Chp 27: Magnetic Field and Magnetic Forces - III

Goals for Chapter 27

- To study magnets and the forces they exert on each other
- To calculate the force that a magnetic field exerts on a
- moving charge
- To contrast magnetic field lines with electric field lines
- To analyze the motion of a charged particle in a magnetic field
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- To see applications of magnetism in physics and chemistry
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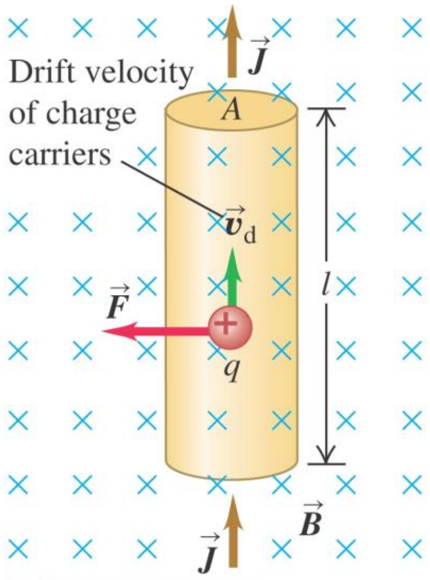
- To analyze magnetic forces on current-carrying conductors
- To study the behavior of current loops in a magnetic field

The magnetic force on a current-carrying conductor



What makes an electric motor work? Within the motor are conductors that carry currents (that is, whose charges are in motion), as well as magnets that exert forces on the moving charges. Hence there is a magnetic force along the length of each current-carrying conductor, and these forces make the motor turn.

$$\vec{F} = q\vec{v} \times \vec{B} \Longrightarrow F = qv_d B$$

The number of charges per unit volume is n
 a segment of conductor with length l has volume Al and
contains a number of charges equal to nAl.

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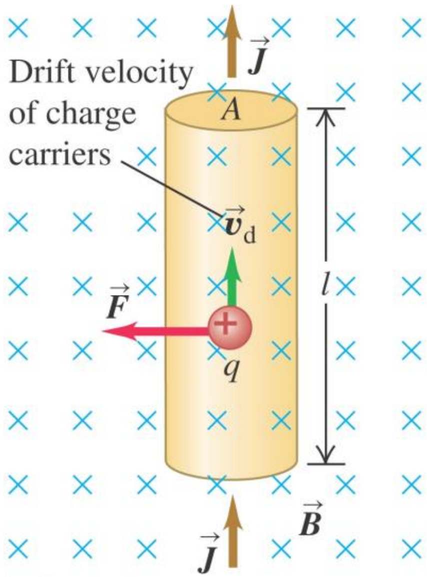

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a segment of conductor with length *l* has volume *Al* and contains a number of charges equal to *nAl*.

$$\Rightarrow F = (nAl)(qv_d B) = (nqv_d A)(lB)$$ and current density $J = nqv_d$

JA is total current ➔ $F = IlB$

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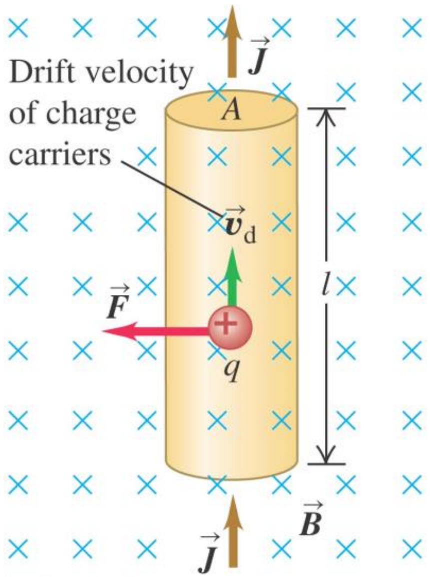

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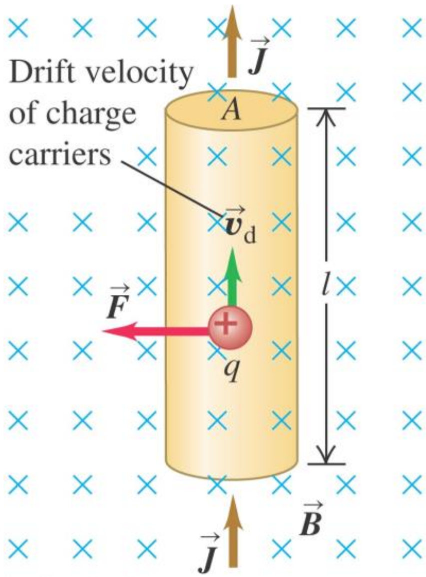

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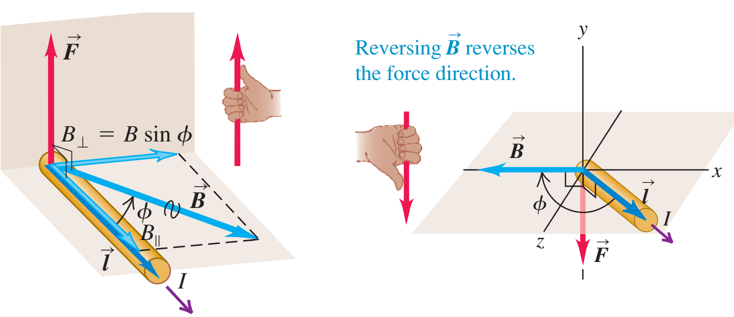
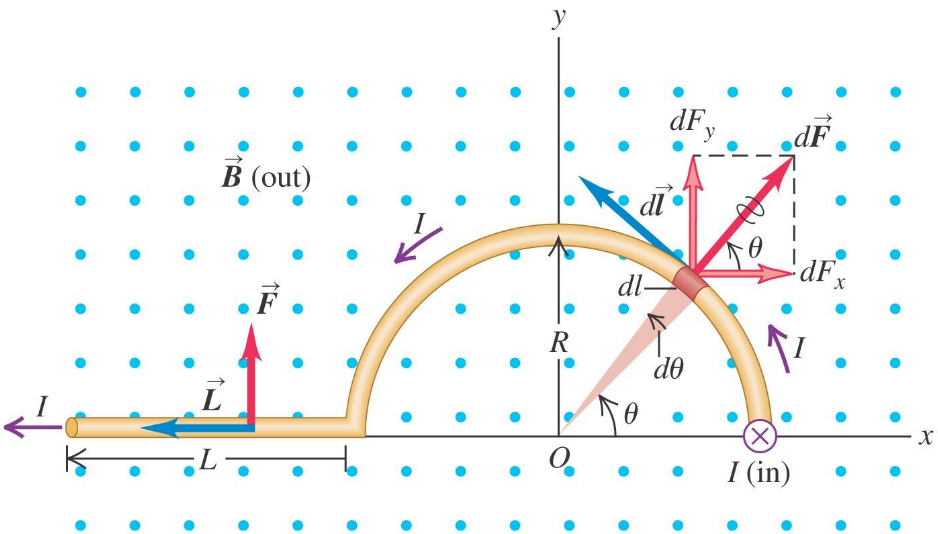
$\Rightarrow$ $\vec{F} = I\vec{l} \times \vec{B}$ (magnetic force on a straight wire segment)



$d\vec{F} = I\,d\vec{l} \times \vec{B}$ (magnetic force on an infinitesimal wire section)

Magnetic force on a curved conductor



$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

the magnetic field B is uniform and perpendicular to the plane of the figure, pointing out of the page. The conductor, carrying current I to the left, has three segments: (1) a straight segment with length L perpendicular to the plane of the figure, (2) a semicircle with radius R, and (3) another straight segment with length L parallel to the x-axis . Find the total magnetic force on this conductor.

(1) Segment 1 $\quad \vec{L} = -L\hat{k} \quad \Rightarrow \quad \vec{F}_1 = I\vec{L} \times \vec{B} = \mathbf{0}$

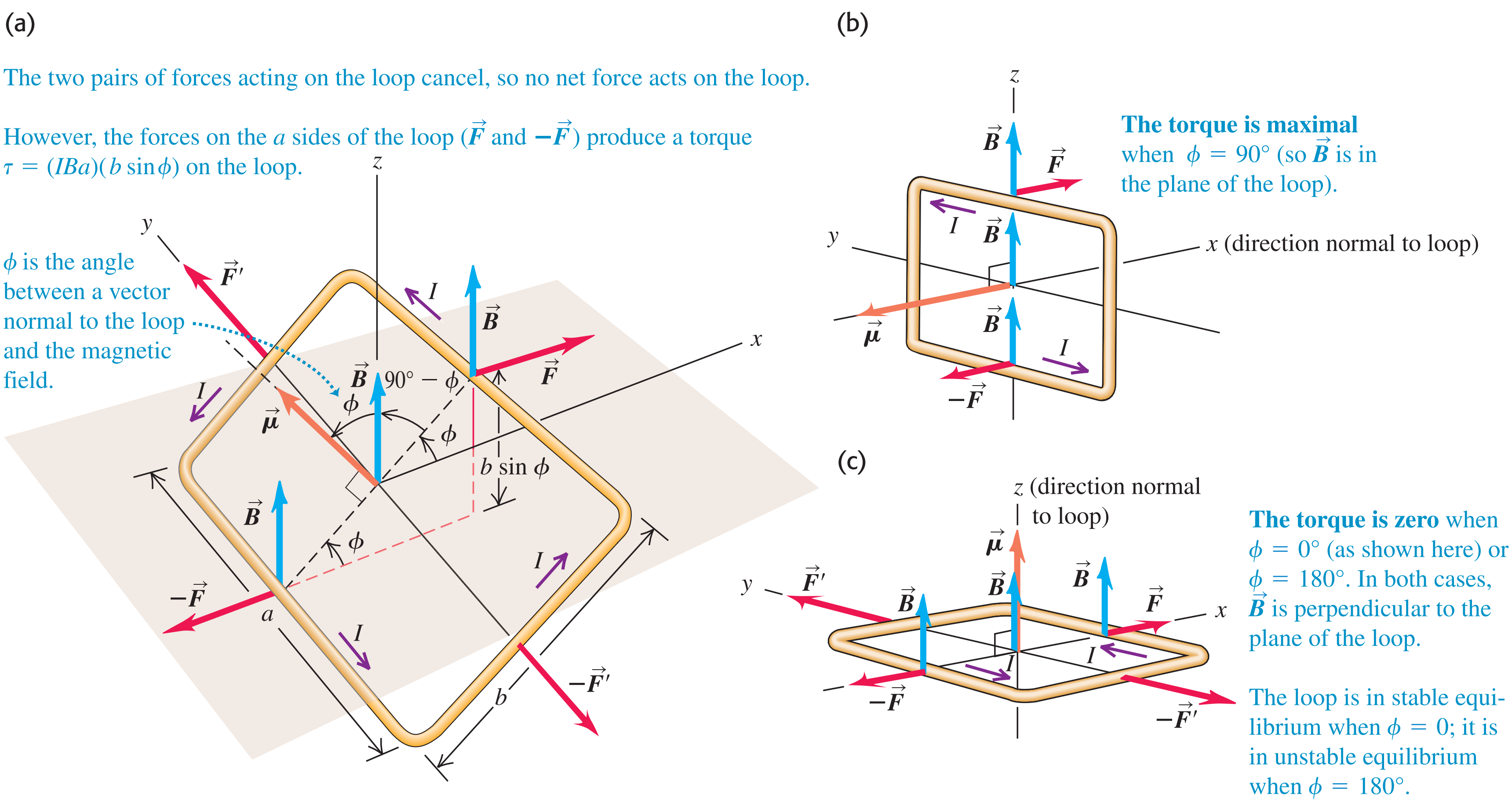
(3) Segment 3 $\quad \vec{L} = -L\hat{\imath} \quad \Rightarrow \quad \vec{F}_3 = I\vec{L} \times \vec{B} = I(-L\hat{\imath}) \times (B\hat{k}) = ILB\hat{\jmath}$

(2) Segment 2 $\quad d\vec{l} \times \vec{B}$ is radially outward $\quad \Rightarrow \quad dF_{2x} = IR\,d\theta\,B\cos\theta \qquad dF_{2y} = IR\,d\theta\,B\sin\theta$

$$F_{2x} = IRB\int_0^{\pi}\cos\theta\,d\theta = 0 \qquad F_{2y} = IRB\int_0^{\pi}\sin\theta\,d\theta = 2IRB \qquad \vec{F}_2 = 2IRB\hat{\jmath}$$

$$\Rightarrow \quad \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 + 2IRB\hat{\jmath} + ILB\hat{\jmath} = IB(2R + L)\hat{\jmath}$$

Force and torque on a current loop



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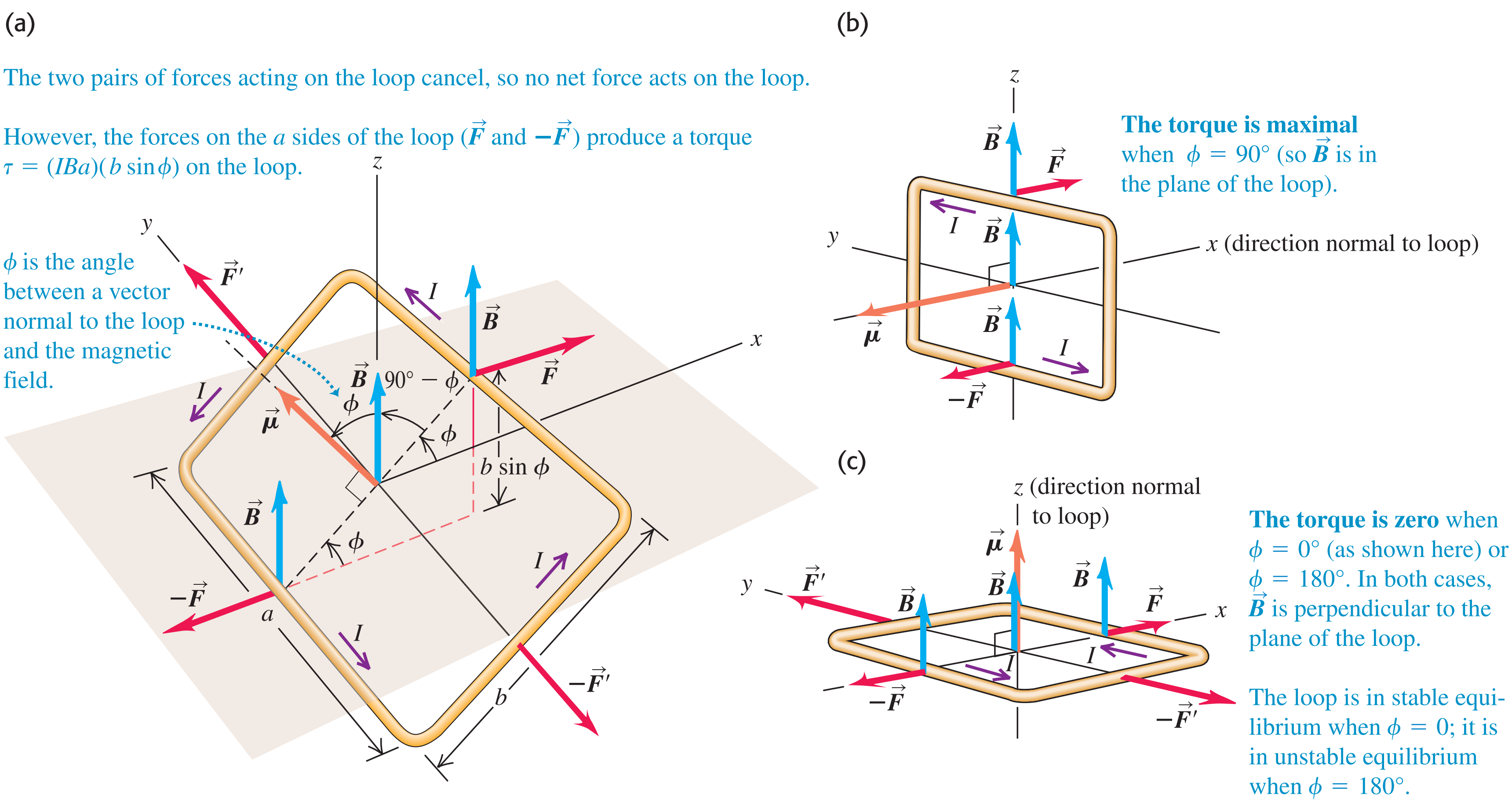
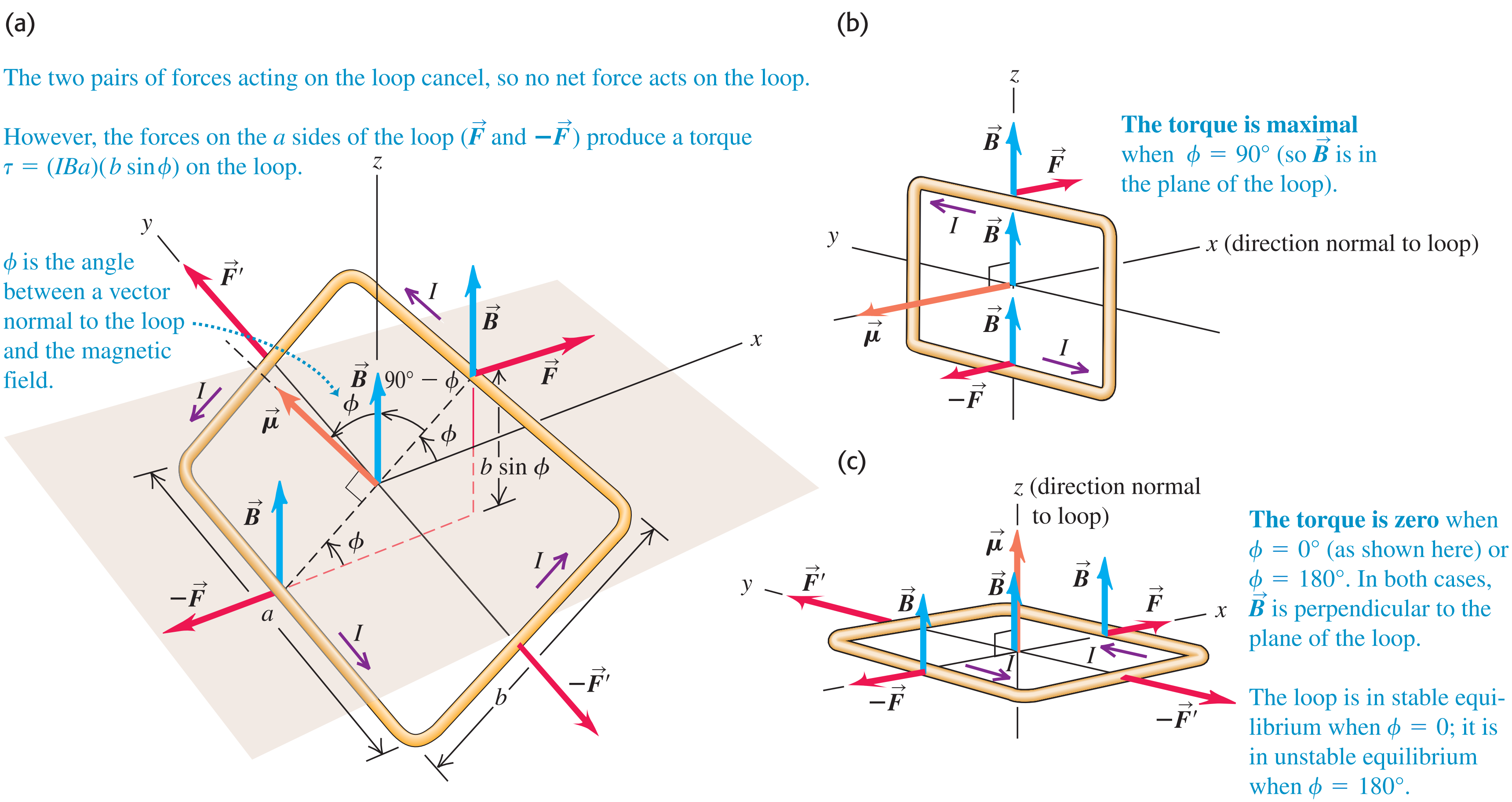


$$F = IaB \qquad \tau = 2F(b/2)\sin\phi = (IBa)(b\sin\phi)$$

$$\tau = IBA\sin\phi \qquad \text{(magnitude of torque on a current loop)}$$

The product IA is called the **magnetic dipole moment** or **magnetic moment** $\mu = IA$

Force and torque on a current loop



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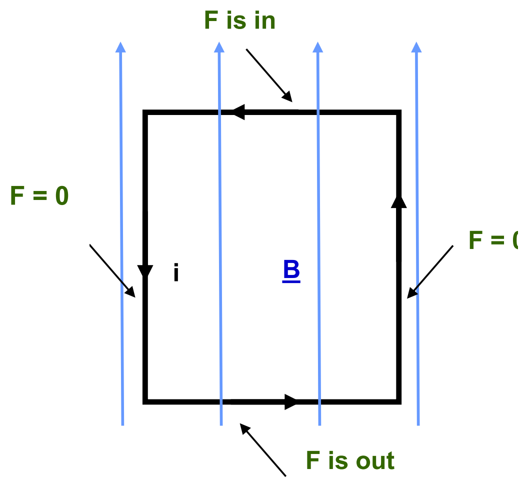
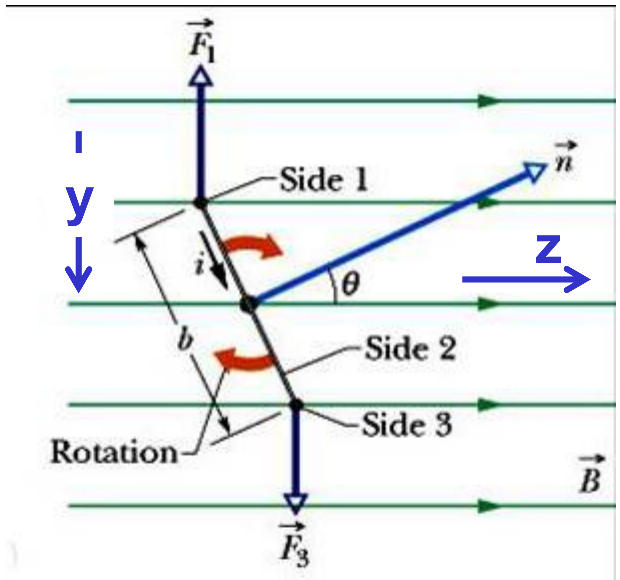
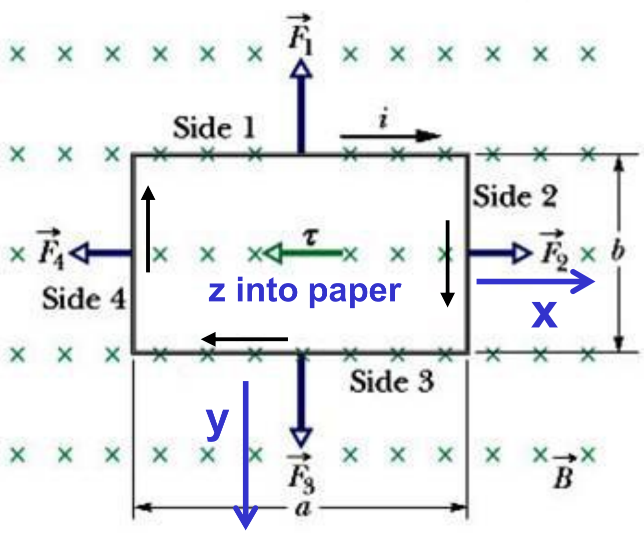
A current-carrying loop experiences A torque (but zero net force)





Net torque $\tau = iabB\sin(\theta)$



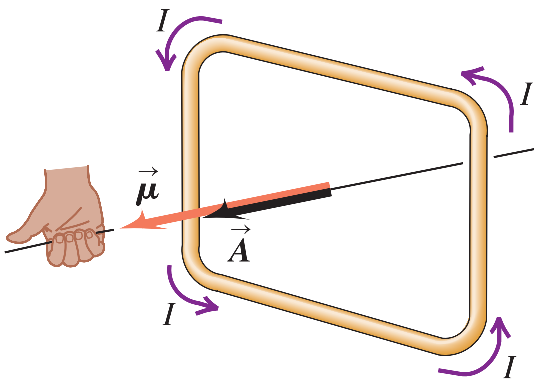
$|F_2| = |F_4| =$ i.b.Bcos(θ): Forces cancel, same line of action → zero torque

- Apply $\vec{F}_m = i\vec{L} \times \vec{B}$ to each side of the loop
- $|F_1| = |F_3| =$ iaB: Forces cancel again but net torque is not zero!
- Moment arms for F_1 & F_3 equal b.sin(q)/2
- Force F_1 produces CW torque equal to t_1 = i.a.B.b.sin(q)/2
- Same for F_3
- Torque vector is down into paper along –x rotation axis

Magnetic moment

$\vec{\mu} = I\vec{A}$ is a vector equation.

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (vector torque on a current loop)



Potential Energy for a Magnetic Dipole

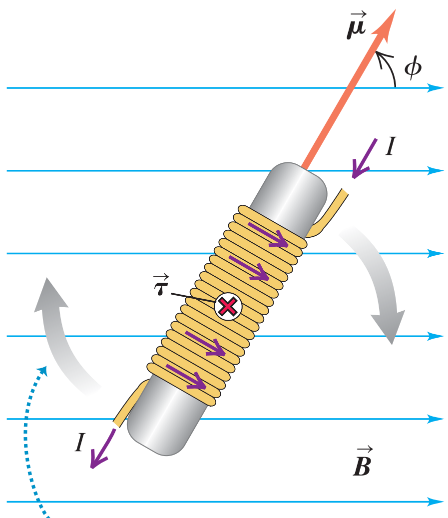
When a magnetic dipole changes orientation in a magnetic field, the field does work on it.
In an infinitesimal angular displacement $d\phi$, the work dW is given by $\tau d\phi$

The torque on an electric dipole $\vec{\tau} = \vec{p} \times \vec{E}$

➔ Corresponding potential energy $U = -\vec{p} \cdot \vec{E}$

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\phi \quad \text{(potential energy for a magnetic dipole)}$$

Magnetic Torque: Loops and Coils

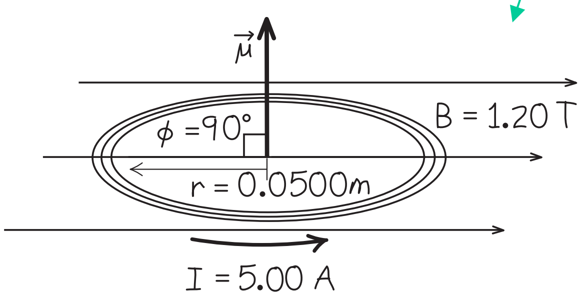


The torque tends to make the solenoid rotate clockwise in the plane of the page, aligning magnetic moment $\vec{\mu}$ with field $\vec{B}$.

$$\tau = NIAB\sin\phi$$

Ex: Magnetic torque on a circular coil

A circular coil 0.0500 m in radius, with 30 turns of wire, lies in a horizontal plane. It carries a counterclockwise (as viewed from above) current of 5.00 A. The coil is in a uniform 1.20-T magnetic field directed toward the right. Find the magnitudes of the magnetic moment and the torque on the coil.



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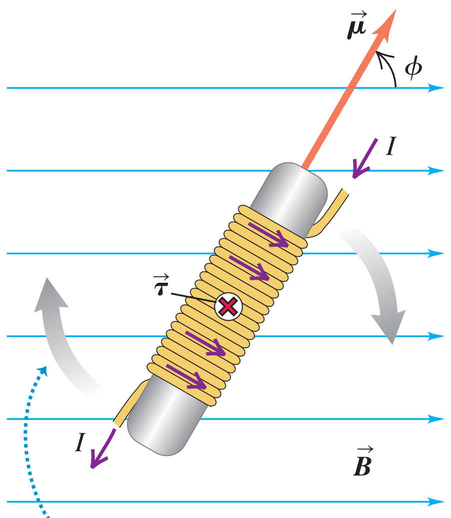

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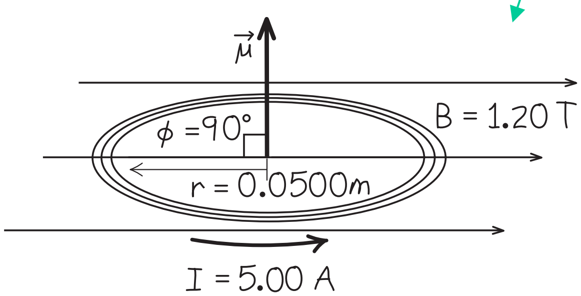
$$A = \pi r^2$$



$$\mu_{\text{total}} = NIA = 30(5.00\ \text{A})\pi(0.0500\ \text{m})^2 = 1.18\ \text{A}\cdot\text{m}^2$$

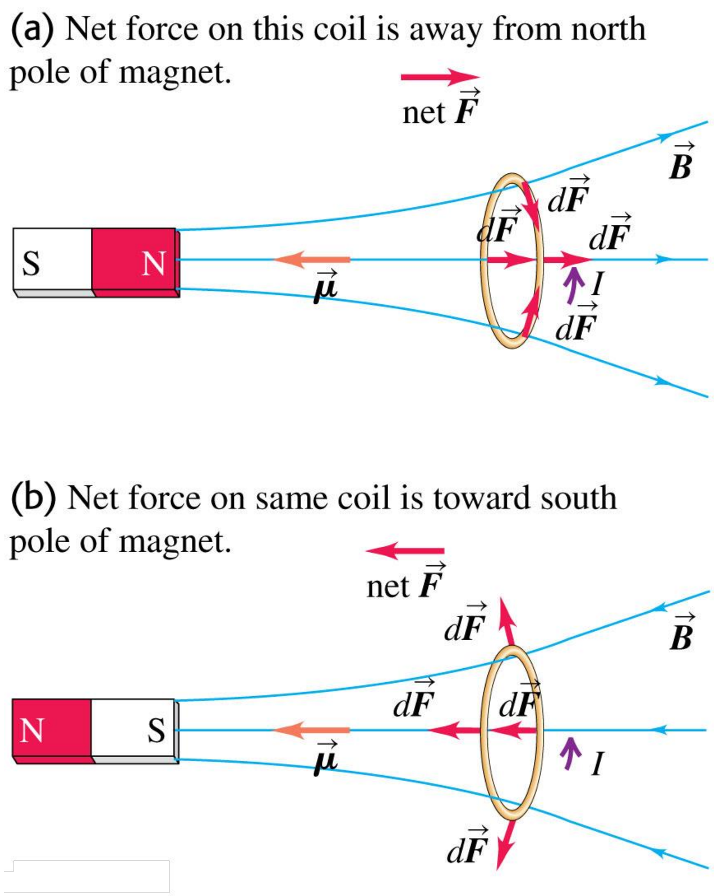
$$\tau = \mu_{\text{total}}B\ \sin\ \phi = (1.18\ \text{A}\cdot\text{m}^2)(1.20\ \text{T})(\sin\ 90°)$$
$$= 1.41\ \text{N}\cdot\text{m}$$

The torque tends to rotate the right side of the coil down and the left side up, into a position where the normal to its plane is parallel to ***B***.

$$\Delta U = U_2 - U_1 = -\mu B\cos\phi_2 - (-\mu B\cos\phi_1) = -\mu B(\cos\phi_2 - \cos\phi_1)$$

$$= -(1.18\ \text{A}\cdot\text{m}^2)(1.20\ \text{T})(\cos 0° - \cos 90°) = -1.41\ \text{J}$$

Magnetic Dipole in a Nonuniform Magnetic Field



Forces on current loops in a nonuniform *B* field. In each case the axis of the bar magnet is perpendicular to the plane of the loop and passes through the center of the loop.

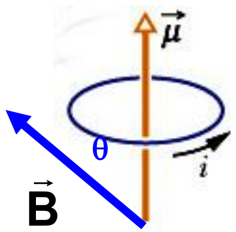
We have seen that a current loop (that is, a magnetic dipole) experiences zero net force in a uniform magnetic field.
But in a nonuniform B field the net force on the loop is *not* zero.

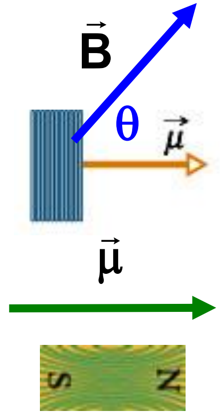
$$d\vec{F} = I\, d\vec{l} \times \vec{B}$$

(a) the force on a segment of the loop has both a radial component and a component to the right. When these forces are summed to find the net force *F* on the loop, the radial components cancel so that the net force is to the right, away from the magnet.
(b) Net force to the left

Current loops are basic magnetic dipoles

Represent loop as a vector





$$\text{Magnetic dipole moment} \equiv \vec{\mu} \equiv N\,i\,A\,\hat{n}$$

$$\text{Dimensions } [\mu] = \text{ampere-m}^2 = \frac{\text{Newton-m}}{\text{Tesla}} = \frac{\text{Joule}}{\text{Tesla}}$$

$N = \text{number of turns in the loop}$

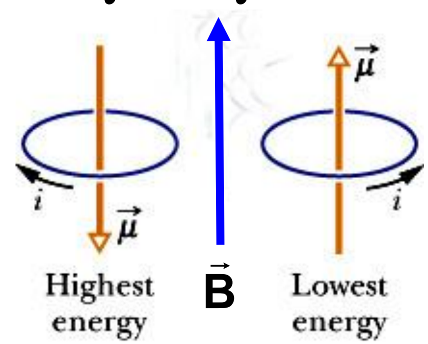
$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad A \equiv \text{area of loop} = ab$$

$$\tau = \mu B \sin(\theta)$$

Magnetic dipole moment $\underline{\mu}$ measures
- **strength of response to external $\underline{B}$ field**
- **strength of loop as source of a dipole field**

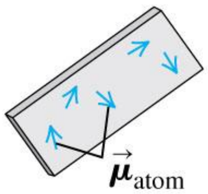
	ELECTRIC DIPOLE	MAGNETIC DIPOLE
MOMENT	$\lvert p \rvert \equiv qd$	$\lvert \mu \rvert \equiv NiA$
TORQUE	$\vec{\tau}_e = \vec{p} \times \vec{E}$	$\vec{\tau}_m = \vec{\mu} \times \vec{B}$
POTENTIAL ENERGY	$U_e = -\vec{p} \circ \vec{E}$	$U_m = -\vec{\mu} \circ \vec{B}$

$$U_M = \int \tau d\theta = \int \mu B \sin(\theta) d\theta$$



How magnets work

(a) Unmagnetized iron: magnetic moments are oriented randomly.



Think of an electron as being like a spinning ball of charge. In this analogy the circulation of charge around the spin axis is like a current loop, and so the electron has a net magnetic moment. (This analogy, while helpful, is inexact; an electron isn't really a spinning sphere. A full explanation of the origin of an electron's magnetic moment involves quantum mechanics, which is beyond our scope here.)

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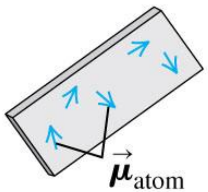
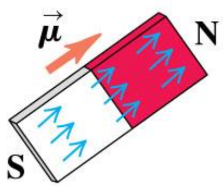
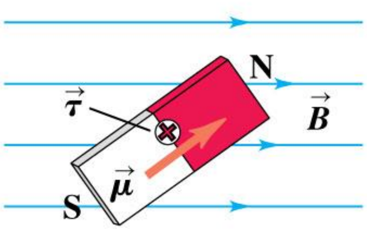


(b) In a bar magnet, the magnetic moments are aligned.



(c) A magnetic field creates a torque on the bar magnet that tends to align its dipole moment with the $\vec{B}$ field.



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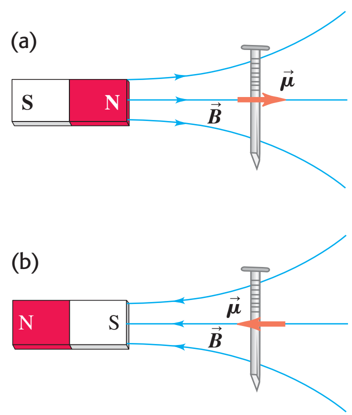
But in an iron bar magnet the magnetic moments of many of the atoms are parallel, and there is a substantial net magnetic moment



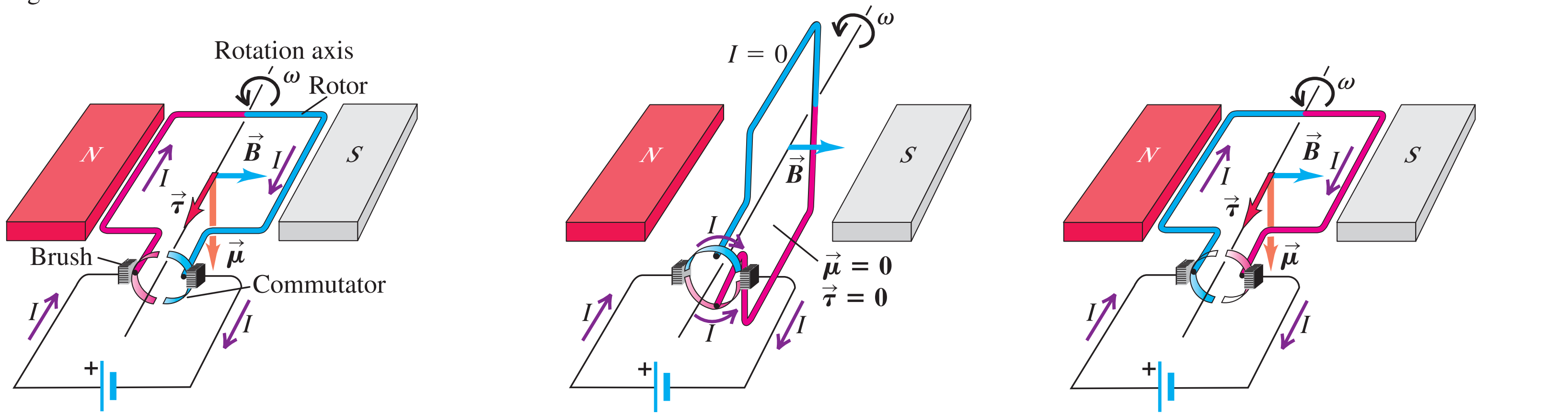
A bar magnet attracts an unmagnetized iron nail in two steps. First, the B field of the bar magnet gives rise to a net magnetic moment in the nail. Second, because the field of the bar magnet is not uniform, this magnetic dipole is attracted toward the magnet. The attraction is the same whether the nail is closer to (a) the magnet's north pole or (b) the magnet's south pole.

The direct-current motor

(a) Brushes are aligned with commutator segments.

(b) Rotor has turned 90°.

(c) Rotor has turned 180°.



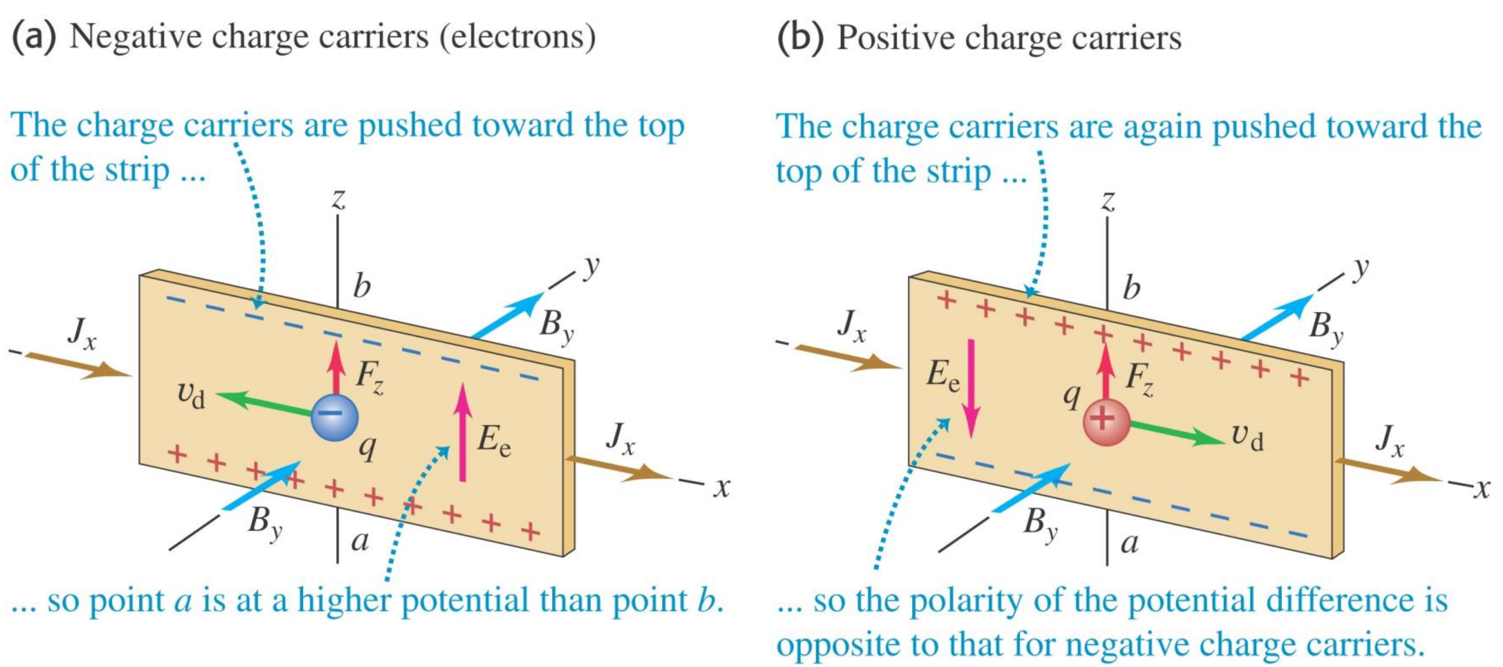
- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

In a motor a magnetic torque acts on a current-carrying conductor, and electric energy is converted to mechanical energy.
The rotor is a current loop with a magnetic moment $\boldsymbol{\mu}$. The rotor lies between opposing poles of a permanent magnet, so there is a magnetic field $\boldsymbol{B}$ that exerts a torque $\boldsymbol{\tau}$ on the rotor. $\vec{\tau} = \vec{\mu} \times \vec{B}$
If the current through the rotor were constant, the rotor would now be in its equilibrium orientation; it would simply oscillate around this orientation. But here's where the commutator comes into play; each brush is now in contact with *both* segments of the commutator. There is no potential difference between the commutators, so at this instant no current flows through the rotor, and the magnetic moment is zero. The rotor continues to rotate counterclockwise because of its inertia, and current again flows through the rotor…

The Hall Effect



If the charge carriers are electrons, an excess negative charge accumulates at the upper edge of the strip, leaving an excess positive charge at its lower edge. This accumulation continues until the resulting transverse electrostatic field E becomes large enough to cause a force (magnitude qE_e) that is equal and opposite to the magnetic force (magnitude qv_dB). After that, there is no longer any net transverse force to deflect the moving charges. This electric field causes a transverse potential difference between opposite edges of the strip, called the *Hall voltage* or the *Hall emf*.

$$qE_z + qv_dB_y = 0 \qquad \text{or} \qquad E_z = -v_dB_y \qquad J_x = nqv_d$$

$$\boxed{nq = \frac{-J_xB_y}{E_z} \qquad \text{(Hall effect)}}$$