Chp 23: Electric Potential

Goals for Chapter 23

- To calculate the electric potential energy of a group of charges

- To know the significance of electric potential

- To calculate the electric potential due to a collection of charges

- To use equipotential surfaces to understand electric potential

- To calculate the electric field using the electric potential

In a gravitational field, **different altitudes** have different amounts of **gravitational potential**.

Similarly, an **imbalance of charge** between different places in an electric field gives these places **different amounts of electric potential**.

A difference in altitude causes **a current of water** to flow.

A difference in electric potential causes **a current of electricity** to flow.

A **voltage** (potential difference) applied across the two ends in a conductor will **cause a current to flow through it**.

Typical conductors offer some **resistance** to this flow of current.

If the **resistance remains constant**, then the **current remains proportional** to the applied **voltage**.

6TH CENTURY BCE

The ancient Greeks give **amber** an **electric charge** by rubbing it with rabbit fur, using its attraction to move light objects.

2ND CENTURY BCE

Chinese scholars use shards of **magnetic lodestone** as simple direction-finders.

1600

English physician and physicist William Gilbert publishes *De Magnete* (*On the Magnet*), the first systematic work on **electricity** and **magnetism** since antiquity. He coins the new Latin word *electrica*, from the Greek for amber (*elektron*).

1745

German cleric Ewald Georg von Kleist and Dutch scientist Pieter van Musschenbroek invent the **Leyden jar** as a way of **storing electric charge**.

1747

Benjamin Franklin develops his **one-fluid theory** of electricity, in which he introduces the idea of **positive and negative charge**.

1785

Charles-Augustin de Coulomb discovers his law for determining the **attractive or repulsive force** between two electrically charged objects.

1800

Alessandro Volta demonstrates the first electric pile, or **battery**, which provides continuous **electric current** for the first time.

1820

Danish physicist Hans Christian Ørsted discovers that a wire carrying an electric current produces a **magnetic field**.

1825

French physicist André-Marie Ampère provides a mathematical derivation of the **magnetic force** between two parallel wires carrying an electric current.

1827

German physicist Georg Ohm publishes his **law** establishing the relationship between **current**, **voltage**, and **resistance**.

1831

Michael Faraday generates an **electric current** from a changing magnetic field to discover **induction**.

1865

James Clerk Maxwell combines all knowledge of electricity and magnetism in a few **equations**.

1882

American inventor Thomas Edison's first **electricity generating plant** starts producing in London.

1911

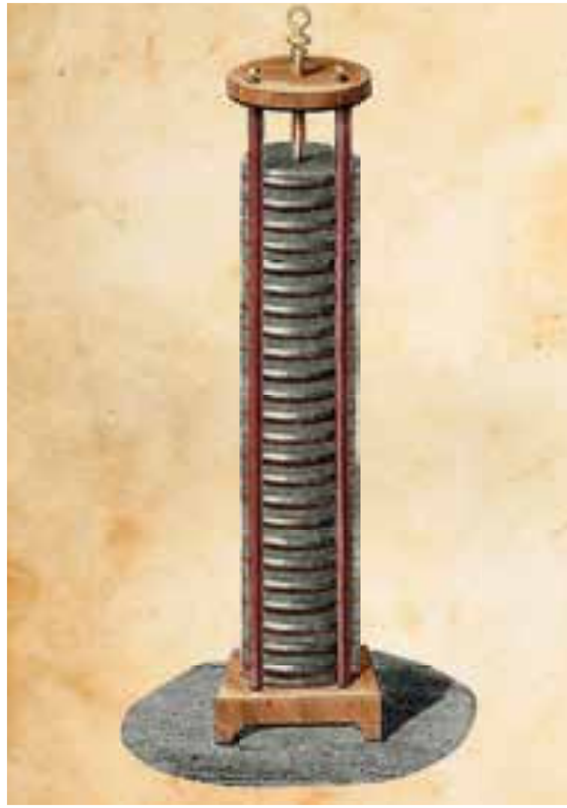
Dutch physicist Heike Kamerlingh Onnes discovers **superconductivity** in mercury chilled to near absolute zero.

1999

American chemist Chad Mirkin invents **nanolithography**, which "writes" nanocircuitry on **silicon wafers**.

In 1780, Italian physician Luigi Galvani, had noticed that when he touched a (dead) frog's leg with two dissimilar metals, or applied an electrical spark to it, the leg twitched. He presumed that the source of this motion was the frog's body and deduced it contained an electrical fluid. Volta performed similar experiments, but without animals, eventually coming up with the theory that the dissimilarity of the metals in the circuit was the source of the electricity.

Volta's simple electrochemical cell consists of two metal pieces (electrodes), separated by a salt solution (an electrolyte). Where each metal meets the electrolyte, a chemical reaction takes place, creating "charge carriers" called ions (atoms that have gained or lost electrons, and so are negatively or positively charged). Oppositely charged ions appear at the two electrodes. Because unlike charges attract each other, separating positive and negative charges requires energy (just as holding apart the opposite poles of two magnets does). This energy comes from chemical reactions in the cell. When the cell is connected to an external circuit, the energy that was "stored" in the potential difference appears as the electrical energy that drives the current around the circuit.



See the videos..

Alessandro Volta

Alessandro Volta was born into an aristocratic family in 1745 in Como, Italy. Volta was aged just seven when his father died. His relatives steered his education towards the Church, but he undertook his own studies in electricity and communicated his ideas to prominent scientists.

Following Volta's early publications on electricity, he was appointed to teach in Como in 1774. The following year, he developed the electrophorus (an instrument for generating electric charge), and in 1776 he discovered methane. Volta became professor of physics at Pavia in 1779. There he engaged in friendly rivalry with Luigi Galvani in Bologna. Volta's doubts about Galvani's ideas of "animal electricity" led to his invention of the voltaic pile. Honoured by both Napoleon and the emperor of Austria, Volta was a wealthy man in his later years, and died in 1827.

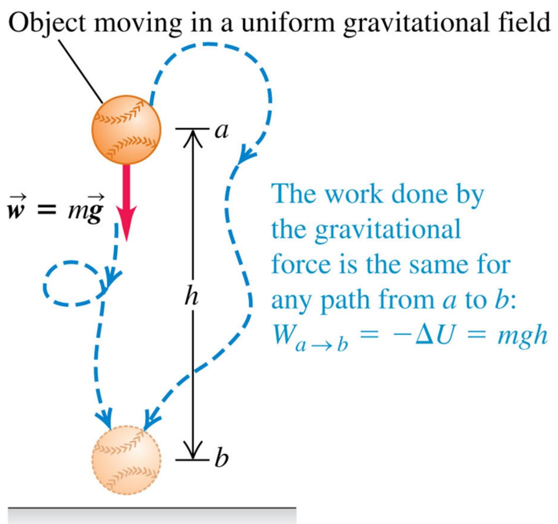
Electric Potential Energy

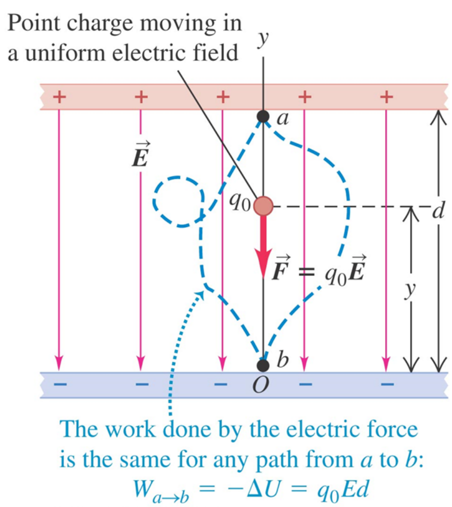
Let's begin by reviewing three essential points from FIZ 101 Chapters 6 and 7.
First, when a force F acts on a particle that moves from point a to point b, the work $W_{a \to b}$ done by the force is given by a line integral:

$$W_{a \to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F \cos\phi \, dl \qquad \text{(work done by a force)}$$

Second, if the force F is conservative, as we defined the term in Section 7.3, the work done by F can always be expressed in terms of a ***potential energy U***.
When the particle moves from a point where the potential energy is U_a to a point where it is U_b the change in potential energy is $U_b - U_a$ and the work $W_{a \to b}$ done by the force is

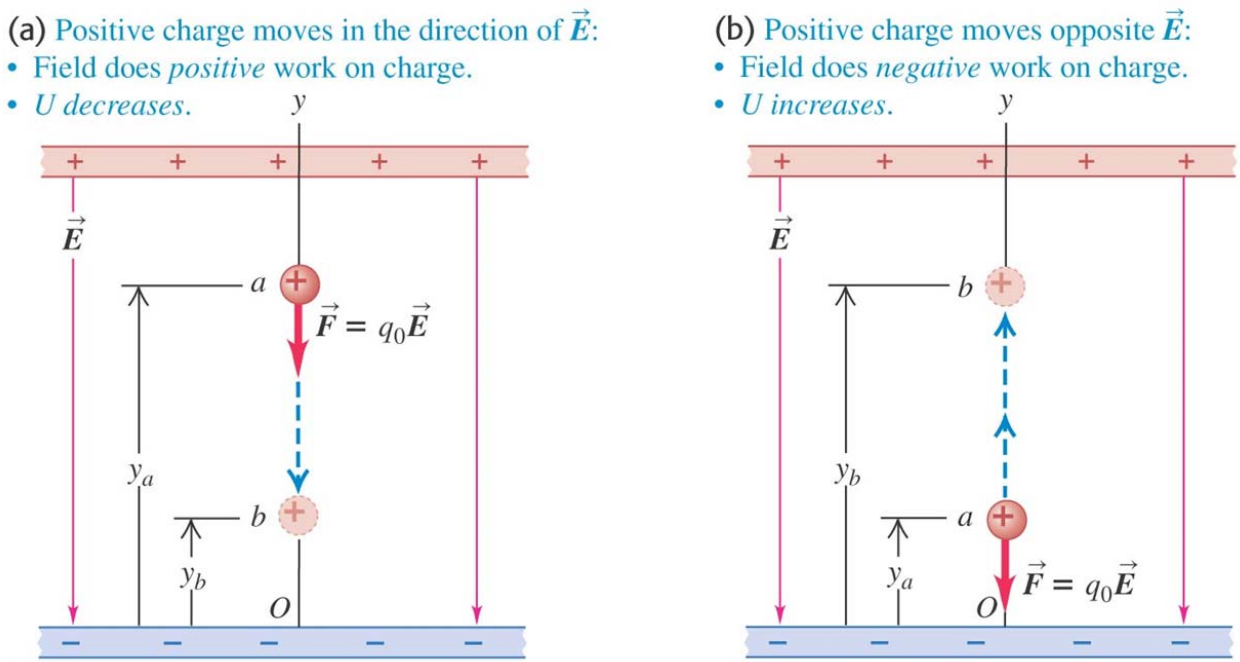
$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U \qquad \text{(work done by a conservative force)}$$





A positive charge moving in a uniform field

If the positive charge moves in the direction of the field, the potential energy *decreases*, but if the charge moves opposite the field, the potential energy *increases*.



the work done by the electric field is the product of the force magnitude and the component of displacement in the (downward) direction of the force:

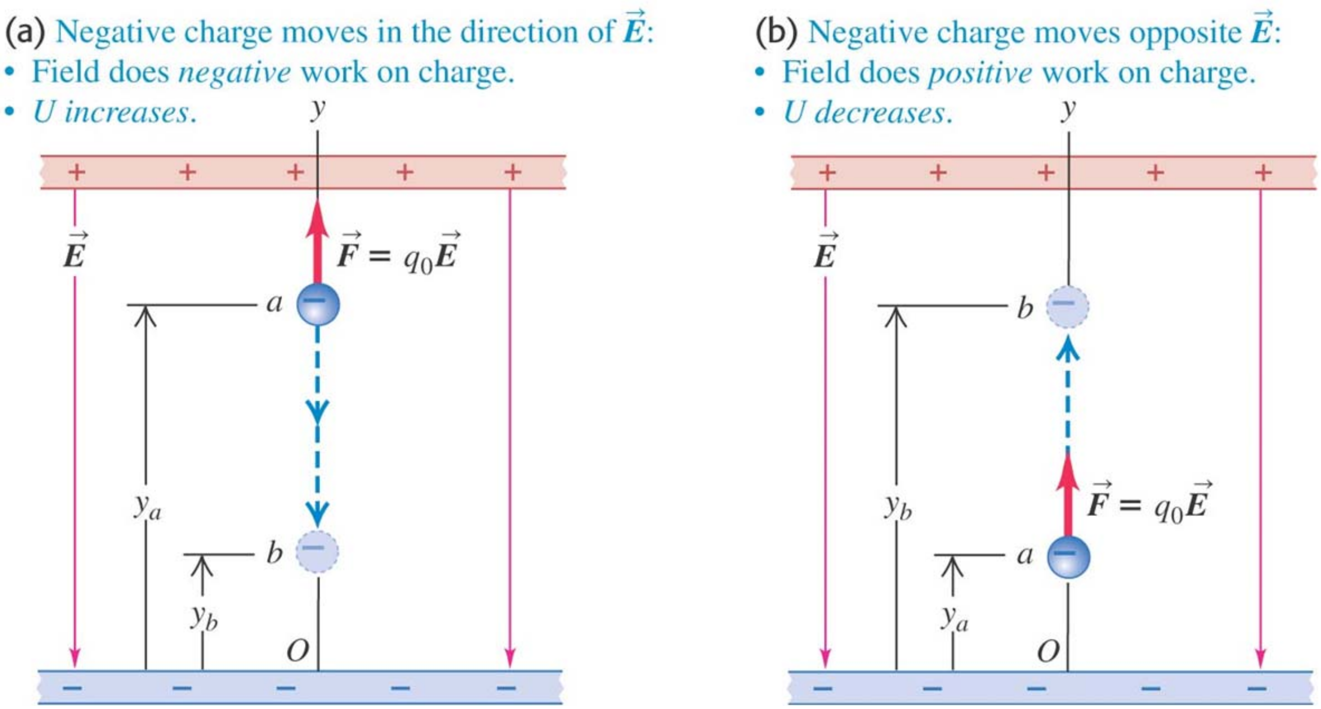
$$W_{a\to b} = Fd = q_0Ed$$

Therefore the potential energy is $U = q_0Ey$

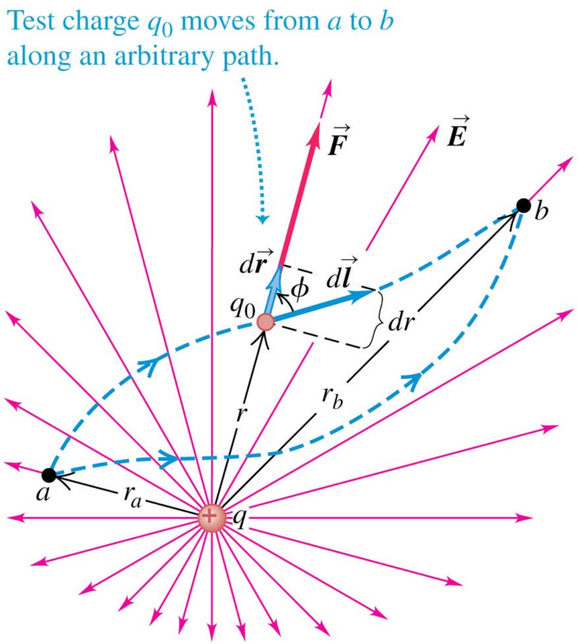
$$W_{a\to b} = -\Delta U = -(U_b - U_a) = -(q_0Ey_b - q_0Ey_a) = q_0E(y_a - y_b)$$

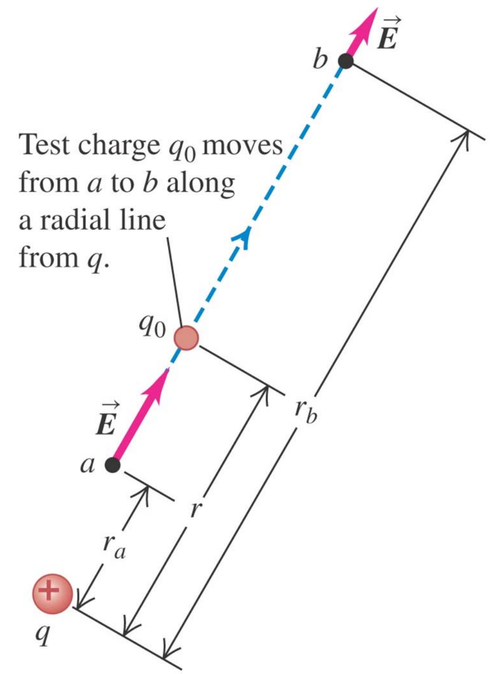
A negative charge moving in a uniform field

If the negative charge moves in the direction of the field, the potential energy *increases*, but if the charge moves opposite the field, the potential energy *decreases*.



Electric potential energy of two point charges





The electric potential is the same whether q_0 moves in a radial line (right figure) or along an arbitrary path (left figure).
The force on q_0 is given by Coulomb's law, and its radial component is $F_r = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$

$$W_{a\to b} = \int_{r_a}^{r_b} F\cos\phi\, dl = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\cos\phi\, dl$$

$$\cos\phi\, dl = dr.$$

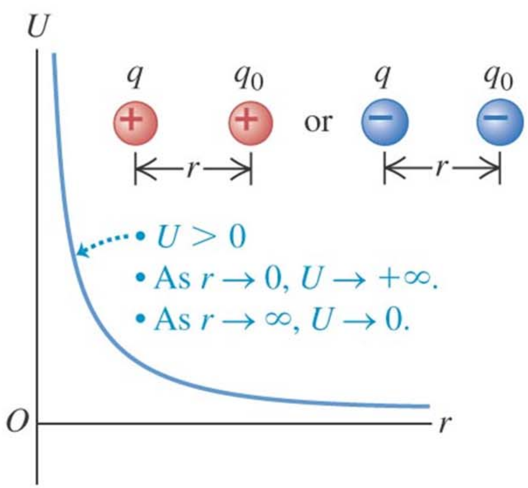
➔ $$W_{a\to b} = \int_{r_a}^{r_b} F_r dr = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} dr = \frac{qq_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

The work done by the electric force for this particular path depends only on the endpoints.

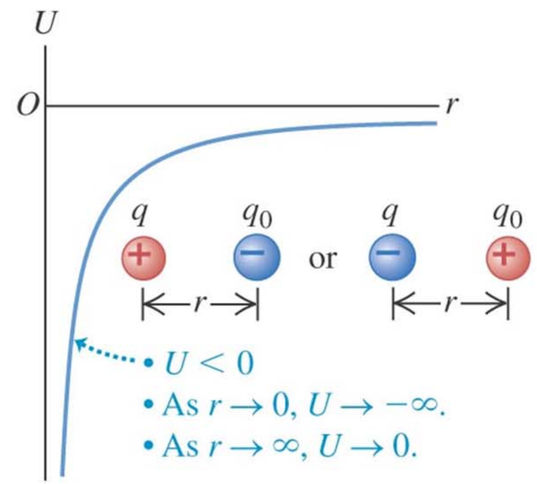
Graphs of the potential energy

The sign of the potential energy depends on the signs of the two charges.

(a) q and q_0 have the same sign.

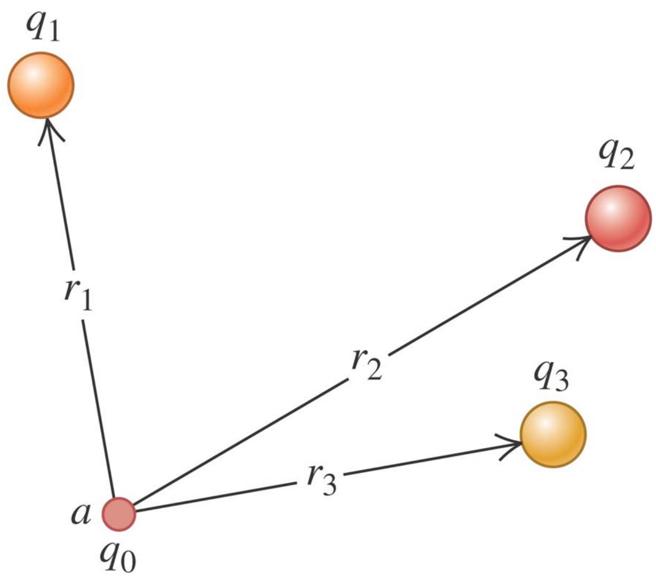


(b) q and q_0 have opposite signs.



$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$ (electric potential energy of two point charges q and q_0)

Electrical potential with several point charges



$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$ (point charge q_0 and collection of charges q_i)

If we start with charges q_1, q_2, q_3 . . . all separated from each other by infinite distances and then bring them together so that the distance between q_i and q_j is r_{ij} the total potential energy U is the sum of the potential energies of interaction for each pair of charges. We can write this as

$$U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

This sum extends over all pairs of charges; we don't let $i = j$ (because that would be an interaction of a charge with itself), and we include only terms with $i < j$ to make sure that we count each pair only once. Thus. to account for the interaction between q_3 and q_4, we include a term with $i = 3$ and $j = 4$ but not a term with $i = 4$ and $j = 3$.

Electric potential

Potential is *potential energy per unit charge.*

$$V = \frac{U}{q_0}$$

We can think of the potential difference between points *a* and *b* in either of two ways. The potential of *a* with respect to *b* ($V_{ab} = V_a - V_b$) equals:

- the work done by the electric force when a *unit* charge moves from *a* to *b.*
- the work that must be done to move a *unit* charge slowly from *b* to *a* against the electric force.

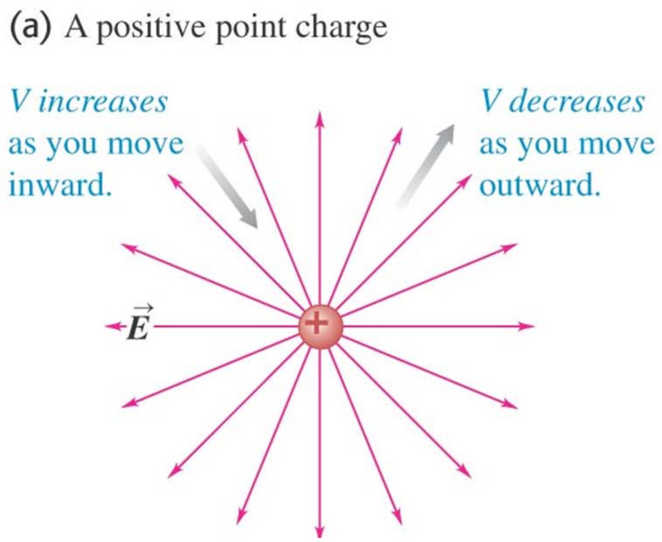
$$1\ \mathrm{V} = 1\ \text{volt} = 1\ \mathrm{J/C} = 1\ \text{joule/coulomb}$$

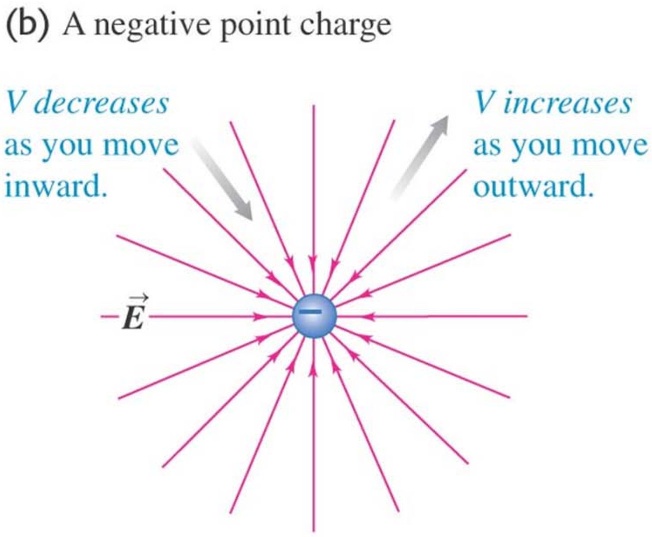
$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b$$

The voltage of this battery equals the difference in potential $V_{ab} = V_a - V_b$ between its positive terminal (point a) and its negative terminal (point b).

Calculating electric potential

If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*.





$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \quad \text{(potential due to a point charge)}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i} \quad \text{(potential due to a collection of point charges)}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} \quad \text{(potential due to a continuous distribution of charge)}$$

Calculating electric potential from the electric field

When we are given a collection of point charges, it is usually easy to calculate the potential V. But in some problems in which the electric field is known or can be found easily, it is easier to determine V from E. The force F on a test charge q_0 can be written as $F = q_0 E$. Therefore the work done by the electric force as the test charge moves from a to b is given by

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi \, dl$$ (potential difference as an integral of $\vec{E}$) $\leftrightarrow$ $$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}$$

Electron Volts

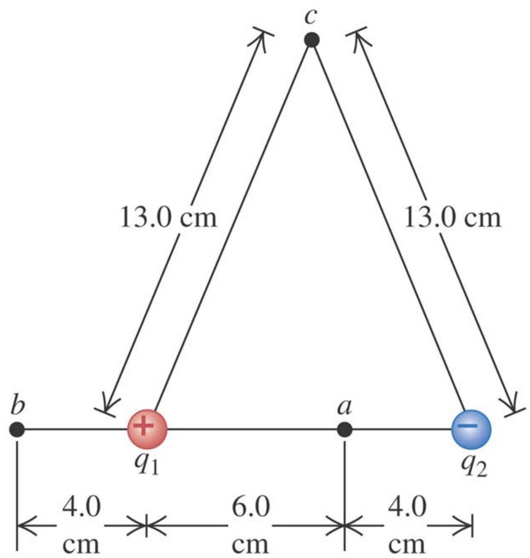
The magnitude e of the electron charge can be used to define a unit of energy that is useful in many calculations with atomic and nuclear systems. When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a the change in the potential energy V is

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

If the charge q equals the magnitude e of the electron charge, 1.602×10^{-19} C, and the potential difference is $V_{ab} = 1$ V, the change in energy is 1 electron volt:

$$U_a - U_b = (1.602\times10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602\times10^{-19}\,\mathrm{J} \quad \rightarrow \quad 1\,\mathrm{eV} = 1.602\times10^{-19}\,\mathrm{J}$$

Potential due to two point charges



An electrric dipole consists of two point charges, q_1 = + 12 nC and q_2 = -12 nC, placed 10 cm apart. Compute the potentials at points a, b, and c by adding the potentials due to either charge.

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

At point (a) the potential due to the positive charge q_1 is

$$\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = (9.0\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{12\times10^{-9}\ \mathrm{C}}{0.060\ \mathrm{m}}$$

$$= 1800\ \mathrm{N\cdot m/C} = 1800\ \mathrm{J/C} = 1800\ \mathrm{V}$$

And the potential due to the negative charge q_2 is

$$\frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} = (9.0\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(-12\times10^{-9}\ \mathrm{C})}{0.040\ \mathrm{m}} = -2700\ \mathrm{N\cdot m/C} = -2700\ \mathrm{J/C} = -2700\ \mathrm{V}$$

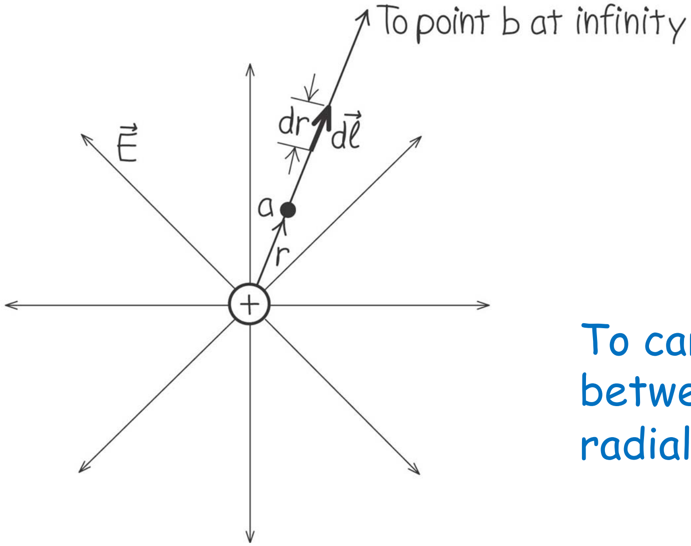
$\rightarrow$ $V_a = 1800\ \mathrm{V} + (-2700\ \mathrm{V}) = -900\ \mathrm{V}$

Similarly at point (b) the potential ➔ $V_b = 2700\ \mathrm{V} + (-770\ \mathrm{V}) = 1930\ \mathrm{V}$

$$V_c = 830\ \mathrm{V} + (-830\ \mathrm{V}) = 0$$

And at point (c) ➔

Finding potential by integration



By integrating the electric field find the potential at a distance r from a point charge q.

$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{l}$$

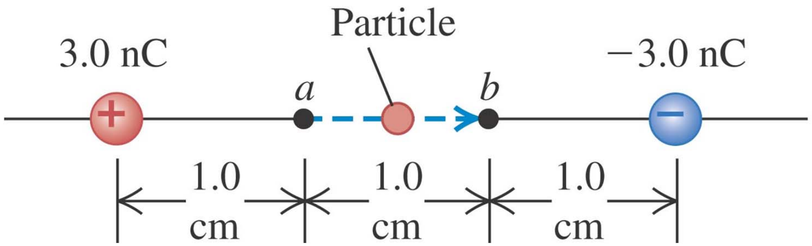
To carry out the integral, we can choose any path we like between points a and b. The most convenient path is a straight radial line

$$\Rightarrow \quad V - 0 = \int_r^\infty E\,dr = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2}dr = -\frac{q}{4\pi\epsilon_0 r}\Bigg|_r^\infty = 0 - \left(-\frac{q}{4\pi\epsilon_0 r}\right) \quad \Rightarrow \quad V = \frac{q}{4\pi\epsilon_0 r}$$

Alternatively

$$V - 0 = V = \int_r^\infty \vec{E}\cdot d\vec{l} = \int_r^\infty \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\cdot\hat{r}\,dr = \int_r^\infty \frac{q}{4\pi\epsilon_0 r^2}dr \quad \Rightarrow \quad V = \frac{q}{4\pi\epsilon_0 r}$$

Moving through a potential difference



a dust particle with mass $m = 5.0 \times 10^{-9}$ kg and charge $q_0 = 2.0$ nC starts from rest at point a and moves in a straight line to point b. What is its speed v at point b?

$$K_a + U_a = K_b + U_b$$

$$\left.\begin{array}{l} K_a = 0 \text{ and } K_b = \frac{1}{2}mv^2 \\ U_a = q_0V_a \text{ and } U_b = q_0V_b \end{array}\right\} \quad 0 + q_0V_a = \frac{1}{2}mv^2 + q_0V_b \quad \rightarrow \quad v = \sqrt{\frac{2q_0(V_a - V_b)}{m}}$$

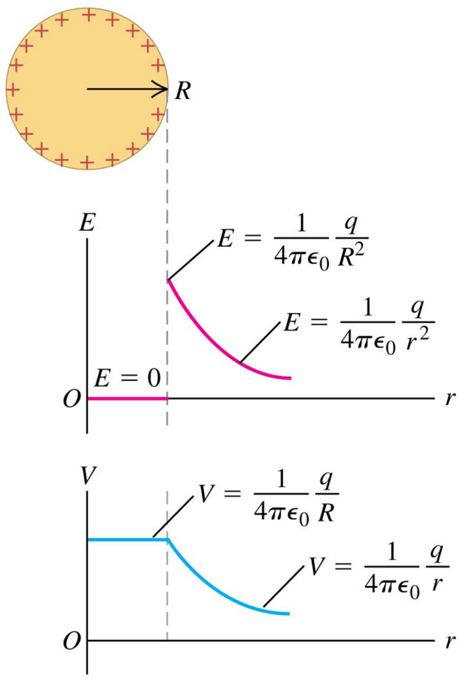
$$\boxed{V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}} \quad \rightarrow \quad \left\{\begin{array}{l} V_a = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \times \left(\frac{3.0 \times 10^{-9}\ \mathrm{C}}{0.010\ \mathrm{m}} + \frac{(-3.0 \times 10^{-9}\ \mathrm{C})}{0.020\ \mathrm{m}}\right) = 1350\ \mathrm{V} \\ V_b = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \times \left(\frac{3.0 \times 10^{-9}\ \mathrm{C}}{0.020\ \mathrm{m}} + \frac{(-3.0 \times 10^{-9}\ \mathrm{C})}{0.010\ \mathrm{m}}\right) = -1350\ \mathrm{V} \end{array}\right.$$

$$V_a - V_b = (1350\ \mathrm{V}) - (-1350\ \mathrm{V}) = 2700\ \mathrm{V}$$

$$\rightarrow \quad v = \sqrt{\frac{2(2.0 \times 10^{-9}\ \mathrm{C})(2700\ \mathrm{V})}{5.0 \times 10^{-9}\ \mathrm{kg}}} = 46\ \mathrm{m/s}$$

Calculating electric potential

A charged conducting sphere



We used Gauss's law to find the electric field at all points for this charge distribution. We can use that result to determine the potential at all points.

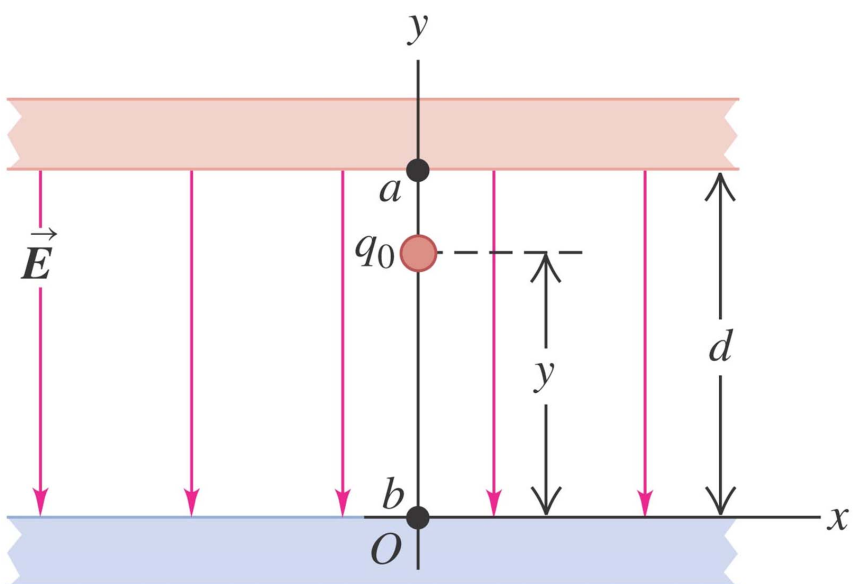
We know that the potential at a point outside the sphere at a distance r from its center is the same as the potential due to a point charge q at the center:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

The potential at surface of the sphere is $V_{surface} = q/4\pi\epsilon_0 R.$

Inside the sphere, E is zero everywhere; otherwise, charge would move within the sphere. Hence if a test charge moves from any point to any other point inside the sphere, no work is done on that charge. This means that the potential is the same at every point inside the sphere.

Oppositely charged parallel plates



Find the potential at any height y between the two oppositely charged parallel plates

$$V(y) = \frac{U(y)}{q_0} = \frac{q_0 E y}{q_0} = Ey$$

We have chosen U(y), and therefore V(y), to be zero at point b, where y = 0. Even if we choose the potential to be different from zero at b, it is still true that

$$V(y) - V_b = Ey$$

The potential decreases as we move in the direction of E from the upper to the lower plate. At point a, where y = d and V(y) = V_a,

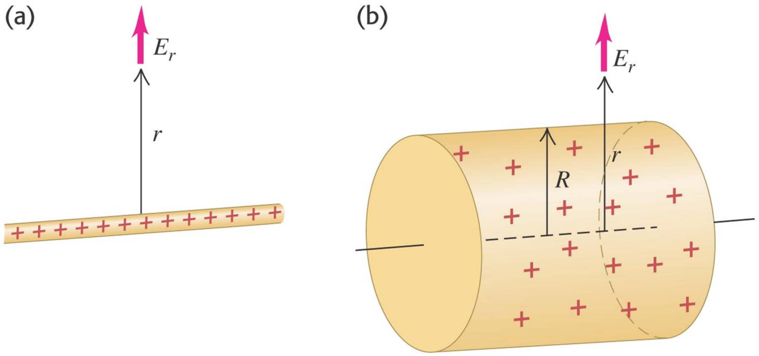
$$V_a - V_b = Ed \quad \text{and} \quad E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}$$

Before we derived that: $E = \sigma / \epsilon_0$

$$E = V_{ab}/d \rightarrow \quad \sigma = \frac{\epsilon_0 V_{ab}}{d}$$

The surface charge density on the positive plate is directly proportional to the potential difference between the plates, and its value σ can be determined by measuring V_{ab} .

An infinite line charge or conducting cylinder



Find the potential at a distance r from a very long line of chargewith linear charge density (charge per unit length) λ

We know that the electric field at a distance r from a long straight-line charge is:

$$E_r = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

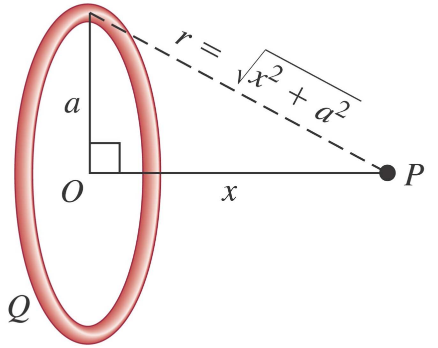
➔ $$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E_r dr = \frac{\lambda}{2\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_b}{r_a}$$

If we take point *b* at infinity and set $V_b = 0$, we find that V_a is infinite: $$V_a = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{\infty}{r_a} = \infty$$

To get around this difficulty, remember that we can define V to be zero at any point we like. We set $V_b = 0$ at point b at an arbitrary radial distance r_0

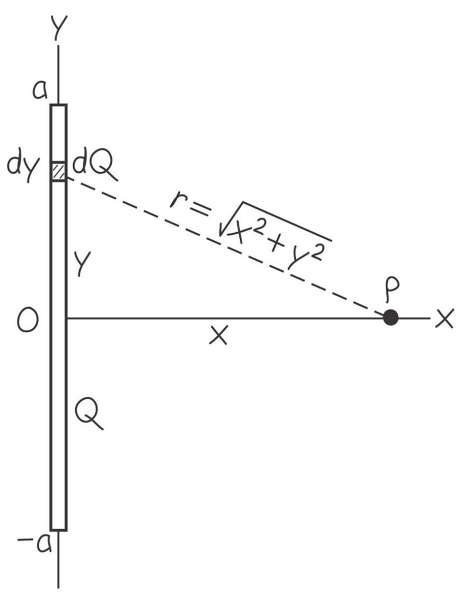
➔ $$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_0}{r}$$ At any point r > R ➔ $$V = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{R}{r}$$

A ring of charge



$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+a^2}}\int dq = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

A finite line of charge



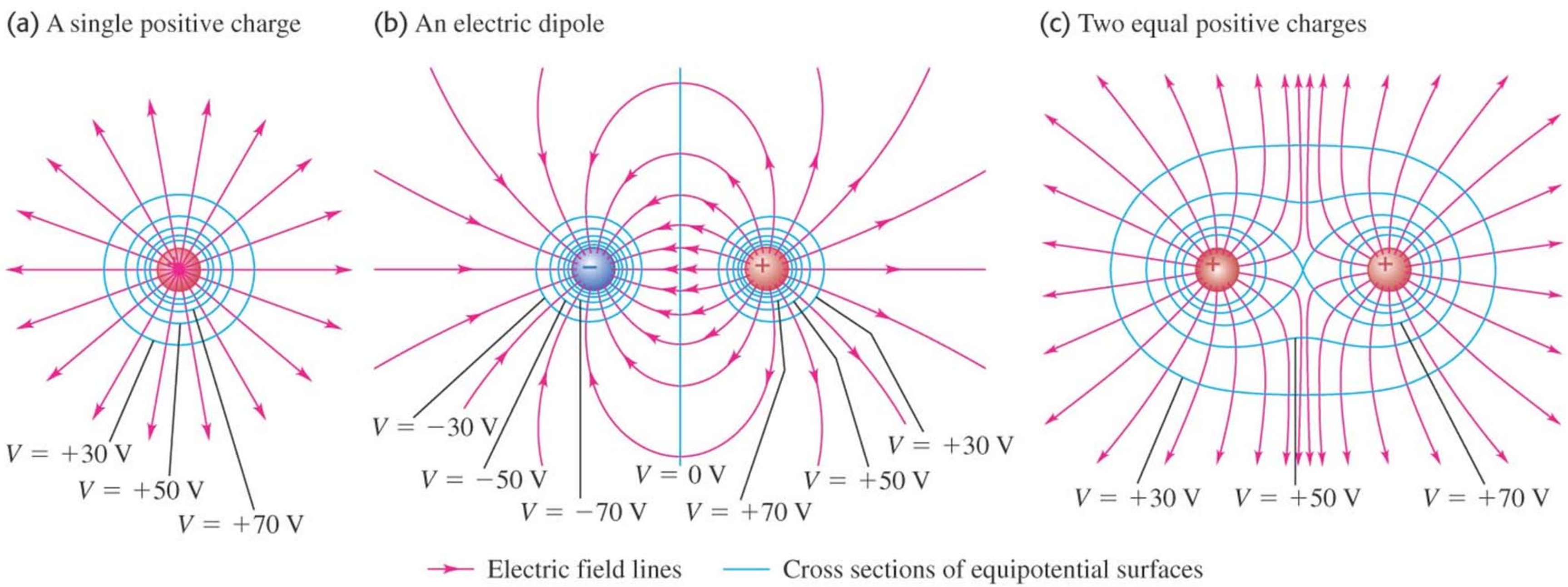
As we have seen before, the element of charge *dQ* corresponding to an element of length *dy* on the rod is given by *dQ = (Q/2a)dy.* The distance from *dQ* to *P* is r and the contribution *dV* that it makes to the potential at P is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\frac{dy}{\sqrt{x^2+y^2}} \quad \rightarrow \quad V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\int_{-a}^{a}\frac{dy}{\sqrt{x^2+y^2}}$$

$$\rightarrow \quad V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2a}\ln\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right)$$

Equipotential surfaces and field lines

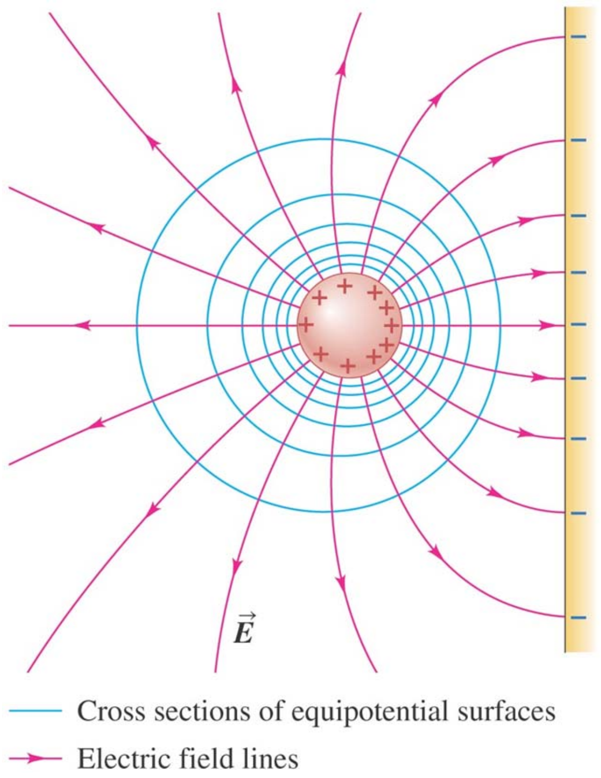
An *equipotential surface* is a surface on which the electric potential is the same at every point.
Field lines and equipotential surfaces are always mutually perpendicular.

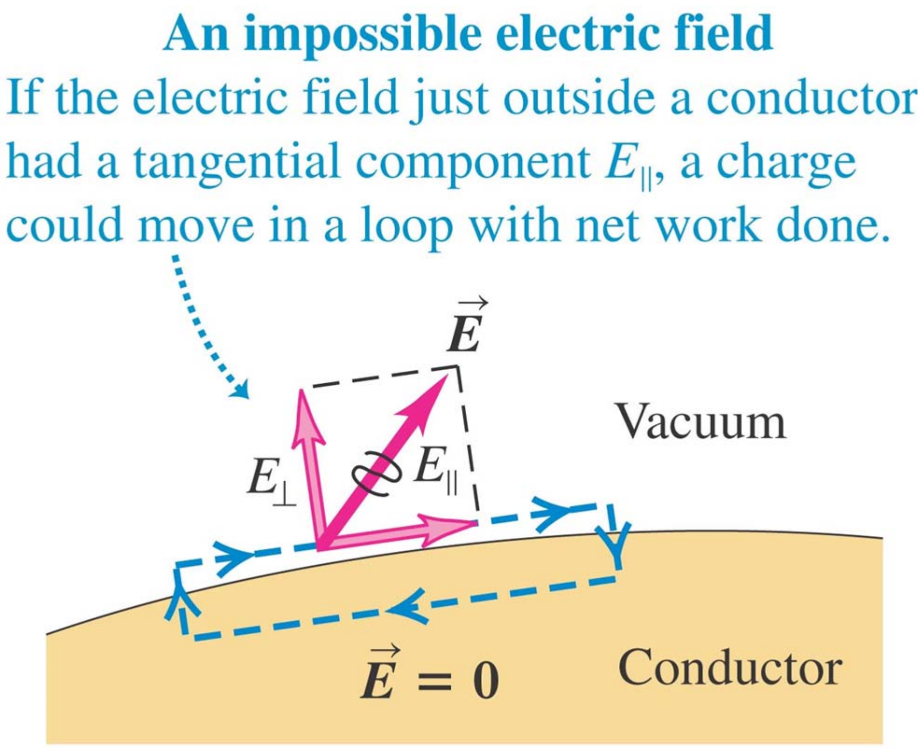


Equipotentials and conductors

When all charges are at rest:

- ✓ the electric field just outside a conductor is always perpendicular to the surface
- ✓ the entire solid volume of a conductor is at the same potential.
- ✓ the surface of a conductor is always an equipotential surface.





Potential gradient

Electric field and potential are closely related.

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

If we know E at various points, we can use this equation to calculate potential differences. In this section we show how to turn this around; if we know the potential V at various points, we can use it to determine E. Regarding V as a function of the coordinates (x,y, z) of a point in space, we will show that the components of E are directly related to the partial derivatives of V with respect to x, y and z.

$$V_a - V_b = \int_b^a dV = -\int_a^b dV$$

By definition $V_a - V_b = \int_b^a dV = -\int_a^b dV \rightarrow -\int_a^b dV = \int_a^b \vec{E} \cdot d\vec{l} \rightarrow -dV = \vec{E} \cdot d\vec{l}$

$$\left.\begin{aligned} d\vec{l} &= \hat{\imath}\, dx + \hat{\jmath}\, dy + \hat{k}\, dz \\ \vec{E} &= \hat{\imath}\, E_x + \hat{\jmath}\, E_y + \hat{k}\, E_z \end{aligned}\right\} \quad -dV = E_x dx + E_y dy + E_z dz$$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \qquad \text{(components of } \vec{E} \text{ in terms of } V\text{)}$$

$$\vec{E} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) \qquad (\vec{E} \text{ in terms of } V) \quad \leftrightarrow \quad \vec{E} = -\vec{\nabla} V$$

Potential gradiant