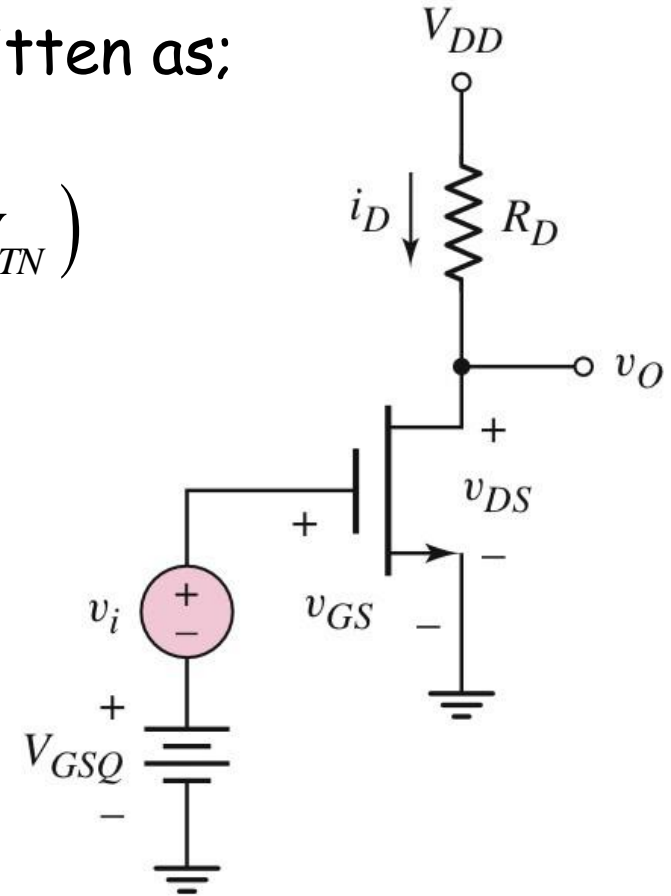
The transconductance can also be written as;

$$g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS} = V_{GSQ} = \text{const.}} = 2K_n (V_{GSQ} - V_{TN})$$

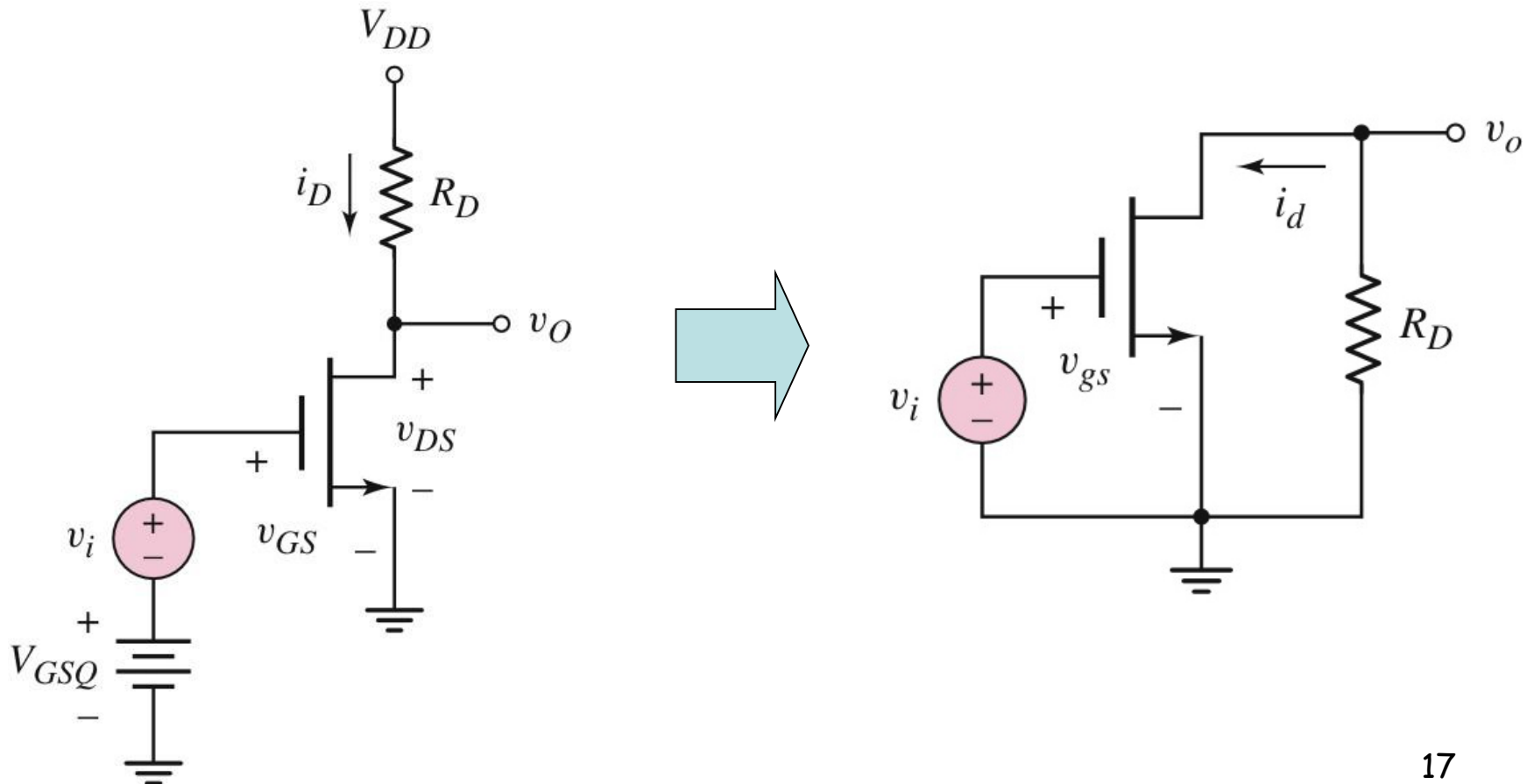
$$I_D = K_n (V_{GSQ} - V_{TN})^2$$

which gives us;

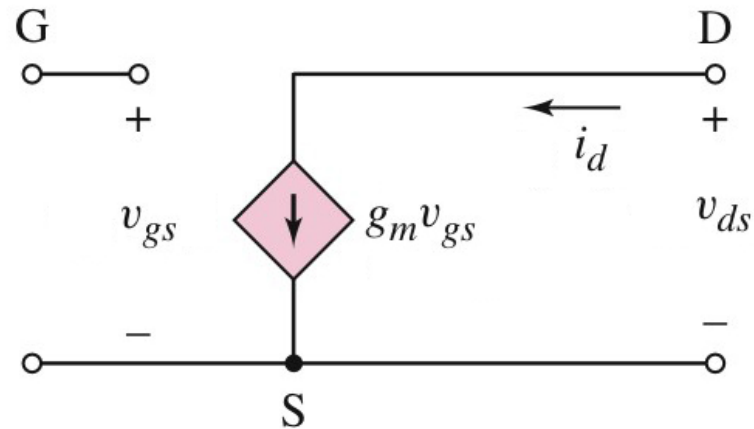
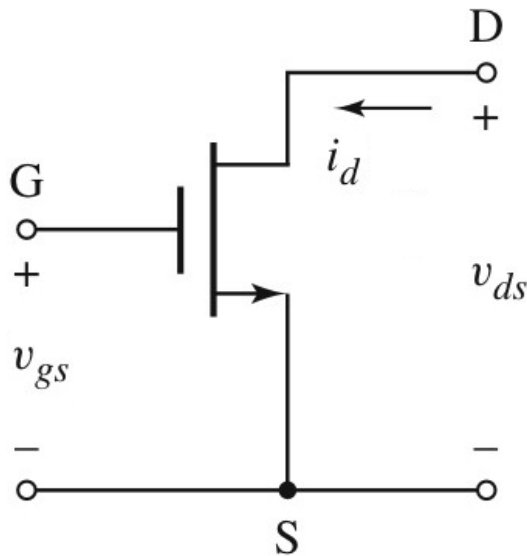
$$g_m = 2\sqrt{K_n I_{DQ}}$$



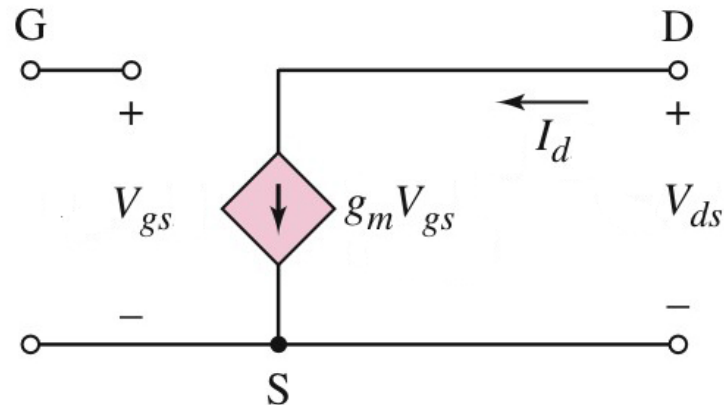
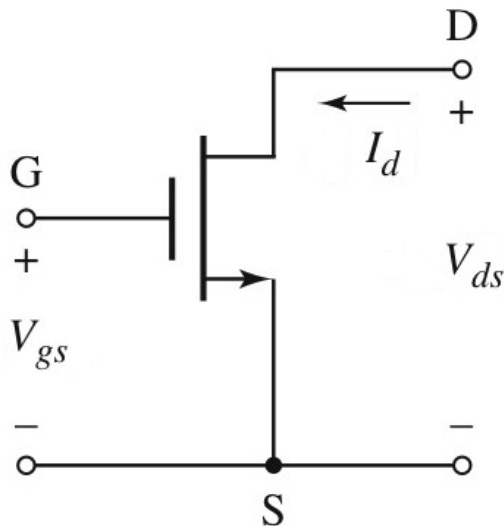
In the **ac equivalent circuit**, all **dc sources are set to zero** and V_{DD} is considered to be at signal ground.



If we neglect the effect of the **output resistance r_o** , the MOSFET and its small-signal equivalent circuit are as follows (in terms of instantaneous ac values):



The small-signal equivalent circuit in terms of phasor components (note the notation used):



$$v(t) = 6 * \cos(120\pi + 30^\circ)$$

$$i(t) = 2 * \cos(120\pi + 30^\circ)$$

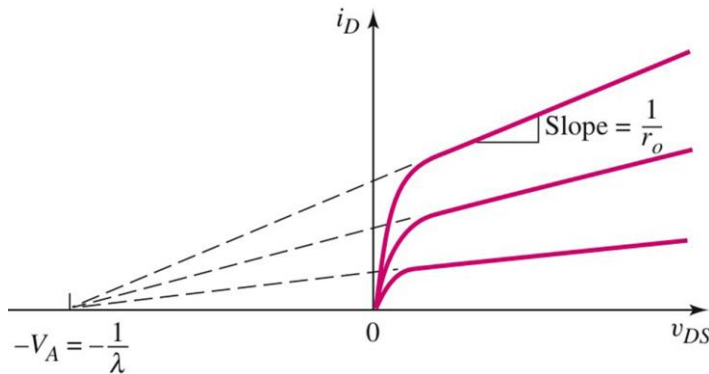
$$V = 6 \angle 30^\circ$$

which is the phasor of $v(t)$

$$I = 2 \angle 30^\circ$$

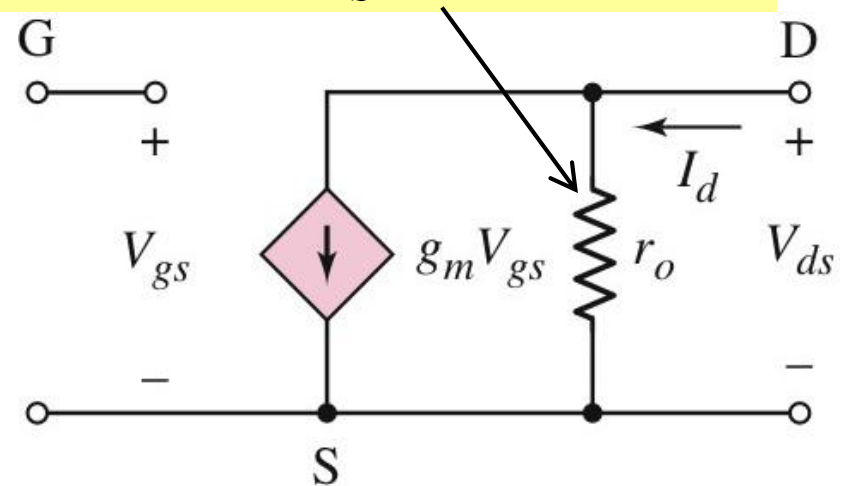
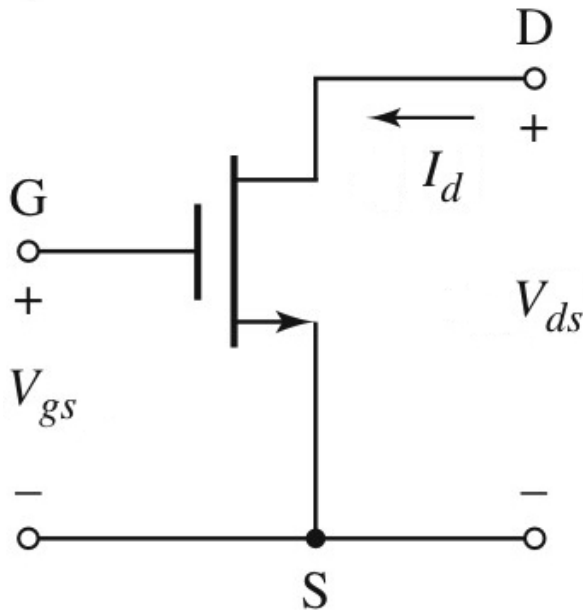
the phasor of $i(t)$.

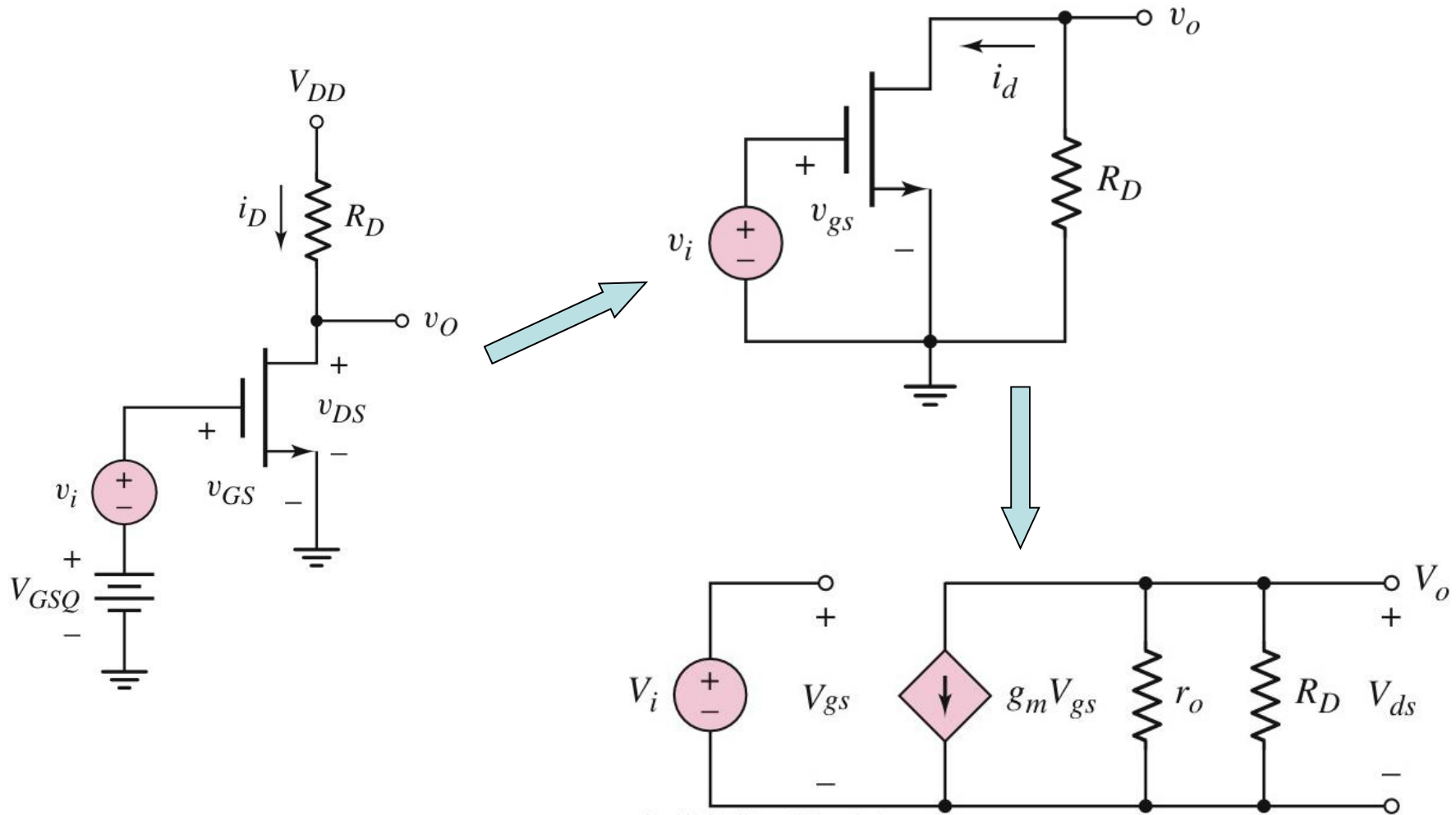
The small-signal equivalent circuit with r_o taken into account.



$$i_D = K_n (v_{GS} - V_{TN})^2 (1 + \lambda v_{DS})$$

$$r_o = \left. \frac{\partial i_D}{\partial v_{DS}} \right|_{v_{GS}=V_{GSQ}=\text{const.}}^{-1} \cong [\lambda I_{DQ}]^{-1}$$





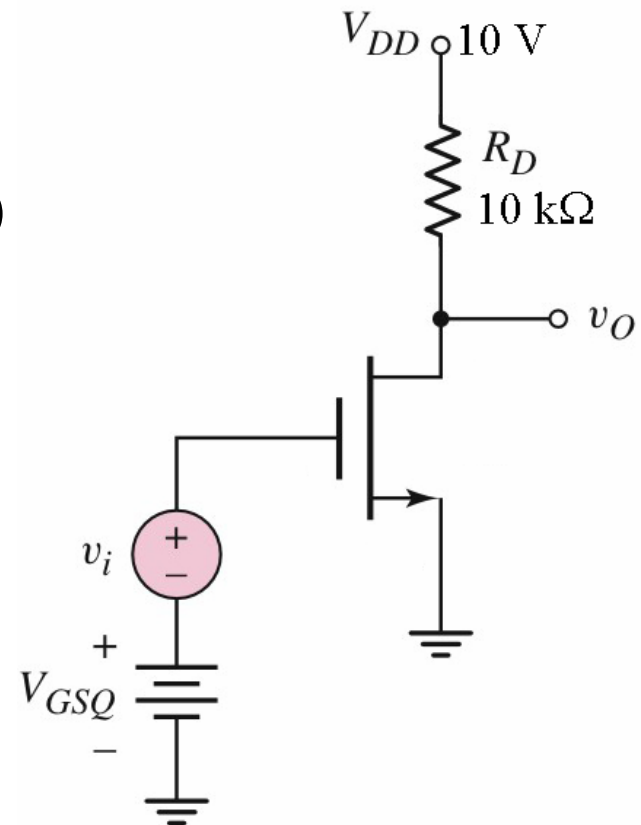
The MOSFET Amplifier Example 1

The MOSFET in the figure has the following parameters;

$$V_{TN} = 2 \text{ V}, \quad K_n = 0.5 \text{ mA/V}^2, \quad \text{and} \quad \lambda = 0$$

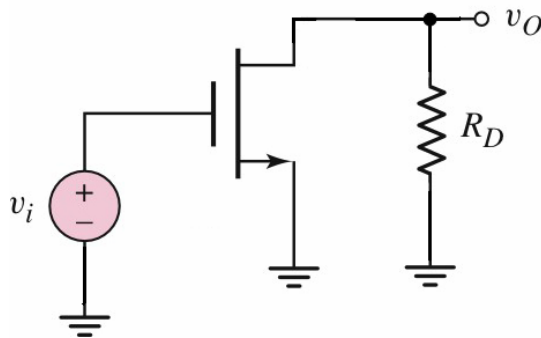
The transistor is biased for $I_{DQ} = 0.4 \text{ mA}$.

- (a) Draw the ac equivalent circuit.
- (b) Draw the small-signal equivalent circuit.
- (c) Determine the small-signal voltage gain A_v .

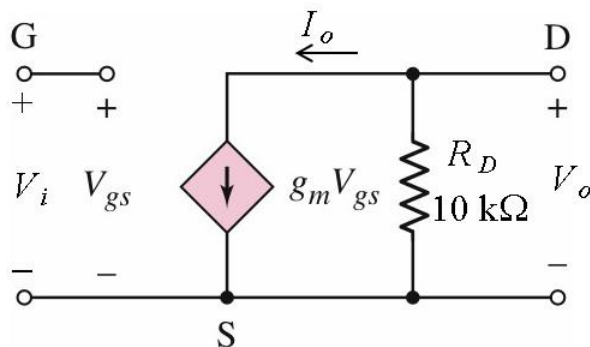


The MOSFET Amplifier Example 1 - Solution

(a) For the ac equivalent circuit, all dc voltage sources are set to zero;

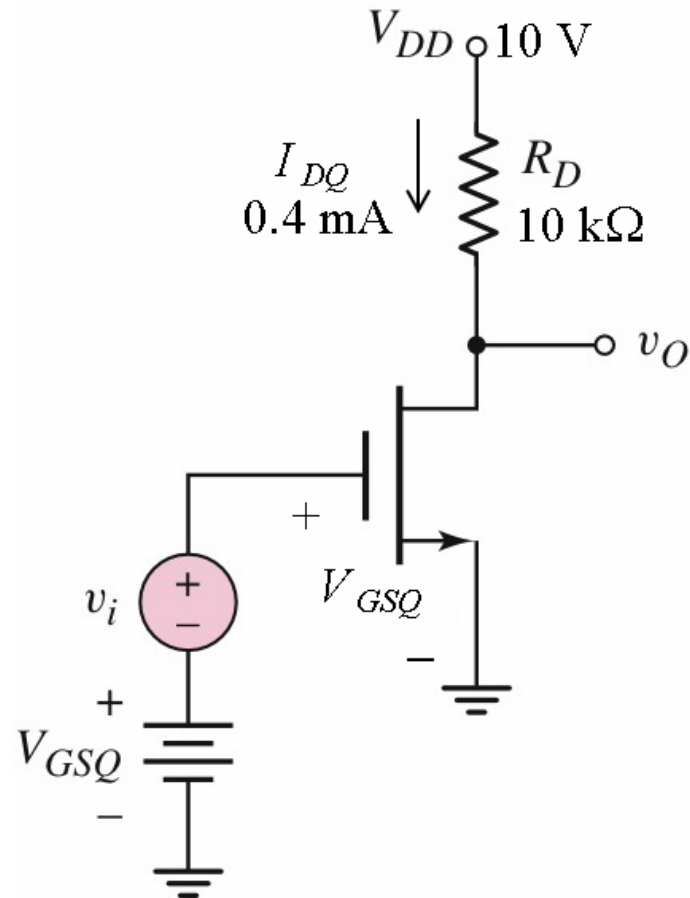


(b) Since $\lambda = 0$, the output resistance r_o is omitted in the small-signal equivalent circuit;



The MOSFET Amplifier Example 1 - Solution (cont'd)

(c) $g_m = 2\sqrt{K_n I_{DQ}}$
 $= 2 \times 10^{-3} \sqrt{0.5 \times 0.4}$
 $= 0.894 \text{ mA/V}$



The MOSFET Amplifier Example 1 - Solution (cont'd)

At the output terminal;

$$V_o = -g_m V_{gs} R_D$$

At the input terminal;

$$V_i = V_{gs}$$

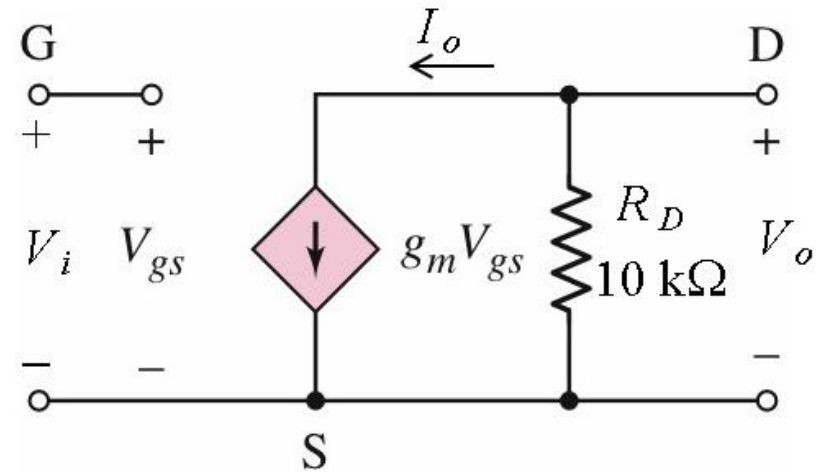
Substituting for V_{gs} ;

$$V_o = -g_m R_D V_i$$

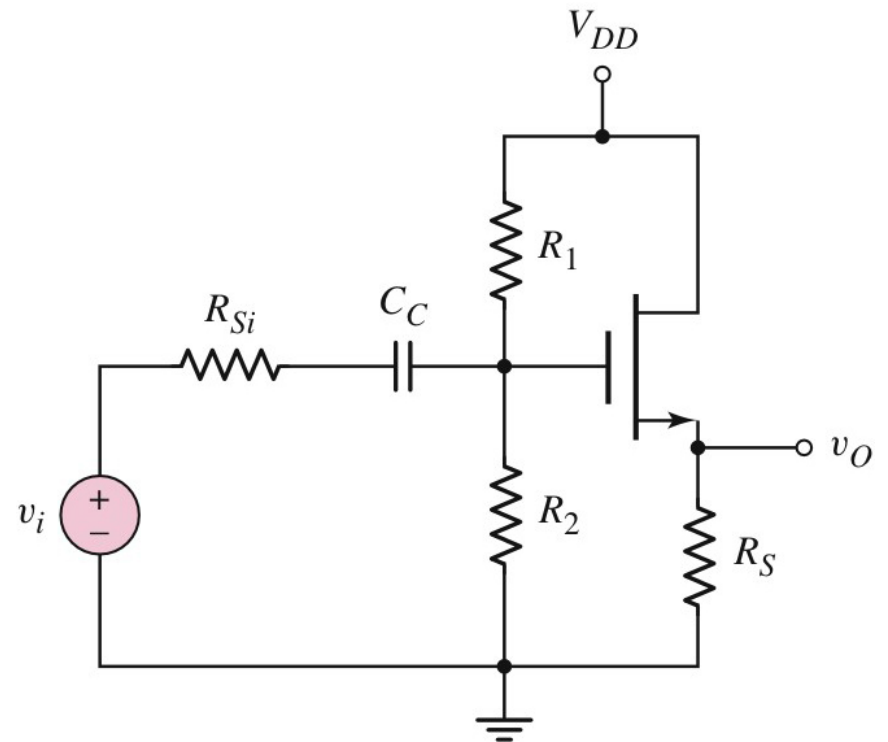
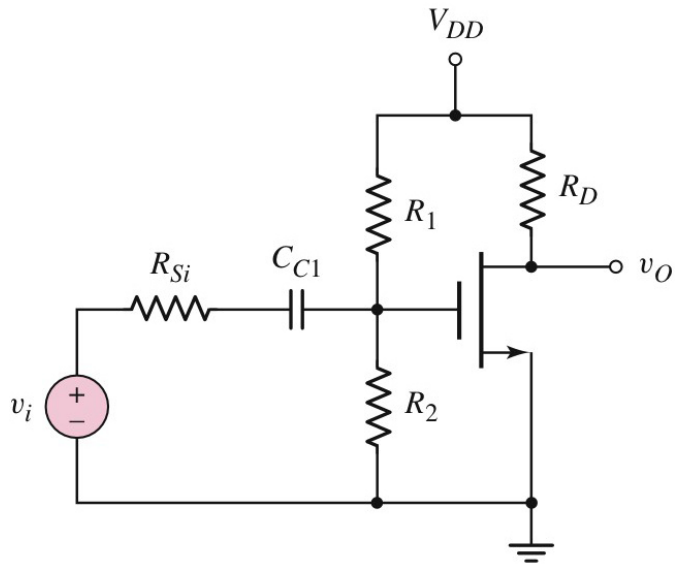
The small-signal voltage gain is;

$$A_v \equiv \frac{V_o}{V_i} = -g_m R_D$$

$$A_v = -g_m R_D = -0.894 \times 10 = -8.94 \text{ V/V}$$



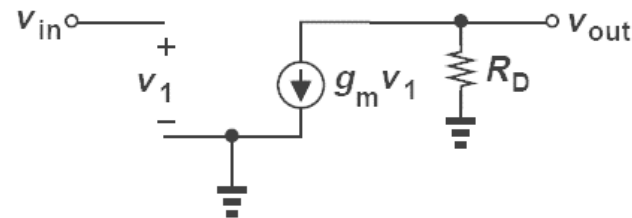
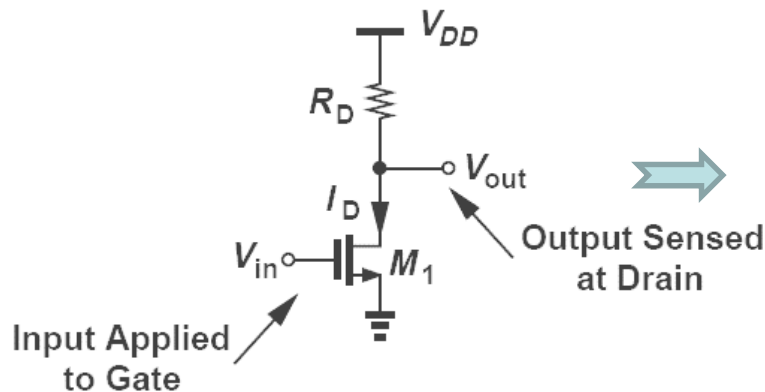
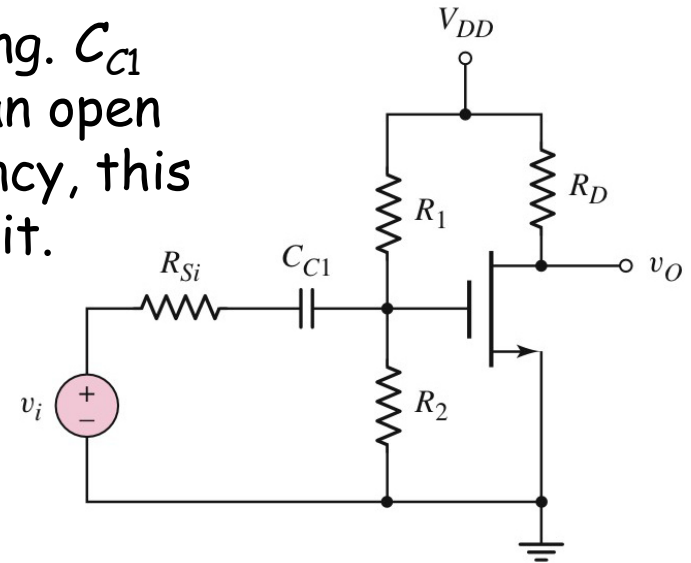
Amplifier Circuits



Common-source

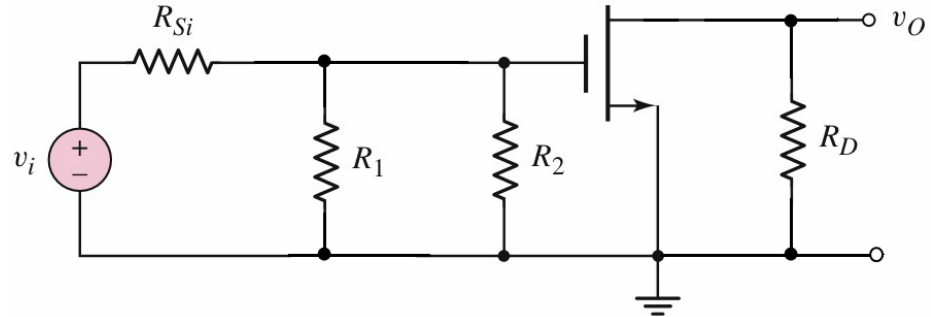
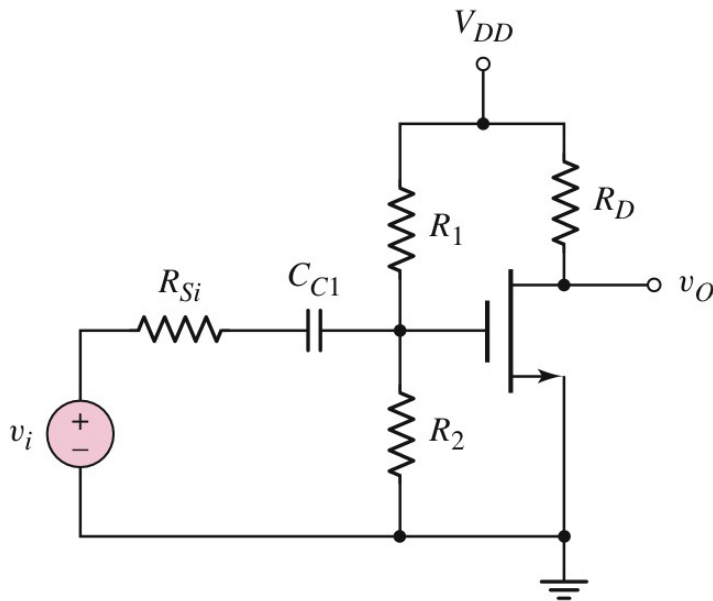
R_1 and R_2 establish the transistor biasing. C_{C1} couples the signal to the input acts as an open circuit for dc voltage. At signal frequency, this capacitor is considered as a short circuit.

The source is grounded and is common to both input and output - hence the name common-source.



Common-source

Since the coupling capacitor is considered as a short circuit at signal frequency and all dc sources are set to zero under ac condition, the ac equivalent circuit for the amplifier may be drawn as follows;

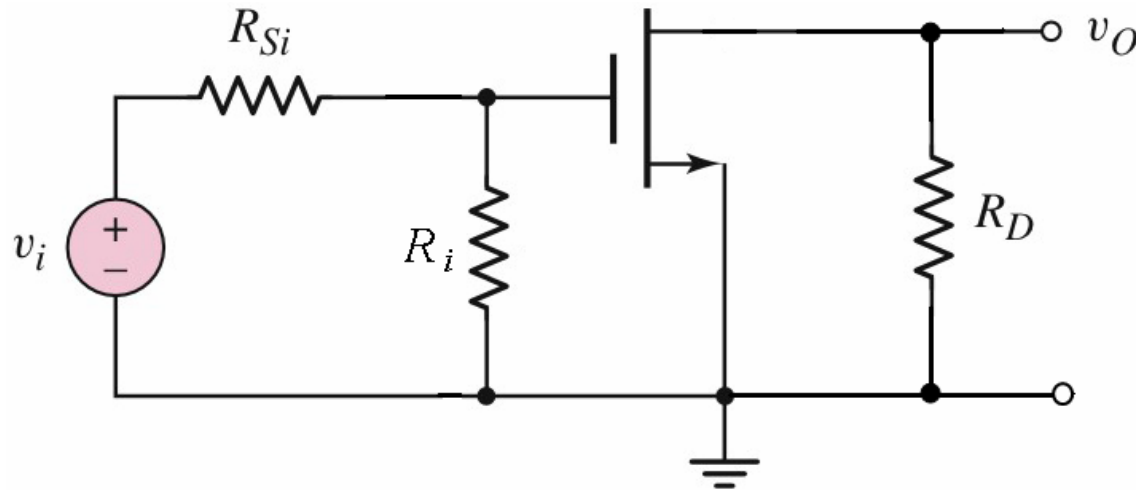


Common-source

The parallel combination of R_1 and R_2 may be replaced by R_i where;

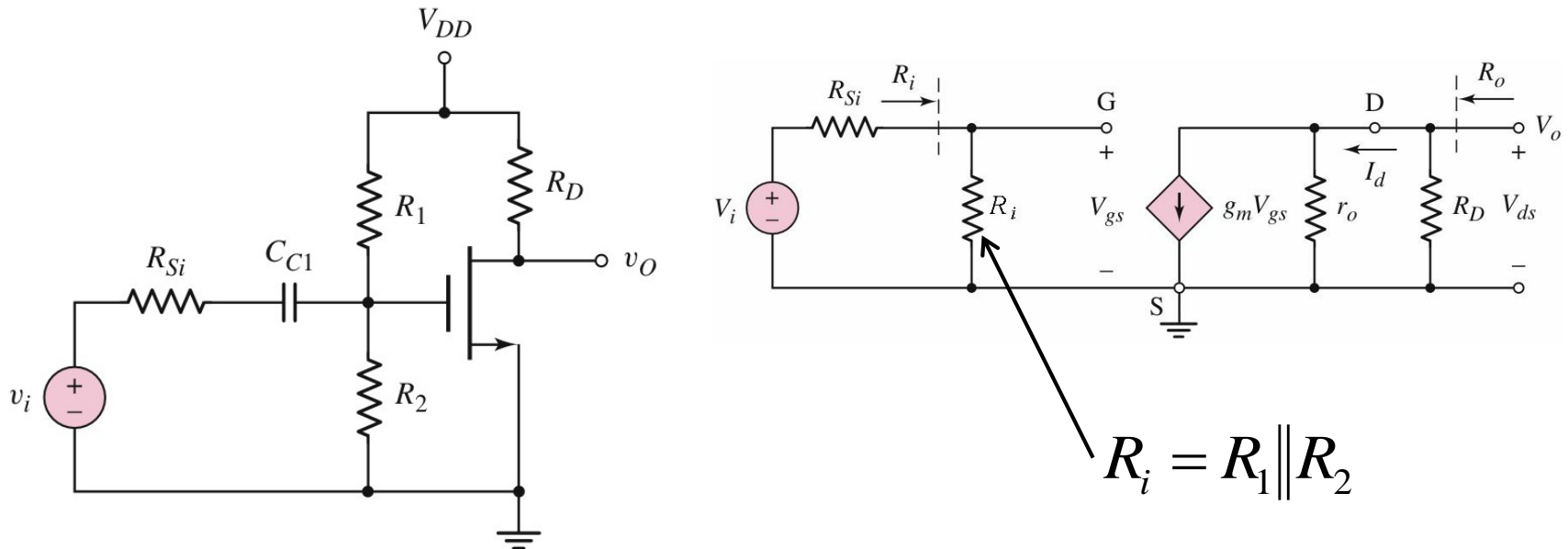
$$R_i = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

R_i is the **input resistance** of the amplifier.



Common-source

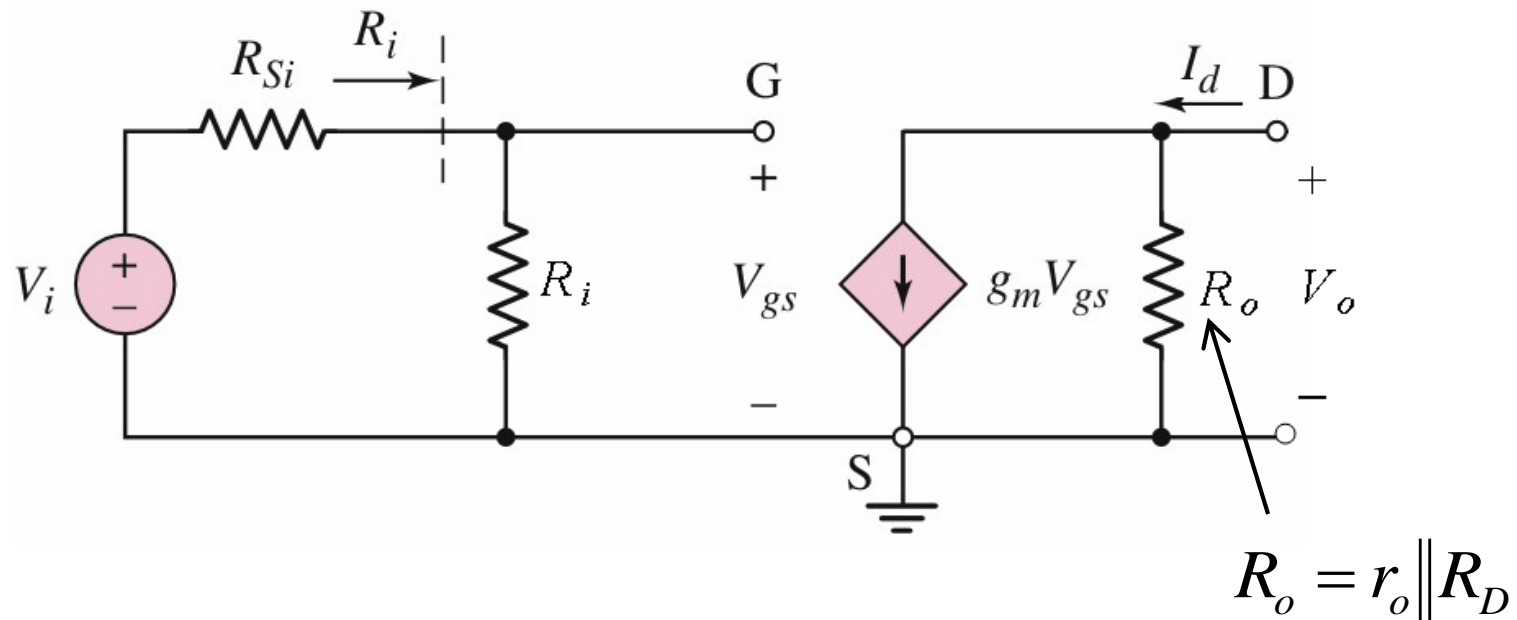
The small-signal equivalent circuit may be drawn as shown below;



In the above circuit, the **output resistance of the transistor, r_o** , is finite and taken into account.

Common-source

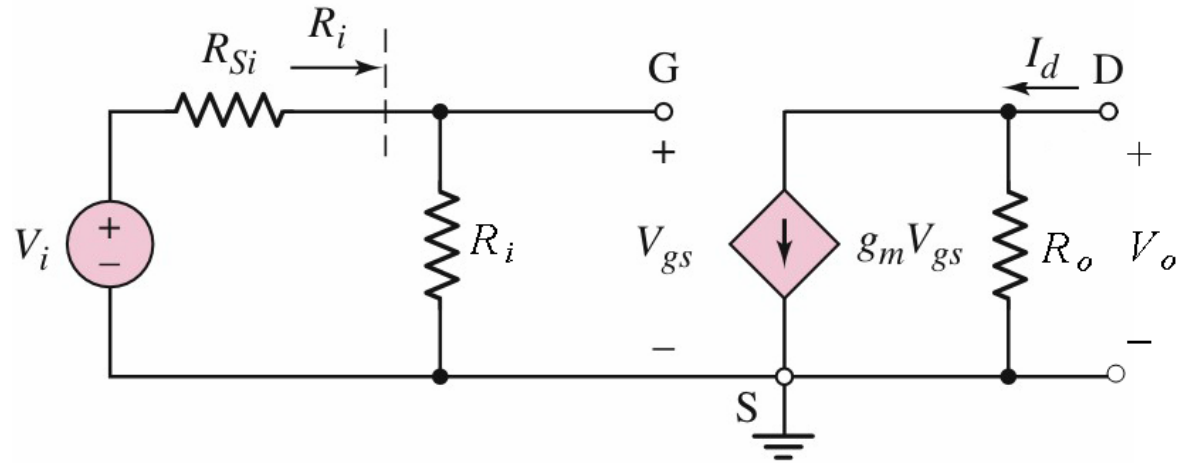
Since R_D and the output resistance of the transistor, r_o , are in parallel, the small-signal equivalent circuit may be redrawn as shown below.



Common-source

Using this equivalent circuit, at the output terminal;

$$V_o = -g_m V_{gs} R_o$$



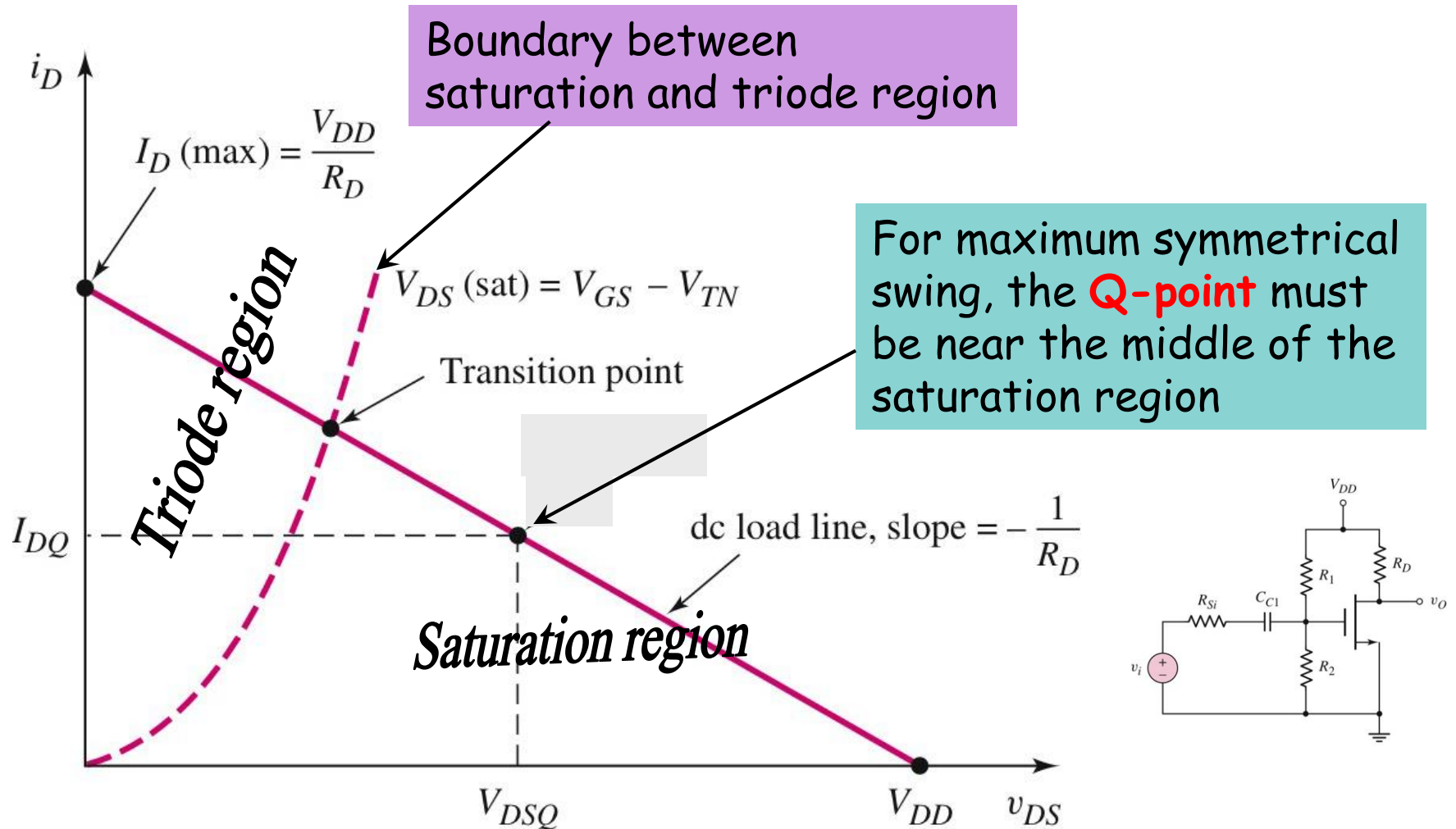
At the input terminal;

$$V_{gs} = V_i \left(\frac{R_i}{R_{Si} + R_i} \right) \quad \Rightarrow \quad V_o = -g_m V_i \left(\frac{R_i}{R_{Si} + R_i} \right) R_o$$

Hence the voltage gain is;

$$A_v \equiv \frac{V_o}{V_i} = -g_m \left(\frac{R_i R_o}{R_{Si} + R_i} \right)$$

Common-source

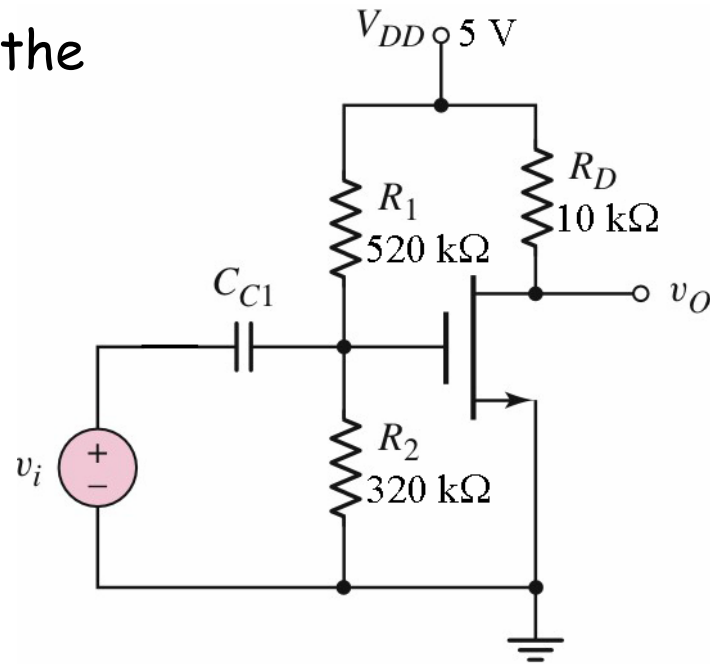


The transistor in the figure has the following parameters;

$$V_{TN} = 0.8 \text{ V}, \quad K_n = 0.2 \text{ mA/V}^2, \\ \text{and } \lambda = 0$$

Find the following;

- (a) g_m and r_o ;
- (b) A_{v_i} ;
- (c) R_i and R_o .



(a)

$$V_{GS} = V_G = V_{DD} \left(\frac{R_2}{R_1 + R_2} \right)$$

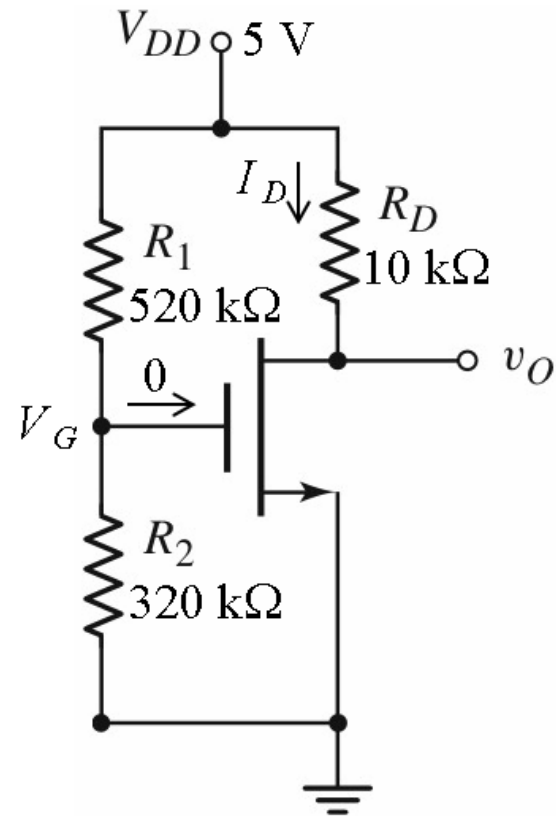
$$= 5 \left(\frac{320}{520 + 320} \right) = \mathbf{1.905\text{ V}}$$

$$g_m = 2K_n (V_{GS} - V_{TN})$$

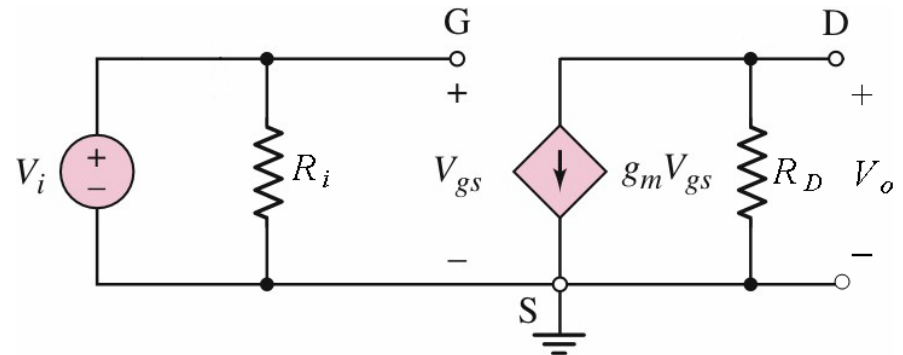
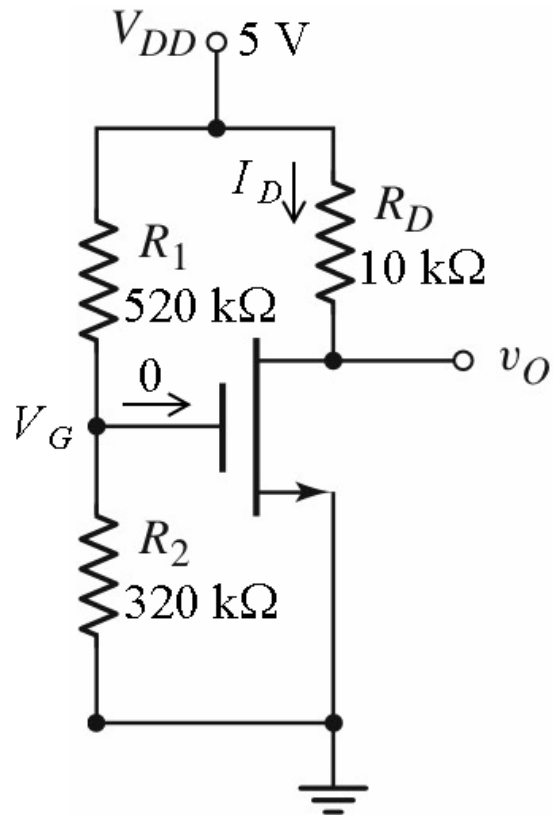
$$= 2 \times 0.2 \times 10^{-3} (1.905 - 0.8)$$

$$= \mathbf{0.442\text{ mA/V}}$$

$$r_o = \frac{1}{\lambda I_D} = \infty$$

The circuit under dc condition

(b)

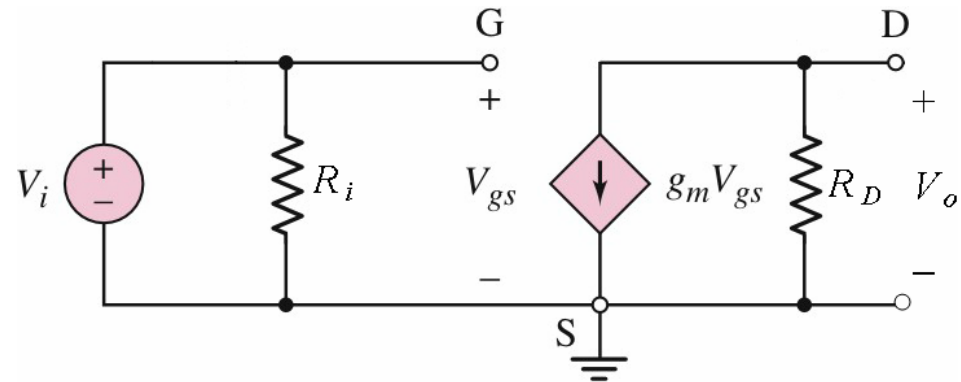
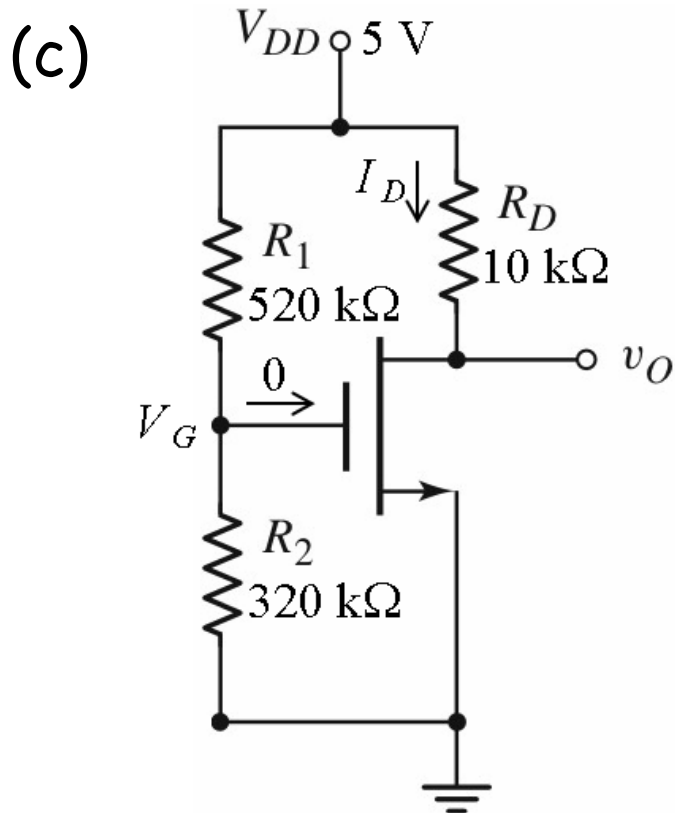


$$V_i = V_{gs}$$

$$V_o = -g_m V_{gs} R_D$$

$$A_v = \frac{V_o}{V_i} = -g_m R_D$$

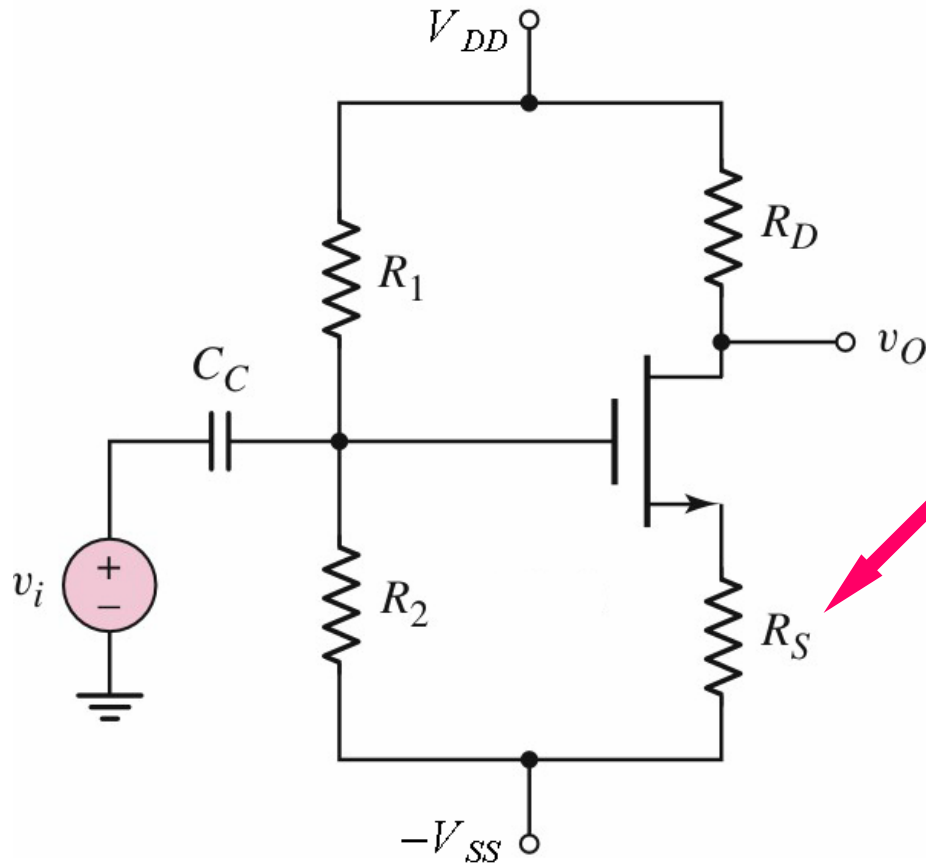
$$= -0.442 \times 10 = -4.42 \text{ V/V}$$



$$R_o = R_D = 10\text{ k}\Omega$$

$$\begin{aligned}
 R_i &= R_1 \parallel R_2 \\
 &= 520 \parallel 320 \\
 &= 198\text{ k}\Omega
 \end{aligned}$$

Common-source with R_S



The source resistor R_S tends to stabilize the Q-point against variations in transistor parameters.

However, the inclusion of this resistor reduces the amplifier gain.

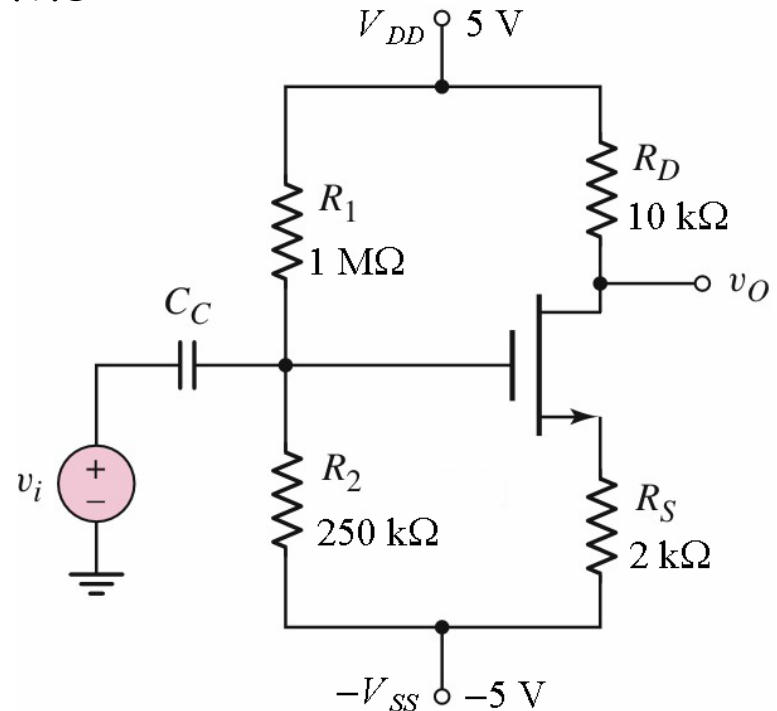
Common-source with R_S Example 2

The transistor in the figure has the following parameters;

$$V_{TN} = 0.6 \text{ V}, \quad K_n = 0.5 \text{ mA/V}^2, \\ \text{and } \lambda = 0$$

(a) Determine the Q-point of the amplifier.

(b) Find the small-signal voltage gain



The circuit under dc condition

The gate voltage:

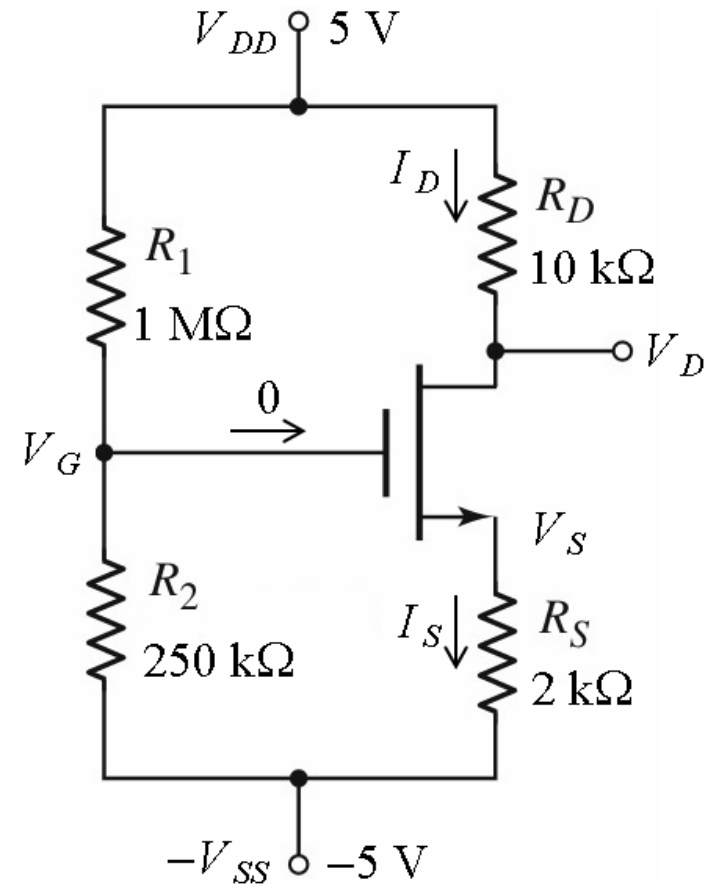
$$V_G = 10 \left(\frac{0.25}{1 + 0.25} \right) - 5 = -3 \text{ V}$$

Voltage drop on R_2

$$V_S = -5 + I_D R_S = -5 + 2 \times 10^3 I_D$$

$$V_{GS} = V_G - V_S = 2 - 2 \times 10^3 I_D$$

$$\therefore I_D = (1 - 0.5 V_{GS}) 10^{-3}$$



$$(1) \quad I_D = (1 - 0.5V_{GS})10^{-3}$$

The circuit under dc condition

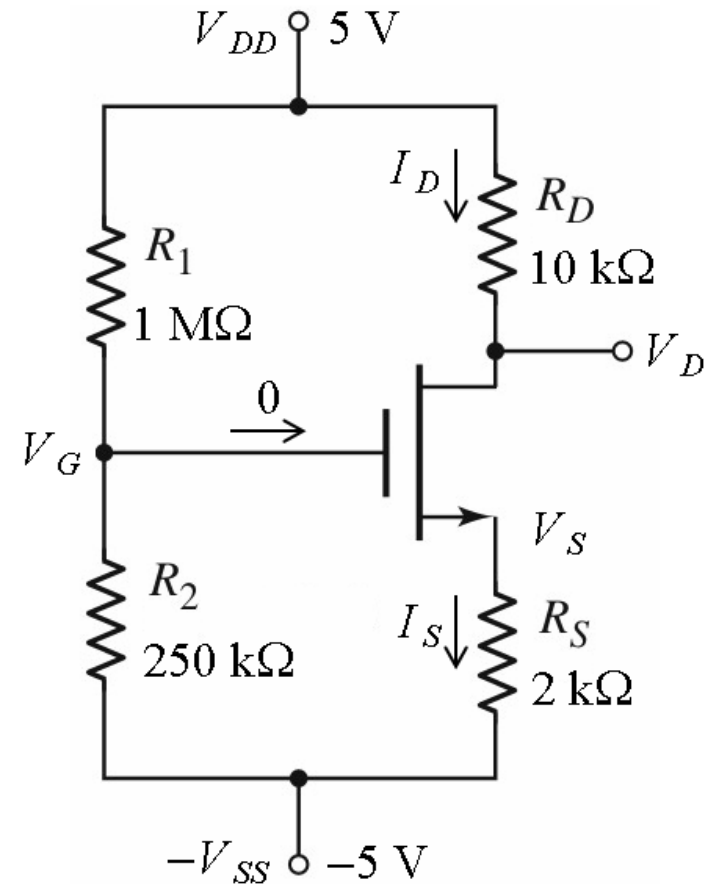
$$(2) \quad I_D = K_n(V_{GS} - V_{TN})^2 \\ = (0.5V_{GS}^2 - 0.6V_{GS} + 0.18)10^{-3}$$

Substituting for I_D and rearranging, we have;

$$(1 - 0.5V_{GS})10^{-3} \\ = (0.5V_{GS}^2 - 0.6V_{GS} + 0.18)10^{-3}$$

or;

$$V_{GS}^2 - 0.2V_{GS} - 1.64 = 0$$



Quadratic Formula Calculator

$$x^2 + 4x + 3 = 0$$

Calculate it!

Use quadratic formula with $a=1$, $b=4$, $c=3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{4}}{2}$$

$$x = -1 \text{ or } x = -3$$

Solving the quadratic equation, gives us;

$$V_{GS}^2 - 0.2V_{GS} - 1.64 = 0$$

$$V_{GS} = \frac{0.2 \pm \sqrt{0.2^2 + 4 \times 1.64}}{2}$$

$$= 1.385 \text{ V}$$

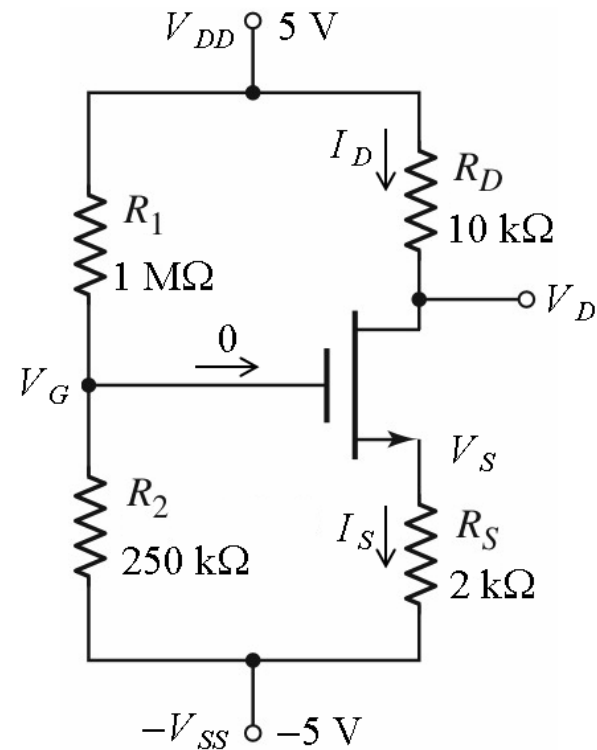
$$V_{GS} = -1.18 \text{ (second root)}$$

Substituting for V_{GS} in;

$$I_D = K_n (V_{GS} - V_{TN})^2$$

we obtain; $I_D = 0.308 \text{ mA}$

The circuit under dc condition



$$V_D = 5 - I_D R_D = 5 - 0.308 \times 10 \\ = 1.92 \text{ V}$$

$$V_S = -5 + I_D R_S = -5 + 0.308 \times 2 \\ = -4.384 \text{ V}$$

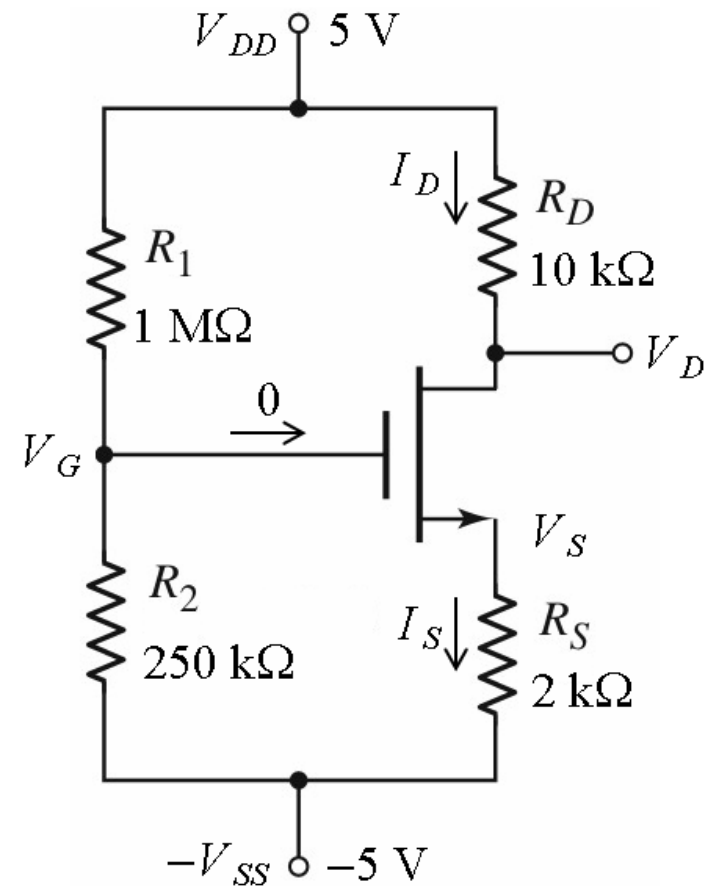
$$V_{DS} = V_D - V_S = 1.92 + 4.384 \\ = 6.304 \text{ V}$$

$$V_{TN} = 0.6 \text{ V} \quad V_{GS} = 1.385 \text{ V}$$

$$V_{DS} > V_{GS} - V_{TN}$$

Saturation region

The circuit under dc condition



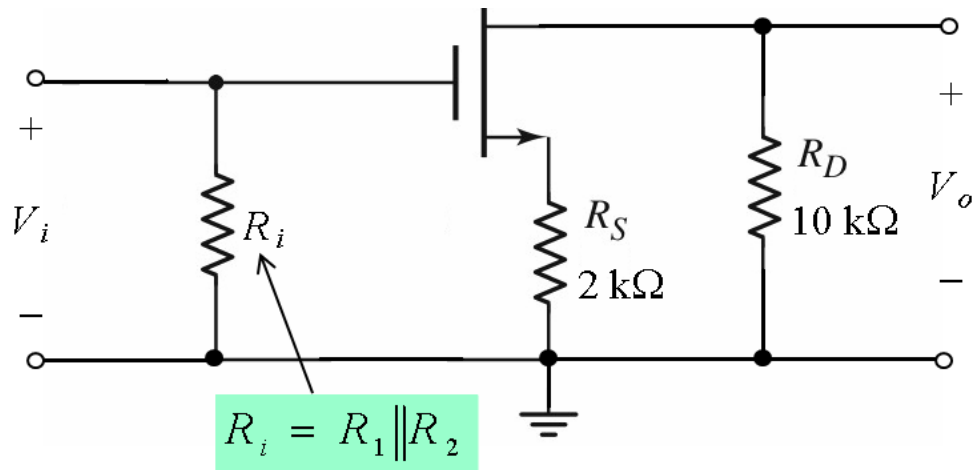
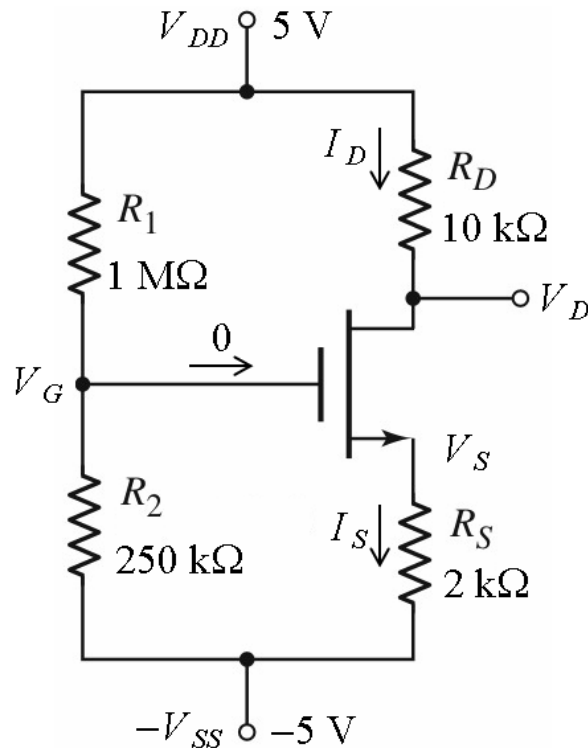
The Q-point:

$$I_{DQ} = 0.308 \text{ mA}; \quad V_{DSQ} = 6.304 \text{ V}$$

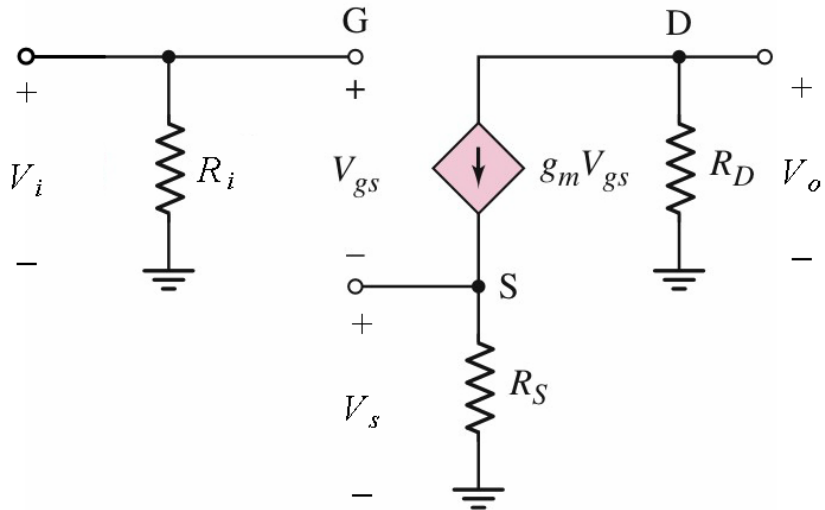
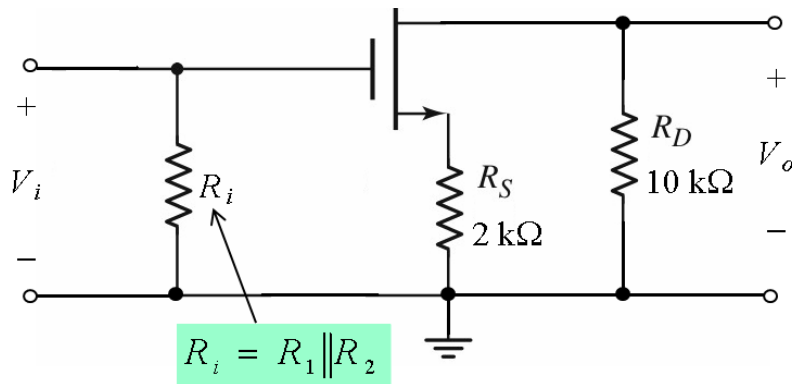
The transconductance is;

$$\begin{aligned} g_m &= 2K_n (V_{GSQ} - V_{TN}) \\ &= 2 \times 0.5 \times 10^{-3} (1.385 - 0.6) \\ &= 0.785 \text{ mA/V} \end{aligned}$$

The ac equivalent circuit may be drawn as follows;



The small-signal equivalent circuit may be drawn as follows;

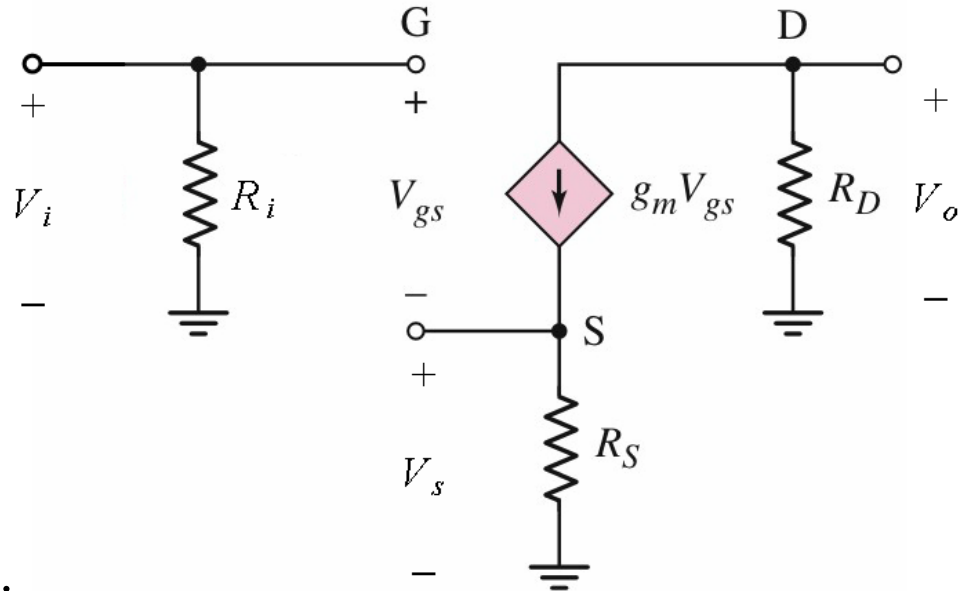


$$\begin{aligned} V_i &= V_{gs} + V_s \\ &= V_{gs} + g_m V_{gs} R_s \end{aligned}$$

$$V_o = -g_m V_{gs} R_D$$

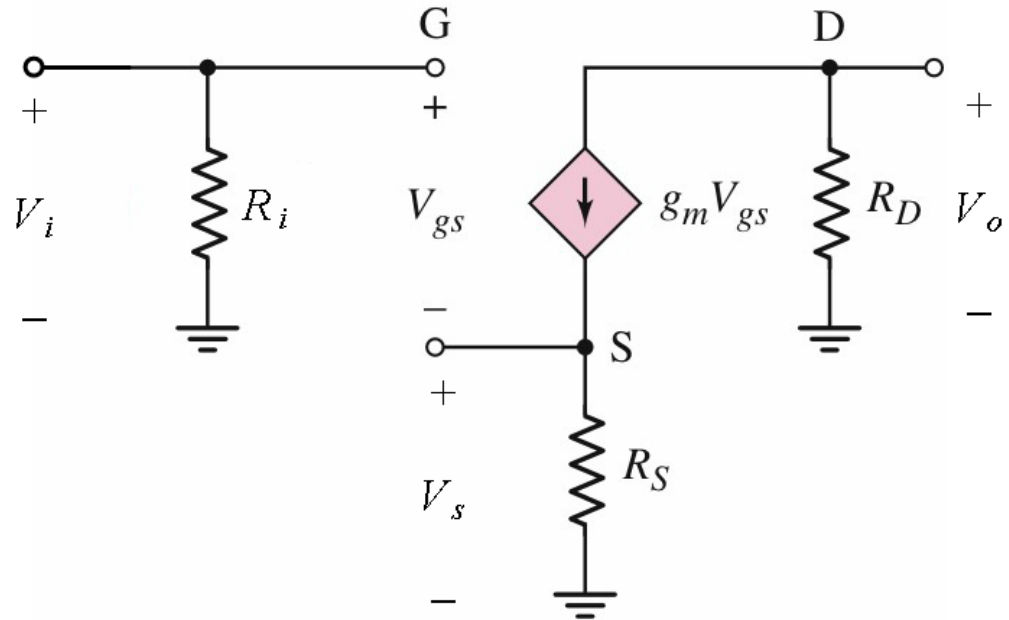
The small-signal voltage gain is;

$$A_v = \frac{V_o}{V_i} = \frac{-g_m V_{gs} R_D}{V_{gs} + g_m V_{gs} R_S} = -\frac{g_m R_D}{1 + g_m R_S}$$



Substituting values;

$$A_v = -\frac{0.785 \times 10}{1 + 0.785 \times 2}$$
$$= -3.05 \text{ V/V}$$



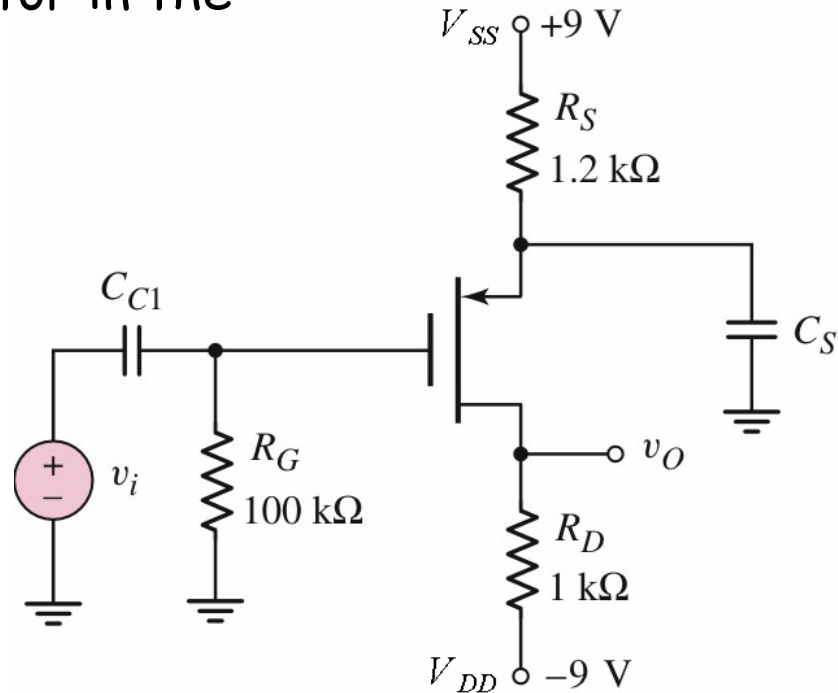
The parameters for the transistor in the figure are;

$$K_p = 2 \text{ mA/V}^2, V_{TP} = -2 \text{ V, and}$$

$$\lambda = 0.01 \text{ V}^{-1}$$

(a) Determine the Q-point

(c) Calculate the small-signal voltage gain A_v .



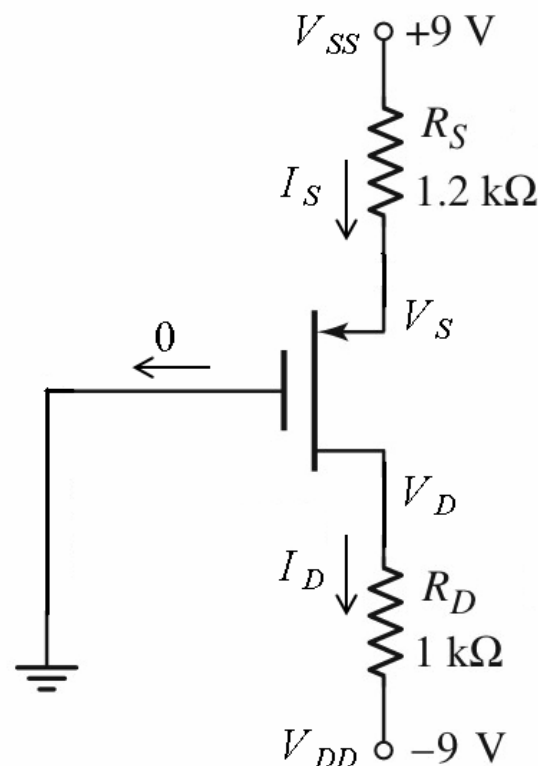
Since I_G is assumed to be zero,
 G is at the ground potential
 and $I_S = I_D$.

$$\begin{aligned}
 \star \quad I_D &= K_p (V_{SG} + V_{TP})^2 \\
 &= 2 \times 10^{-3} (V_{SG} - 2)^2 \\
 &= 2 \times 10^{-3} (V_{SG}^2 - 4V_{SG} + 4)
 \end{aligned}$$

$$\begin{aligned}
 V_{SG} &= 9 - I_D R_S \\
 &= 9 - 1.2 \times 10^3 I_D
 \end{aligned}$$

$$\star \quad I_D = (7.5 - 0.833 V_{SG}) 10^{-3}$$

The circuit under dc condition



Substituting for I_D ;

$$(7.5 - 0.833V_{SG})10^{-3} \\ = 2 \times 10^{-3}(V_{SG}^2 - 4V_{SG} + 4)$$

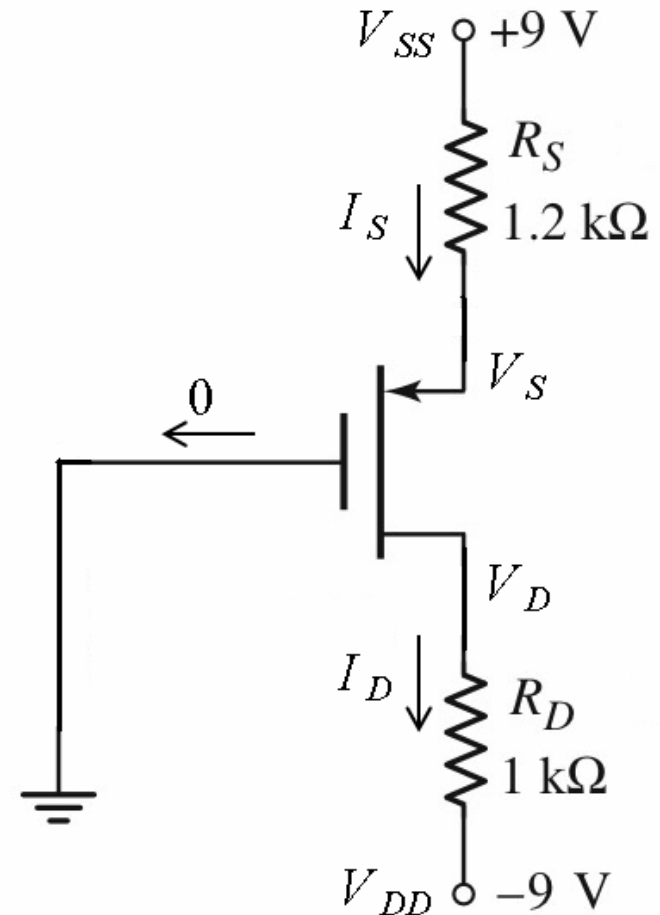
or;

$$V_{SG}^2 - 3.583V_{SG} + 0.25 = 0$$

Solving the equation for V_{SG} , we obtain;

$$V_{SG} = 3.512 \text{ V}$$

The circuit under dc condition



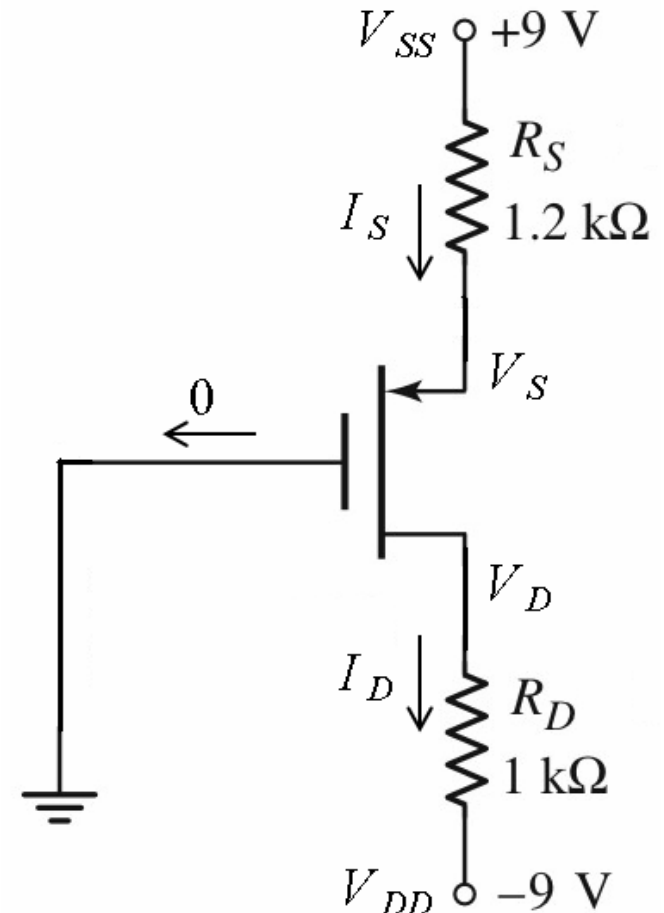
The circuit under dc condition

$$V_{SG} = 3.512 \text{ V}$$

$$I_D = I_S = \frac{9 - 3.512}{1.2 \times 10^3} = 4.57 \text{ mA}$$

$$\begin{aligned} V_D &= 4.57 \times 10^{-3} \times 1 \times 10^3 - 9 \\ &= -4.427 \text{ V} \end{aligned}$$

$$\begin{aligned} V_{SD} &= V_S - V_D \\ &= 3.512 + 4.427 \\ &= 7.94 \text{ V} \end{aligned}$$

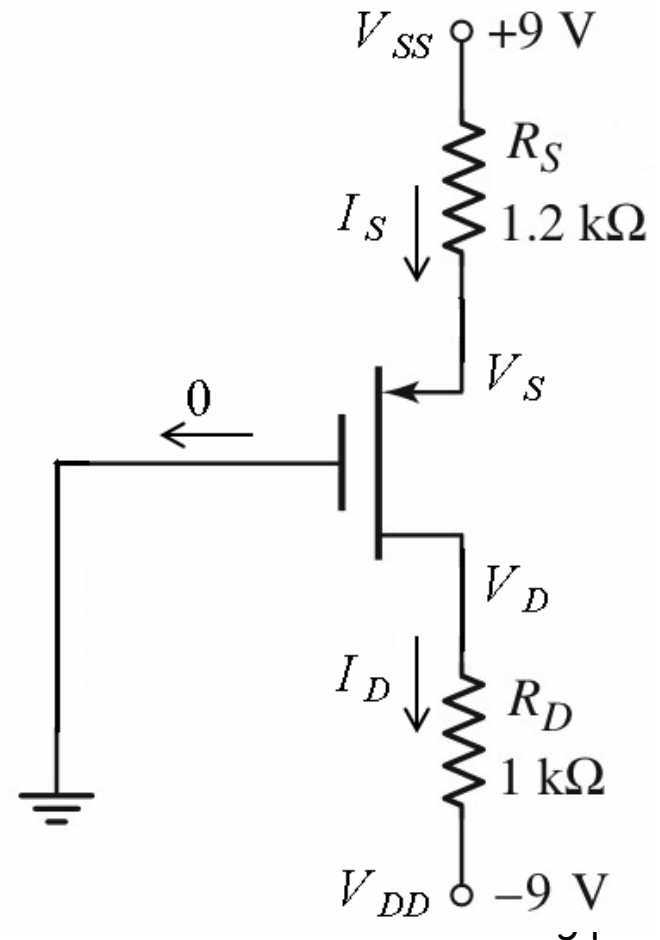


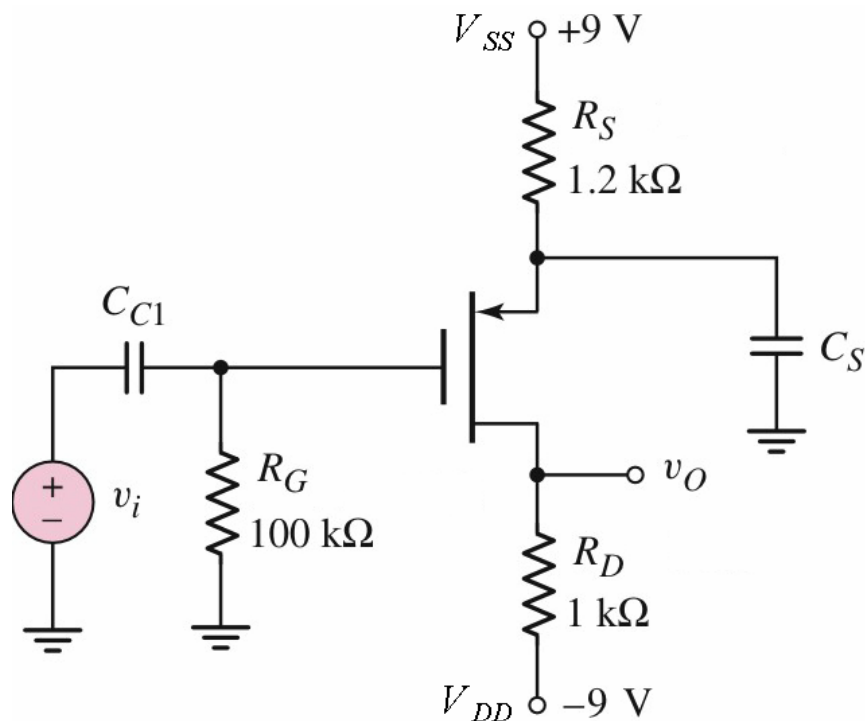
(a)

The Q-point is;

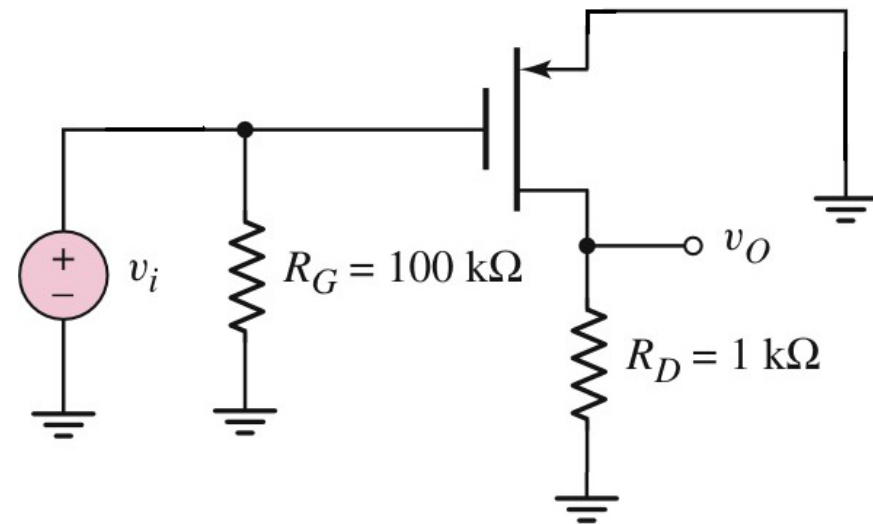
$$I_{DQ} = 4.57 \text{ mA}, V_{SDQ} = 7.94 \text{ V}$$

$$\begin{aligned} g_m &= 2K_p (V_{SG} + V_{TP}) \\ &= 2(3.512 - 2) \\ &= 6.048 \text{ mA/V} \end{aligned}$$

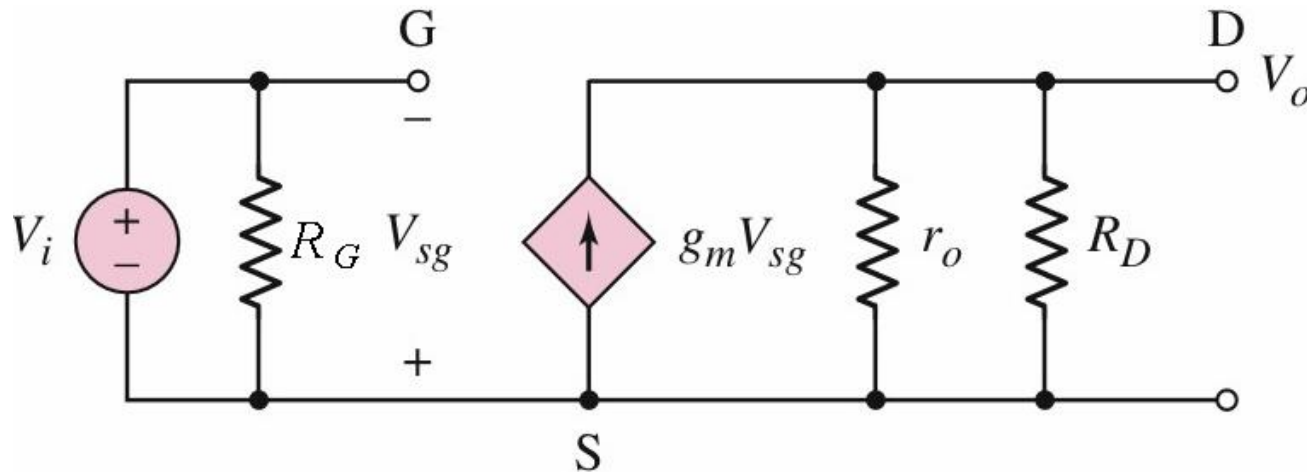
The circuit under dc condition



The ac equivalent circuit

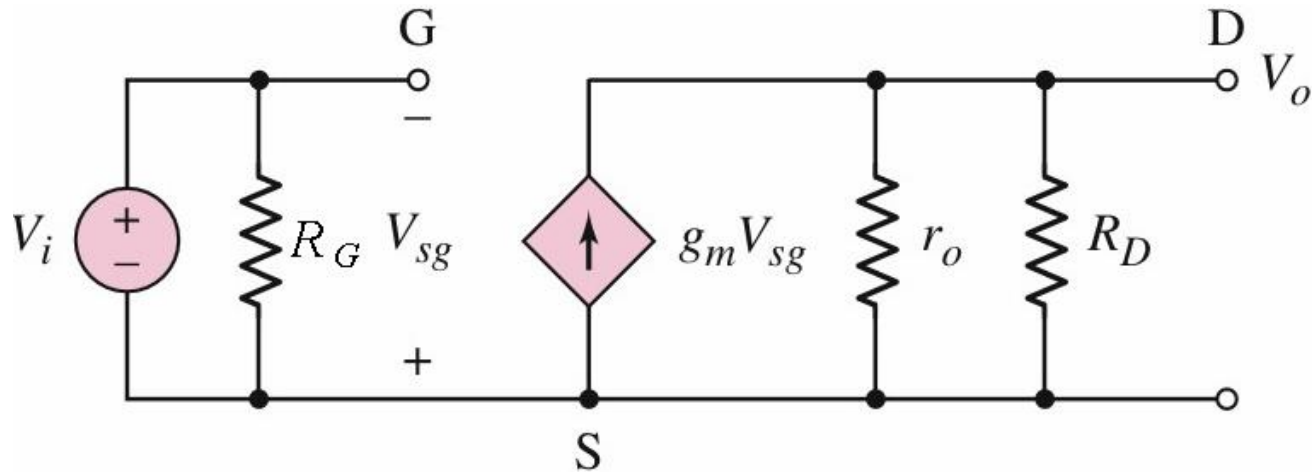


The small-signal equivalent circuit may be drawn as follows;



Note that r_o is included in the equivalent circuit because the value of λ is given.

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.01 \times 4.57 \times 10^{-3}} = 218.8 \text{ k}\Omega$$



$$r_o = 218.8 \text{ k}\Omega$$

$$R_D = 1 \text{ k}\Omega$$

$$r_o // R_D \cong R_D$$

$$A_v \equiv \frac{V_o}{V_i} \cong -g_m R_D = -6.05 \times 10^{-3} \times 1 \times 10^3$$

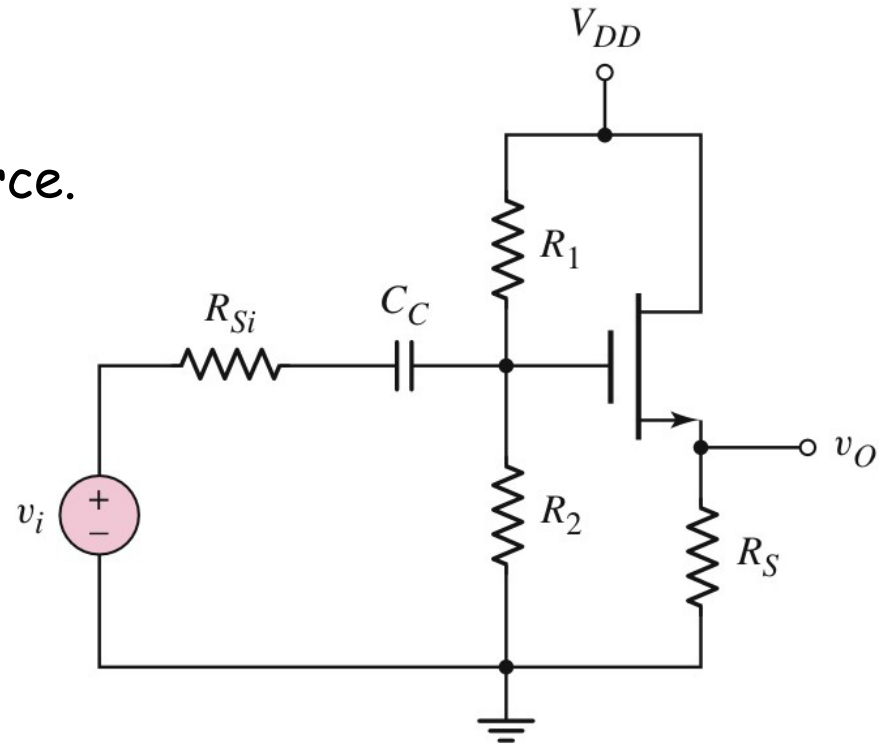
$$A_v = -6.05 \text{ V/V}$$

Common-drain

Also known as **source-follower**

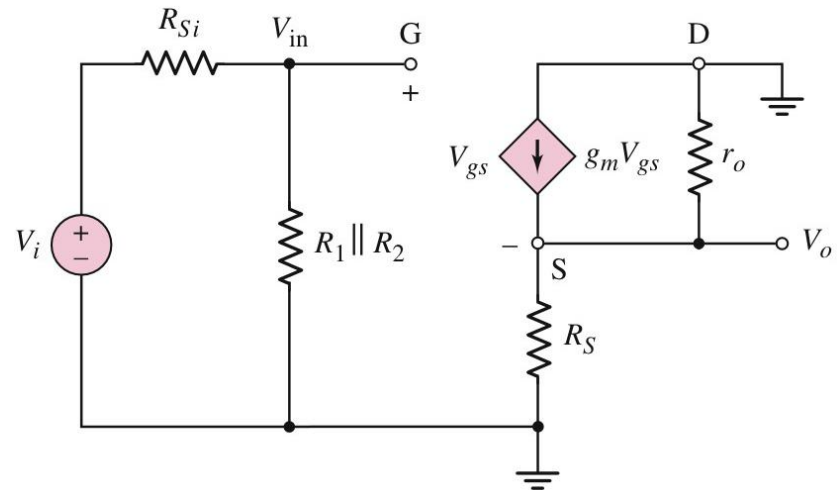
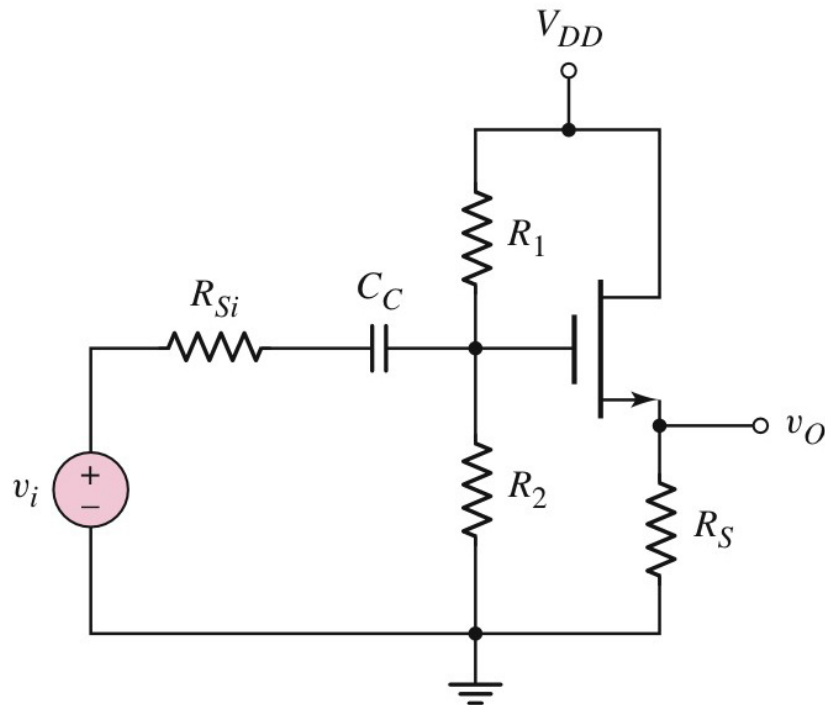
The output is taken from the source.

The drain is connected to V_{DD} which is the signal ground and becomes the common terminal for the input and output, hence the name common-drain.



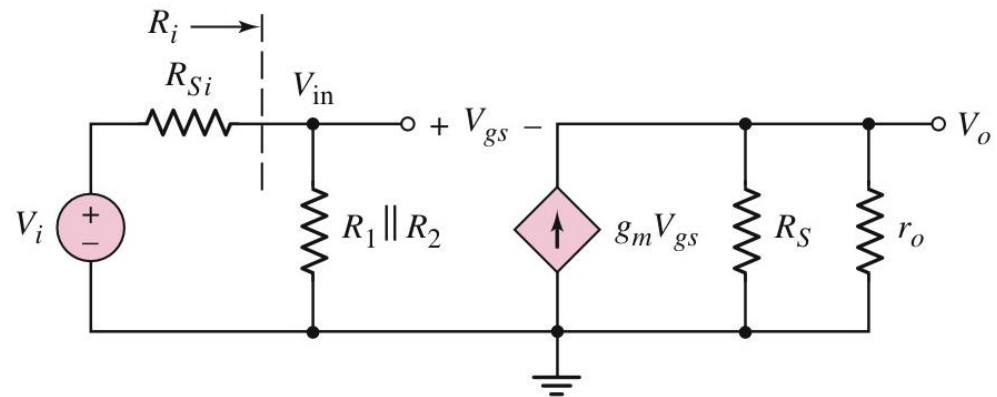
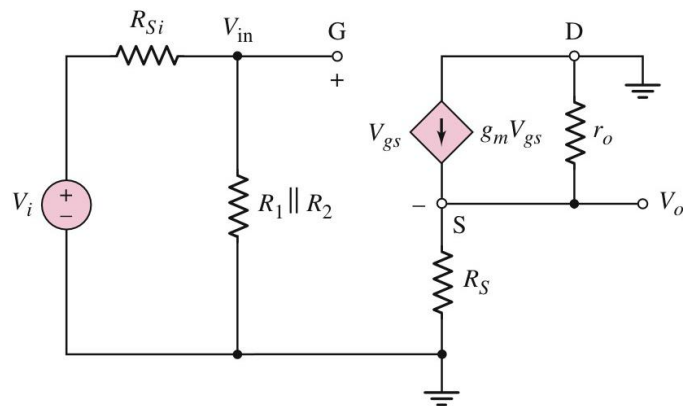
Common-drain

The small-signal analysis may be performed using the following small-signal equivalent circuit.



Common-drain - Voltage gain A_v

The small-signal equivalent circuit may be redrawn as follows;

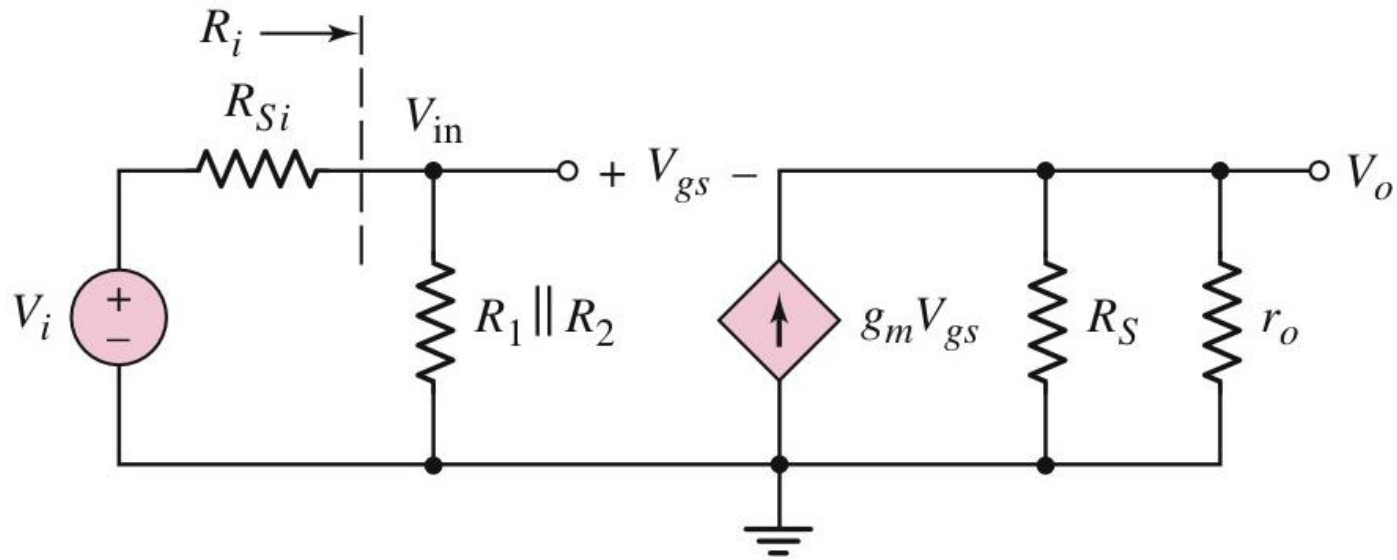


$$V_o = g_m V_{gs} (R_S \parallel r_o)$$



$$\frac{V_o}{V_{gs}} = g_m (R_S \parallel r_o)$$

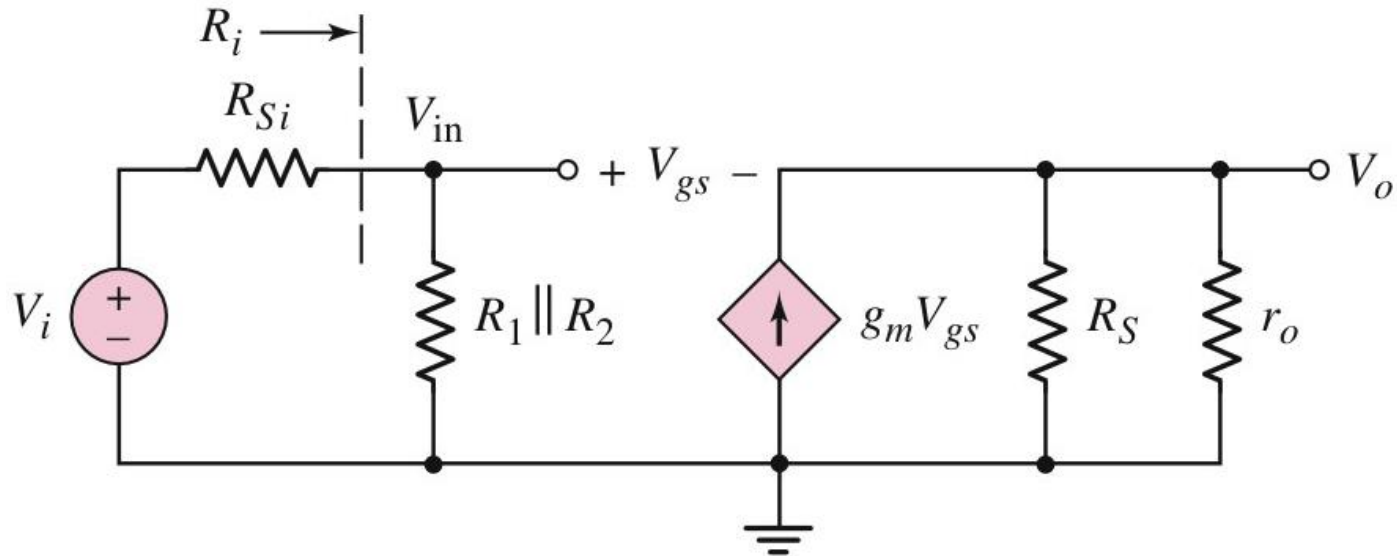
Common-drain - Voltage gain A_v



$$V_{in} = V_{gs} + V_o = V_{gs} + g_m V_{gs} (R_S \parallel r_o) \quad \Rightarrow \quad V_{gs} = \frac{V_{in}}{1 + g_m (R_S \parallel r_o)}$$

$$V_o = g_m V_{gs} (R_S \parallel r_o)$$

Common-drain - Voltage gain A_v



At the input side;

$$V_{in} = V_i \left(\frac{R_i}{R_{Si} + R_i} \right) \quad \text{where} \quad R_i = R_1 \parallel R_2$$

Common-drain - Voltage gain A_v

$$V_{gs} = \frac{V_{in}}{1 + g_m (R_S \parallel r_o)} \quad \text{and} \quad V_{in} = V_i \left(\frac{R_i}{R_{Si} + R_i} \right)$$

Combining the two equations, gives us;

$$V_{gs} = \frac{R_i}{[1 + g_m (R_S \parallel r_o)](R_{Si} + R_i)} V_i$$

Hence;

$$\frac{V_{gs}}{V_i} = \frac{R_i}{[1 + g_m (R_S \parallel r_o)](R_{Si} + R_i)}$$

Common-drain - Voltage gain A_v

$$\frac{V_o}{V_{gs}} = g_m (R_S \parallel r_o)$$

$$\frac{V_{gs}}{V_i} = \frac{R_i}{[1 + g_m (R_S \parallel r_o)](R_{Si} + R_i)}$$

The small-signal voltage gain is;

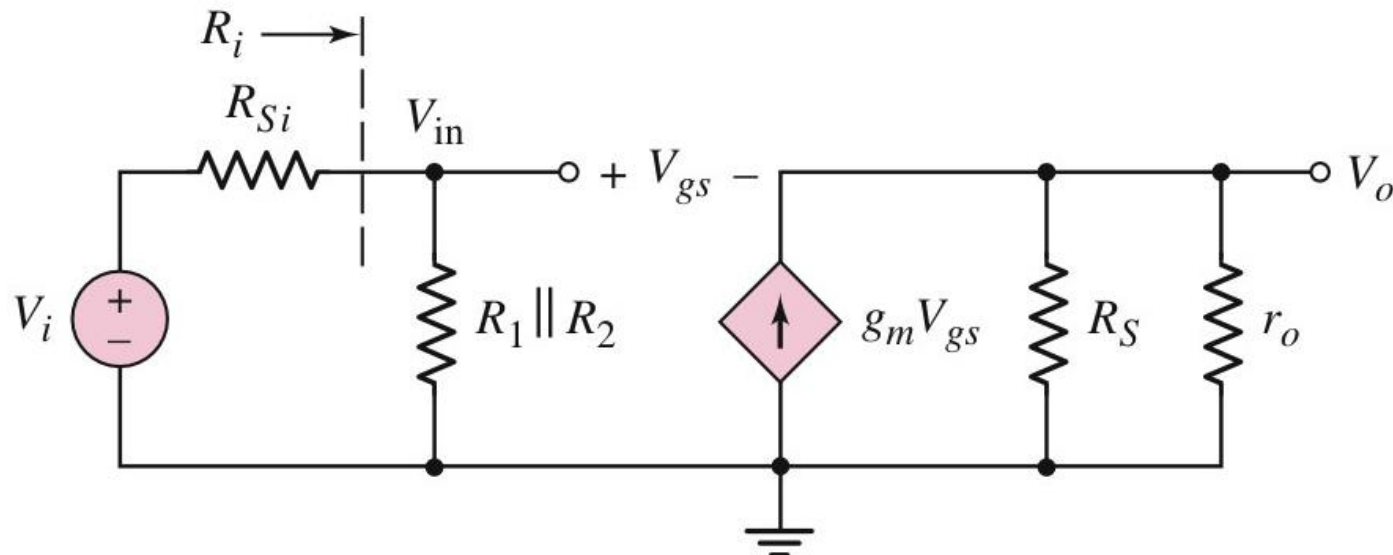
$$\begin{aligned} A_v &= \frac{V_o}{V_i} = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_i} \\ &= \frac{g_m (R_S \parallel r_o)}{1 + g_m (R_S \parallel r_o)} \left(\frac{R_i}{R_{Si} + R_i} \right) \end{aligned}$$

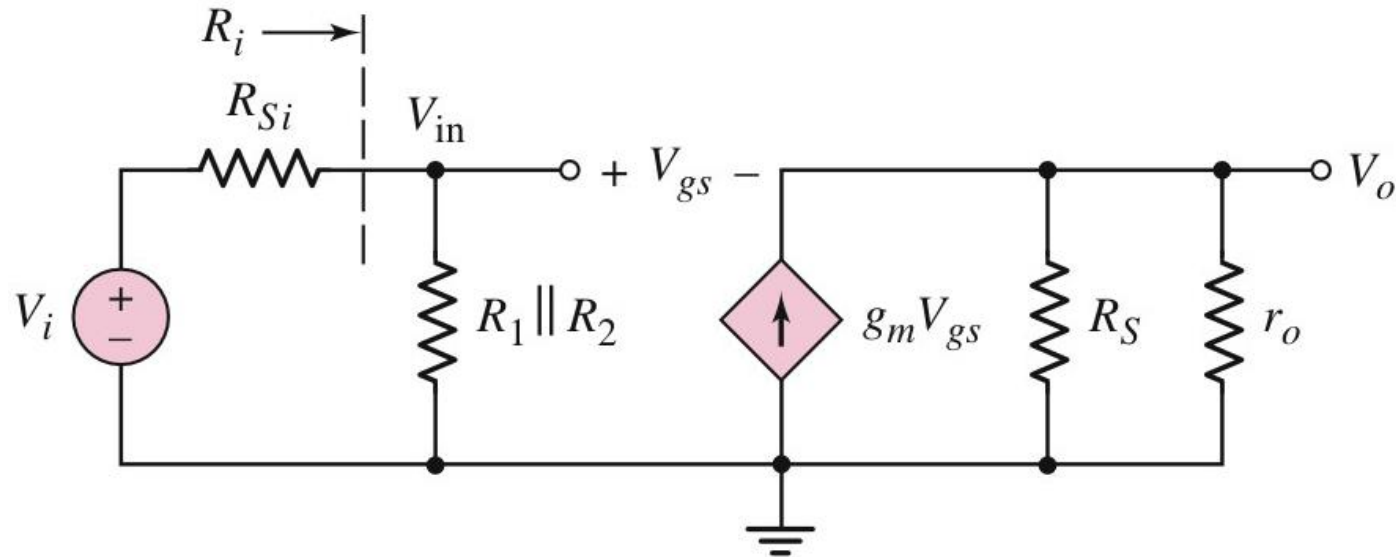
Common-drain - Voltage gain A_v

The small-signal voltage gain also can be written as;

$$A_v = \frac{(R_S \parallel r_o)}{1/g_m + (R_S \parallel r_o)} \left(\frac{R_i}{R_{Si} + R_i} \right)$$

Note that there is no phase shift



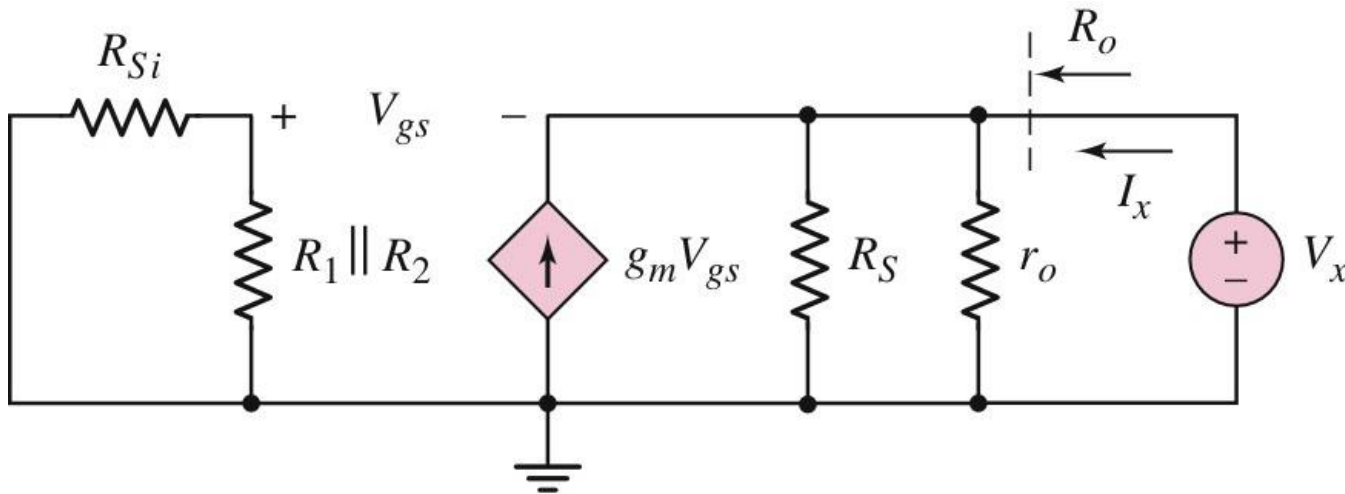
Common-drain - Input resistance R_i 

The input resistance of the common-drain amplifier is;

$$R_i = (R_1 \parallel R_2)$$

Common-drain - Output resistance R_o

To calculate the output resistance, set all the independent voltage source to zero and apply a test voltage V_x at the output terminal as shown below;

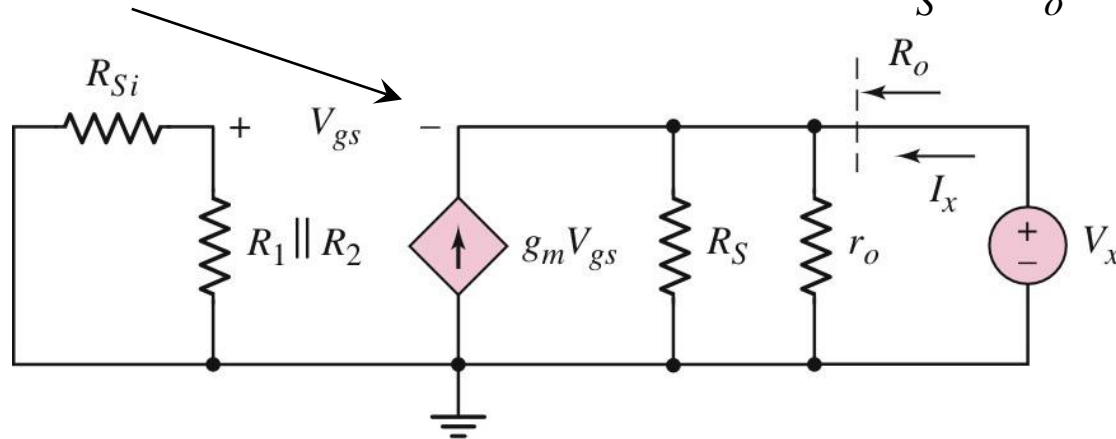


$$R_o = \frac{V_x}{I_x}$$

Common-drain - Output resistance R_o

Applying KCL at this node gives the equation

$$I_x + g_m V_{gs} = \frac{V_x}{R_S} + \frac{V_x}{r_o}$$



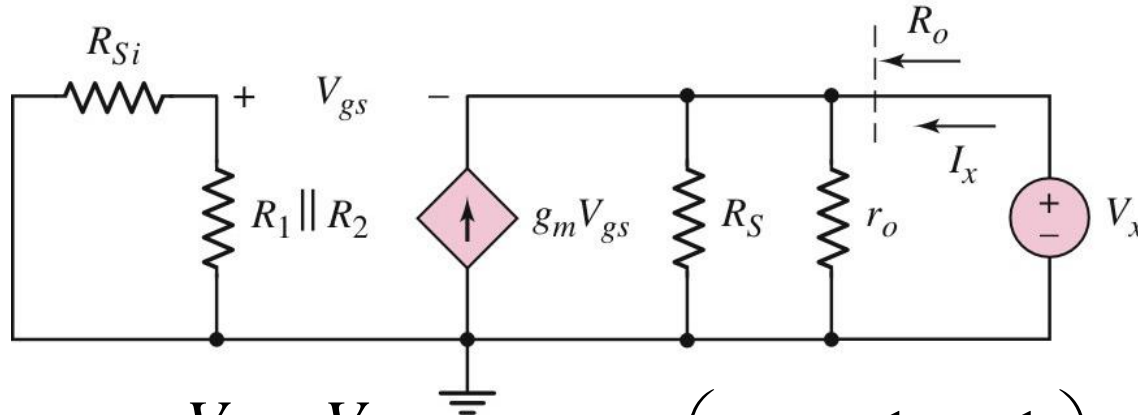
Since there is no current in the input side, we have;

$$V_{gs} = -V_x$$

Substituting for V_{gs} , we have;

$$I_x - g_m V_x = \frac{V_x}{R_S} + \frac{V_x}{r_o}$$

Common-drain - Output resistance R_o



$$\text{Or; } I_x = g_m V_x + \frac{V_x}{R_S} + \frac{V_x}{r_o} \quad I_x = \left(g_m + \frac{1}{R_S} + \frac{1}{r_o} \right) V_x$$

$$\frac{I_x}{V_x} = \frac{1}{R_o} = \left(g_m + \frac{1}{R_S} + \frac{1}{r_o} \right)$$

The output resistance is: $R_o = \left(1 / g_m \parallel R_S \parallel r_o \right)$

The transistor parameters in the figure are as follows;

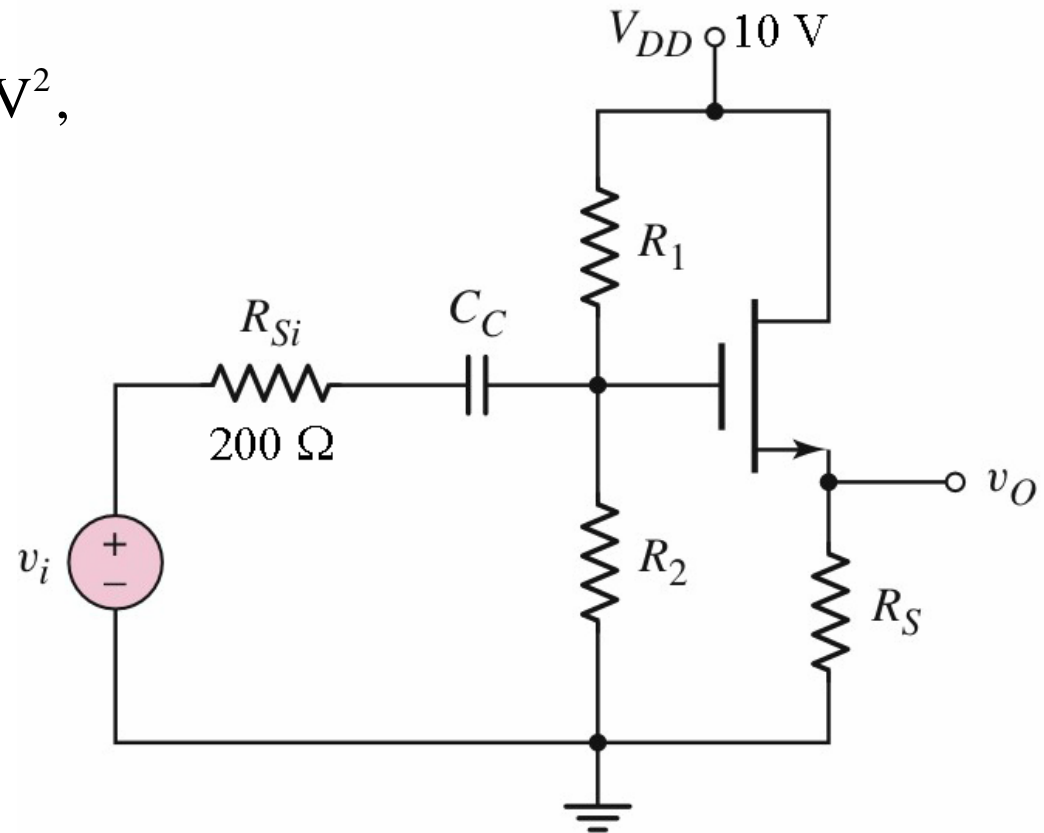
$$V_{TN} = +0.8 \text{ V}, \quad K_n = 1 \text{ mA/V}^2, \\ \text{and } \lambda = 0.015 \text{ V}^{-1}$$

Design the circuit such that;

$$R_1 + R_2 = 400 \text{ k}\Omega,$$

$$I_{DQ} = 1.5 \text{ mA}$$

$$\text{and } V_{DSQ} = 5 \text{ V}$$



Determine the small-signal voltage gain

$$R_1 + R_2 = 400 \text{ k}\Omega, I_{DQ} = 1.5 \text{ mA} \\ \text{and } V_{DSQ} = 5 \text{ V}$$

$$V_S = V_{DD} - V_{DS} = 10 - 5 = 5 \text{ V}$$

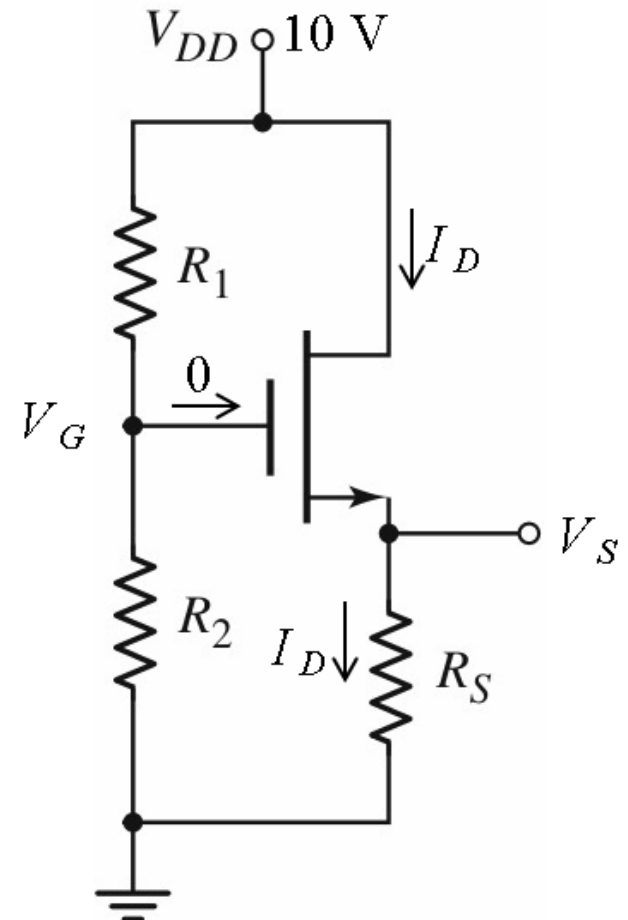
$$R_S = \frac{V_S}{I_D} = \frac{5}{1.5 \times 10^{-3}} = 3.33 \text{ k}\Omega$$

$$V_G = V_{GS} + V_S = V_{GS} + 5$$

$$V_{GS} + 5 = 10 \left(\frac{R_2}{R_1 + R_2} \right) = \frac{R_2}{40 \times 10^3}$$

$$V_{GS} = \frac{R_2}{40 \times 10^3} - 5$$

The dc equivalent circuit;



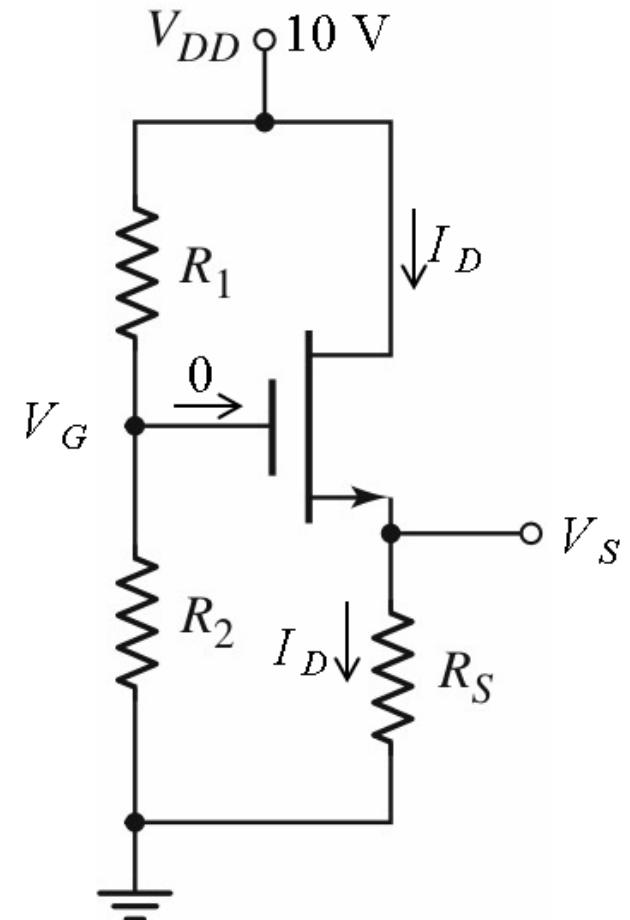
The dc equivalent circuit;

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$1.5 = 1 \left[\left(\frac{R_2}{40 \times 10^3} - 5 \right) - V_{TN} \right]^2$$

$$= \left[\left(\frac{R_2}{40 \times 10^3} - 5 \right) - 0.8 \right]^2$$

$$= \left[\frac{R_2}{40 \times 10^3} - 5.8 \right]^2$$



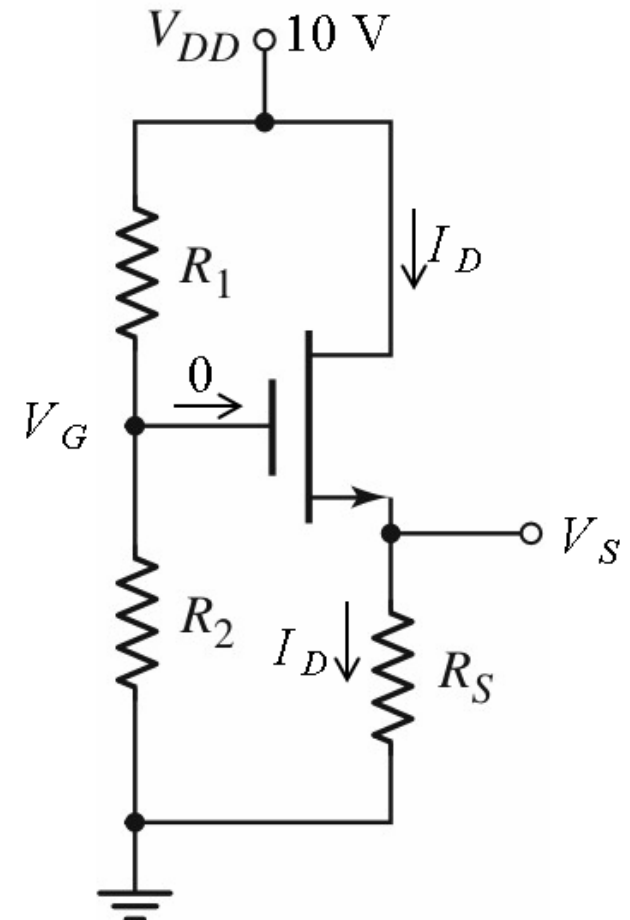
The dc equivalent circuit;

$$\frac{R_2}{40 \times 10^3} - 5.8 = \pm \sqrt{1.5} = \pm 1.225$$

$$R_2 = (\pm 1.225 + 5.8) 40 \times 10^3$$

$$R_2 = 281 \text{ k}\Omega$$

$$R_2 = 183 \text{ k}\Omega$$



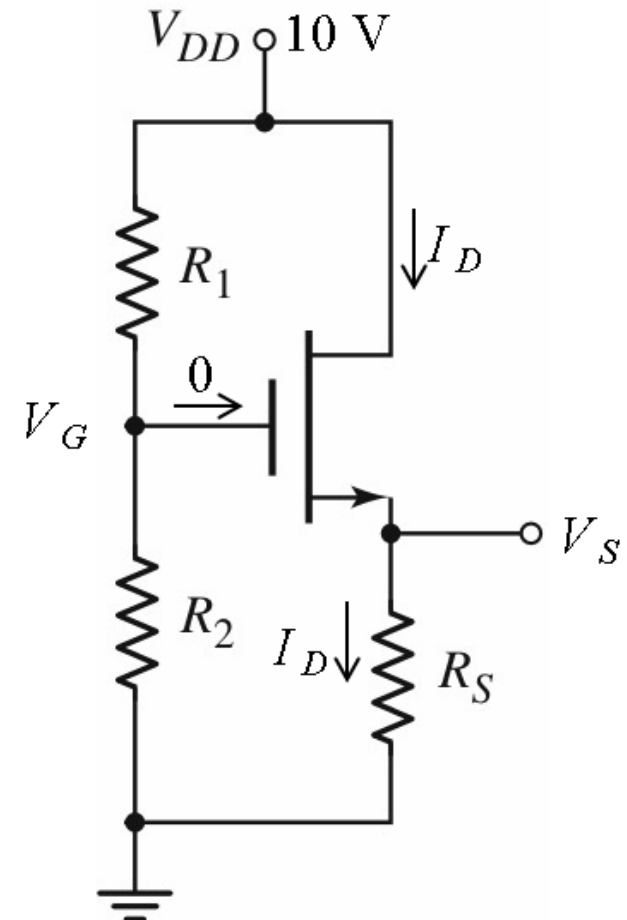
If $R_2 = 281 \text{ k}\Omega$

$$R_1 = 400 - 281 = 119 \text{ k}\Omega$$

$$V_G = 10 \left(\frac{281}{119 + 281} \right) = 7.025 \text{ V}$$

$$V_{GS} = V_G - V_S = 7.025 - 5 = 2.025 \text{ V}$$

The dc equivalent circuit:



The dc equivalent circuit;

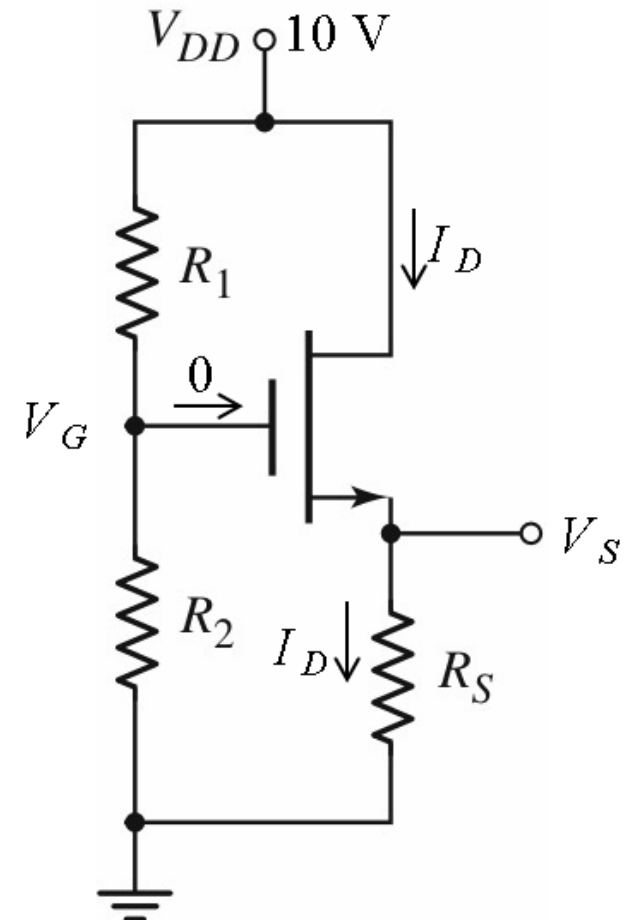
If $R_2 = 183 \text{ k}\Omega$

$$R_1 = 400 - 183 = 217 \text{ k}\Omega$$

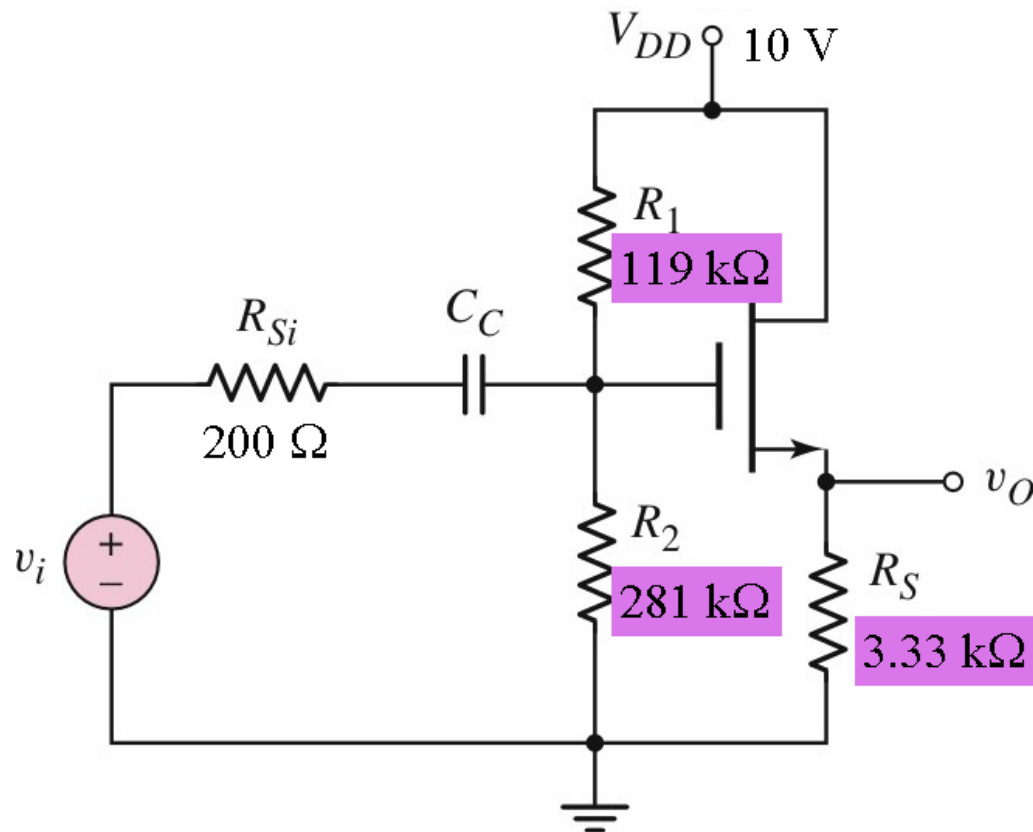
$$V_G = 10 \left(\frac{183}{217 + 183} \right) = 4.575 \text{ V}$$

$$V_{GS} = V_G - V_S = 4.575 - 5 = -0.425 \text{ V}$$

Unrealistic - negative V_{GS} .



The circuit with all the designed values;



The transconductance is;

$$\begin{aligned} g_m &= 2K_n (V_{GS} - V_{TN}) \\ &= 2 \times 10^{-3} (2.025 - 0.8) \\ &= 2.45 \text{ mA/V} \end{aligned}$$

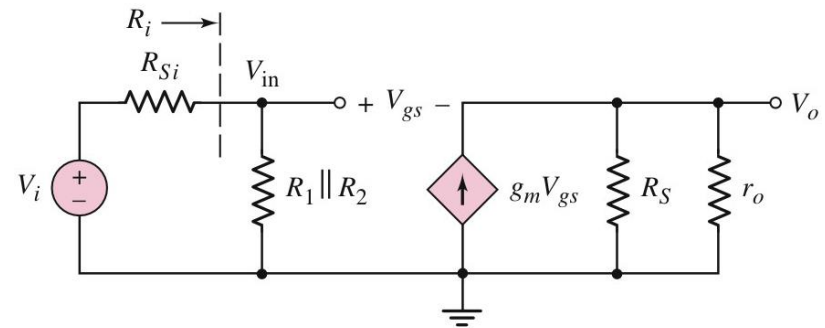
The output resistance of the transistor;

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.015 \times 1.5 \times 10^{-3}} = 44.44 \text{ k}\Omega$$

The small-signal voltage of the amplifier is;

$$A_v = \frac{(R_S \parallel r_o)}{1/g_m + (R_S \parallel r_o)} \left(\frac{R_i}{R_{Si} + R_i} \right)$$

$$= \frac{(3.33 \parallel 44.44)}{1/2.45 \times 10^{-3} + (3.33 \parallel 44.44)} \left(\frac{119 \parallel 281}{0.2 + 119 \parallel 281} \right)$$



$$A_v = 0.882 \text{ V/V}$$

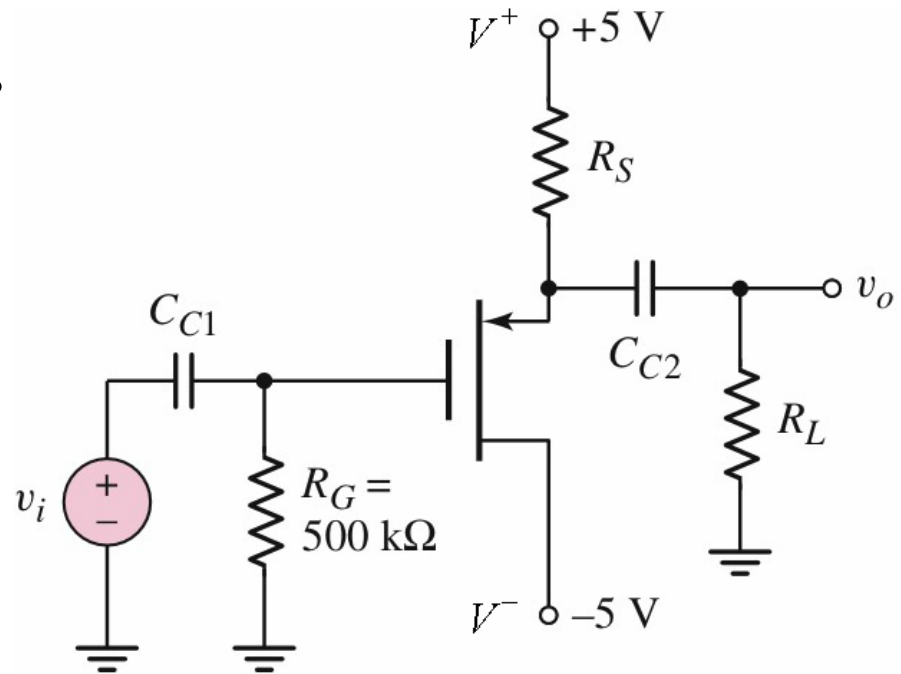
The transistor in the figure has the following parameters;

$$V_{TP} = -2 \text{ V}, \quad K_p = 2 \text{ mA/V}^2,$$

$$\text{and } \lambda = 0.02 \text{ V}^{-1}$$

Design the circuit such that
 $I_{DQ} = 3 \text{ mA}$.

Determine the open-circuit
small-signal **voltage gain** and
output resistance



What value of R_L will result in a 10% reduction in gain?

Since $I_D = 3 \text{ mA}$;

$$V_S = 5 - 3 \times 10^{-3} R_S$$

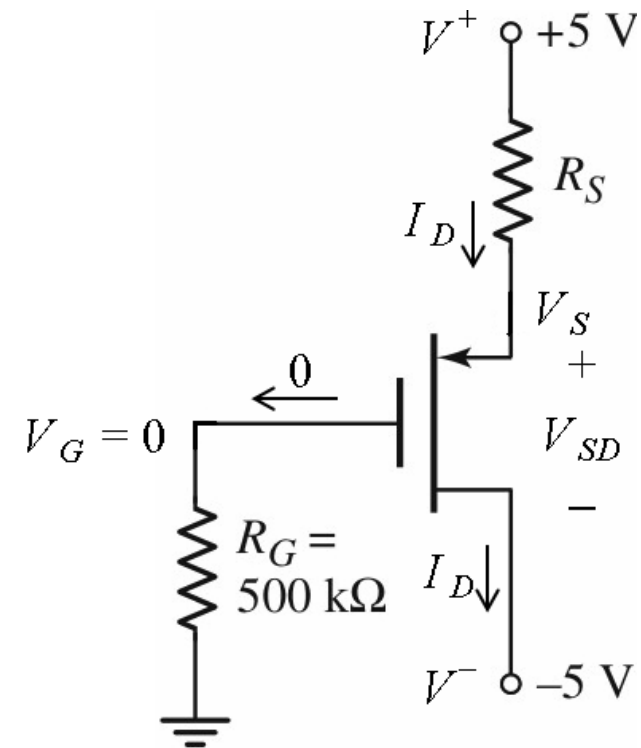
Because G is at ground potential, we have;

$$V_{SG} = V_S = 5 - 3 \times 10^{-3} R_S$$

The equation for the drain current is;

$$I_D = K_p (V_{SG} + V_{TP})^2$$

The circuit under dc condition



Substituting values;

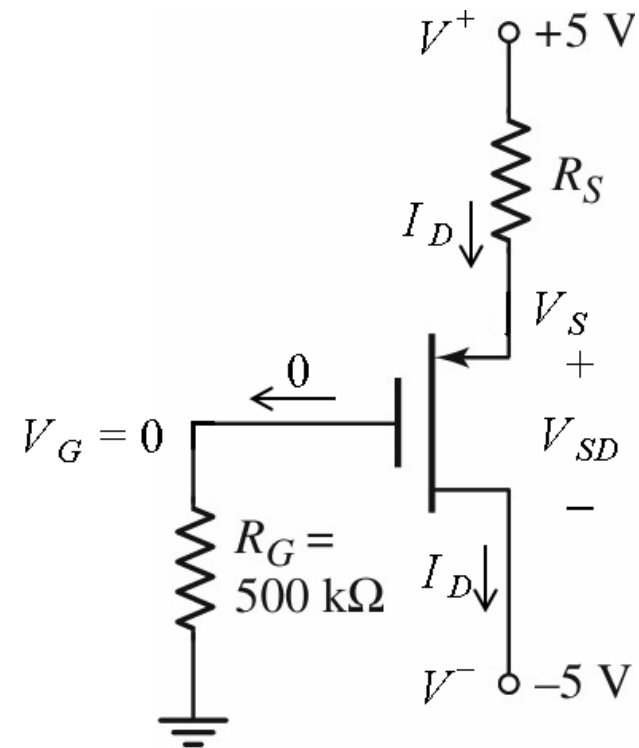
$$3 = 2(5 - 3 \times 10^{-3} R_S - 2)^2$$

Or;

$$R_S = \frac{3 \pm \sqrt{1.5}}{3} \times 10^3$$

The above equation gives us two values;

$$R_S = 1.41 \text{ k}\Omega \quad \text{or} \quad R_S = 592 \text{ }\Omega$$



For $R_S = 1.41 \text{ k}\Omega$

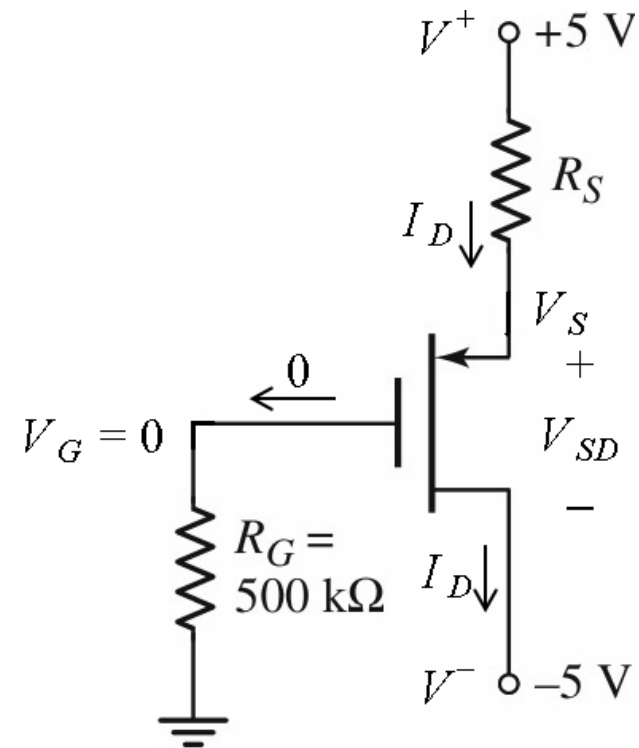
$$V_{SG} = 5 - 3 \times 10^{-3} \times 1.41 \times 10^3 \\ = 0.77 \text{ V}$$

(unrealistic because $V_{SG} < |V_{TP}|$)

For $R_S = 592 \text{ }\Omega$

$$V_{SG} = 5 - 3 \times 10^{-3} \times 0.592 \times 10^3 \\ = 3.24 \text{ V}$$

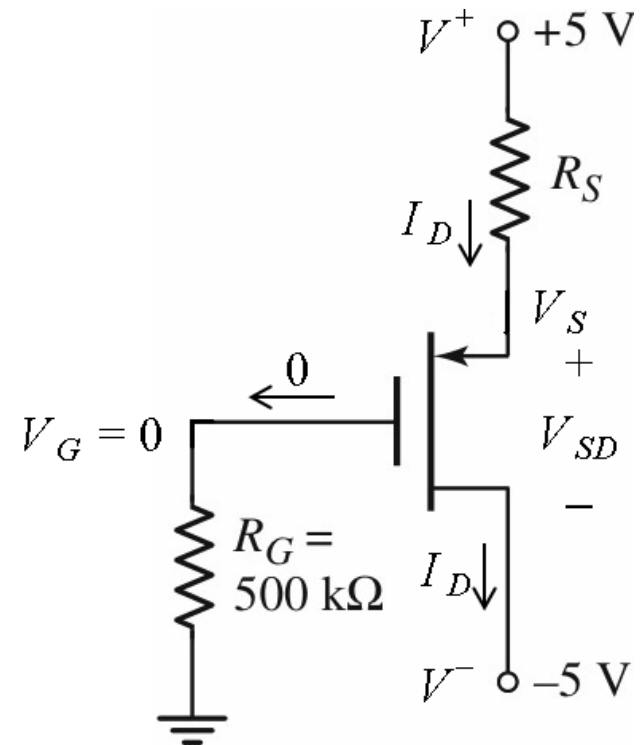
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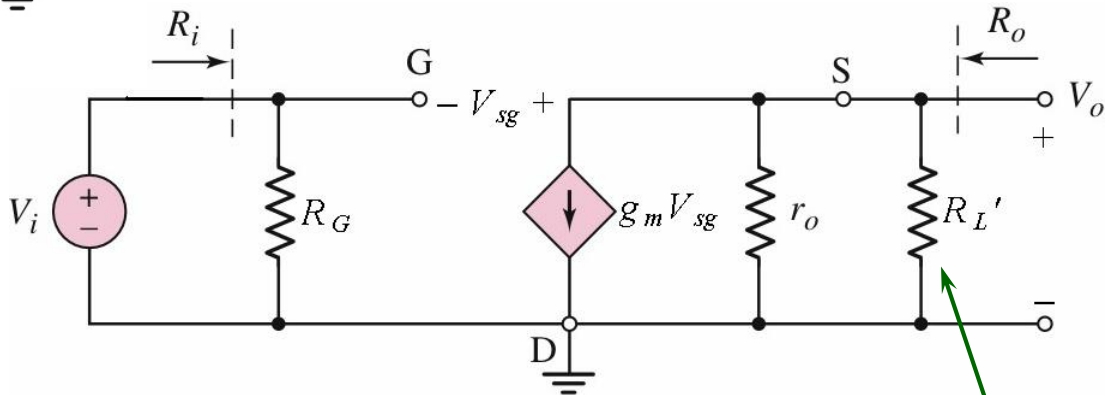
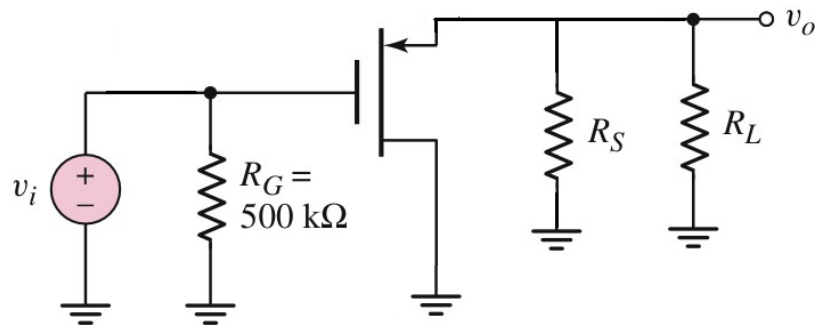


The transconductance is;

$$\begin{aligned} g_m &= 2K_p(V_{SG} + V_{TP}) \\ &= 2 \times 2 \times 10^{-3}(3.24 - 2) \\ &= 4.96 \text{ mA/V} \end{aligned}$$

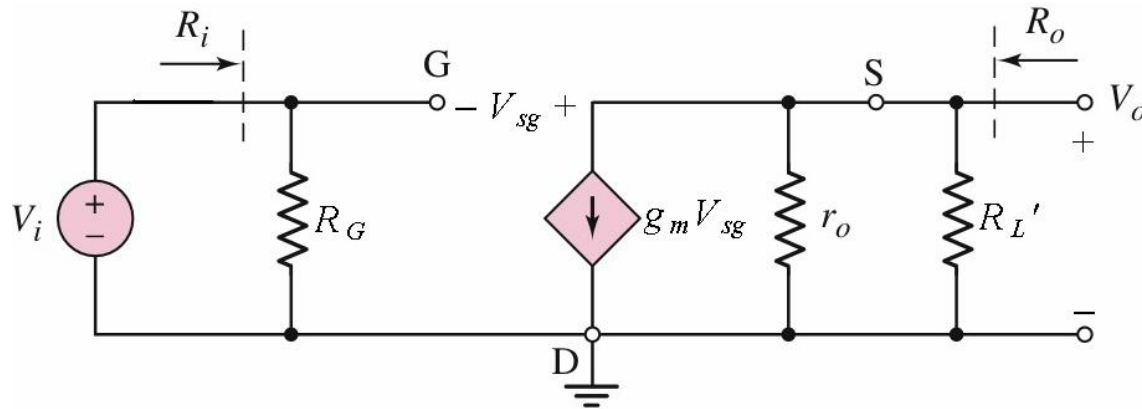
$$\begin{aligned} r_o &= \frac{1}{\lambda I_D} = \frac{1}{0.02 \times 3 \times 10^{-3}} \\ &= 16.6 \text{ k}\Omega \end{aligned}$$





$$R_L' = R_S \parallel R_L$$

$$V_o = -g_m V_{sg} (r_o \parallel R_L') \quad \Rightarrow \quad V_{sg} = -\frac{V_o}{g_m (r_o \parallel R_L')}$$



$$V_i = -V_{sg} + V_o = \frac{V_o}{g_m(r_o \parallel R_L')} + V_o = \left[\frac{1 + g_m(r_o \parallel R_L')}{g_m(r_o \parallel R_L')} \right] V_o$$

$$A_v = \frac{V_o}{V_i} = \left[\frac{g_m(r_o \parallel R_L')}{1 + g_m(r_o \parallel R_L')} \right]$$

$$A_v = \frac{V_o}{V_i} = \left[\frac{g_m (r_o \parallel R_L')}{1 + g_m (r_o \parallel R_L')} \right]$$

$$\left. \begin{array}{l} g_m = 4.96 \text{ mA/V} \\ r_o = 16.7 \text{ k}\Omega \end{array} \right\} \text{ (As calculated previously)}$$

$$\left. \begin{array}{l} R_L' = R_S \parallel R_L = R_S \\ = 0.592 \text{ k}\Omega \end{array} \right\} \text{ (Since } R_L = \infty \text{ – open circuit)}$$

The open-circuit small-signal voltage gain is;

$$A_v(\text{open - circuit}) = \left[\frac{4.96 \times 10^{-3} \times 592}{1 + 4.96 \times 10^{-3} \times 592} \right]$$

$$A_v(\text{open - circuit}) = 0.746 \text{ V/V}$$

The voltage gain after a reduction of 10% is;

$$\begin{aligned} A_v' &= 0.9 A_v(\text{open - circuit}) \\ &= 0.9 \times 0.746 \\ &= 0.671 \text{ V/V} \end{aligned}$$

Substituting for A_v in the equation;

$$A_v = \frac{V_o}{V_i} = \left[\frac{g_m (r_o \parallel R_L')}{1 + g_m (r_o \parallel R_L')} \right]$$

we have;

$$0.671 = \left[\frac{g_m (r_o \parallel R_L')}{1 + g_m (r_o \parallel R_L')} \right]$$

Substituting the value for g_m gives us;

$$0.671 = \left[\frac{4.96 \times 10^{-3} (r_o \parallel R_L')}{1 + 4.96 \times 10^{-3} (r_o \parallel R_L')} \right]$$

Solving the above equation, we obtain;

$$r_o \parallel R_L' = \frac{1}{2.432 \times 10^{-3}} = 411 \Omega$$

This equation may be written as;

$$\frac{r_o R_L'}{r_o + R_L'} = 411 \Omega$$

Or;

$$R_L' = \frac{411 r_o}{r_o - 411}$$

Substituting values;

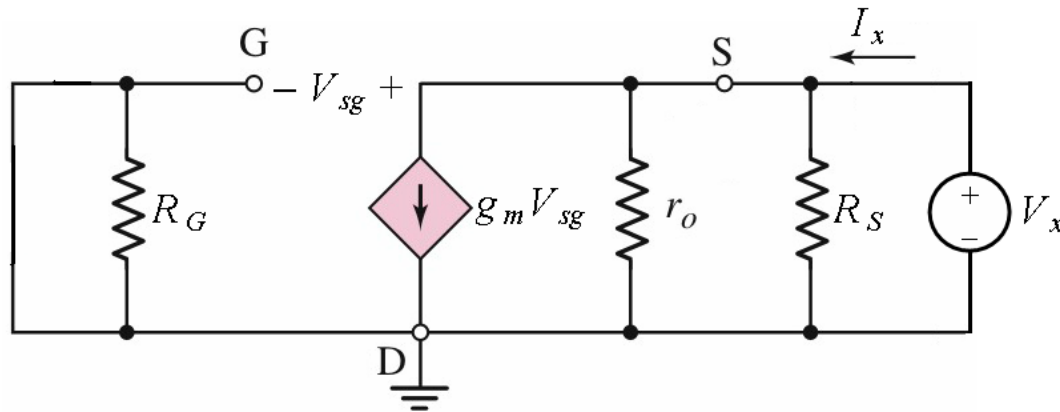
$$R_L' = \frac{411 \times 16.7 \times 10^3}{16.7 \times 10^3 - 411} = 421 \Omega$$

$$R_L' = \frac{R_S R_L}{R_S + R_L} = 421 \Omega$$

$$R_L' = \frac{R_S R_L}{R_S + R_L} = 421 \, \Omega$$

Solving the equation for R_L gives us;

$$R_L = 1.46 \, \text{k}\Omega$$



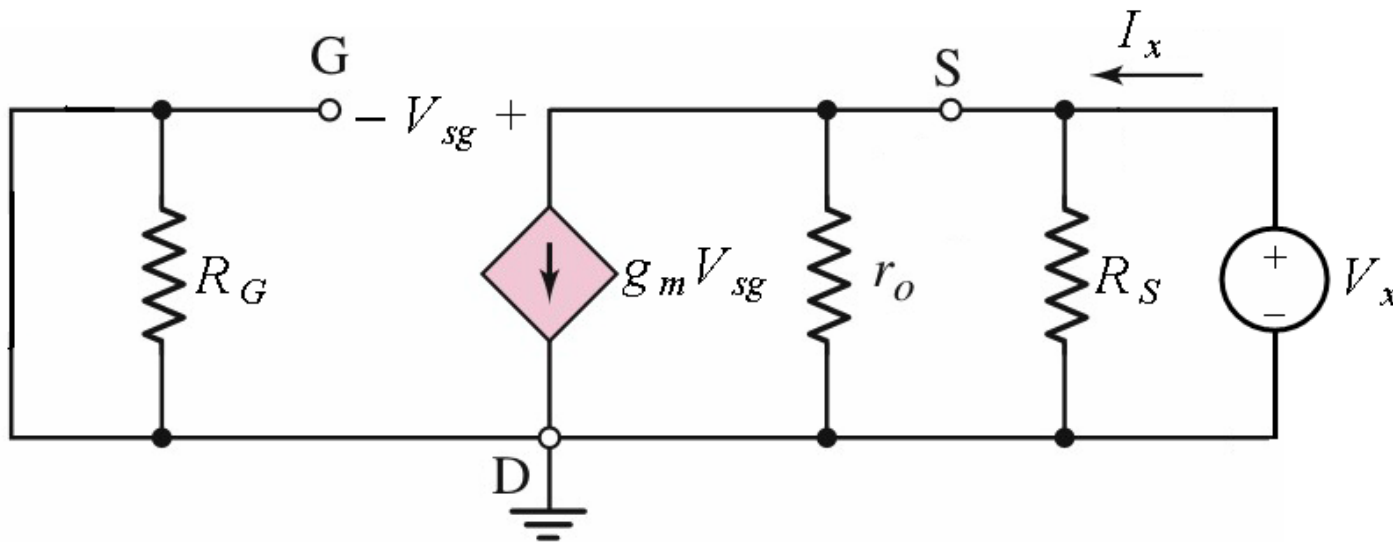
Using the equivalent circuit above, the output resistance R_o is given by the expression;

$$\frac{1}{R_o} = \left(g_m + \frac{1}{R_S} + \frac{1}{r_o} \right)$$

Substituting values;

$$\frac{1}{R_o} = \left(4.96 \times 10^{-3} + \frac{1}{0.592 \times 10^3} + \frac{1}{16.7 \times 10^3} \right)$$

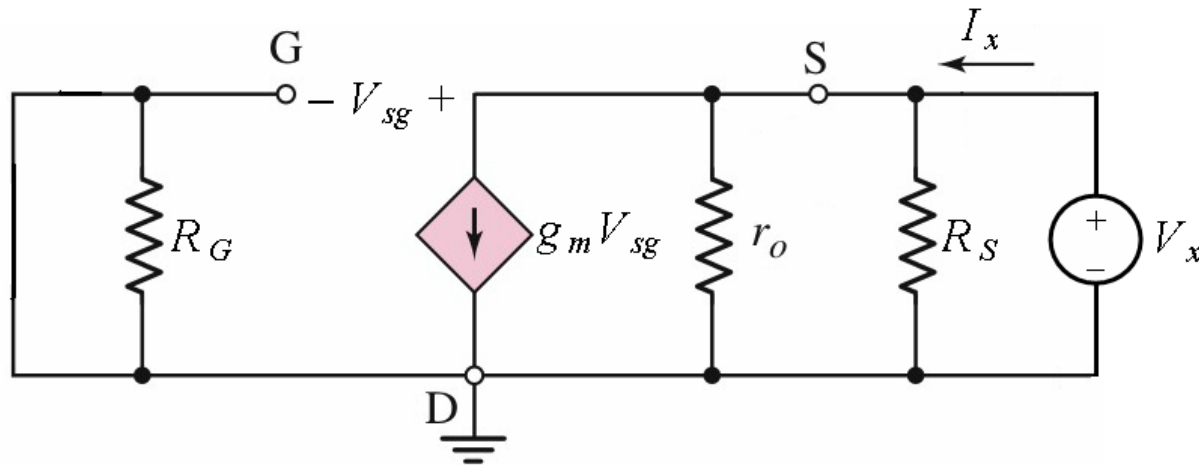
$$= 6.709 \times 10^{-3}$$



The output resistance is;

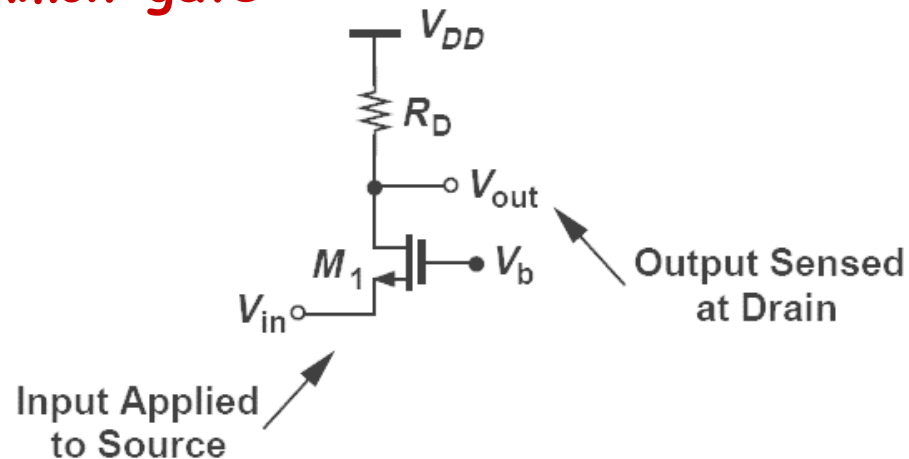
$$R_o = \frac{1}{6.709 \times 10^3}$$

$$R_o = 149 \, \Omega$$



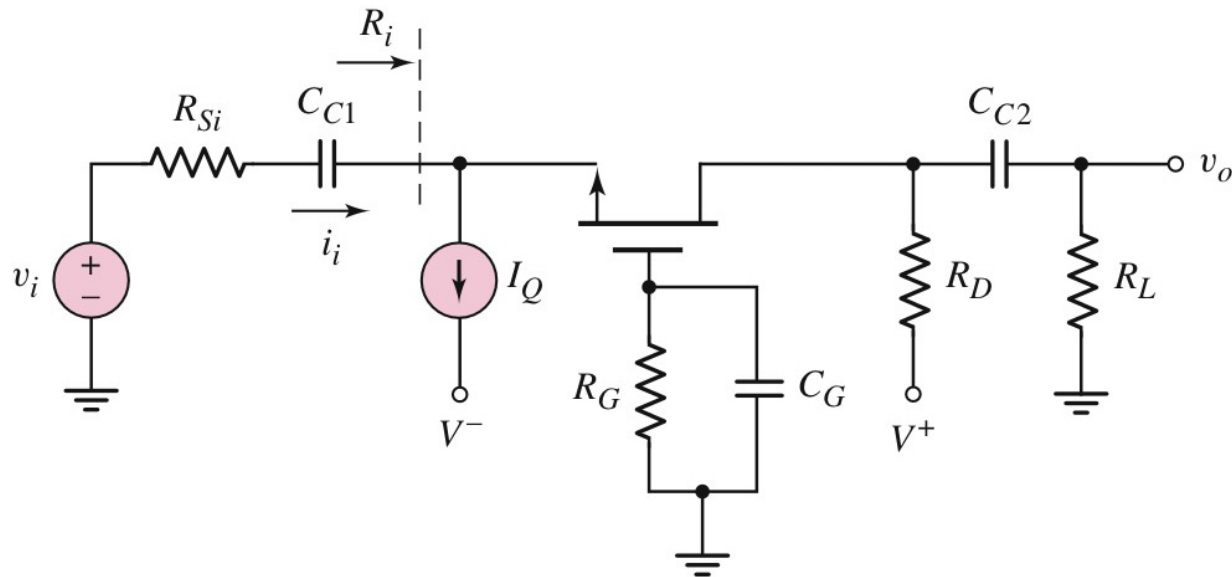
Common-gate

The input signal is applied to the source and gate terminal is grounded. The output is taken between the drain and gate terminals. The gate is therefore common to both the input and output and hence the name "**common-gate**"



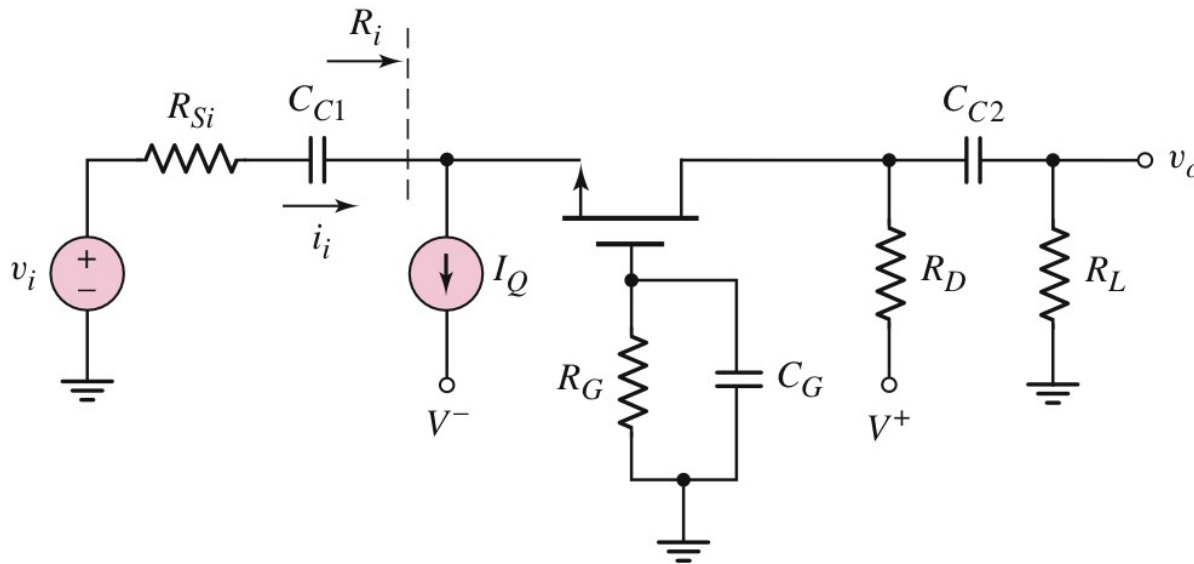
Common-gate

C_{C1} and C_{C2} are the coupling capacitors for input and output respectively. In the following figure, the DC bias current is obtained from the current source I_Q .



Common-gate

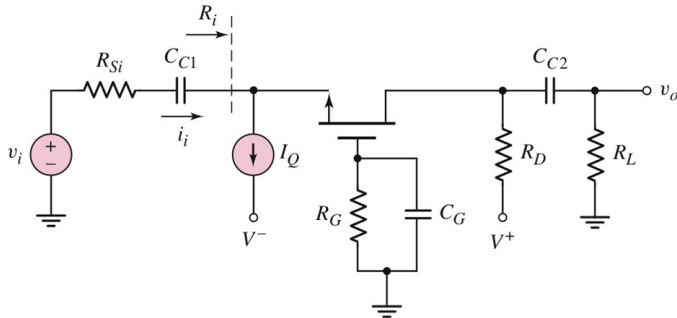
The dc analysis is the same as that of previous configurations.



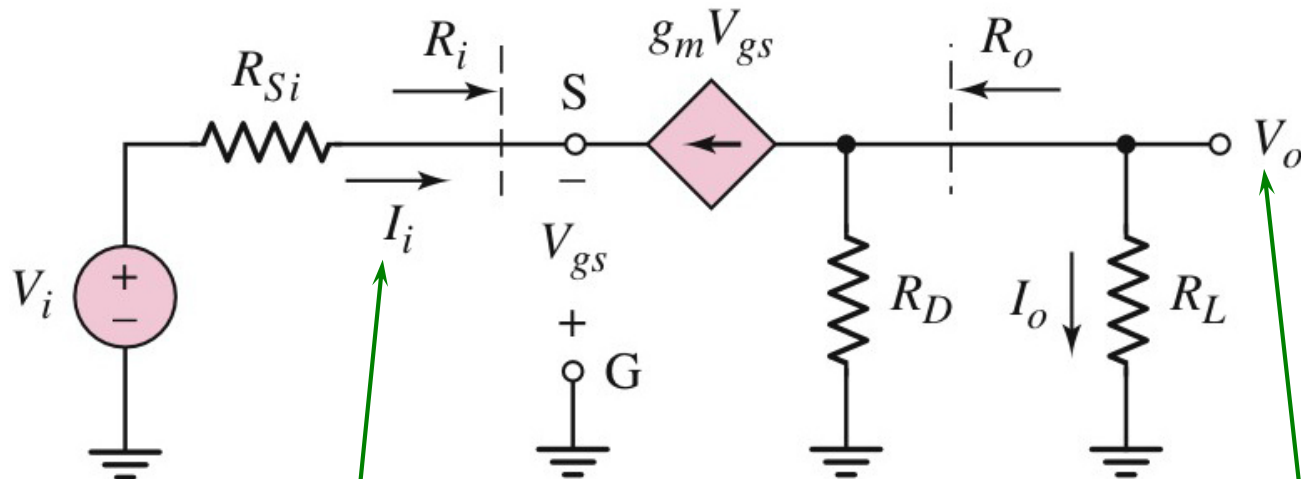
MOSFET

Basic Amplifier Configurations

Common-gate - small-signal voltage gain



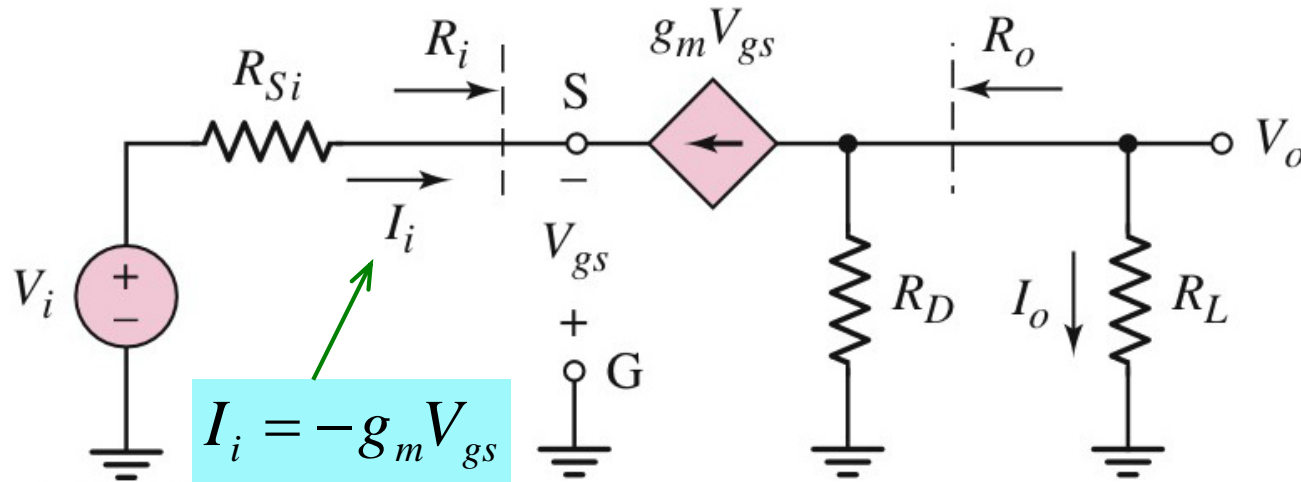
The gains can be determined using the following equivalent circuit;



$$I_i = -g_m V_{gs}$$

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

Common-gate - small-signal voltage gain



Applying KVL at the input, we obtain;

$$V_i = I_i R_{Si} + V_{sg} = I_i R_{Si} - V_{gs}$$

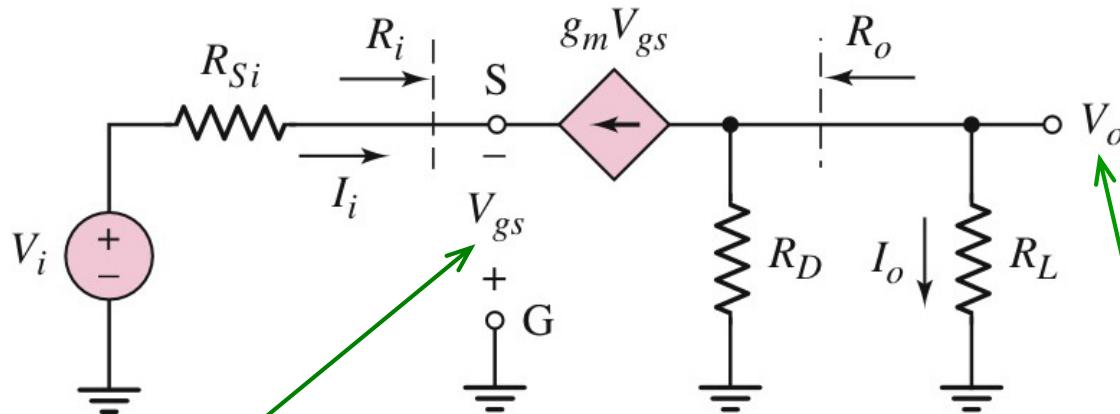
Substituting for I_i , we obtain;

$$V_i = -g_m V_{gs} R_{Si} - V_{gs}$$

Common-gate - small-signal voltage gain

$$V_i = -g_m V_{gs} R_{Si} - V_{gs}$$

Re-arranging, gives us;
$$V_{gs} = \frac{-V_i}{1 + g_m R_{Si}}$$



$$V_{gs} = \frac{-V_i}{(1 + g_m R_{Si})}$$

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

Common-gate - small-signal voltage gain

Substituting for V_{gs} in the expression for V_o , we obtain;

$$V_o = -g_m \left(\frac{-V_i}{(1 + g_m R_{Si})} \right) (R_D \parallel R_L)$$

$$V_{gs} = \frac{-V_i}{(1 + g_m R_{Si})}$$

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

The small-signal voltage gain is;

$$A_v = \frac{V_o}{V_i} = \frac{g_m (R_D \parallel R_L)}{1 + g_m R_{Si}}$$

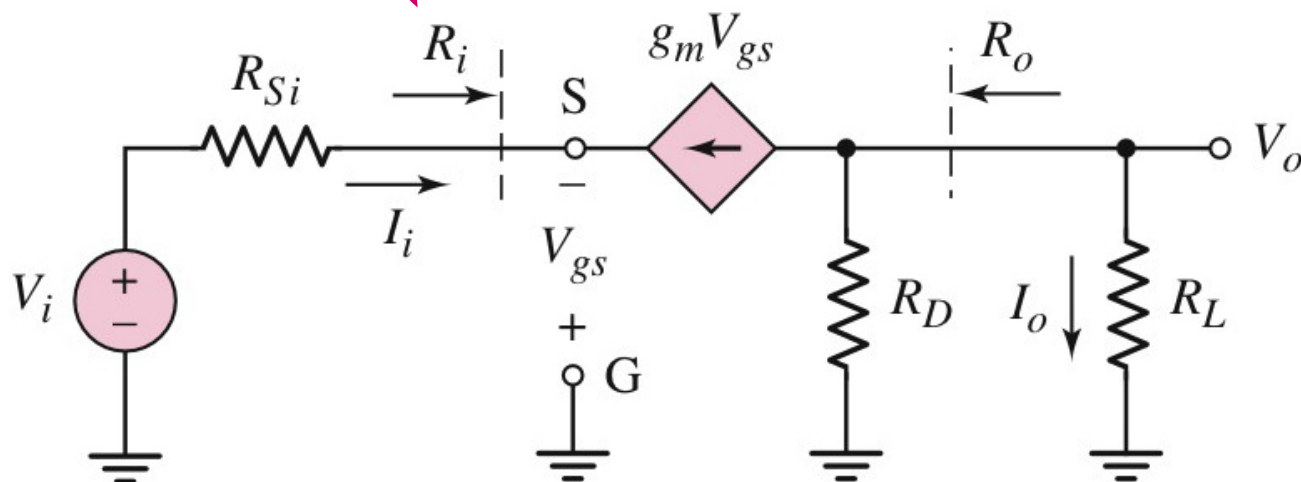
Common-gate - input resistance

Since; $I_i = -g_m V_{gs}$

the input resistance becomes;

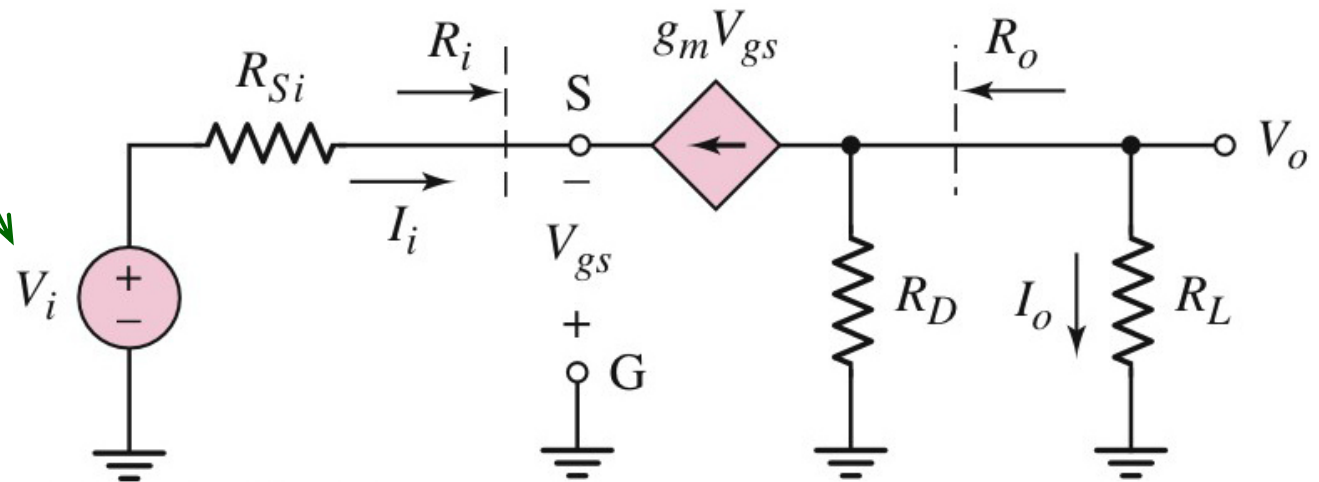
$$R_i = \frac{-V_{gs}}{I_i}$$

$$R_i = \frac{1}{g_m}$$

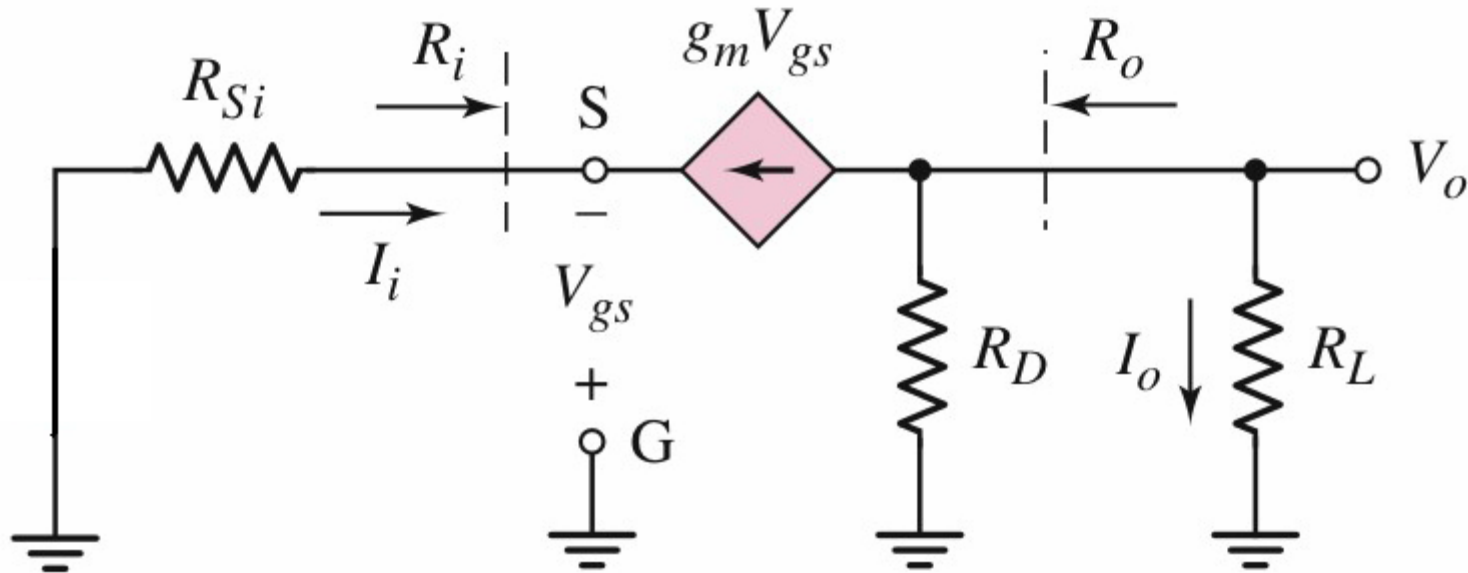


Common-gate - output resistance

The output resistance may be found by setting $V_i = 0$



Common-gate - output resistance



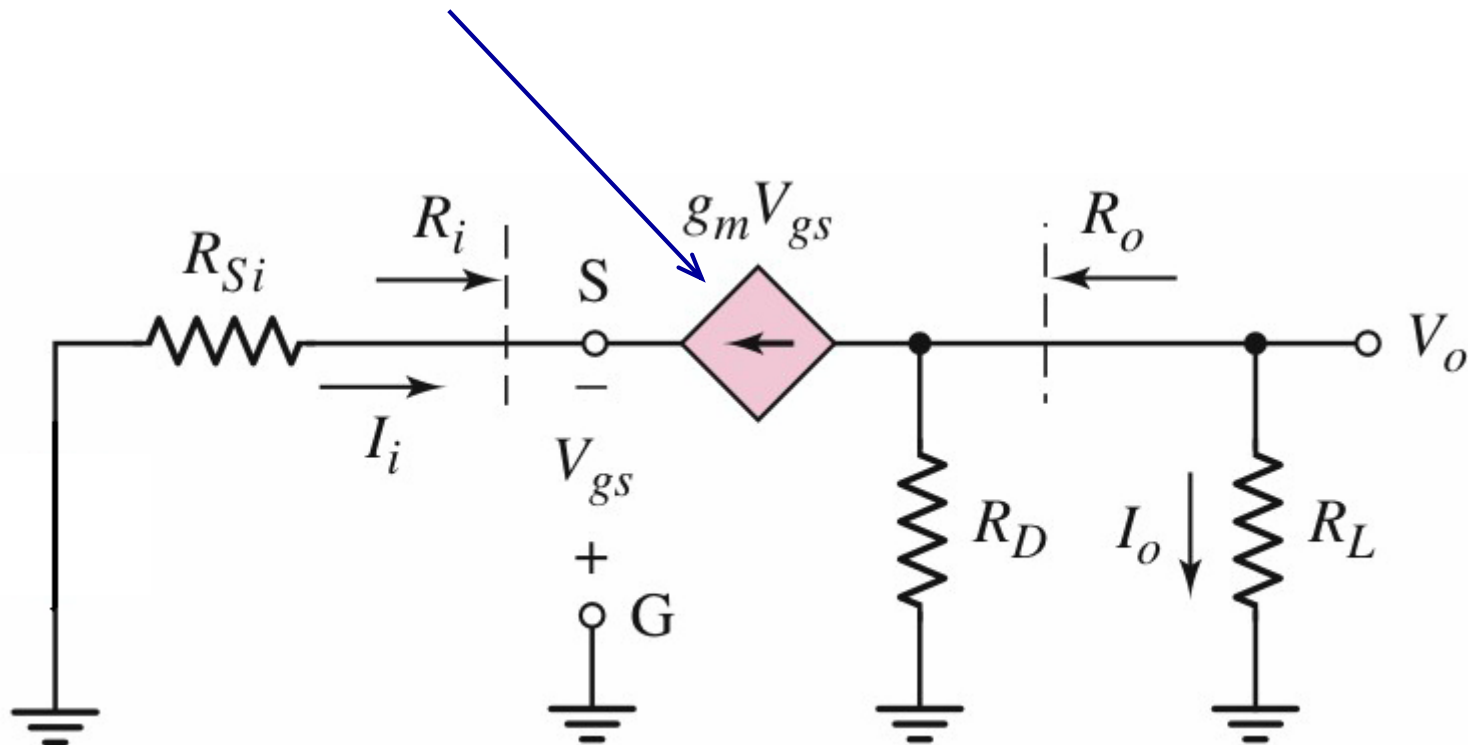
When V_i is set to zero;

$$V_{gs} = -g_m V_{gs} R_{Si}$$

which only holds if $V_{gs} = 0$

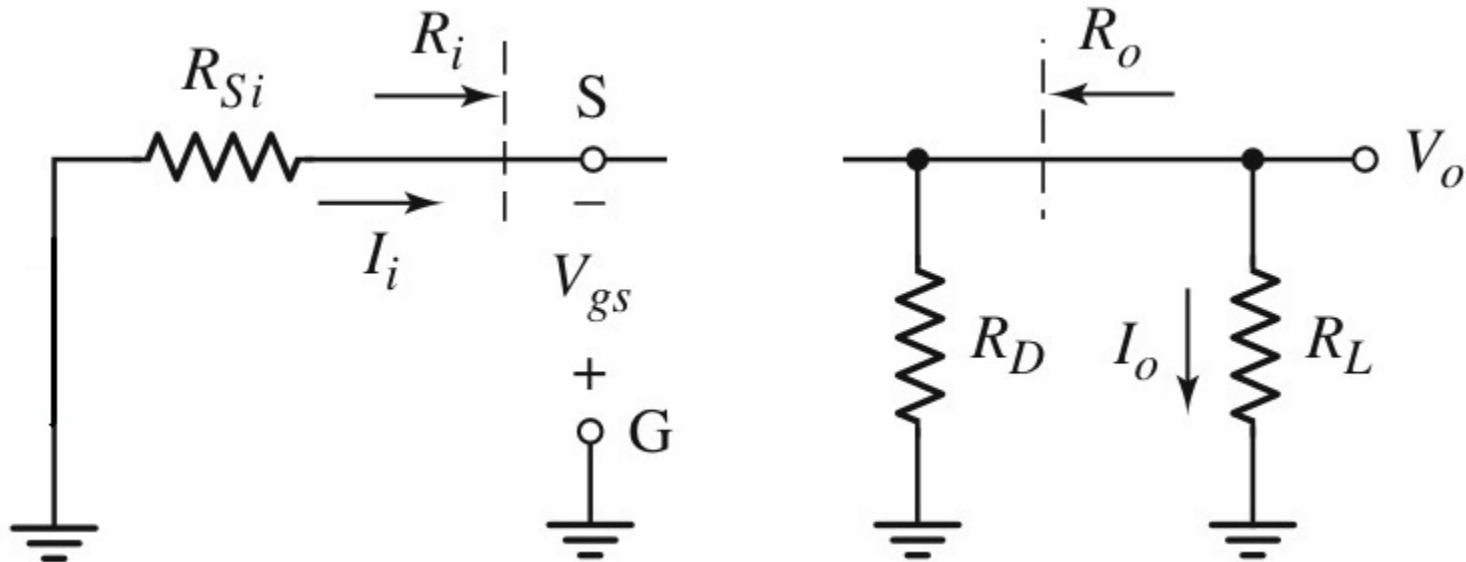
Common-gate - output resistance

When $V_{gs} = 0$, the controlled current source $g_m V_{gs}$ is open



Common-gate - output resistance

The equivalent circuit becomes as shown below;



Hence the output resistance is;

$$R_o = R_D$$

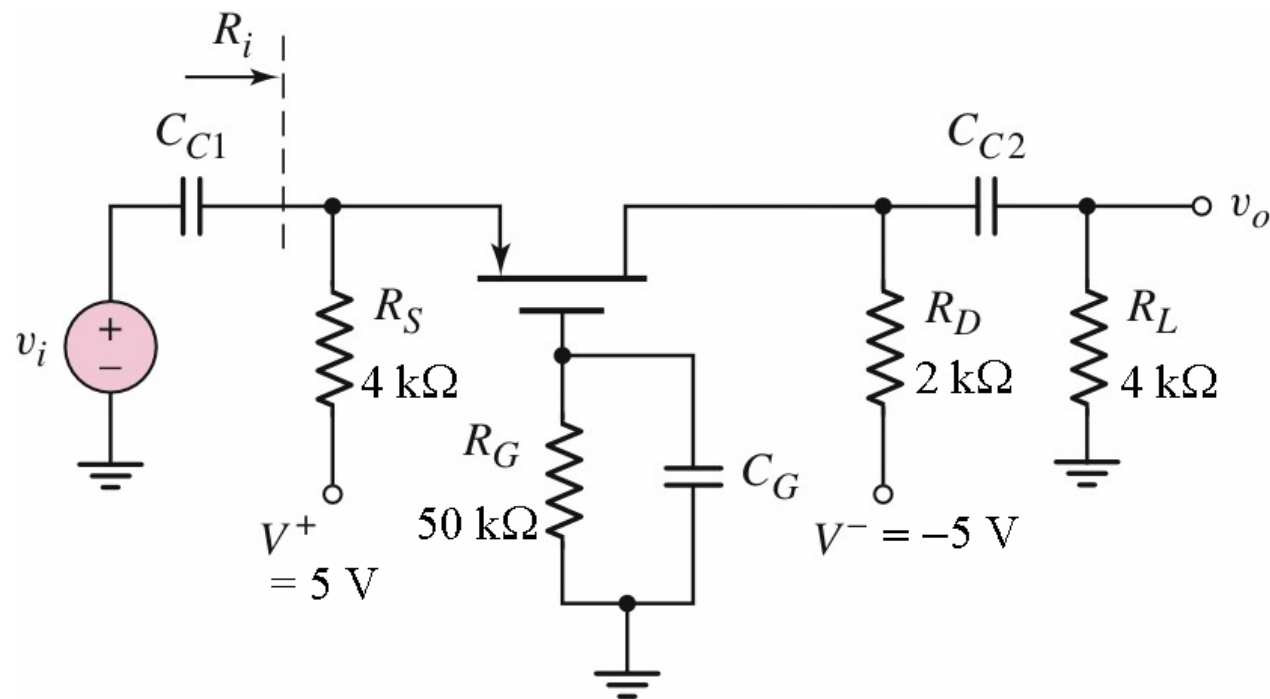
The transistor parameters in the following figure are;

$$K_p = 1 \text{ mA/V}^2, V_{TP} = -0.8 \text{ V}, \lambda = 0$$

(a) Draw the small-signal equivalent

(b) Determine the small-signal voltage gain A_v .

(c) Find the input resistance R_i .

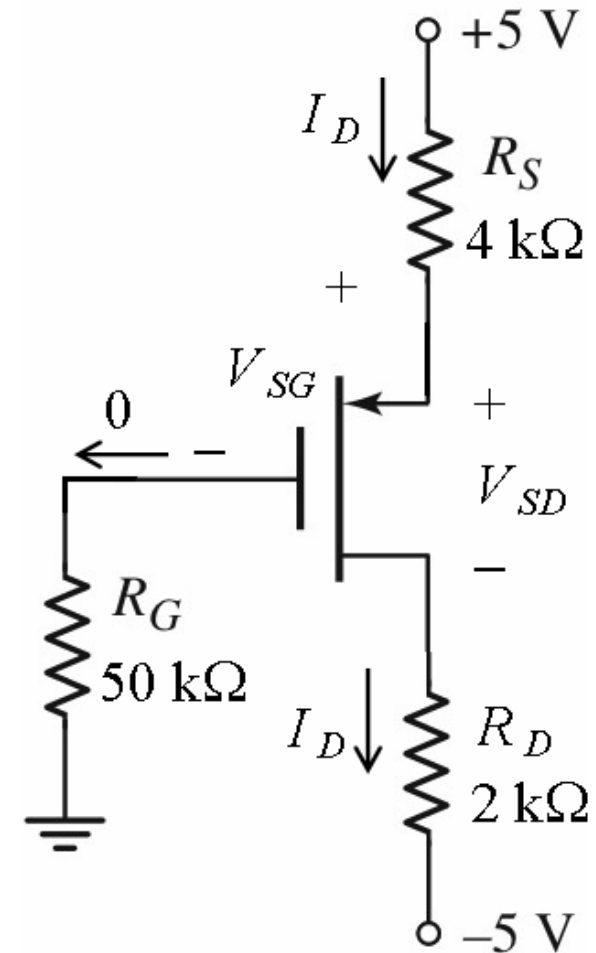


DC analysis

May be performed using the dc equivalent circuit shown.

$$\begin{aligned} I_D &= K_p (V_{SG} + V_{TP})^2 \\ &= 10^{-3} (V_{SG} - 0.8)^2 \end{aligned}$$

$$I_D = 10^{-3} (V_{SG}^2 - 1.6V_{SG} + 0.64)$$

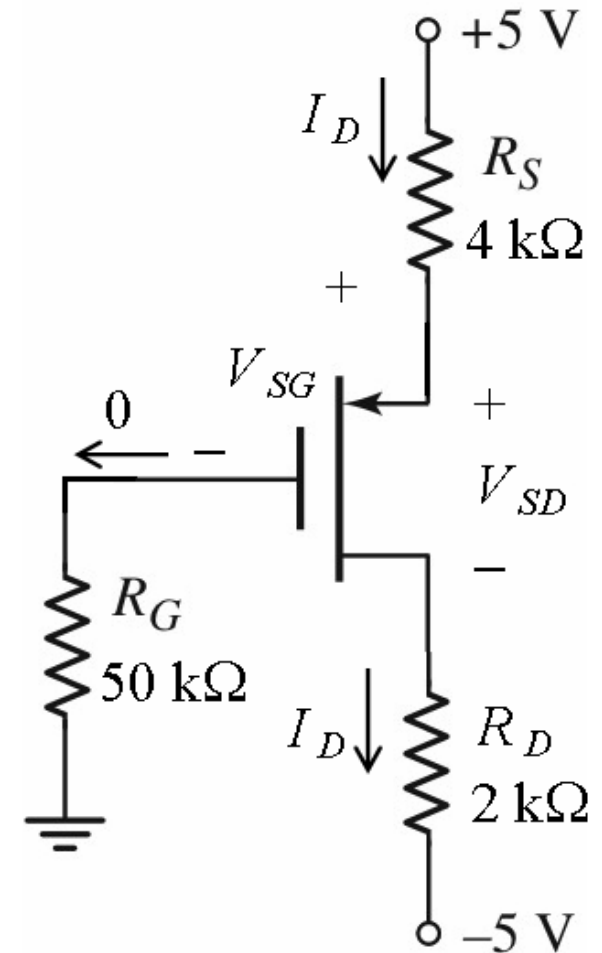


From the dc equivalent circuit;

$$\begin{aligned} V_{SG} = V_S &= 5 - I_D R_S \\ &= 5 - 4 \times 10^3 I_D \end{aligned}$$

Rearranging, we have;

$$I_D = (1.25 - 0.25 V_{SG}) 10^{-3}$$



Substituting for I_D :

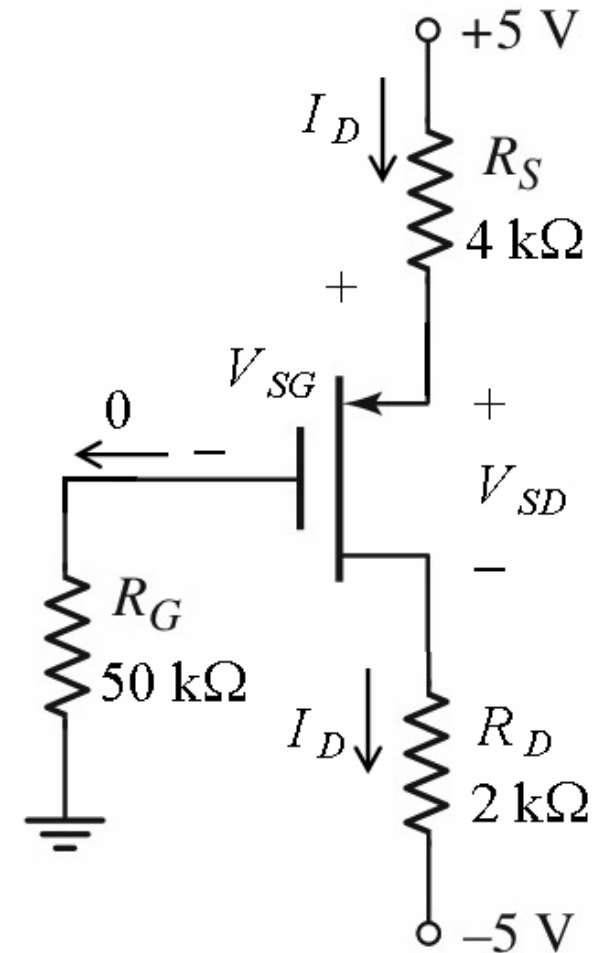
$$1.25 - 0.25V_{SG} = V_{SG}^2 - 1.6V_{SG} + 0.64$$

or;

$$V_{SG}^2 - 1.35V_{SG} - 0.61 = 0$$

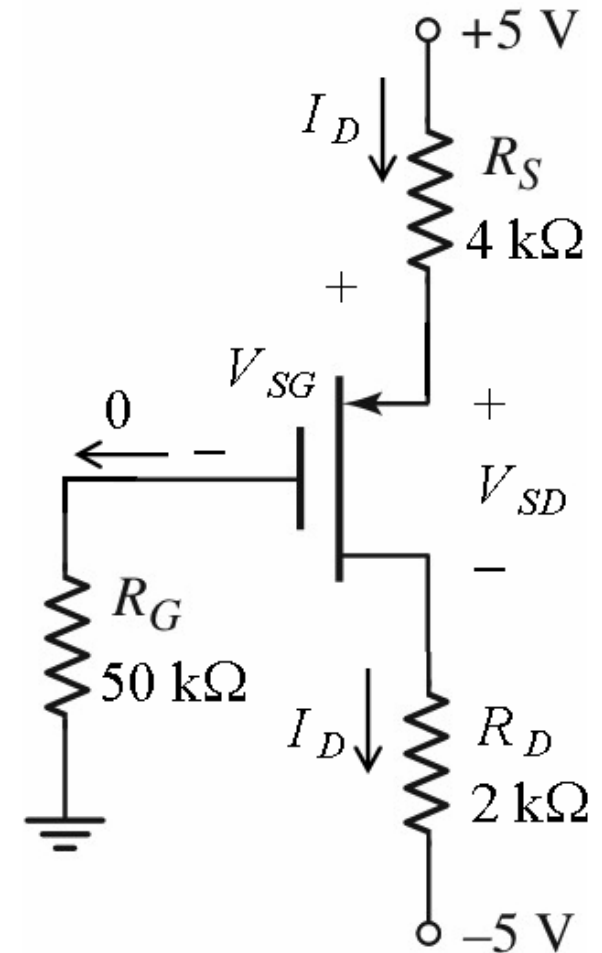
Solving the equation, we have;

$$V_{SG} = 1.71 \text{ V}$$

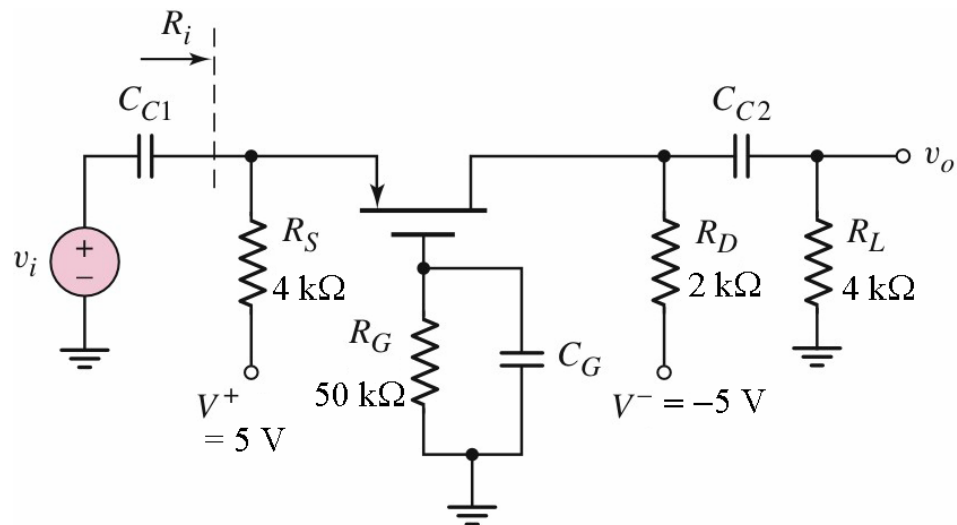
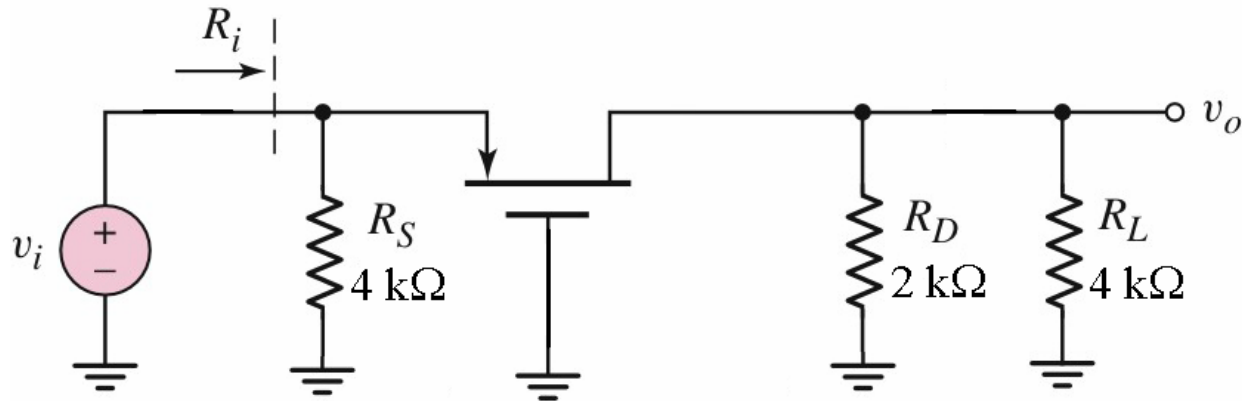


The transconductance is;

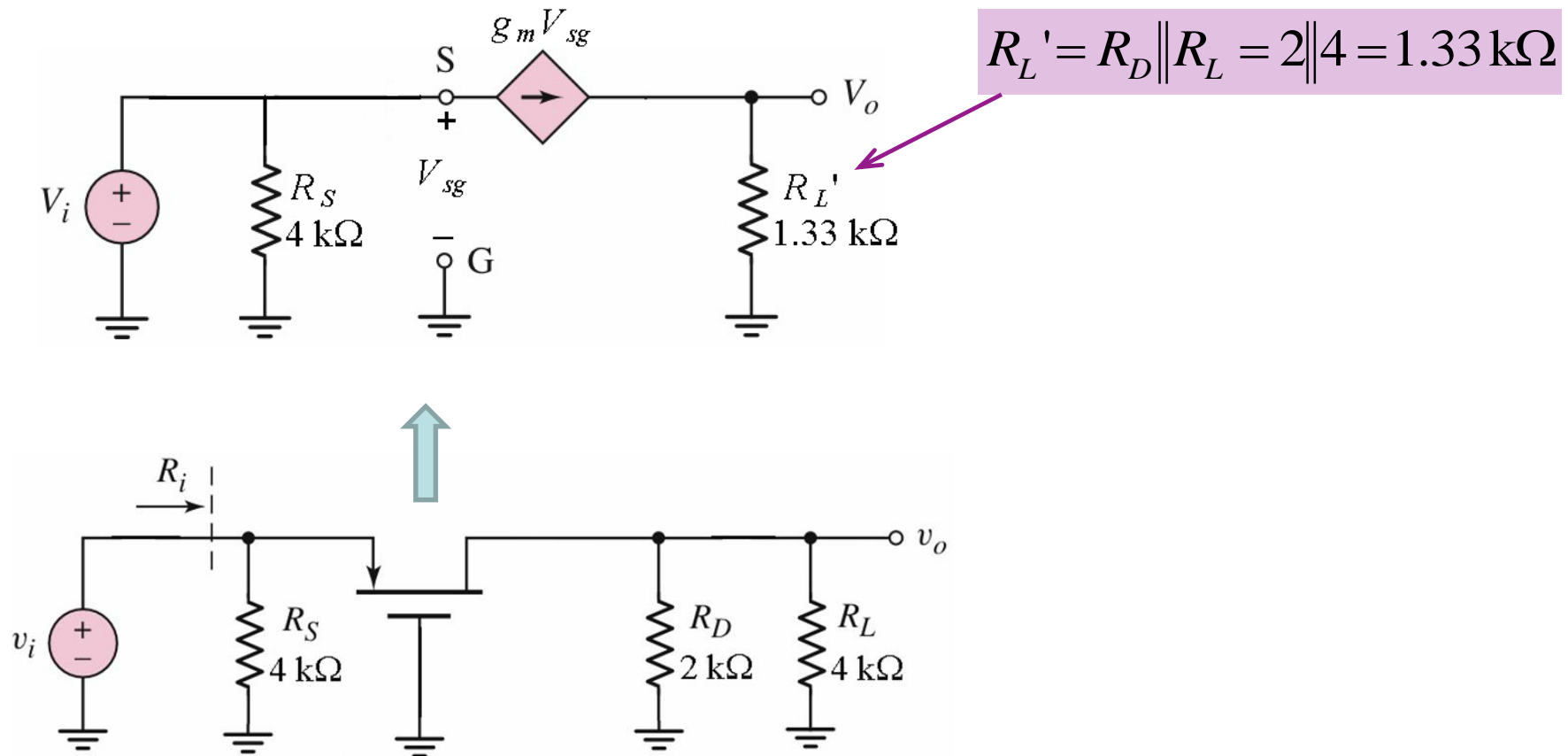
$$\begin{aligned}g_m &= 2K_p(V_{SG} + V_{TP}) \\&= 2 \times 10^{-3}(1.71 - 0.8) \\&= 1.81 \text{ mA/V}\end{aligned}$$

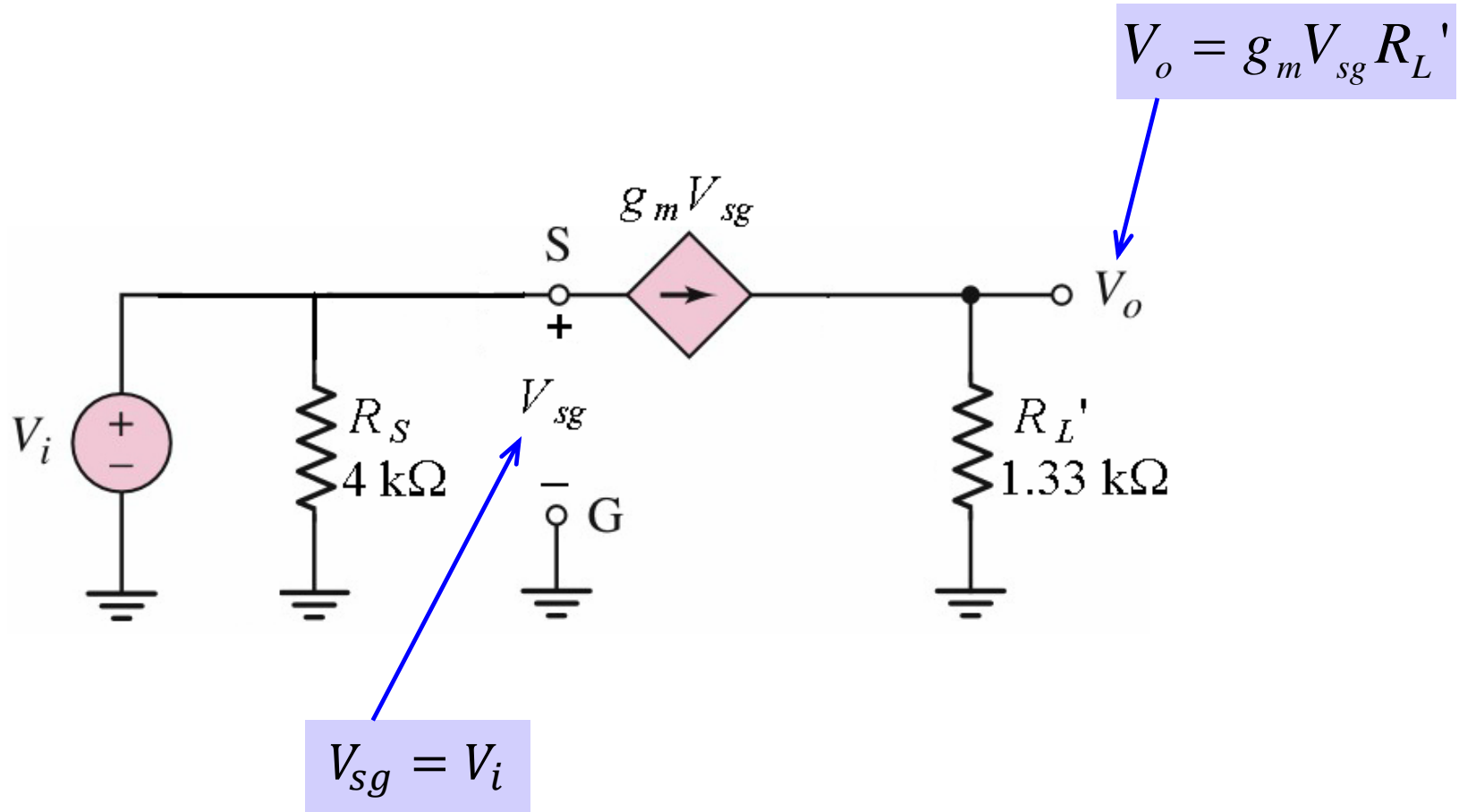


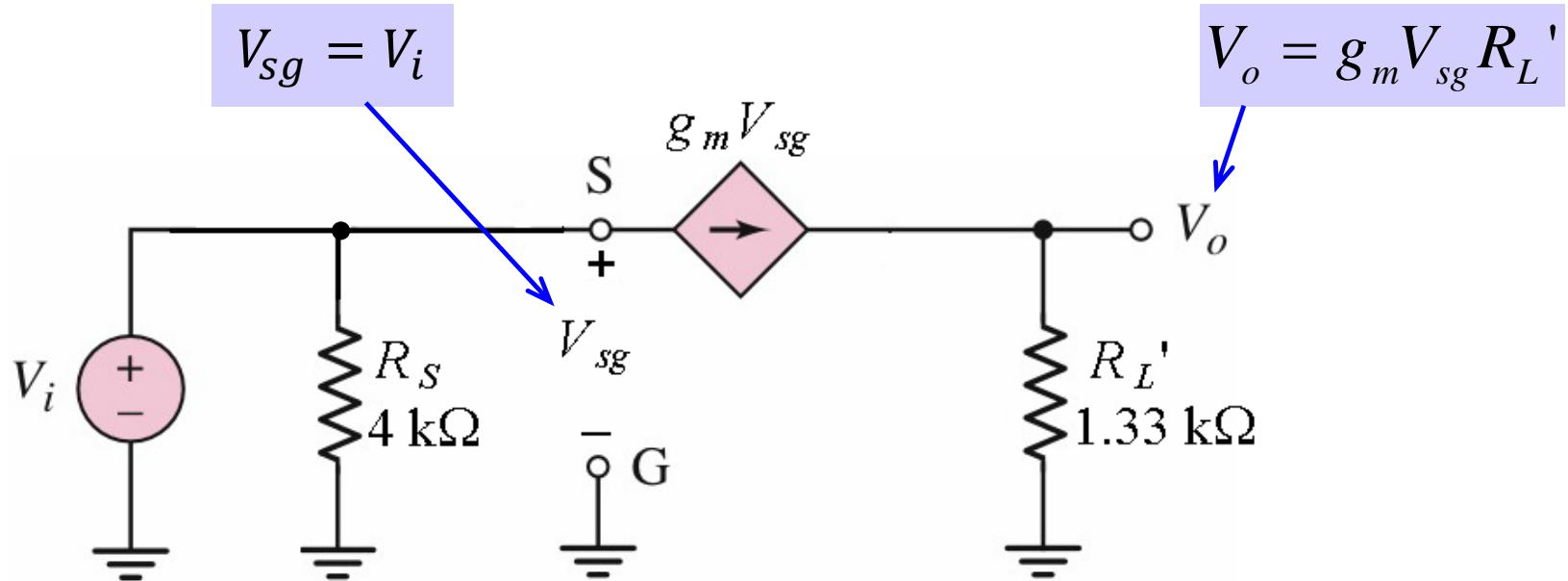
The ac equivalent circuit is as follows;



Replacing the transistor with its small-signal model, we have the following circuit;





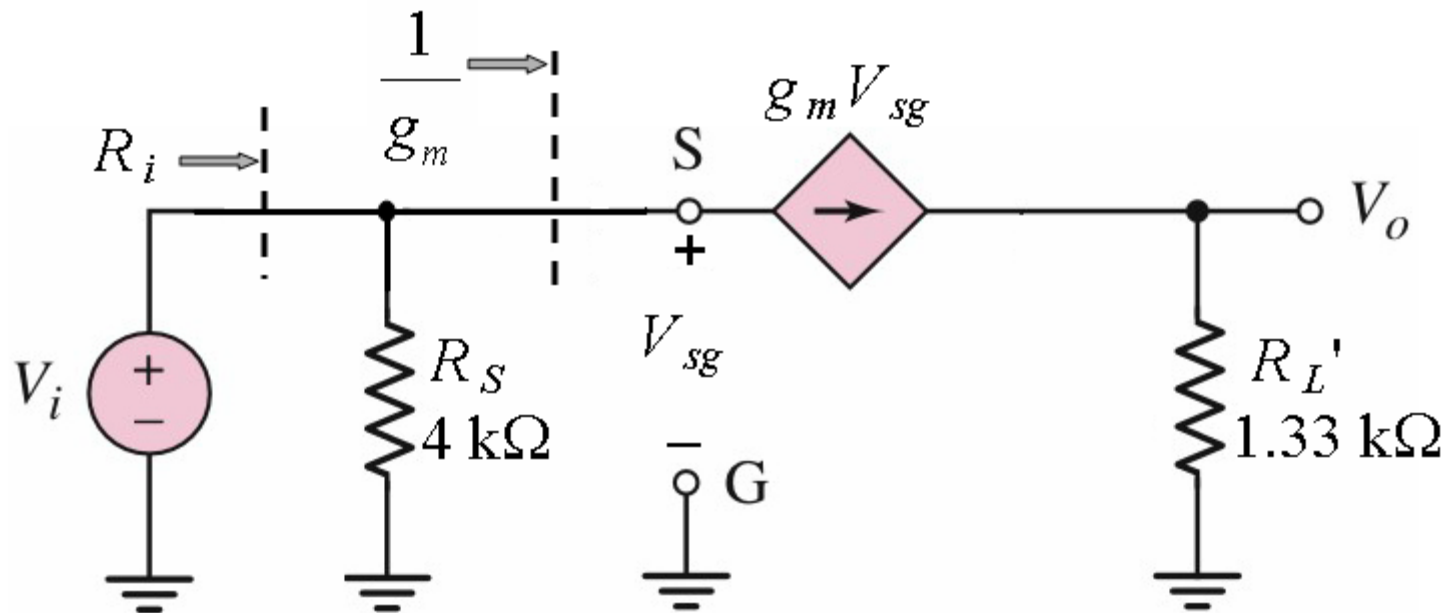


$$V_o = g_m V_{sg} R_L' = g_m V_i R_L'$$

The small-signal voltage gain is $A_v = \frac{V_o}{V_i} = g_m R_L'$

Substituting values;

$$A_v = 1.81 \times 1.33 = 2.41$$



The input resistance is;

$$R_i = R_S \parallel \frac{1}{g_m} = 4 \parallel \frac{1}{1.81 \times 10^{-3}} = 0.485 \text{ k}\Omega$$