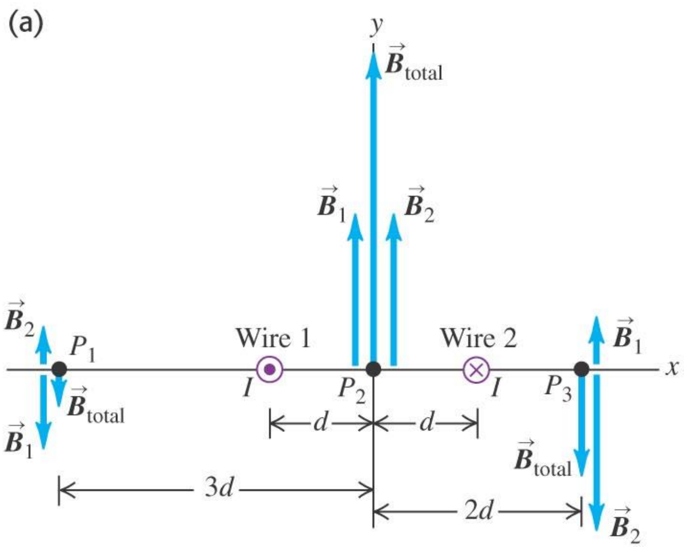
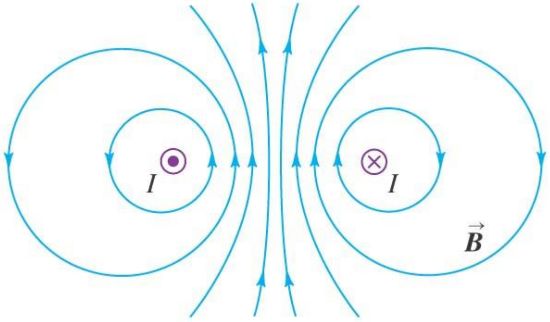
Chp 28: Sources of Magnetic Field - (II)

Magnetic fields of long wires





two long, straight, parallel wires perpendicular to the *xy*-plane, each carrying a current *I* but in opposite directions. (a) Find *B* at points *P1*, *P2*, and *P3*. (b) Find an expression for *B* at any point on the *x*-axis to the right of wire 2.

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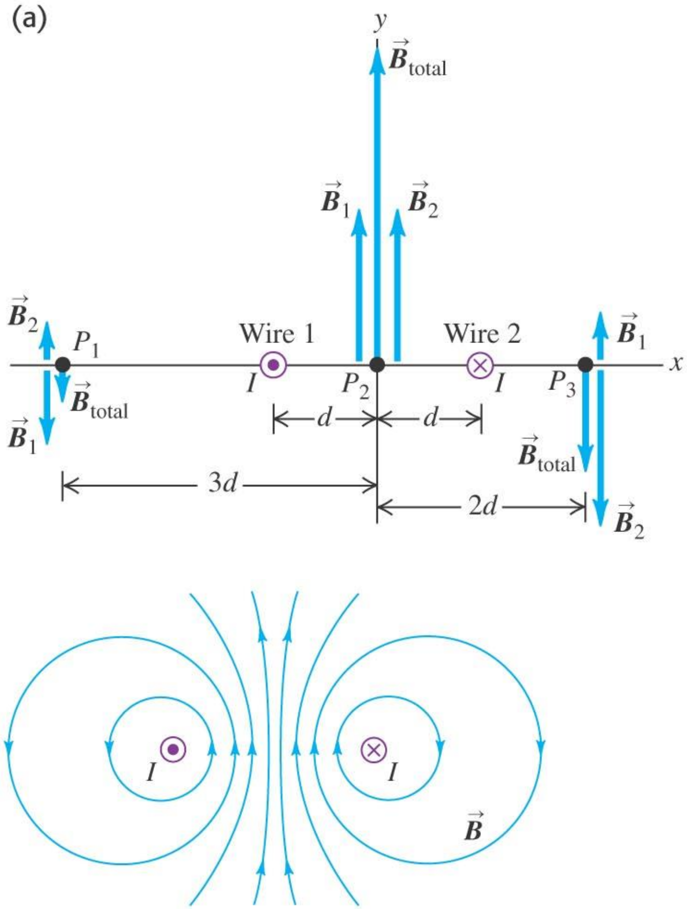

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(a) at point P_1 is a distance 2*d* from wire 1 and a distance 4*d* from wire 2 ➔ $B_1 = \mu_0 I/2\pi(2d) = B_1 = \mu_0 I/4\pi d$ and $B_2 = B1 = \mu_0 I/8\pi d$

$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I}{4\pi d}\hat{j} + \frac{\mu_0 I}{8\pi d}\hat{j} = -\frac{\mu_0 I}{8\pi d}\hat{j}$$

Point P_2 is at a distance d from both wires

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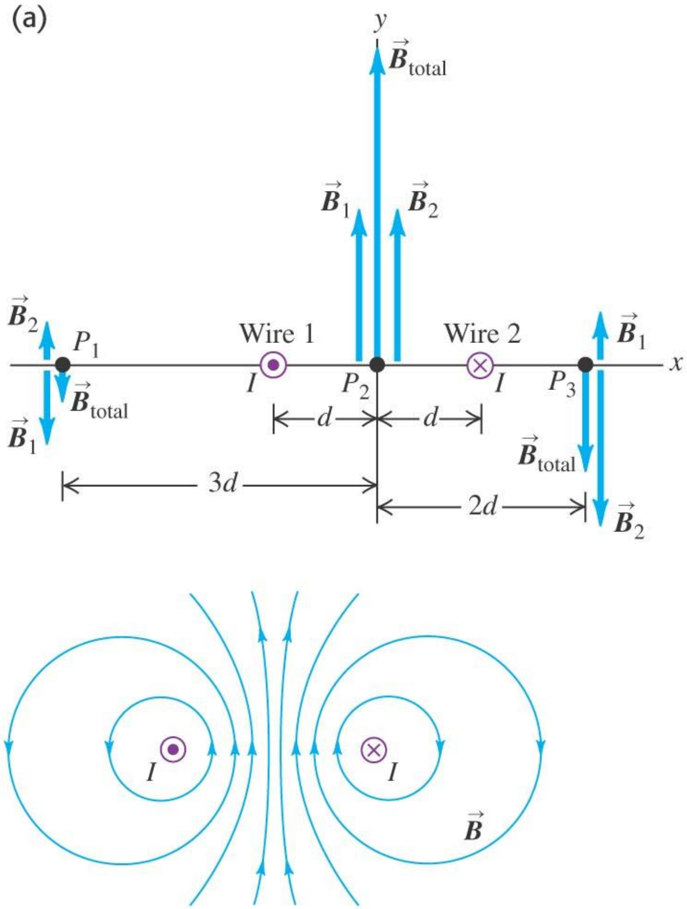

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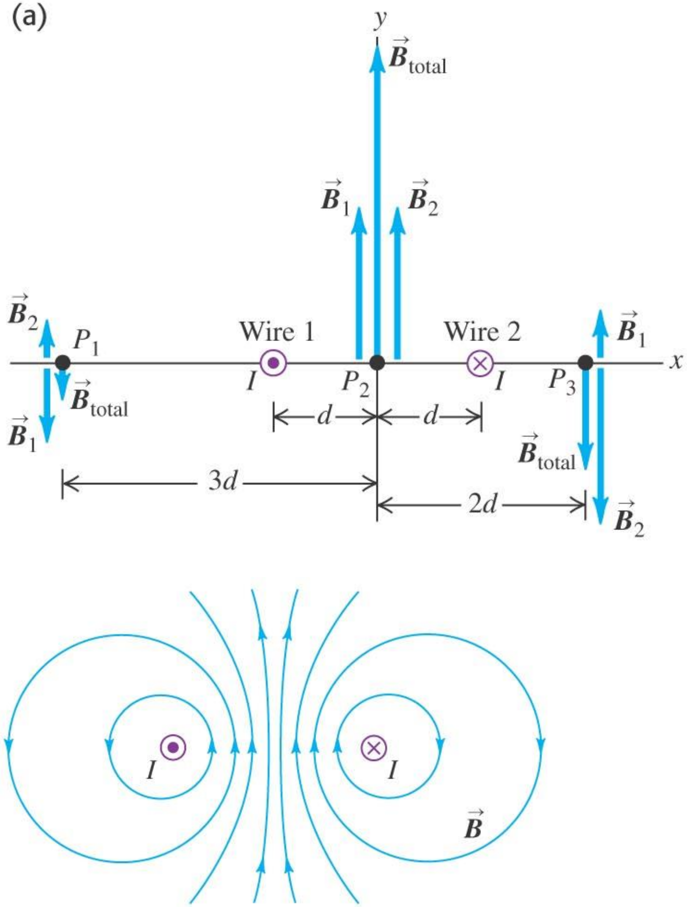
Magnetic fields of long wires



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$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi(x + d)}\hat{j} - \frac{\mu_0 I}{2\pi(x - d)}\hat{j} = -\frac{\mu_0 I d}{\pi(x^2 - d^2)}\hat{j}$$

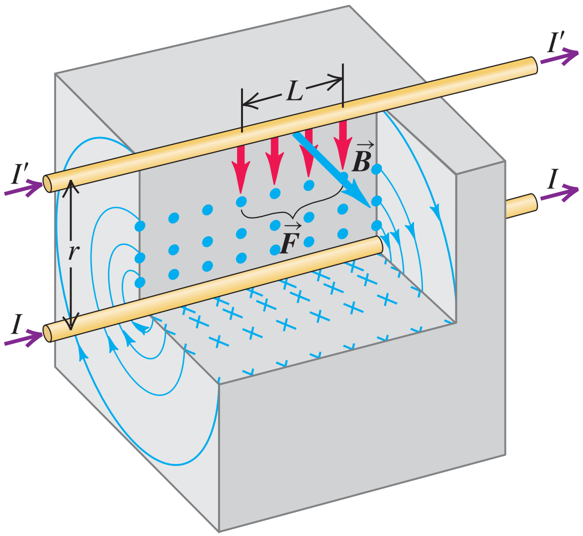
Force between parallel conductors

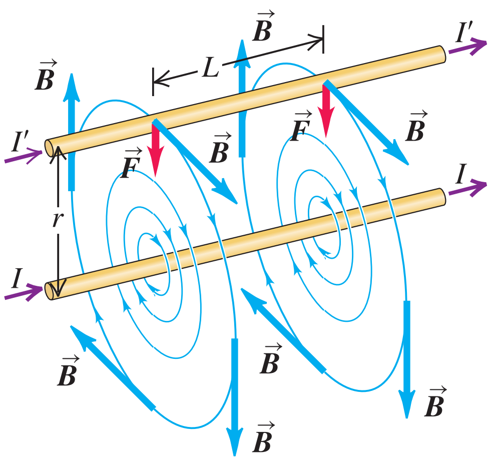
The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.

$$B = \frac{\mu_0 I}{2\pi r} \qquad \vec{F} = I'\vec{L} \times \vec{B}$$

$$\Rightarrow \quad F = I'LB = \frac{\mu_0 I I' L}{2\pi r}$$





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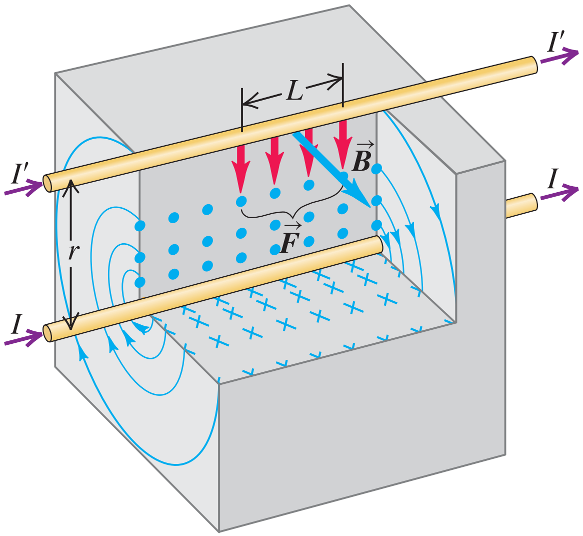

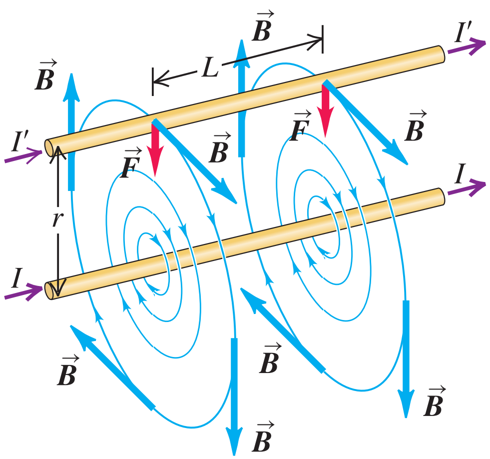

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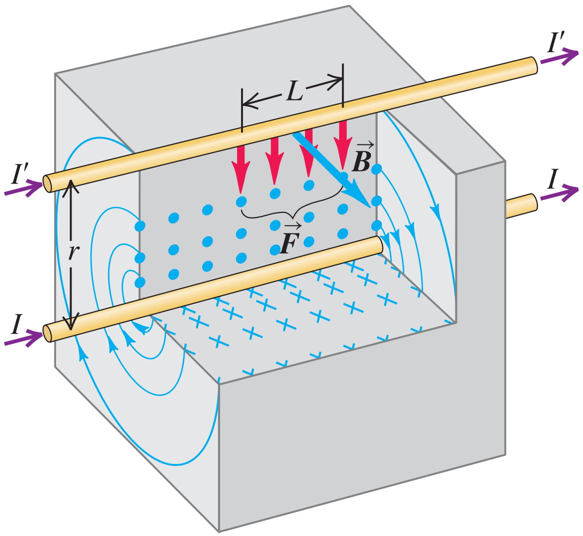

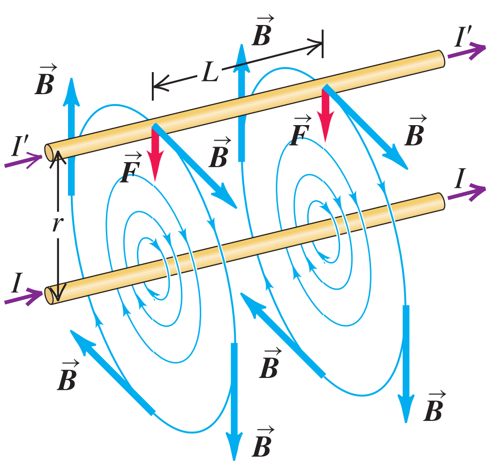

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Magnetic Forces and Defining the Ampere

one ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly $2\text{x}10^{-7}$ newtons per meter of length.

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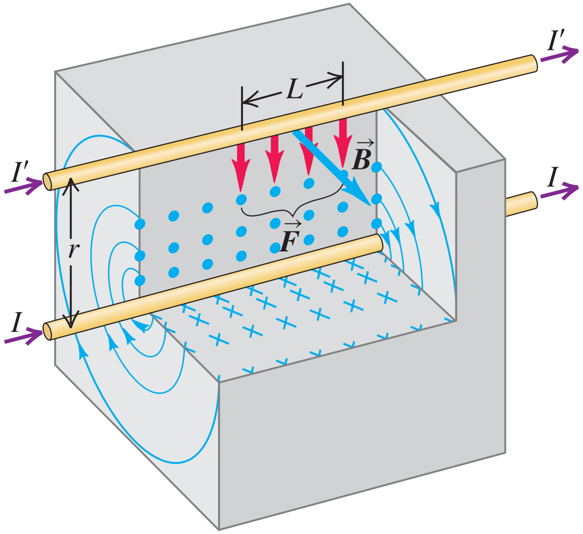

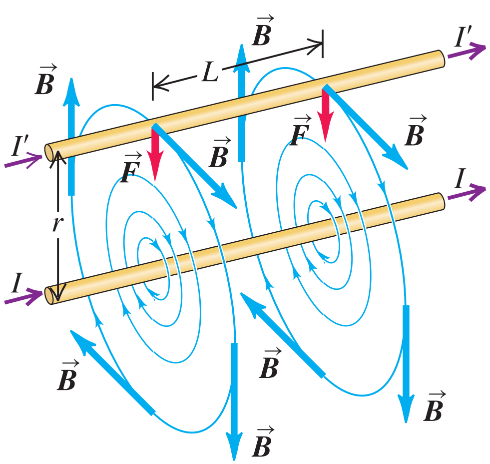

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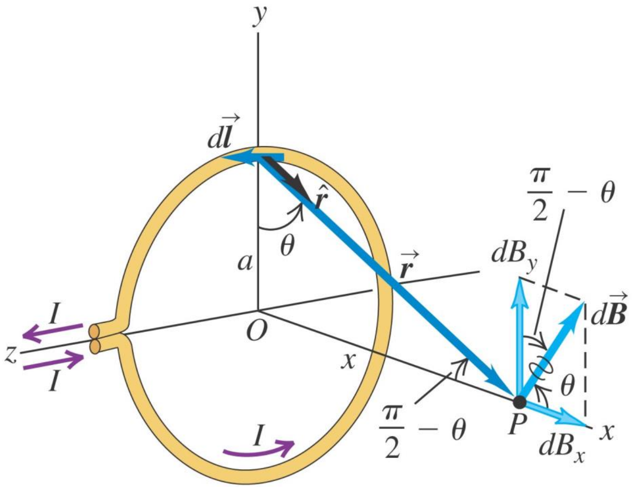
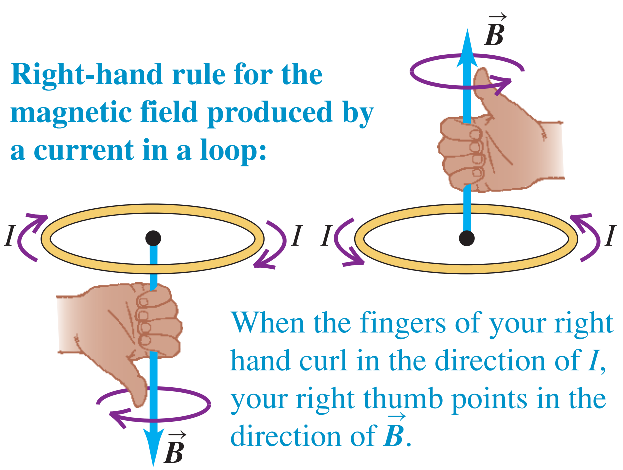
$$\Leftrightarrow \quad \mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

Magnetic field of a circular current loop





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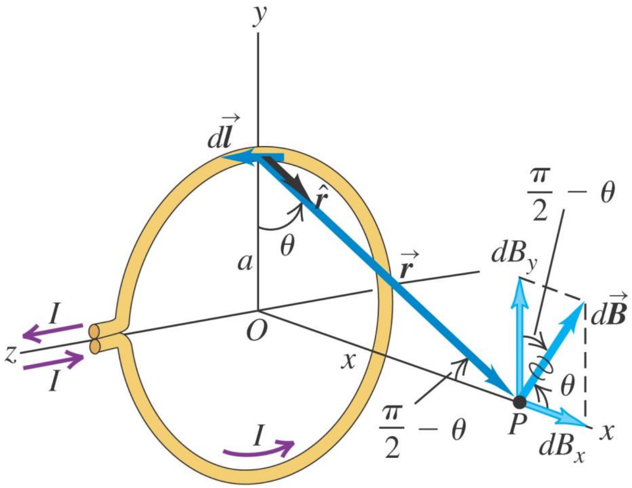
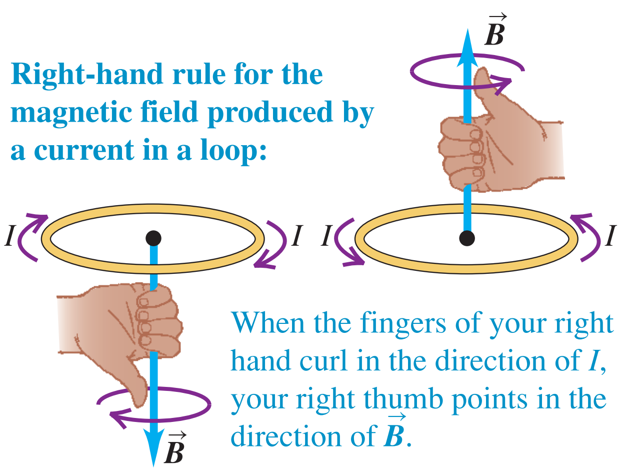


$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2} \qquad r^2 = x^2 + a^2$$

$$\Rightarrow \quad dB = \frac{\mu_0 I}{4\pi} \frac{dl}{(x^2 + a^2)}$$



$\Rightarrow$

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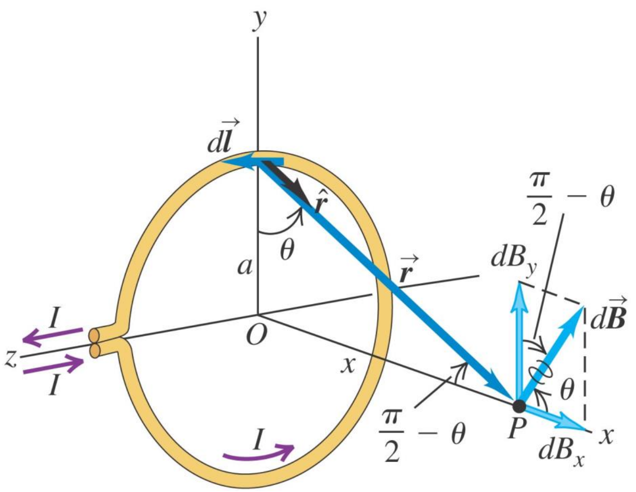

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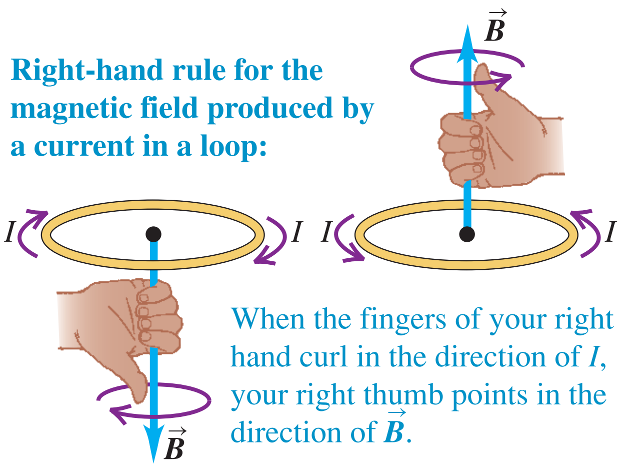

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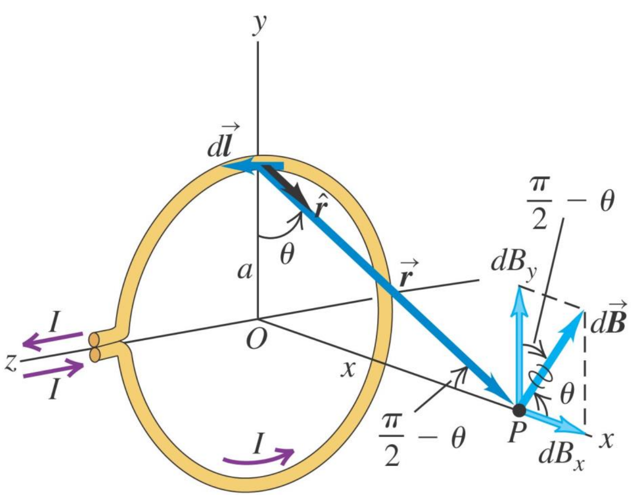

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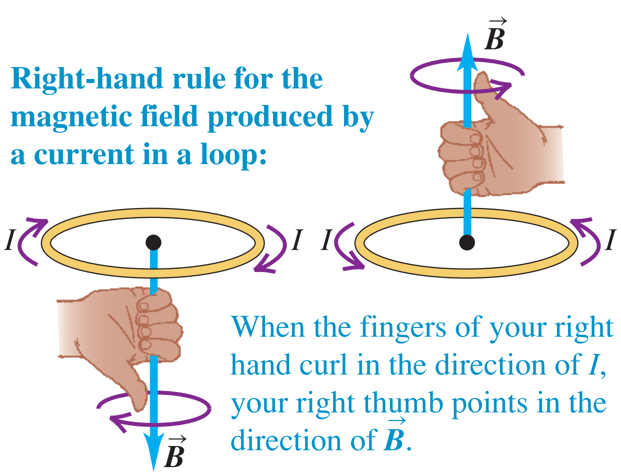
$$\int dl = 2\pi a$$



Integral of dB_y cancels out ➔ $B_y = 0$

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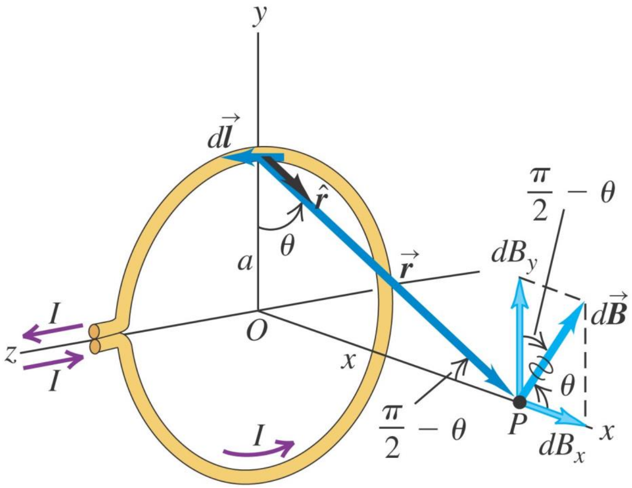

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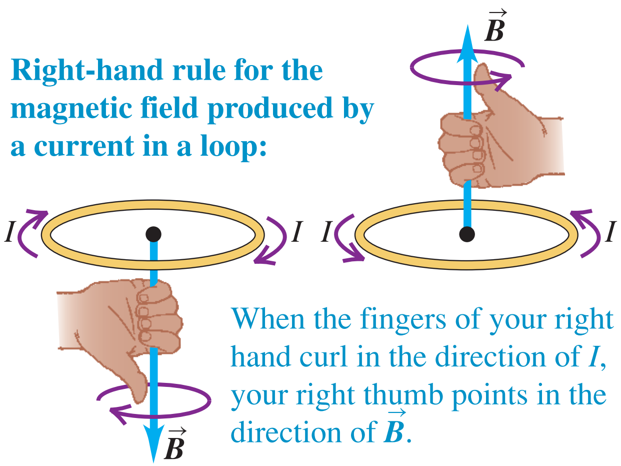
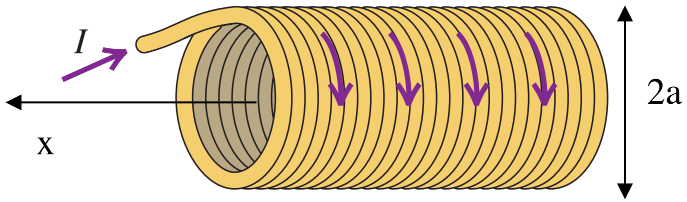


$$\Rightarrow \quad B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \qquad \text{(on the axis of a circular loop)}$$

Magnetic Field on the Axis of a Coil



A solenoid with N turns

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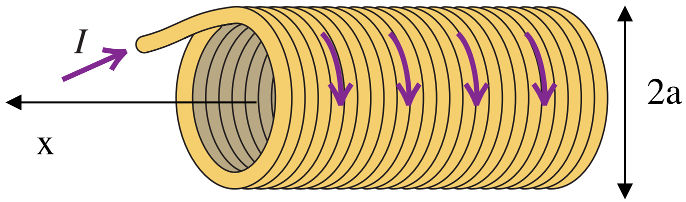

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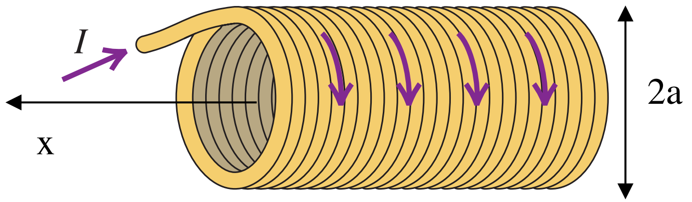

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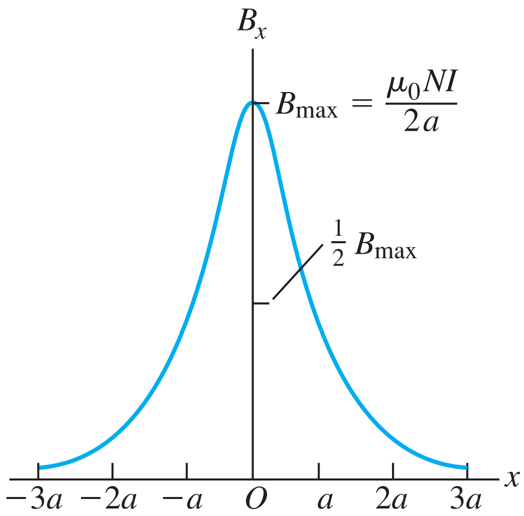

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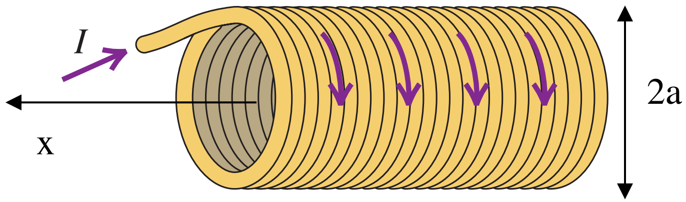

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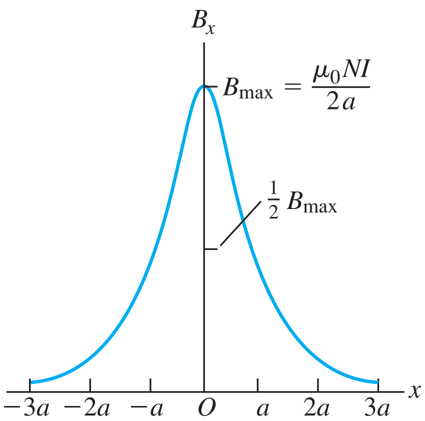

magnetic dipole moment (or *magnetic moment*) $\boldsymbol{\mu}$ of a current-carrying loop to be equal to IA, where $A = \pi a^2$ is the cross- sectional area of the loop.

N loops ➔ $\boldsymbol{\mu} = NIA = NI\pi a^2$

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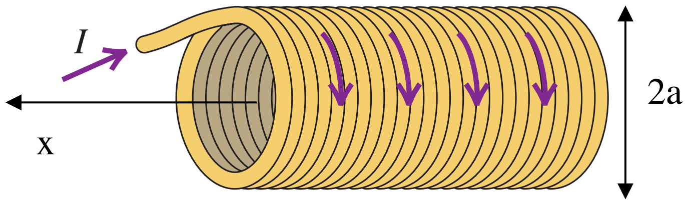

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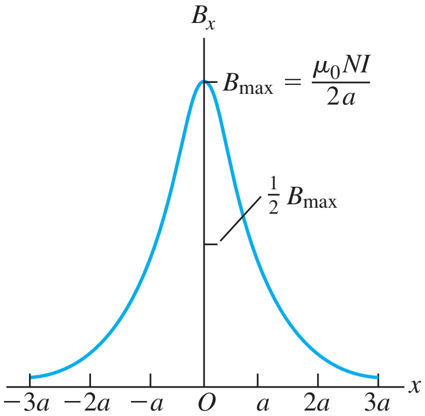

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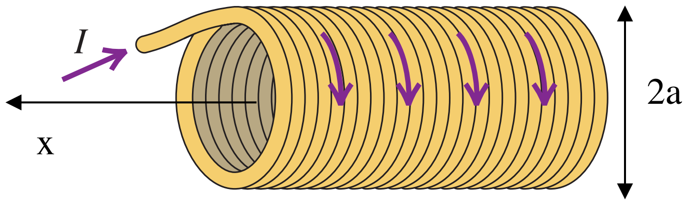

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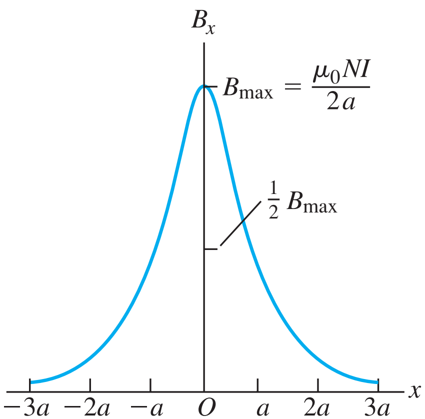

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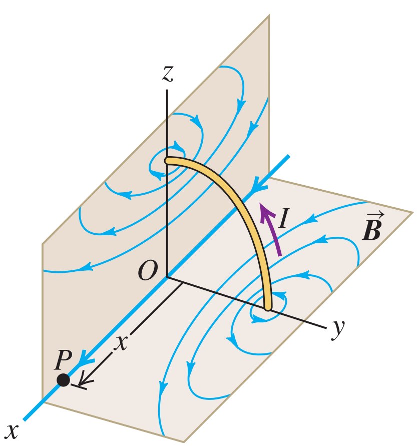
Example: A coil with 100 loops, radius 60 cm carries 5 A current.
(a) What is B along the axis at a distance 80 cm from the center?
(b) along the axis what distance from the center B is 1/8 as great as it is at the center?



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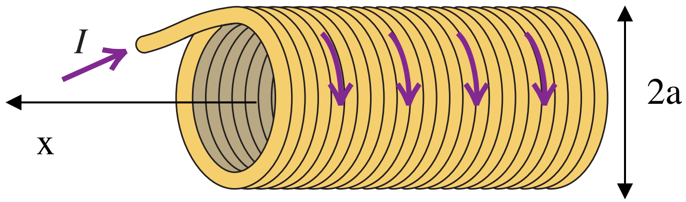

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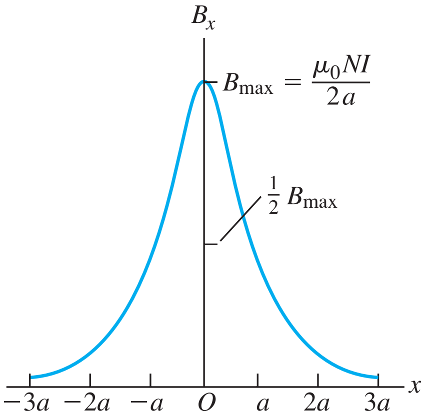

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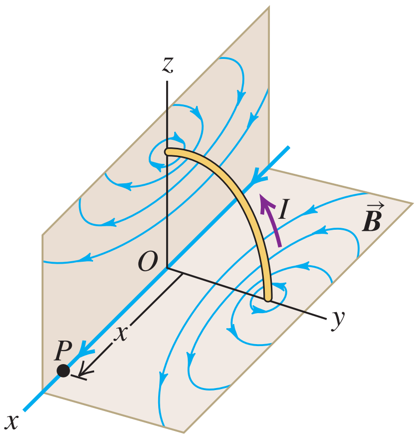

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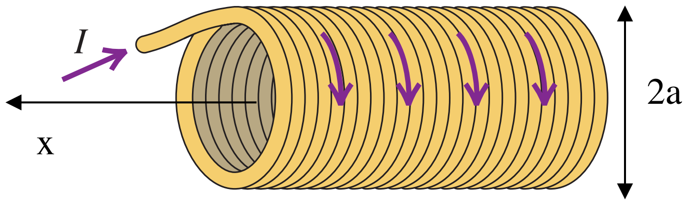
(a) $$B_x = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(100)(5.0\ \mathrm{A})(0.60\ \mathrm{m})^2}{2[(0.80\ \mathrm{m})^2 + (0.60\ \mathrm{m})^2]^{3/2}} = 1.1 \times 10^{-4}\,\mathrm{T}$$

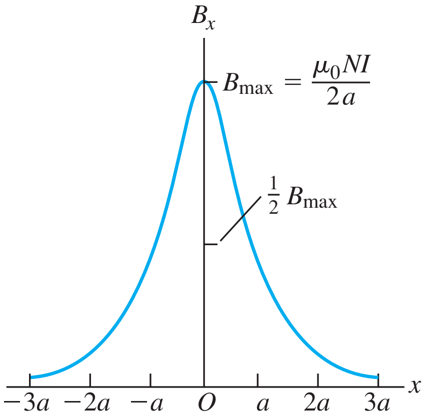
Magnetic Field on the Axis of a Coil



A solenoid with N turns

$$B_x = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$$

$$B_x = \frac{\mu_0 N I}{2a} \quad \text{(at the center of } N \text{ circular loops)}$$



magnetic dipole moment (or *magnetic moment*) μ of a current-carrying loop to be equal to IA, where $A = \pi a^2$ is the cross- sectional area of the loop.

N loops ➔ $\mu = NIA = NI\pi a^2$

$$B_x = \frac{\mu_0 \mu}{2\pi(x^2 + a^2)^{3/2}}$$

Example: A coil with 100 loops, radius 60 cm carries 5 A current.
(a) What is B along the axis at a distance 80 cm from the center?
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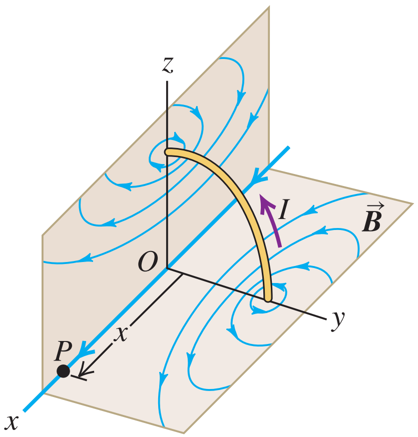


(a) $$B_x = \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(100)(5.0\ \mathrm{A})(0.60\ \mathrm{m})^2}{2[(0.80\ \mathrm{m})^2 + (0.60\ \mathrm{m})^2]^{3/2}} = 1.1 \times 10^{-4}\,\mathrm{T}$$

(b) $$\frac{1}{(x^2 + a^2)^{3/2}} = \frac{1}{8}\frac{1}{(0^2 + a^2)^{3/2}} \quad \Rightarrow \quad x = \pm\sqrt{3}a = \pm 1.04\ \mathrm{m}$$

Ampere's Law

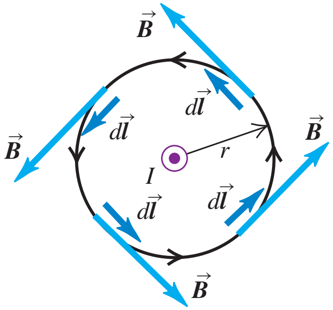
similar to Gauss law but Ampere's law is formulated not in terms of magnetic flux, but rather in terms of the *line integral* of ***B*** around a closed path, denoted by

$$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}}$$

Ampere's Law

(a) Integration path is a circle centered on the conductor; integration goes around the circle counterclockwise.

Result: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$



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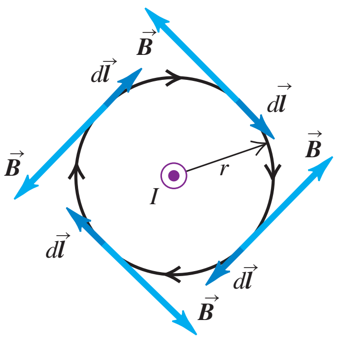
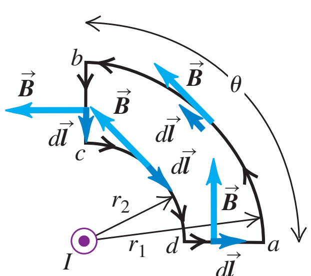
(b) Same integration path as in (a), but integration goes around the circle clockwise.

Result: $\oint \vec{B} \cdot d\vec{l} = -\mu_0 I$



(c) An integration path that does not enclose the conductor

Result: $\oint \vec{B} \cdot dl = 0$



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$$\oint \vec{B} \cdot d\vec{l} = \oint B_{\parallel} dl = B_1 \int_a^b dl + (0) \int_b^c dl + (-B_2) \int_c^d dl + (0) \int_d^a dl$$

$$= \frac{\mu_0 I}{2\pi r_1}(r_1 \theta) + 0 - \frac{\mu_0 I}{2\pi r_2}(r_2 \theta) + 0 = 0$$

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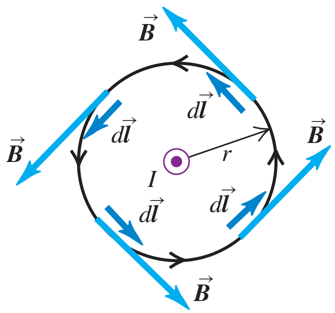

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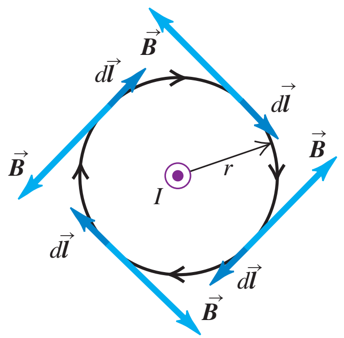

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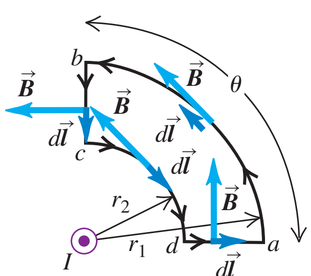

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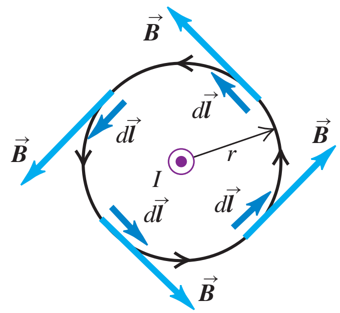

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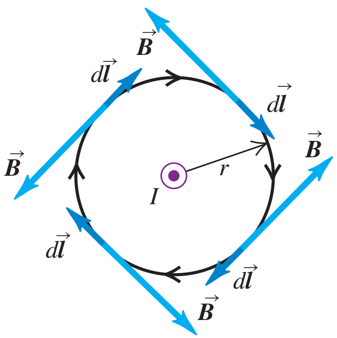

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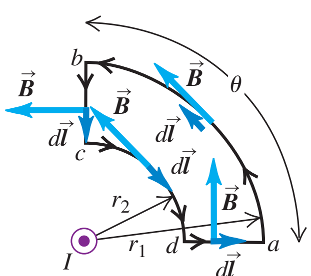

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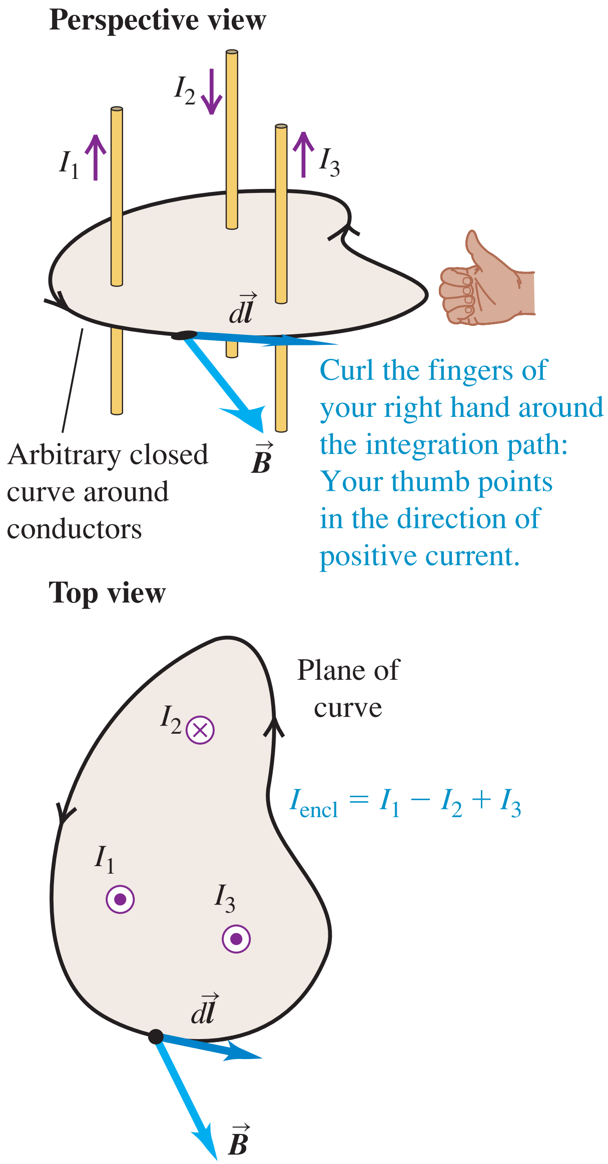
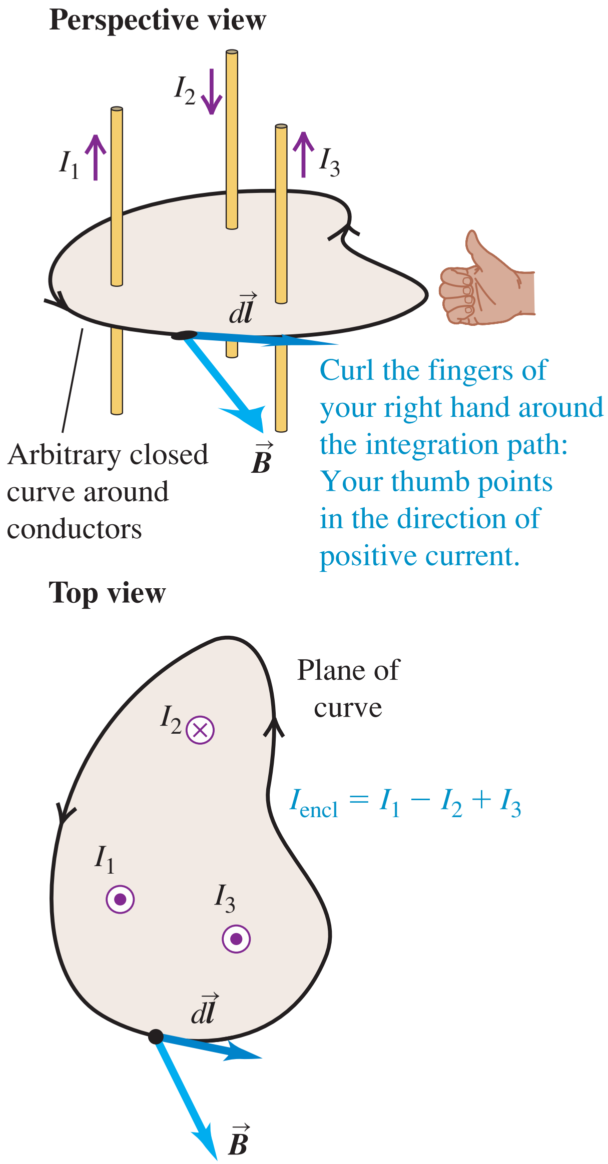
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \qquad \text{(Ampere's law)}$$

Current inside the closed path

Applications of Ampere's Law



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.



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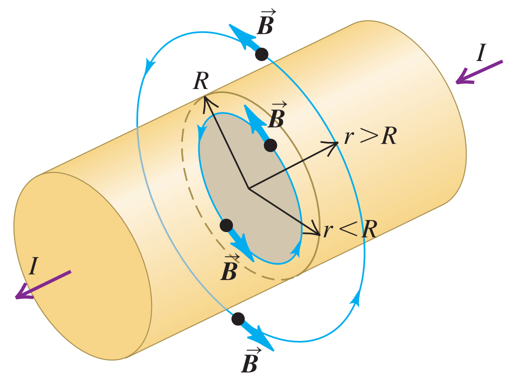
Applications of Ampere's Law

- Field of a long cylindrical conductor



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$$J = I/\pi R^2$$

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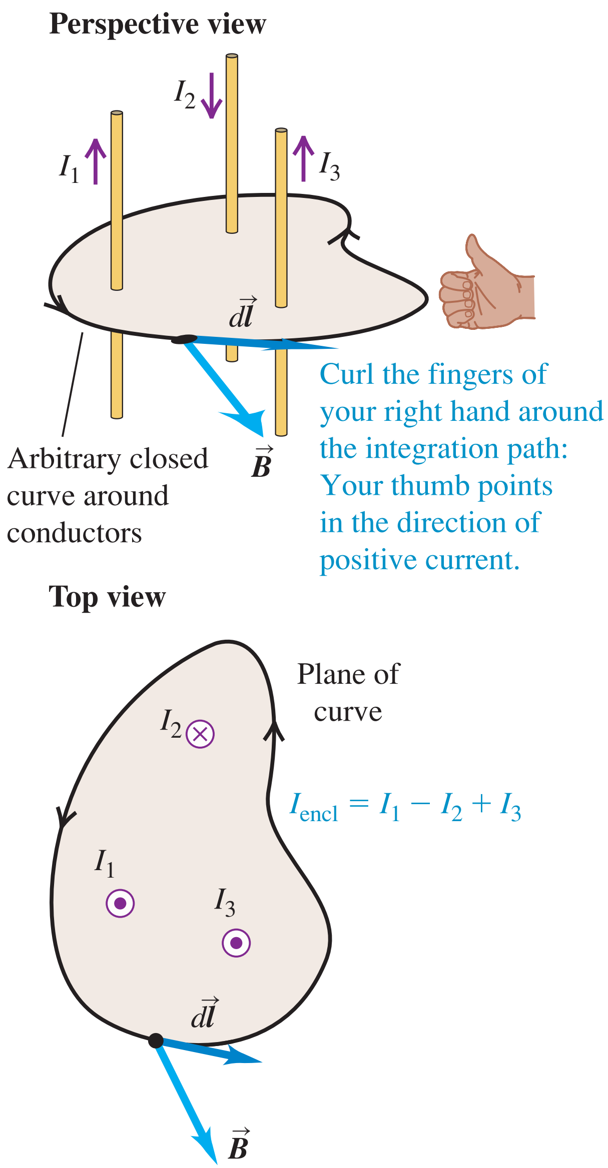

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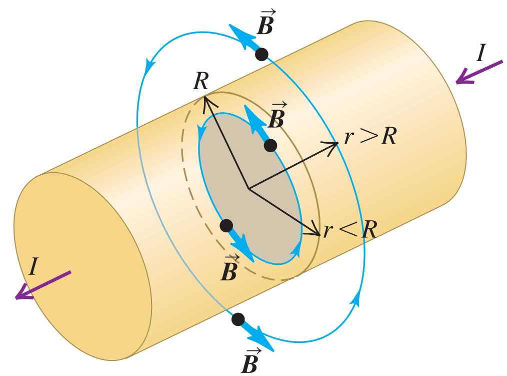

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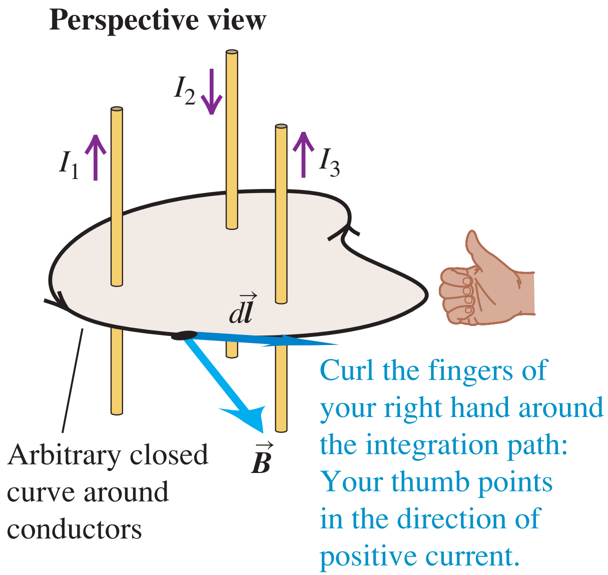

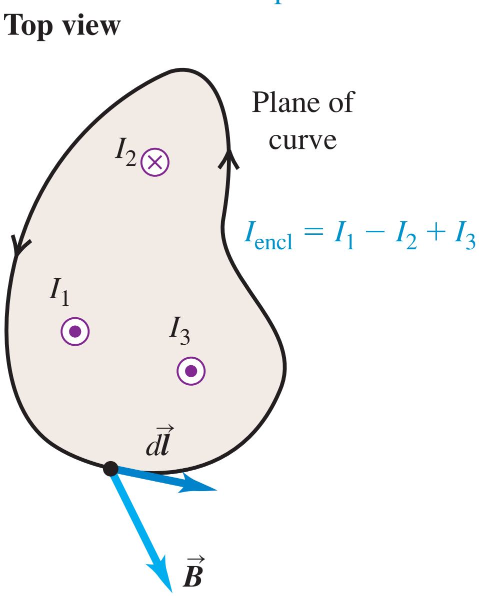

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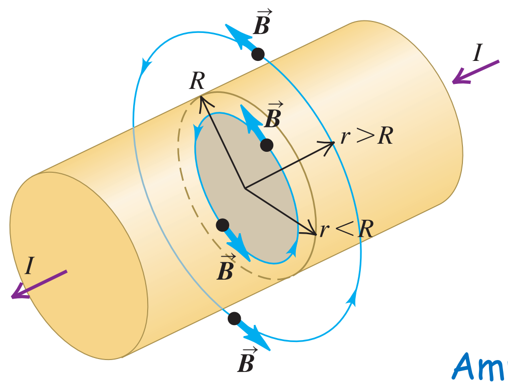

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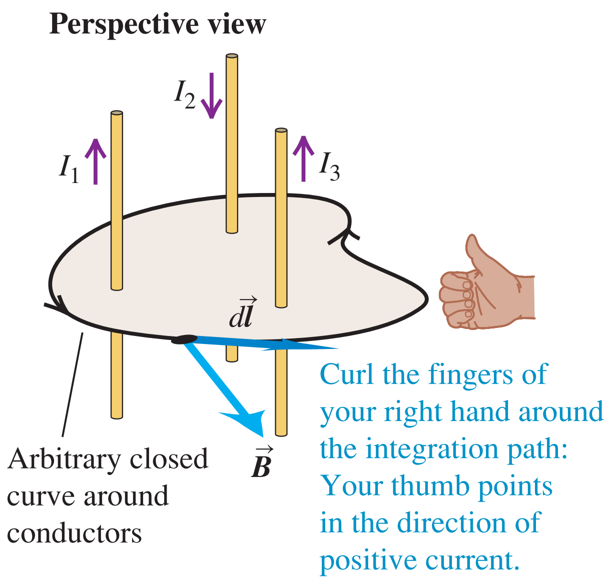

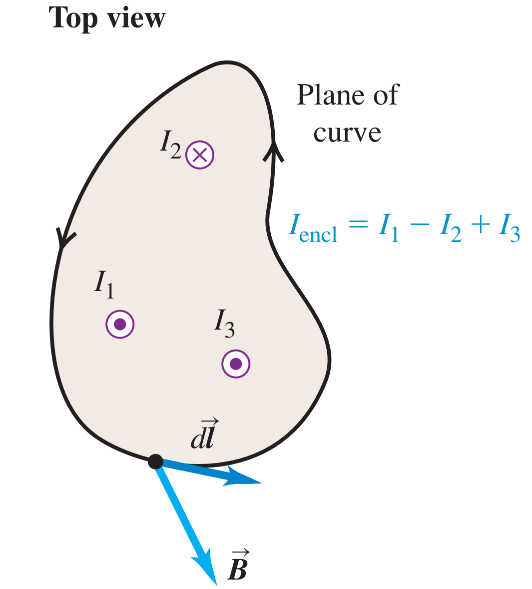

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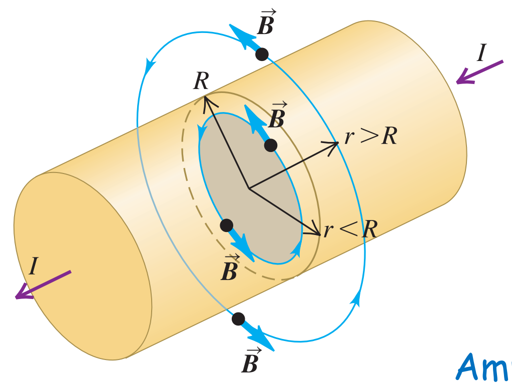

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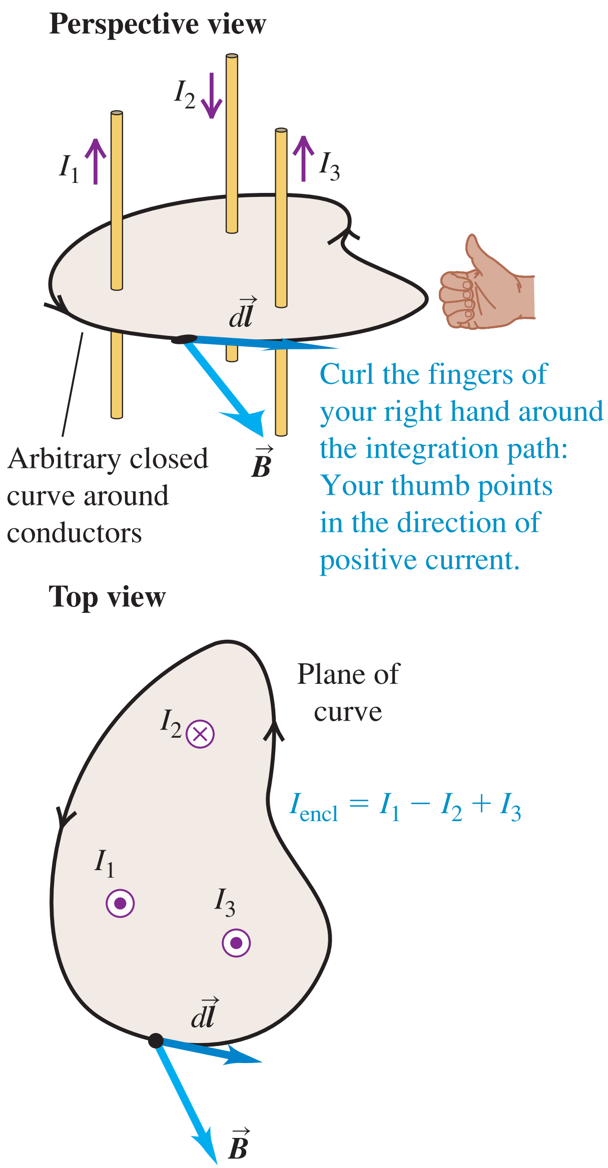

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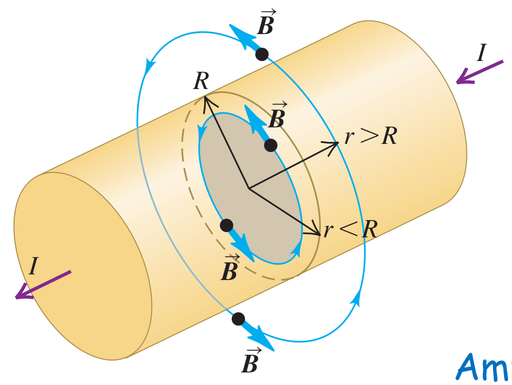

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