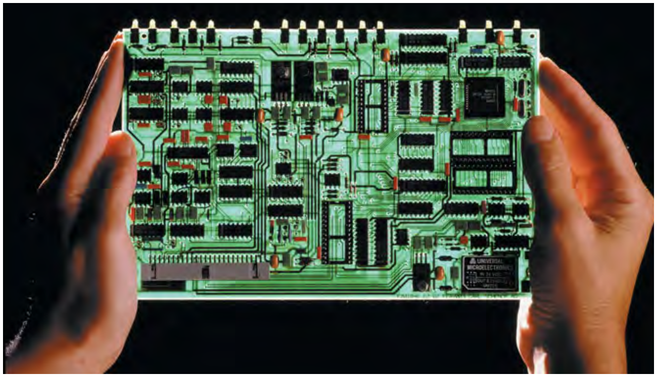
Chp 26: Direct Current Circuits

Goals for Chapter 26

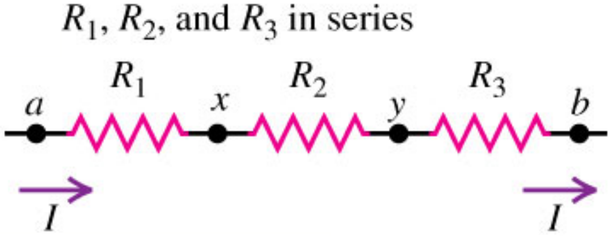
- To analyze circuits having resistors in series and parallel
- To apply Kirchhoff's rules to multiloop circuits
- To learn how to use various types of meters in a circuit
- To analyze circuits containing capacitors and resistors
- To study power distribution in the home

Resistors in series and parallel

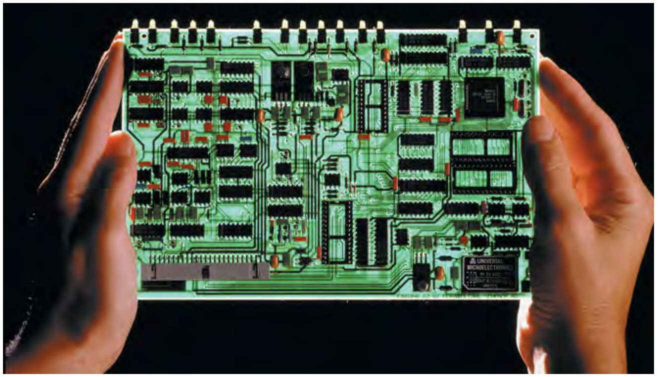


•Resistors are in *series* if they are connected one after the other so the current is the same in all of them.

•The *equivalent resistance* of a series combination is the *sum* of the individual resistances: $R_{eq} = R_1 + R_2 + R_3 + \ldots$



Resistors in series and parallel



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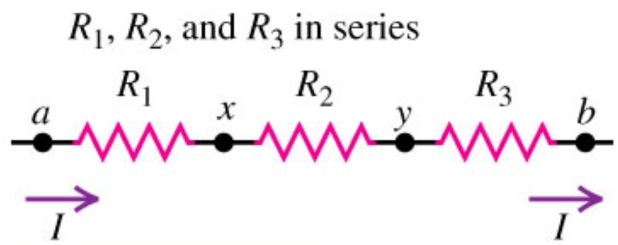
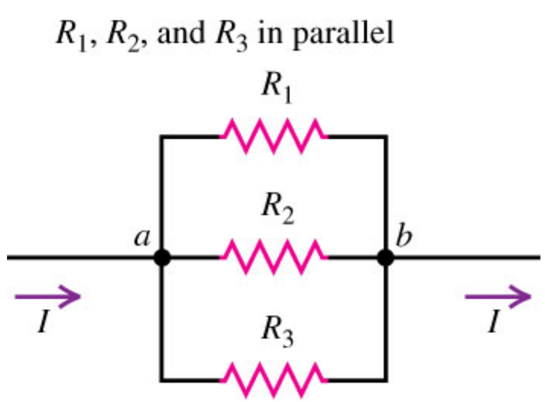
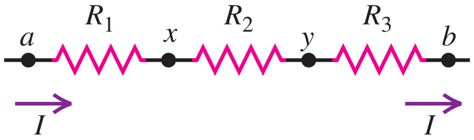


Resistors are in *parallel* if they are connected so that the potential difference must be the same across all of them (see right figure below).
The *equivalent resistance* of a parallel combinaton is given by $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + ...$



(a) R_1, R_2, and R_3 in series



$$V_{ax} = IR_1 \qquad V_{xy} = IR_2 \qquad V_{yb} = IR_3$$

$$\Rightarrow \quad V_{ab} = V_{ax} + V_{xy} + V_{yb} = I(R_1 + R_2 + R_3)$$

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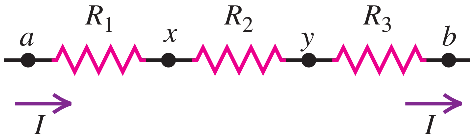

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$$\frac{V_{ab}}{I} = R_1 + R_2 + R_3 \qquad R_{eq} = \frac{V_{ab}}{I} \quad \Rightarrow \quad R_{eq} = R_1 + R_2 + R_3$$

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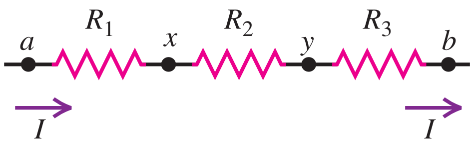

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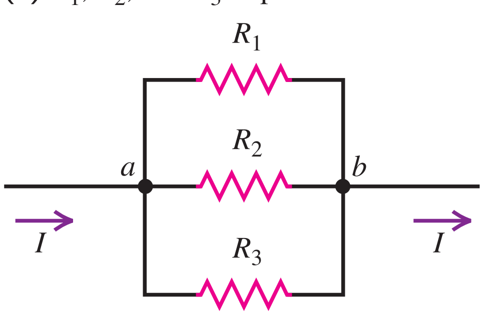
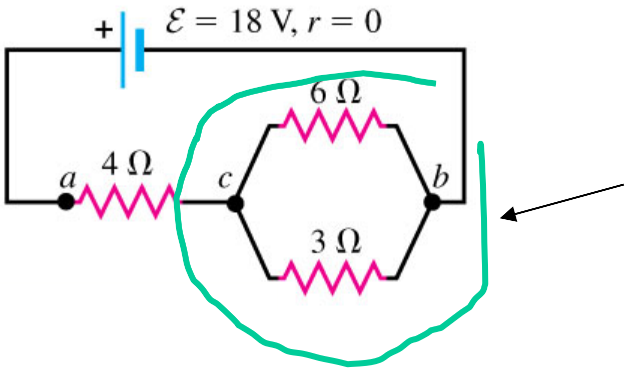


$$I_1 = \frac{V_{ab}}{R_1} \qquad I_2 = \frac{V_{ab}}{R_2} \qquad I_3 = \frac{V_{ab}}{R_3}$$

$$I = I_1 + I_2 + I_3 = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

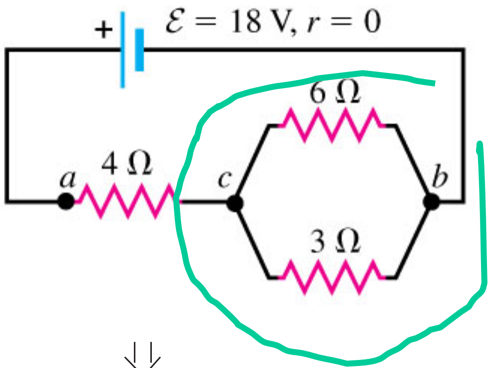
$$\rightarrow \quad \frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Equivalent resistance



(a) $$\frac{1}{R_{6\ \Omega+3\ \Omega}} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega} = \frac{1}{2\ \Omega}$$

Equivalent resistance



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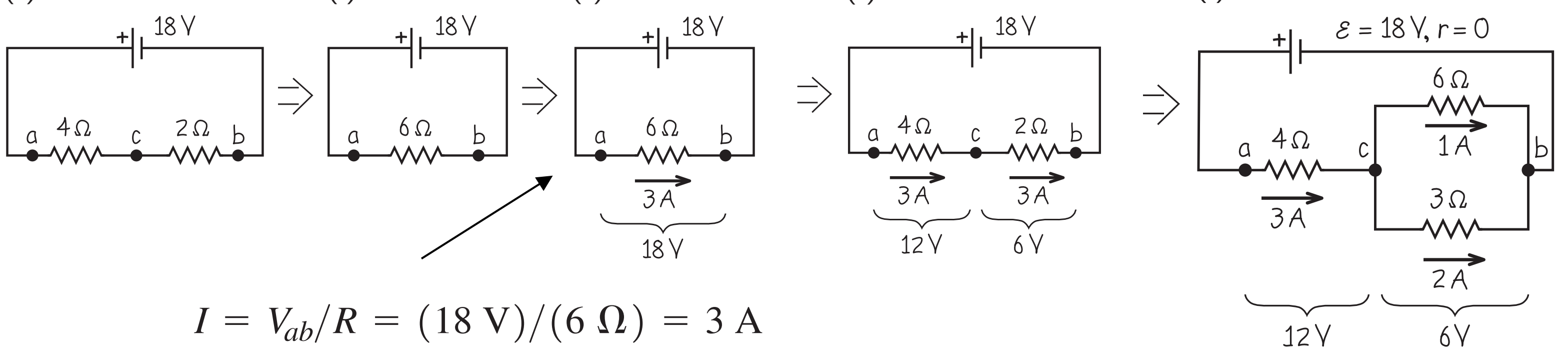
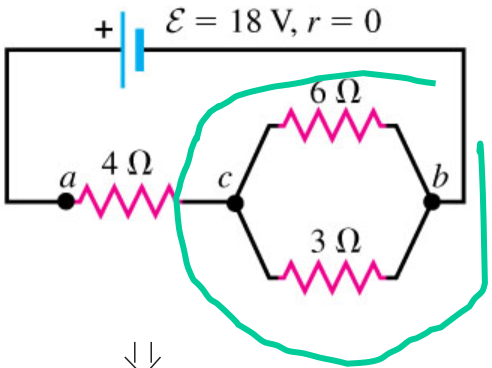


$$I = V_{ab}/R = (18\text{ V})/(6\ \Omega) = 3\text{ A}$$

Equivalent resistance



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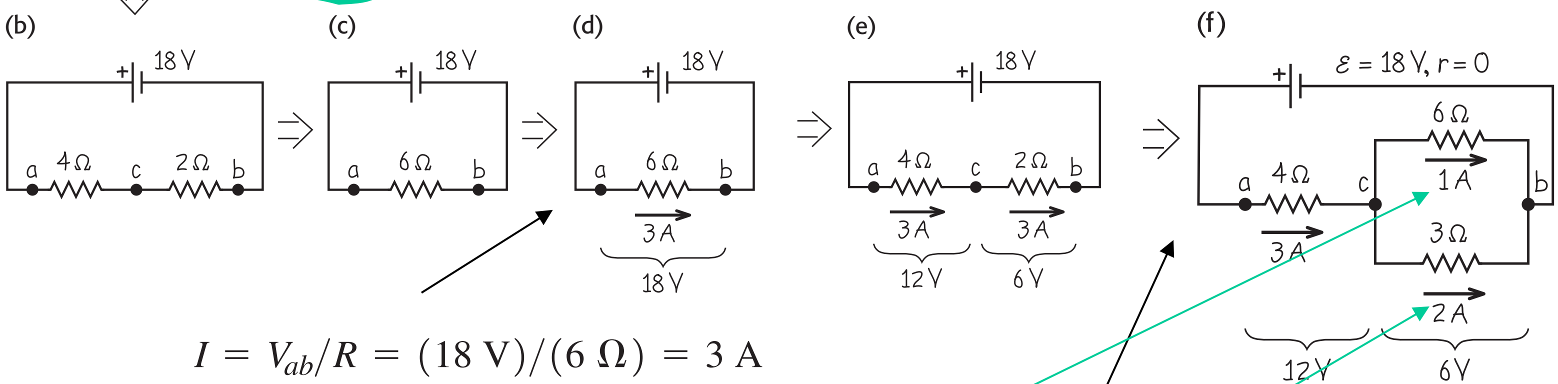


$$I = V_{ab}/R = (18\ \text{V})/(6\ \Omega) = 3\ \text{A}$$

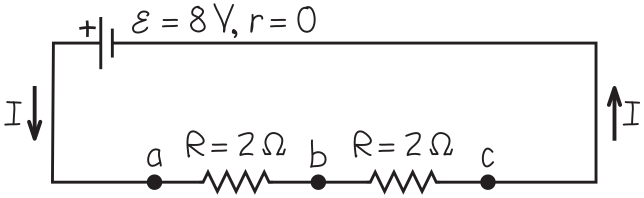
$$\rightarrow\ V_{cb} = IR = (3\ \text{A})(2\ \Omega) = 6\ \text{V}.$$

$$I = V_{cb}/R \begin{cases} (6\ \text{V})/(6\ \Omega) = 1\ \text{A} \\ (6\ \text{V})/(3\ \Omega) = 2\ \text{A} \end{cases}$$

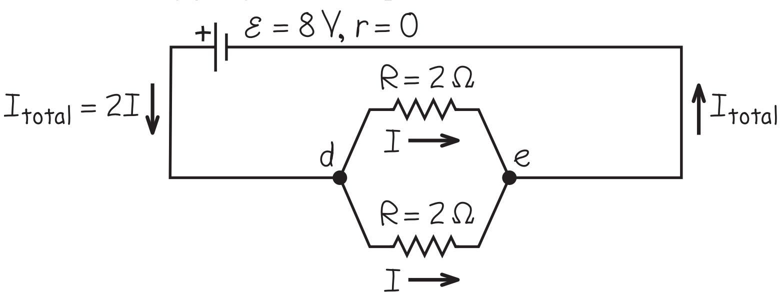
Series versus parallel combinations

Two identical light bulbs, each with resistance $R = 2\ \Omega$, are connected to a source with $\mathcal{E} = 8$ V and negligible internal resistance. Find the current through each bulb, the potential difference across each bulb, and the power delivered to each bulb and to the entire network if the bulbs are connected (a) in series and (b) in parallel. (c) Suppose one of the bulbs burns out; that is, its filament breaks and current can no longer flow through it. What happens to the other bulb in the series case? In the parallel case?

(a) Light bulbs in series



(b) Light bulbs in parallel



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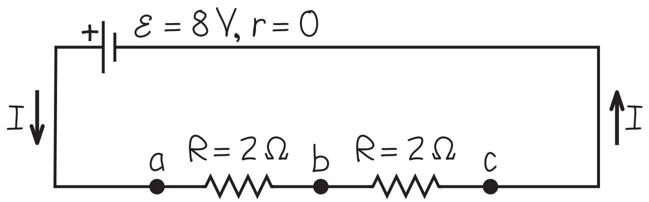
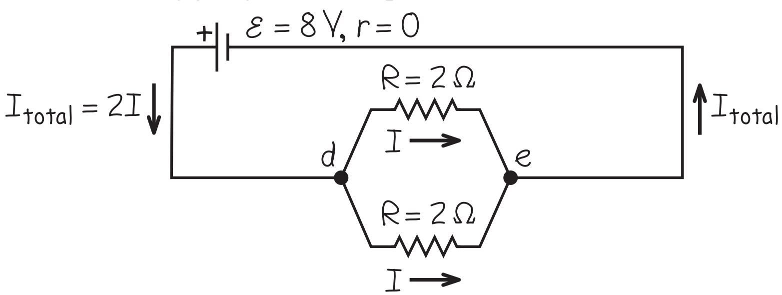


$$I = \frac{V_{ac}}{R_{eq}} = \frac{8\ \text{V}}{4\ \Omega} = 2\ \text{A}$$

$$V_{ab} = V_{bc} = IR = (2\ \text{A})(2\ \Omega) = 4\ \text{V}$$

$$\Rightarrow \quad P = I^2R = (2\ \text{A})^2(2\ \Omega) = 8\ \text{W}$$

(b) Light bulbs in parallel



$$I = \frac{V_{de}}{R} = \frac{8\ \text{V}}{2\ \Omega} = 4\ \text{A}$$

$$\Rightarrow \quad P = I^2R = (4\ \text{A})^2(2\ \Omega) = 32\ \text{W}$$

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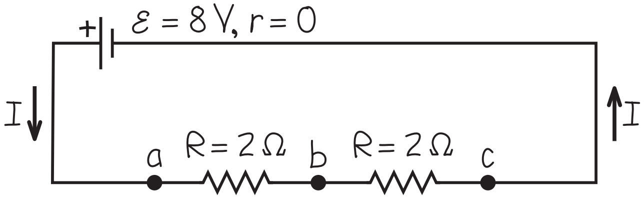

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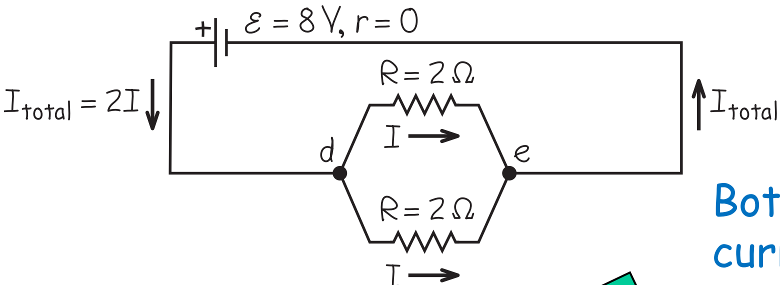

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So, is it free energy?

Both the potential difference across each bulb and the current through each bulb are twice as great. Hence the power delivered to each bulb is *four* times greater, and each bulb is brighter. The total power delivered to the parallel network is $P_{total} = 2P = 64$ W, four times greater

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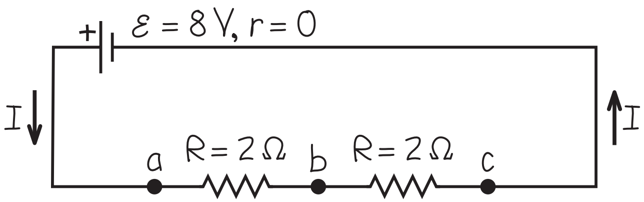

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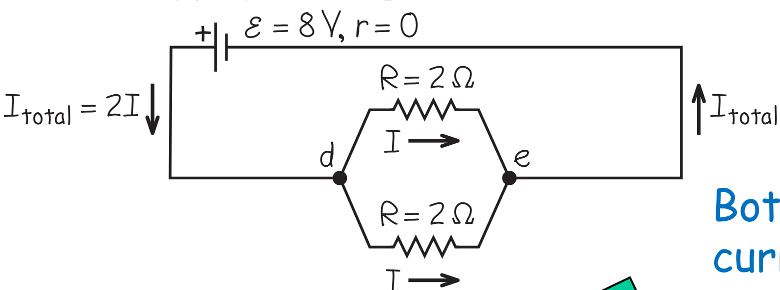

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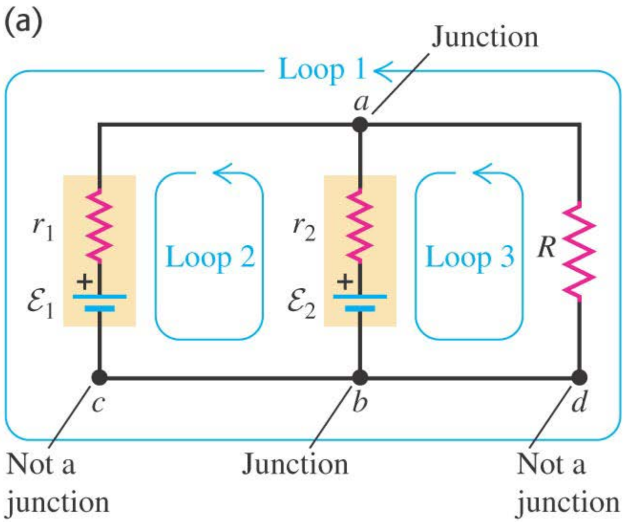
No it is not free energy: energy is extracted from the source four times more rapidly in the parallel case than in the series case. If the source is a battery, it will be used up four times as fast.

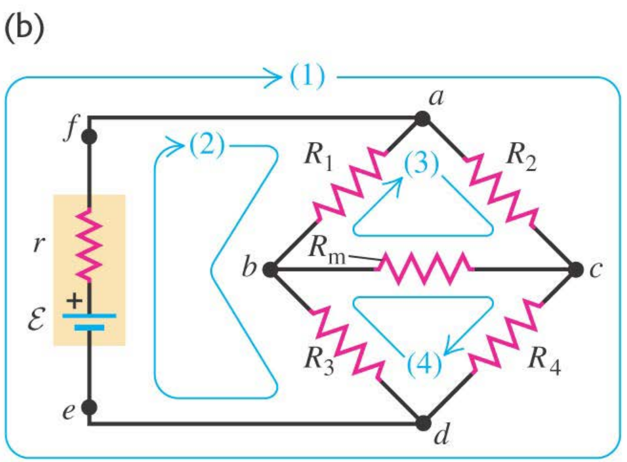
Kirchhoff's Rules

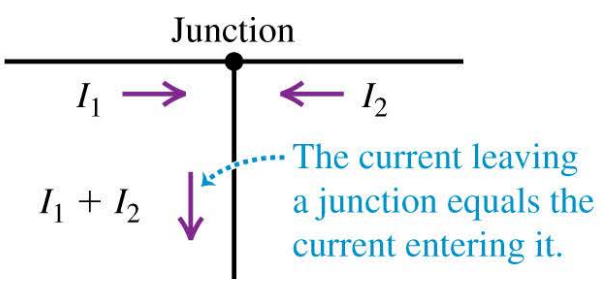


- *A junction* is a point where three or more conductors meet.
- *A loop* is any closed conducting path.

Kirchhoff's junction rule: *The algebraic sum of the currents into any junction is zero.*

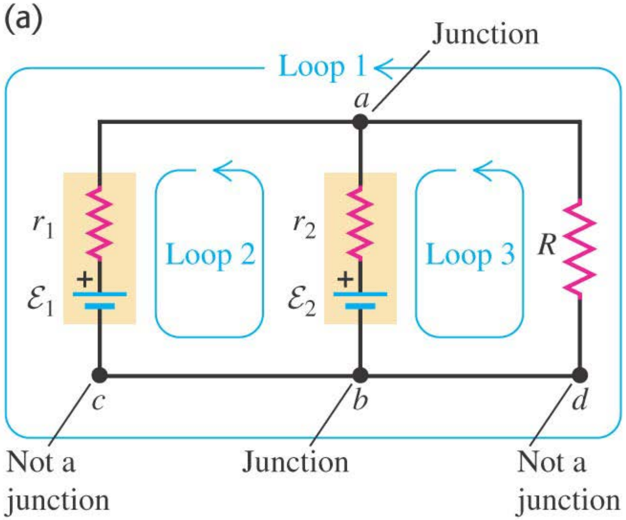
$$\sum I = 0 \qquad \text{(junction rule, valid at any junction)}$$



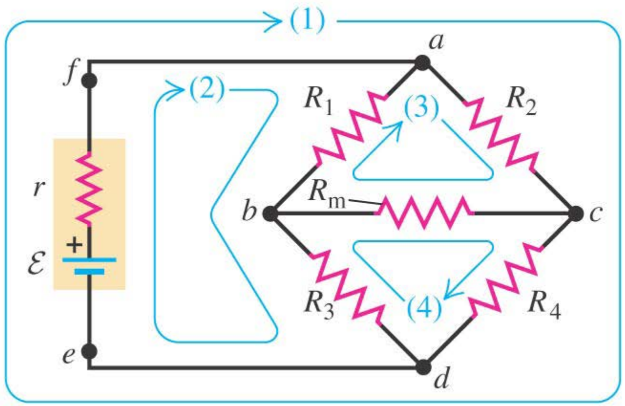


Kirchhoff's Rules

(a)



(b)



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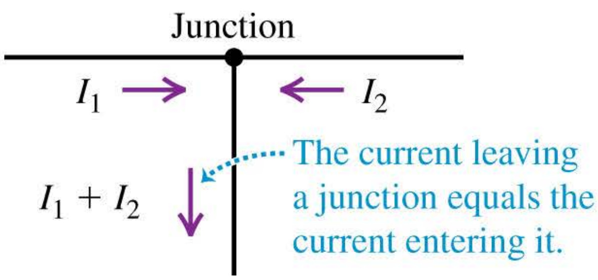
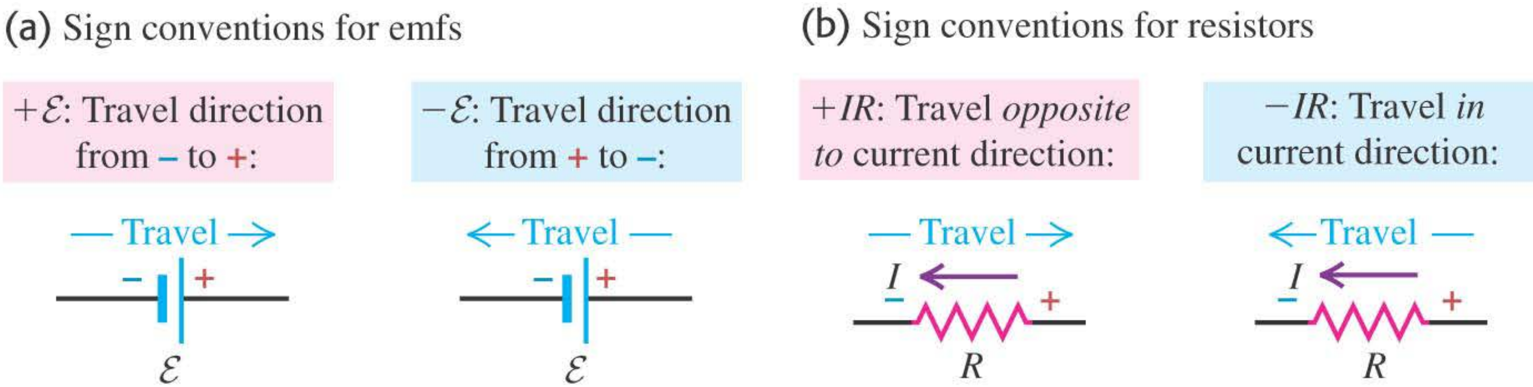


Kirchhoff's loop rule: *The algebraic sum of the potential differences in any loop,* including those associated with emfs and those of resistive elements, *must equal zero.*

$$\sum V = 0 \qquad \text{(loop rule, valid for any closed loop)}$$

Sign convention for the loop rule



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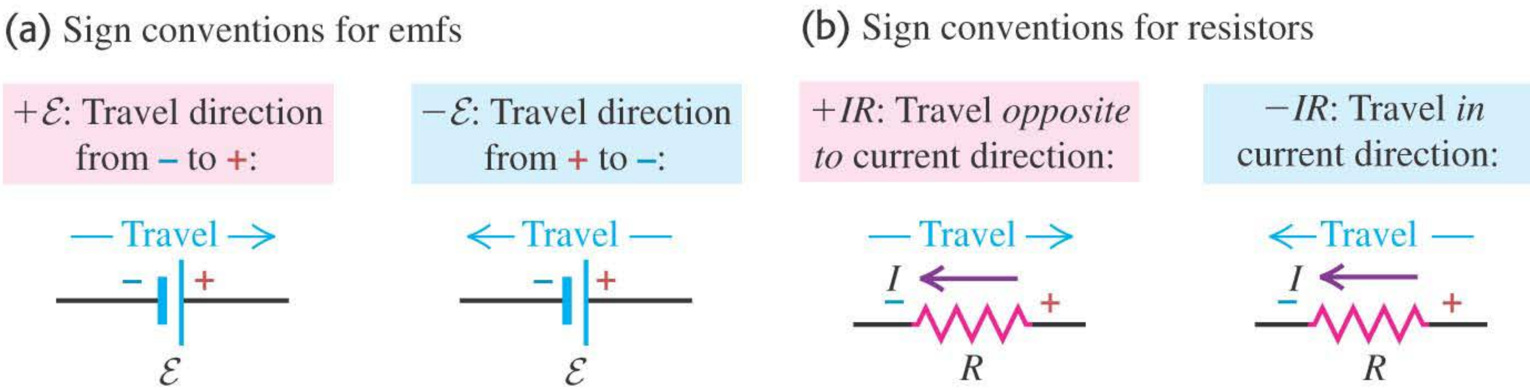

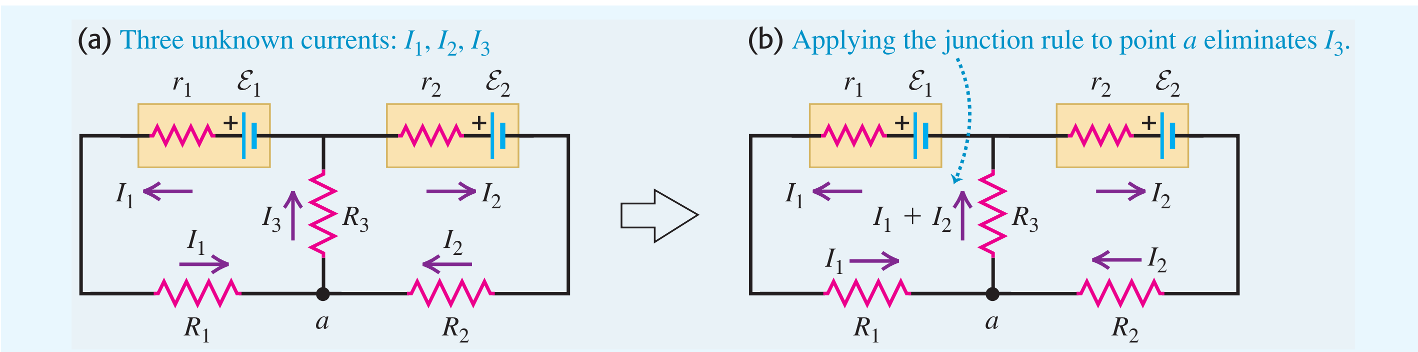

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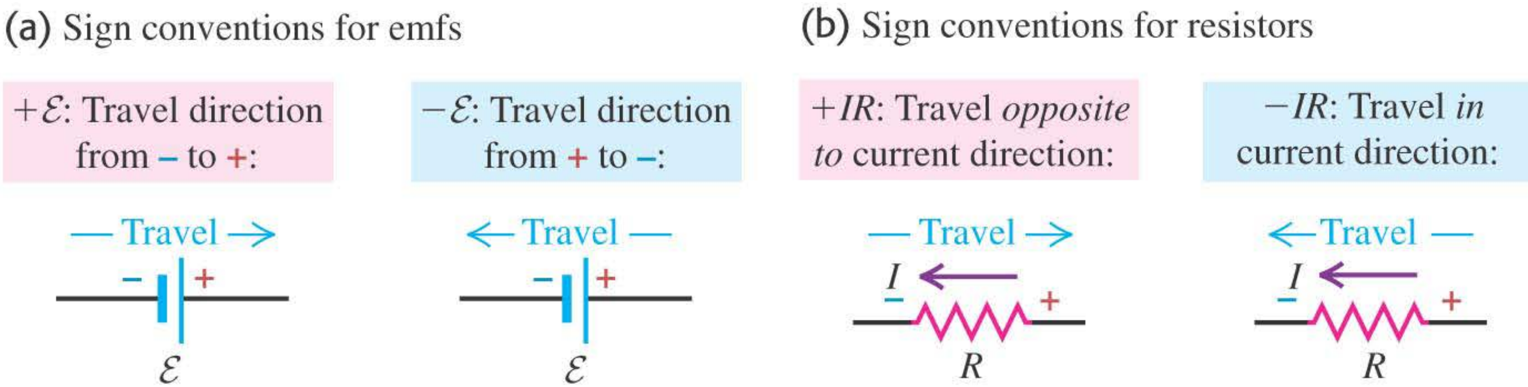
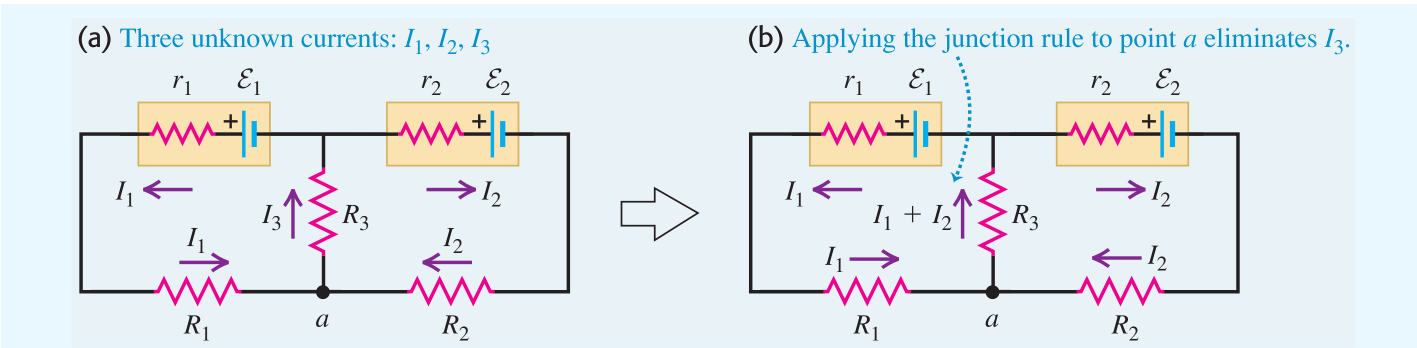
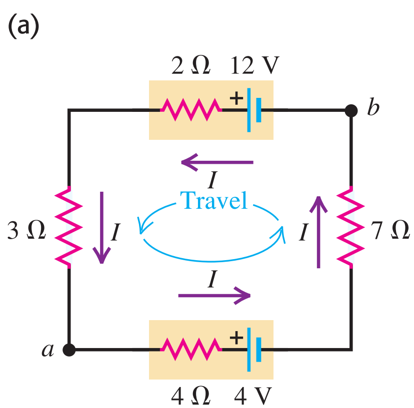


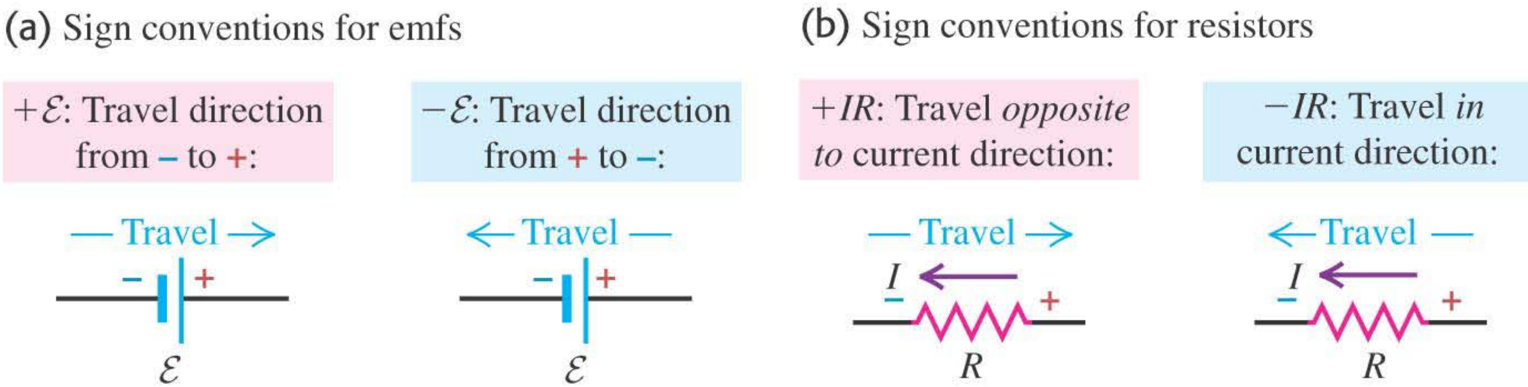


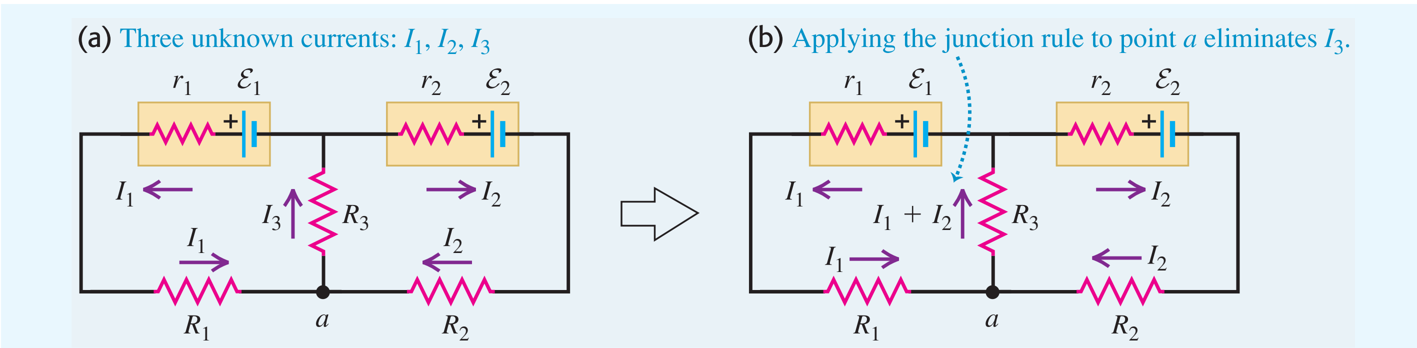


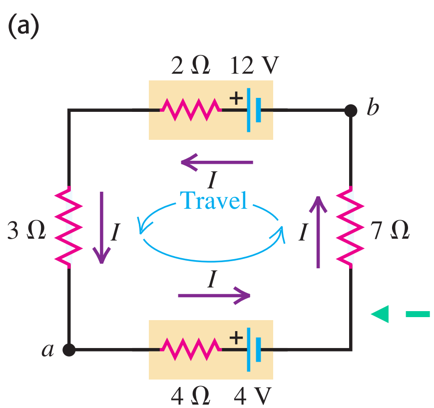
$$-I(4\ \Omega) - 4\ \text{V} - I(7\ \Omega) + 12\ \text{V} - I(2\ \Omega) - I(3\ \Omega) = 0$$

$\rightarrow$ $8\ \text{V} = I(16\ \Omega)$ $\rightarrow$ $I = 0.5\ \text{A}$

Sign convention for the loop rule







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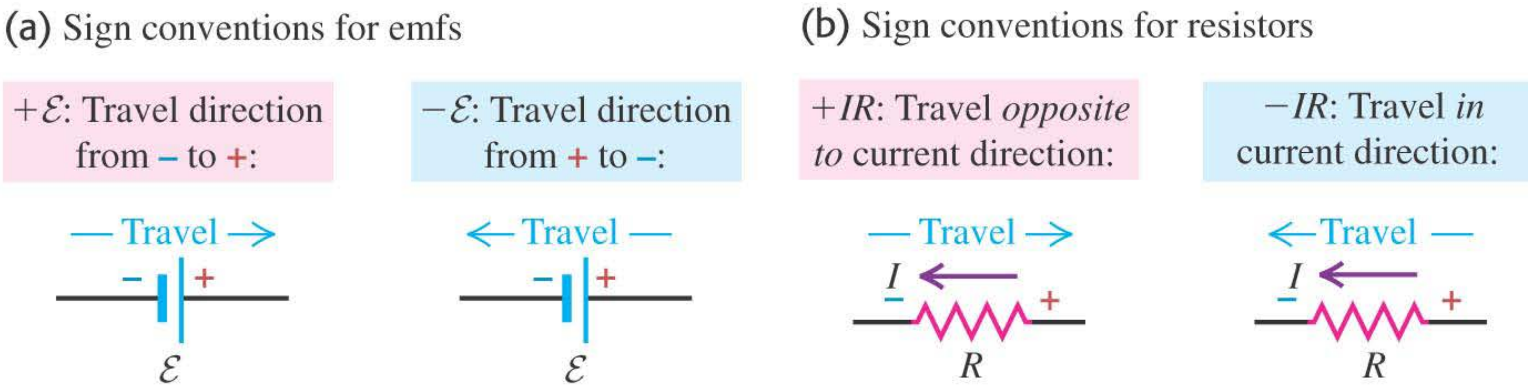
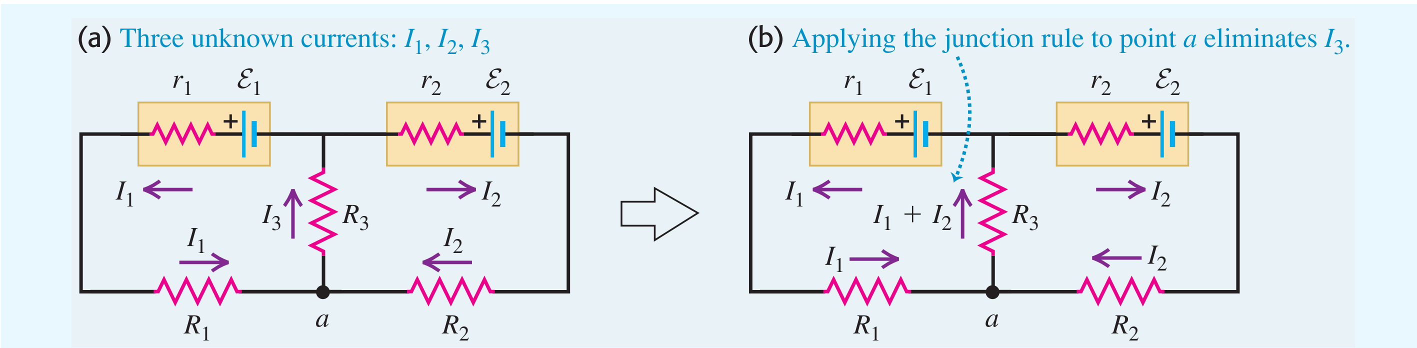
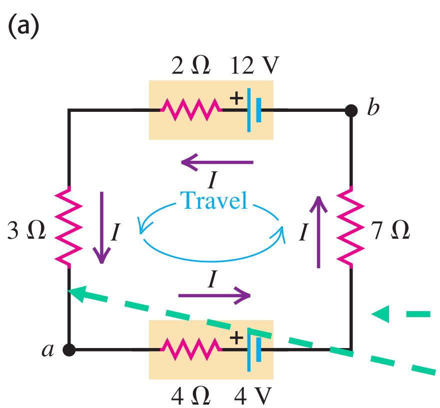
$$V_{ab} = (0.5\ \text{A})(7\ \Omega) + 4\ \text{V} + (0.5\ \text{A})(4\ \Omega)$$
$$= 9.5\ \text{V}$$

Sign convention for the loop rule







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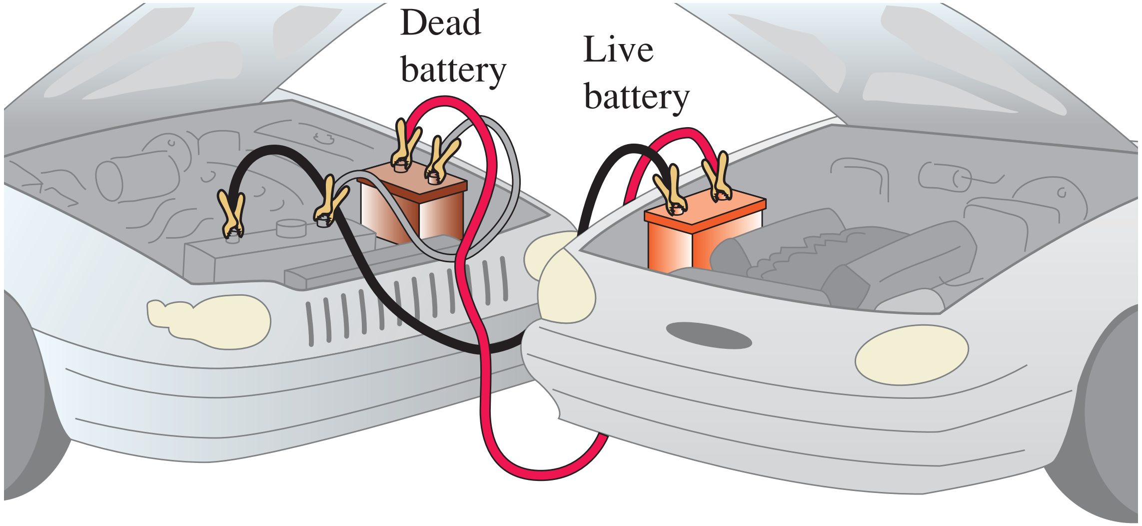
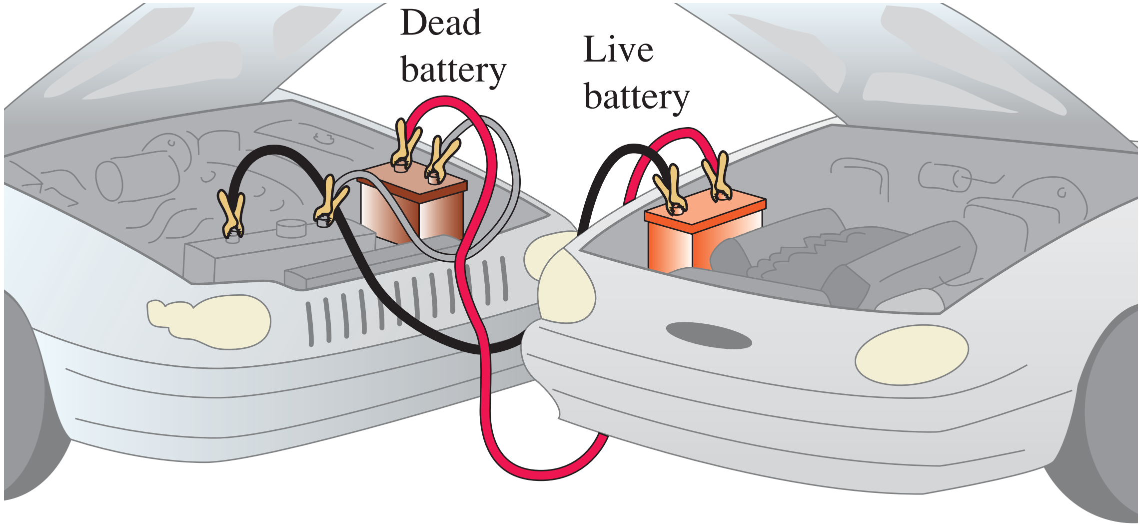
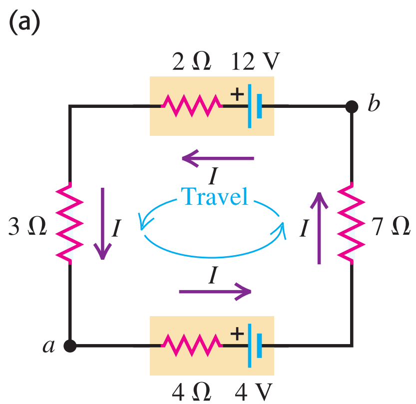
$$V_{ab} = 12\ \text{V} - (0.5\ \text{A})(2\ \Omega) - (0.5\ \text{A})(3\ \Omega) = 9.5\ \text{V}$$

"jump-starting" a car



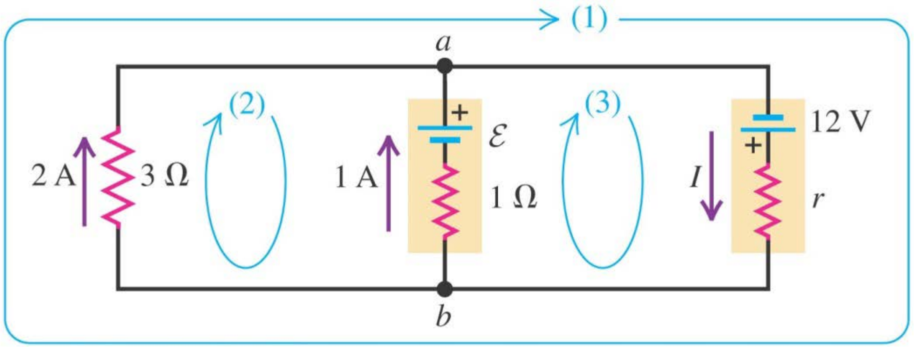
"jump-starting" a car



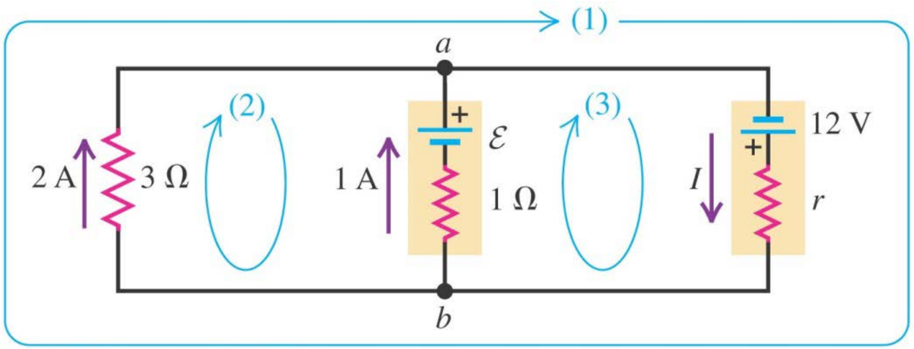


The circuit shown in Fig. (a) is much like that used when a fully charged 12-V storage battery (in a car with its engine running) is used to "jump-start" a car with a run-down battery. The run-down battery is slightly recharged in the process. The 3-Ω and 7-Ω resistors in figure represent the resistances of the jumper cables and of the conducting path through the automobile with the run-down battery.

Charging a battery



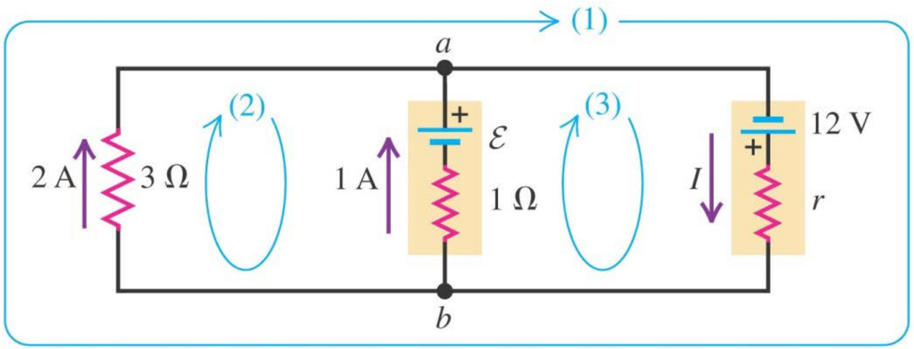
Charging a battery



In the circuit a 12-V power supply with unknown internal resistance r is connected to a run-down rechargeable battery with unknown emf ε and internal resistance 1 Ω and to an indicator light bulb of resistance 3 Ω carrying a current of 2 A. The current through the run-down battery is 1 A in the direction shown. Find r, ε, and the current I.

Is this assumption correct?

Charging a battery



In the circuit a 12-V power supply with unknown internal resistance *r* is connected to a run-down rechargeable battery with unknown emf $\mathcal{E}$ and internal resistance 1 Ω and to an indicator light bulb of resistance 3 Ω carrying a current of 2 A. The current through the run-down battery is 1 A in the direction shown. Find *r*, $\mathcal{E}$, and the current *I*.

Is this assumption correct?

Junction rule at (a)

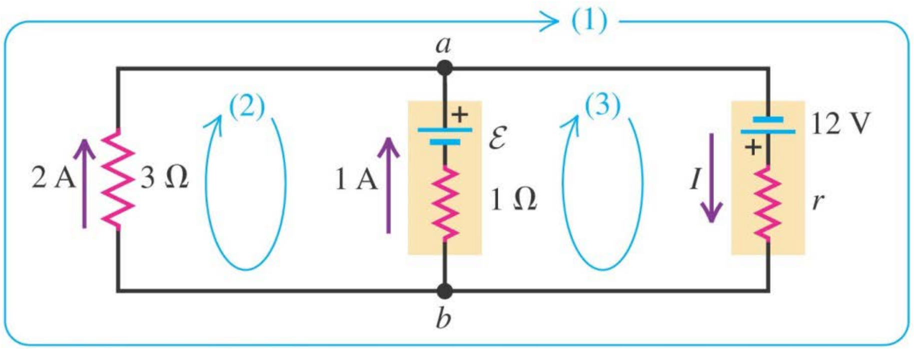
$-I + 1\ \text{A} + 2\ \text{A} = 0$ ➔ $I = 3\ \text{A}$

Following loop (1) ➔ $12\ \text{V} - (3\ \text{A})r - (2\ \text{A})(3\ \Omega) = 0$ ➡ $r = 2\ \Omega$

Following loop (2) ➔ $-\mathcal{E} + (1\ \text{A})(1\ \Omega) - (2\ \text{A})(3\ \Omega) = 0$ ➡ $\mathcal{E} = -5\ \text{V}$

The assumption of the polarity of $\mathcal{E}$ was incorrect!

Charging a battery



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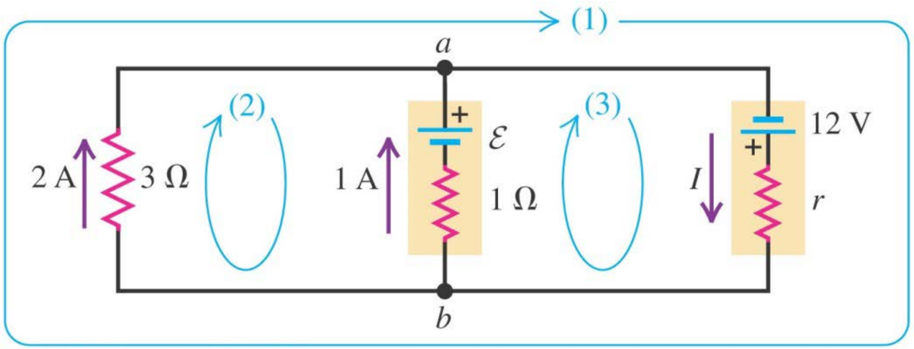
Checking result by following loop (3)

$12\ \text{V} - (3\ \text{A})(2\ \Omega) - (1\ \text{A})(1\ \Omega) + \mathcal{E} = 0$ ➔ $\mathcal{E} = -5\ \text{V}$

Charging a battery



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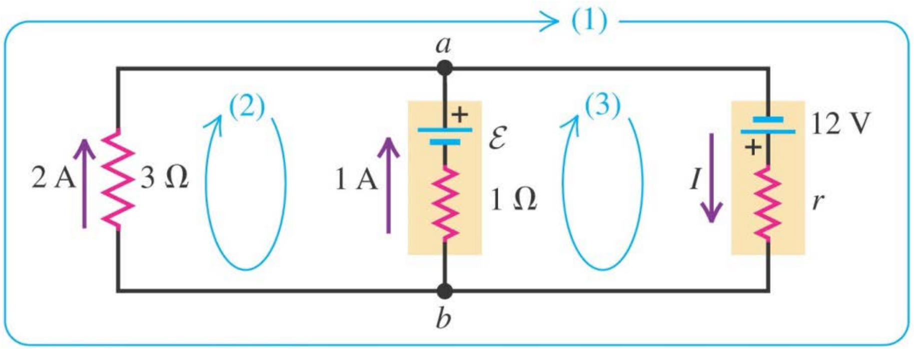
Additional check: $V_{ba} = V_b - V_a$

$$V_{ba} = V_b - V_a \begin{cases} (2\text{ A})(3\ \Omega) = 6\text{ V} \\ +12\text{ V} - (3\text{ A})(2\ \Omega) = +6\text{ V} \\ -(-5\text{ V}) + (1\text{ A})(1\ \Omega) = +6\text{ V} \end{cases}$$

Power in a battery-charging circuit



find the power delivered by the 12-V power supply and by the battery being recharged, ($\mathcal{E}$=5V) and find the power dissipated in each resistor.

- the power delivered *from* an emf to a circuit is $\mathcal{E}I$
- the power delivered to a resistor from a circuit is $V_{ab}I = I^2R$

$$P_{\text{supply}} = \mathcal{E}_{\text{supply}} I_{\text{supply}} = (12\ \text{V})(3\ \text{A}) = 36\ \text{W}$$

$$P_{r-\text{supply}} = I_{\text{supply}}{}^2 r_{\text{supply}} = (3\ \text{A})^2(2\ \Omega) = 18\ \text{W}$$

$$P_{\text{net}} = 36\ \text{W} - 18\ \text{W} = 18\ \text{W}$$

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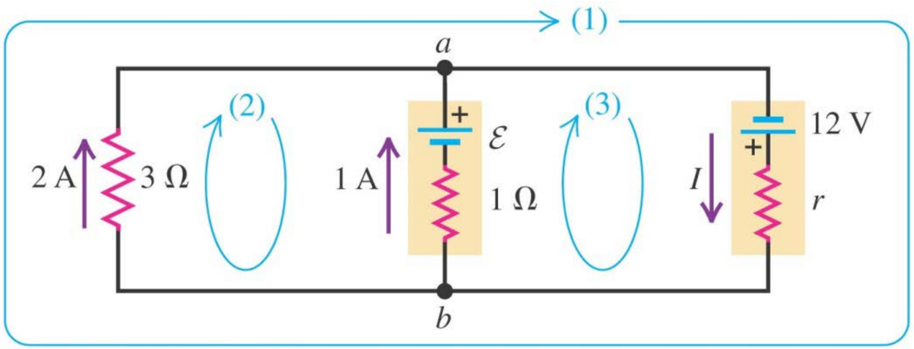

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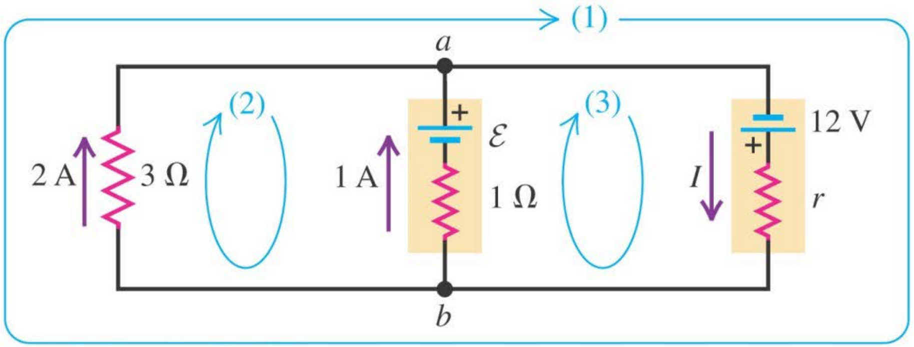

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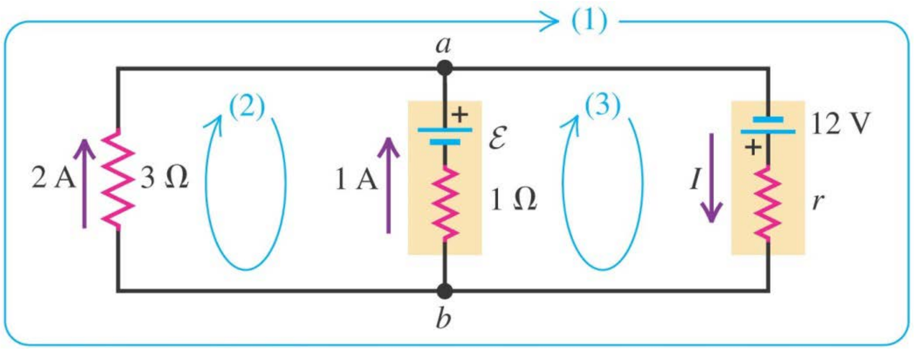

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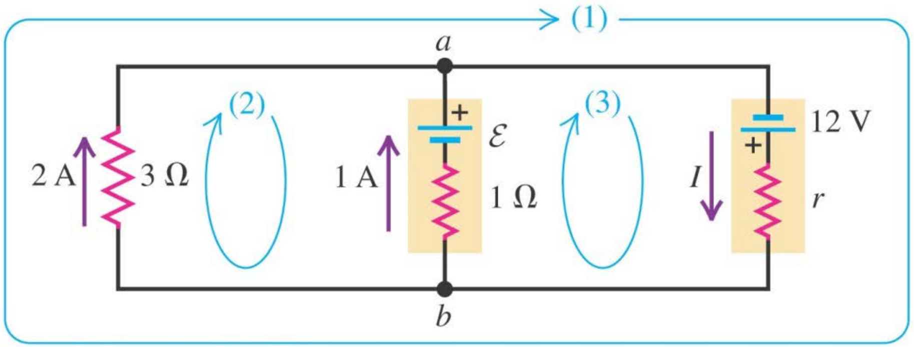

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Power output of the battery being charged: $P_{\text{emf}} = \mathcal{E} I_{\text{battery}} = (-5\ \text{V})(1\ \text{A}) = -5\ \text{W}$

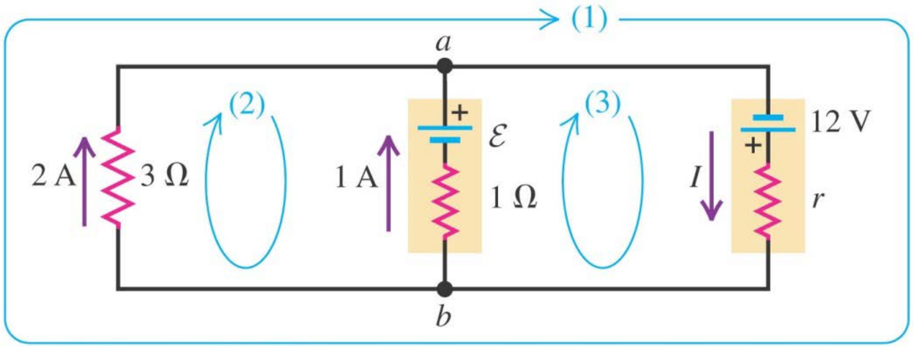
$$P_{r\text{-battery}} = I_{\text{battery}}{}^2 r_{\text{battery}} = (1\ \text{A})^2(1\ \Omega) = 1\ \text{W}$$

The assumption of the polarity of $\mathcal{E}$ was incorrect!

We are storing energy as we charge it

➔ Total power input to the battery is 1W + 5W = 6 W

Power in a battery-charging circuit



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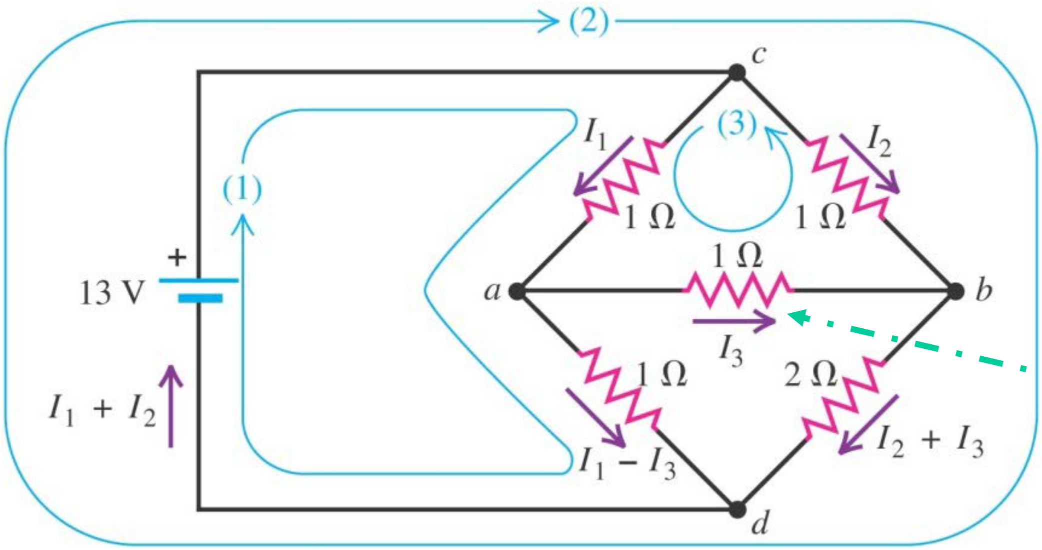
Power dissipated in the light bulb (3Ω) ➔ $P_{\text{bulb}} = I_{\text{bulb}}{}^2 R_{\text{bulb}} = (2\ \text{A})^2(3\ \Omega) = 12\ \text{W}$

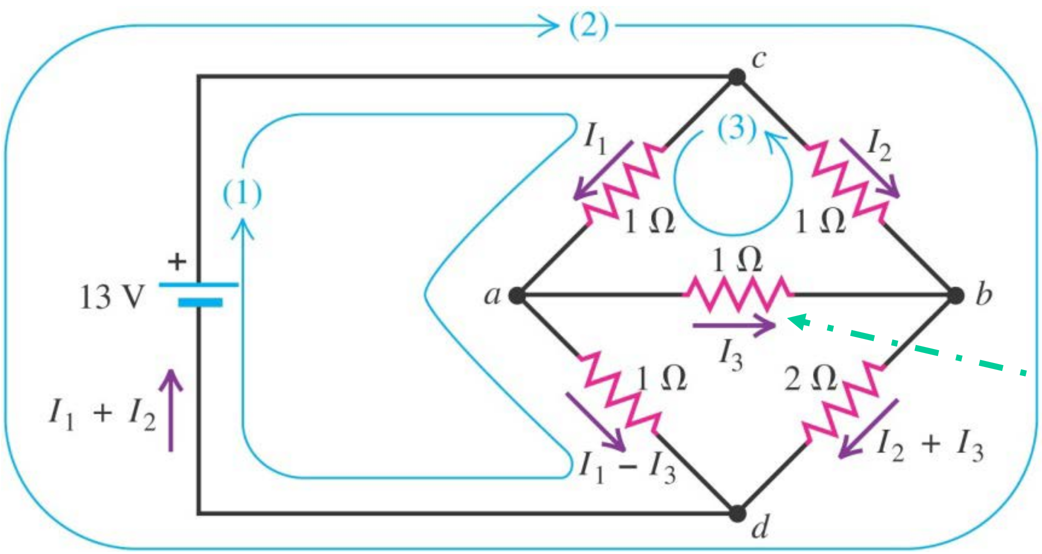
A complex network



Find the current in each resistor and the equivalent resistance of the network of five resistors. (5 unknowns)

Polarity is not important in the beginning

A complex network



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Applying loop rule:

$$13\ \text{V} - I_1(1\ \Omega) - (I_1 - I_3)(1\ \Omega) = 0 \quad (1)$$

$$-I_2(1\ \Omega) - (I_2 + I_3)(2\ \Omega) + 13\ \text{V} = 0 \quad (2)$$

$$-I_1(1\ \Omega) - I_3(1\ \Omega) + I_2(1\ \Omega) = 0 \quad (3)$$

One way to solve these simultaneous equations is to solve Eq. (3) for I_2, obtaining $I_2 = I_1 + I_3$, and then substitute this expression into Eq. (2) to eliminate I_2. We then have

$$13\ \text{V} = I_1(2\ \Omega) - I_3(1\ \Omega) \quad (1')$$

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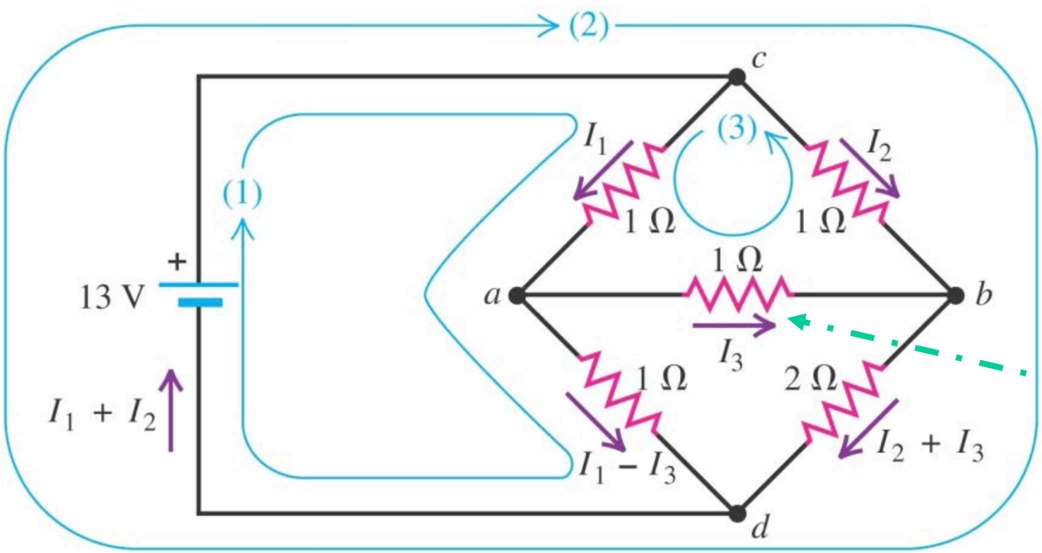

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Now we can eliminate I_3 by multiplying Eq. (1') by 5 and adding the two equations. We obtain

$$78\text{ V} = I_1(13\ \Omega) \qquad I_1 = 6\text{ A} \qquad \Rightarrow \qquad R_{eq} = \frac{13\text{ V}}{11\text{ A}} = 1.2\ \Omega$$

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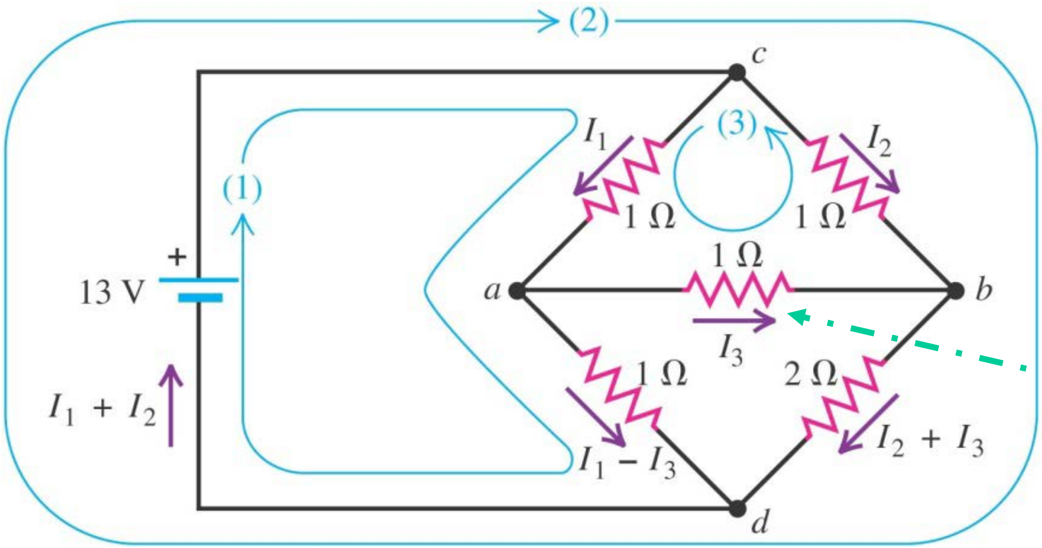

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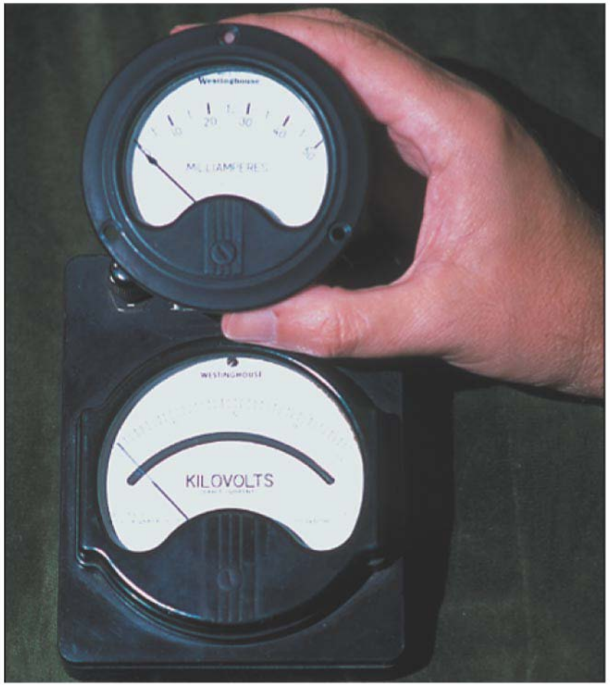
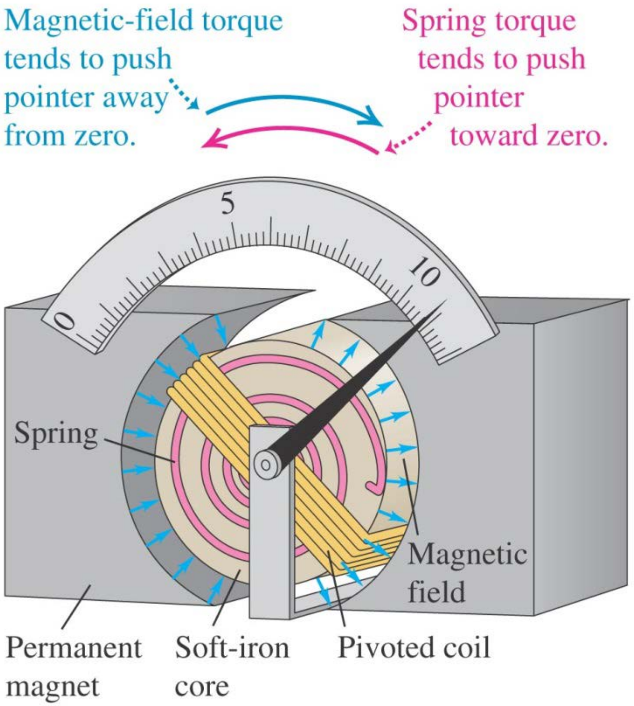
$$I_3 = -1\text{ A}.$$ Polarity is wrong $$\Rightarrow \quad |I_3|R = (1\text{ A})(1\ \Omega) = 1\text{ V}$$

Hence V_{ab} = -1 V, and the potential at *a* is 1 V less than at point *b*.

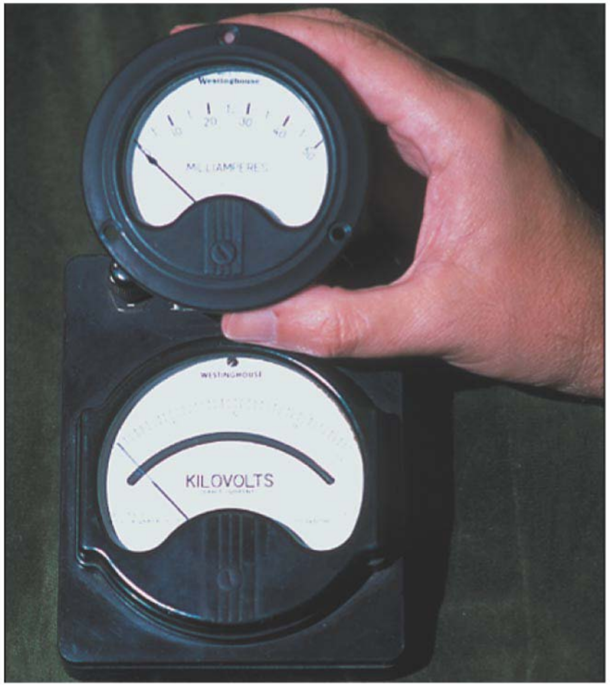
Electrical Measuring Instruments



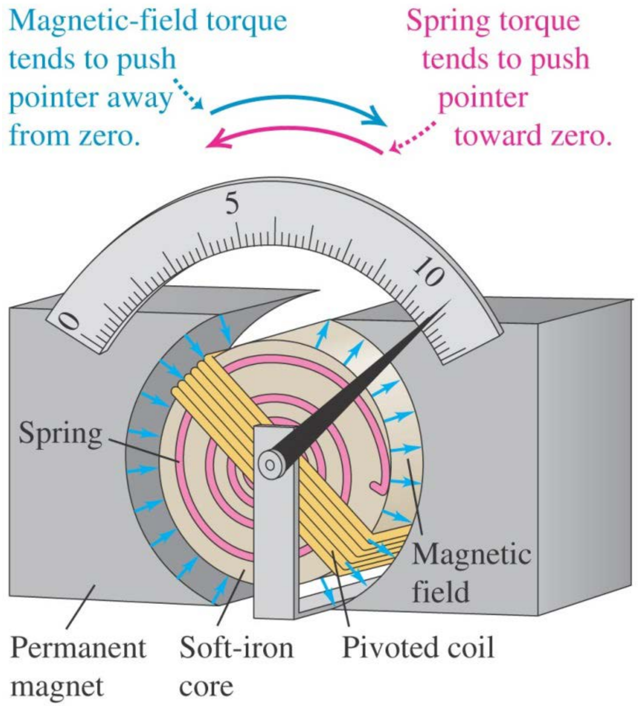


- A *d'Arsonval galvanometer* measures the current through it
- Many electrical instruments, such as ammeters and voltmeters, use a galvanometer in their design.

Electrical Measuring Instruments



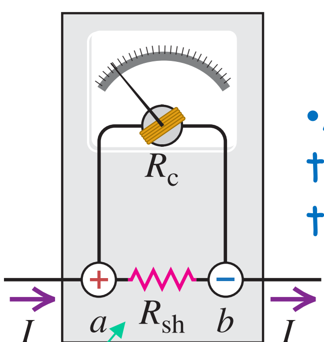
(a) Moving-coil ammeter



(b) Moving-coil voltmeter

•A *d'Arsonval galvanometer* measures the current through it

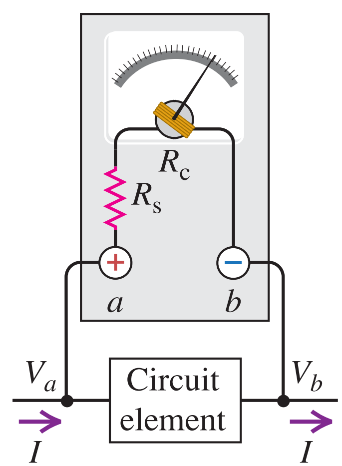
•Many electrical instruments, such as ammeters and voltmeters, use a galvanometer in their design.



Shunt resistor

•An *ammeter* measures the current passing through it.

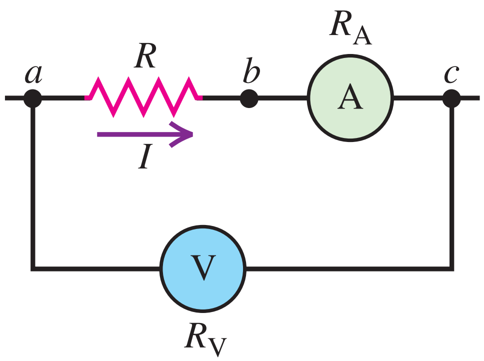
$$I_{fs}R_c = (I_a - I_{fs})R_{sh}$$



voltmeter measures the potential difference between two points.

$$V_V = I_{fs}(R_c + R_s)$$

Ammeters and Voltmeters in Combination



The power input P to any circuit element is the product of the potential difference across it and the current through it: $P = V_{ab} I$. In principle, the most straightforward way to measure R or P is to measure V_{ab} and I simultaneously. Ammeter A reads the current I in the resistor R. Voltmeter V, however, reads the *sum* of the potential difference V_{ab} across the resistor and the potential difference V_{bc} across the ammeter.

Voltmeter reads 12 V, ammeter reads 0.1 A, R_V = 10 kΩ, R_A = 2 Ω

$V_{bc} = IR_A = (0.100\ A)(2\ \Omega) = 0.2\ V \rightarrow V_{ab} = IR = V - V_{bc} = 11.8\ V \rightarrow R = 118\ \Omega$

$\Rightarrow$ $P = V_{ab}I = (11.8\ \text{V})(0.100\ \text{A}) = 1.18\ \text{W}$

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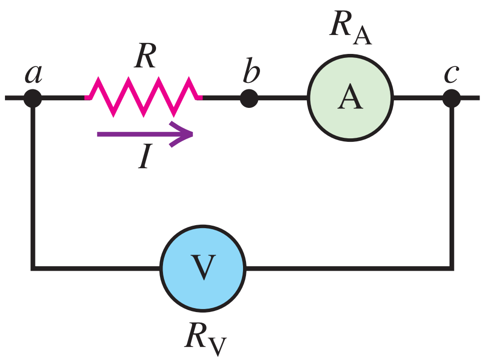

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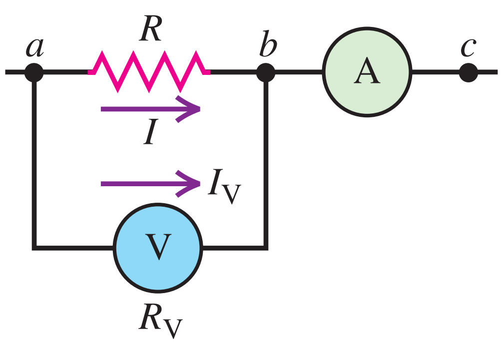


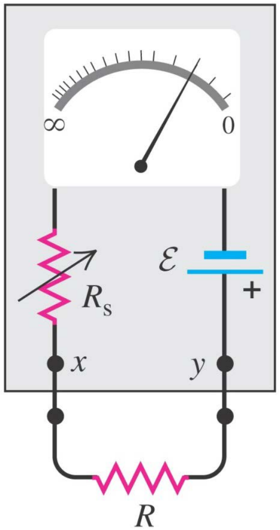
If we transfer the voltmeter terminal from c to b, as in figure, then the voltmeter reads the potential difference V_{ab} correctly, but the ammeter now reads the *sum* of the current I in the resistor and the current I_V in the voltmeter. Either way, we have to correct the reading of one instrument or the other unless the corrections are small enough to be negligible.

$V_{ab} = 12\ V;\ I_A = I + I_V$ $\qquad I_V = V/R_V = (12.0\ \text{V})/(10{,}000\ \Omega) = 1.20\ \text{mA}$

Actual current in the resistor $I = I_A - I_V = 0.1\ A - 0.0012\ A = 0.0988\ A$

$$R = \frac{V_{ab}}{I} = \frac{12.0\ \text{V}}{0.0988\ \text{A}} = 121\ \Omega \qquad P = V_{ab}I = (12.0\ \text{V})(0.0988\ \text{A}) = 1.19\ \text{W}$$

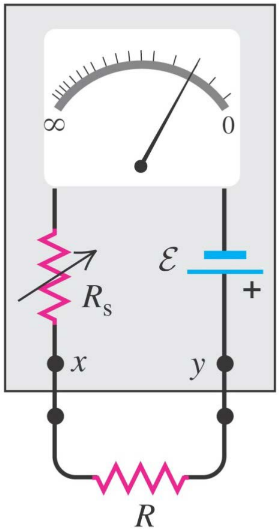
Ohmmeters and potentiometers



ohmmeter is designed to measure resistance

The series resistance R_s is variable; it is adjusted so that when terminals x and y are short-circuited (that is, when $R = 0$), the meter deflects full scale. When nothing is connected to terminals x and y, so that the circuit between x and y is *open* (that is, when $R \rightarrow \infty$), there is no current and hence no deflection. For any intermediate value of R the meter deflection depends on the value of R, and the meter scale can be calibrated to read the resistance R directly.

Ohmmeters and potentiometers



ohmmeter is designed to measure resistance

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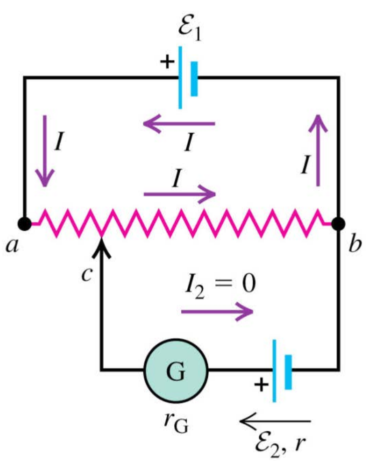
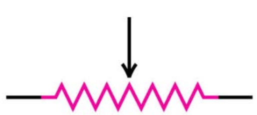


potentiometer measures the emf of a source without drawing any current from the source.

To determine the value of $\boldsymbol{\varepsilon}_2$, contact c is moved until a position is found at which the galvanometer shows no deflection; this corresponds to zero current passing through $\boldsymbol{\varepsilon}_2$. With I_2 = 0, Kirchhoff's loop rule gives

$$\mathcal{E}_2 = IR_{cb}$$

(b) Circuit symbol for potentiometer (variable resistor)

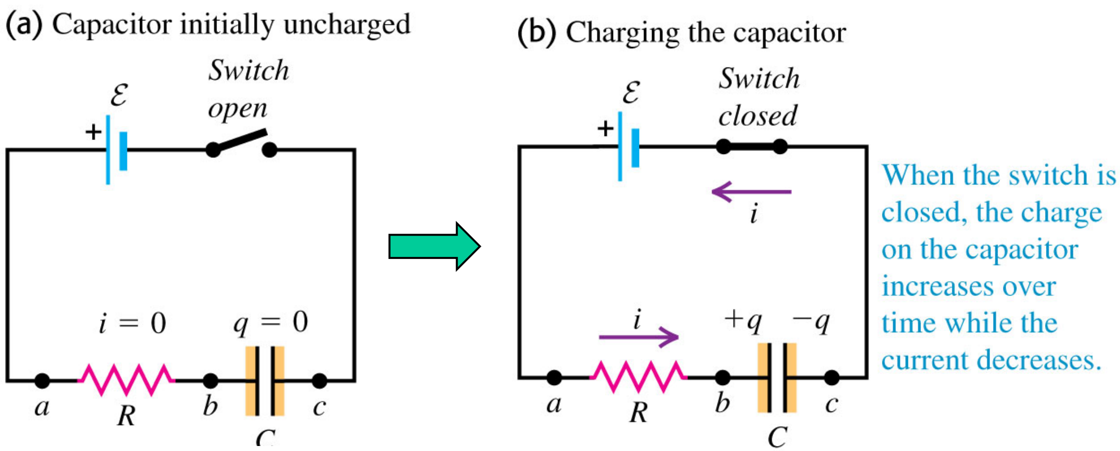


if the resistance wire is uniform, R_{cb} is proportional to the length of wire between c and b. We calibrate the device by replacing $\boldsymbol{\varepsilon}_2$ by a source of known emf; then any unknown emf $\boldsymbol{\varepsilon}_2$ can be found by measuring the length of wire cb for which I_2 = 0

R-C Circuits

In the circuits we have analyzed up to this point, we have assumed that all the emfs and resistances are *constant* (time independent) so that all the potentials, currents, and powers are also independent of time. But in the simple act of charging or discharging a capacitor we find a situation in which the currents, voltages, and powers *do* change with time.

Charging a capacitor



at $t = 0$ v_{bc} across it is zero

across the resistor R, $v_{ab} = \mathcal{E}$

$$I_0 = v_{ab}/R = \mathcal{E}/R$$

As the capacitor charges, its voltage v_{bc} increases and the potential difference v_{ab} decreases corresponding to a decrease in current.

$$v_{ab} = iR \qquad v_{bc} = \frac{q}{C} \;\Rightarrow\; \mathcal{E} - iR - \frac{q}{C} = 0 \;\Rightarrow\; i = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

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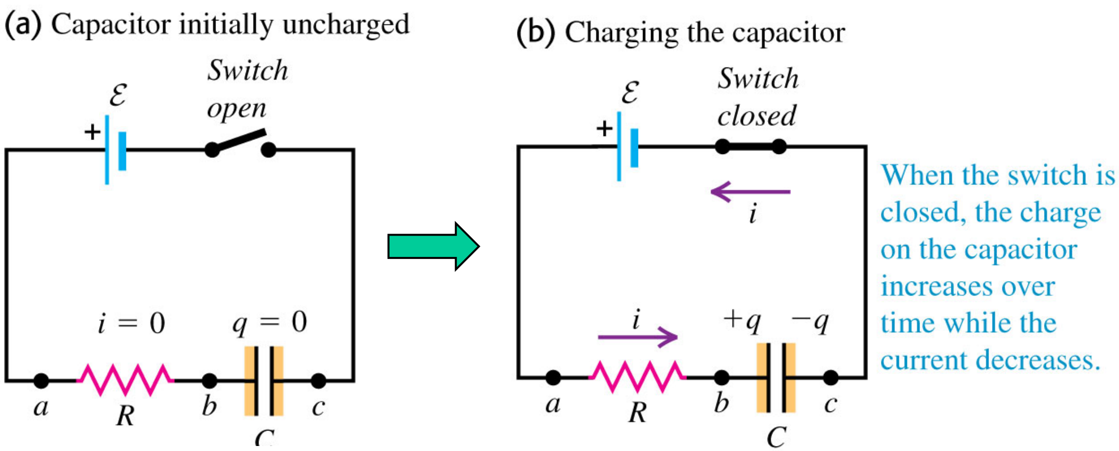

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at $t \to \infty$ $i = 0$, $q = Q_f$ *(finalcharge)* $\Rightarrow$ $\dfrac{\mathcal{E}}{R} = \dfrac{Q_f}{RC}$ $\qquad Q_f = C\mathcal{E}$

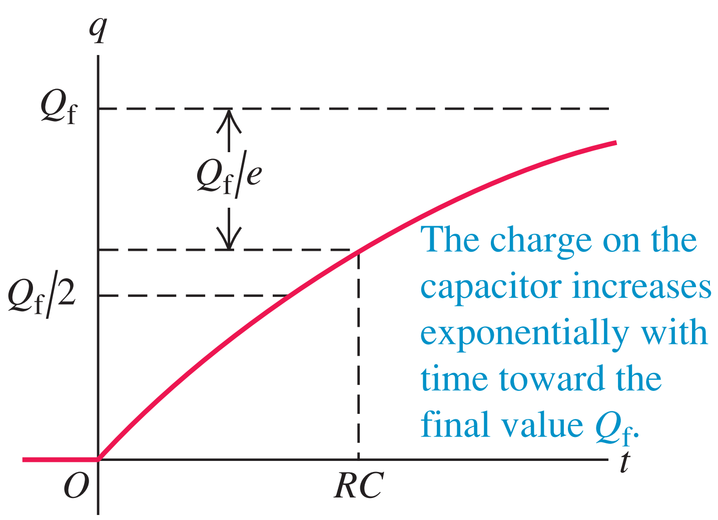
$$i = dq/dt \;\Rightarrow\; \frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC} = -\frac{1}{RC}(q - C\mathcal{E}) \;\Rightarrow\; \frac{dq}{q - C\mathcal{E}} = -\frac{dt}{RC}$$

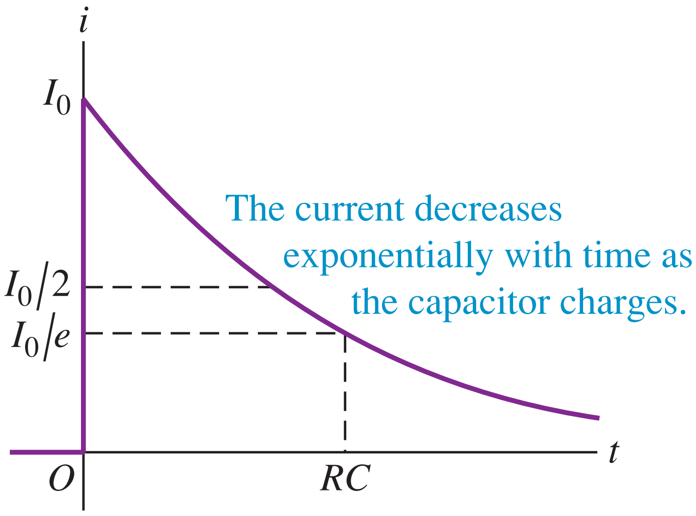
$$\int_0^q \frac{dq'}{q' - C\mathcal{E}} = -\int_0^t \frac{dt'}{RC} \;\Rightarrow\; \ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC} \;\Rightarrow\; \boxed{q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC})}$$

Time constant

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC}) \quad \text{(R-C circuit, charging capacitor)}$$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad \text{(R-C circuit, charging capacitor)}$$

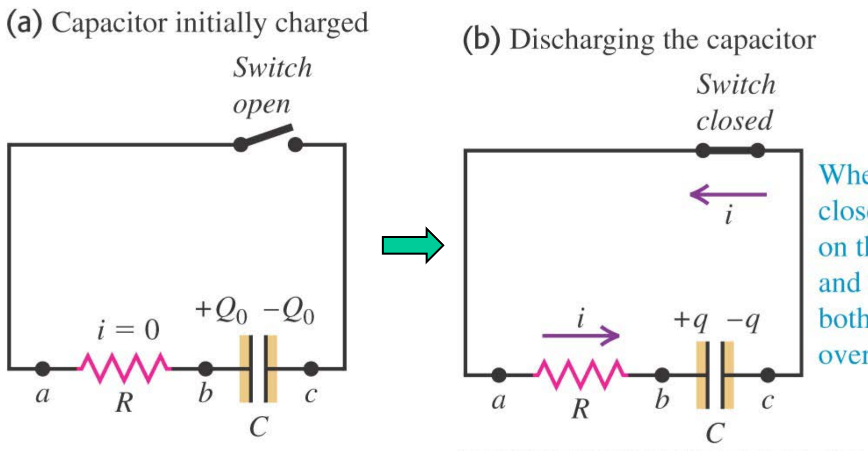




After a time equal to RC, the current in the R-C circuit has decreased to $1/e$ (about 0.368) of its initial value. At this time, the capacitor charge has reached $(1 - 1/e)$ = 0.632 of its final value $Q_f = C\mathcal{E}$. The product RC is therefore a measure of how quickly the capacitor charges. We call RC the **time constant**, or the **relaxation time**, of the circuit, denoted by τ:

$$\tau = RC \qquad \text{(time constant for } R\text{-}C \text{ circuit)}$$

Discharging a capacitor



When the switch is closed, the charge on the capacitor and the current both decrease over time.

at $t = 0$ $q = Q_0$

Applying Kirchhoff's loop rule:

$$-iR - \frac{q}{C} = 0$$

$$\Rightarrow \quad i = \frac{dq}{dt} = -\frac{q}{RC}$$

initial current: $I_0 = -Q_0/RC$

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\frac{1}{RC}\int_0^t dt' \quad \Rightarrow \quad \ln\frac{q}{Q_0} = -\frac{t}{RC}$$

$$q = Q_0 e^{-t/RC} \qquad (R\text{-}C \text{ circuit, discharging capacitor})$$

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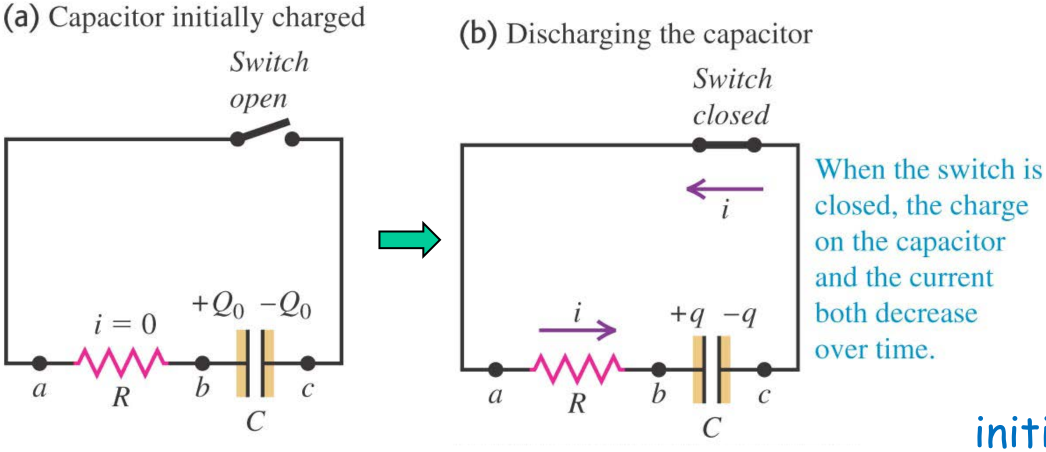

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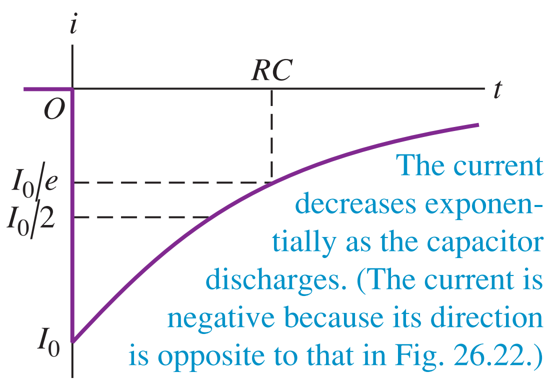
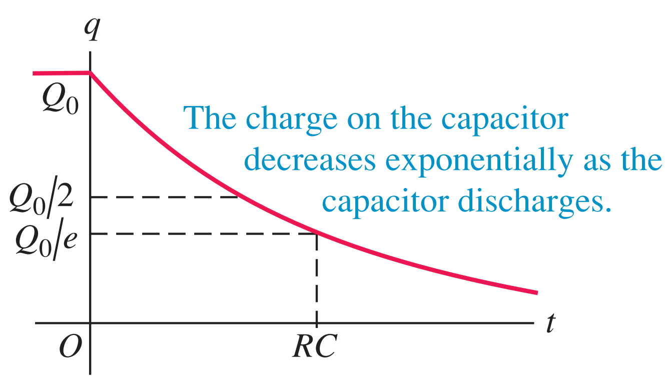
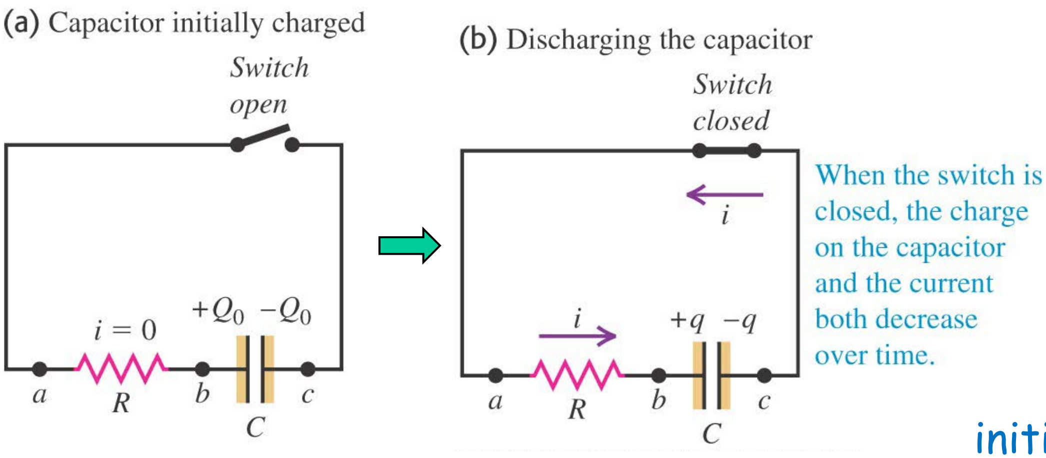




Discharging a capacitor



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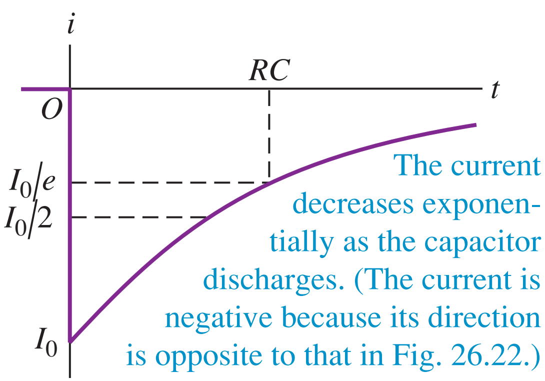

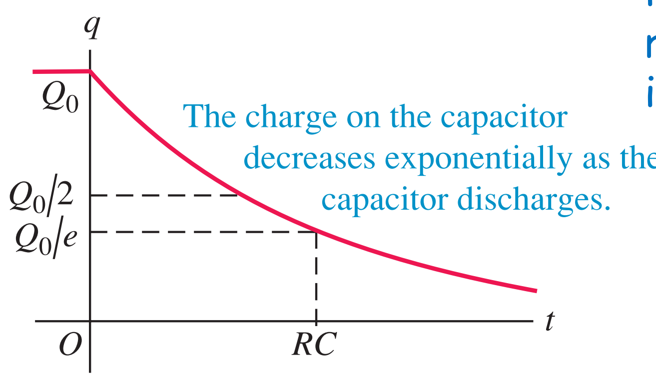

Instantenous rate at which electrical energy is dissipated in the resistor is i^2R and the rate at which energy is stored in the capacitor is $iv_{bc} = iq/C$

$$\mathcal{E}i = i^2R + \frac{iq}{C}$$

power εi supplied by the battery is dissipated in the resistor and is stored in the capacitor.

Exercices:

1-) Discharging Example: A 2 μ F capacitor is charged and then connected in series with a resistance *R*. The original potential across it drops to 1/4 of it's starting value in 2 seconds. What is the value of the resistance?

2-) Charging Example: How many time constants does it take for an initially uncharged capacitor in an *RC* circuit to become 99% charged?

3) Charging a 100 μF capacitor in series with a 10,000 Ω resistor, using EMF ε = 5 V.

a) How long after voltage is applied does $V_{cap}(t)$ reach 4 volts?

b) What's the current through R at t = 2 sec?