Chp 30: Inductance - (III)

Energy in an R-L circuit

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$$i = I_0 e^{-(R/L)t} \qquad U = \tfrac{1}{2}Li^2 \quad \Rightarrow \quad U = \tfrac{1}{2}L{I_0}^2 e^{-2(R/L)t} = U_0 e^{-2(R/L)t}$$

$U_0 = \frac{1}{2}L{I_0}^2$ is the energy at the initial time $t = 0$. When $t = 2.3\tau = 2.3L/R$

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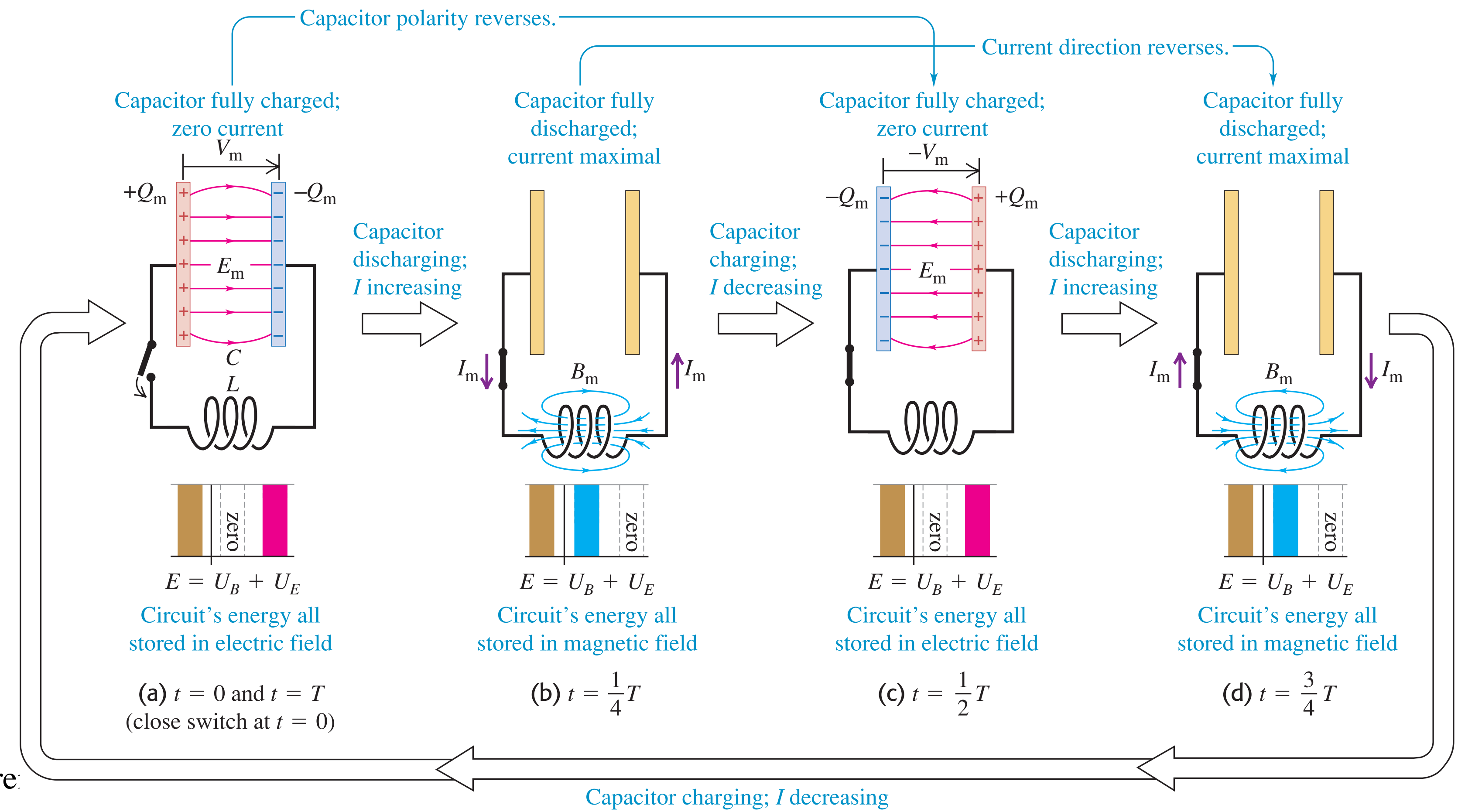
$U_0 = \frac{1}{2}LI_0^2$ is the energy at the initial time $t = 0$. When $t = 2.3\tau = 2.3L/R$

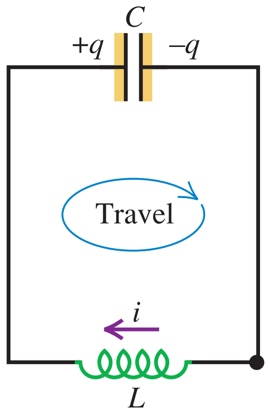
$$\Rightarrow \quad U = U_0 e^{-2(2.3)} = U_0 e^{-4.6} = 0.010U_0$$

That is, only 0.010 or 1.0% of the energy initially stored in the inductor remains, so 99.0% has been dissipated in the resistor.

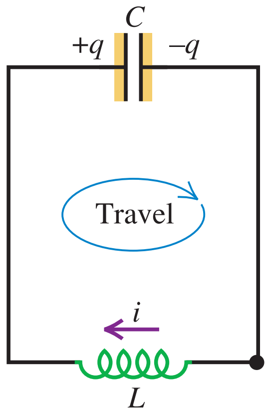
The L-C Circuit

A circuit containing an inductor and a capacitor shows an entirely new mode of behavior, characterized by *oscillating* current and charge. This is in sharp contrast to the *exponential* approach to a steady-state situation that we have seen with both *R-C* and *R-L* circuits. In the ***L-C* circuit** we charge the capacitor to a potential difference V_m and initial charge $Q = CV_m$ on its left-hand plate and then close the switch. The capacitor begins to discharge through the inductor. Because of the induced emf in the inductor, the current cannot change instantaneously it starts at zero and eventually builds up to a maximum value I_m. During this buildup the capacitor is discharging ➔ ***electrical oscilation***

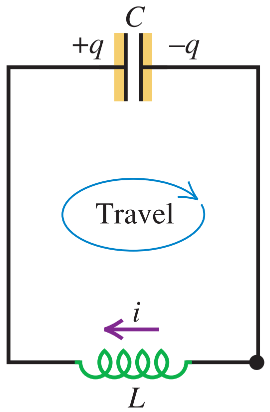




$$-L\frac{di}{dt} - \frac{q}{C} = 0$$



$$-L\frac{di}{dt} - \frac{q}{C} = 0 \quad \Rightarrow \quad \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$



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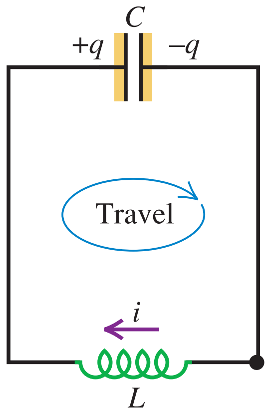
Similar to the equation we derived for simple harmonic motion

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$



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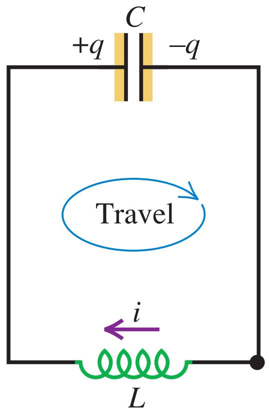
$$x = A\cos(\omega t + \phi) \quad \Leftrightarrow \quad q = Q\cos(\omega t + \phi)$$

$$\boxed{\omega = \sqrt{\frac{1}{LC}}}$$

angular frequency of oscillation in an L-C circuit



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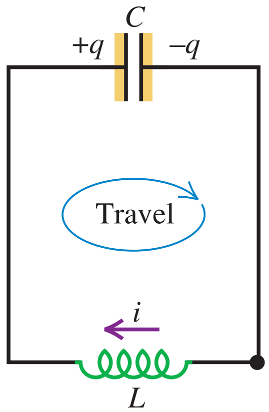

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Mass-Spring System	**Inductor-Capacitor Circuit**
Kinetic energy $= \frac{1}{2}m{v_x}^2$	Magnetic energy $= \frac{1}{2}Li^2$
Potential energy $= \frac{1}{2}kx^2$	Electric energy $= q^2/2C$
$\frac{1}{2}m{v_x}^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$	$\frac{1}{2}Li^2 + q^2/2C = Q^2/2C$
$v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$	$i = \pm\sqrt{1/LC}\sqrt{Q^2 - q^2}$
$v_x = dx/dt$	$i = dq/dt$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{1}{LC}}$
$x = A\cos(\omega t + \phi)$	$q = Q\cos(\omega t + \phi)$

An oscillating circuit

A 300-V dc power supply is used to charge a 25-μF capacitor. After the capacitor is fully charged, it is disconnected from the power supply and connected across a 10-mH inductor. The resistance in the circuit is negligible. Find

(a) the frequency and period of oscillation of the circuit.

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We must calculate the magnetic energy U_B (stored in the inductor) and the electric energy U_E (stored in the capacitor) at two times during the *L*-*C* circuit oscillation.

$$U_B = \tfrac{1}{2}Li^2 \qquad U_E = q^2/2C$$

(a) At $t = 0 \Rightarrow i = 0$ $\quad U_B = \tfrac{1}{2}Li^2 = 0 \quad U_E = \dfrac{Q^2}{2C} = \dfrac{(7.5 \times 10^{-3}\ \mathrm{C})^2}{2(25 \times 10^{-6}\ \mathrm{F})} = 1.1\ \mathrm{J}$

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(a) At $t = 1.2\ ms \rightarrow$ we have found $i = 10\ A$ and $q = -5.5\mathrm{x}10^{-3}C$

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(10 \times 10^{-3}\ \mathrm{H})(-10\ \mathrm{A})^2 = 0.5\ \mathrm{J}$$

$$\Rightarrow \quad U_E = \frac{q^2}{2C} = \frac{(-5.5 \times 10^{-3}\ \mathrm{C})^2}{2(25 \times 10^{-6}\ \mathrm{F})} = 0.6\ \mathrm{J}$$

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(a) At *t* = *1.2 ms* ➔ we have found *i* = 10 *A* and *q* = -5.5x10^{-3}*C*

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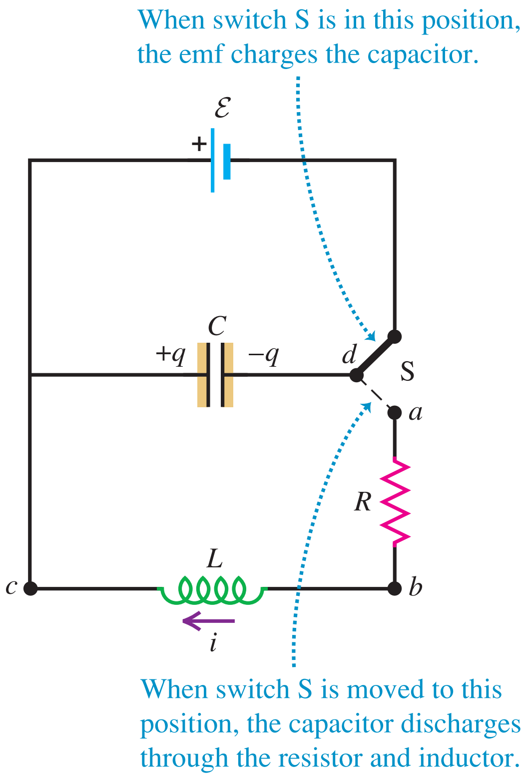
The magnetic and electric energies are the same at $t = 3T/8 = 0.375T$.
We saw that the time considered in part (b), $t = 1.2$ ms, equals $0.38T$; this is slightly later than $0.375T$, so U_B is slightly less than U_E.
At *all* times the *total* energy $E = U_B + U_E$ has the same value, 1.1 J.
An *L-C* circuit without resistance is a conservative system; no energy is dissipated.

The *L-R-C* series circuit



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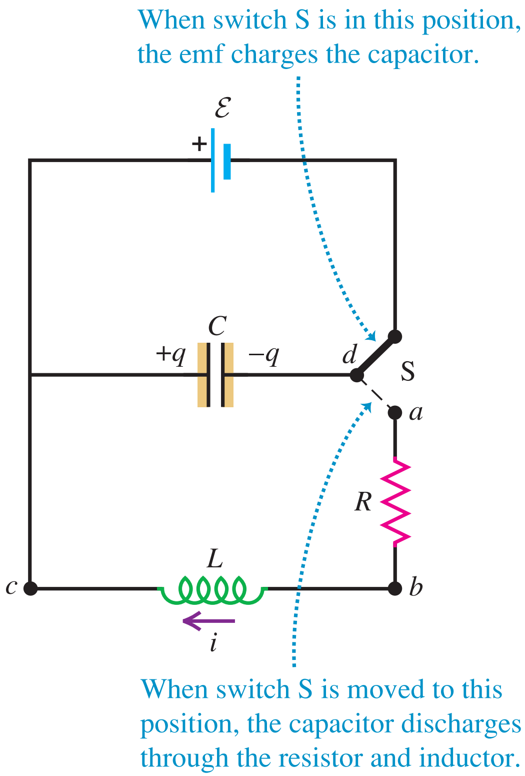

As before, the capacitor starts to discharge as soon as the circuit is completed. But because of I^2R losses in the resistor, the magnetic-field energy acquired by the inductor when the capacitor is completely discharged is *less* than the original electric-field energy of the capacitor. In the same way, the energy of the capacitor when the magnetic field has decreased to zero is still smaller, and so on.

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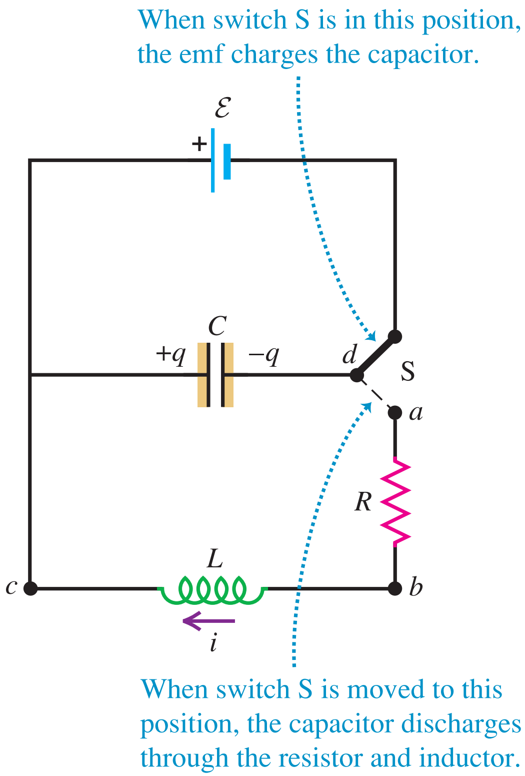

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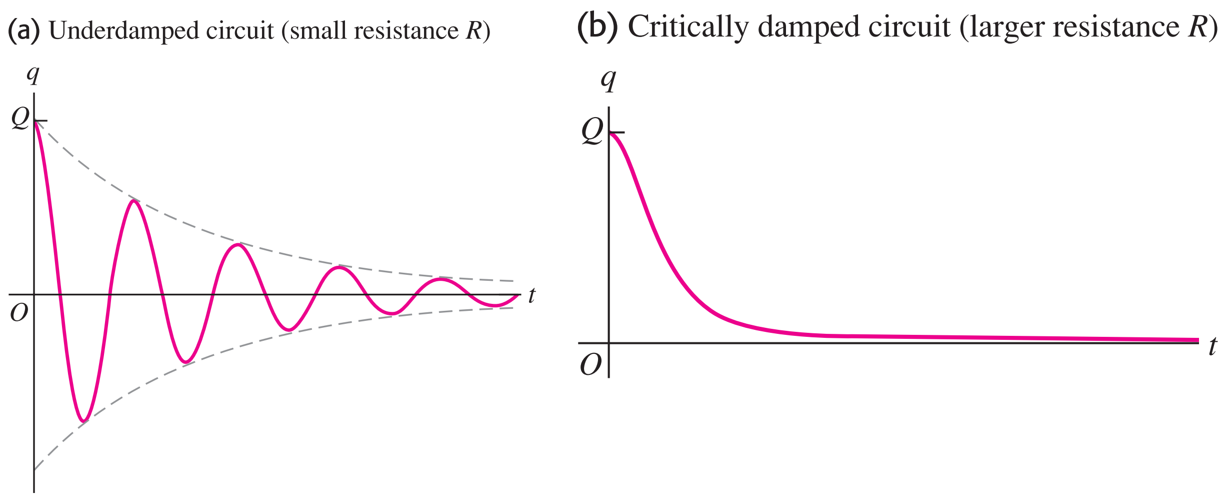

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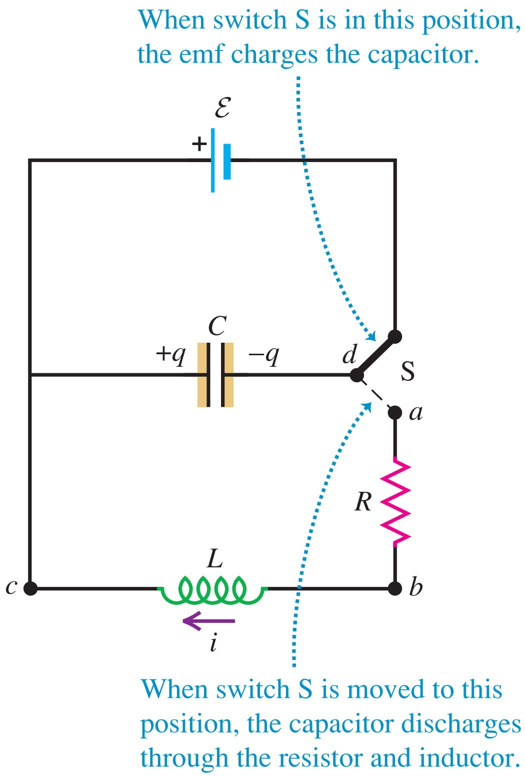

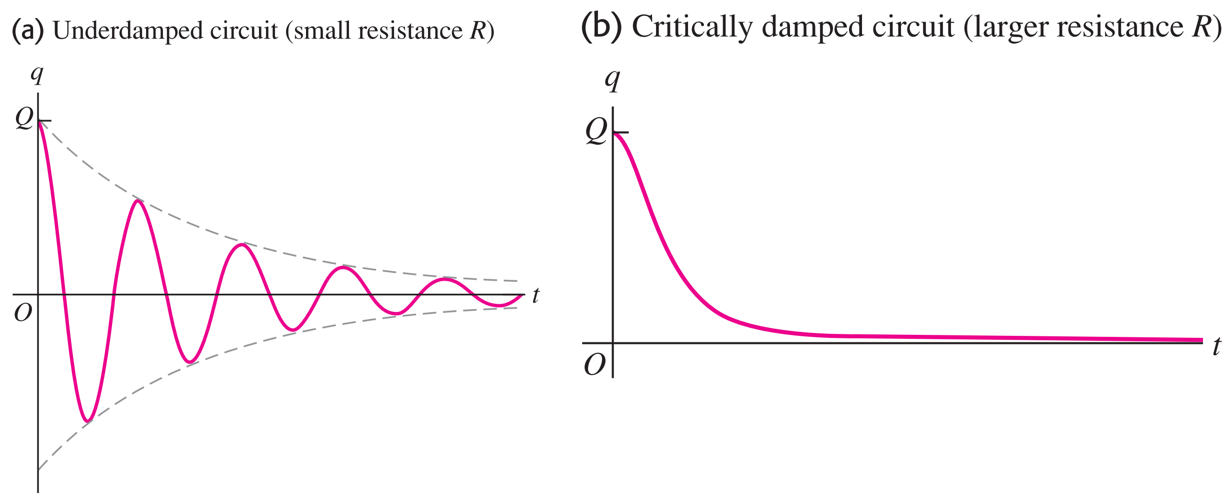

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$$q = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right)$$

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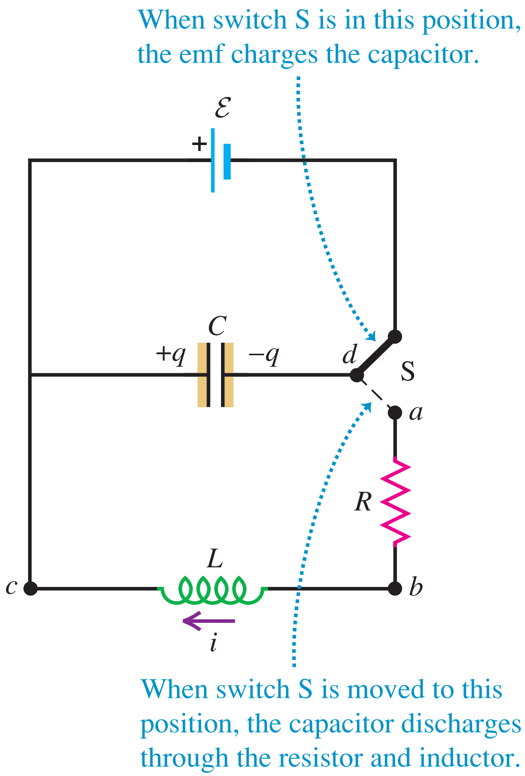

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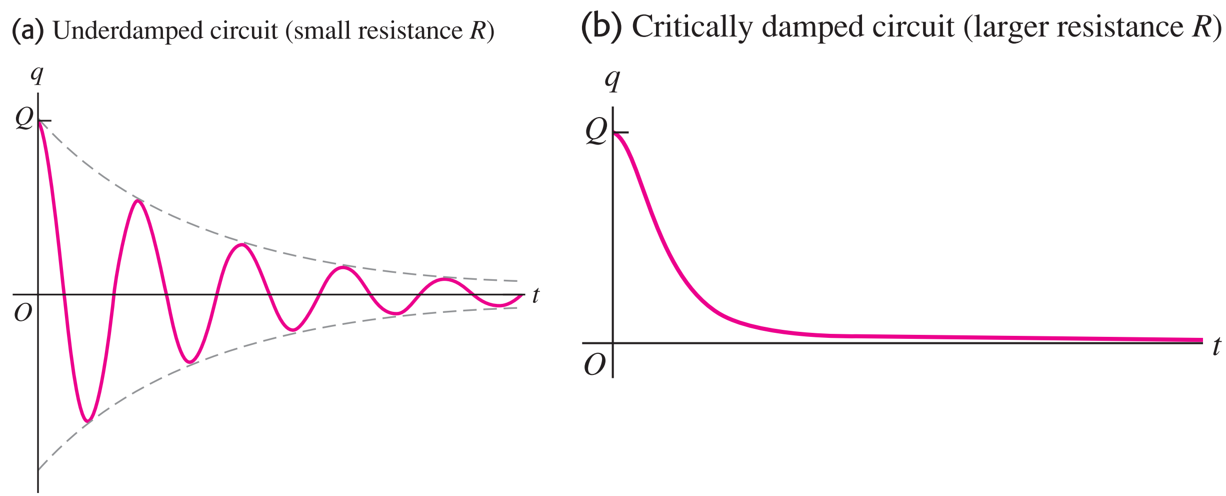

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$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

$$q = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \phi\right)$$

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad \text{(underdamped } L\text{-}R\text{-}C \text{ series circuit)}$$