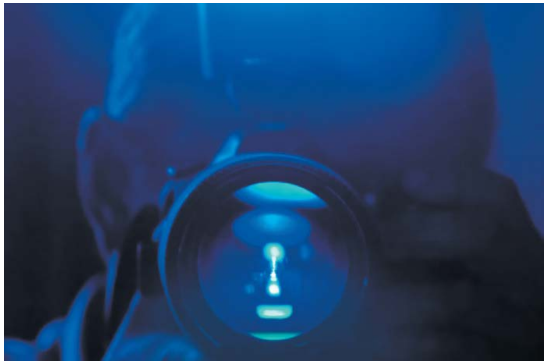
Chp 24: Capacitance and Dielectrics

Goals for Chapter 24

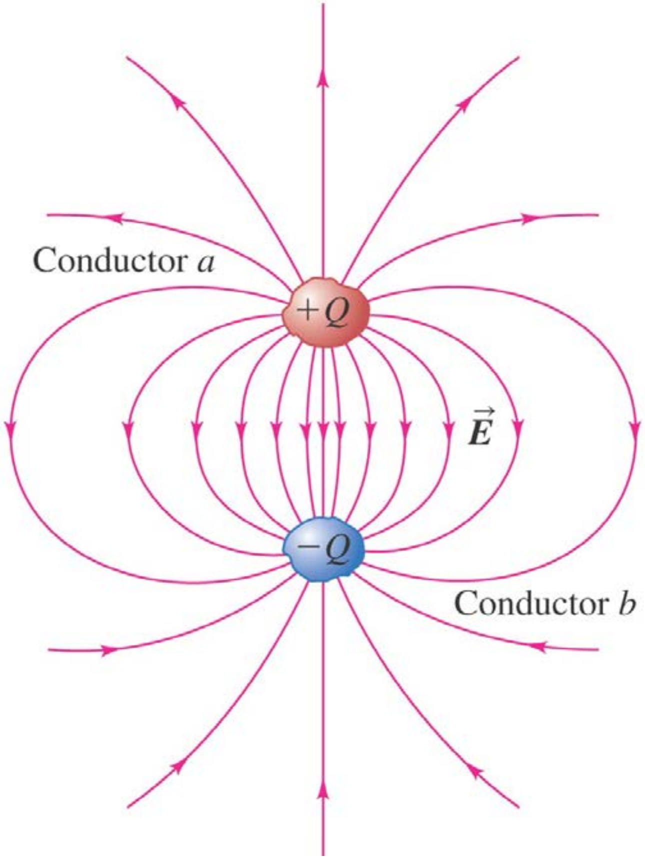
- To understand capacitors and calculate capacitance
- To analyze networks of capacitors
- To calculate the energy stored in a capacitor
- To examine dielectrics and how they affect capacitance

Capacitors and Capacitance



Capacitors are devices that store electric potential energy. The energy of a capacitor is actually stored in the electric field.

In circuit diagrams a capacitor is represented by either of these symbols:



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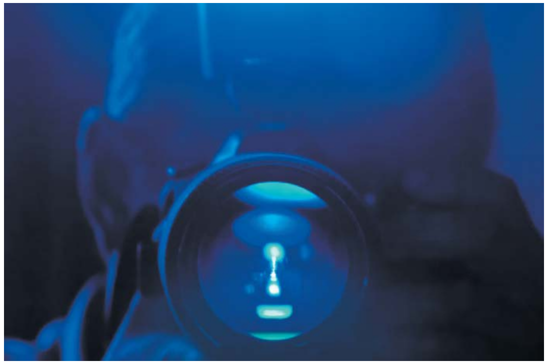

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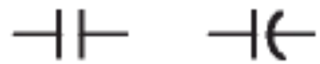
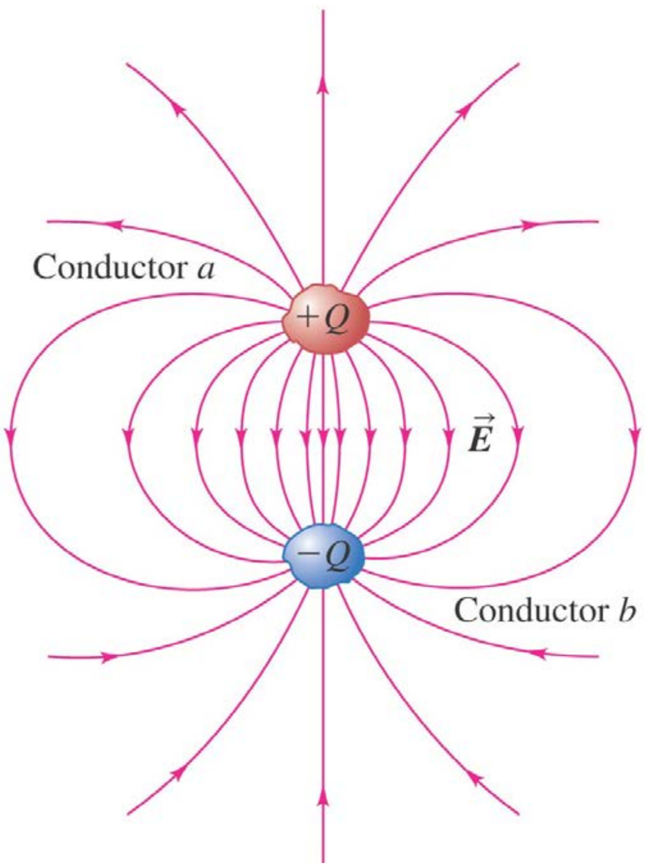




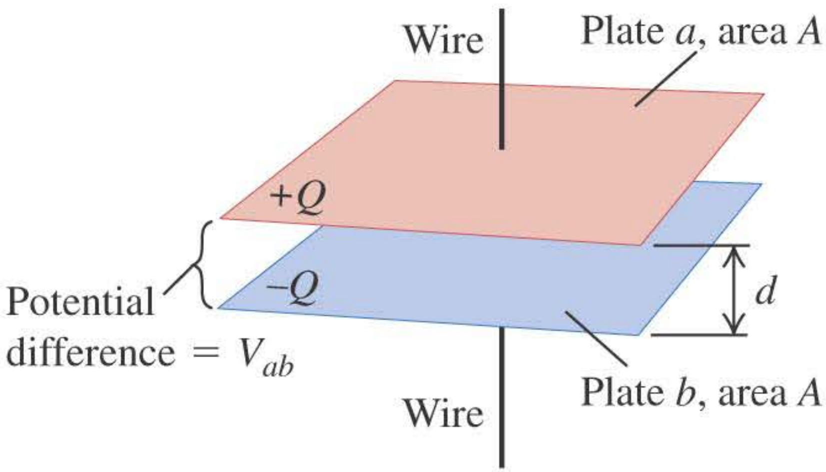
- Any two conductors separated by an insulator form a *capacitor*
- The definition of capacitance is $C = Q/V_{ab}$

$$C = \frac{Q}{V_{ab}} \qquad \text{(definition of capacitance)}$$

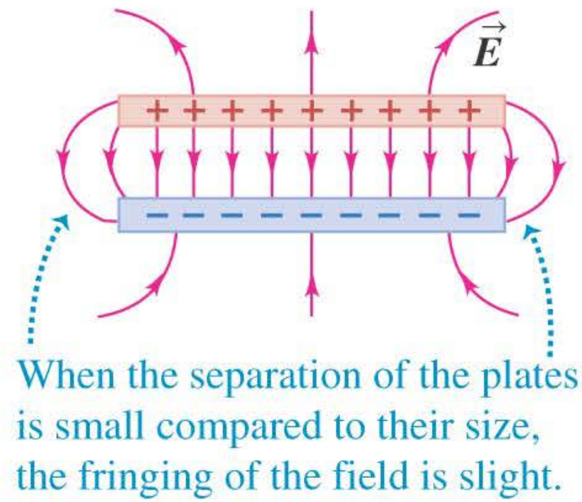
$$1\ \mathrm{F} = 1\ \text{farad} = 1\ \mathrm{C/V} = 1\ \text{coulomb/volt}$$

Parallel-plate capacitor

(a) Arrangement of the capacitor plates



(b) Side view of the electric field $\vec{E}$



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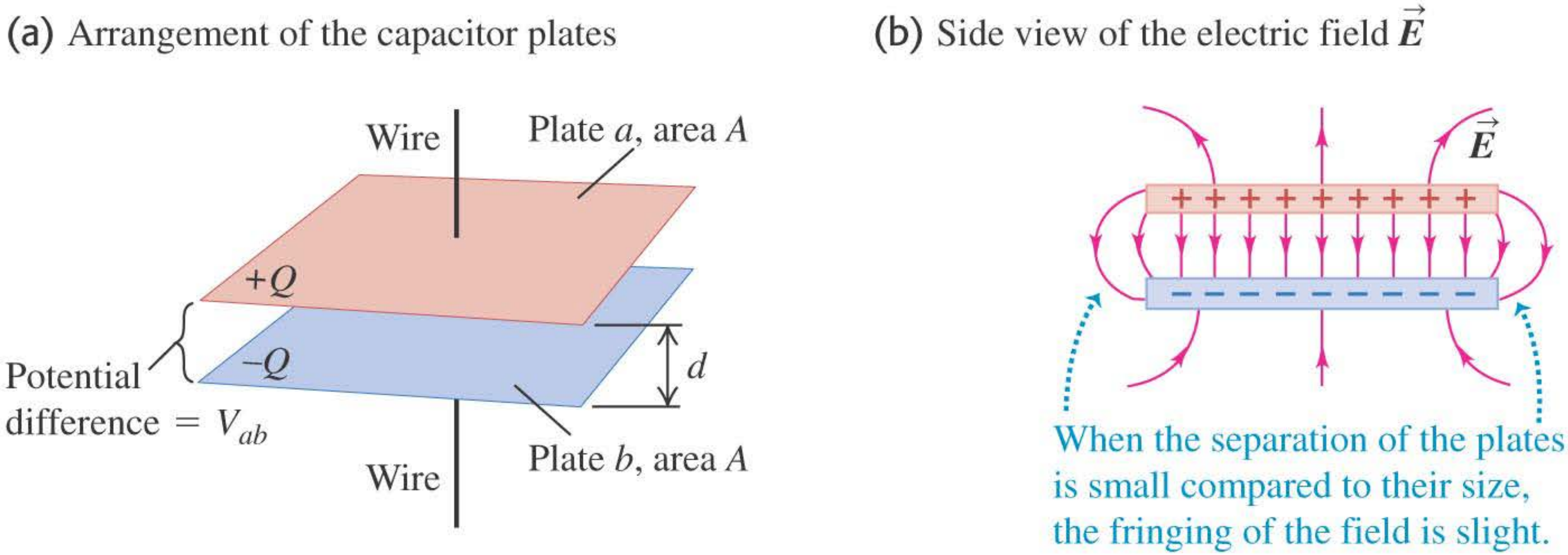

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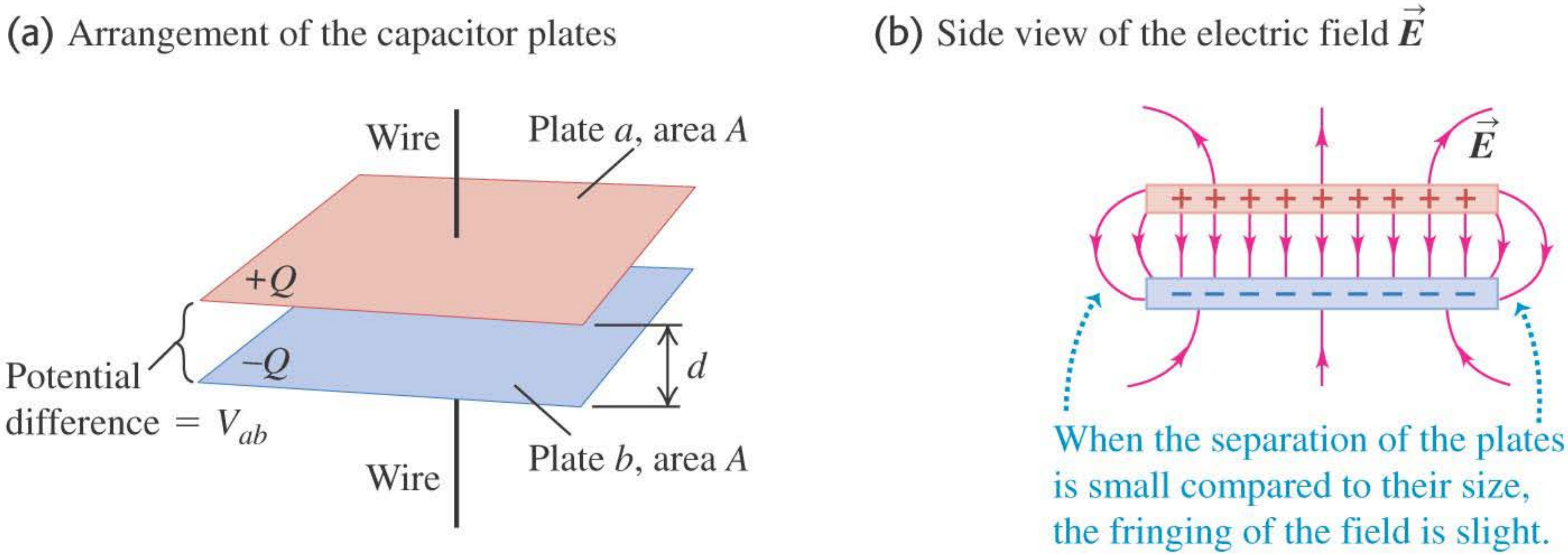

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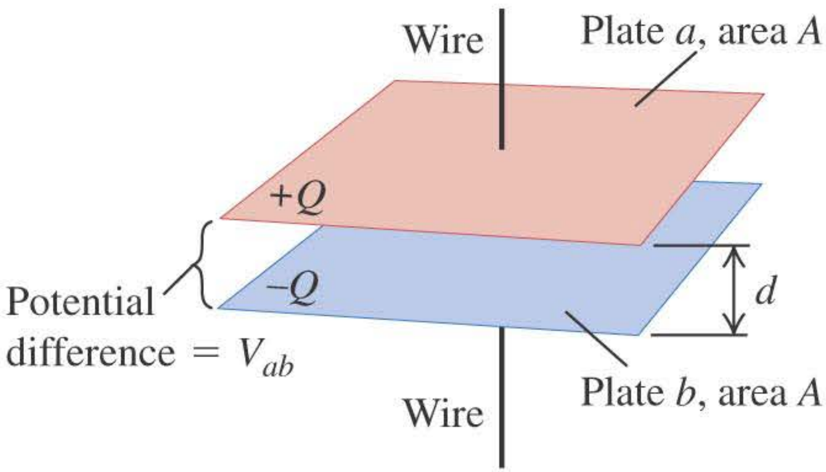

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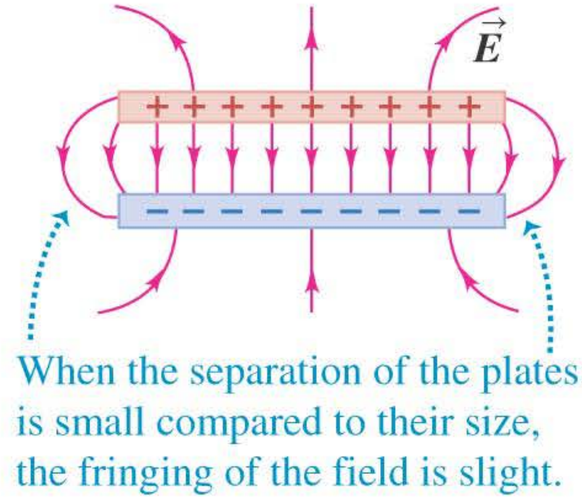

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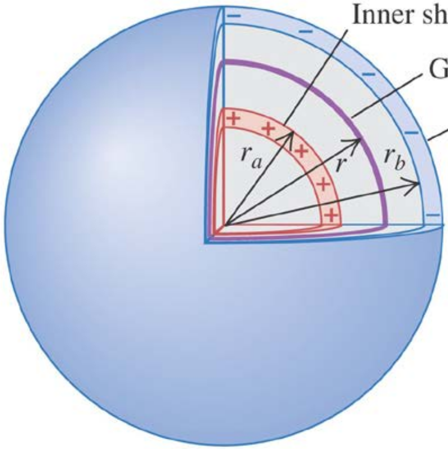
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \rightarrow V_{ab} = Ed = \frac{1}{\epsilon_0}\frac{Qd}{A} \rightarrow C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

(capacitance of a parallel-plate capacitor in vacuum)

$$1\,\mathrm{F} = 1\,\mathrm{C^2/N \cdot m} = 1\,\mathrm{C^2/J} \qquad \epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F/m}$$

Spherical capacitor



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \rightarrow (E)(4\pi r^2) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$V_{ab} = V_a - V_b = \frac{Q}{4\pi\epsilon_0 r_a} - \frac{Q}{4\pi\epsilon_0 r_b} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{Q}{4\pi\epsilon_0}\frac{r_b - r_a}{r_a r_b}$$

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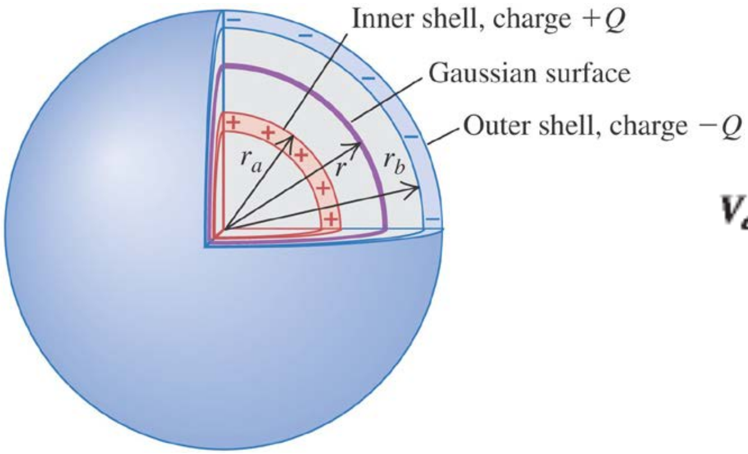

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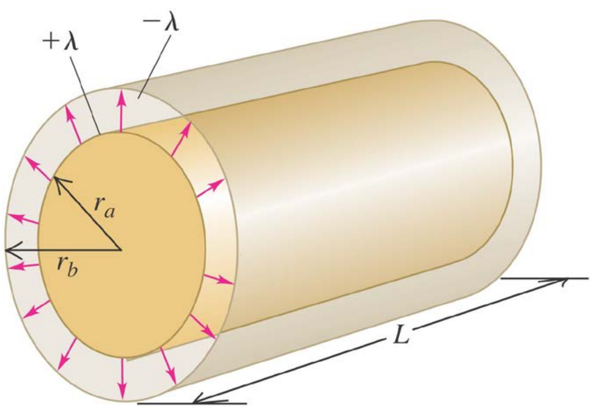
A cylindrical capacitor



$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r} \rightarrow V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$

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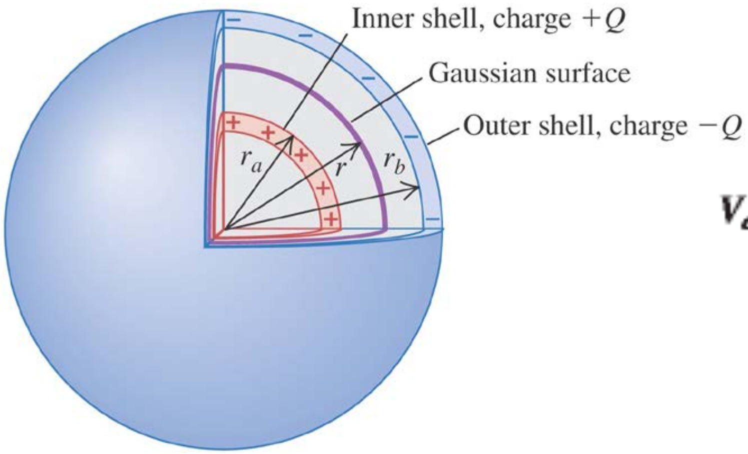

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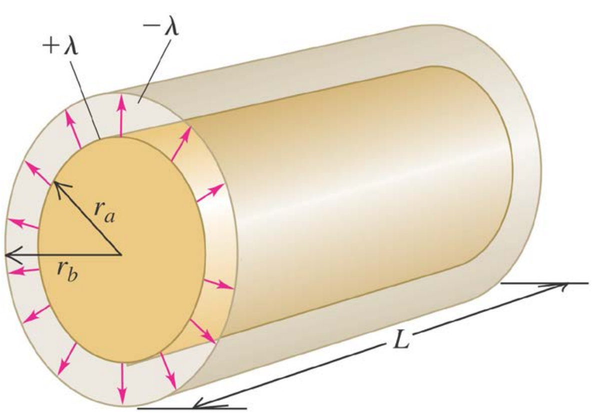
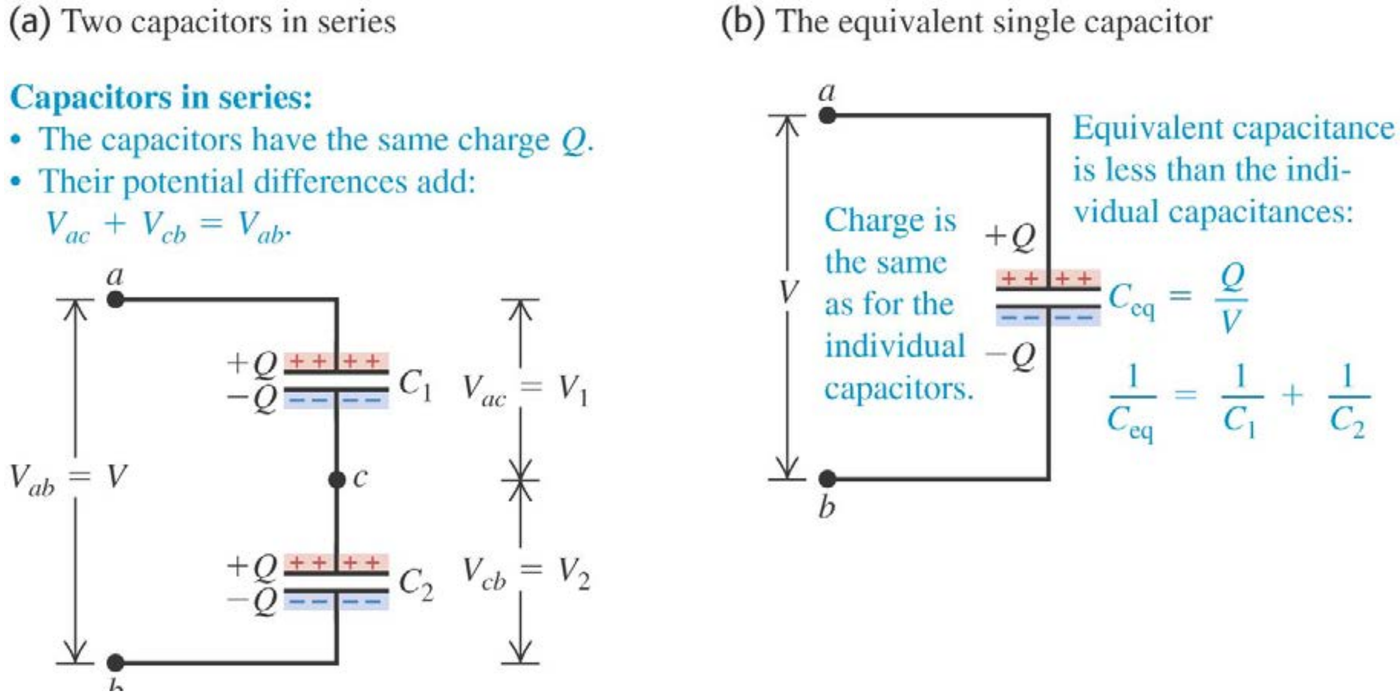


$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r} \rightarrow V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$$

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\dfrac{\lambda}{2\pi\epsilon_0} \ln \dfrac{r_b}{r_a}} = \frac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$$

$$\rightarrow \quad \frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$$

Capacitors in series



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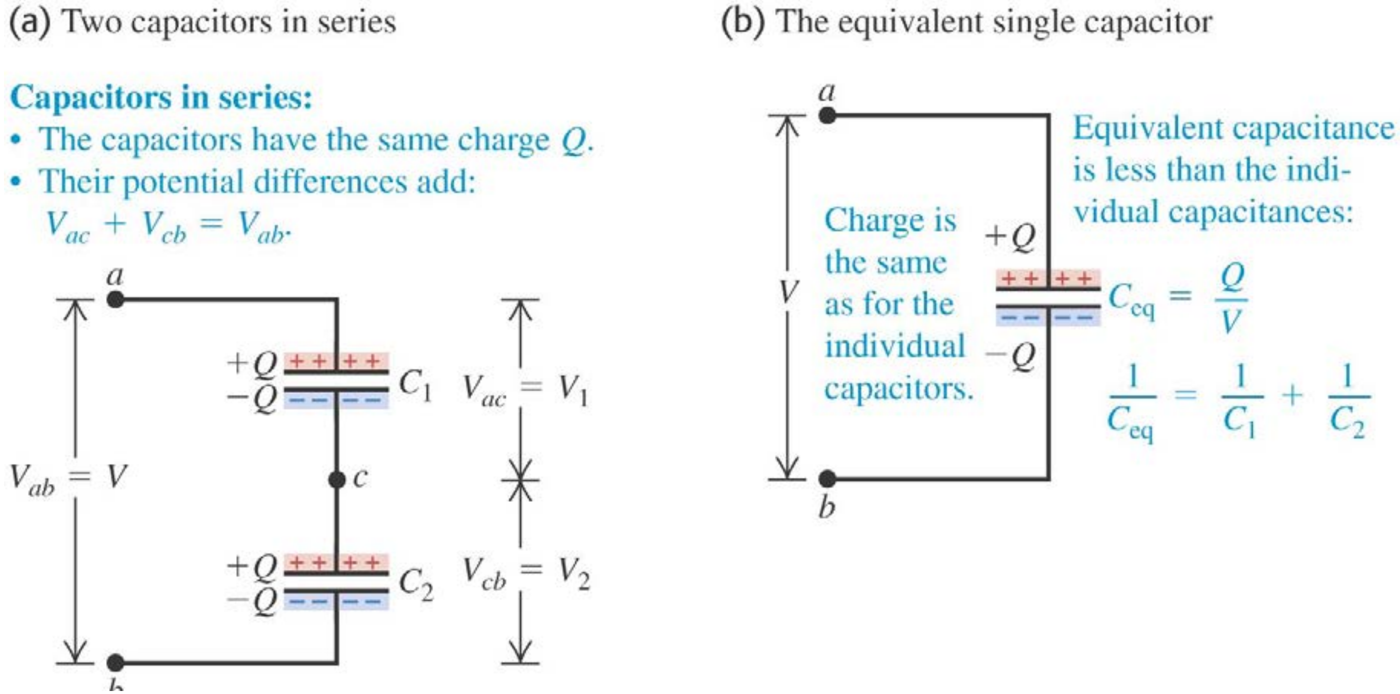
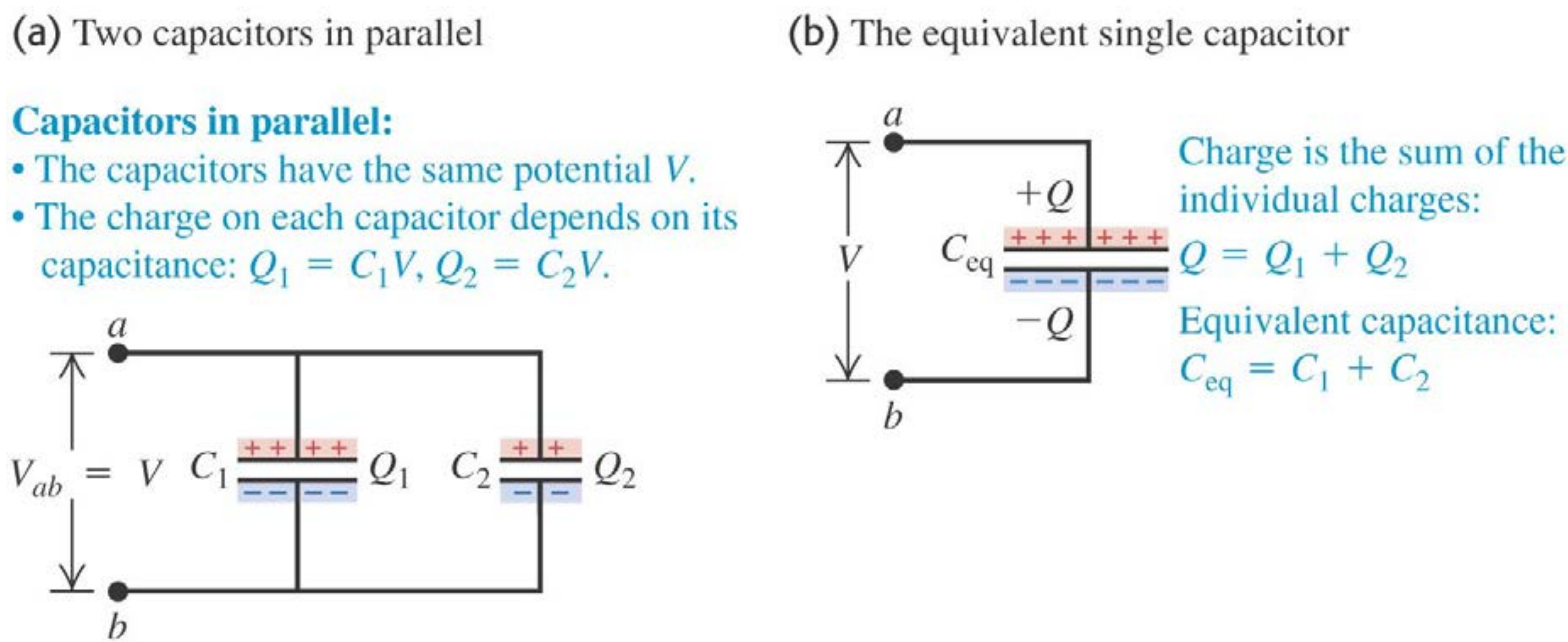


$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2} \quad \rightarrow \quad V_{ab} = V = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \quad \rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(capacitors in series)}$$

Capacitors in parallel

Capacitors are connected in *parallel* between *a* and *b* if the potential difference *Vab* is the same for all the capacitors.



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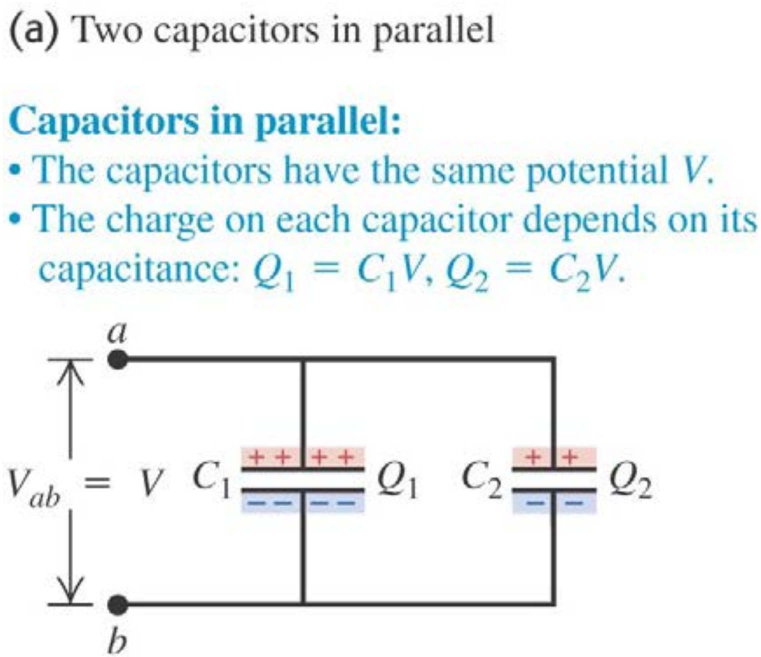

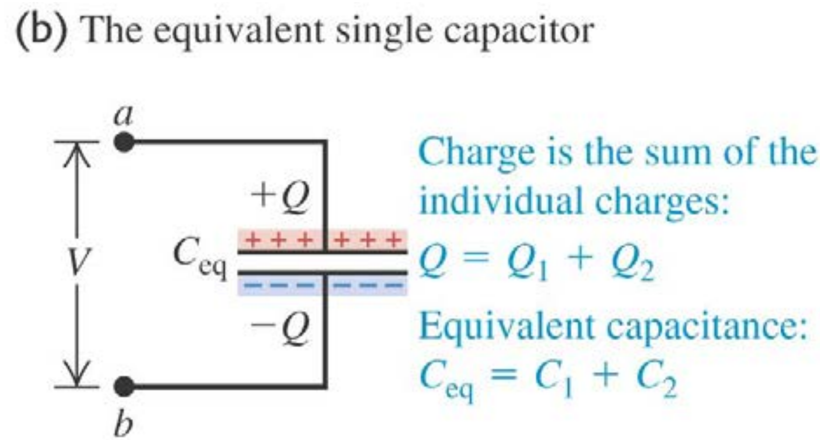

$$Q_1 = C_1V \quad \text{and} \quad Q_2 = C_2V \quad \rightarrow \quad Q = Q_1 + Q_2 = (C_1 + C_2)V$$

$$\rightarrow \quad \frac{Q}{V} = C_1 + C_2 \quad \rightarrow \quad C_{eq} = C_1 + C_2$$

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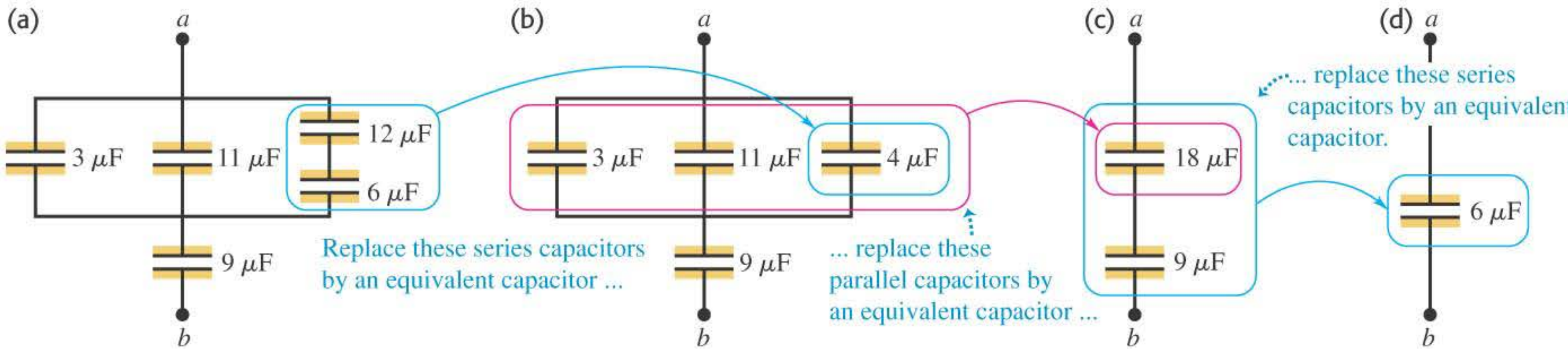
Calculations of capacitance



These capacitors are neither all in series nor all in parallel. We can, however, identify portions of the arrangement that *are* either in series or parallel.
We first replace the 12-μF and 6- μ F series combination by its equivalent capacitance *C′*

$$\frac{1}{C'} = \frac{1}{12\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}} \qquad C' = 4\ \mu\text{F}$$

Now we see three capacitors in parallel, and we replace them with their equivalent capacitance *C″*

$$C'' = 3\ \mu\text{F} + 11\ \mu\text{F} + 4\ \mu\text{F} = 18\ \mu\text{F}$$

➔ $$\frac{1}{C_{\text{eq}}} = \frac{1}{18\ \mu\text{F}} + \frac{1}{9\ \mu\text{F}} \qquad C_{\text{eq}} = 6\ \mu\text{F}$$

Energy stored in a capacitor

- The electric potential energy stored in a charged capacitor is just equal to the amount of work required to charge it.
- When the capacitor is dis- charged, this stored energy is recovered as work done by electrical forces.
- We can calculate the potential energy *U* of a charged capacitor by calculating the work *W* required to charge it. Suppose that when we are done charging the capacitor, the final charge is *Q* and the final potential difference is *V*.

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Let q and v be the charge and potential difference, respectively, at an intermediate stage during the charging process; then $v = q/C$. At this stage the work dW required to transfer an additional element of charge dq is

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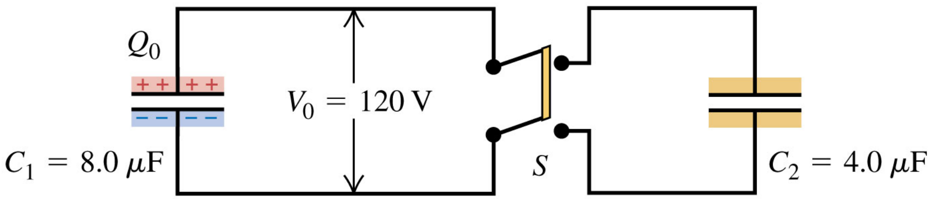
Electric-Field Energy

$$u = \text{Energy density} = \frac{\frac{1}{2}CV^2}{Ad} \rightarrow$$

$$u = \tfrac{1}{2}\epsilon_0 E^2$$ (electric energy density in a vacuum)

Some examples of capacitor energy



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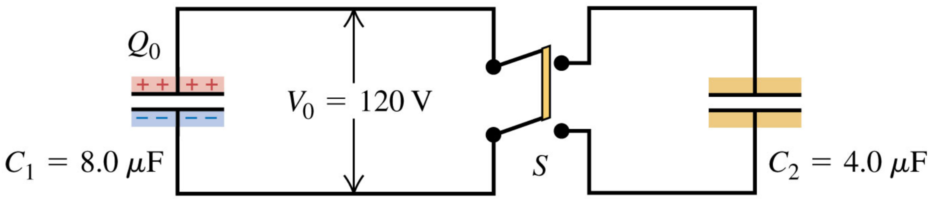

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(b) What is the energy stored in C_1?
(c) Capacitor $C_2 = 4.0\ \mu\text{F}$ is initially uncharged. We close switch S. After charge no longer flows, what is the potential difference across each capacitor, and what is the charge on each capacitor?
(d) What is the final energy of the system?

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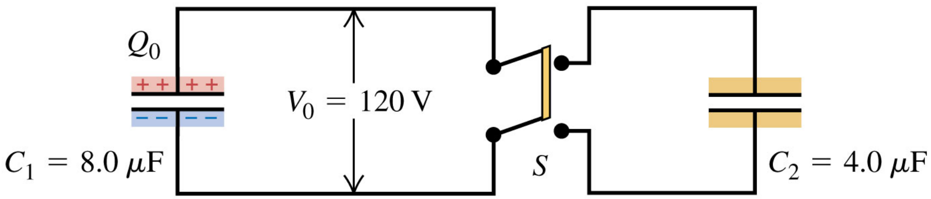

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(a) The initial charge Q_0 on C_1 is

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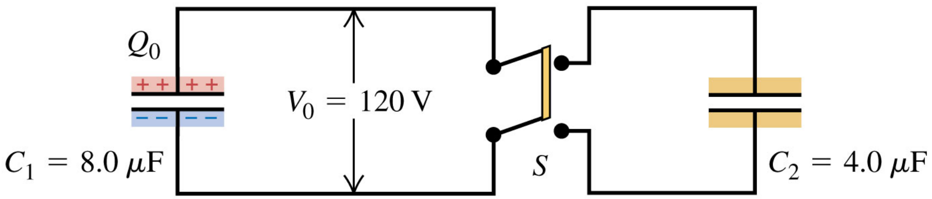

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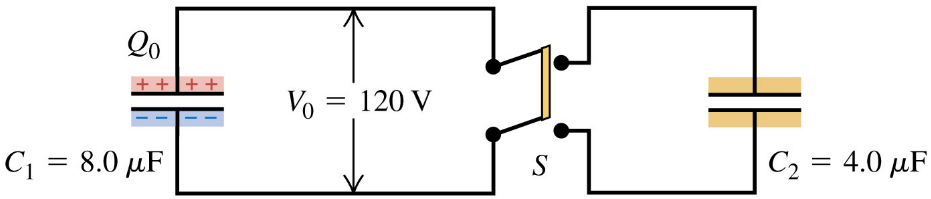

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(c) When we close the switch ➔ $Q_0 = Q_1 + Q_2$

$$\rightarrow \quad V = \frac{Q_0}{C_1 + C_2} = \frac{960\ \mu\text{C}}{8.0\ \mu\text{F} + 4.0\ \mu\text{F}} = 80\ \text{V} \quad \rightarrow \quad Q_1 = 640\ \mu\text{C} \qquad Q_2 = 320\ \mu\text{C}$$

(d) Final energy of the system
$$U_{\text{final}} = \tfrac{1}{2} Q_1 V + \tfrac{1}{2} Q_2 V = \tfrac{1}{2} Q_0 V$$
$$= \tfrac{1}{2}(960 \times 10^{-6}\ \text{C})(80\ \text{V}) = 0.038\ \text{J}$$