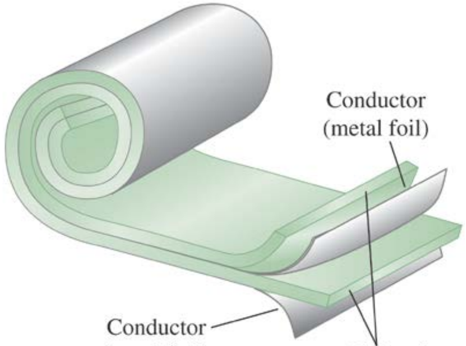
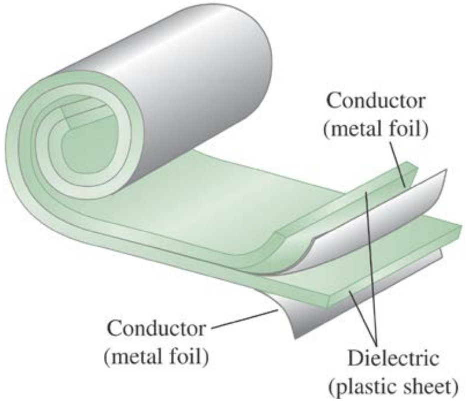
Chp 24: Capacitance and Dielectrics

II. part



Dielectrics

A *dielectric* is a nonconducting material. Most capacitors have dielectric between their plates. Dielectric *increases* the maximum potential differences ➔ *increases* capacitance and the energy density by a factor K. The *dielectric constant* of the material is $K = C/C_0 > 1$.



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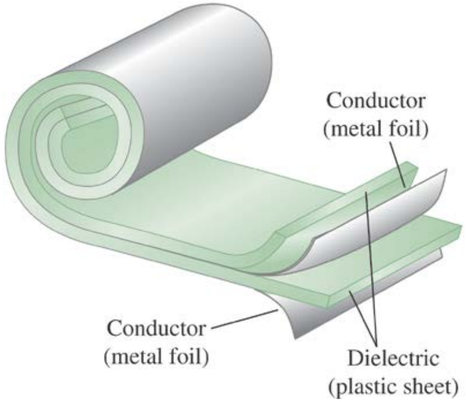
$$Q = C_0 V_0 = CV \quad \rightarrow \quad V = \frac{V_0}{K}$$



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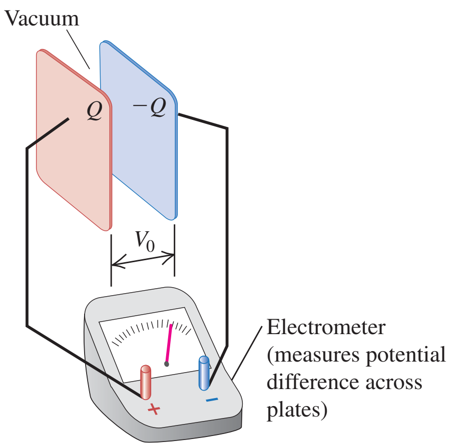

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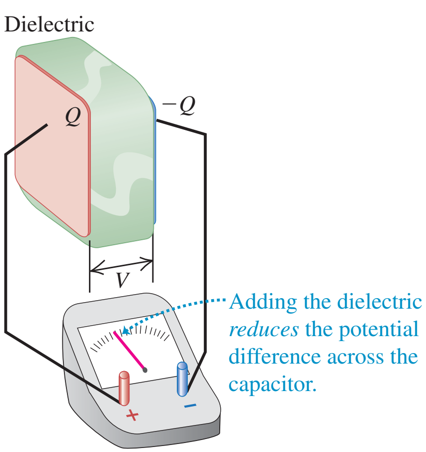

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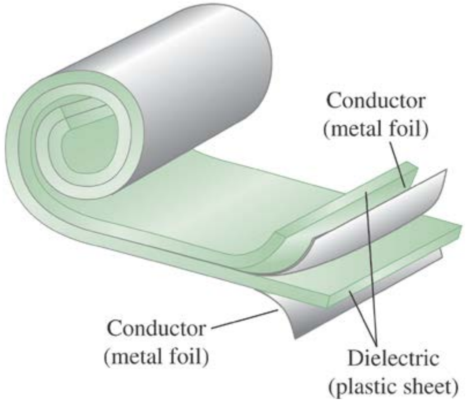

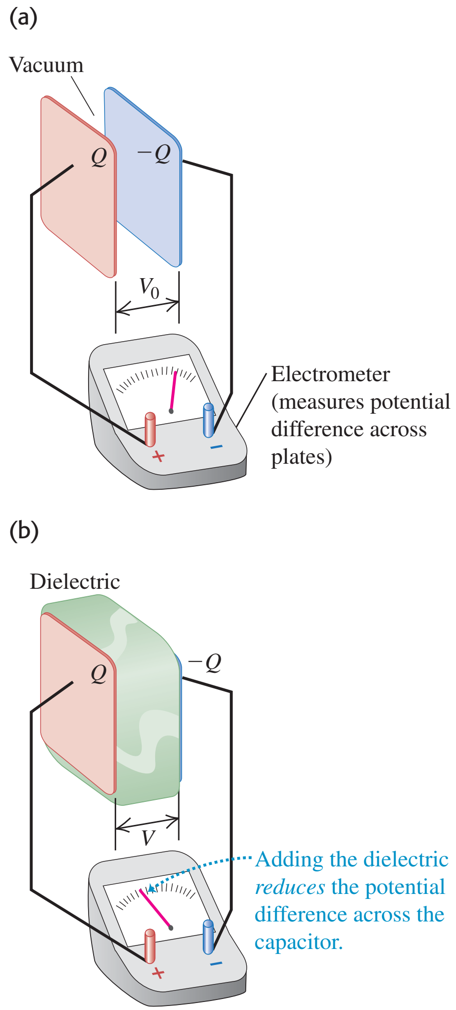

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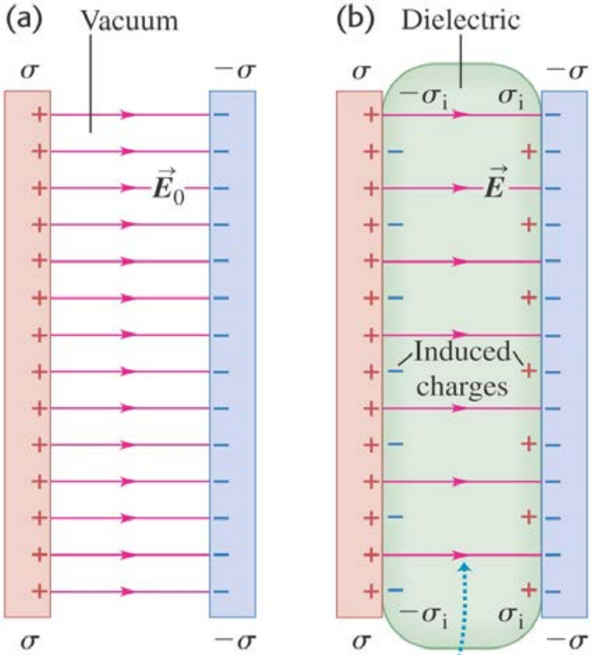
Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Induced Charge and Polarization



For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.

When a dielectric material is inserted between the plates while the charge is kept constant, the potential difference between the plates decreases by a factor K. Therefore the electric field between the plates must decrease by the same factor. If E_0 is the vacuum value and E is the value with the dielectric, then

$$E = \frac{E_0}{K} \qquad (\text{when } Q \text{ is constant})$$

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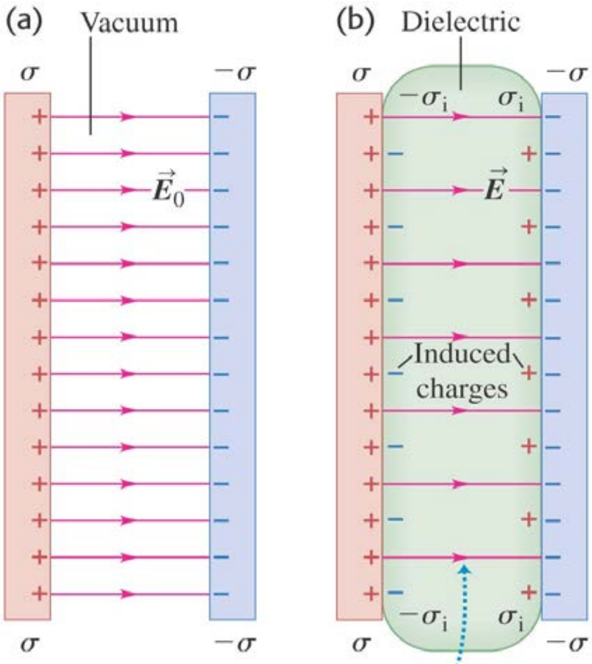

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Since the electric-field magnitude is smaller when the dielectric is present, the surface charge density (which causes the field) must be smaller as well. The surface charge on the conducting plates does not change, but an *induced* charge of the opposite sign appears on each surface of the dielectric. The dielectric was originally electrically neutral and is still neutral; the induced surface charges arise as a result of *redistribution* of positive and negative charge within the dielectric material, a phenomenon called **polarization.**

$$E = \sigma_{\text{net}}/\epsilon_0 \rightarrow \quad E_0 = \frac{\sigma}{\epsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\epsilon_0} \quad \rightarrow \quad E = \frac{\sigma}{\epsilon}$$

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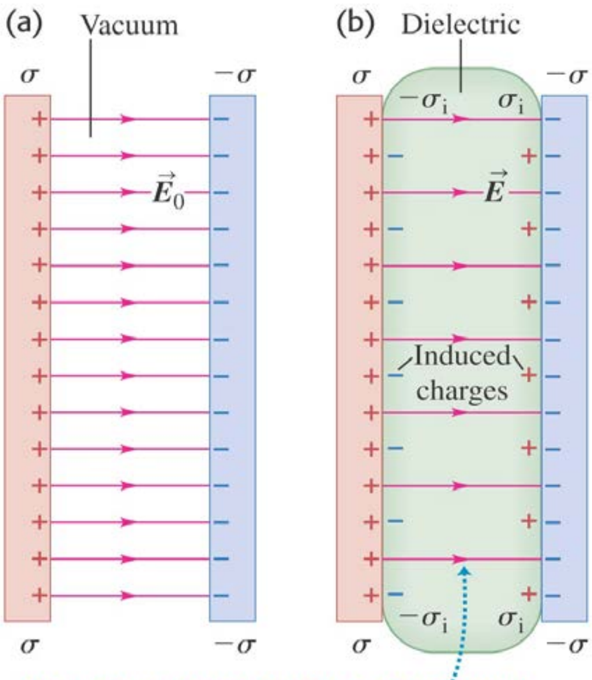

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$$\rightarrow \quad \sigma_i = \sigma\left(1 - \frac{1}{K}\right) \qquad \text{(induced surface charge density)}$$

The product $K\varepsilon_0$ is called the **permittivity** of the dielectric, denoted by ε:

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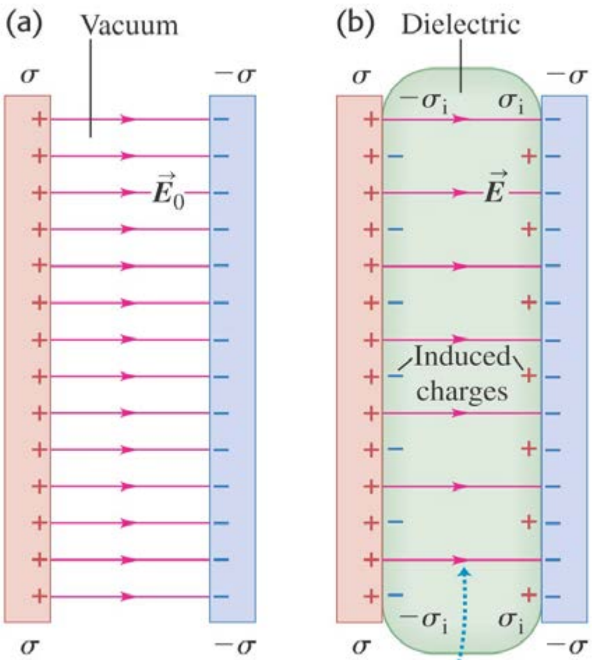

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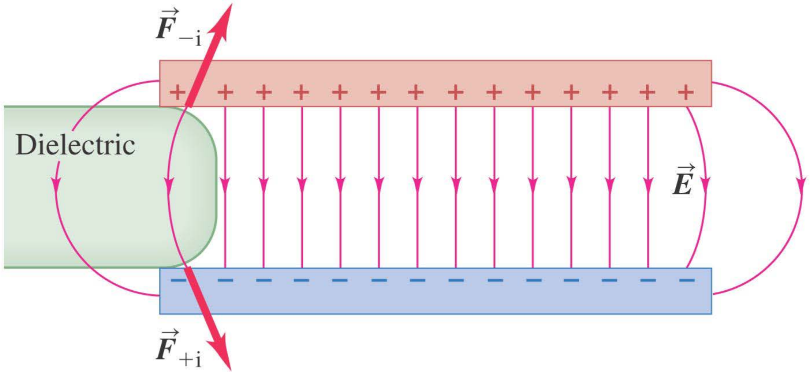
$$\epsilon = K\epsilon_0$$

$$C = KC_0 = K\epsilon_0\frac{A}{d} = \epsilon\frac{A}{d} \qquad \text{(parallel-plate capacitor, dielectric between plates)}$$

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \qquad \text{(electric energy density in a dielectric)}$$

Examples with and without a dielectric



Each parallel plates have an area of 2000 cm^2 and are 1 cm apart. We connect the capacitor to a power supply, charge it to a potential difference V_0 = 3000 V and disconnect the power supply. We then insert a sheet of insulating plastic material between the plates, completely filling the space between them. We find that the potential difference decreases to 100 kV while the charge on each capacitor plate remains constant. Find the energy stored in the electric field of a capacitor and the energy density, both before and after the dielectric sheet is inserted.

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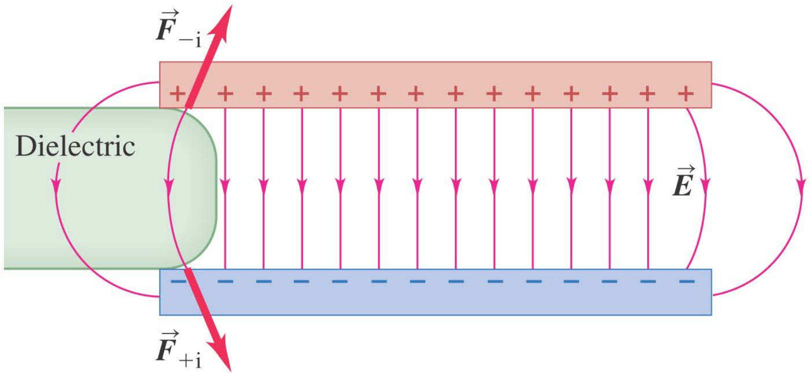

$$V = \frac{V_0}{K}$$

➔ K = 3 kV/1 kV = 3

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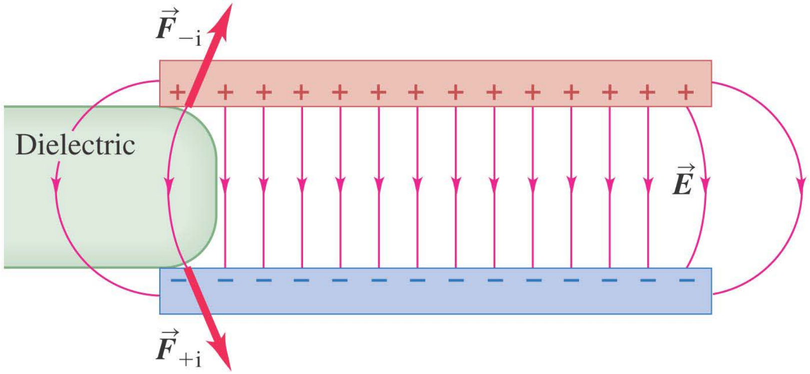

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$$C_0 = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \text{F/m}) \frac{2.00 \times 10^{-1}\ \text{m}^2}{1.00 \times 10^{-2}\ \text{m}}$$
$$= 1.77 \times 10^{-10}\ \text{F} = 177\ \text{pF}$$

$$Q = C_0 V_0 = (1.77 \times 10^{-10}\ \text{F})(3.00 \times 10^3\ \text{V})$$
$$= 5.31 \times 10^{-7}\ \text{C} = 0.531\ \mu\text{C}$$

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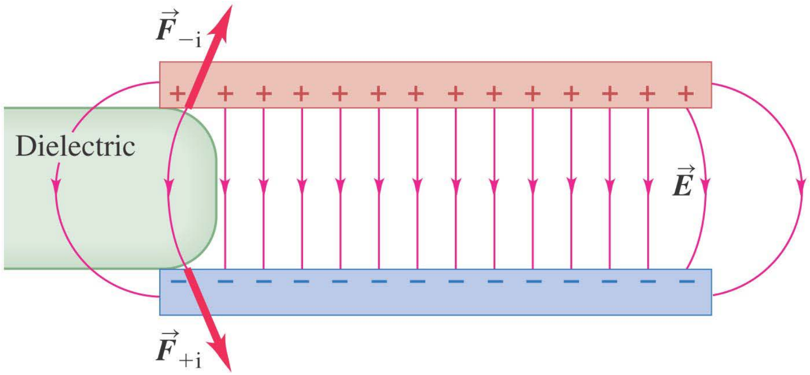

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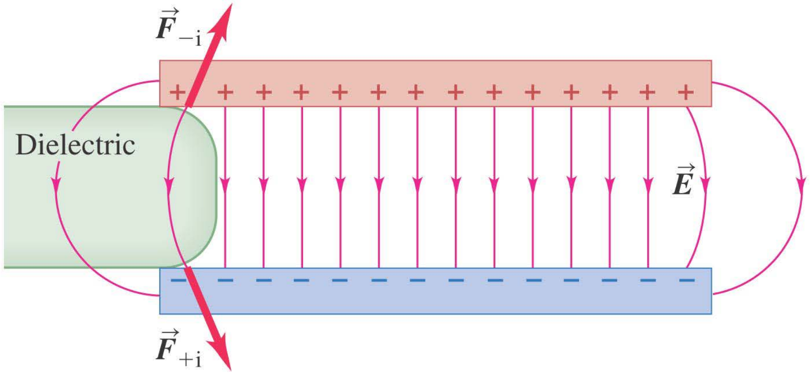

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After the dielectric inserted

$$E = \frac{V}{d} = \frac{1000\ \mathrm{V}}{1.00 \times 10^{-2}\ \mathrm{m}} = 1.00 \times 10^{5}\ \mathrm{V/m}$$

$$U_0 = \tfrac{1}{2} C_0 V_0^2 = \tfrac{1}{2}(1.77 \times 10^{-10}\ \mathrm{F})(3000\ \mathrm{V})^2 = 7.97 \times 10^{-4}\ \mathrm{J}$$

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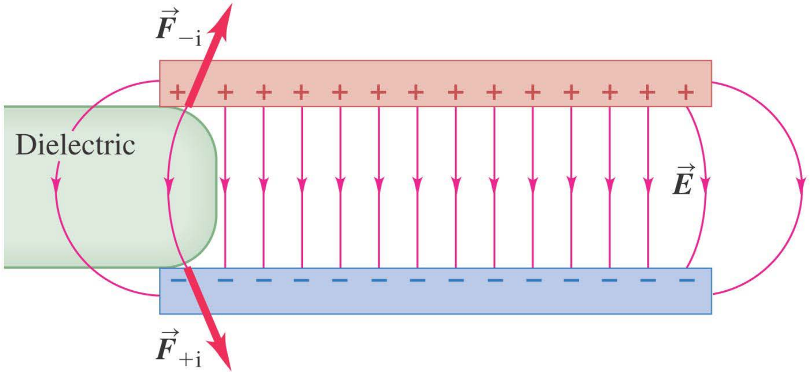

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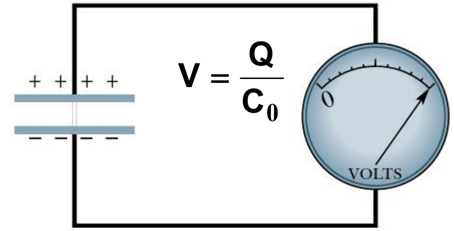
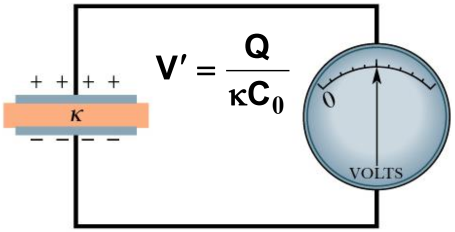
What happens as you insert a dielectric?

Initially, charge capacitor C_0 to voltage V, charge Q, field E_{net}.



- With battery detached insert dielectric
- Q remains constant, E_{net} is reduced
- Voltage (fixed Q) drops to V'

Dielectric reduced E_{net} and V



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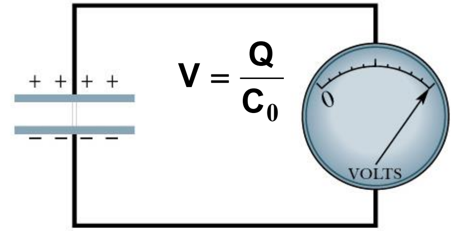

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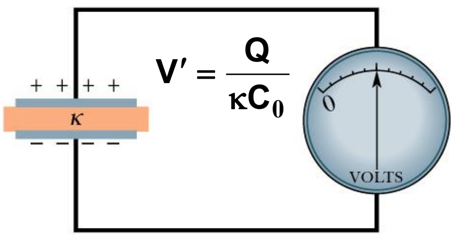

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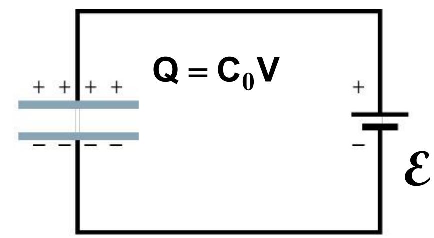
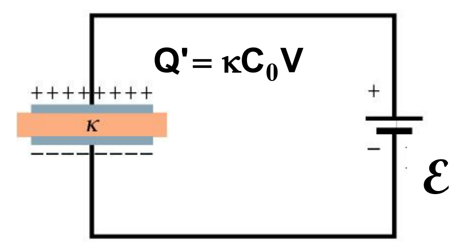


- With battery attached, insert dielectric.
- E_{net} and V are momentarily reduced but battery maintains voltage E
- Charge flows to the capacitor as dielectric is inserted until V and E_{net} are back to original values.



V = a constant

Q' > Q

Dielectric breakdown

•If the electric field is strong enough, *dielectric breakdown* occurs and the dielectric becomes a conductor. This occurs when the electric field is so strong that electrons are ripped loose from their molecules and crash into other molecules, liberating even more electrons. This avalanche of moving charge forms a spark or arc discharge. Lightning is a dramatic example of dielectric breakdown in air (3×10^6 V/m).

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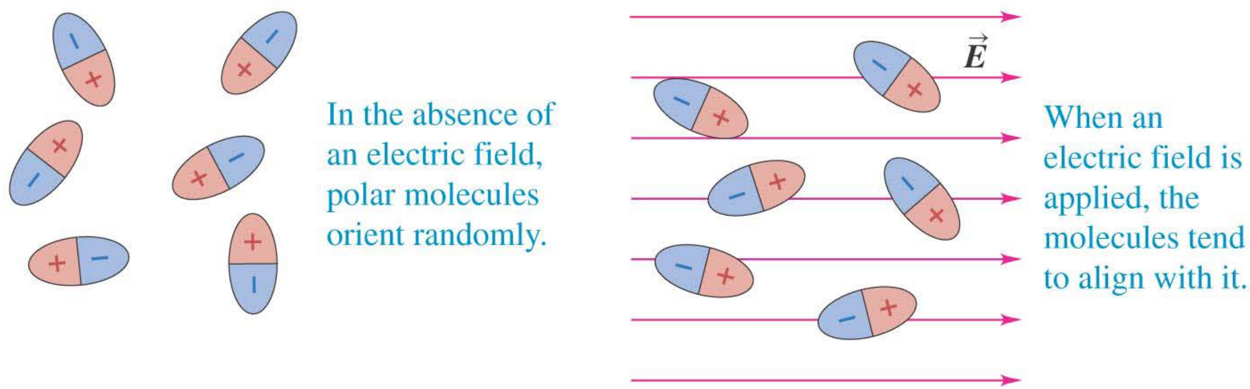
• The *dielectric strength* is the maximum electric field the material can withstand before breakdown occurs.

• Table below shows the dielectric strength of some insulators.

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

Molecular model of induced charge - I

Previously we discussed **induced surface charges** on a *dielectric* in an electric field. Now let's look at how these surface charges can arise. If the material were a *conductor*, the answer would be simple. Conductors contain charge that is free to move, and when an electric field is present, some of the charge redistributes itself on the surface so that there is no electric field inside the conductor. But an **ideal dielectric has *no* charges that are free to move**, so surface charge occur due to polarization.



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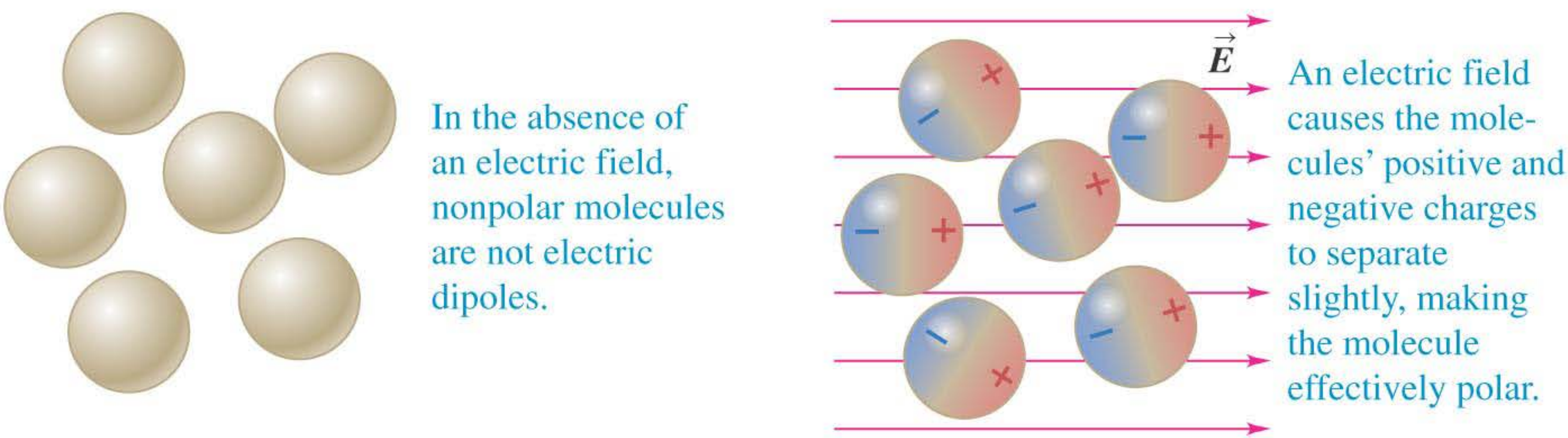
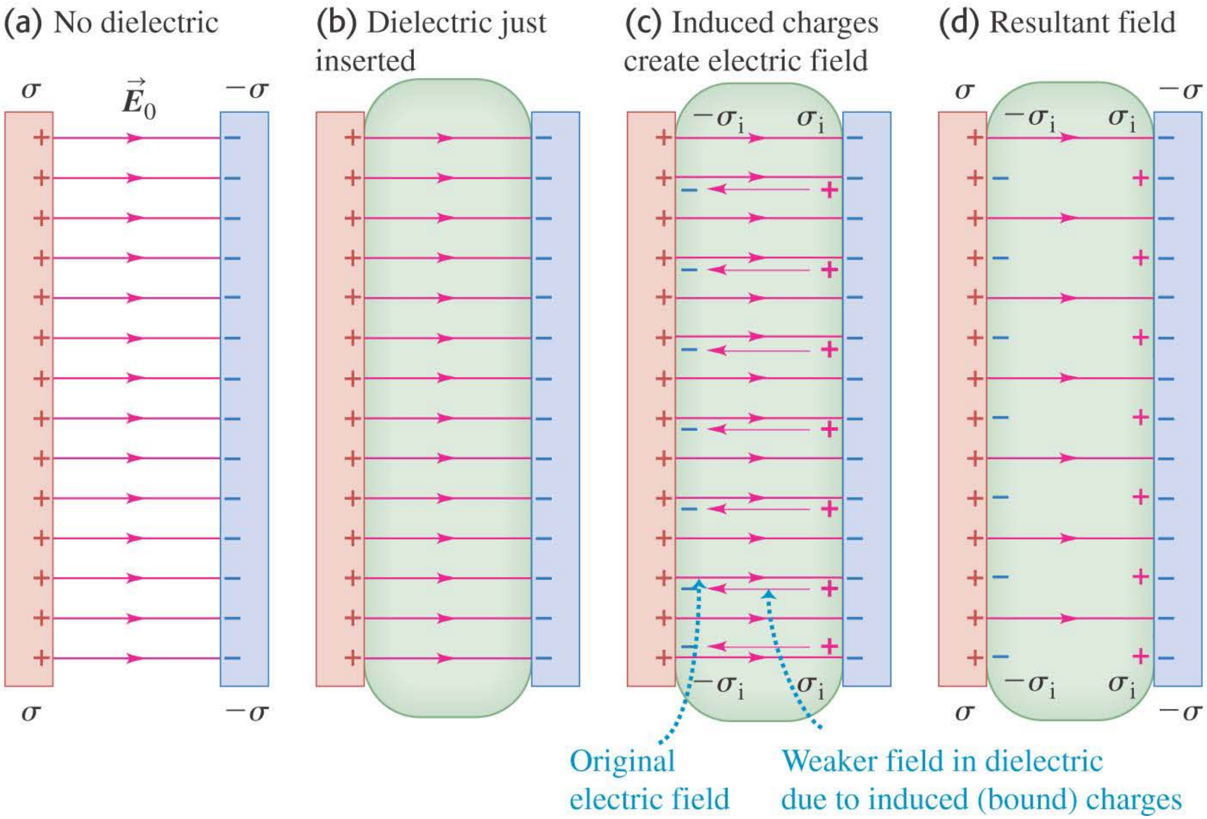
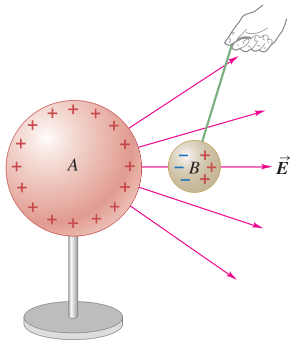


Molecular model of induced charge - II

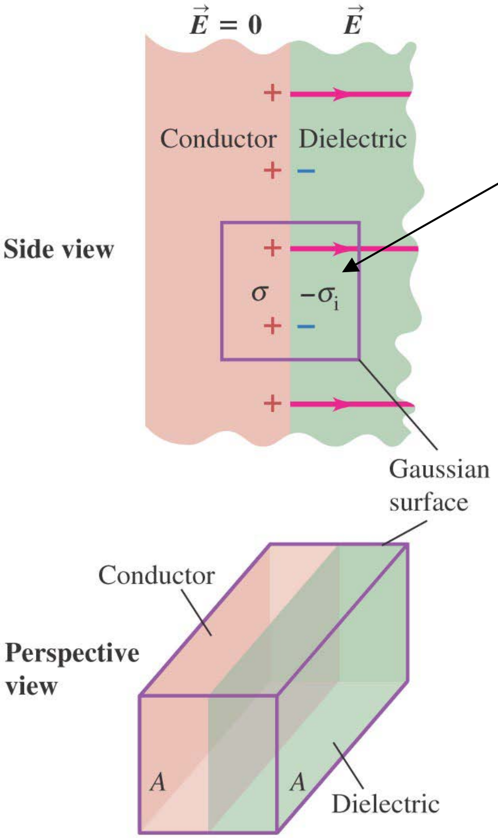
Figure below shows *polarization* of the dielectric and how the induced charges reduce the magnitude of the resultant electric field.





A neutral sphere *B* in the radial electric field of a positively charged sphere *A* is attracted to the charge because of polarization.

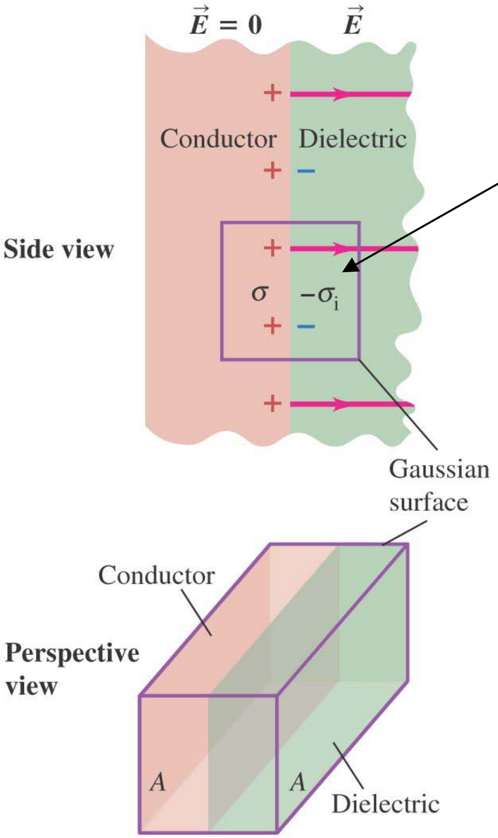
Gauss's law in dielectrics



$$Q_{\text{encl}} = (\sigma - \sigma_i)A$$

Gauss's Law ➔ $$EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0}$$

Gauss's law in dielectrics



$$Q_{encl} = (\sigma - \sigma_i)A$$

Gauss's Law ➔ $$EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0}$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right) \quad \text{or} \quad \sigma - \sigma_i = \frac{\sigma}{K}$$

➔ $$EA = \frac{\sigma A}{K\epsilon_0} \quad \text{or} \quad KEA = \frac{\sigma A}{\epsilon_0}$$

$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{encl\text{-}free}}{\epsilon_0} \qquad \text{(Gauss's law in a dielectric)}$$