

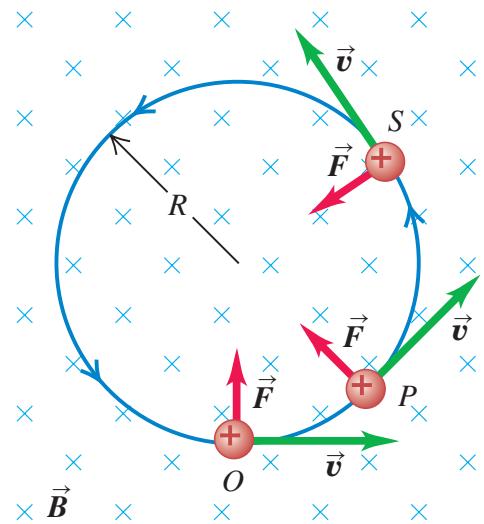
Chp 27: Magnetic Field and Magnetic Forces -II

Goals for Chapter 27

- To study magnets and the forces they exert on each other
- To calculate the force that a magnetic field exerts on a moving charge
- To contrast magnetic field lines with electric field lines
- To analyze the motion of a charged particle in a magnetic field
- To see applications of magnetism in physics and chemistry
- To analyze magnetic forces on current-carrying conductors
- To study the behavior of current loops in a magnetic field

Motion of charged particles in a magnetic field

A charge moving at right angles to a uniform \vec{B} field moves in a circle at constant speed because \vec{F} and \vec{v} are always perpendicular to each other.

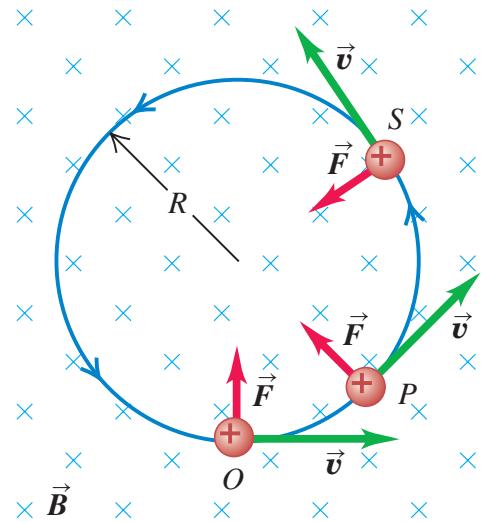


$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

- A particle with positive charge q is moving with velocity v in a uniform magnetic field B directed into the plane of the figure. The vectors v and B are **perpendicular**, so the magnetic force $F = qvB$ and a direction as shown in the figure.

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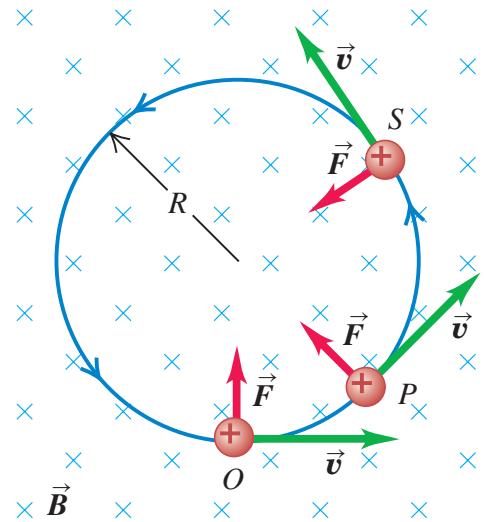


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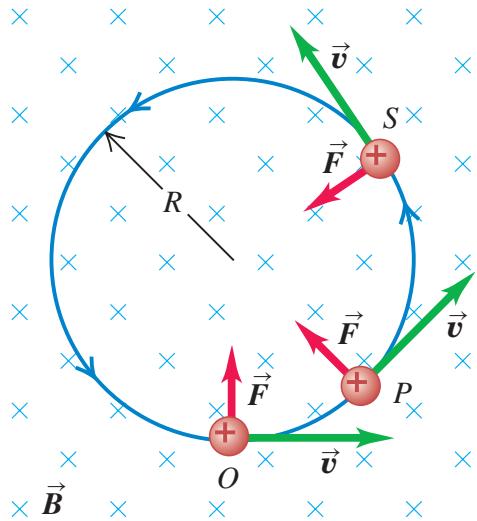


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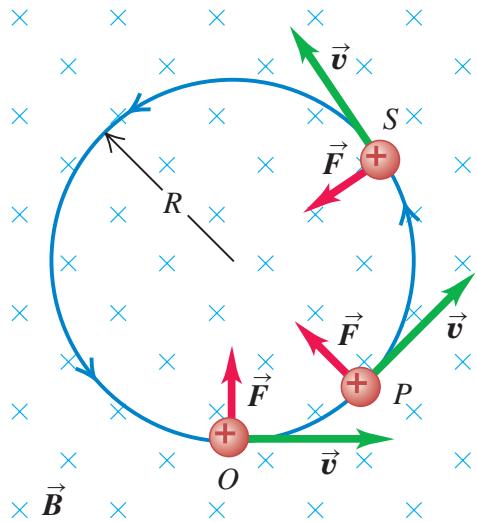


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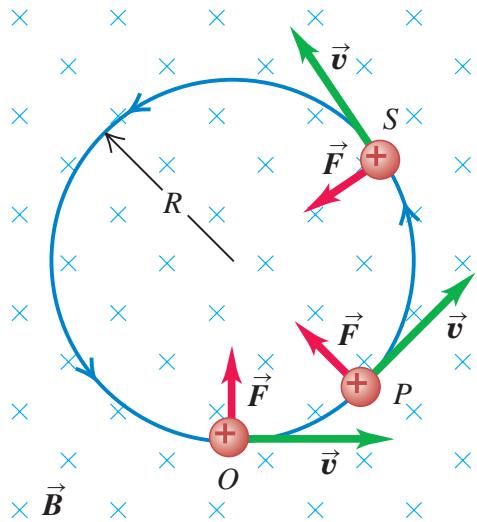
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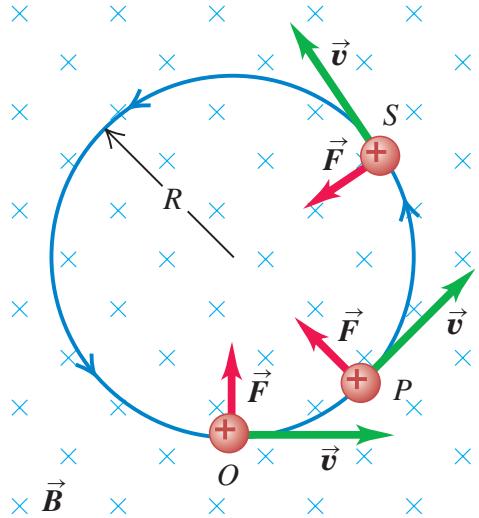
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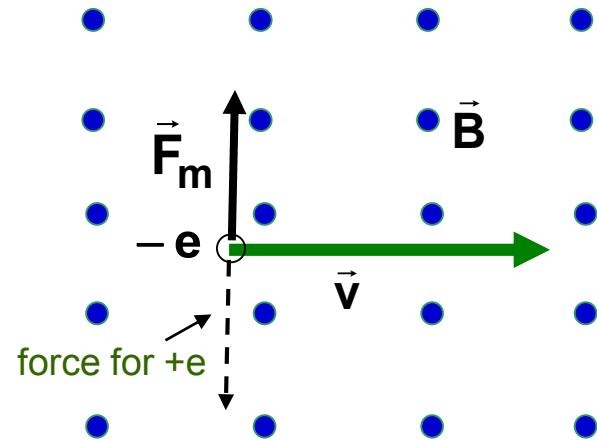
Newton's 2'nd law: $F = |q|vB = m\frac{v^2}{R}$

→ $R = \frac{mv}{|q|B}$ (radius of a circular orbit in a magnetic field)

angular speed: $\omega = \frac{v}{R} = v\frac{|q|B}{mv} = \frac{|q|B}{m}$

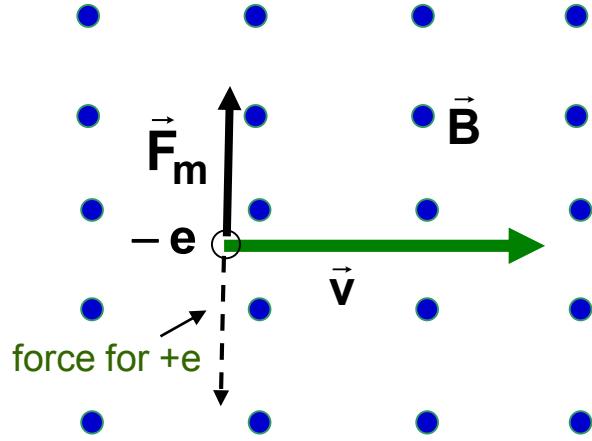
number of revolution per unit time: $f = \omega/2\pi$
(independent of radius)

A numerical example



- Electron beam moving in plane of sketch
- $v=10^7$ m/s along+x
- $B=10^{-3}$ T.out of page along+y

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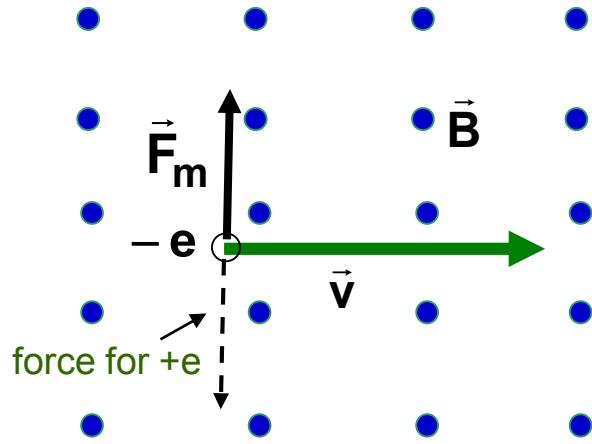
- (a) Find the Force on the electron
(b) Find the acceleration

$$\vec{F}_m = -e \vec{v} \times \vec{B} = -1.6 \times 10^{-19} \times 10^7 \times 10^{-3} \times \sin(90^\circ) \hat{k}$$

$$\vec{F}_m = -1.6 \times 10^{-15} \hat{k}$$

Negative sign means force is opposite to result of using the RH rule

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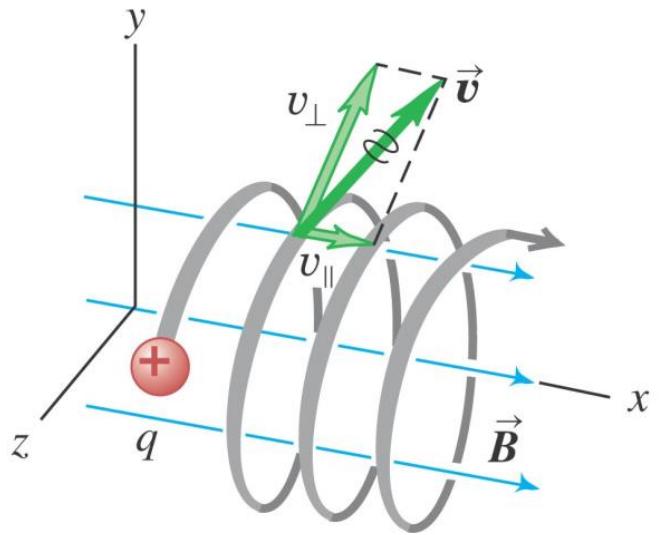
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$$\vec{a} = \frac{\vec{F}_m}{m_e} = \frac{-1.6 \times 10^{-15} \text{ N}}{9 \times 10^{-31} \text{ Kg}} \hat{k} = -1.76 \times 10^{+15} \hat{k} [\text{m/s}^2]$$

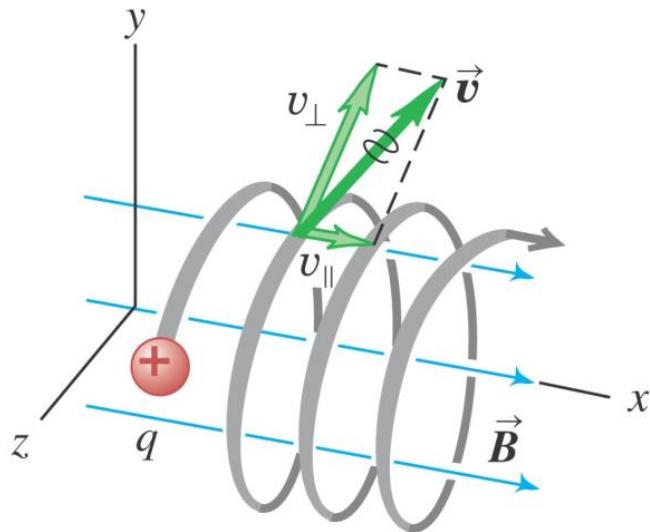
Direction is the same as that of the force

Helical motion



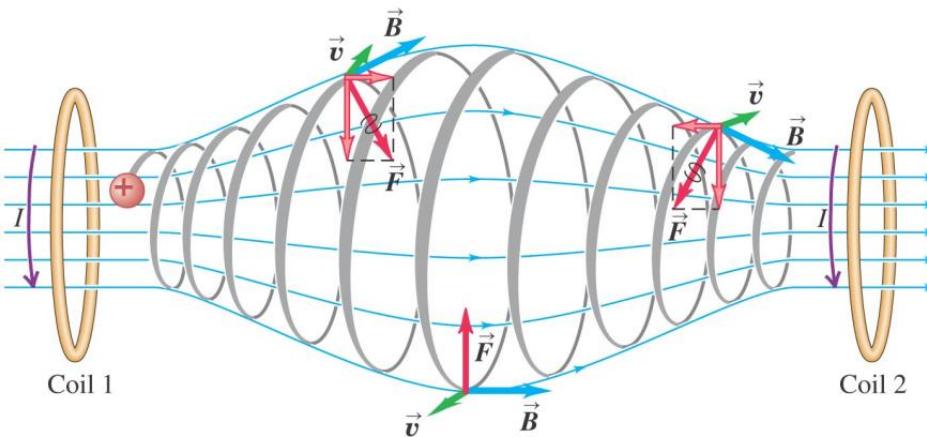
- If the particle has velocity components parallel to and perpendicular to the field, its path is a helix.

Helical motion

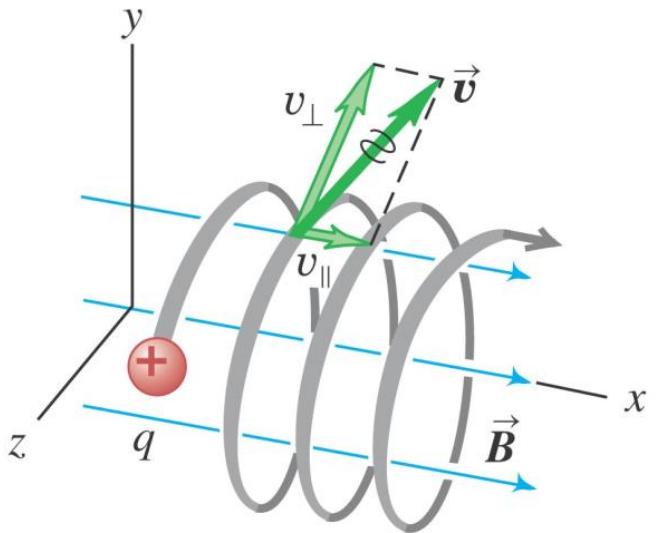


- If the particle has velocity components parallel to and perpendicular to the field, its path is a helix.
- The speed and kinetic energy of the particle remain constant.

A nonuniform magnetic field

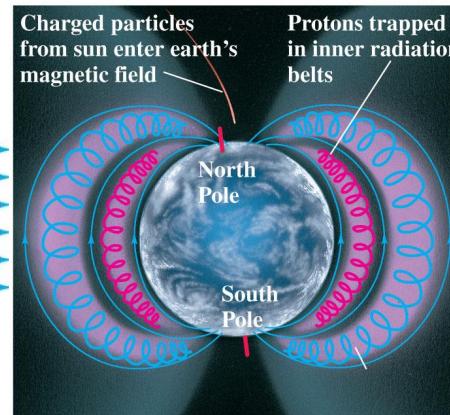
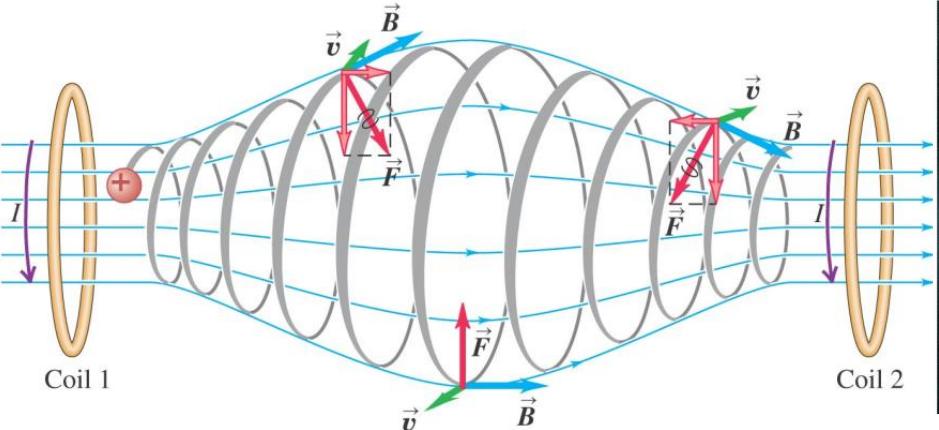


Helical motion

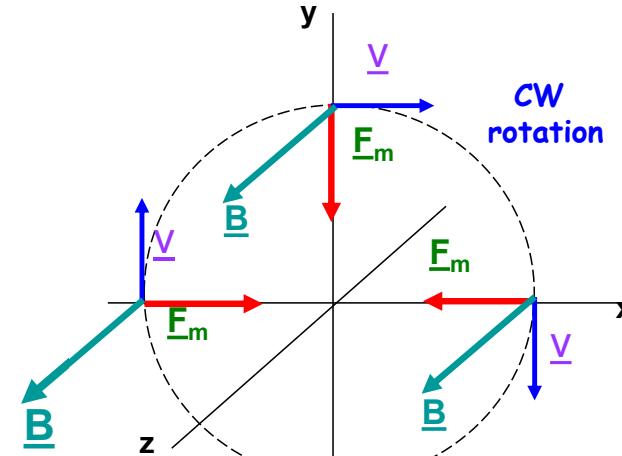
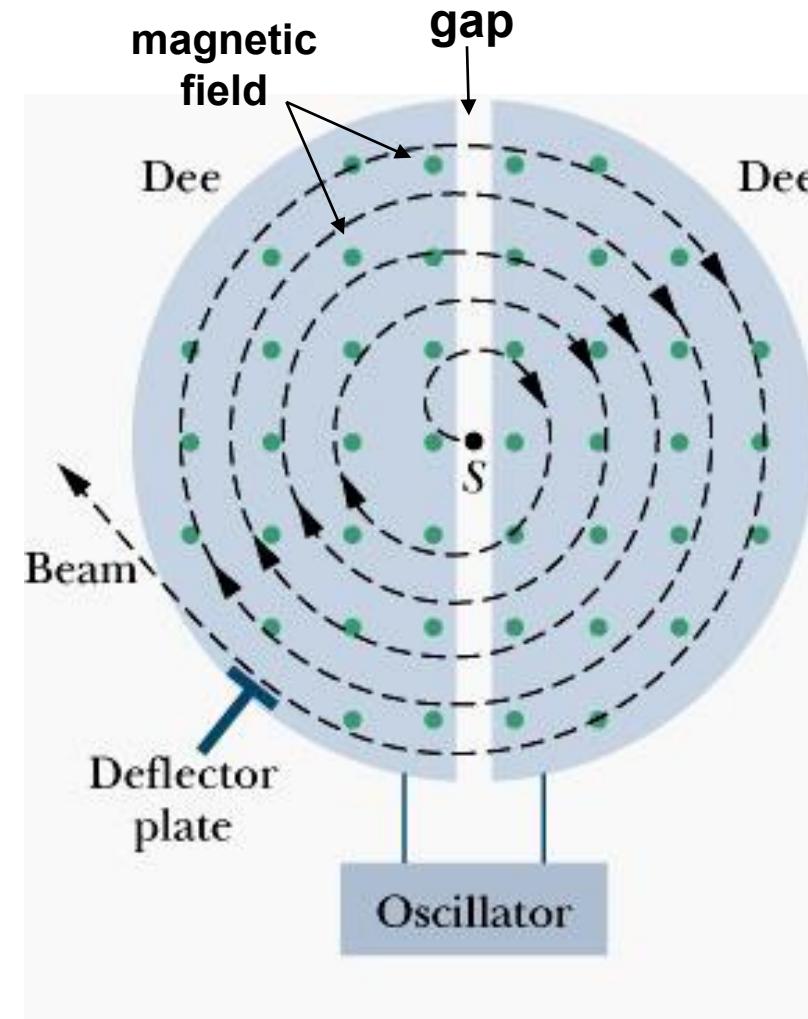


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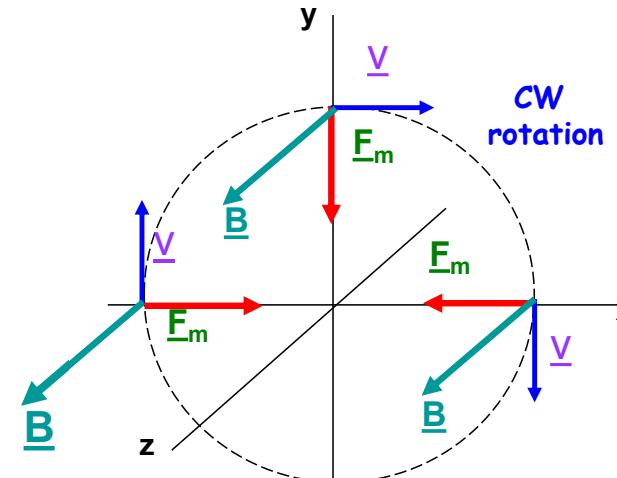
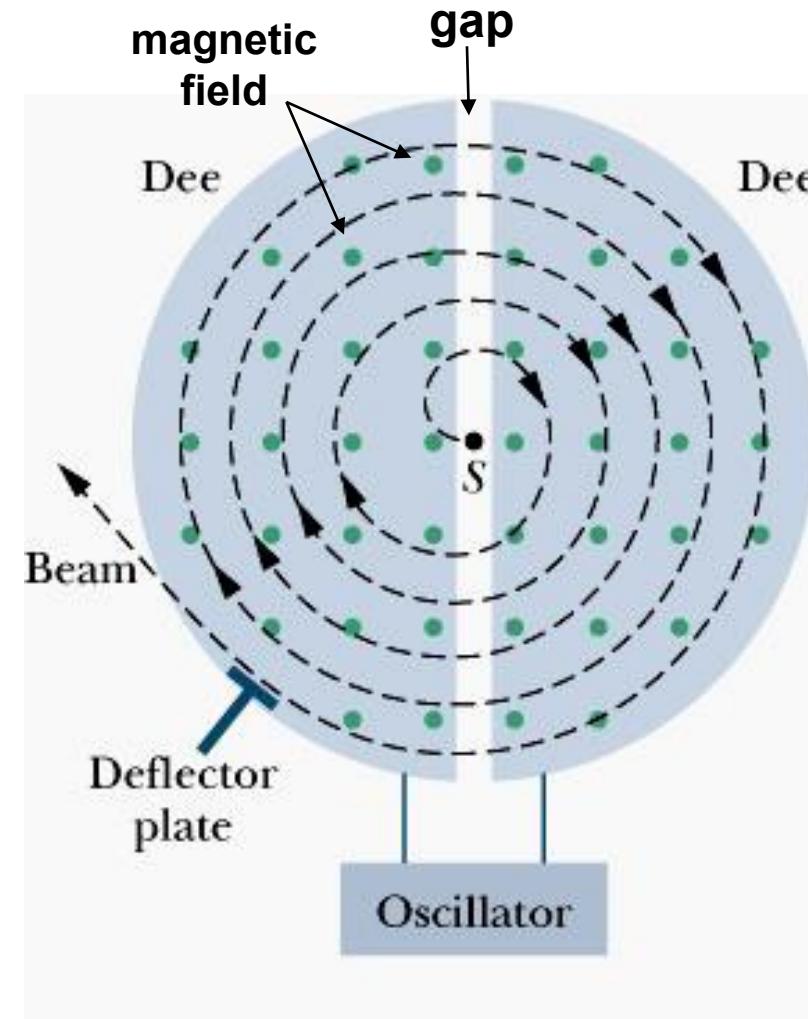
Cyclotron Frequency: Charged Particles Circle at Constant Speed in a Uniform Magnetic Field (used in Early nuclear physics research, Biomedical applications)



$$r = \frac{mv}{qB}$$

$$\omega_c = \frac{2\pi}{\tau_c} = \frac{qB}{m}$$

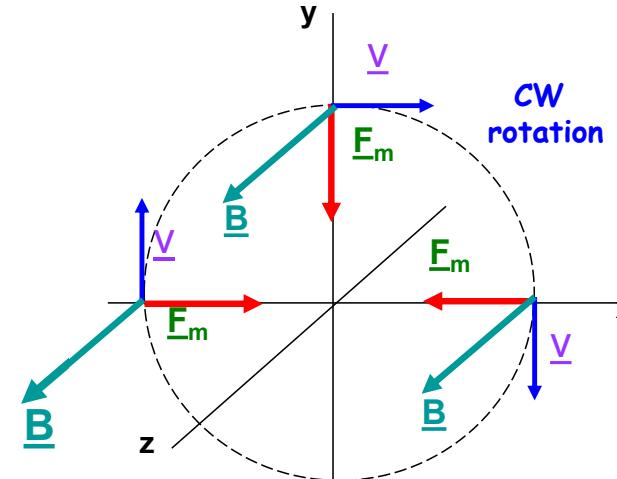
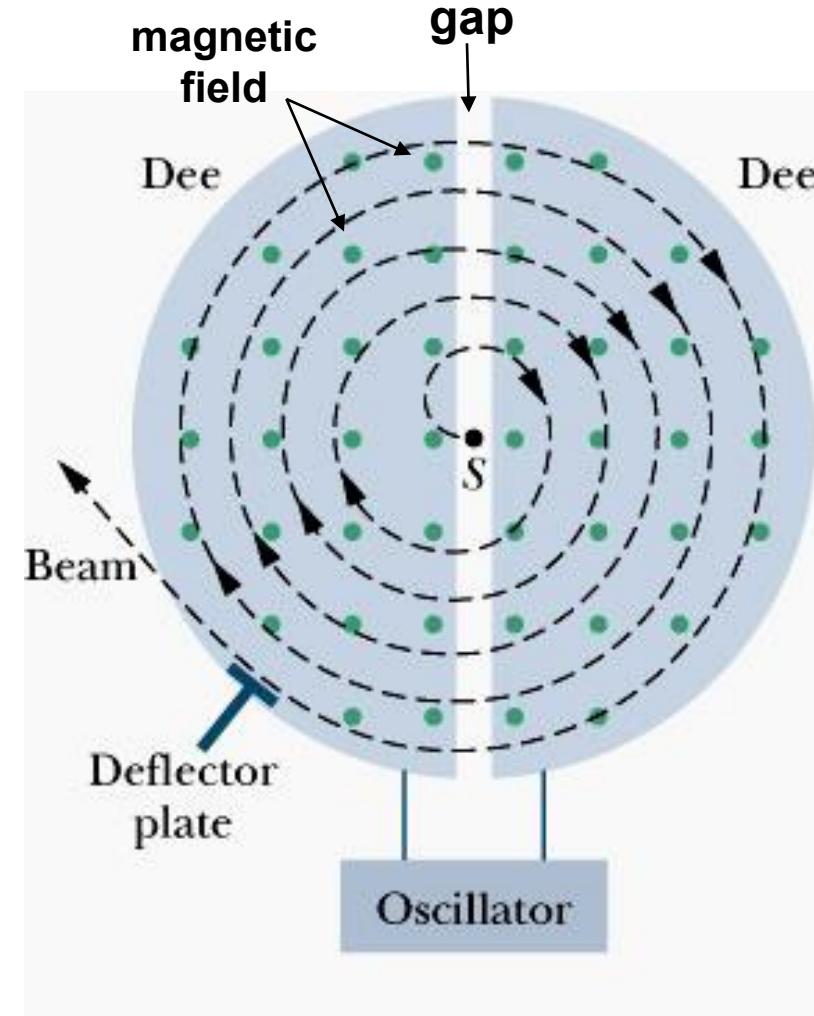
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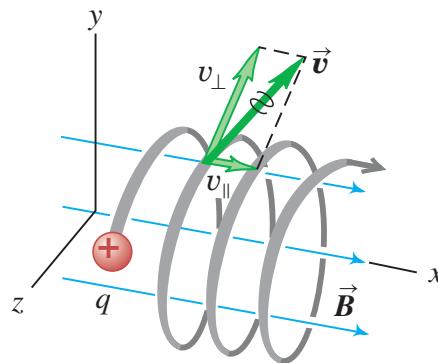
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- Frequency does not depend on speed or radius

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Ex: Helical particle motion in a magnetic field

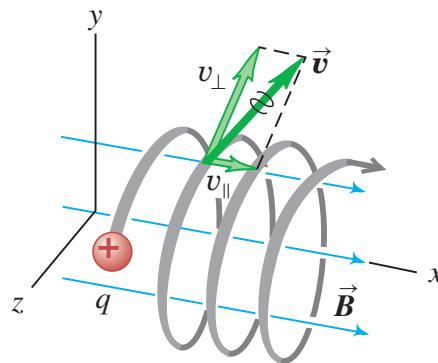


a proton ($q = 1.60 \times 10^{-19}$, $m = 1.67 \times 10^{-27}$ kg) and the uniform, 0.500-T magnetic field is directed along the x -axis.

At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s. Only the magnetic force acts on the proton.

- At $t = 0$, find the force on the proton and its acceleration.
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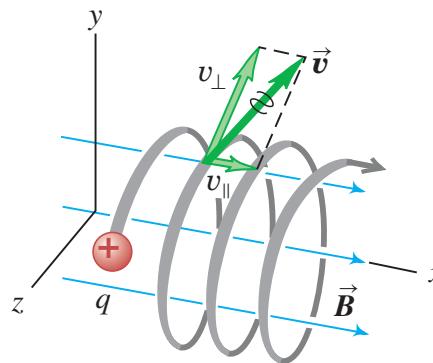
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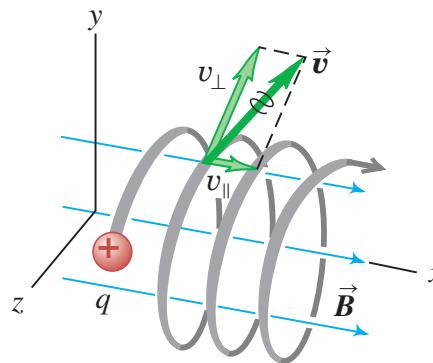
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$$\begin{aligned}\vec{B} &= B\hat{i} \text{ and } \vec{v} = v_x\hat{i} + v_z\hat{k} \quad (\text{a}) \rightarrow \vec{F} = q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_z\hat{k}) \times B\hat{i} = qv_z B\hat{j} \\ &= (1.60 \times 10^{-19} \text{ C})(2.00 \times 10^5 \text{ m/s})(0.500 \text{ T})\hat{j} \\ &= (1.60 \times 10^{-14} \text{ N})\hat{j}\end{aligned}$$

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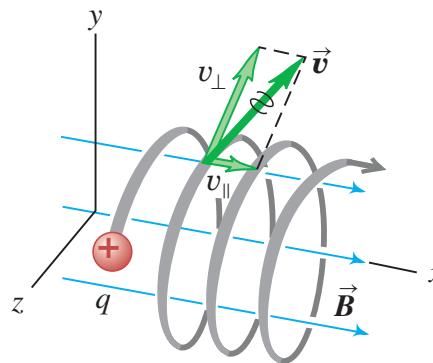
$$\rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{1.60 \times 10^{-14} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} \hat{j} = (9.58 \times 10^{12} \text{ m/s}^2)\hat{j}$$

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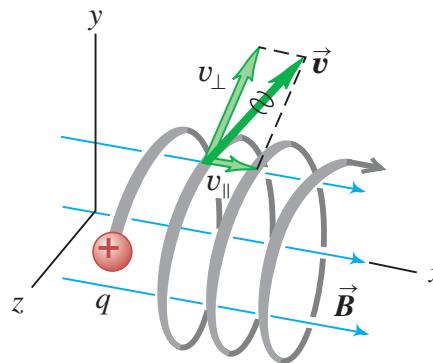
$$(\text{b}) \quad R = \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 4.18 \times 10^{-3} \text{ m} = 4.18 \text{ mm}$$

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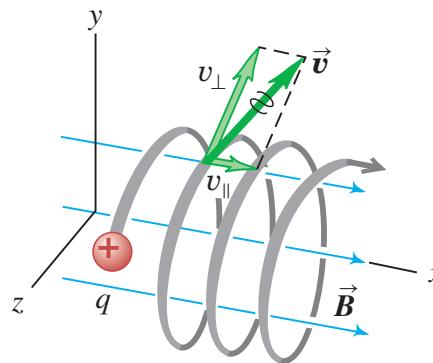
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$$(\text{b}) \quad R = \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 4.18 \times 10^{-3} \text{ m} = 4.18 \text{ mm}$$

$$\rightarrow \omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 4.79 \times 10^7 \text{ rad/s}$$

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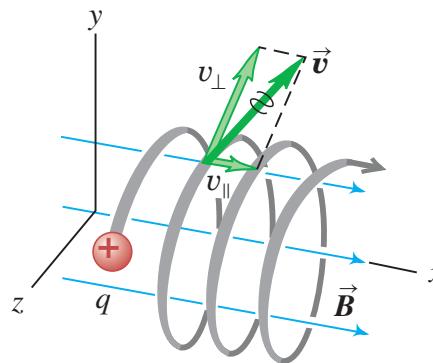
$$\rightarrow T = 2\pi/\omega = 2\pi/(4.79 \times 10^7 \text{ s}^{-1}) = 1.31 \times 10^{-7} \text{ s}$$

a proton ($q = 1.60 \times 10^{-19}$, $m = 1.67 \times 10^{-27}$ kg) and the uniform, 0.500-T magnetic field is directed along the x -axis.

At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s. Only the magnetic force acts on the proton.

- (a) At $t = 0$, find the force on the proton and its acceleration.
- (b) Find the radius of the resulting helical path, the angular speed of the proton, and the *pitch* of the helix (the distance traveled along the helix axis per revolution).

Ex: Helical particle motion in a magnetic field



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{B} = B\hat{i} \text{ and } \vec{v} = v_x\hat{i} + v_z\hat{k} \quad (\text{a}) \rightarrow \vec{F} = q\vec{v} \times \vec{B} = q(v_x\hat{i} + v_z\hat{k}) \times B\hat{i} = qv_z B \hat{j}$$

$$= (1.60 \times 10^{-19} \text{ C})(2.00 \times 10^5 \text{ m/s})(0.500 \text{ T})\hat{j}$$

$$= (1.60 \times 10^{-14} \text{ N})\hat{j}$$

$$\rightarrow \vec{a} = \frac{\vec{F}}{m} = \frac{1.60 \times 10^{-14} \text{ N}}{1.67 \times 10^{-27} \text{ kg}}\hat{j} = (9.58 \times 10^{12} \text{ m/s}^2)\hat{j}$$

$$(\text{b}) \quad R = \frac{mv_z}{|q|B} = \frac{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} = 4.18 \times 10^{-3} \text{ m} = 4.18 \text{ mm}$$

$$\rightarrow \omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})}{1.67 \times 10^{-27} \text{ kg}} = 4.79 \times 10^7 \text{ rad/s}$$

$$\rightarrow T = 2\pi/\omega = 2\pi/(4.79 \times 10^7 \text{ s}^{-1}) = 1.31 \times 10^{-7} \text{ s} \rightarrow v_x T = (1.50 \times 10^5 \text{ m/s})(1.31 \times 10^{-7} \text{ s}) = 0.0197 \text{ m} = 19.7 \text{ mm}$$

a proton ($q = 1.60 \times 10^{-19}$, $m = 1.67 \times 10^{-27}$ kg) and the uniform, 0.500-T magnetic field is directed along the x -axis.

At $t = 0$ the proton has velocity components $v_x = 1.50 \times 10^5$ m/s, $v_y = 0$, and $v_z = 2.00 \times 10^5$ m/s. Only the magnetic force acts on the proton.

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Charged particle in both E and B fields

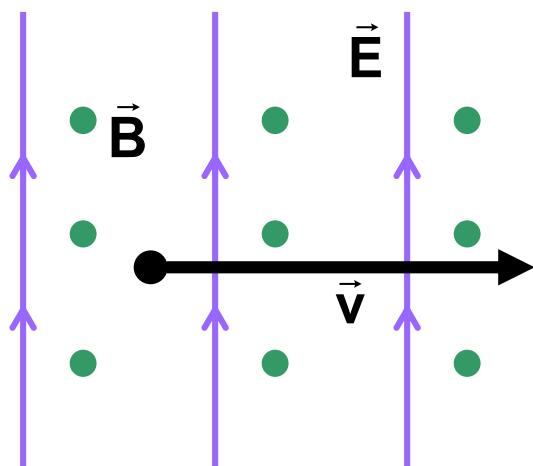
Add the electrostatic
and magnetic forces

$$\vec{F}_{\text{tot}} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$\vec{F}_{\text{tot}} = m\vec{a}$$

also

Example: Crossed E and B fields

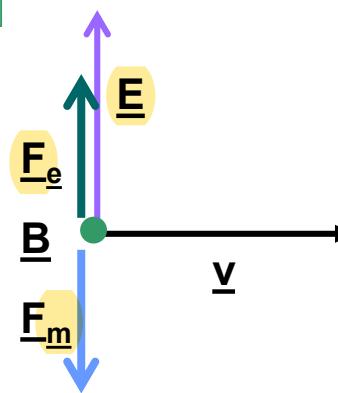


- \vec{B} out of paper
- \vec{E} up and normal to \vec{B}
- + charge
- \vec{v} normal to both \vec{E} & \vec{B}

$$\vec{F}_e = q\vec{E} \quad (\text{up})$$
$$\vec{F}_m = q\vec{v} \times \vec{B} \quad (\text{down})$$

CONVENTION
● OUT X IN

FBD of q



Equilibrium when...

$$\vec{F}_{\text{tot}} = 0$$

$$q\vec{E} = q\vec{v}\vec{B}$$

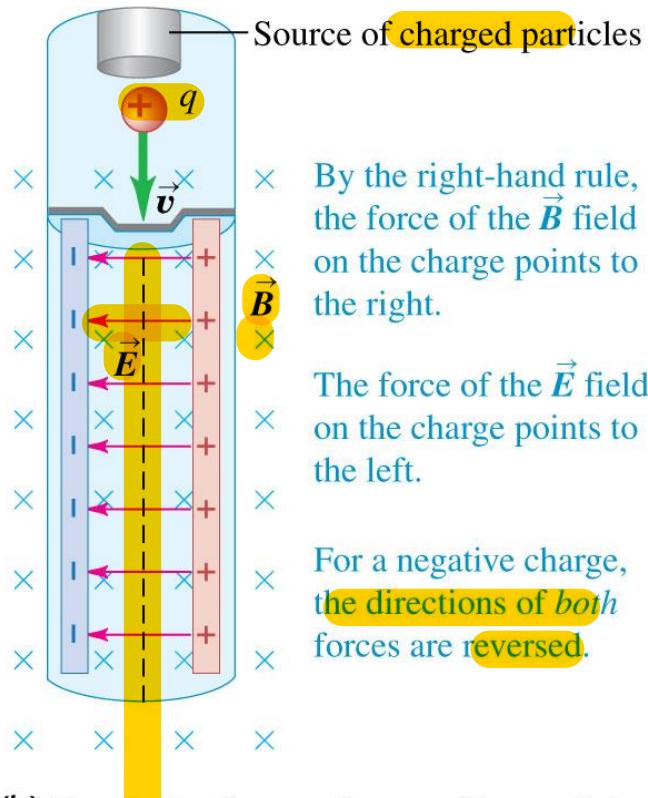
$$\vec{v} = |\vec{E}| / |\vec{B}|$$

OPPOSED
FORCES

- Independent of charge q
- Only particles having speed $v = E/B$ pass through undeflected.

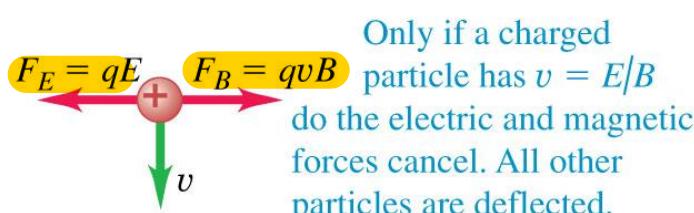
Velocity selector

(a) Schematic diagram of velocity selector



- × By the right-hand rule, the force of the \vec{B} field on the charge points to the right.
- × The force of the \vec{E} field on the charge points to the left.
- × For a negative charge, the directions of both forces are reversed.

(b) Free-body diagram for a positive particle



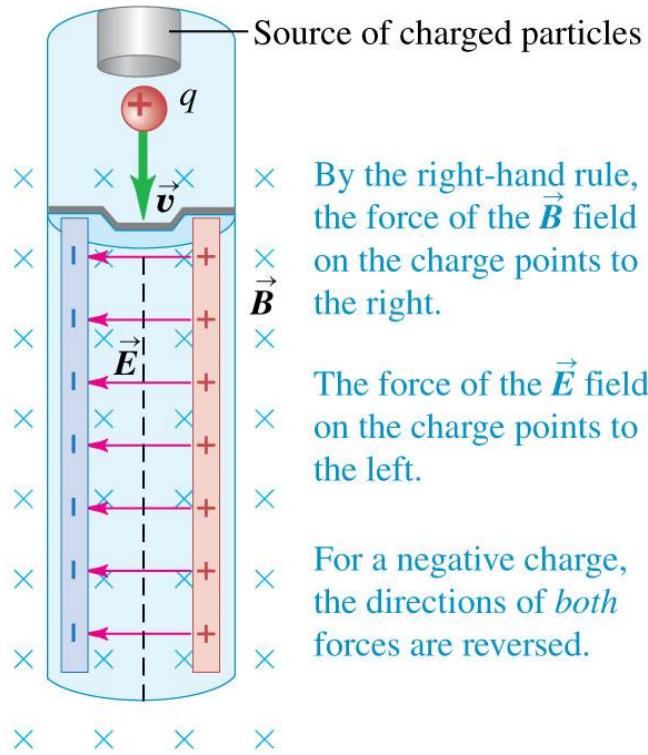
Only if a charged particle has $v = E/B$ do the electric and magnetic forces cancel. All other particles are deflected.

- A velocity selector uses perpendicular electric and magnetic fields to select particles of a specific speed from a beam.

- Only particles having speed $v = E/B$ pass through undeflected.

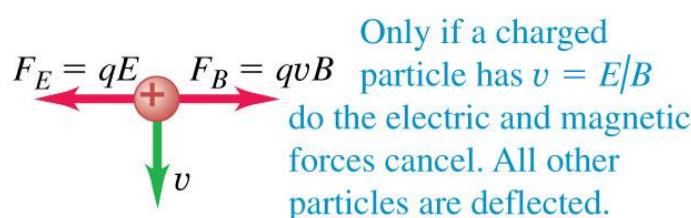
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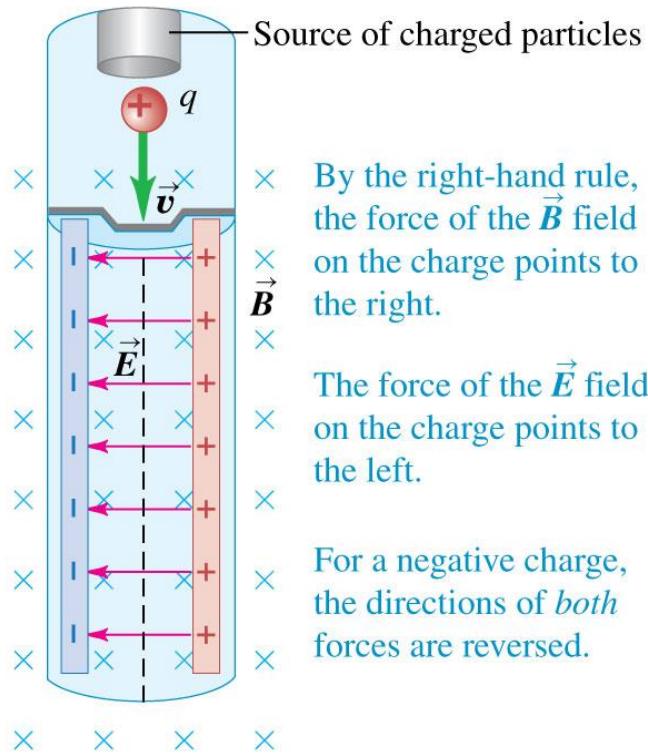
- Only particles having speed $v = E/B$ pass through undeflected.

In a beam of charged particles produced by a heated cathode or a radioactive material, not all particles move with the same speed. If we can create a beam in which all the particle speeds are the same, a specific speed can be selected from the beam using an arrangement of electric and magnetic fields

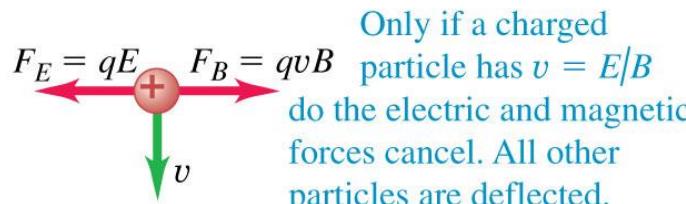
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

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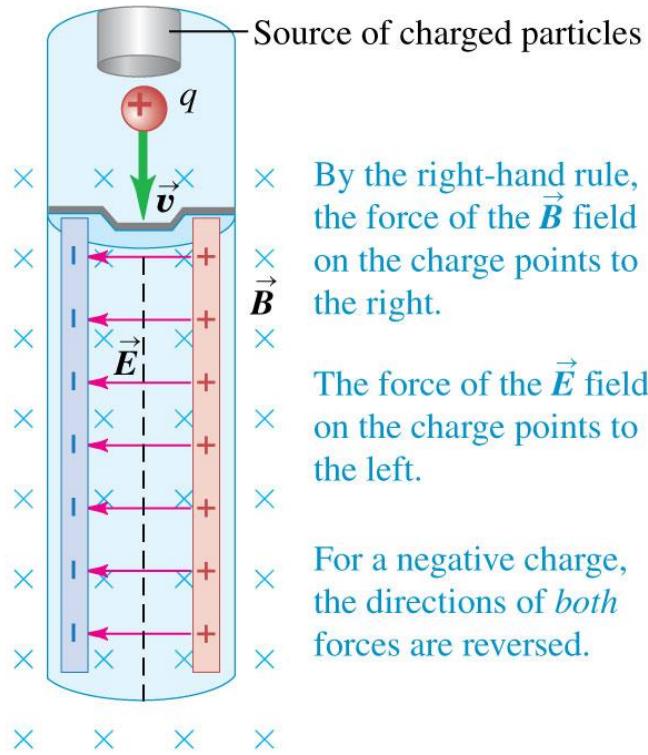
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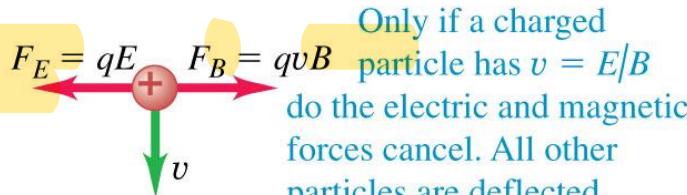
$$\sum F_y = 0 \rightarrow -qE + qvB = 0$$

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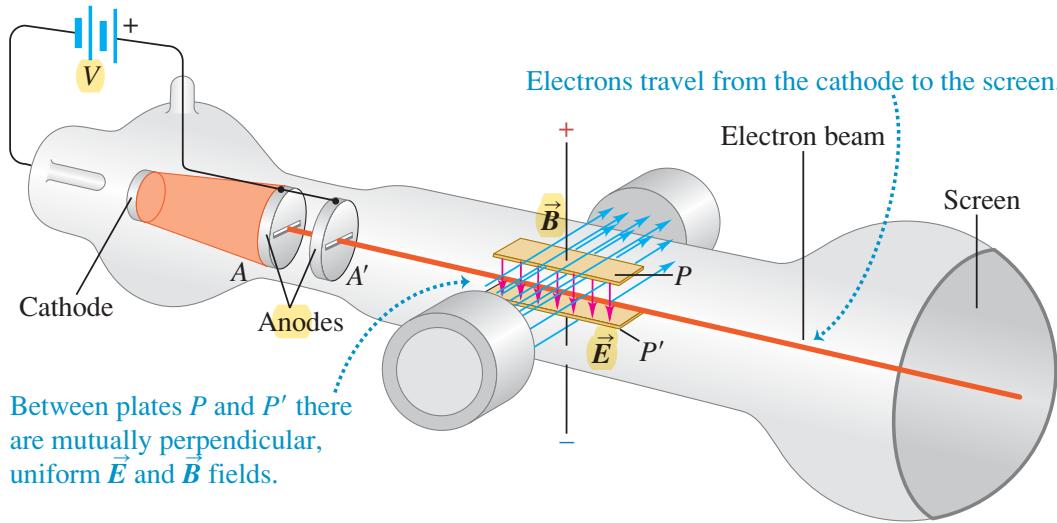
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\sum F_y = 0 \rightarrow -qE + qvB = 0$$

$$v = \frac{E}{B}$$

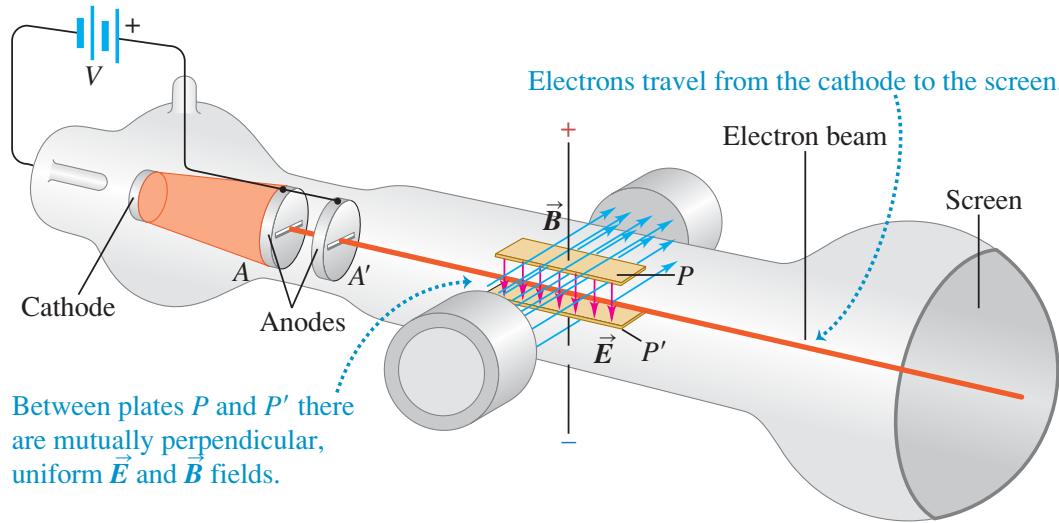
Only particles with speeds equal to E/B can pass through without being deflected by the fields

Thomson's e/m Experiment



For his experiment carried out in 1897 at the Cavendish Laboratory in Cambridge, England, J.J. Thomson used the apparatus shown in the figure

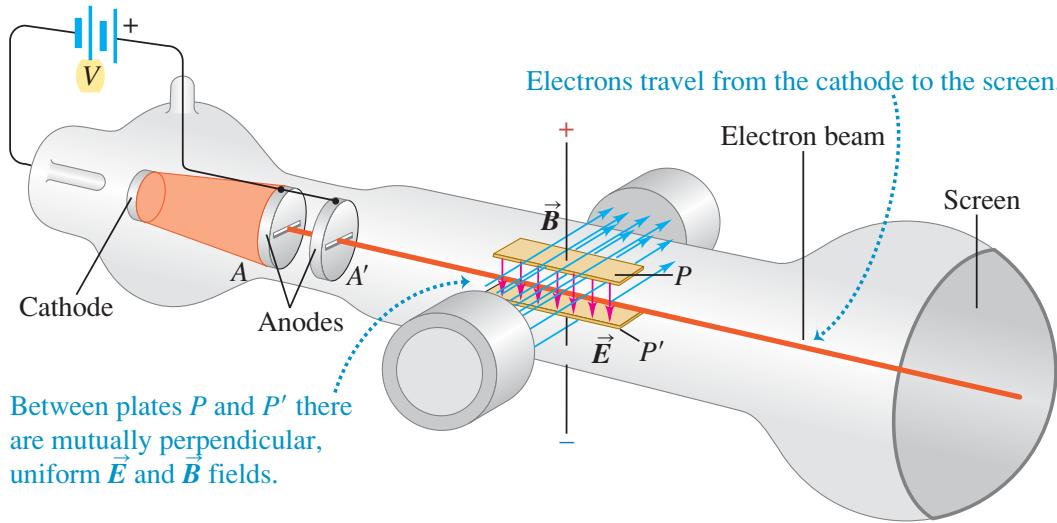
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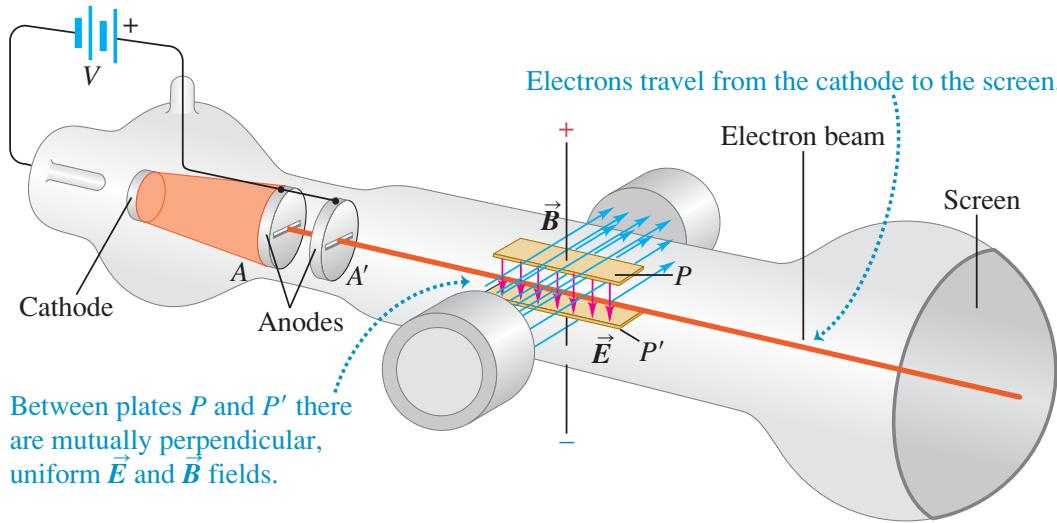
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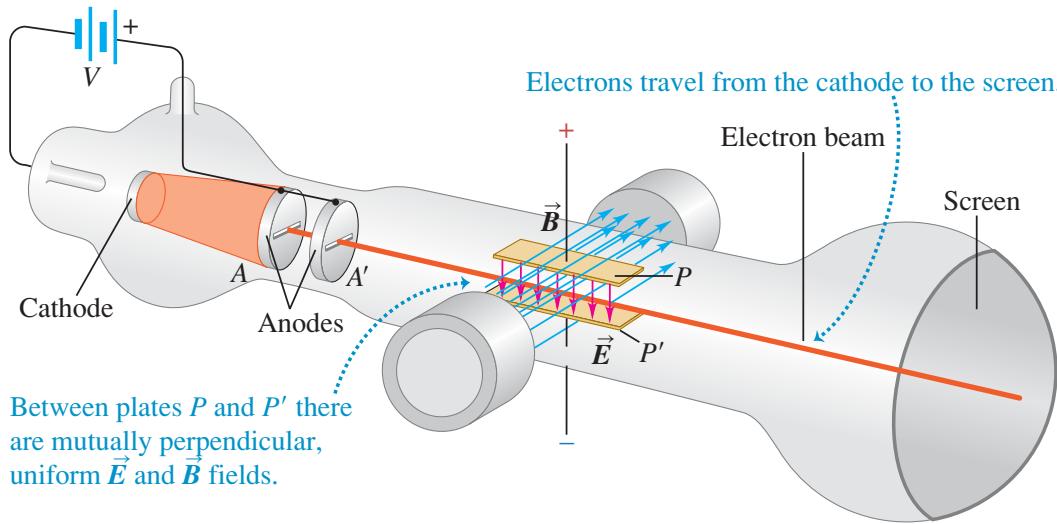


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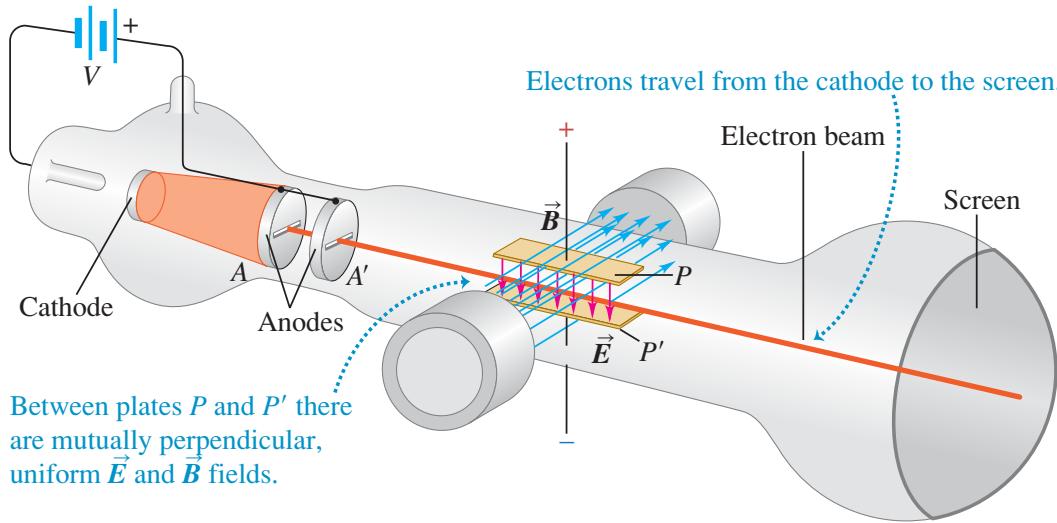
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$$\rightarrow \quad \frac{e}{m} = \frac{E^2}{2VB^2}$$

Thomson's e/m Experiment



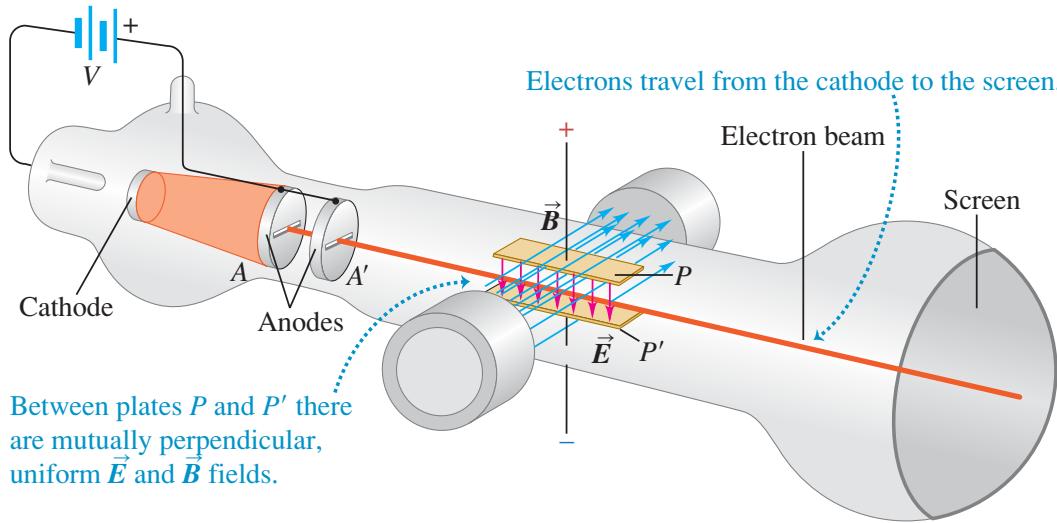
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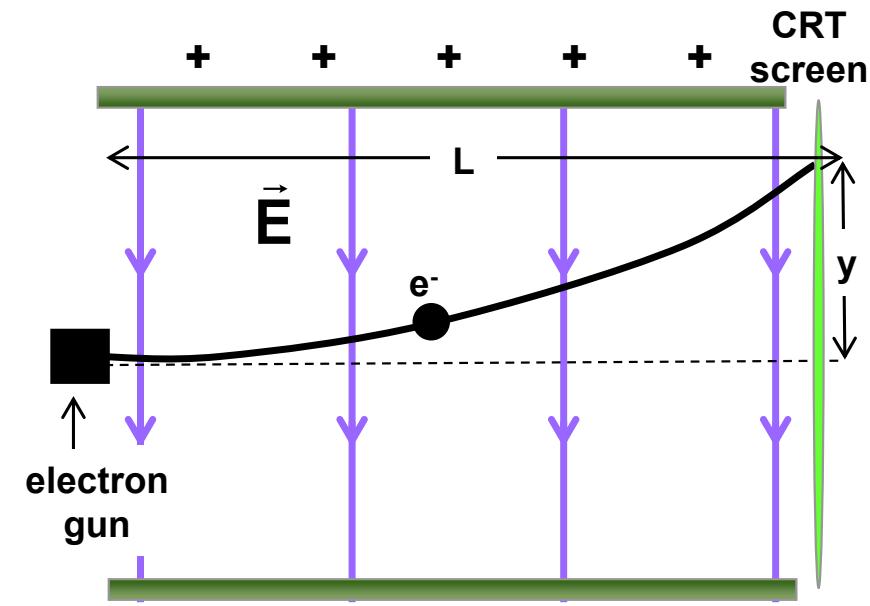
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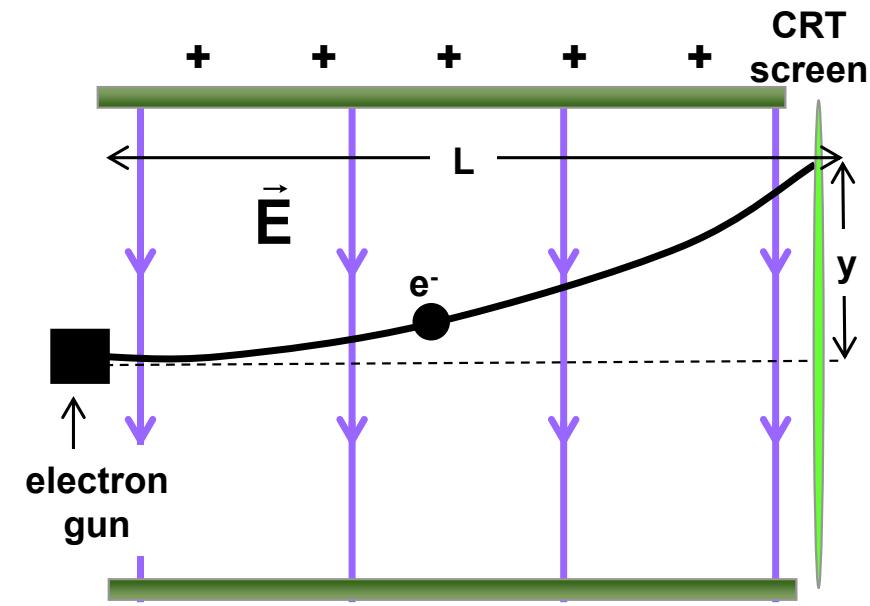
$$\rightarrow \quad m = 9.10938215(45) \times 10^{-31} \text{ kg}$$

Another approach of Thomson's e/m Experiment



First: . Apply E field only in $-y$ direction
. y = deflection of beam from center of the screen (position for $E = 0$)

Another approach of Thomson's e/m Experiment



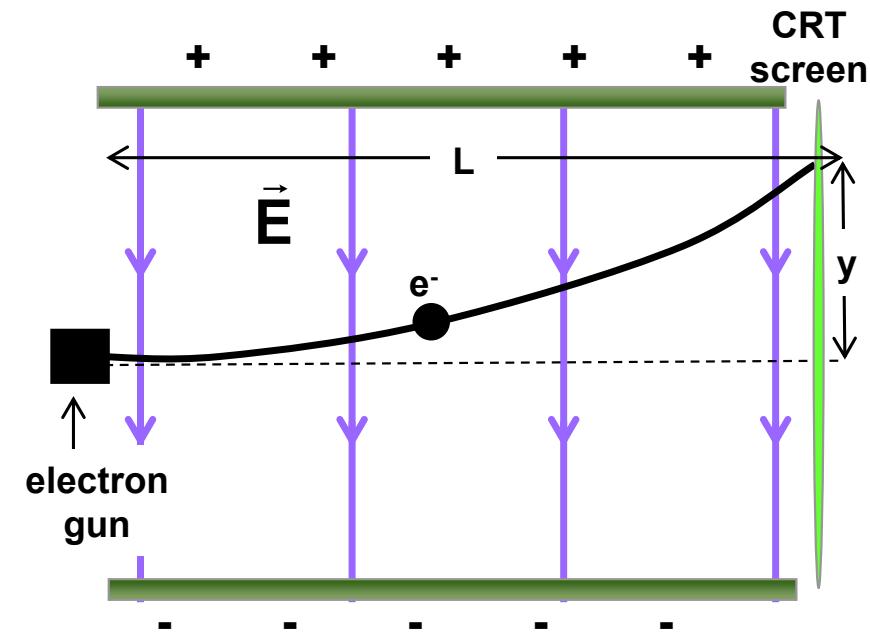
First: . Apply E field only in -y direction
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$$F_y = qE = ma_y$$
$$q = -e$$



$$\frac{e}{m} = \frac{a_y}{E}$$

Another approach of Thomson's e/m Experiment



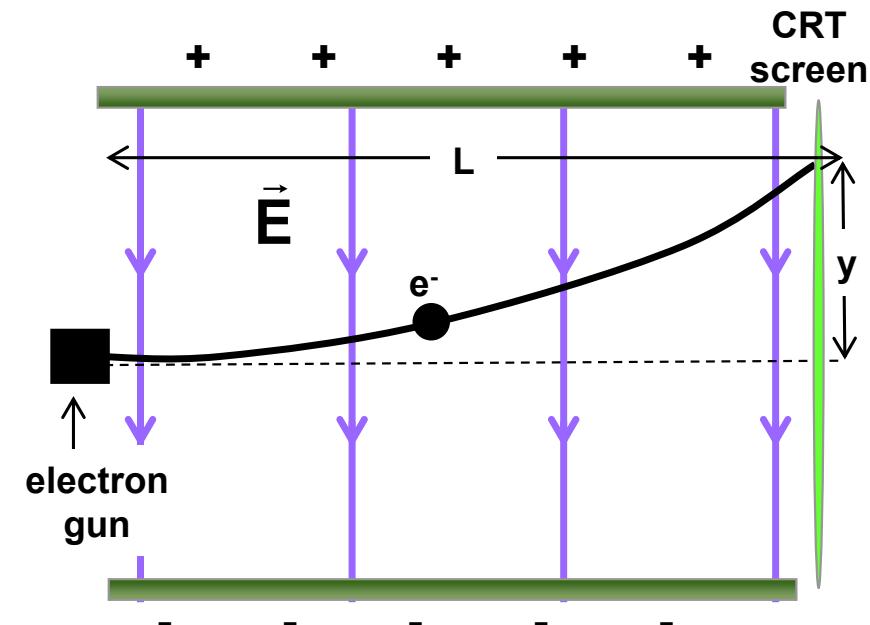
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measure $y = \frac{1}{2} a_y t^2$
note that $t = L / v_x$

Next: . Find a_y by measuring y and flight time $t = L / v_x$ (constant)

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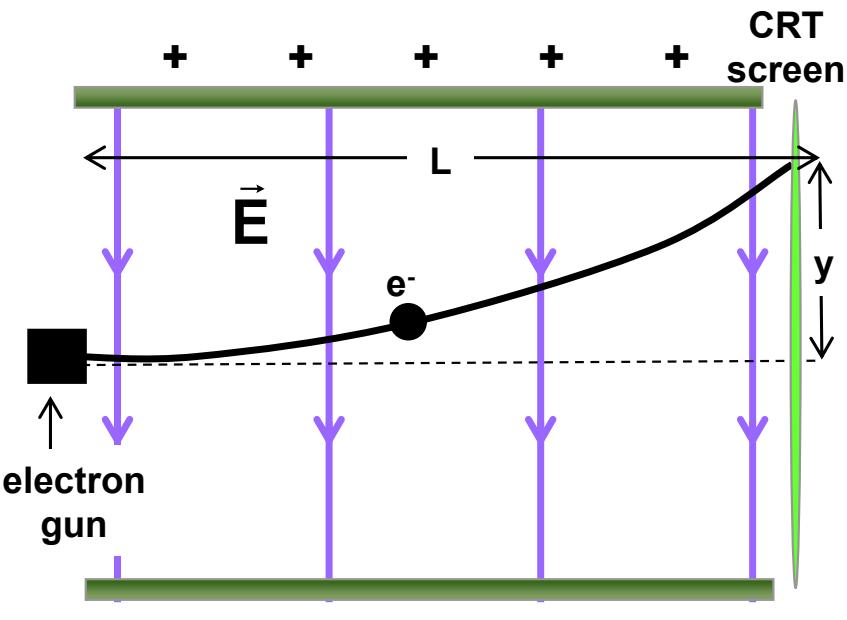
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Use crossed B and E fields to measure v_x

Let B point into the page, perpendicular to both E and V_x

F_M points along $-y$, opposite to FE (negative charge)

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$$\vec{F}_M = -e\vec{v} \times \vec{B} \quad \vec{F}_E = -e\vec{E}$$

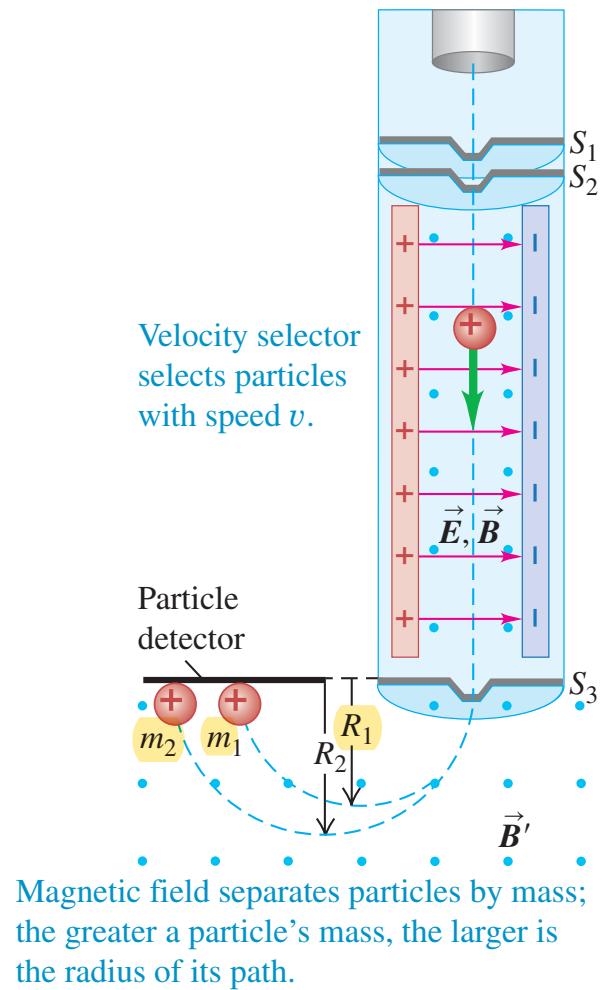
at equilibrium $v_x = E / B$

Adjust B until beam deflection = 0 (F_E cancels F_M) to find v_x and time t

$$t = \frac{BL}{E} \Rightarrow a_y = \frac{2y}{t^2} = 2y \frac{E^2}{B^2 L^2}$$

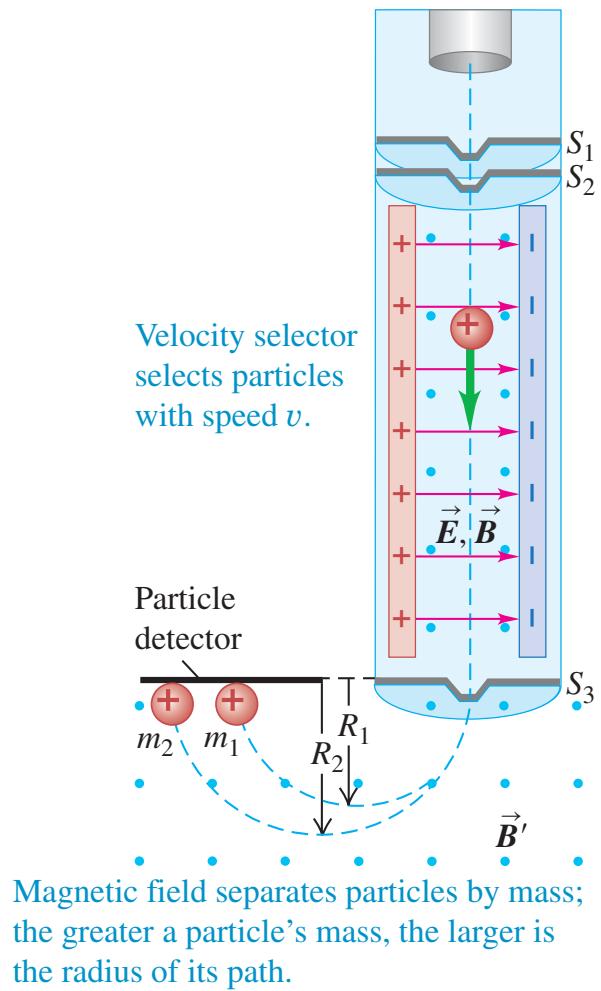
$$\frac{e}{m} = 2y \frac{E}{B^2 L^2} = 1.76 \times 10^{11} \text{ C/kg}$$

Mass Spectrometers



Techniques similar to Thomson's e/m experiment can be used to measure masses of ions and thus measure atomic and molecular masses. In 1919, Francis Aston (1877-1945), a student of Thomson's, built the first of a family of instruments called **mass spectrometers**

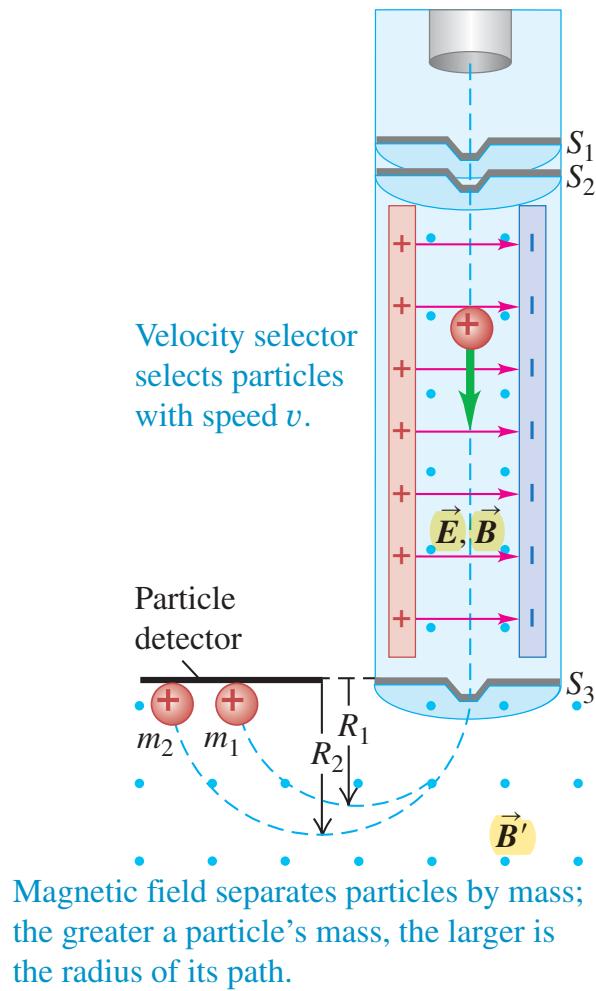
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Positive ions from a source pass through the slits S_1 and S_2 , forming a narrow beam. Then the ions pass through a velocity selector with crossed E and B fields, as we have described, to block all ions except those with speeds v equal to E/B .

Mass Spectrometers



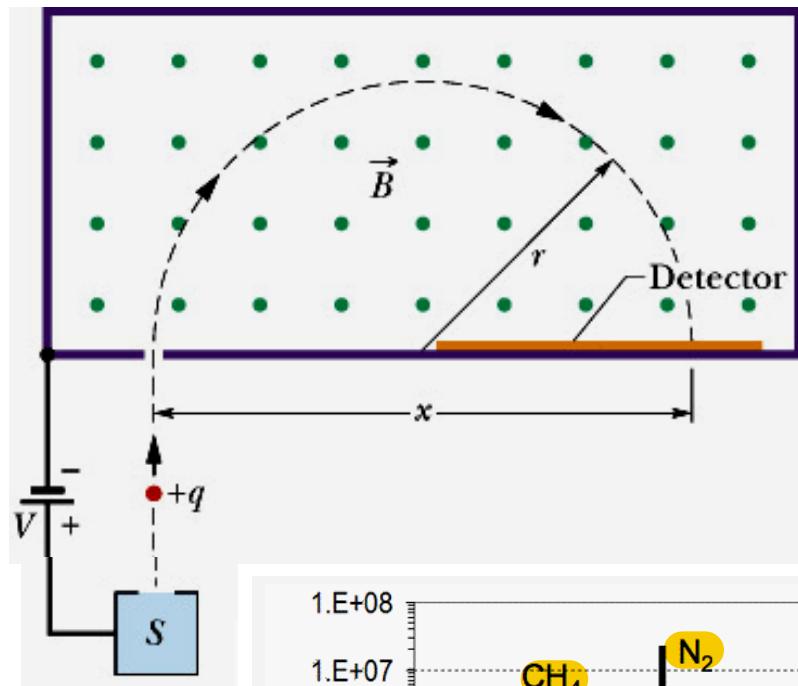
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Finally, the ions pass into a region with a magnetic field B' perpendicular to the figure, where they move in circular arcs with radius R determined by

$$R = mv/qB'$$

Mass spectrometer Separates particles with different charge/mass ratios



$$v = \sqrt{\frac{2qV}{m}}$$

$$r = \frac{mv}{qB}$$

ionized atoms or
molecules accelerated
through potential V

ionized isotopes or
molecules are separated
forming a “spectrum”

$$r = \frac{1}{B} \sqrt{\frac{2mV}{q}} \quad \rightarrow \quad \boxed{\frac{q}{m} = \frac{2V}{B^2 r^2}}$$

