Chapter 13

Project Management

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Learning Objectives

Students will be able to:

1. Understand how to plan, monitor, and control projects with the use of PERT.
2. Determine earliest start, earliest finish, latest start, latest finish, and slack times for each activity along with the total project completion time.
3. Reduce total project time at the least total cost by crashing the network using manual or linear programming techniques.
4. Understand the important role of software in project management.

Chapter Outline

13.1 Introduction

13.2 PERT

13.3 PERT/COST

13.4 Critical Path Method

13.5 Other Topics in Project Management

Introduction

- Project management can be used to manage complex projects.
- The first step in planning and scheduling a project is to develop the work breakdown structure.
- This involves identifying the activities that must be performed in the project. Each detail and each activity may be broken into its most basic components.
- The time, cost, resource requirements, predecessors, and person(s) responsible are identified.

Project Management

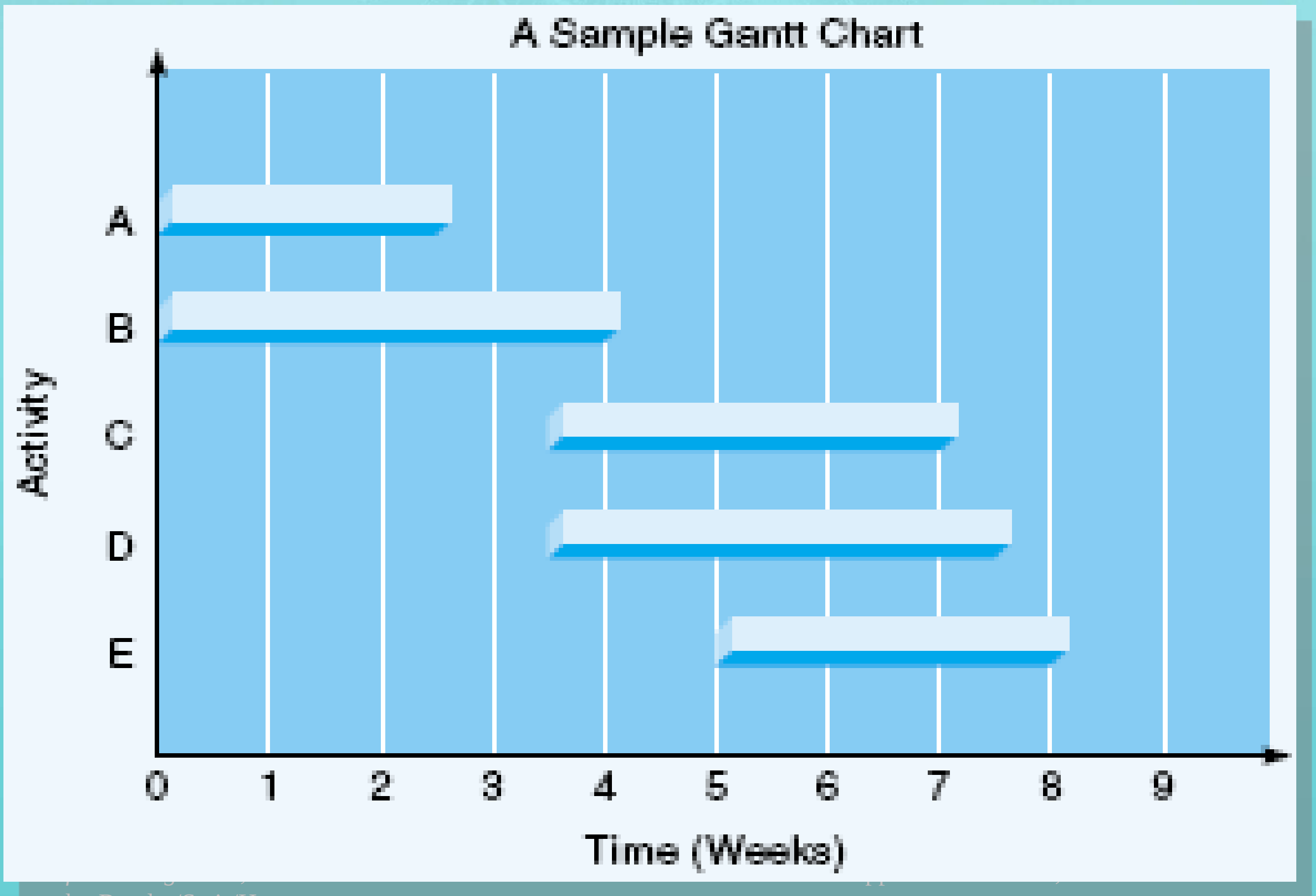
Definition:

It is nothing more (or less) than knowing what the status of a project is:

- when it should be done,
- how much (and if) it has slipped from the original schedule,
- what the bottlenecks are, and
- what you might drop to save some time.

Project Management Models: History

- One of the earliest techniques was the *Gantt chart* *(Used by US Navy)*.
- This type of chart shows the start and finish times of one or more activities, as shown below:



Project Planning, Controlling and Scheduling

Project Planning:

1. Setting goals.
2. Defining the project.
3. Tying needs into timed project activities.
4. Organizing the team.

Project Scheduling:

1. Tying resources to specific activities.
2. Relating activities to each other.
3. Updating and revising on regular basis.

Before Project

Project Controlling:

1. Monitoring resources, costs, quality and budgets.
2. Revising and changing plans.
3. Shifting resources to meet demands.

During Project

Project Management Models: Today

- PERT
- PERT/Cost
- Critical Path Method (CPM)

PERT and CPM

- The *program evaluation and review technique* (PERT) and the *critical path method* (CPM) are two popular quantitative analysis techniques that help managers plan, schedule, monitor, and control large and complex projects.
- They were developed because there was a critical need for a better way to manage.

Framework of PERT and CPM

There are six steps common to both PERT and CPM.

1. Define the project and all of its significant activities or tasks.
2. Develop the relationships among the activities. Decide which activities must precede others.
3. Draw the network connecting all of the activities.
4. Assign time and/or cost estimates to each activity.
5. Compute the longest time path through the network; this is called the *critical path*.
6. Use the network to help plan, schedule, monitor, and control the project.

Framework of PERT and CPM *(continued)*

- The critical path is important because activities on the critical path can delay the entire project.
- *PERT* is probabilistic, whereas *CPM* is deterministic.
- Almost any large project can be subdivided into a series of smaller activities or tasks that can be analyzed with *PERT*.
- Projects can have thousands of specific activities and it is important to be able to answer many associated questions.

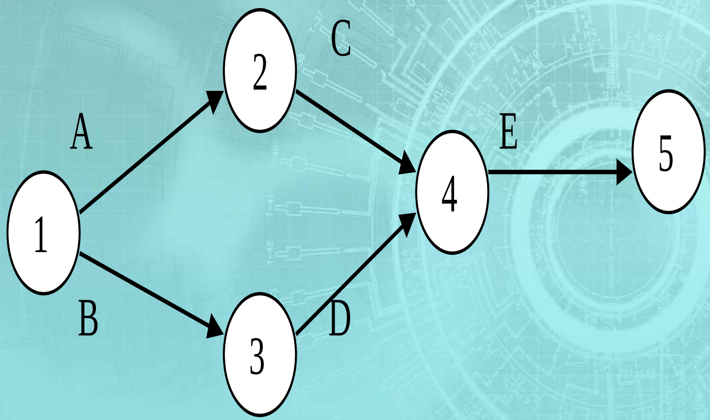
Questions answered by PERT

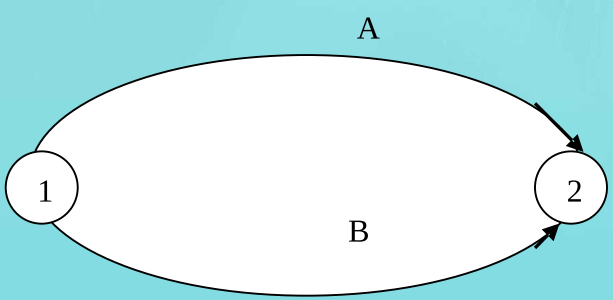
1. When will the entire project be completed?
2. What are the *critical* activities or tasks in the project, that is, the ones that will delay the entire project if they are late?
3. Which are the *non-critical* activities, that is, the ones that can run late without delaying the entire project's completion?
4. What is the probability that the project will be completed by a specific date?

Questions answered by PERT *(continued)*

5. At any particular date, is the project on schedule, behind schedule, or ahead of schedule?
6. On any given date, is the money spent equal to, less than, or greater than the budgeted amount?
7. Are there enough resources available to finish the project on time?
8. If the project is to be finished in a shorter amount of time, what is the best way to accomplish this at the least cost?

network





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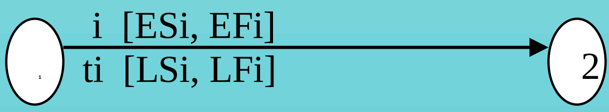
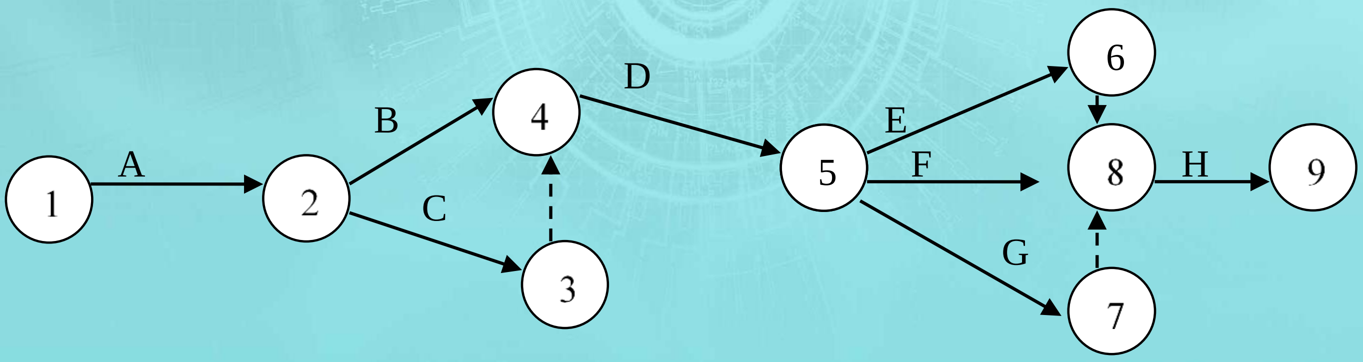


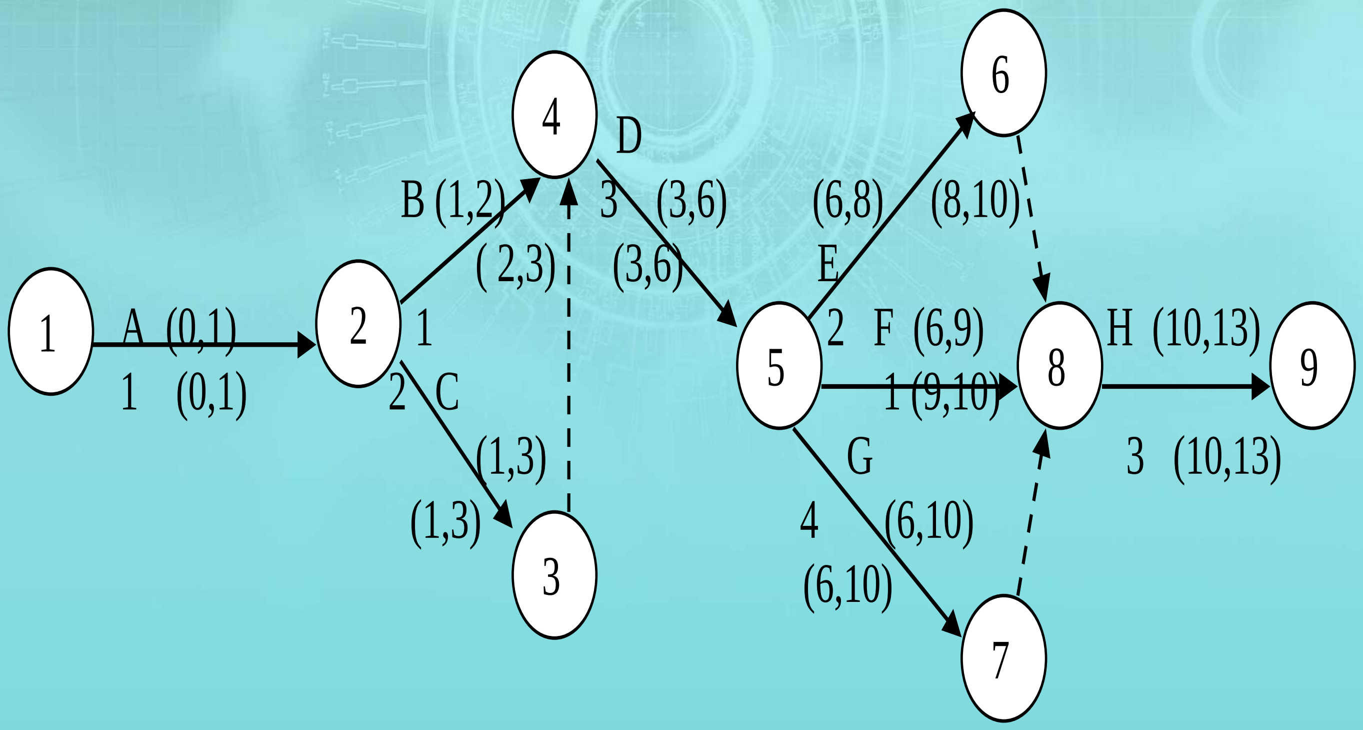
Example

Activity	Preceding activity	Expected completion time (week)
A	-	1
B	A	1
C	A	2
D	B,C	3
E	D	2
F	D	1
G	D	4
H	E, F,G	3

Network diagram



Completion times



Linear prog. for finding the completion time.

Xi: nodes i=1…9 showing the complition time of activities until that node

$\text{Min } Z = X9$

$X1 \geq 0$

$X2 \geq X1+1$

$X3 \geq X2+2$

$X4 \geq X2+1$

$X5 \geq X4+3$

$X5 \geq X3$

$X6 \geq X5+2$

$X7 \geq X5+4$

$X8 \geq X5+1$

$X8 \geq X6$

$X8 \geq X7$

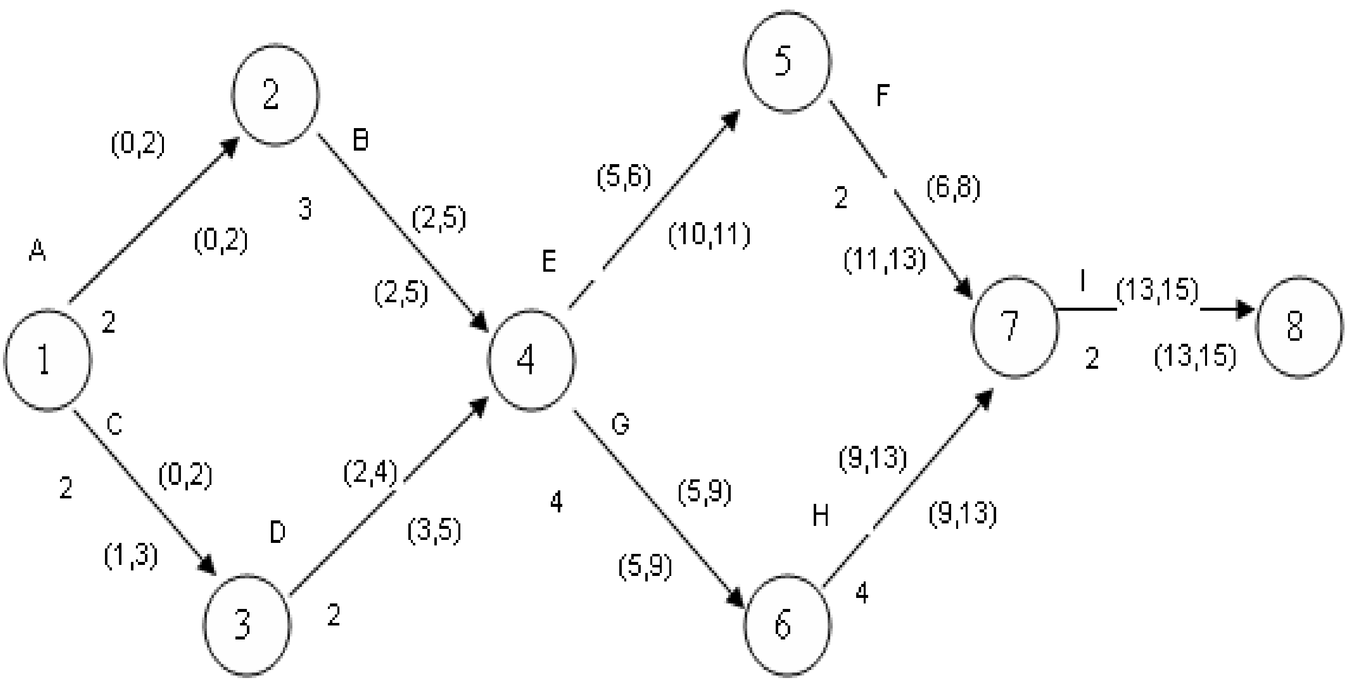
$X9 \geq X8+3$

example

Activity	Predecessor activity	normal time	normal cost	crash time	crash cost
A	-	2	120	0,8	180
B	A	3	400	1	780
C	-	2	170	0,4	220
D	C	2	240	0,6	360
E	B,D	1	150	0,2	480
F	E	2	350	0,6	600
G	B,D	4	410	2	740
H	G	4	540	1,8	900
I	F,H	2	600	0,4	1000

Example (cont)

- Network diagram
- Min completion time
- Critical path
- Write LP model that finds the way to coplete the project in 12 months with minimum crash cost
- Make the budget arrangement that will be necessary for each month


1
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A
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C
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(0,2)
(1,3)
2
(2,4)
(3,5)
2
(5,6)
(10,11)
(6,8)
(11,13)
2
(5,9)
(5,9)
4
(9,13)
(9,13)
4
(13,15)
(13,15)
2

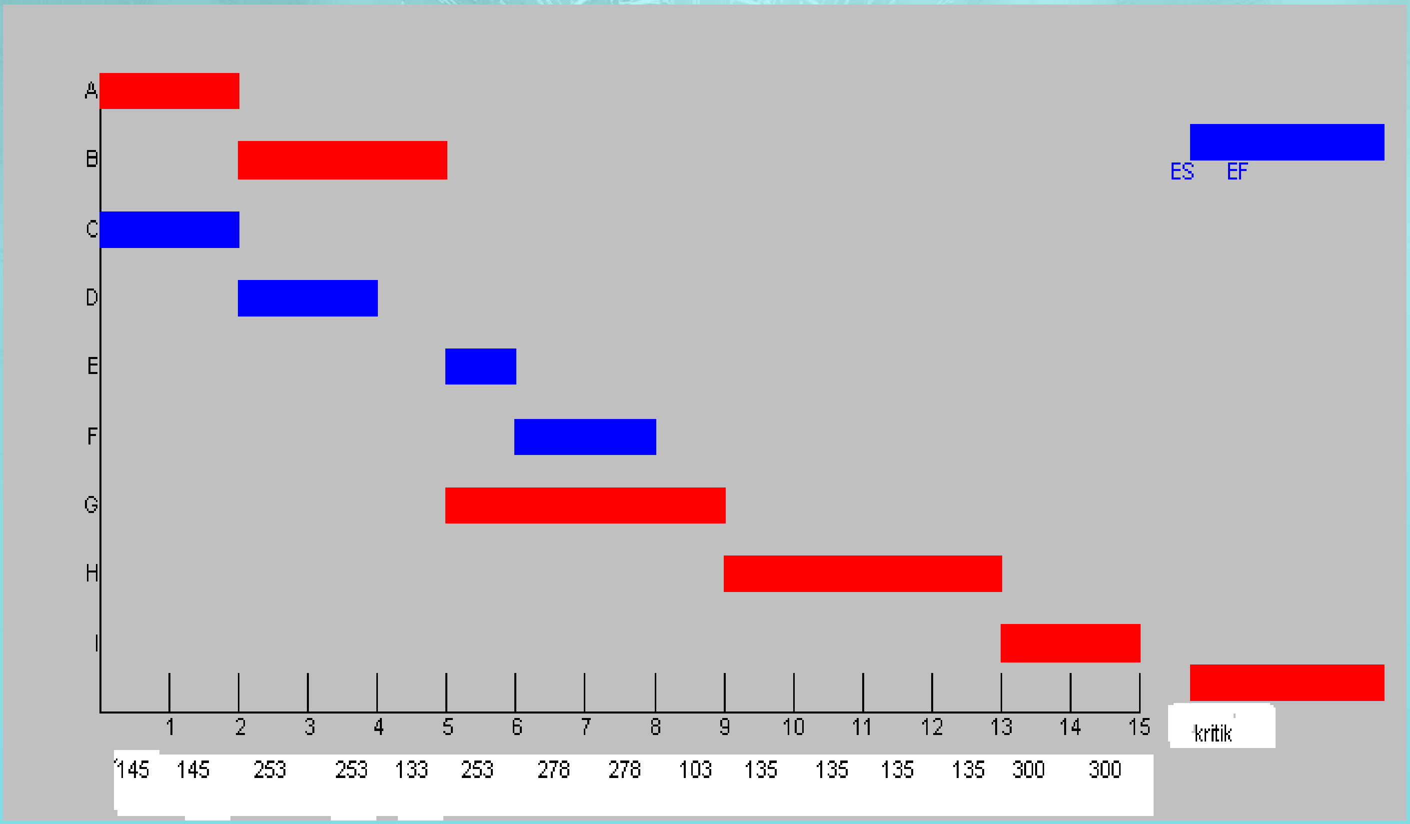
- **Xi: time node, X1,...,X8**
- **Y_i: crash time on the activity (i), Y_A,...,Y_I**
- **Crash cost(montly)=(crash cost – normal cost)/ (normal time – crash time)**

Montly crash cost for activity A is 50 thousand $.

$$\text{Min } Z = 50\,Y_A + 190\,Y_B + 31{,}25\,Y_C + 85{,}71\,Y_D + 412{,}5\,Y_E + 178{,}57\,Y_F + 165\,Y_G + 163{,}64\,Y_H + 250\,Y_I$$

$X1 \geq 0$
$X2 \geq X1+2-Y_A$
$X3 \geq X1+2-Y_C$
$X4 \geq X2+3-Y_B$
$X4 \geq X3+2-Y_D$
$X5 \geq X4+1-Y_E$
$X6 \geq X4+4-Y_G$
$X7 \geq X5+2-Y_F$
$X7 \geq X6+4-Y_H$
$X8 \geq X7+2-Y_I$
$X8 \leq 12$

$Y_A \leq 1{,}2$
$Y_B \leq 2$
$Y_C \leq 1{,}6$
$Y_D \leq 1{,}4$
$Y_E \leq 0{,}8$
$Y_F \leq 1{,}4$
$Y_G \leq 2$
$Y_H \leq 2{,}2$
$Y_I \leq 1{,}6$


A
B
C
D
E
F
G
H
I
ES
EF
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
kritik
145 145 253 253 133 253 278 278 103 135 135 135 135 300 300

General Foundry Example of PERT

Background:

- General Foundry, Inc., a metal works plant has long been trying to avoid the expense of installing air pollution control equipment.
- The local environmental protection group has recently given the foundry 16 weeks to install a complex air filter system on its main smokestack.
- General Foundry was warned that it will be forced to close unless the device is installed in the allotted period.
- They want to make ensure that installation of the filtering system progresses smoothly and on time.

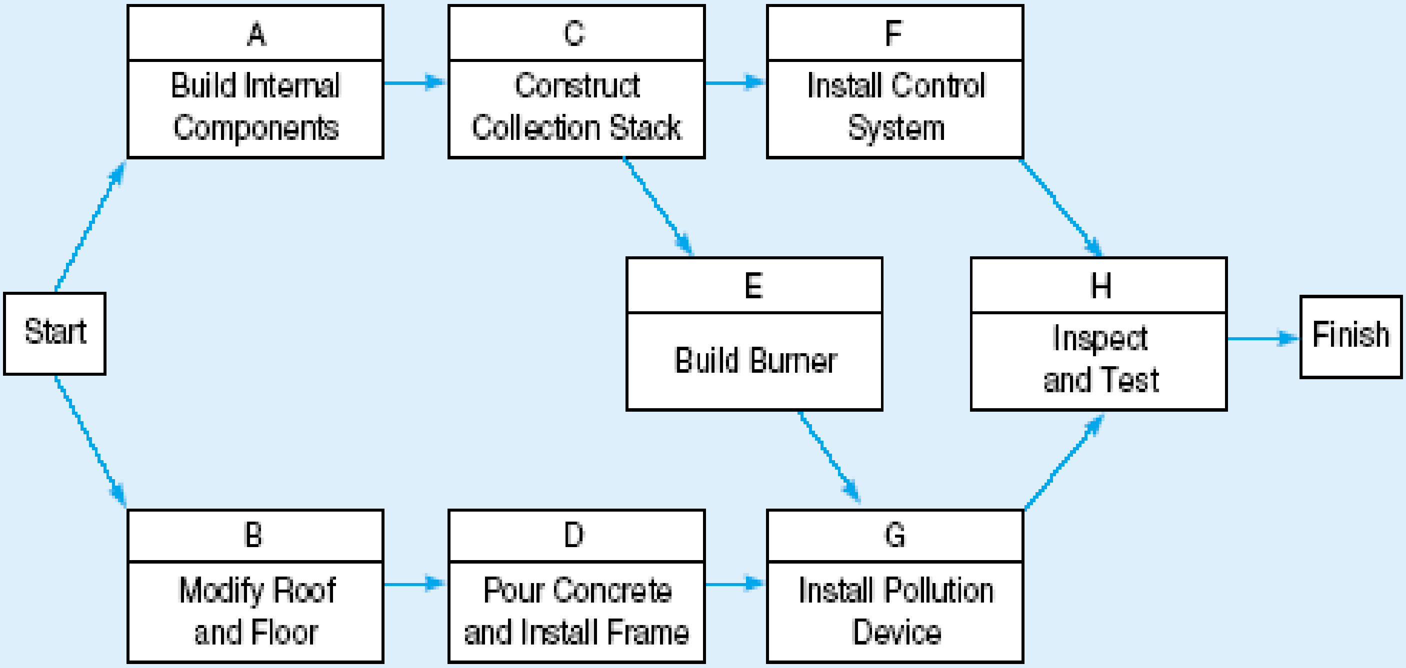
Steps One and Two:

1. Define the project and all project activities.
2. Immediate predecessors are determined in the second step.

ACTIVITY	DESCRIPTION	IMMEDIATE PREDECESSORS
A	Build internal components	—
B	Modify roof and floor	—
C	Construct collection stack	*A*
D	Pour concrete and install frame	*B*
E	Build high-temperature burner	*C*
F	Install control system	*C*
G	Install air pollution device	*D, E*
H	Inspect and test	*F, G*

Step Three

3. Activities and events are drawn and connected.



Step Four

4. The fourth step is to assign activity times.

- Time is usually given in units of weeks.
- Without solid historical data, managers are often uncertain as to activity times.
- The developers of PERT thus employed a probability distribution based on three time estimates for each activity:
  - ✓ **Optimistic time**
  - ✓ **Pessimistic time**
  - ✓ **Most likely time**

Step Four *(continued)*

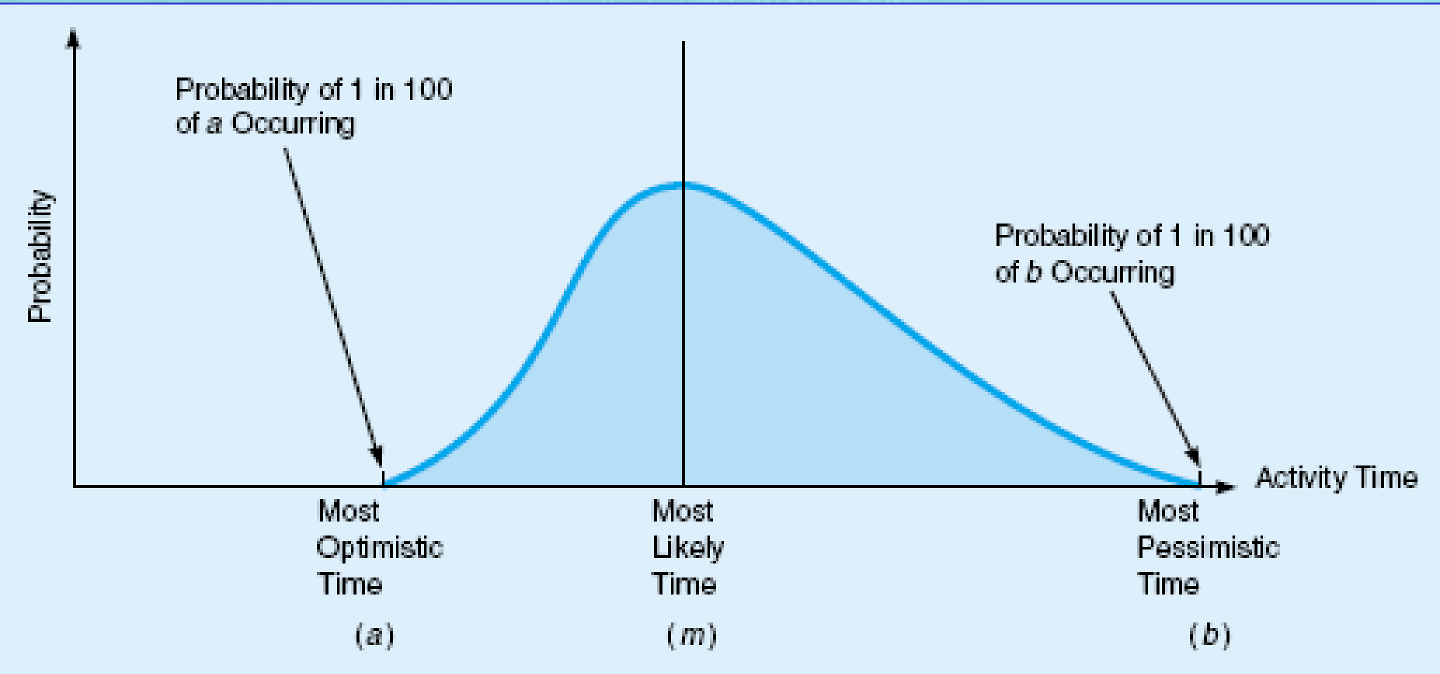
Optimistic time (**a**) = time an activity will take if everything goes as well as possible. There should be only a small probability (say, 1/1000) of this occurring.

Pessimistic time (**b**) = time an activity would take assuming very unfavorable conditions. There should also be only a small probability that the activity will really take this long.

Most likely time (**m**) = most realistic time estimate to complete the activity.

Beta Probability

- The beta probability distribution is often used.
- Following is a Beta Probability Distribution with Three Time Estimates:



Calculations with PERT

Expected time for activity

$$t = \frac{a + 4m + b}{6}$$

Variance for activity

$$\sigma^2 = \left(\frac{b - a}{6} \right)^2$$

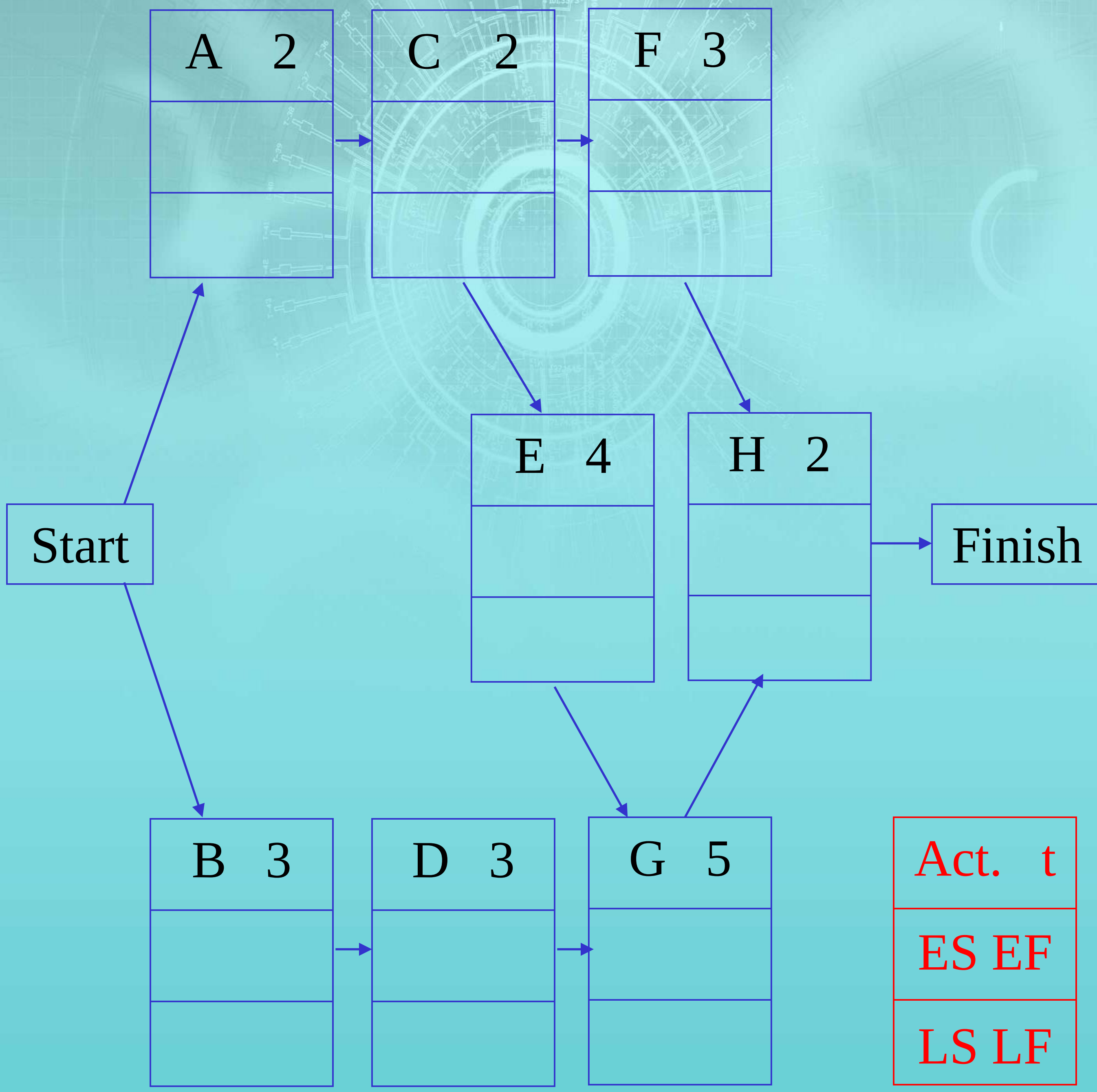
General Foundry, Inc. Time Estimates

Activity	*a*	*m*	*b*	*E(t)*	σ^2
A	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{1}{9}$
B	2	3	4	3	$\left(\frac{4-2}{6}\right)^2 = \frac{1}{9}$
C	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{1}{9}$
D	2	4	6	4	$\left(\frac{6-2}{6}\right)^2 = \frac{4}{9}$
E	1	4	7	4	$\left(\frac{7-1}{6}\right)^2 = \frac{9}{9}$
F	1	2	9	3	$\left(\frac{9-1}{6}\right)^2 = \frac{16}{9}$
G	3	4	11	5	$\left(\frac{11-3}{6}\right)^2 = \frac{16}{9}$
H	1	2	3	2	$\left(\frac{3-1}{6}\right)^2 = \frac{1}{9}$

Step Five

The fifth step is to compute the longest path through the network— the critical path.

General Foundry, Inc.
PERT Network - with E(t)



Finding Critical Path

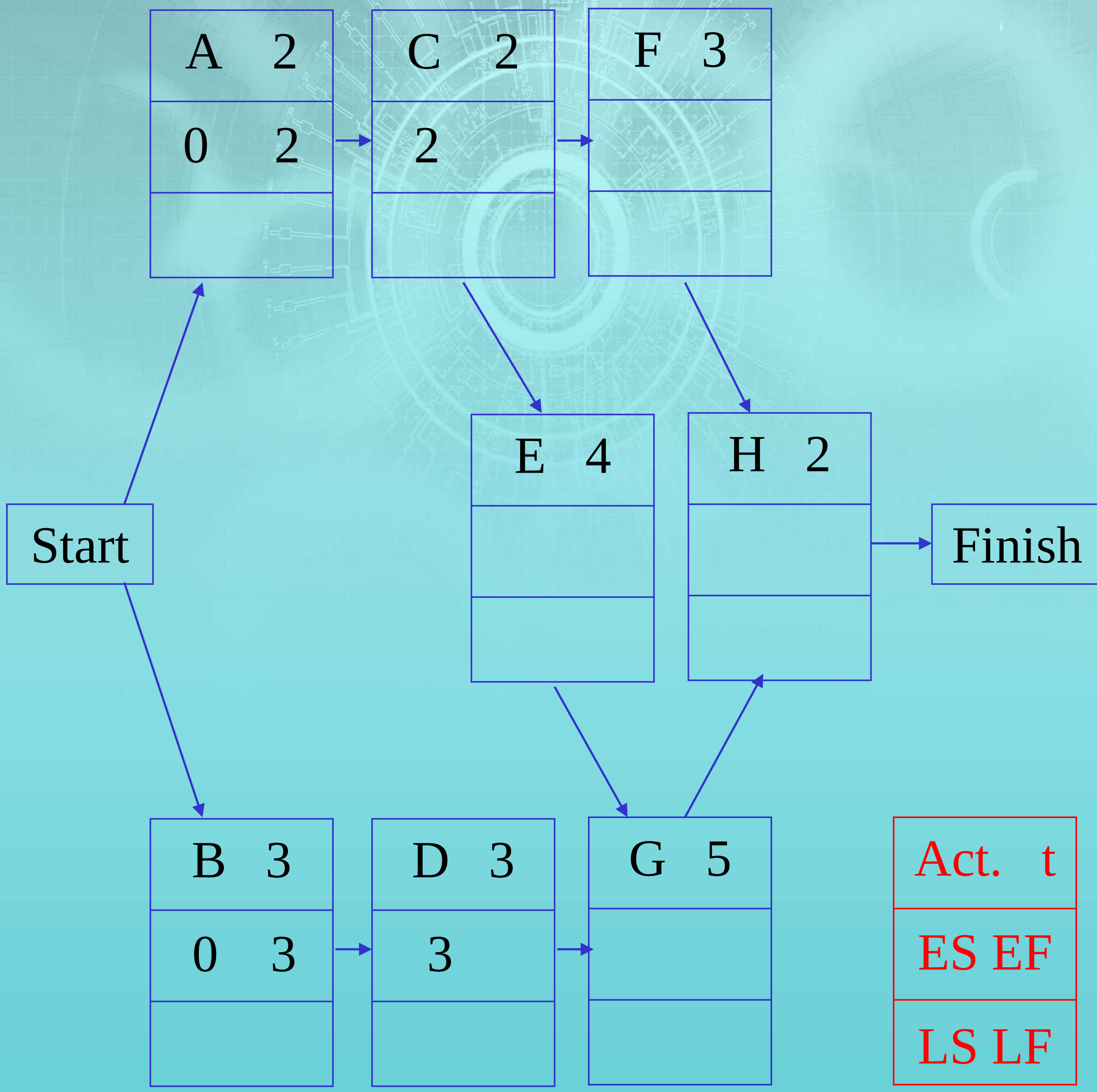
To find the critical path, need to determine the following quantities for each activity in the network:

1. Earliest start time (*ES*): the earliest time an activity can begin without violation of immediate predecessor requirements.
2. Earliest finish time (*EF*): the earliest time at which an activity can end.
3. Latest start time (*LS*): the latest time an activity can begin without delaying the entire project.
4. Latest finish time (*LF*): the latest time an activity can end without delaying the entire project.

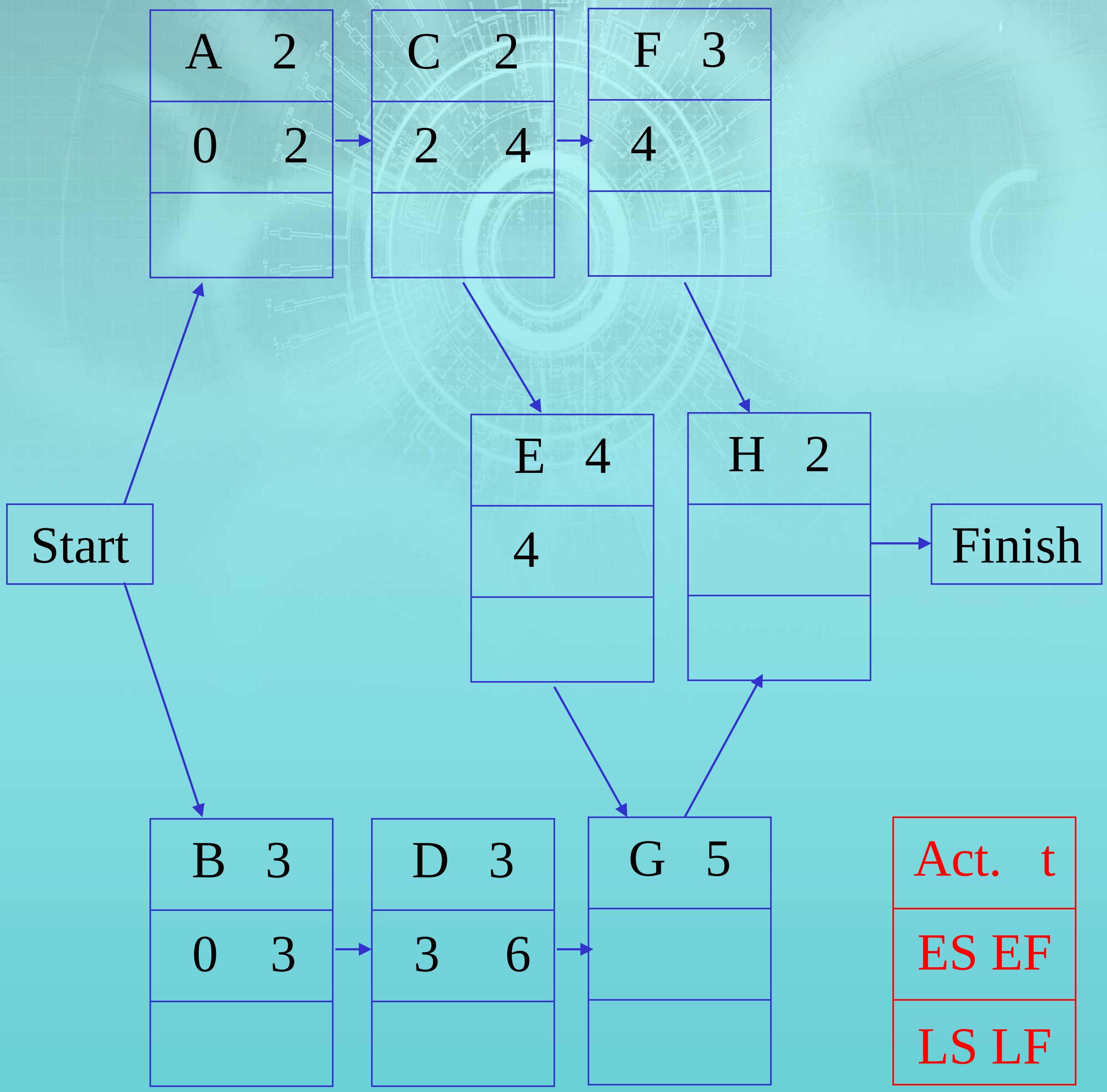
Earliest Times – Calculations

- EF = ES + t
- ES = Largest EF of Immediate Predecessors

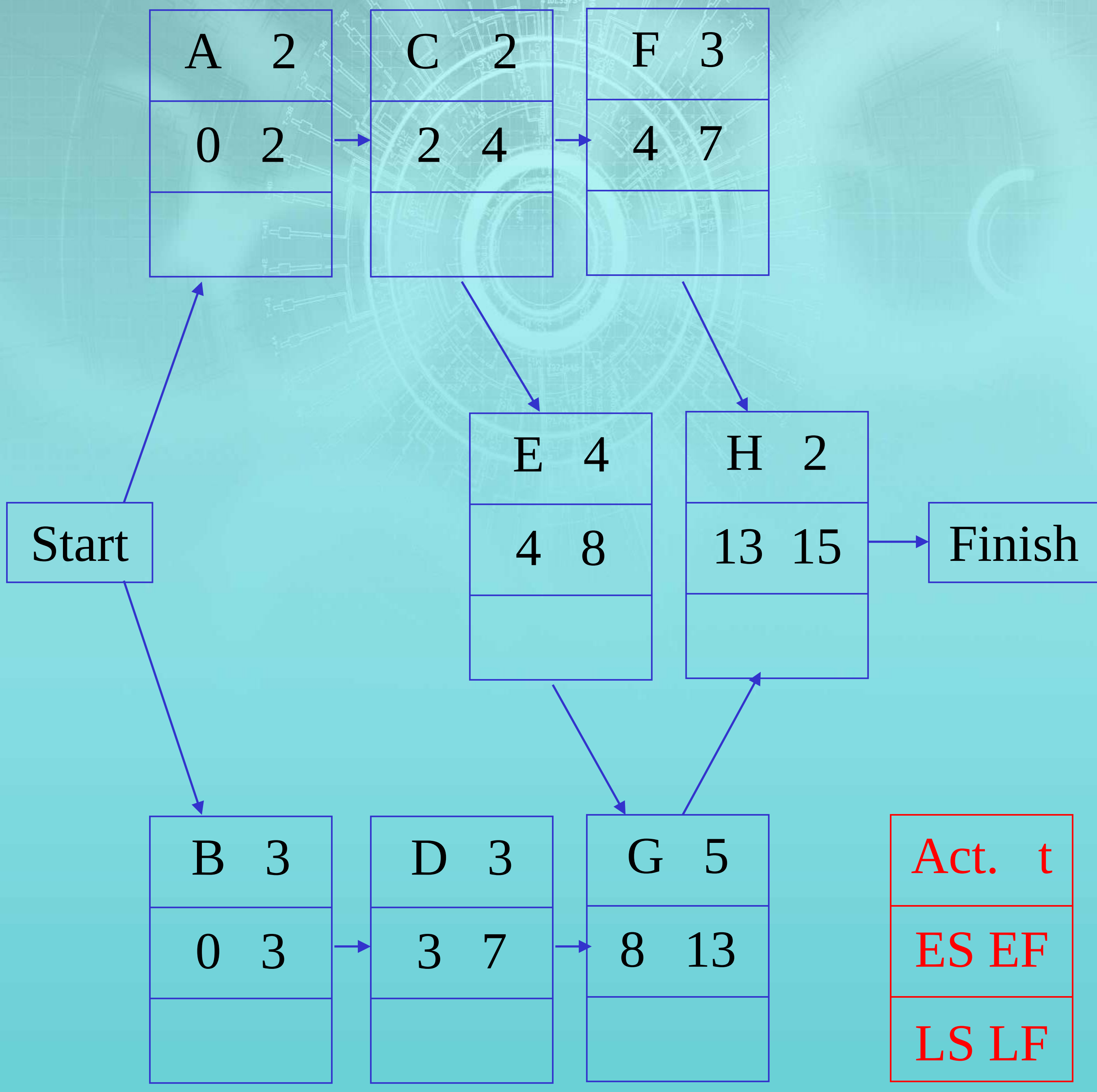
General Foundry, Inc. PERT Network - with E(t)



General Foundry, Inc.
PERT Network - with E(t)



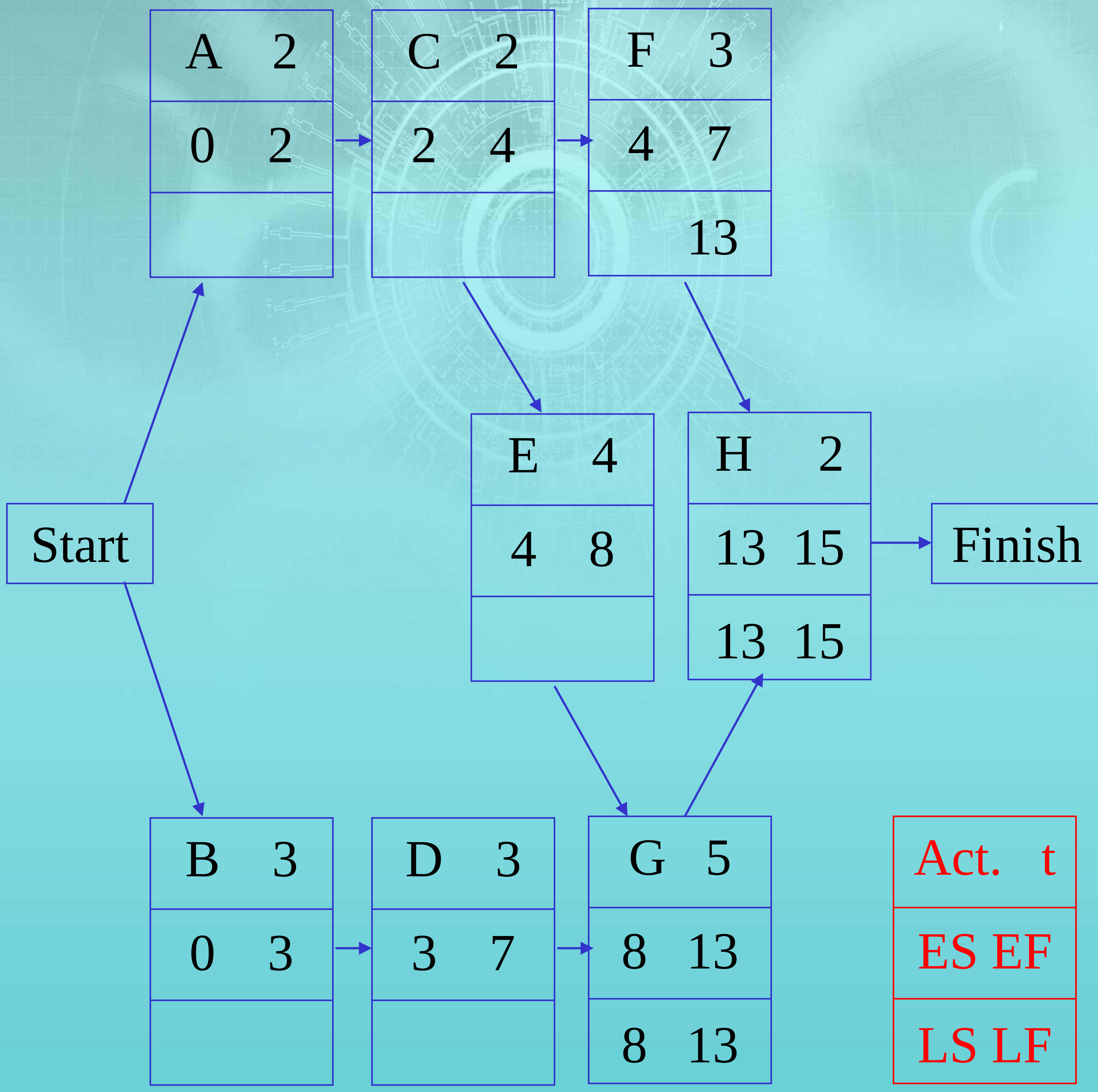
General Foundry, Inc. PERT Network ES/EF, LS/LF



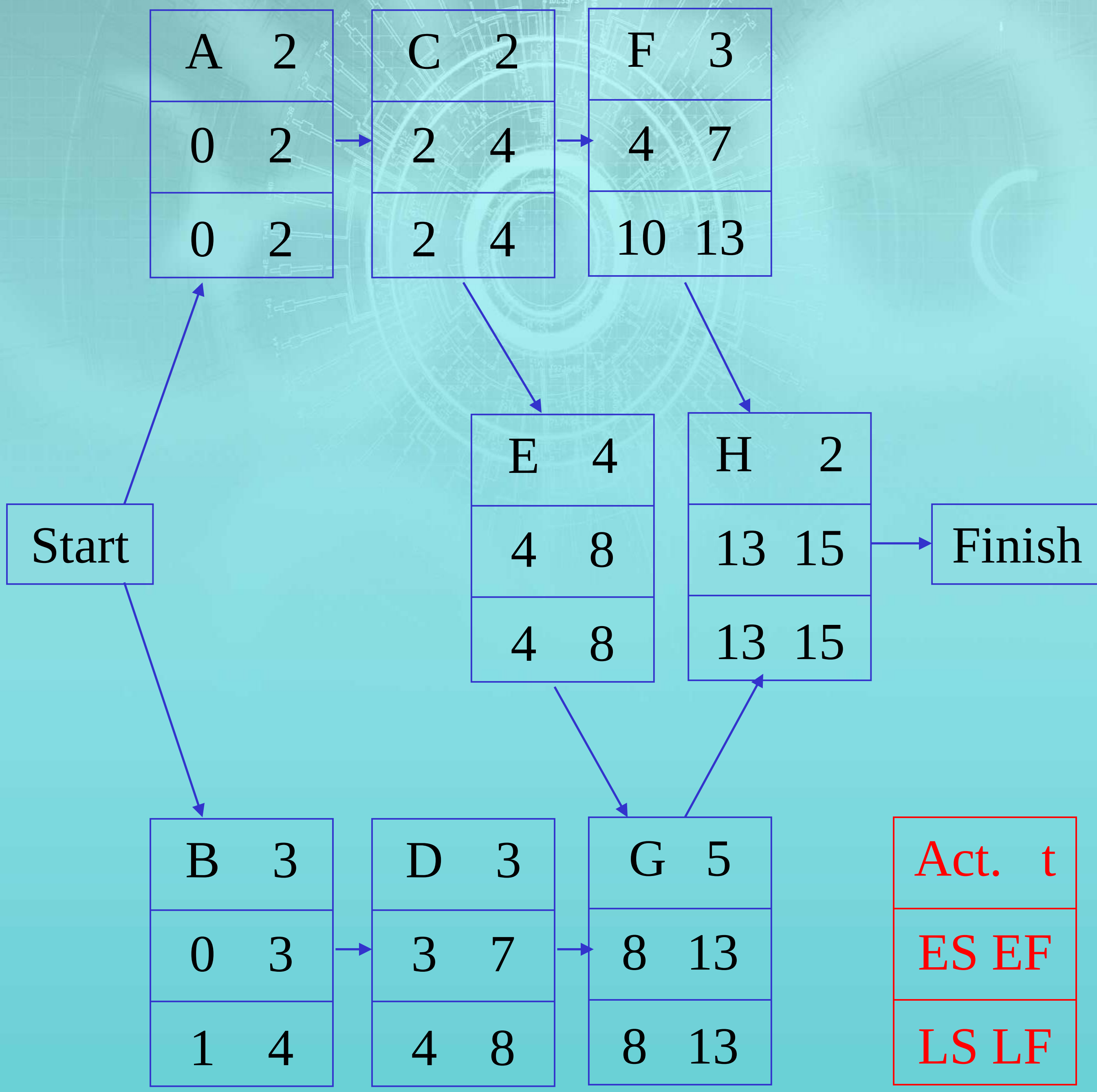
Latest Times – Calculations

- LS = LF - t
- LF = Smallest LS of activities that immediately follow

General Foundry, Inc. PERT Network ES/EF, LS/LF



General Foundry, Inc. PERT Network ES/EF, LS/LF



Slack

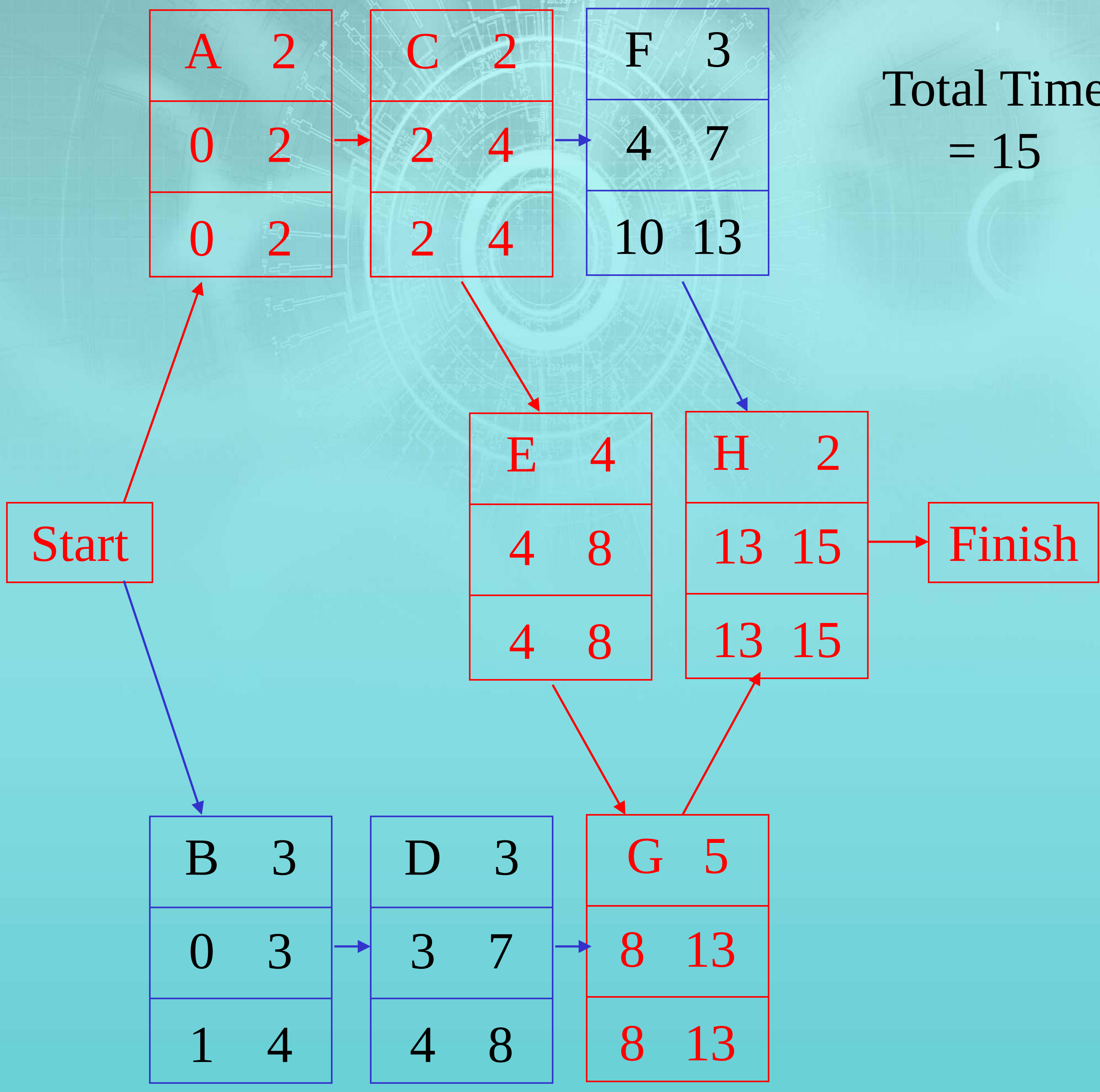
Slack indicates how long an activity may be delayed without delaying the entire project.

$$\text{Slack} = LS - ES$$
$$= LF - EF$$

General Foundry Schedule & Slacks

Activity	ES	EF	LS	LF	LS-ES	On Critical Path?
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
G	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes

General Foundry, Inc.
Critical Path



Project Variance

- The term "project variance" refers to the critical path variance since the time to complete the project is the time to complete the critical path. This is found by summing the variances of the activities on the critical path.

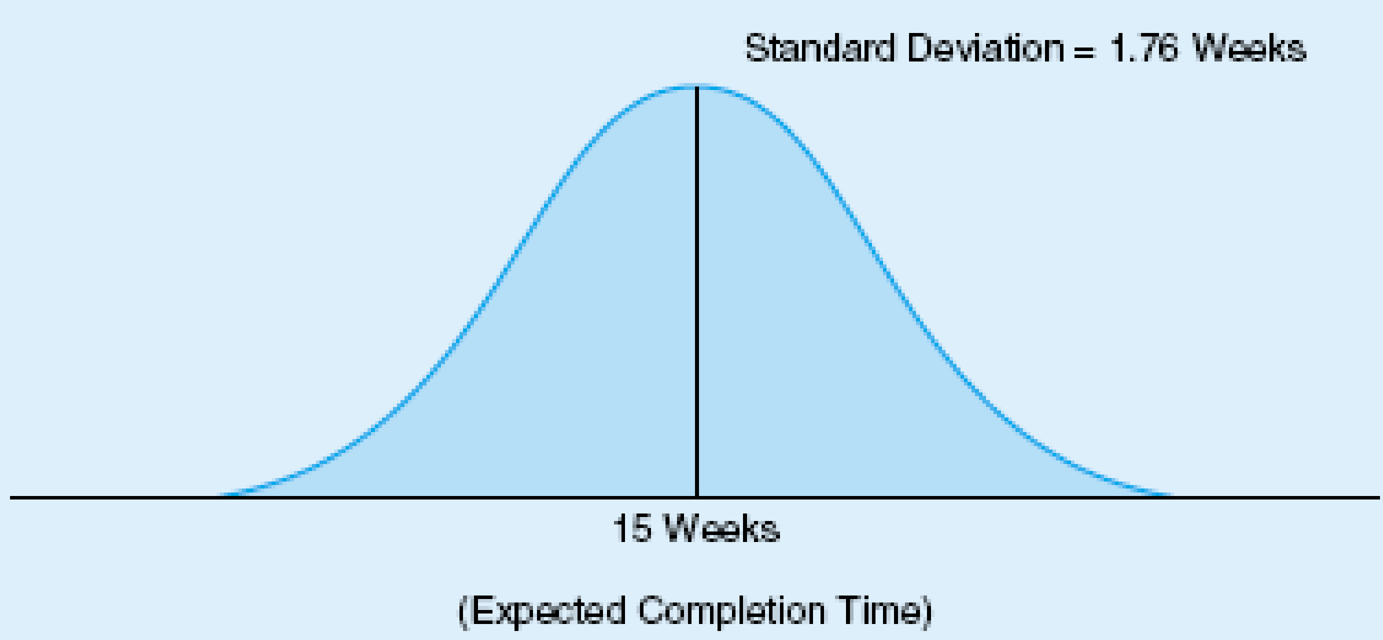
General Foundry, Inc. Project Variance

$$\sigma_{CP}^2 = \sigma_A^2 + \sigma_C^2 + \sigma_E^2 + \sigma_G^2 + \sigma_H^2$$

$$\sigma_{CP}^2 = \frac{4}{36} + \frac{4}{36} + \frac{36}{36} + \frac{64}{36} + \frac{4}{36} = 3.111$$

Probability Distribution for Project Completion Times

Computing the standard deviation



We know that the standard deviation is just the square root of the variance, so

$$\text{project standard deviation} = \sigma_T = \sqrt{\text{project variance}}$$

$$= \sqrt{3.11} = 1.76 \text{ weeks}$$

What PERT Was Able to Provide

- The project's expected completion date is 15 weeks.
- There is a 71.6% chance that the equipment will be in place within the 16-week deadline.
- PERT can easily find the probability of finishing by any date they are interested in.
- Five activities (*A*, *C*, *E*, *G*, *H*) are on the critical path.

What PERT Was Able to Provide *(continued)*

- If any one of them is delayed for any reason, the entire project will be delayed.
- Three activities (*B*, *D*, *F*) are not critical but have some slack time built in.
- This means that they can borrow from their resources, if needed, possibly to speed up the entire project.
- A detailed schedule of activity starting and ending dates has been made available.

Impact of a Change in Time for a Critical Path Activity

ACTIVITY TIME	SUCCESSOR ACTIVITY	PARALLEL ACTIVITY	PREDECESSOR ACTIVITY
Earliest start	Increase (decrease)	No change	No change
Earliest finish	Increase (decrease)	No change	No change
Latest start	Increase (decrease)	Increase (decrease)	No change
Latest finish	Increase (decrease)	Increase (decrease)	No change
Slack	No change	Increase (decrease)	No change

If the time it takes to complete activity *G* increases, there will be an increase in the earliest start, earliest finish, latest start, and latest finish times for all successor activities.

PERT/COST

- Although PERT is an excellent method of monitoring and controlling project length, it does not consider another very important factor, project *cost*.
- *PERT/Cost* is a modification of PERT that allows a manager to plan, schedule, monitor, and control cost as well as time.
- Using PERT/Cost to plan, schedule, monitor, and control project cost helps accomplish the sixth and final step of PERT.

Pert/Cost *(continued)*

- Two issues to be considered
  1. Investigating how costs can be planned and scheduled.
  2. Next, how costs can be monitored and controlled.

Planning and Scheduling Project Costs: Budgeting Process

- The overall approach in the budgeting process of a project is to determine how much is to be spent every week or month.
- This is accomplished in four basic budgeting steps.

Four Steps of the Budgeting Process

1. Identify all costs associated with each of the activities.
   - Then add these costs together to get one estimated cost or budget for each activity.
2. If you are dealing with a large project, several activities can be combined into larger work packages.
   - A *work package* is simply a logical collection of activities.

Four Steps of the Budgeting Process *(continued)*

3. Convert the budgeted cost per activity into a cost per time period.

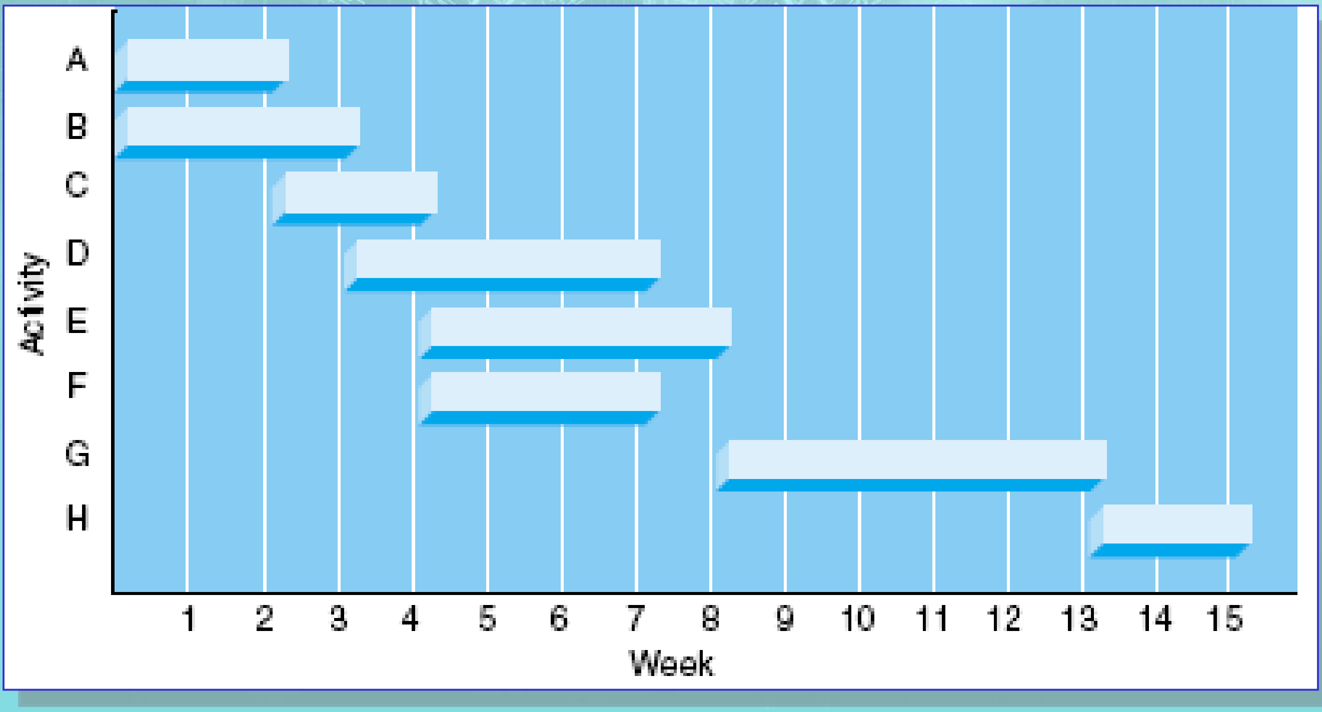
- To do this, we assume that the cost of completing any activity is spent at a uniform rate over time.
- Thus, if the budgeted cost for a given activity is $48,000 and the activity's expected time is four weeks, the budgeted cost per week is $12,000 (= $48,000/4 weeks).

Four Steps of the Budgeting Process - *continued*

4. Using the earliest and latest start times, find out how much money should be spent during each week or month to finish the project by the date desired.

An example of Budgeting for General Foundry follows next:

Gantt Chart for General Foundry (Example)

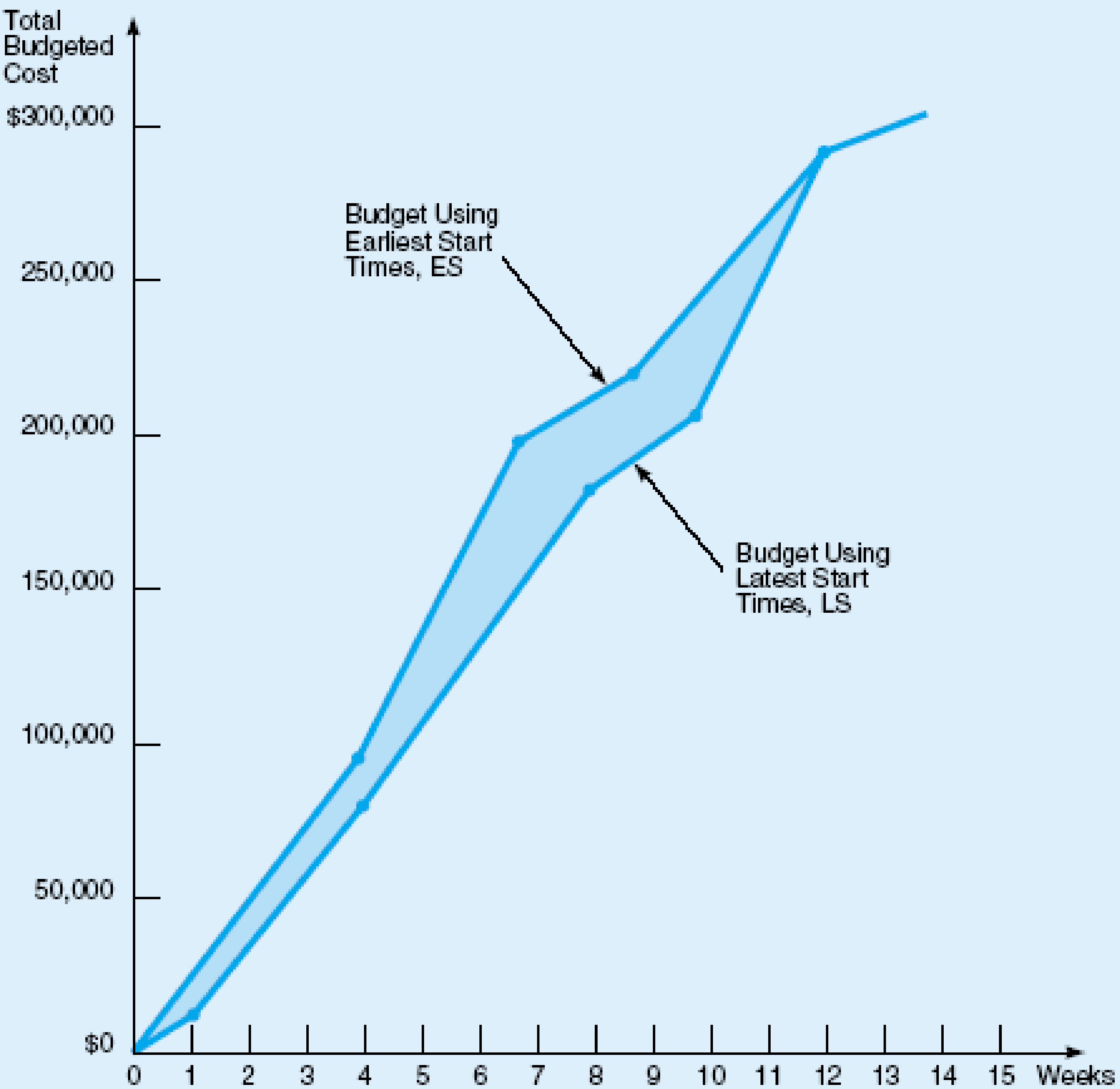


Activity Cost for General Foundry, Inc.

Activity	ES	LS	t	Total budgeted cost	Budgeted cost per week
A	0	0	2	22,000	11,000
B	0	1	3	30,000	10,000
C	2	2	2	26,000	13,000
D	3	4	4	48,000	12,000
E	4	4	4	56,000	14,000
F	4	10	3	30,000	10,000
G	8	8	5	80,000	16,000
H	13	13	2	16,000	8,000

The weekly budget for the project is developed from the data in above table. The earliest start time for activity *A*, for example, is 0. Because *A* takes 2 weeks to complete, its weekly budget of $11,000 should be spent in weeks 1 and 2.

Budget Ranges for General Foundry



Monitoring Costs

Act.	Budget	% Compl.	Value Compl.	Actual Cost	Difference
A	22	100	22	20	–2
B	30	100	30	36	6
C	26	100	26	26	0
D	48	10	4.8	6	1.2
E	56	20	11.2	20	8.8
F	30	20	6	4	–2
G	80	0	0	0	0
H	16	0	0	0	0

The value of work completed, or the cost ($1,000s) to date for any activity, can be computed as follows:

value of work completed = (% of work completed) x (total activity budget)

Critical Path Method (CPM)

- CPM is a *deterministic* network model.
- This means it assumes that both
  - the time to complete each activity, and
  - the cost of doing so are known with certainty.
- Unlike PERT, it does not employ probability concepts.
- Instead, CPM uses two sets of time and cost estimates for activities:
  1. A normal time and cost and
  2. A crash time and cost.

CPM *(continued)*

- The *normal time* estimate is like PERT's expected time.
- The *normal cost* is an estimate of cost to complete an activity in normal time.
- The *crash time* is the shortest possible activity time.
- *Crash cost* is the cost of completing the activity on a crash or deadline basis.
- The *critical path calculations* for a CPM network follow the same steps as used in PERT.
- All required is to just find the early start times (ES), late start times (LS), early finish (EF), late finish (LF), and slack.

Project Crashing with CPM: Four Steps

1. Find the normal critical path and identify the critical activities.
2. Compute the crash cost per week (or other time period) for all activities in the network.

This process uses the following formula:

$$\text{crash cost/time period} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

Project Crashing with CPM: Four Steps *(continued)*

3. Select the activity on the critical path with the smallest crash cost per week.
   - Crash this activity to the maximum extent possible or to the point at which your desired deadline has been reached.

4. Check to be sure that the critical path you were crashing is still critical.
   - Often, a reduction in activity time along the critical path causes a non-critical path or paths to become critical.
     - ✓ If the critical path is still the longest path through the network, return to step 3.
     - ✓ If not, find the new critical path and return to step 2.

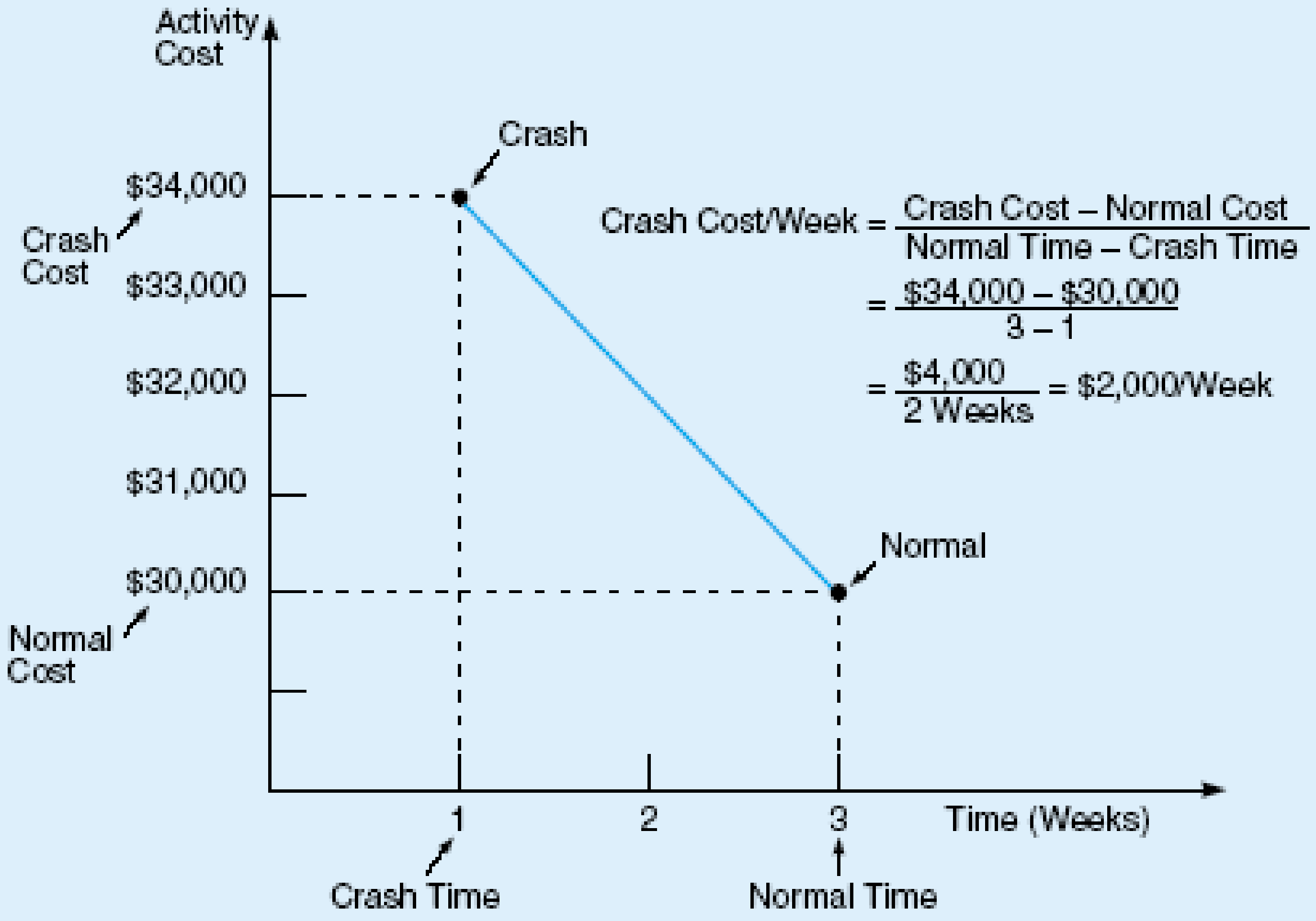
Normal and Crash Data for General Foundry, Inc.

ACTIVITY	TIME (WEEKS) NORMAL	TIME (WEEKS) CRASH	COST ($) NORMAL	COST ($) CRASH
A	2	1	22,000	23,000
B	3	1	30,000	34,000
C	2	1	26,000	27,000
D	4	3	48,000	49,000
E	4	2	56,000	58,000
F	3	2	30,000	30,500
G	5	2	80,000	86,000
H	2	1	16,000	19,000

ACTIVITY	CRASH COST PER WEEK ($)	CRITICAL PATH?
A	1,000	Yes
B	2,000	No
C	1,000	Yes
D	1,000	No
E	1,000	Yes
F	500	No
G	2,000	Yes
H	3,000	Yes

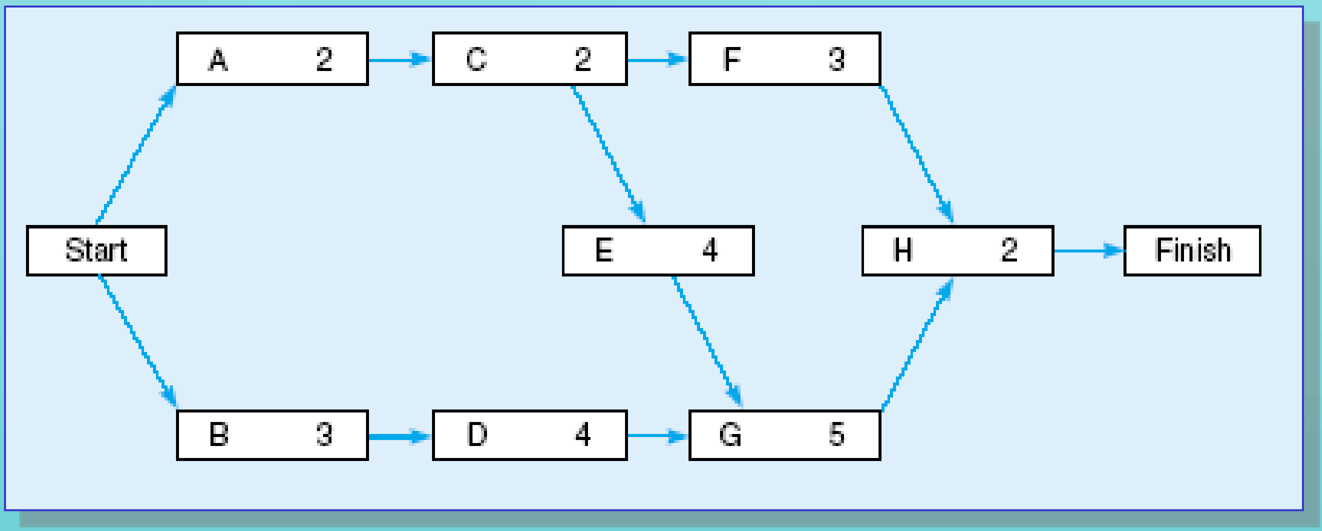
Crash costs are linear. If they are not, adjustments must be made.

Crash and Normal Times and Costs for Activity *B*

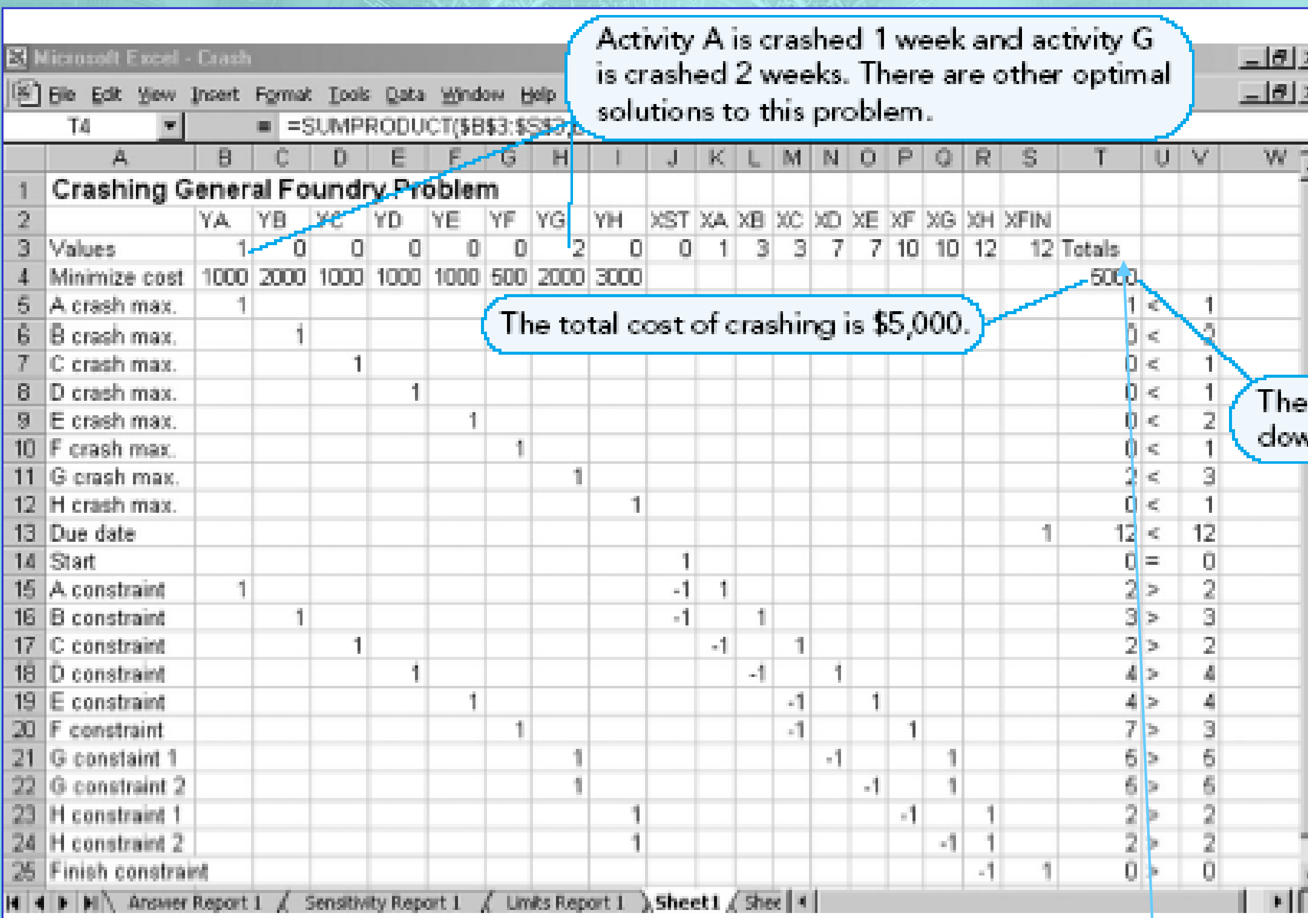


Project Crashing with Linear Programming

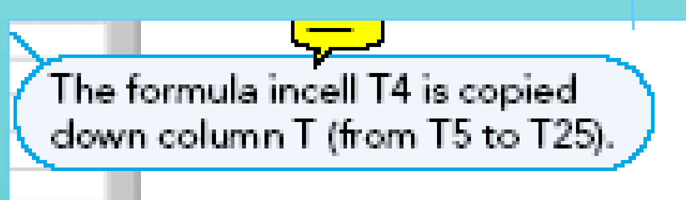
- Linear programming is another approach to finding the best project crashing schedule. We illustrate its use on General Foundry's network.
- The data needed are derived from Normal and Crash Data for General Foundry, Inc. (2 slides back) and General Foundry's Network with Activity Times (shown below):



Solution to Crashing Problem Using Solver in Excel



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Crashing General Foundry Problem																					
2		YA	YB	YC	YD	YE	YF	YG	YH	XST	XA	XB	XC	XD	XE	XF	XG	XH	XFIN			
3	Values	1	0	0	0	0	0	2	0	0	1	3	3	7	7	10	10	12	12	Totals		
4	Minimize cost	1000	2000	1000	1000	1000	500	2000	3000											5000		
5	A crash max.	1																		1	<	1
6	B crash max.		1																	0	<	2
7	C crash max.			1																0	<	1
8	D crash max.				1															0	<	1
9	E crash max.					1														0	<	2
10	F crash max.						1													0	<	1
11	G crash max.							1												2	<	3
12	H crash max.								1											0	<	1
13	Due date																		1	12	<	12
14	Start									1										0	=	0
15	A constraint	1								-1	1									2	>	2
16	B constraint		1							-1		1								3	>	3
17	C constraint			1							-1		1							2	>	2
18	D constraint				1							-1		1						4	>	4
19	E constraint					1							-1		1					4	>	4
20	F constraint						1						-1			1				7	>	3
21	G constaint 1							1						-1			1			6	>	6
22	G constraint 2							1							-1		1			6	>	6
23	H constraint 1								1							-1		1		2	>	2
24	H constraint 2								1								-1	1		2	>	2
25	Finish constraint																	-1	1	0	>	0



Other Topics in Project Management

- Subprojects
- Milestones
- Resource Leveling
- Software

Subprojects

- For extremely large projects, an activity may be made of several smaller sub-activities.
- Each activity might be viewed as a smaller project or a subproject of the original project.
- The person in charge of the activity might wish to create a PERT/CPM chart for managing this subproject.
- Many software packages have the ability to include several levels of subprojects.

Milestones

- Major events in a project are often referred to as *milestones*.
- These are often reflected in *Gantt charts* and PERT charts to highlight the importance of reaching these events.

example

activity	optimistic time	pessimistic time	most likely time
1,2	2	8	5
2,3	3	15	6
2,4	1	3	2
3,5	5	11	8
3,6	2	6	4
4,5	1	9	5
4,6	4	8	6
5,6	2	16	9

Example (cont)

- -draw the network diagram
- -find the expected times of activities
- -find the minimum copletion time of the project
- -what is the probability that this project can be conpleted in 25 weeks and can not be completed in 32 weeks

example

Activity	Predecessor activity	normal time	normal cost	crash time	crash cost
A	-	2	120	0,8	180
B	A	3	400	1	780
C	-	2	170	0,4	220
D	C	2	240	0,6	360
E	B,D	1	150	0,2	480
F	E	2	350	0,6	600
G	B,D	4	410	2	740
H	G	4	540	1,8	900
I	F,H	2	600	0,4	1000

Example (cont)

- Network diagram
- Min completion time
- Critical path
- Write LP model that finds the way to coplete the project in 12 months with minimum crash cost
- Make the budget arrangement that will be necessary for each month

example

Activity	Activity Description	Immediate Predecessors	optimistic time	most likely time	pessimistic time	Crash Time	Normal Cost (1000)	Crash cost (1000)
A	Excavate	—	1	2	3	1	150	280
B	Lay the foundation	*A*	2	3,5	8	2	300	410
C	Put up the rough wall	*B*	6	9	18	7	600	850
D	Put up the roof	*C*	4	5,5	10	4	250	350
E	Install the exterior plumbing	*C*	1	4,5	5	3	400	580
F	Install the interior plumbing	*E*	4	4	10	3	180	250
G	Put up the exterior siding	*D*	5	6,5	11	4	900	1000
H	Do the exterior painting	*E, G*	5	8	17	6	200	380
I	Do the electrical work	*C*	3	7,5	9	5	210	280
J	Put up the wallboard	*F, I*	3	9	9	6	400	500
K	Install the flooring	*J*	4	4	4	3	150	200
L	Do the interior painting	*J*	1	5,5	7	3	250	350
M	Install the exterior fixtures	*H*	1	2	3	1	100	200
N	Install the interior fixtures	*K, L*	5	5,5	9	3	340	500

Example (cont)

- **Activity list for the Reliable Construction Co. Project (for questions 4 - 8)**
- **4. draw the network diagram**
- **5. find the minimum completion time of project by using the expected times of activities**
- **6. find the variance of the project**
- **7. find critical path activities**
- **8. how the project can be completed in 35 weeks with the minimum crash cost? (write a LP model and solve it)**