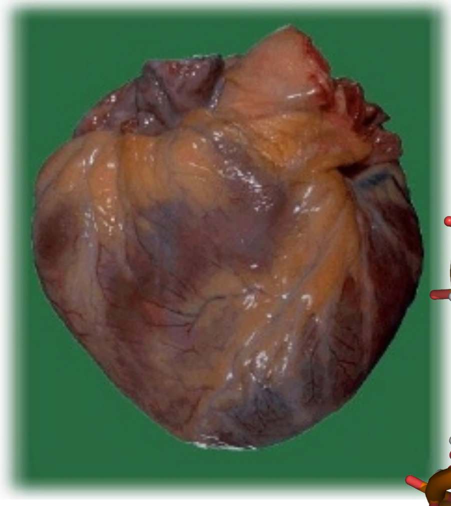
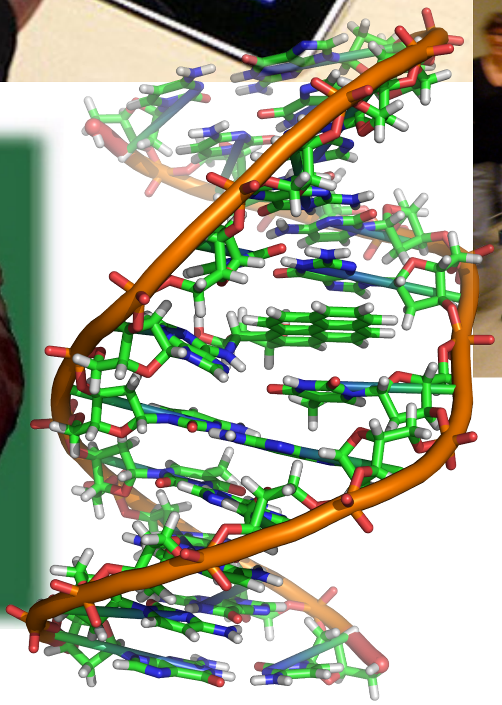
Chp 21: ELECTRIC CHARGE AND ELECTRIC FIELD

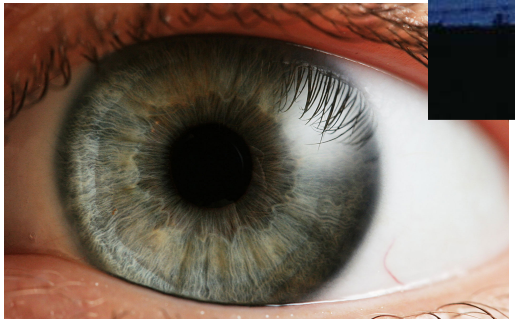
Goals for Chapter 21

- To remember what we learned about vectors
- To study electric charge and charge conservation
- To see how objects become charged
- To calculate the electric force between objects using Coulomb's law
- To learn the distinction between electric force and electric field
- To calculate the electric field due to many charges
- To visualize and interpret electric fields
- To calculate the properties of electric dipoles


Brooklyn Bridge-City Hall Station
Nassau St
Park Row
Church St
140
yards
0.25 M
1.30 min
10:11 am
10:12 am
31 mph
Broadway
tomtom







Reminder: Vector Definitions

E&M uses vectors for fields, vector products for magnetic field and force

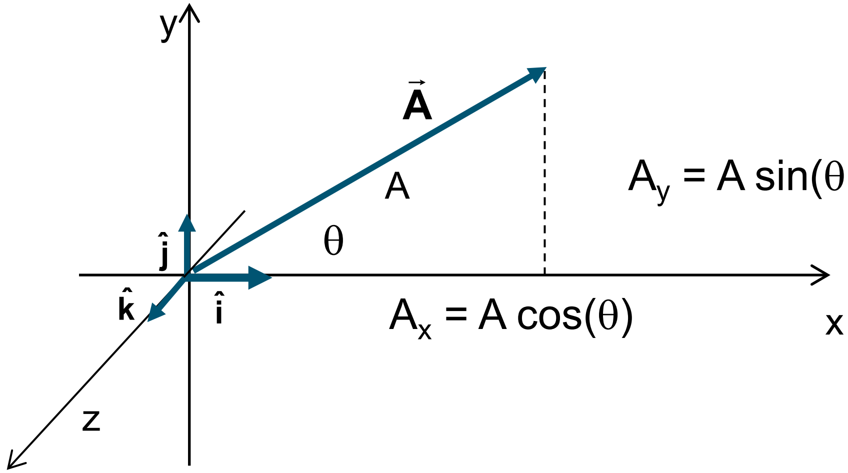
Representations in 2 Dimensions:

- Cartesian (x,y) coordinates

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

- Magnitude & direction

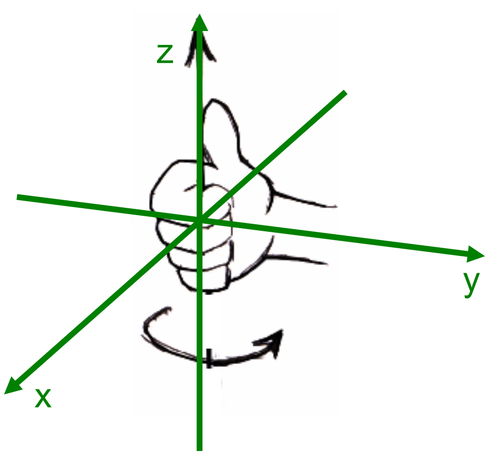
$$A = \sqrt{A_x^2 + A_y^2} \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$



- Addition and subtraction of vectors:

$$\vec{C} = \vec{A} + \vec{B} \text{ means } C_x = A_x + B_x \text{ and } C_y = A_y + B_y$$

$$\vec{C} = -\vec{A} \text{ means } C_x = -A_x \text{ and } C_y = -A_y$$



We always use right-handed coordinate systems.
In three-dimensions the right-hand rule determines which way the positive axes point.
Curl the fingers of your RIGHT HAND so they go from x to y.
Your thumb will point in the positive z direction.

Vectors in 3 dimensions

René Descartes 1596 - 1650

Unit vector (Cartesian) notation:

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

Spherical polar coordinate representati

1 magnitude and 2 directions

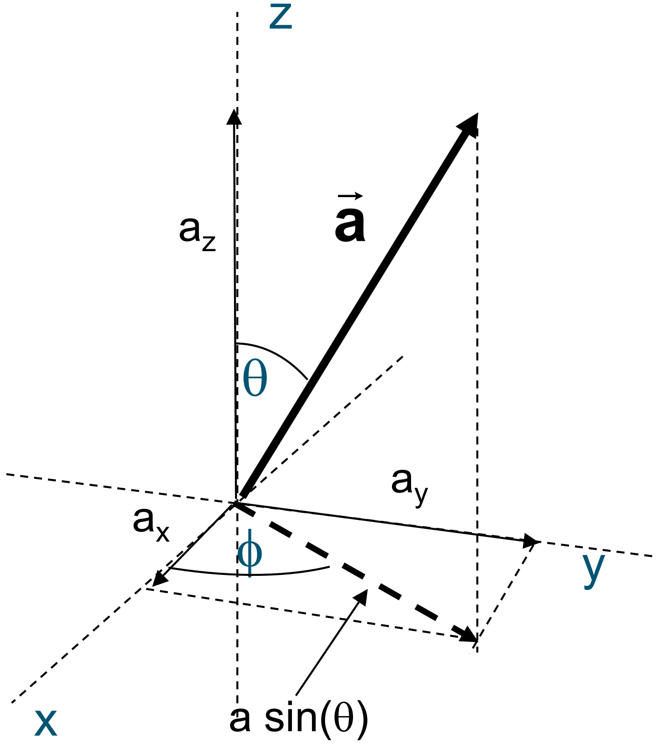
$$\vec{a} \equiv (a, \theta, \phi)$$

Conversion into x, y, z components

$$a_x = a \sin\theta \cos\phi$$
$$a_y = a \sin\theta \sin\phi$$
$$a_z = a \cos\theta$$

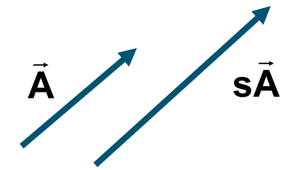
Conversion from x, y, z components

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$
$$\theta = \cos^{-1} a_z / a$$
$$\phi = \tan^{-1} a_y / a_x$$

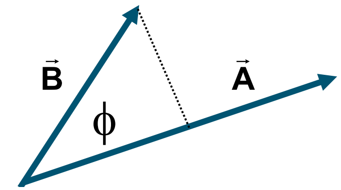


Vector Multiplication

Multiplication of a vector by a scalar: $s\vec{A} = sA_x\hat{i} + sA_y\hat{j}$



Dot product (or Scalar product or Inner product):



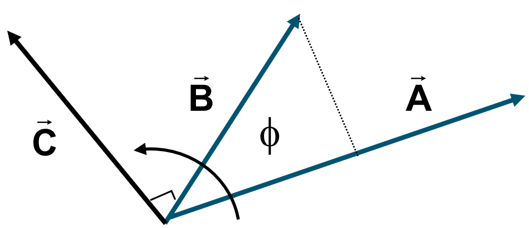
$$\vec{A}\circ\vec{B} \equiv AB\cos(\varphi) = \vec{B}\circ\vec{A} = A_xB_x + A_yB_y + A_zB_z$$

$$\hat{i}\circ\hat{j} = 0, \quad \hat{j}\circ\hat{k} = 0, \quad \hat{i}\circ\hat{k} = 0 \qquad \hat{i}\circ\hat{i} = 1, \quad \hat{j}\circ\hat{j} = 1, \quad \hat{k}\circ\hat{k} = 1$$

Cross product (or Vector product or Outer product):

Vector times vector ➔ another vector perpendicular to the plane of *A* and B

$$\vec{C} = \vec{A}\times\vec{B} = -\vec{B}\times\vec{A} \qquad \textbf{magnitude}: \left|C\right| = AB\sin(\phi)$$

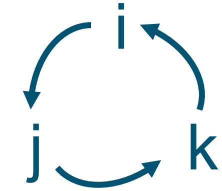


$$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$$
$$s\vec{A}\times\vec{B} = (s\vec{A})\times\vec{B} = \vec{A}\times(s\vec{B})$$
$$(\vec{A}\times\vec{B})\times\vec{C} = \vec{A}\times(\vec{B}\times\vec{C})$$

$$\vec{A}\times\vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k}) \times (B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

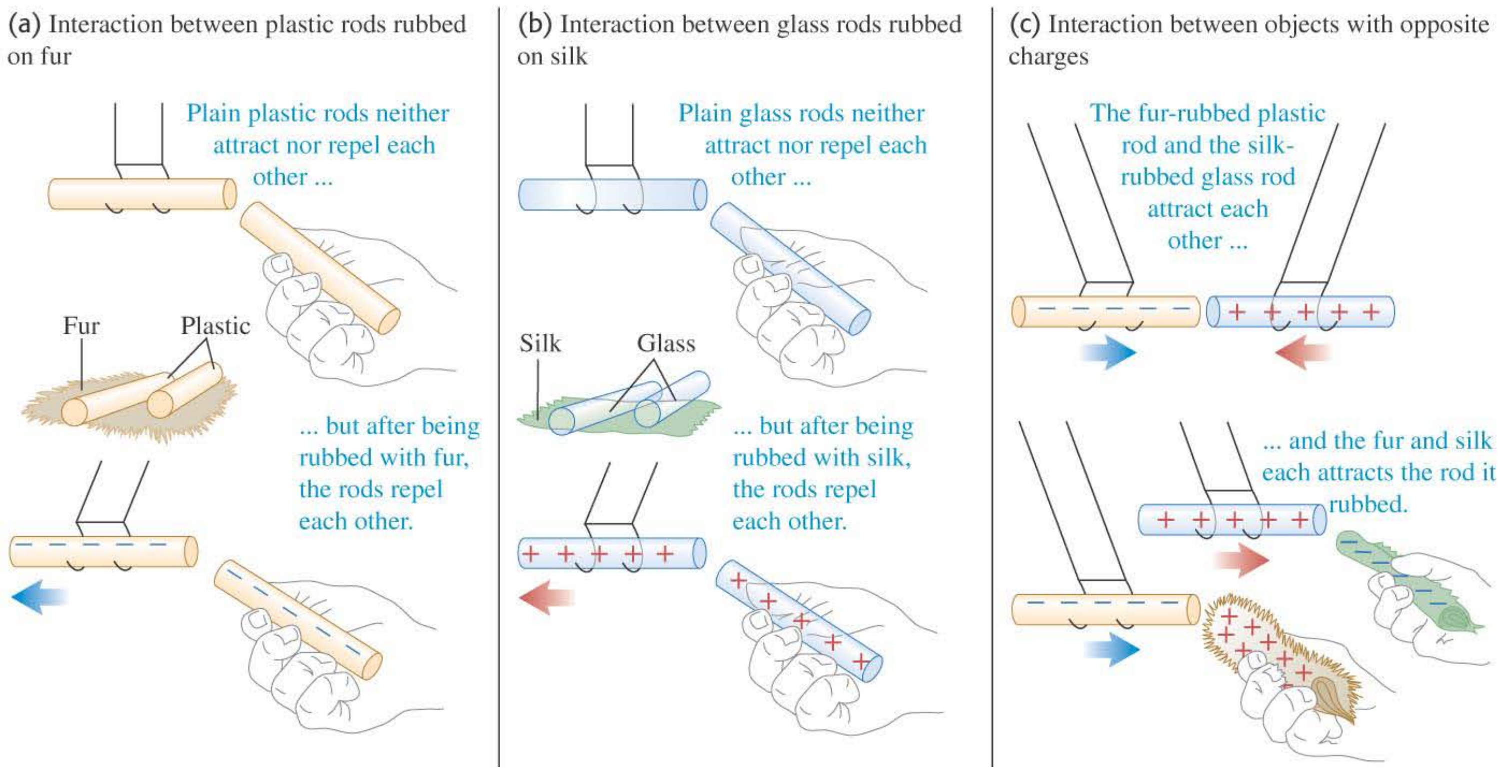
$$\hat{i}\times\hat{j} = \hat{k}, \quad \hat{j}\times\hat{k} = \hat{i}, \quad \hat{i}\times\hat{k} = -\hat{j}$$
$$\hat{i}\times\hat{i} = 0, \quad \hat{j}\times\hat{j} = 0, \quad \hat{k}\times\hat{k} = 0$$



Electric Charge

The ancient Greeks discovered as early as 600 B.C. that after they rubbed amber with wool. the amber could attract other objects. Today we say that the amber has acquired a net electric charge, or has become charged. The word "electric" is derived from the Greek word elektron, meaning amber. When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

- Two positive or two negative charges repel each other. A positive charge and a negative charge attract each other.

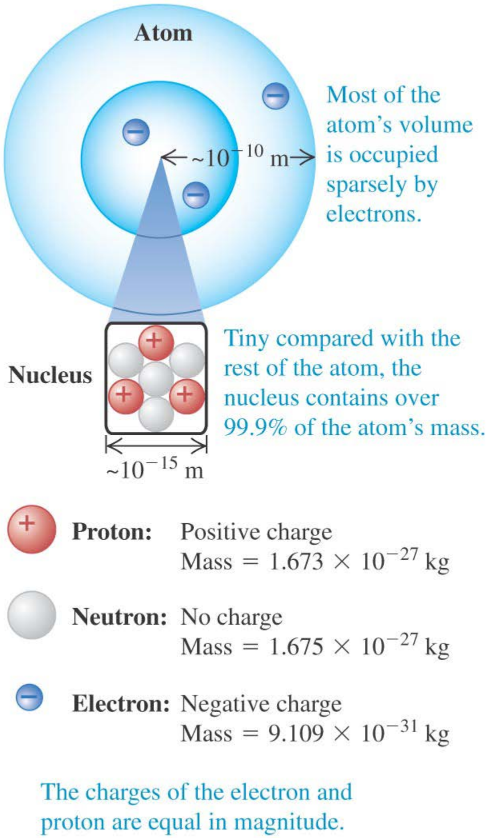


Electric charge and the structure of matter

The particles of the atom are the negative *electron*, the positive *proton*, and the uncharged *neutron*.

Protons and neutrons make up the tiny dense nucleus which is surrounded by electrons (see Figure 21.3 at the right).

The electric attraction between protons and electrons holds the atom together.

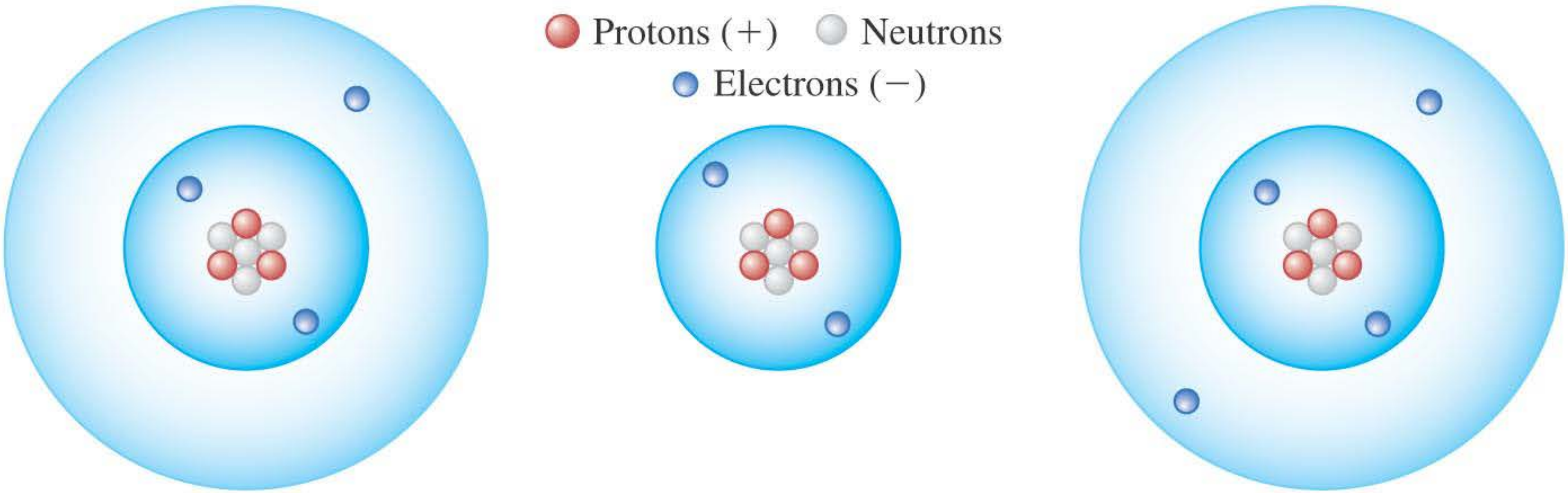


Name	Symbol	Value
Speed of light in vacuum	c	2.99792458×10^{8} m/s
Magnitude of charge of electron	e	$1.602176487(40) \times 10^{-19}$ C
Gravitational constant	G	$6.67428(67) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Planck's constant	h	$6.62606896(33) \times 10^{-34}\ \mathrm{J \cdot s}$
Boltzmann constant	k	$1.3806504(24) \times 10^{-23}$ J/K
Avogadro's number	N_A	$6.02214179(30) \times 10^{23}$ molecules/mol
Gas constant	R	$8.314472(15)\ \mathrm{J/mol \cdot K}$
Mass of electron	m_e	$9.10938215(45) \times 10^{-31}$ kg
Mass of proton	m_p	$1.672621637(83) \times 10^{-27}$ kg
Mass of neutron	m_n	$1.674927211(84) \times 10^{-27}$ kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{Wb/A \cdot m}$
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854187817\ldots \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$
	$1/4\pi\epsilon_0$	$8.987551787\ldots \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$

Atoms and ions

- A neutral atom has the same number of protons as electrons.
- A *positive ion* is an atom with one or more electrons removed.

A *negative ion* has gained one or more electrons.



(a) Neutral lithium atom (Li):

3 protons (3+)
4 neutrons
3 electrons (3−)

Electrons equal protons:
Zero net charge

(b) Positive lithium ion (Li^+):

3 protons (3+)
4 neutrons
2 electrons (2−)

Fewer electrons than protons:
Positive net charge

(c) Negative lithium ion (Li^-):

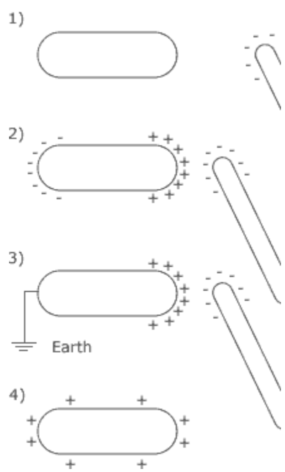
3 protons (3+)
4 neutrons
4 electrons (4−)

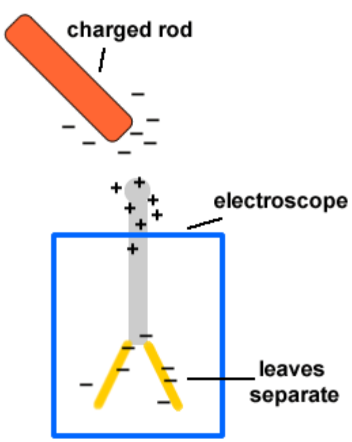
More electrons than protons:
Negative net charge

The universal *principle of charge conservation* states that the algebraic sum of all the electric charges in any closed system is constant.

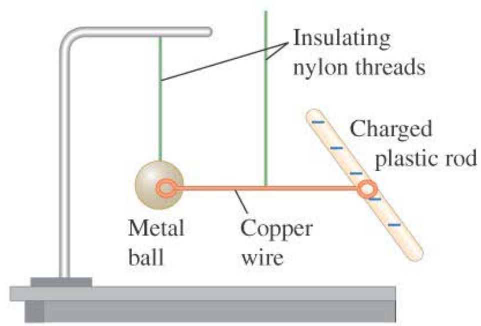
Conductors and insulators

- A *conductor* permits the easy movement of charge through it. An *insulator* does not.
- Most metals are good conductors, while most nonmetals are insulators.
- *Semiconductors* are intermediate in their properties between good conductors and good insulators.



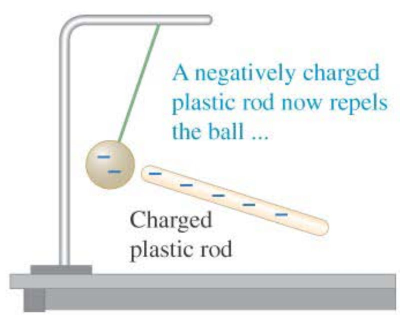


(a)



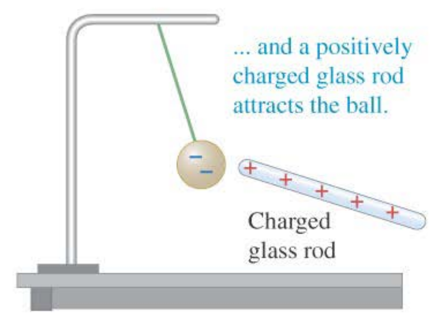
The wire conducts charge from the negatively charged plastic rod to the metal ball.

(b)



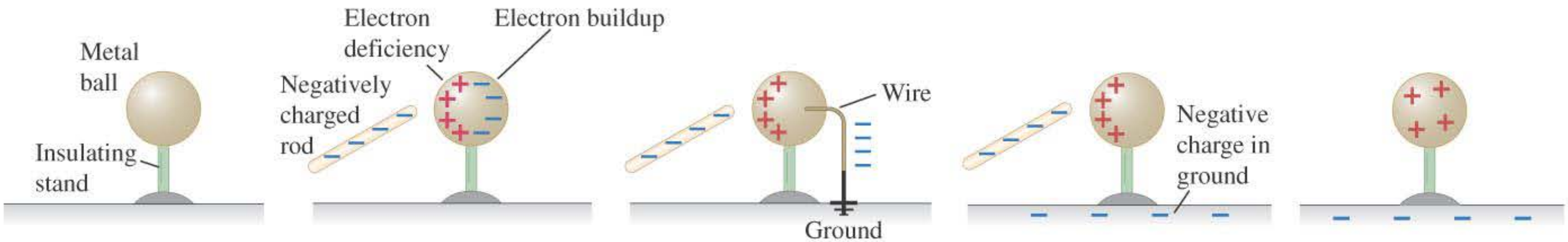
(c)

(c)



Charging by induction

In Figure below, the negative rod is able to charge the metal ball without losing any of its own charge. This process is called charging by *induction*.



(a) Uncharged metal ball

(b) Negative charge on rod repels electrons, creating zones of negative and positive **induced charge**.

(c) Wire lets electron build-up (induced negative charge) flow into ground.

(d) Wire removed; ball now has only an electron-deficient region of positive charge.

(e) Rod removed; electrons rearrange themselves, ball has overall electron deficiency (net positive charge).

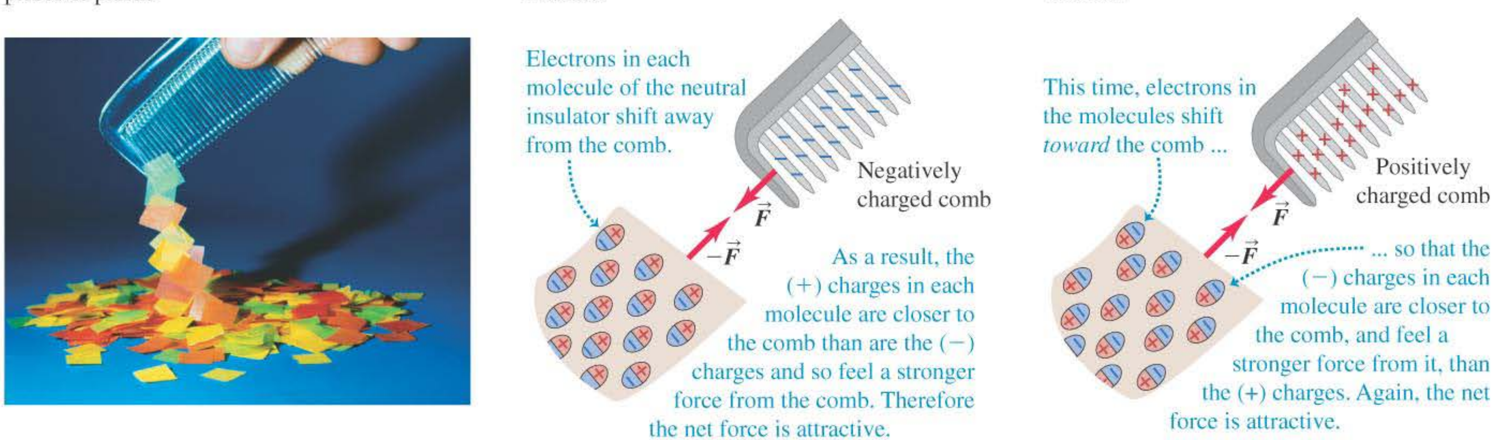
Electric forces on uncharged objects

The charge within an insulator can shift slightly due to polarization.
As a result, two neutral objects can exert electric forces on each other

(a) A charged comb picking up uncharged pieces of plastic

(b) How a negatively charged comb attracts an insulator

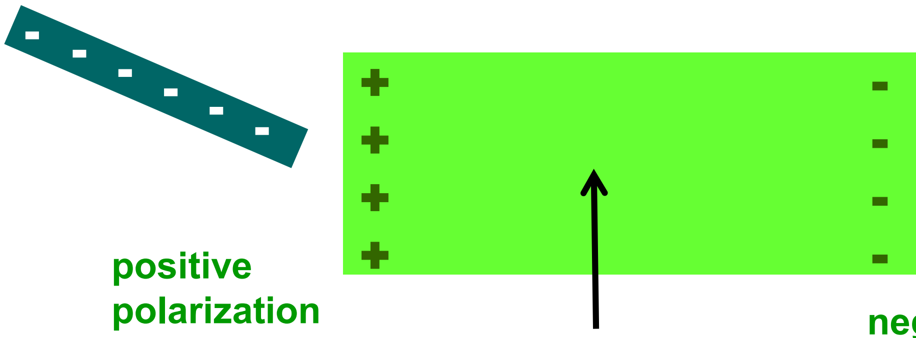
(c) How a positively charged comb attracts an insulator



Insulators: Charges not free to move

insulators can be induced to polarize by nearby charges

Polarization means charge separates but does not leave home



positive polarization charge

field inside material is smaller than it would be in vacuum

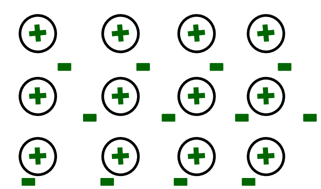
negative polarization charge

THE *DIELECTRIC CONSTANT* MEASURES MATERIALS' ABILITY TO *POLARIZE*

Conductors: charges free to move

SOLID CONDUCTORS

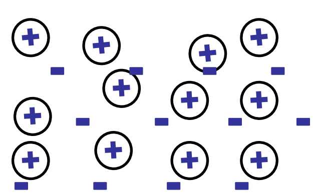
metals for example



- Regular lattice of fixed + ions
- conduction band free electrons wander when E field is applied

LIQUID & GASEOUS CONDUCTORS

sea water, humans, hot plasmas

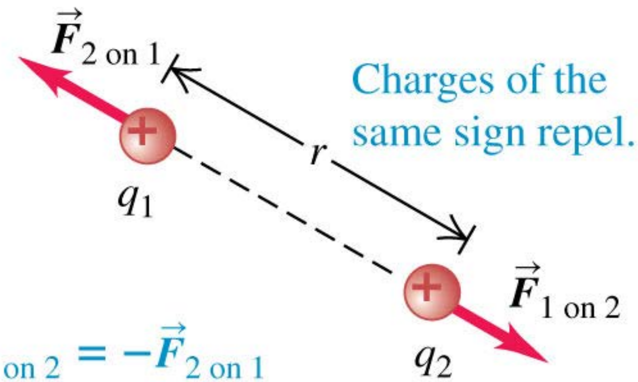


- Electrons and ions both free to move independently
- Normally random motion
- Electrons & ions move in opposite directions in an E field.

Coulomb's law

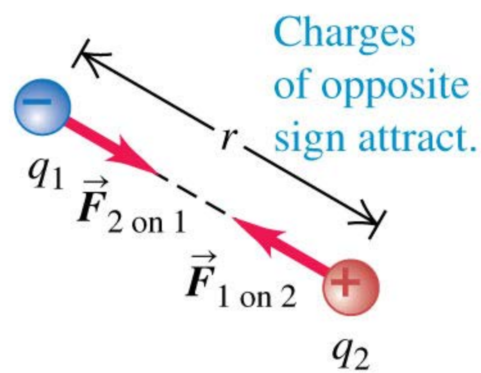
Coulomb's Law: The magnitude of the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

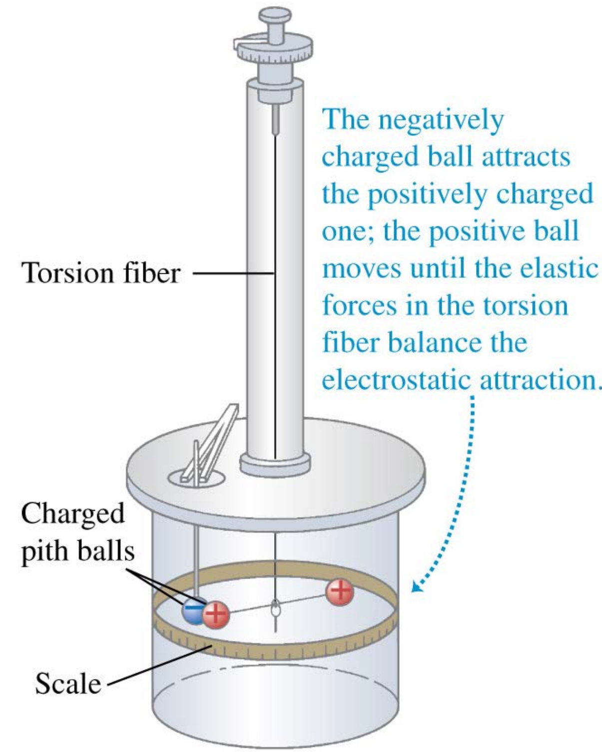
$$F = k\frac{|q_1 q_2|}{r^2}$$

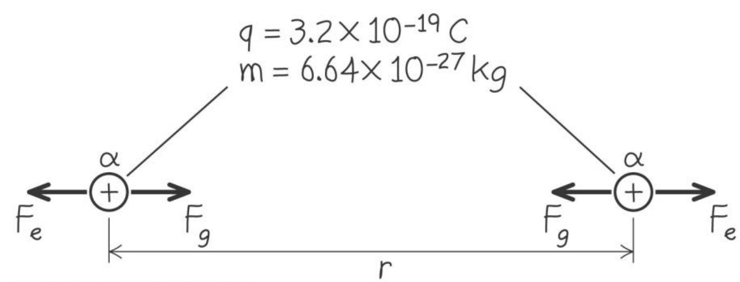


$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$$

$$F_{1\text{ on }2} = F_{2\text{ on }1} = k\frac{|q_1 q_2|}{r^2}$$







$$F = k|q_1q_2|/r^2 = (1/4\pi\varepsilon_0)|q_1q_2|/r^2$$

$$k = 8.987551787 \times 10^9\ \mathrm{N \cdot m^2/C^2} \cong 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

Speed of light c = 2.99792458x 10^8 m/s.
The numerical value of k is defined in terms of c to be precisely

$$k = (10^{-7}\ \mathrm{N \cdot s^2/C^2})c^2$$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$$ (Coulomb's law: force between two point charges)

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

the most fundamental unit of charge is the magnitude of the charge of an electron or a proton. which is denoted bye. The most precise value available is:

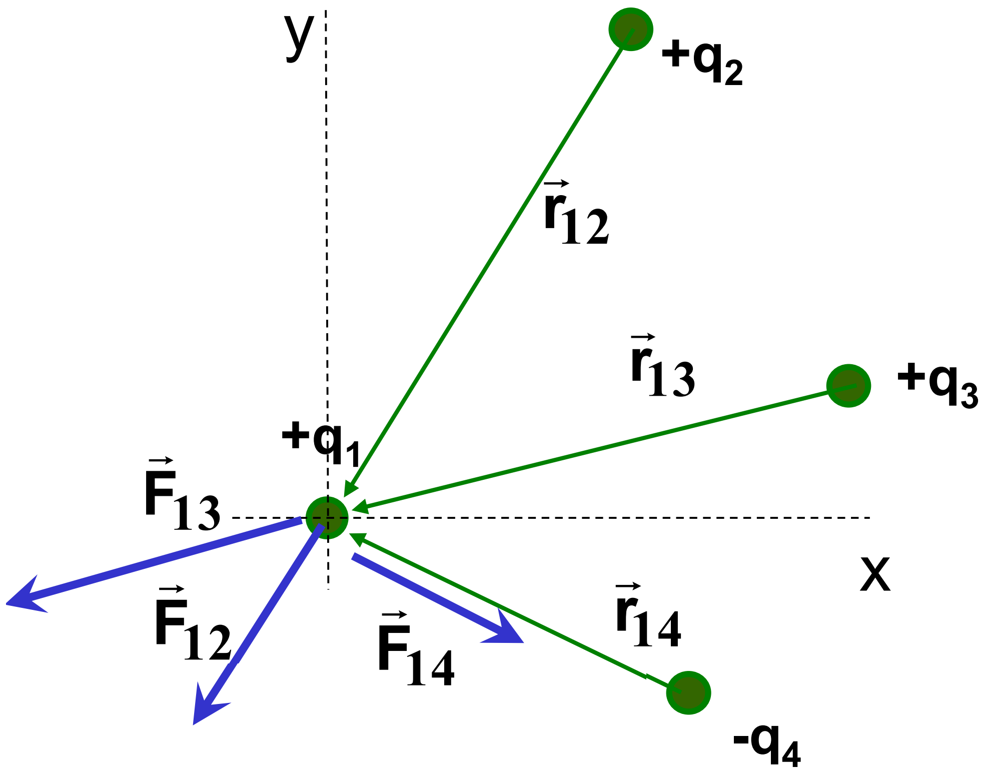
e= 1.60217653(14) X 10^{-19} C

comparing the electric and gravitational forces.

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} \qquad F_g = G\frac{m^2}{r^2}$$

$$\frac{F_e}{F_g} = \frac{1}{4\pi\epsilon_0 G}\frac{q^2}{m^2} = \frac{9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}}\frac{(3.2 \times 10^{-19}\ \mathrm{C})^2}{(6.64 \times 10^{-27}\ \mathrm{kg})^2}$$
$$= 3.1 \times 10^{35}$$

Superposition of forces

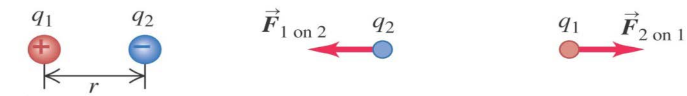


NET force on q_1

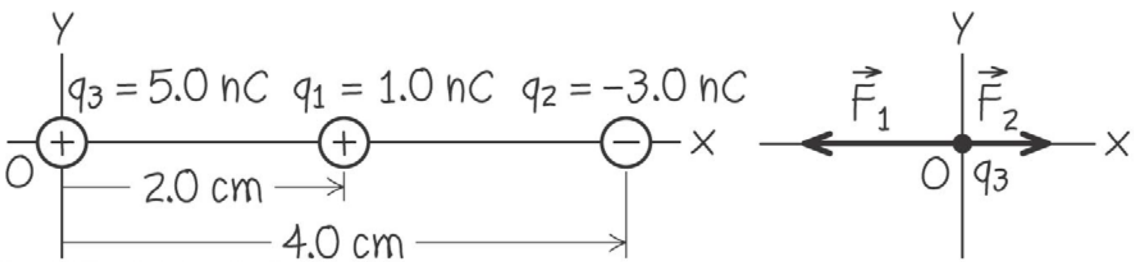
$$\vec{F}_{net\,on\,1} = \sum_{i=2}^{n} \vec{F}_{1,i} = \vec{F}_{1,2} + \vec{F}_{1,3} + \vec{F}_{1,4+\ldots}$$

Vector addition of electric forces on a line

(a) The two charges (b) Free-body diagram for charge q_2 (c) Free-body diagram for charge q_1



(a) Our diagram of the situation (b) Free-body diagram for q_3



$$F_{1\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_3|}{r^2}$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.020\ \text{m})^2}$$
$$= 1.12 \times 10^{-4}\ \text{N} = 112\ \mu\text{N}$$

The magnitude $F_{2\text{ on }3}$ of the force of q_2 on q_3 is

$$F_{2\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_2q_3|}{r^2}$$
$$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.0 \times 10^{-9}\ \text{C})(5.0 \times 10^{-9}\ \text{C})}{(0.040\ \text{m})^2}$$
$$= 8.4 \times 10^{-5}\ \text{N} = 84\ \mu\text{N}$$

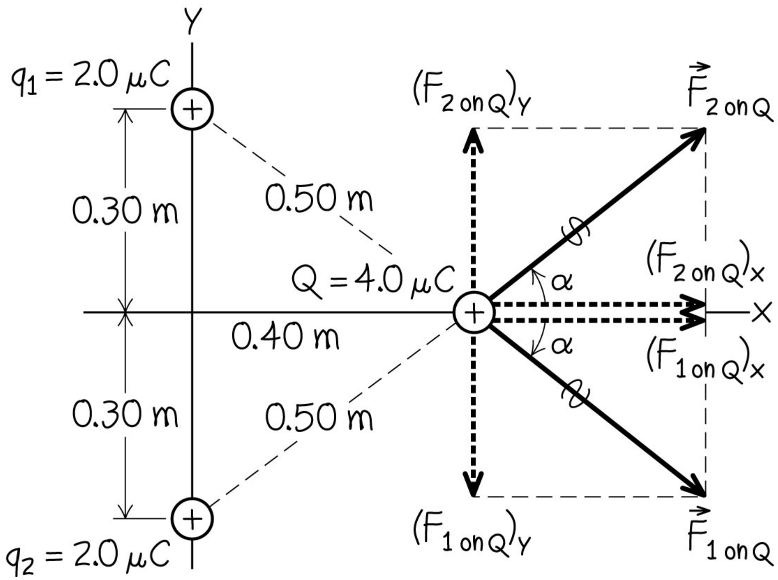
This force has a positive x-component because q_3 is attracted (that is, pulled in the positive x-direction) by q_2. The sum of the x-components is

$$F_x = -112\ \mu\text{N} + 84\ \mu\text{N} = -28\ \mu\text{N}$$

There are no y- or z-components. Thus the total force on q_3 is directed to the left, with magnitude $28\ \mu\text{N} = 2.8 \times 10^{-5}$ N.

Vector addition of electric forces

Example 21.4 shows that we must use vector addition when adding electric forces. Follow this example using Figure 21.14 below.



the force on Q due to the upper charge q_1. From Coulomb's law the magnitude F of this force is

$$F_{1\,\mathrm{on}\,Q} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(4.0 \times 10^{-6}\ \mathrm{C})(2.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2}$$

$$= 0.29\ \mathrm{N}$$

$$(F_{1\,\mathrm{on}\,Q})_x = (F_{1\,\mathrm{on}\,Q})\cos\alpha = (0.29\ \mathrm{N})\frac{0.40\ \mathrm{m}}{0.50\ \mathrm{m}} = 0.23\ \mathrm{N}$$

$$(F_{1\,\mathrm{on}\,Q})_y = -(F_{1\,\mathrm{on}\,Q})\sin\alpha = -(0.29\ \mathrm{N})\frac{0.30\ \mathrm{m}}{0.50\ \mathrm{m}} = -0.17\ \mathrm{N}$$

$$F_x = 0.23\ \mathrm{N} + 0.23\ \mathrm{N} = 0.46\ \mathrm{N}$$

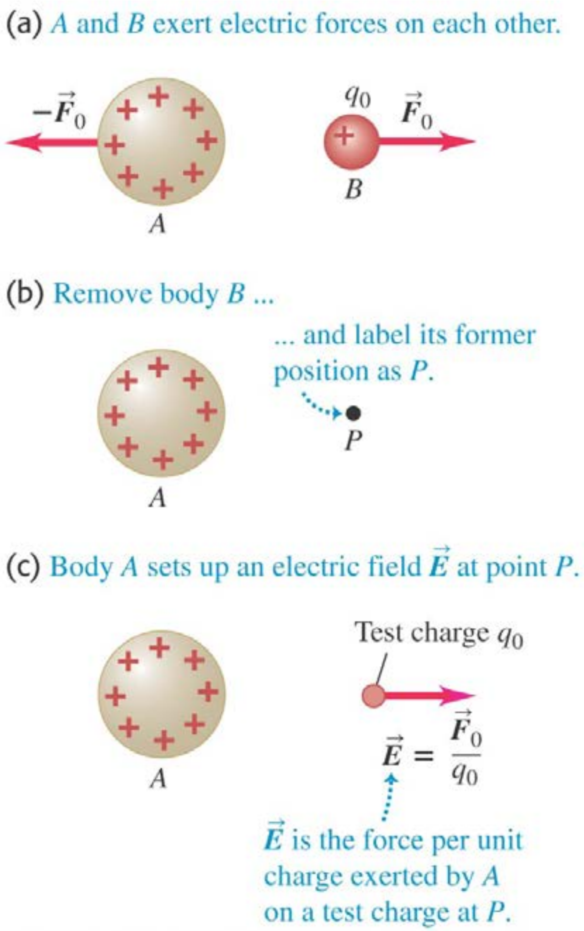
$$F_y = -0.17\ \mathrm{N} + 0.17\ \mathrm{N} = 0$$

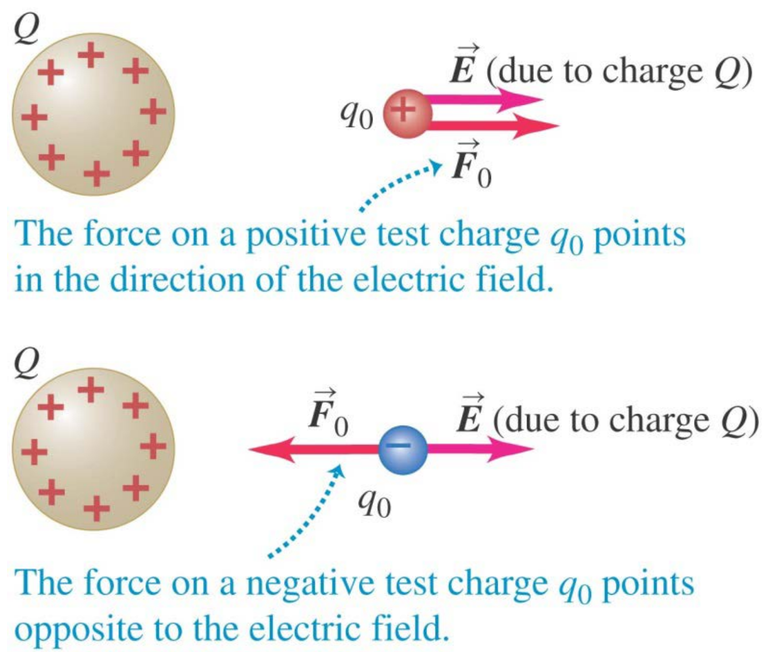
The total force on Q is in the +x-direction, with magnitude 0.46 N.

Electric field

- A charged body produces an *electric field* in the space around it (see Figure below at the lower left).
- We use a small *test charge* q_0 to find out if an electric field is present (see Figure below at the lower right).

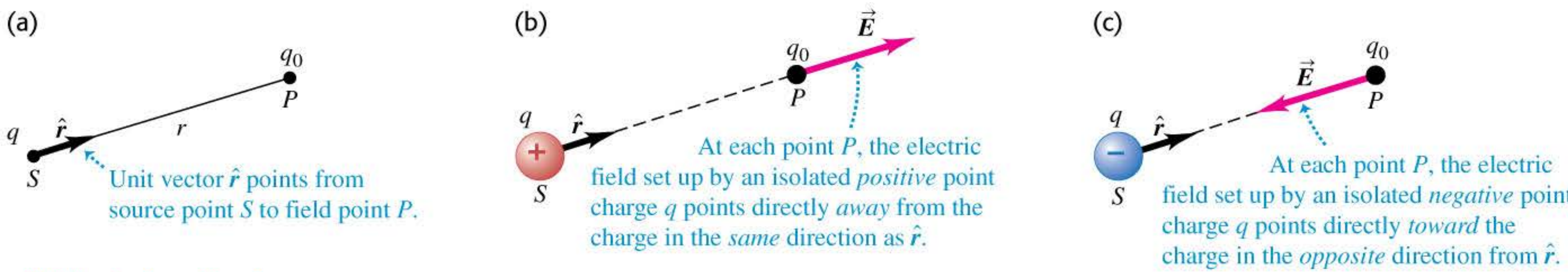
The electric force on a charged body is exerted by the electric field created by other charged bodies.



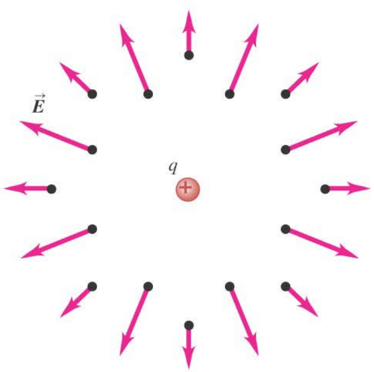


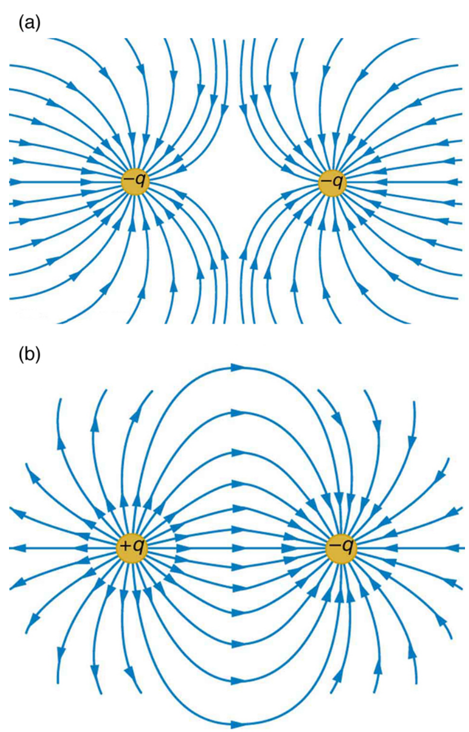
$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

Definition of the electric field



(a) The field produced by a positive point charge points *away from* the charge.

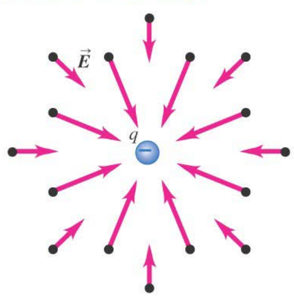




$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad \text{(electric field of a point charge)}$$

$$\vec{F}_0 = q_0\vec{E} \qquad \text{(force exerted on a point charge } q_0 \text{ by an electric field } \vec{E}\text{)}$$

(b) The field produced by a negative point charge points *toward* the charge.



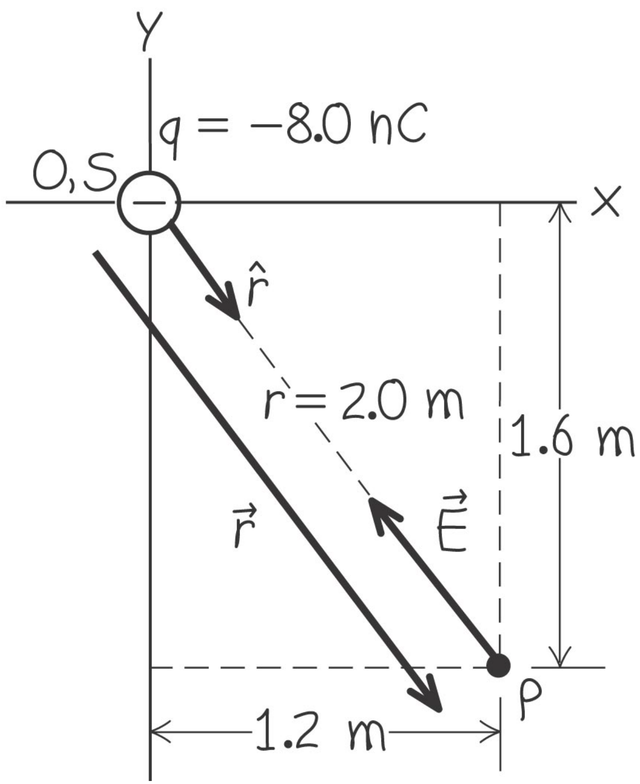
Similar to gravitational field:

$$\vec{F}_g = m_0\vec{g} \qquad \vec{g} = \frac{\vec{F}_g}{m_0}$$

$$\vec{E} = \vec{F}/q = \left(\frac{kQq}{r^2}\hat{r}\right)\frac{1}{q}$$

Electric-field vector of a point charge

Example 21.6 shows the vector nature of the electric field.



$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\,\text{m})^2 + (-1.6\,\text{m})^2} = 2.0\,\text{m}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{r}$$

$$= \frac{(1.2\,\text{m})\hat{\imath} + (-1.6\,\text{m})\hat{\imath}}{2.0\,\text{m}} = 0.60\hat{\imath} - 0.80\hat{\jmath}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

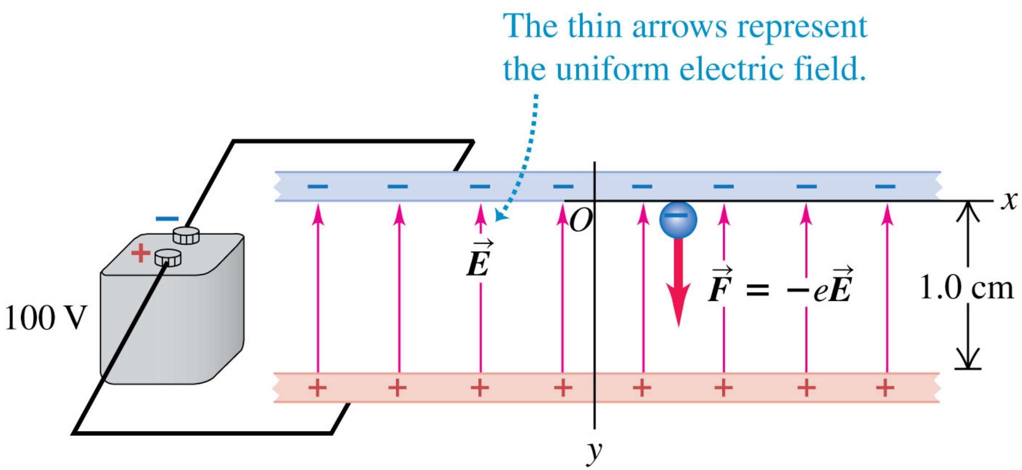
$$= (9.0 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-8.0 \times 10^{-9}\,\text{C})}{(2.0\,\text{m})^2}(0.60\hat{\imath} - 0.80\hat{\jmath})$$

$$= (-11\,\text{N/C})\hat{\imath} + (14\,\text{N/C})\hat{\jmath}$$

Since q is negative, E points from the field point to the charge (the source point).
in the direction opposite to the direction of the unit vector of r.

Electron in a uniform field

Normally Electric field changes with distance. But as an approximation, we can assume that it is constant at a short distance as in the Example 21.7



An electron has charge $-e = -1.60 \times 10^{-9}$ C and mass $m = 9.11 \times 10^{-31}$ kg.
If the plates are horizontal and separated by 1.0 cm and the plates are connected to a 100-volt battery the magnitude of the field is $E = 1.00 \times 10^4$ N/C.
(a) If an electron is released from rest at the upper plate, what is its acceleration?
(b) What speed and kinetic energy does the electron acquire while traveling 1.0 cm to the lower plate?
(c) How much time is required for it to travel this distance?

$$a_y = \frac{F_y}{m} = \frac{-eE}{m} = \frac{(-1.60 \times 10^{-19}\,\mathrm{C})(1.00 \times 10^4\,\mathrm{N/C})}{9.11 \times 10^{-31}\,\mathrm{kg}}$$
$$= -1.76 \times 10^{15}\,\mathrm{m/s^2}$$

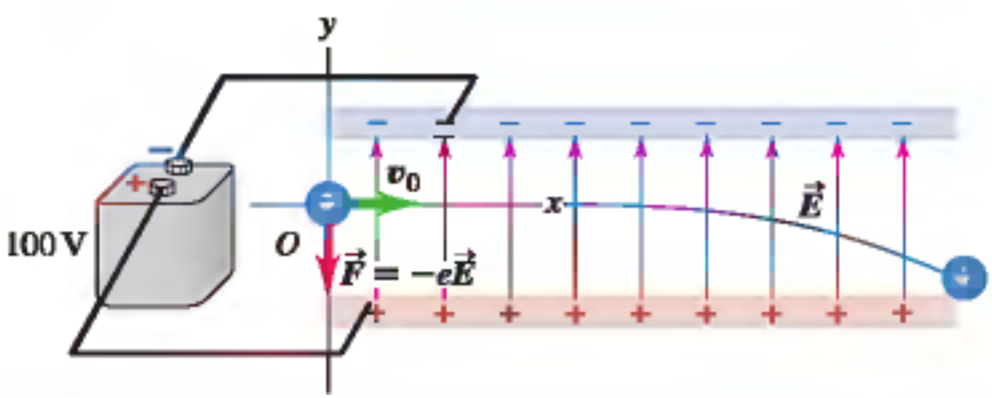
$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$|v_y| = \sqrt{2a_y y} = \sqrt{2(-1.76 \times 10^{15}\,\mathrm{m/s^2})(-1.0 \times 10^{-2}\,\mathrm{m})}$$
$$= 5.9 \times 10^6\,\mathrm{m/s}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31}\,\mathrm{kg})(5.9 \times 10^6\,\mathrm{m/s})^2$$
$$= 1.6 \times 10^{-17}\,\mathrm{J}$$

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{(-5.9 \times 10^6\,\mathrm{m/s}) - (0\,\mathrm{m/s})}{-1.76 \times 10^{15}\,\mathrm{m/s^2}}$$
$$= 3.4 \times 10^{-9}\,\mathrm{s}$$

An electron trajectory



The parabolic trajectory of an electron in a uniform electric field.

$a_x = 0$ $\qquad a_y = (-e)E/m$

$x_0 = y_0 = 0, v_{0x} = v_0,$ and $v_{0y} = 0$

$$x = v_0 t \quad \text{and} \quad y = \frac{1}{2}a_y t^2 = -\frac{1}{2}\frac{eE}{m}t^2$$

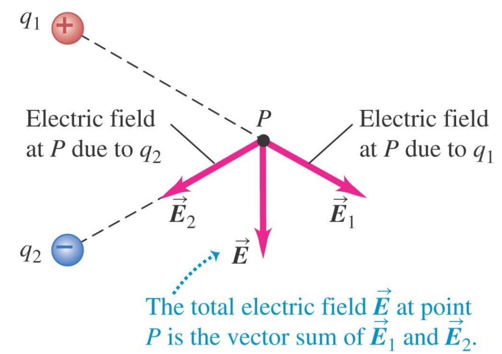
$$y = -\frac{1}{2}\frac{eE}{mv_0^2}x^2$$

This is the equation of a parabola, just like the trajectory of a projectile launched horizontally in the earth's gravitational field For a given initial velocity of the electron, the curvature of the trajectmy depends on the field magnitude E.
If we reverse the signs of the charges on the two plates in Figure, the direction of E reverses and the electron trajectory will curve up, not down. Hence we can "steer" the elec-tron by varying the charges on the plates. The electric field between charged conducting plates can be used in this way to con-trol the trajectoryy of electron beams in oscilloscopes.

Superposition of electric fields

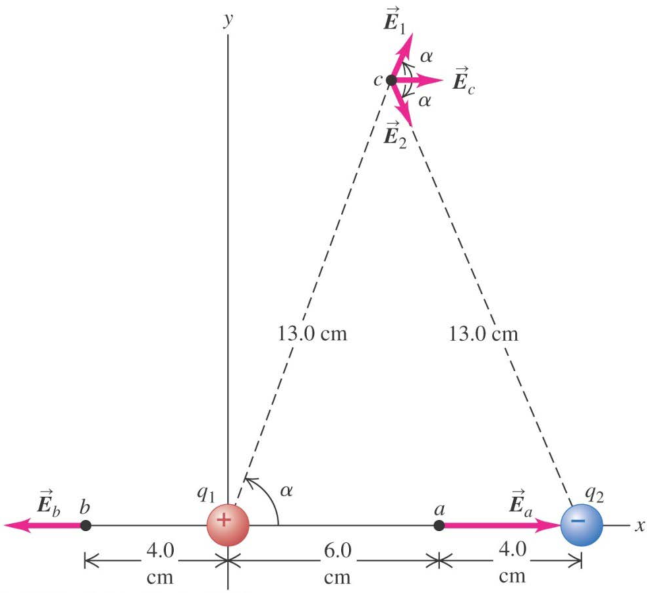
The total electric field at a point is the vector sum of the fields due to all the charges present.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... = \sum_{i=1}^{N} \vec{E}_i = \sum_{i=1}^{N} \frac{kq_i}{r_i^2}\hat{r}_i.$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2.$$

Follow Example 21.9 for an electric dipole



$$E_{1x} = 3.0 \times 10^4\ \mathrm{N/C} \qquad E_{1y} = 0$$
$$E_{2x} = 6.8 \times 10^4\ \mathrm{N/C} \qquad E_{2y} = 0$$

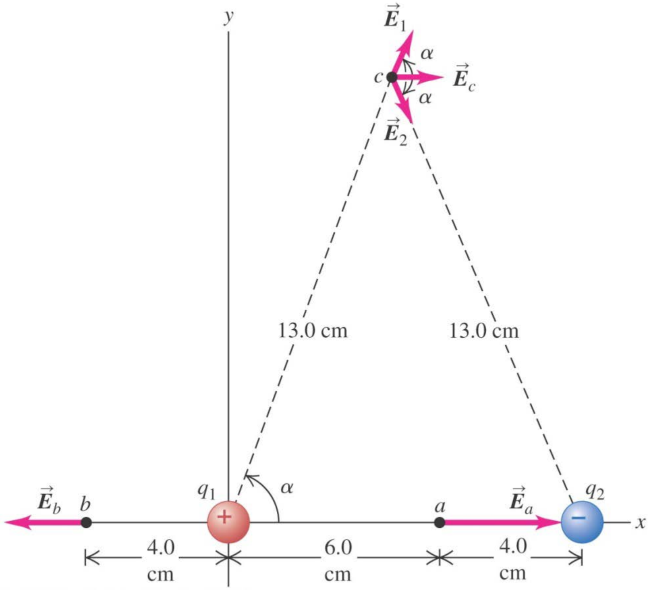
At point a:

$$\vec{E}_a = \vec{E}_1 + \vec{E}_2$$

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{12 \times 10^{-9}\ \mathrm{C}}{(0.060\ \mathrm{m})^2}$$
$$= 3.0 \times 10^4\ \mathrm{N/C}$$
$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} = (9.0 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{12 \times 10^{-9}\ \mathrm{C}}{(0.040\ \mathrm{m})^2}$$
$$= 6.8 \times 10^4\ \mathrm{N/C}$$

$$(E_a)_x = E_{1x} + E_{2x} = (3.0 + 6.8) \times 10^4\ \mathrm{N/C}$$
$$(E_a)_y = E_{1y} + E_{2y} = 0$$

$\rightarrow$ $$\vec{E}_a = (9.8 \times 10^4\ \mathrm{N/C})\hat{\imath}$$



At point b:

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r^2} = (9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{12 \times 10^{-9}\,\mathrm{C}}{(0.040\,\mathrm{m})^2}$$
$$= 6.8 \times 10^4\,\mathrm{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r^2} = (9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{12 \times 10^{-9}\,\mathrm{C}}{(0.140\,\mathrm{m})^2}$$
$$= 0.55 \times 10^4\,\mathrm{N/C}$$

$$E_{1x} = -6.8 \times 10^4\,\mathrm{N/C} \qquad E_{1y} = 0$$
$$E_{2x} = 0.55 \times 10^4\,\mathrm{N/C} \qquad E_{2y} = 0$$
$$(E_b)_x = E_{1x} + E_{2x} = (-6.8 + 0.55) \times 10^4\,\mathrm{N/C}$$
$$(E_b)_y = E_{1y} + E_{2y} = 0$$

$$\rightarrow \quad \vec{E}_b = (-6.2 \times 10^4\,\mathrm{N/C})\hat{\imath}$$

At point c:

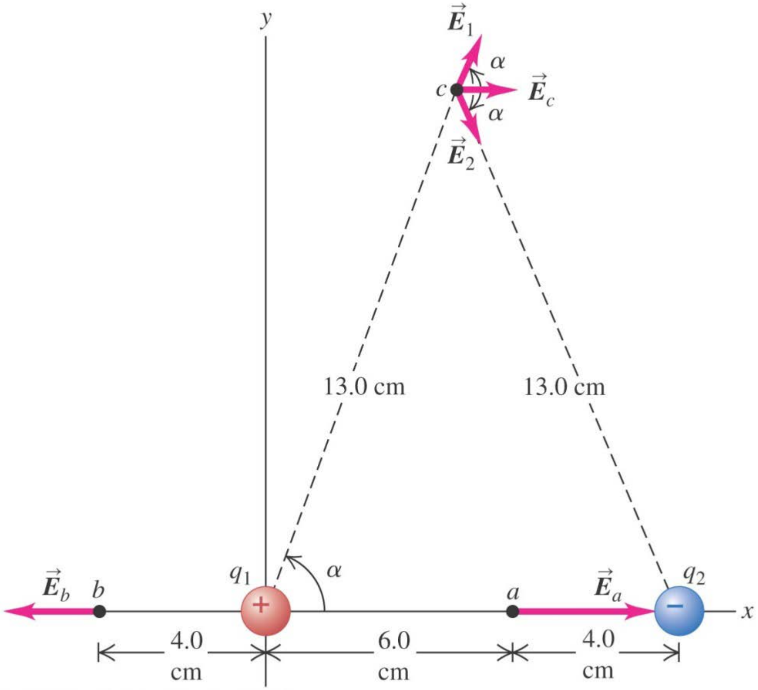
$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{12 \times 10^{-9}\,\mathrm{C}}{(0.130\,\mathrm{m})^2}$$
$$= 6.39 \times 10^3\,\mathrm{N/C}$$

$$E_{1x} = E_{2x} = E_1\cos\alpha = (6.39 \times 10^3\,\mathrm{N/C})\left(\frac{5}{13}\right)$$
$$= 2.46 \times 10^3\,\mathrm{N/C}$$

$$(E_c)_x = E_{1x} + E_{2x} = 2(2.46 \times 10^3\,\mathrm{N/C}) = 4.9 \times 10^3\,\mathrm{N/C}$$
$$(E_c)_y = E_{1y} + E_{2y} = 0$$

$$\rightarrow \quad \vec{E}_c = (4.9 \times 10^3\,\mathrm{N/C})\hat{\imath}$$

Alternatively for point c:



$$\vec{r}_1 = r\cos\alpha\hat{\imath} + r\sin\alpha\hat{\jmath}$$

$$\hat{r}_1 = \frac{\vec{r}_1}{r} = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$$

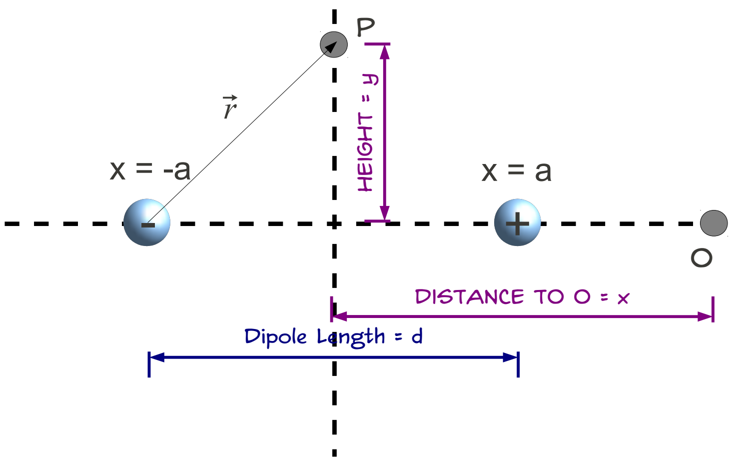
$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}\hat{r}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath})$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}\hat{r}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2}(-\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath})$$

$$\begin{aligned}
\vec{E}_c &= \vec{E}_1 + \vec{E}_2 \\
&= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(\cos\alpha\hat{\imath} + \text{in}\alpha\hat{\jmath}) + \frac{1}{4\pi\epsilon_0}\frac{(-q_1)}{r^2}(-\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}) \\
&= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r^2}(2\cos\alpha\hat{\imath}) \\
&= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12 \times 10^{-9}\ \text{C}}{(0.13\ \text{m})^2}\left[2\left(\frac{5}{13}\right)\right]\hat{\imath} \\
&= (4.9 \times 10^3\ \text{N/C})\hat{\imath}
\end{aligned}$$

➔ $\vec{E}_c = (4.9 \times 10^3\ \text{N/C})\hat{\imath}$

Alternative approach: Dipole Electric Field



Calculate the electric field of the NEGATIVE charge at point P
Calculate the electric field of the POSITIVE charge at point P
Calculate the electric field of the POSITIVE charge at point O
Calculate the electric field of the NEGATIVE charge at point O

On the perpendicular bisector:

Electric field from negative charge (left):

$$\vec{E}_- = k\frac{-q}{(y^2+a^2)}\frac{(a\hat{i}+y\hat{j})}{\sqrt{y^2+a^2}} = -\frac{kq(a\hat{i}+y\hat{j})}{(y^2+a^2)^{3/2}}$$

$$\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$$

Electric field from positive charge (right):

$$\vec{E}_+ = k\frac{q}{(y^2+a^2)}\frac{(-a\hat{i}+y\hat{j})}{\sqrt{y^2+a^2}} = -\frac{kq(a\hat{i}-y\hat{j})}{(y^2+a^2)^{3/2}}$$

$$\vec{E} = \vec{E}_- + \vec{E}_+ = -\frac{2kqa\hat{i}}{(y^2+a^2)^{3/2}}$$

y>>a ➔ $\vec{E} = -\frac{2kqa\hat{i}}{y^3}$

On the axis of the dipole (axial field):

Electric field from negative charge (left):

$$\vec{E}_- = k\frac{-q}{(x+a)^2}\frac{(x+a)\hat{i}}{(x+a)} = -\frac{kq\hat{i}}{(x+a)^2}$$

Electric field from positive charge (right):

$$\vec{E}_+ = k\frac{q}{(x-a)^2}\frac{(x-a)\hat{i}}{(x-a)} = \frac{kq\hat{i}}{(x-a)^2}$$

$$\vec{E} = \vec{E}_- + \vec{E}_+ = \frac{4kqax\hat{i}}{(x+a)^2(x-a)^2}$$

x>>a ➔ $\vec{E} = \frac{4kqa\hat{i}}{x^3}$

Continuous Distributions of Charge

There is a smallest unit of charge, that carried by either of the electron or the proton. All matter is made from a large number of these particles. However, it is impractical to ask anyone to sum over 10^{23} individual point-charges to obtain the total force or field outside of the distribution.
Instead, we need to use calculus to describe such objects. In calculus, we treat the object as a sum over a large number of infinitesimal pieces, and then sum the pieces. For instance, if we have a large collection of charges, each producing a small piece *dE* of the total electric field *E* at some point P, then to obtain the total electric field at point P:

$$\vec{E} = \int d\vec{E} = \int \frac{kdq}{r^2}\hat{r}$$

Each charge element's field $d\vec{E} = (k\,dq/r^2)\hat{r}$ so the above simply performs a sum over those elements. The strategy for solving such problems is the same: identify the field point and the source charges - except this time the source is a continuous distribution of charge. The hard part is to find a way to express the unit vector *r* and distances *r* in terms of coordinates over which we can then integrate.

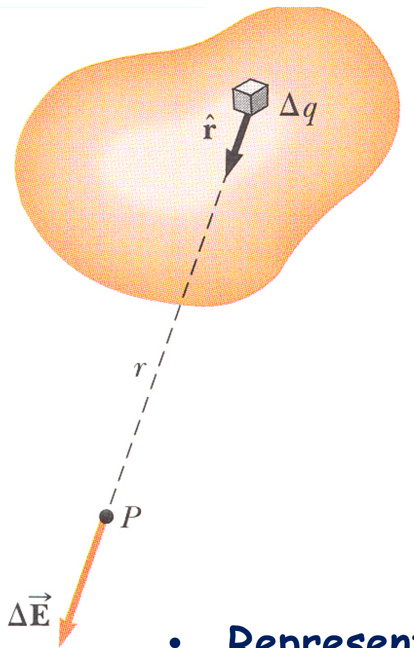
If uniform distribution:

Linear charge density: $\lambda = \Delta q/\Delta x = Q/L$

Surface charge density: $\sigma = \Delta q/\Delta A = Q/A$

Volume charge density: $\rho = \Delta q/\Delta V = Q/V$

Method for finding the electric field at point P (given a known *continuous* charge distribution)



$$\vec{E}_P = \frac{1}{4\pi\varepsilon_0} \lim_{\Delta q \to 0} \sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i \Rightarrow \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2}\hat{r}$$

- Find an expression for *dq*, the "point charge" within a differentially "small" chunk of the distribution

$$dq = \begin{cases} \lambda dl & \text{for a linear distribution} \\ \sigma dA & \text{for a surface distribution} \\ \rho dV & \text{for a volume distribution} \end{cases}$$

- Represent field contributions at P due to a point charge *dq* located anyhwere in the distribution. Use symmetry where possible.

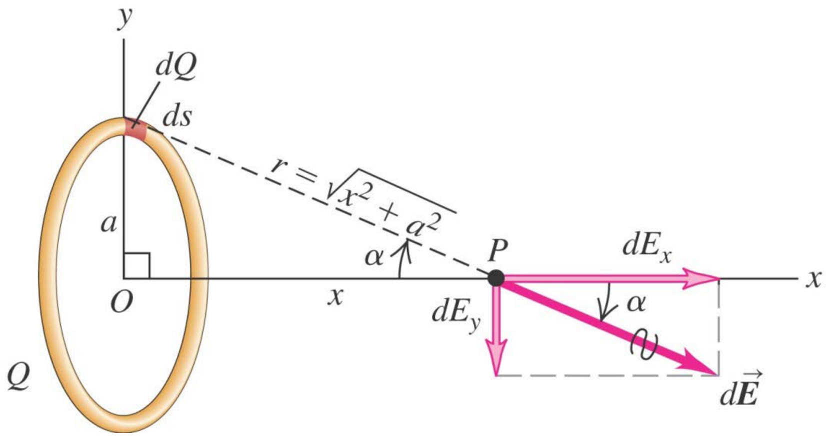
$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

Add up (integrate) the contributions dE over the whole distribution, varying the displacement and direction as needed.
Use symmetry where possible.

$$\vec{E}_P = \int_{\text{dist}} d\vec{E} \qquad \text{(line, surface, or volume integral)}$$

Field of a ring of charge

Follow Example 21.10: A ring-shaped conductor with radius a carries a total positive charge Q uniformly distributed around it. Find the electric field at a point p that lies on the axis of the ring at a distance x from its center.



The magnitude of the Electric field from a ring segment at point P

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2 + a^2}$$

$$\cos\alpha = x/r = x/(x^2 + a^2)^{1/2},$$

$$\rightarrow \quad dE_x = dE\cos\alpha = \frac{1}{4\pi\epsilon_0}\frac{dQ}{x^2 + a^2}\frac{x}{\sqrt{x^2 + a^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{x\,dQ}{(x^2 + a^2)^{3/2}}$$

Total x-component of the Electric field from the ring at point P

$$E_x = \int \frac{1}{4\pi\epsilon_0}\frac{x\,dQ}{(x^2 + a^2)^{3/2}}$$

*due to the symmetry, y-component cancel out.

Since x is constant, we can it out of Integral and the Integral depending only to Q

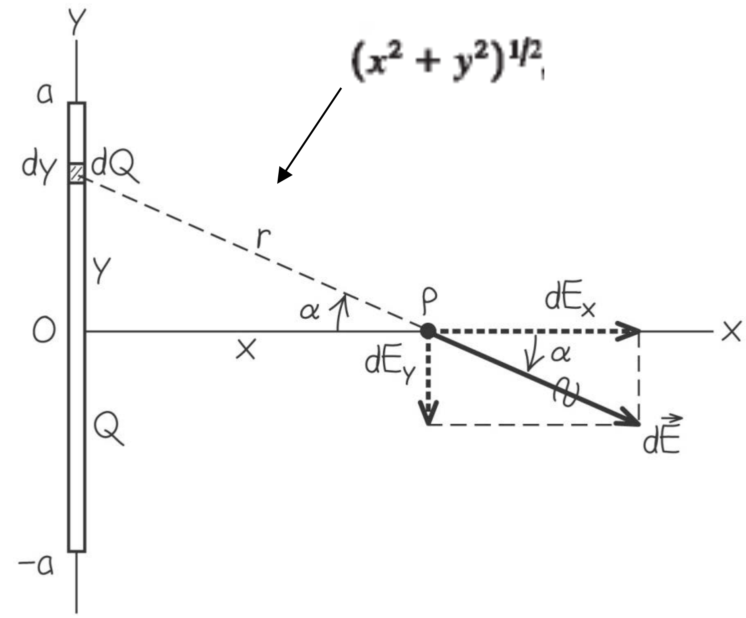
$$\rightarrow \quad \vec{E} = E_x\hat{\imath} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2 + a^2)^{3/2}}\hat{\imath}$$

*in the limit x>>a $\rightarrow$ $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x^2}\hat{\imath}$

The field is the same as if from a point charge!

Field of a charged line segment

Follow Example 21.11; Positive electric charge Q is distributed uniformly along a line with length 2a, lying along the y-axis between y = -a and y = +a. Find the electric field at point P on the x-axis at a distance x from the origin.



We divide the line charge into infinitesimal segments, each of which acts as a point charge; let the length of a typical segment at height *y* be *dy*. If the charge is distributed uniformly, the linear charge density *A* at any point on the line is $\lambda = Q/2a$ (the total charge divided by the total length). Hence the charge *dQ* in a segment of length *dy* is

*due to the symmetry, y-component cancel out.

$$dQ = \lambda dy = \frac{Q dy}{2a}$$ → $$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{Q}{4\pi\epsilon_0}\frac{dy}{2a(x^2+y^2)}$$ $$dE_x = dE\cos\alpha \qquad dE_y = -dE\sin\alpha$$

$$dE_x = \frac{Q}{4\pi\epsilon_0}\frac{x\,dy}{2a(x^2+y^2)^{3/2}}$$ $$E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{2a}\int_{-a}^{a}\frac{dy}{(x^2+y^2)^{3/2}} = \frac{Q}{4\pi\epsilon_0}\frac{1}{x\sqrt{x^2+a^2}}$$

This is a difficult one.. (look at the integral table)

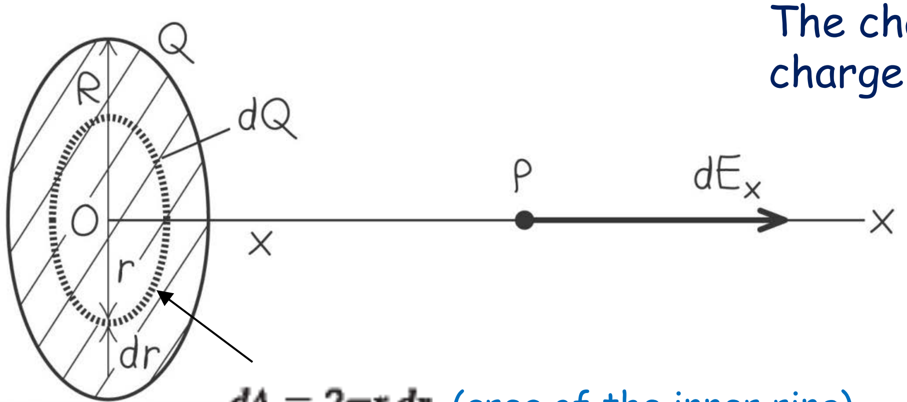
→ $$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x^2/a^2)+1}}\hat{\imath}$$

→ $$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{\imath}$$

At the limit a>>x (infinite line) → $$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 x}\hat{\imath}$$ or $$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Field of a uniformly charged disk

Example 21.12: Find the electric field caused by a disk of radius R with a uniform positive surface charge density (charge per unit area) σ, at a point along the axis of the disk a distance x from its center. Assume that x is positive.



$dA = 2\pi r\,dr$ (area of the inner ring)

The charge per unit area is $\sigma = dQ/dA$, so the charge of the ring is $dQ = \sigma\, dA = \sigma\,(2\pi r dr)$

$$dQ = 2\pi\sigma r\,dr$$

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(2\pi\sigma r\,dr)x}{(x^2+r^2)^{3/2}}$$

To find the total field due to all the rings. we integrate dE_x over r from r = 0 to r = R (not from -R to R):

$$E_x = \int_0^R \frac{1}{4\pi\epsilon_0}\frac{(2\pi\sigma r\,dr)x}{(x^2+r^2)^{3/2}} = \frac{\sigma x}{2\epsilon_0}\int_0^R \frac{r\,dr}{(x^2+r^2)^{3/2}}$$

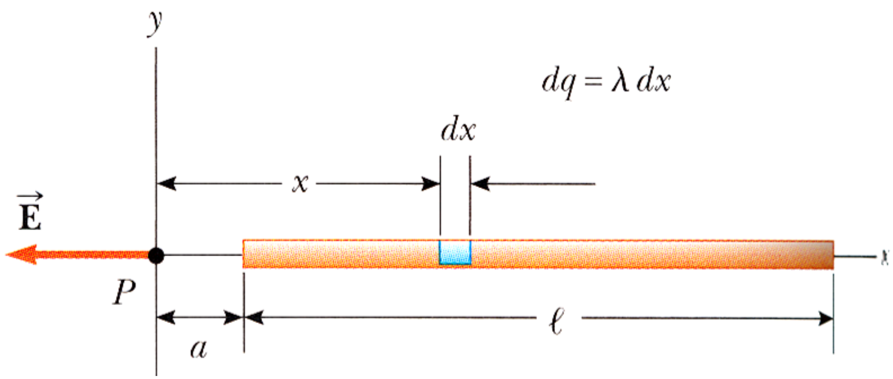
(This is an easy integral, solved by variable changing)

➔ $$E_x = \frac{\sigma x}{2\epsilon_0}\left[-\frac{1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right] = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right]$$

At the limit $R \rightarrow \infty$ (infinite disk)

➔ $$E = \frac{\sigma}{2\epsilon_0}$$

Example: Find electric field on the axis of a charged rod



Rod has length L, uniform positive charge per unit length λ, total charge Q. $\lambda = Q/L$.
Calculate electric field at point P on the axis of the rod a distance a from one end. Field points along x-axis.

$$dq = \lambda dx \qquad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{x^2}$$

Add up contributions to the field from all locations of *dq* along the rod ($x \in [a, L + a]$).

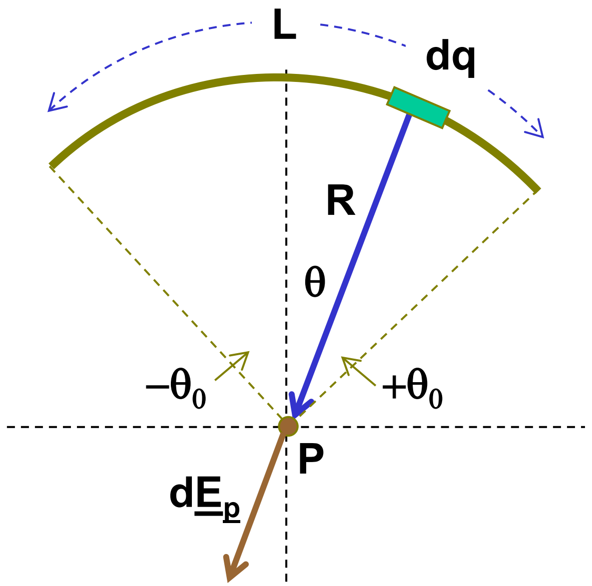
$$E = \int_a^{L+a} \frac{\lambda}{4\pi\varepsilon_0}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\int_a^{L+a}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_a^{L+a} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{L}\left(\frac{1}{a} - \frac{1}{L+a}\right)$$

$$\rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 a(L+a)}$$

Interpret Limiting cases:

- L => 0 rod becomes point charge
- L << a; same, L/a <<1
- L >>a; same, a/L<<1,

Example: Electric field at center of an ARC of charge



Uniform linear charge density $\lambda = Q/L$

➔ $dq = \lambda ds = \lambda R d\theta$

P on symmetry axis at center of arc

➔ Net E is along y axis ➔ need E_y only

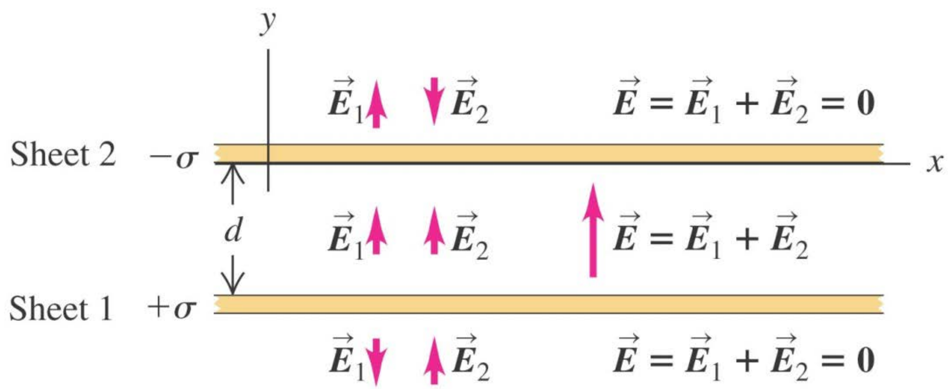
$$\rightarrow \quad \vec{E}_{P,y} = \frac{-1}{4\pi\varepsilon_0}\hat{j}\frac{\lambda}{R^2}R\int_{-\theta_0}^{+\theta_0}\cos(\theta)d\theta = -\frac{\lambda}{4\pi\varepsilon_0}\hat{j}\frac{1}{R}\sin(\theta)\Big|_{-\theta_0}^{+\theta_0}$$

$$\int \cos(\theta)d\theta = \sin(\theta) \quad \text{and} \quad \sin(-\theta) = -\sin(\theta)$$

$$\rightarrow \quad \vec{E}_{P,y} = -2k\frac{\lambda}{R}\sin(\theta_0)\,\hat{j}$$

Field of two oppositely charged infinite sheets

Follow Example 21.13; Two infinite plane sheets are placed parallel to each other, separated by a distance d.The lower sheet has a uniform positive surface charge density σ, and the upper sheet has a uniform negative surface charge density - σ with the same magnitude. Find the electric field between the two sheets, above the upper sheet, and below the lower sheet.

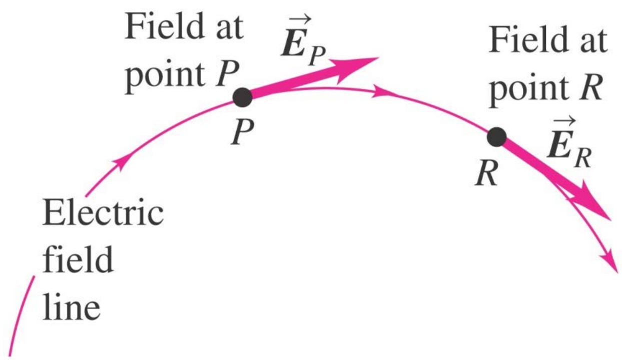


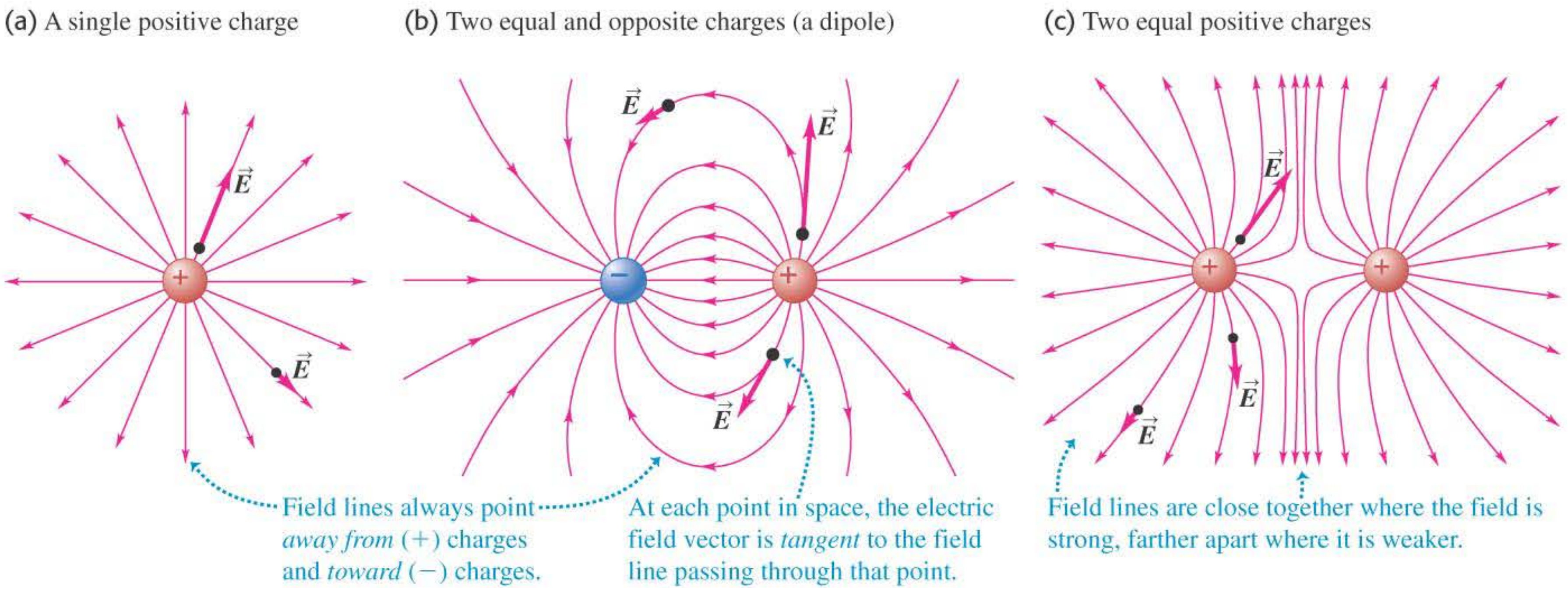
$$E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \begin{cases} 0 & \text{above the upper sheet} \\ \dfrac{\sigma}{\epsilon_0}\hat{j} & \text{between the sheets} \\ 0 & \text{below the lower sheet} \end{cases}$$

Electric field lines

An *electric field line* is an imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point.

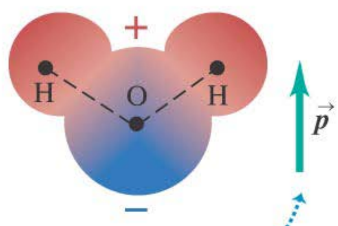




Electric dipoles

- An *electric dipole* is a pair of point charges having equal but opposite sign and separated by a distance.

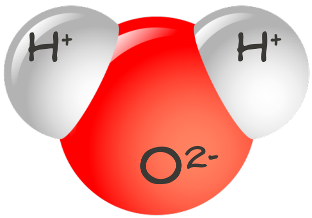
(a) A water molecule, showing positive charge as red and negative charge as blue

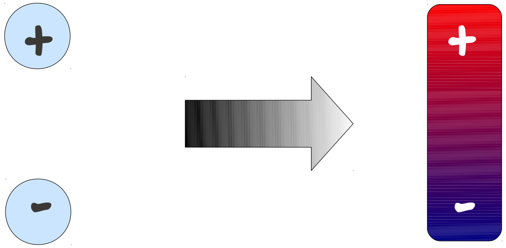


The electric dipole moment $\vec{p}$ is directed from the negative end to the positive end of the molecule.

(b) Various substances dissolved in water

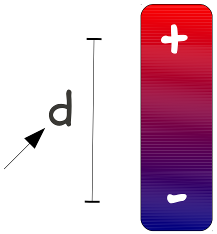
water molecule





Terminology:

$+q$



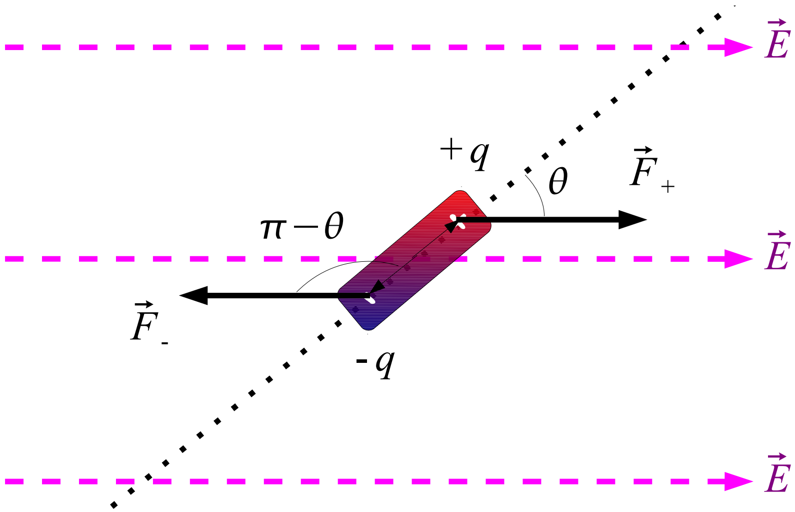
Separation of charges

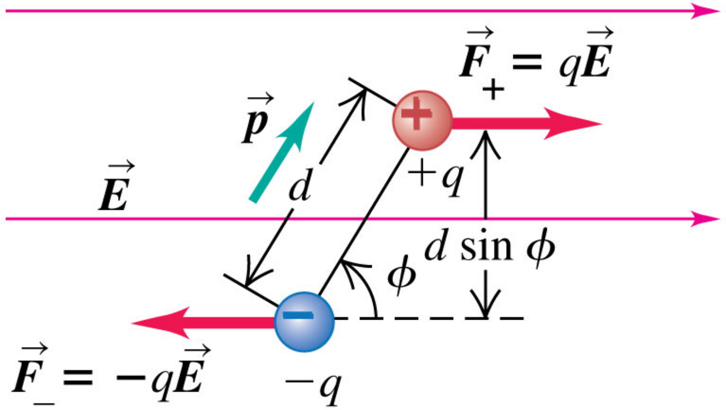
$-q$

"Dipole Moment"

$$\vec{p} = (qd)\,\hat{d}$$

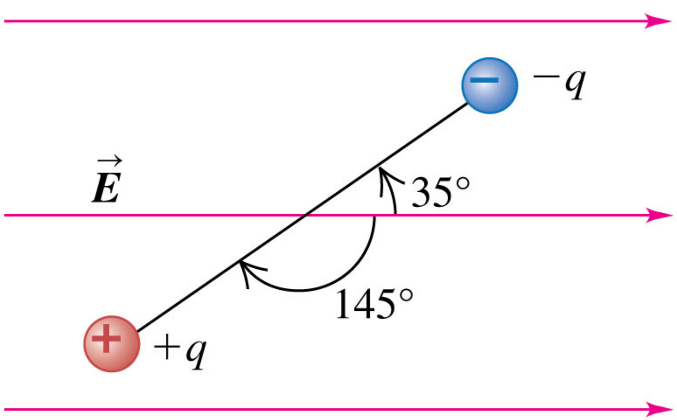
Dipole in electric field: Force and torque on a dipole

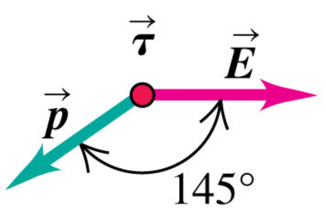




The magnitude of the net torque is twice of the magnitude of either individual torque:

$$\tau = (qE)(d\sin\phi)$$





$p = qd$ (magnitude of electric dipole moment)

$\tau = pE\sin\phi$ (magnitude of the torque on an electric dipole)

$\vec{\tau} = \vec{p} \times \vec{E}$ (torque on an electric dipole, in vector form)

Potential Energy of an Electric Dipole

When a dipole changes direction in an electric field, the electric-field torque does work on it with a corresponding change in potential energy. The work *dW* done by a torque *T* during an infinitesimal displacement $d\phi$ is $dW = T\, d\phi$.

Because the torque is in the direction of decreasing ϕ we must write the torque as

$$\tau = -pE\sin\phi$$

$$\rightarrow \quad dW = \tau\, d\phi = -pE\sin\phi\, d\phi$$

In a finite displacement from ϕ_1 to ϕ_2 the total work done on the dipole is:

$$W = \int_{\phi_1}^{\phi_2} (-pE\sin\phi) = pE\cos\phi_2 - pE\cos\phi_1$$

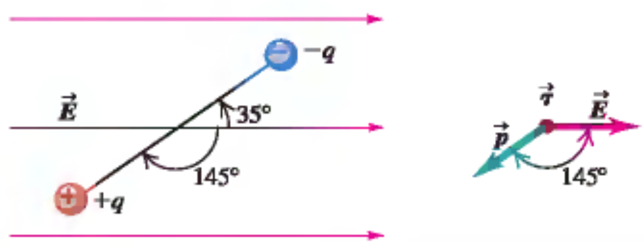
$$W = U_1 - U_2 \quad \rightarrow \quad U(\phi) = -pE\cos\phi$$

$$U = -\vec{p}\cdot\vec{E} \qquad \text{(potential energy for a dipole in an electric field)}$$

The potential energy has its minimum value $U = -pE$ at equilibrium position, where $\phi = 0$

Example 21.14 :an electric dipole in a uniform electric field with magnitude 5.0 x 10^5 N/C directed parallel to the plane of the figure. The charges are 1.6 X 10^{-19} C; both lie in the plane and are separated by 0.125 nm = 0.125 x 10^{-9} m. Find

(a) the net force exerted by the field on the dipole;
(b) the magnitude and direction of the electric dipole moment;
(c) The magnitude and direction of the torque;
(d) the potential energy of the system in the position shown.



(a) Since the field is uniform, the forces on the two charges are equal and opposite. and the total force is zero.

(b) The magnitude *p* of the electric dipole moment. The direction of *p* is from the negative to the positive charge, 145° clockwise from the electric-field direction

$$p = qd = (1.6 \times 10^{-19}\,\mathrm{C})(0.125 \times 10^{-9}\,\mathrm{m})$$
$$= 2.0 \times 10^{-29}\,\mathrm{C \cdot m}$$

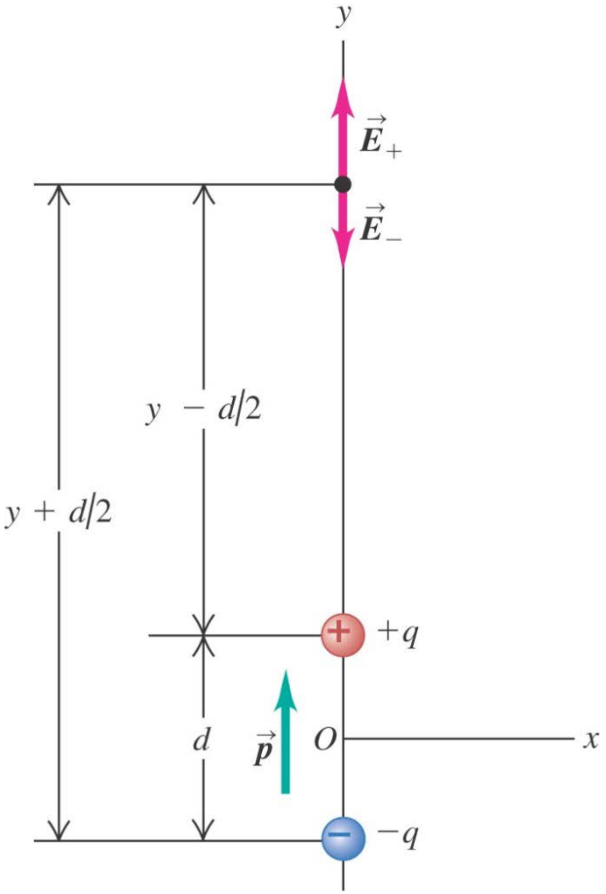
(c) The magnitude *of the torque*
And the direction is out of the page (right hand rule)

$$\tau = pE\sin\phi = (2.0 \times 10^{-29}\,\mathrm{C})(5.0 \times 10^{5}\,\mathrm{N/C})(\sin 145°)$$
$$= 5.7 \times 10^{-24}\,\mathrm{N \cdot m}$$

(d) The potential energy

$$U = -pE\cos\phi = -(2.0 \times 10^{-29}\,\mathrm{C \cdot m})(5.0 \times 10^{5}\,\mathrm{N/C})(\cos 145°)$$
$$= 8.2 \times 10^{-24}\,\mathrm{J}$$

Example 21.15 : find electric field at y-axis



Total E_y at y-axis

$$E_y = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0 y^2}\left[\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right]$$

$$\left(1-\frac{d}{2y}\right)^{-2} \cong 1+\frac{d}{y} \quad \text{and} \quad \left(1+\frac{d}{2y}\right)^{-2} \cong 1-\frac{d}{y}$$

$$\rightarrow \quad E \cong \frac{q}{4\pi\epsilon_0 y^2}\left[1+\frac{d}{y} - \left(1-\frac{d}{y}\right)\right] = \frac{qd}{2\pi\epsilon_0 y^3} = \frac{p}{2\pi\epsilon_0 y^3}$$