Chp 25: Current, Resistance, and Electromotive Force

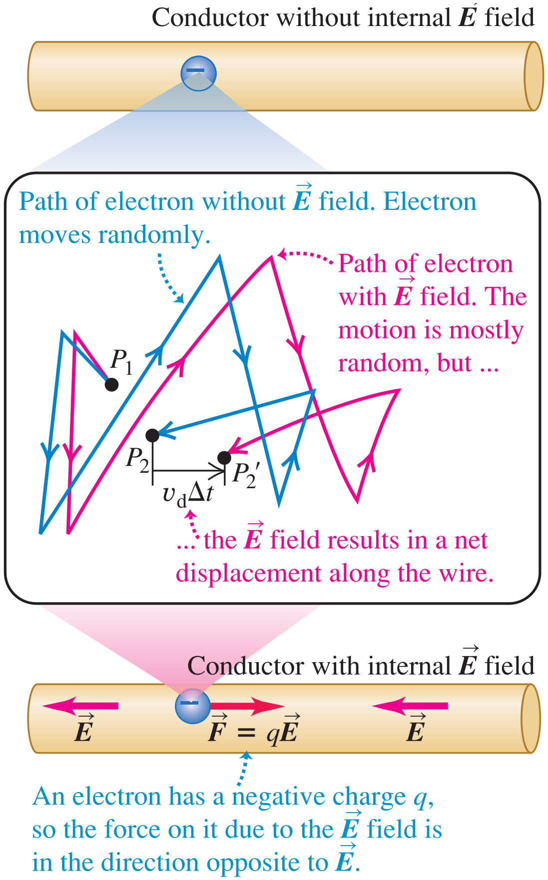
Goals for Chapter 25

- To understand current and how charges move in a conductor
- To understand resistivity and conductivity
- To calculate the resistance of a conductor
- To learn how an emf causes current in a circuit
- To calculate energy and power in circuits

Current

Electric circuits contain ***charges in motion***. Circuits are at the heart of modern devices such as computers, televisions, and industrial power systems.

Current is any motion of charge from one region to another and is defined as $I = dQ/dt$.



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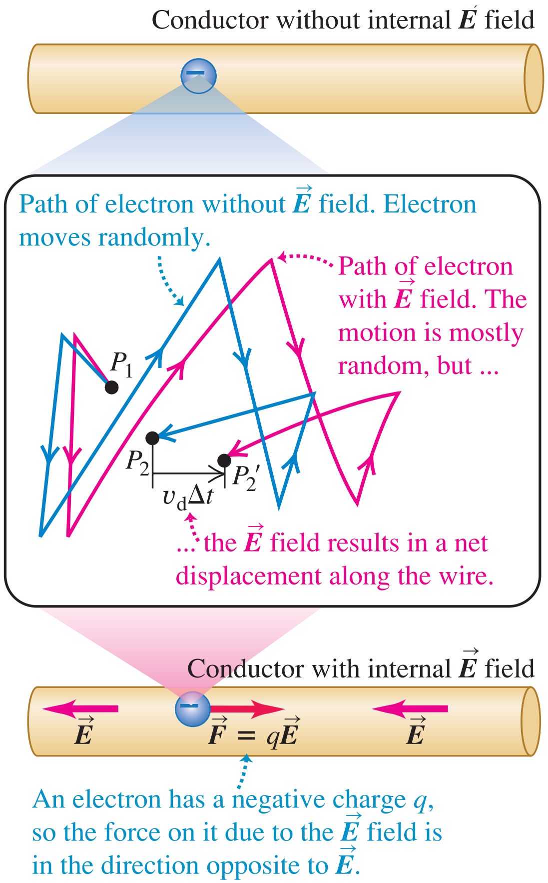
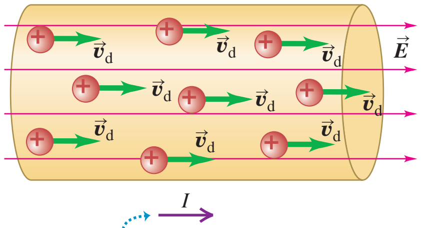


In electrostatic situations the electric field is zero everywhere within the conductor, and there is *no* current. However, this does not mean that all charges within the conductor are at rest. In an ordinary metal some of the electrons are free to move within the conducting material in all directions, somewhat like the molecules of a gas but with much greater speeds, of the order of 10^6 m/s. The electrons nonetheless do not escape from the conducting material, because they are attracted to the positive ions of the material. The motion of the electrons is random, so there is no *net* flow of charge in any direction and hence no current.
If a constant, steady electric field *E* is established inside a conductor, free electrons inside the conducting material are then subjected to a steady force $F = qE$.
A charged particle moving in a *conductor* undergoes frequent collisions with the massive, nearly stationary ions of the material.
This motion described in terms of drift velocity v_d

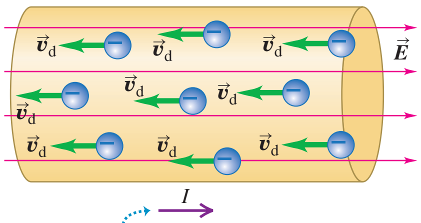
The Direction of Current Flow

(a)



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

$$I = \frac{dQ}{dt} \qquad \text{(definition of current)}$$

The SI unit of current is the **ampere;** one ampere is defined to be *one coulomb per second*
1 A = 1 C/s

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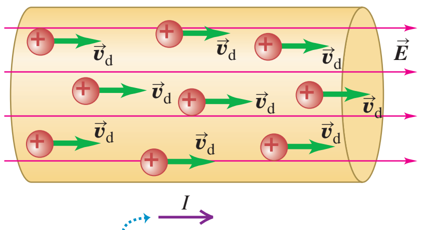

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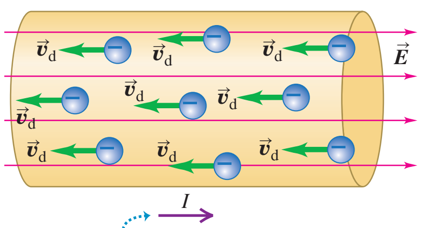

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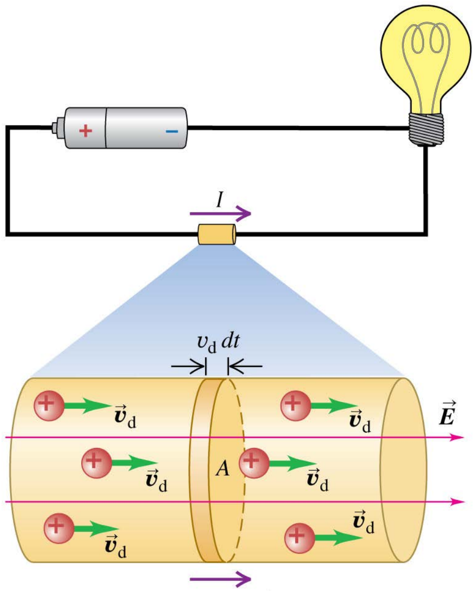
The drift of moving charges through a conductor can be interpreted in terms of work and energy. The electric field *E* does work on the moving charges. The resulting kinetic energy is transferred to the material of the conductor by means of collisions with the ions, which vibrate about their equilibrium positions in the crystalline structure of the conductor. This energy transfer increases the average vibrational energy of the ions and therefore the temperature of the material. Thus much of the work done by the electric field goes into heating the conductor, *not* into making the moving charges move ever faster and faster. This heating is sometimes useful, as in an electric toaster, but in many situations is simply an unavoidable by-product of current flow.

We describe currents as though they consisted entirely of positive charge flow, even in cases in which we know that the actual current is due to electrons. Hence the current is to the right in both Figures. This choice or convention for the direction of current flow is called **conventional current.** While the direction of the conventional current is *not* necessarily the same as the direction in which charged particles are actually moving, we'll find that the sign of the moving charges is of little importance in analyzing electric circuits.

Current, drift velocity, and current density

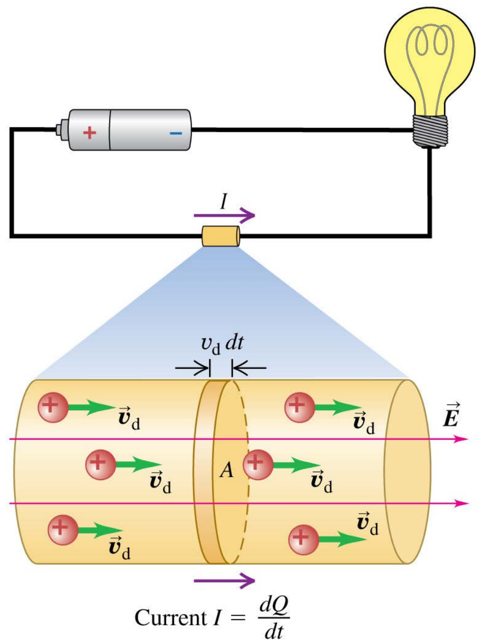


Suppose there are n moving charged particles per unit volume. We call n the **concentration** of particles; its SI unit is m^{-3} .
In a time interval dt, each particle moves a distance $v_d\,dt$.
The volume of the shaded cylinder is $Av_d\,dt$, and the number of particles within it is $nAv_d\,dt$.
the charge dQ that flows out of the end of the cylinder during time dt is

$$dQ = q(nAv_d\,dt) = nqv_dA\,dt$$

➔ $$I = \frac{dQ}{dt} = n|q|v_dA \qquad \text{(general expression for current)}$$

Current, drift velocity, and current density



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The current in the solution is carried by Na^+ and Cl^- ions

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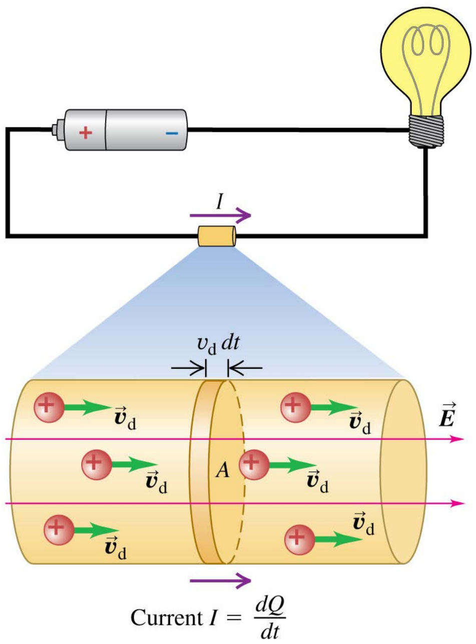

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The current *per unit cross-sectional area* is called the **current density** J:

$$J = \frac{I}{A} = n|q|v_d \qquad (\mathrm{A/m^2})$$

$$\vec{J} = nq\vec{v}_d \qquad \text{(vector current density)}$$

There are *no* absolute value signs here. If q is positive, $\vec{v}_d$ is in the same direction as $\vec{E}$; if q is negative, $\vec{v}_d$ is opposite to $\vec{E}$. In either case, $\vec{J}$ is in the same direction as $\vec{E}$.

Note that current density $\vec{J}$ is a vector but the current I is not!

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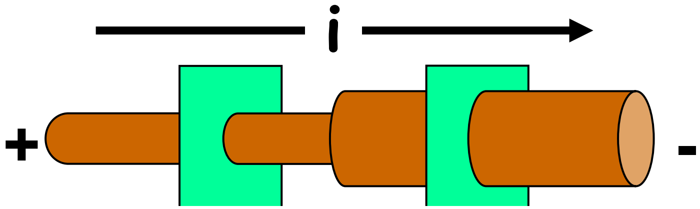
Current is the same across each cross-section of a wire



Current density J may vary
[J] = current/area

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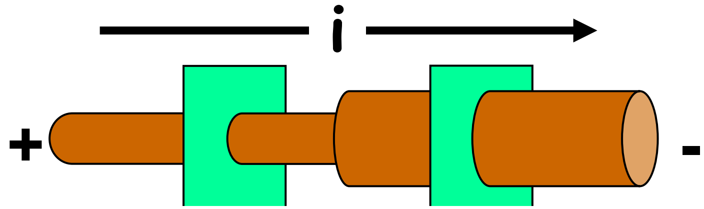

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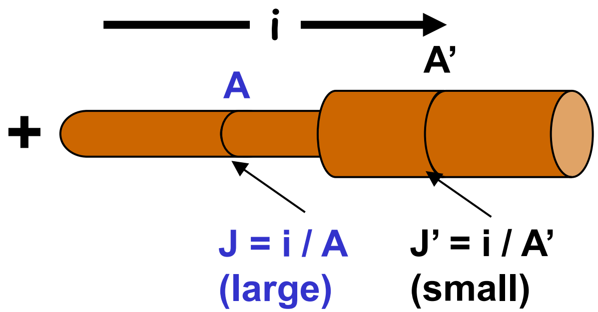
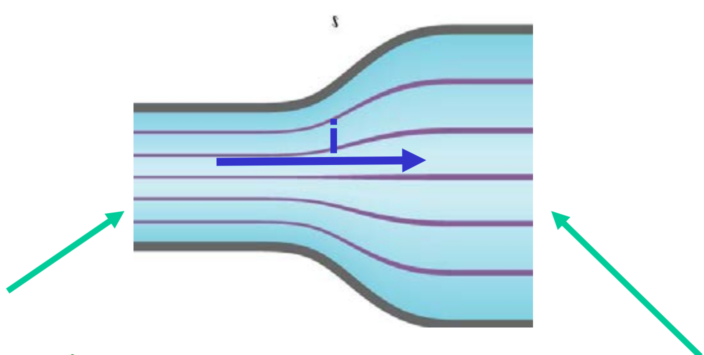
Current density J: Current / Unit Area (Vector)



Same current crosses larger or smaller Surfaces, current density J varies



High current density in this region

Low current density in this region

For uniform density i = J/A or J = i/A

If not uniform:

$$di \equiv \vec{J} \circ \hat{n} dA$$

$$i = \int_{area} \vec{J} \circ d\vec{A}$$

Resistivity (ρ)

The current density J in a conductor depends on the electric field E and on the properties of the material. In general, this dependence can be quite complex. But for some materials, especially metals, at a given temperature, J is nearly *directly proportional* to E, and the ratio of the magnitudes of E and J is constant. This relationship, called Ohm's law, was discovered in 1826 by the German physicist Georg Simon Ohm (1787–1854).

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Resistivities at Room Temperature (20°C)

Substance	ρ ($\Omega \cdot m$)	Substance	ρ ($\Omega \cdot m$)
		Semiconductors	
Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
Copper	1.72×10^{-8}	Pure germanium	0.60
Gold	2.44×10^{-8}	Pure silicon	2300
Aluminum	2.75×10^{-8}	**Insulators**	
Tungsten	5.25×10^{-8}	Amber	5×10^{14}
Steel	20×10^{-8}	Glass	10^{10}–10^{14}
Lead	22×10^{-8}	Lucite	$>10^{13}$
Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
Nichrome	100×10^{-8}	Teflon	$>10^{13}$
		Wood	10^{8}–10^{11}

$$\rho = \frac{E}{J}$$

$$(V/m)/(A/m^2) = V \cdot m/A$$

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$$\rho = \frac{E}{J}$$

$$(\text{V/m})/(\text{A/m}^2) = \text{V} \cdot \text{m/A}$$

1 V/A is called one ohm (Ω) ➔ SI units for ρ is Ω m.

The reciprocal of resistivity is **conductivity (σ)**. Its units are $(\Omega\ \text{m})^{-1}$

Resistivity and temperature

The resistivity of a *metallic* conductor nearly always increases with increasing temperature. As temperature increases, the ions of the conductor vibrate with greater amplitude, making it more likely that a moving electron will collide with an ion; this impedes the drift of electrons through the conductor and hence reduces the current. Over a small temperature range (up to 100 C° or so), the resistivity of a metal can be represented approximately by the equation

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$ (temperature dependence of resistivity)

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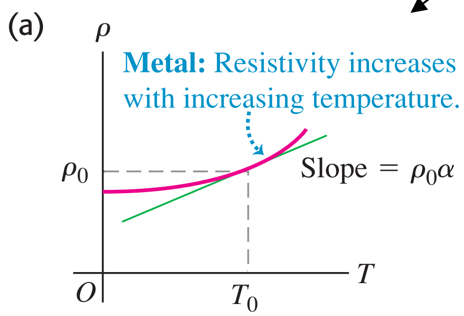

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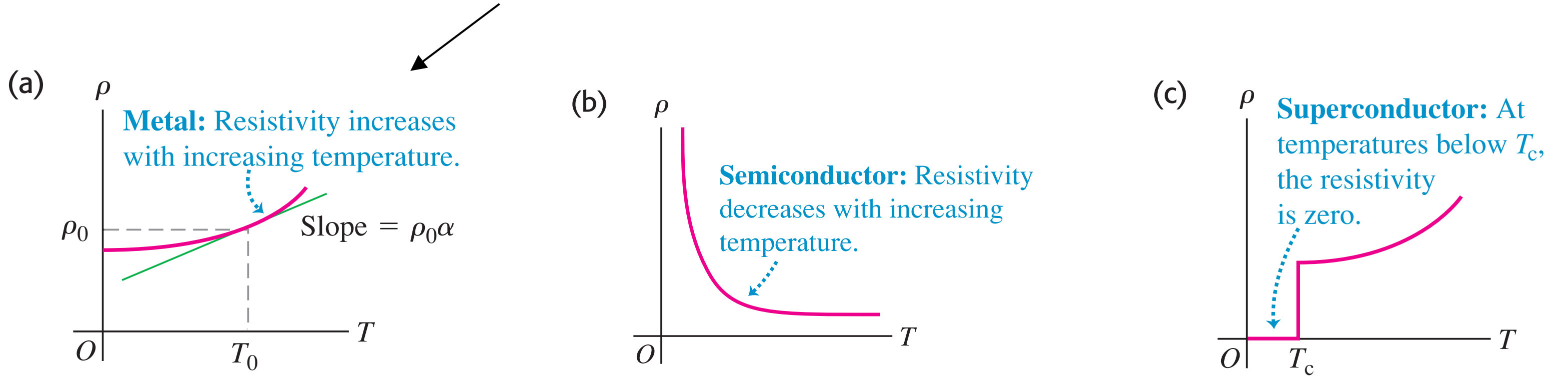

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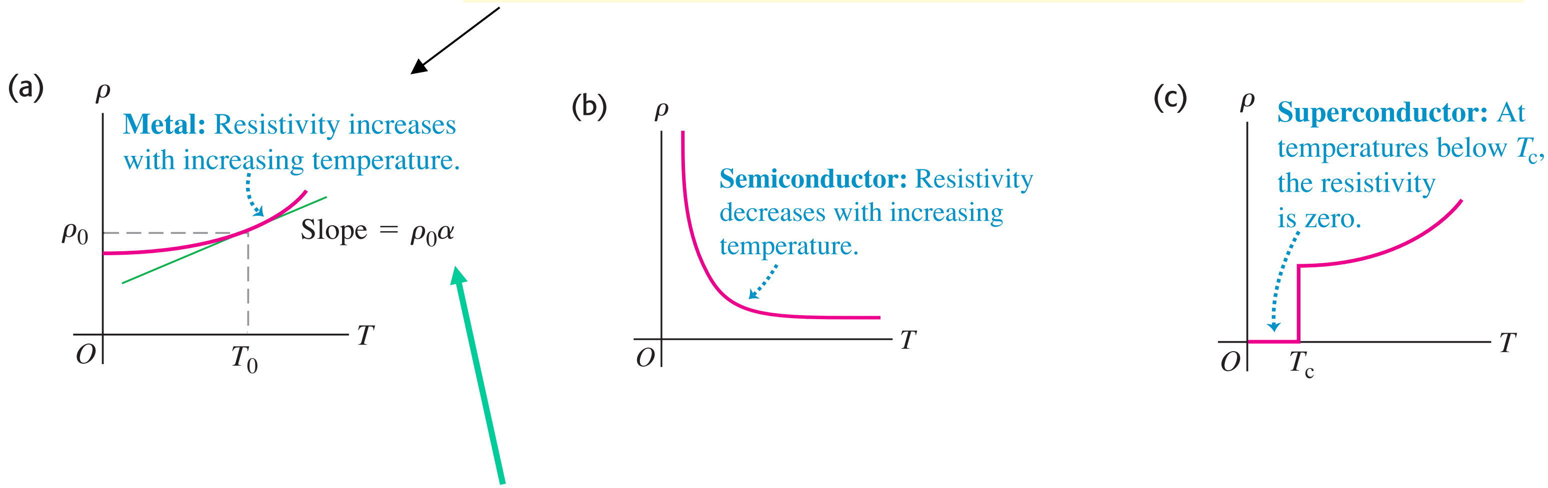
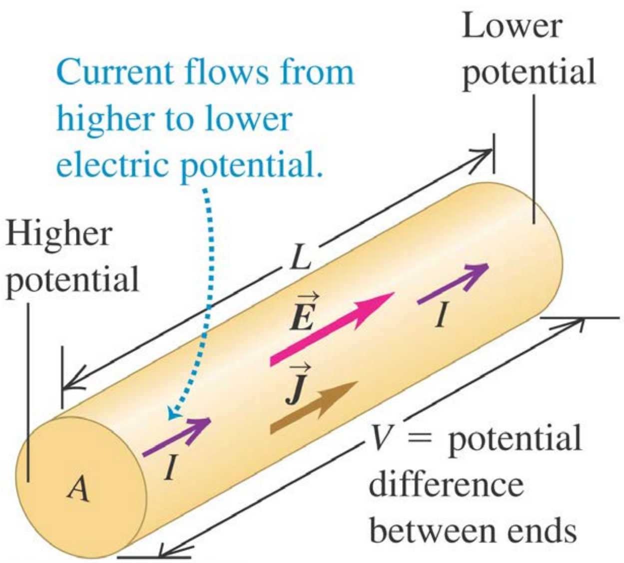


Material	α [(°C)$^{-1}$]	Material	α [(°C)$^{-1}$]
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

Resistance (R)



The *resistance* of a conductor is $R = \rho L/A$
The potential across a conductor is $V = IR$.
If V is directly proportional to I (that is, if R is constant), the equation $V = IR$ is called *Ohm's law.*

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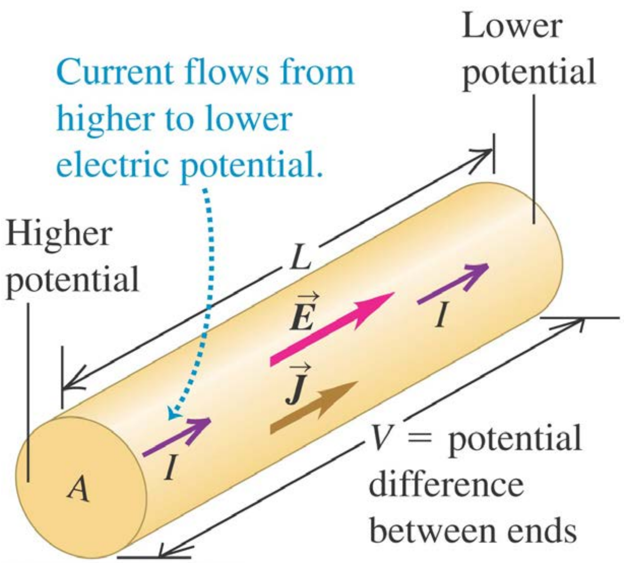

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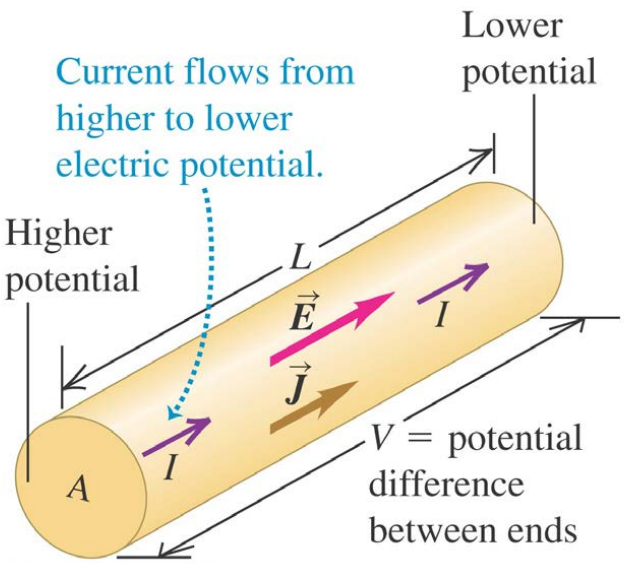

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$$\rightarrow \quad V = \frac{\rho L}{A} I \qquad R = \frac{V}{I} \qquad \boxed{R = \frac{\rho L}{A}}$$

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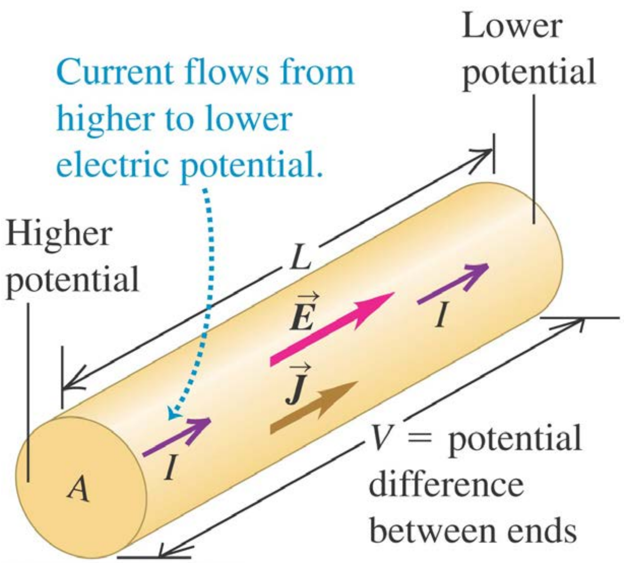

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The *kilohm* (1 k Ω = 10^3 Ω) and the *megohm* (1 M Ω = 10^6 Ω) are also in common use.

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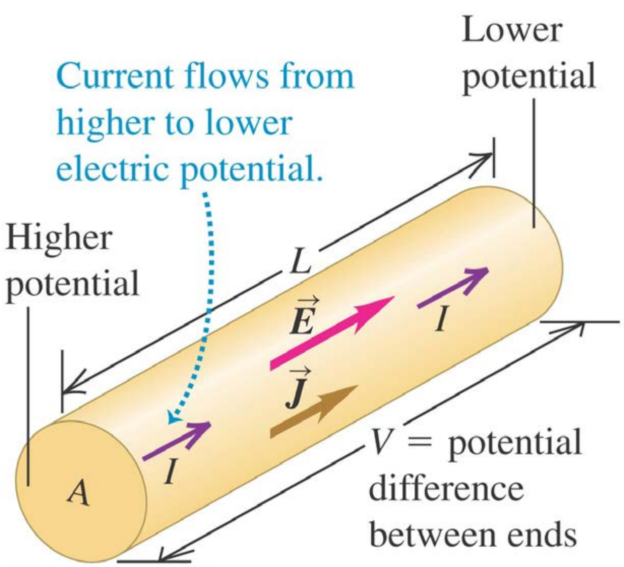

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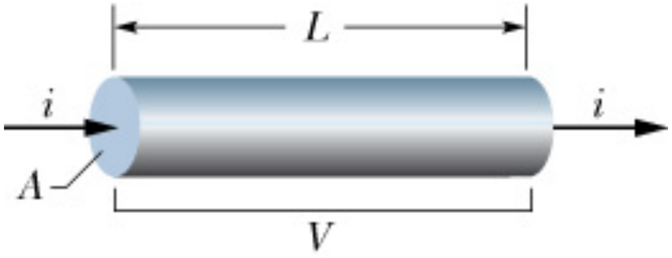
A 100-m length of 12-gauge copper wire, the size usually used in household wiring, has a resistance at room temperature of about 0.5 Ω. A 100-W, 120-V light bulb has a resistance (at operating temperature) of 140 Ω. If the same current I flows in both the copper wire and the light bulb, the potential difference $V = IR$ is much greater across the light bulb, and much more potential energy is lost per charge in the light bulb. This lost energy is converted by the light bulb filament into light and heat. You don't want your household wiring to glow white-hot, so its resistance is kept low by using wire of low resistivity and large cross-sectional area.

Calculating resistance, given the resistivity



resistivity

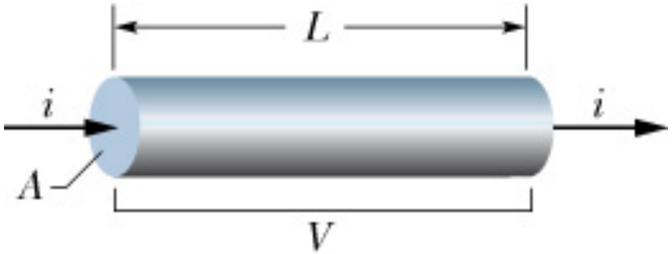
resistance

$$R = \frac{\rho L}{A}$$

proportional to length

inversely proportional to cross section area

Calculating resistance, given the resistivity



resistivity

resistance $R = \dfrac{\rho L}{A}$

proportional to length

inversely proportional to cross section area

EXAMPLE:

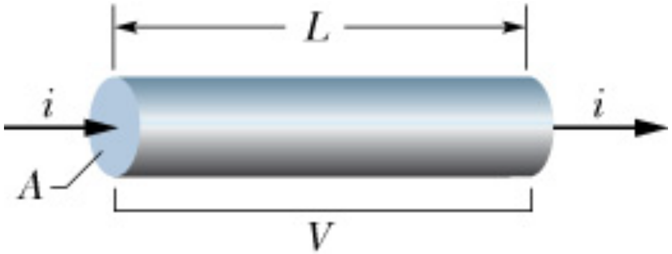
Find R for a 10 m long iron wire, 1 mm in diameter

$$R = \frac{\rho L}{A} = \frac{9.7\text{x}10^{-8}\ \Omega.\text{m x } 10\text{ m}}{\pi \text{ x } (10^{-3}/2)^2\ \text{m}^2} = 1.2\ \Omega$$

Find the potential difference across R if i = 10 A. (Amperes)

$$\Delta V = iR = 12\text{ V}$$

Calculating resistance, given the resistivity



resistivity → proportional to length

resistance → $R = \dfrac{\rho L}{A}$

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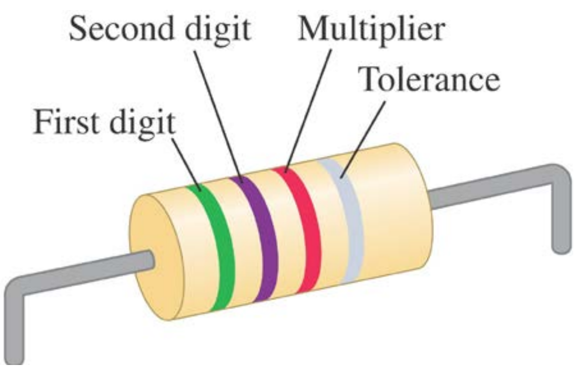
EXAMPLE:

Find resistivity of a wire with R = 50 mΩ,
diameter d = 1 mm, length L = 2 m

$$\rho = \frac{RA}{L} = \frac{50\text{x}10^{-3}\ \Omega \text{ x } 10\text{ m}}{2\text{ m}} \text{ x } \pi\left(10^{-3}/2\right)^2 = 1.96\text{x}10^{-8}\ \Omega.\text{m}$$

Use a table to identify material. Not Cu or Al, possibly an alloy

Resistors are color-coded for easy identification



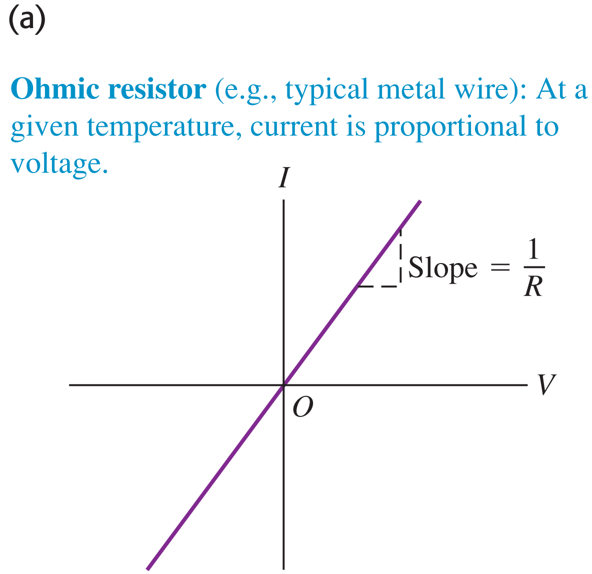
This resistor has a resistance of 5.7 kΩ with a tolerance of ±10%.

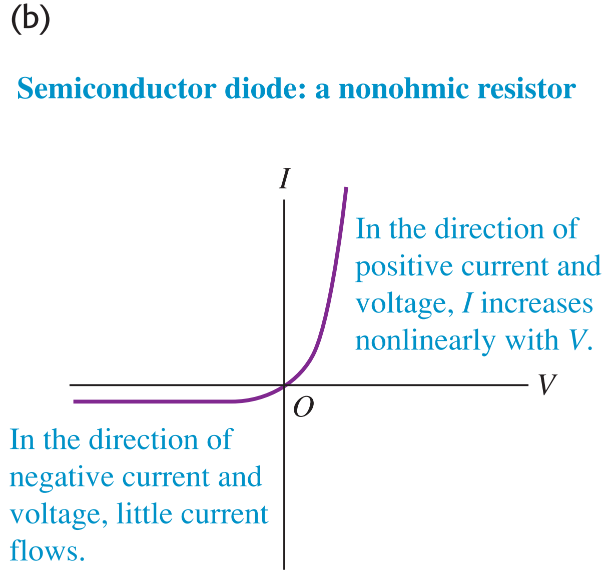
Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Fourth band: no band means ± 20%, a silver band ± 10%, and a gold band ± 5%.

Ohmic and nonohmic resistors





Because the resistivity of a material varies with temperature, the resistance of a specific conductor also varies with temperature. For temperature ranges that are not too great, this variation is approximately a linear relationship,

$$R(T) = R_0[1 + \alpha(T - T_0)]$$

R_0 is the resistance at temperature T_0

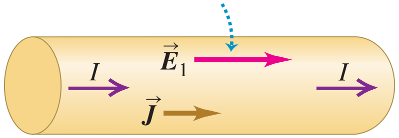
temperature coefficient of resistance

Electromotive force (ε) and circuits

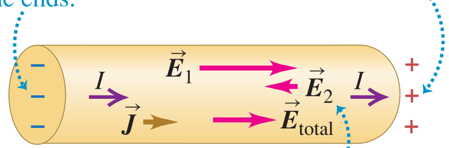
Open circuit:

An *electromotive force* (*emf*) makes current flow. In spite of the name, an emf is *not* a force.

(a) An electric field $\vec{E}_1$ produced inside an isolated conductor causes a current.

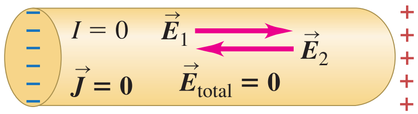


(b) The current causes charge to build up at the ends.



The charge buildup produces an opposing field $\vec{E}_2$, thus reducing the current.

(c) After a very short time $\vec{E}_2$ has the same magnitude as $\vec{E}_1$; then the total field is $\vec{E}_{total} = \mathbf{0}$ and the current stops completely.



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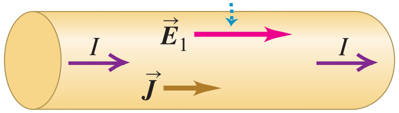

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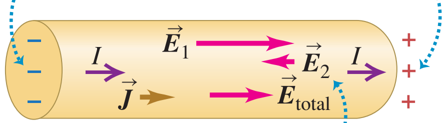

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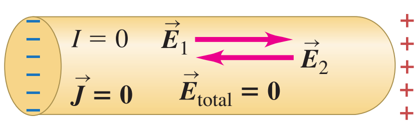

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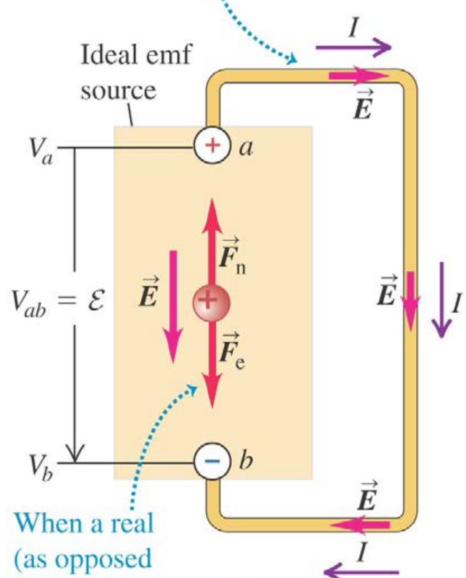
closed circuit:

Potential across terminals creates electric field in circuit, causing charges to move.



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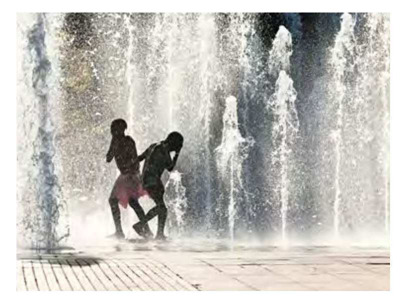

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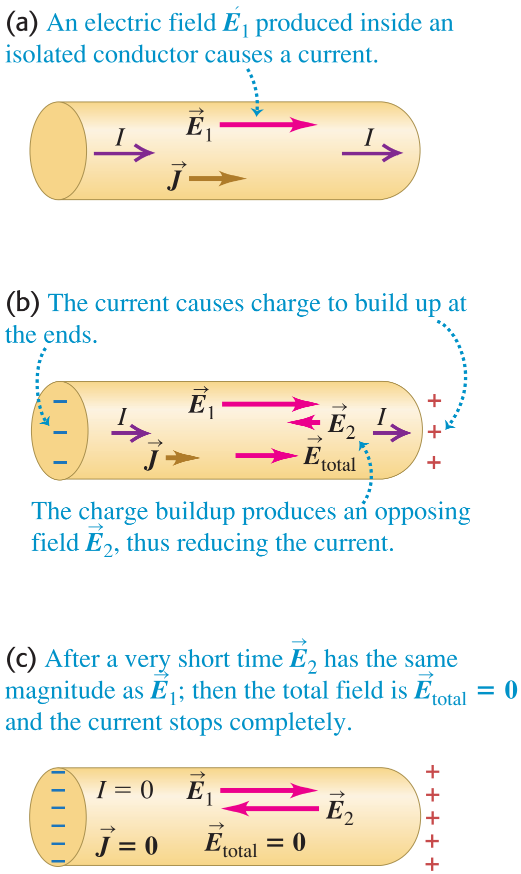

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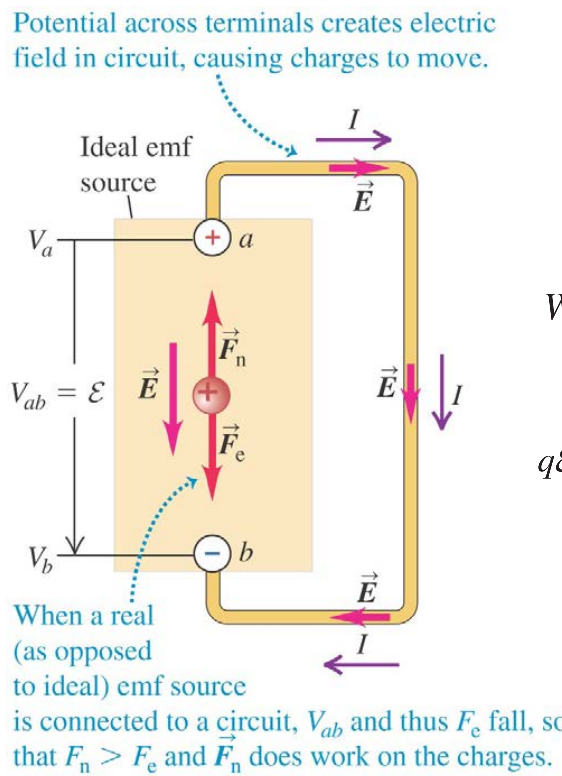

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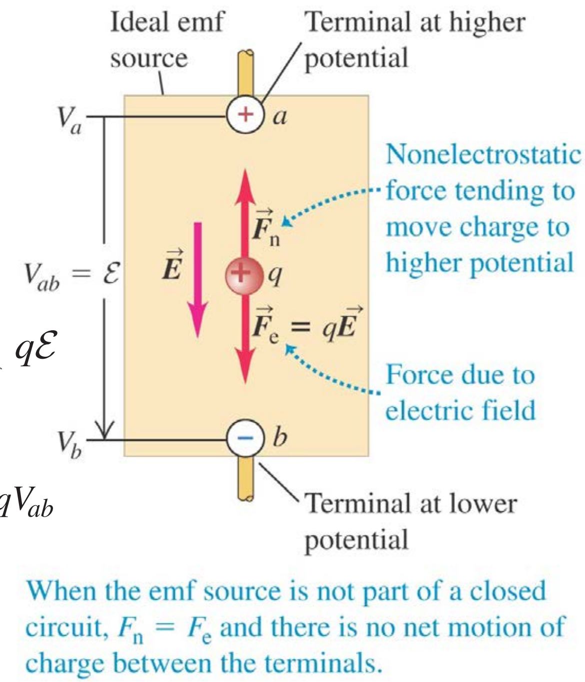

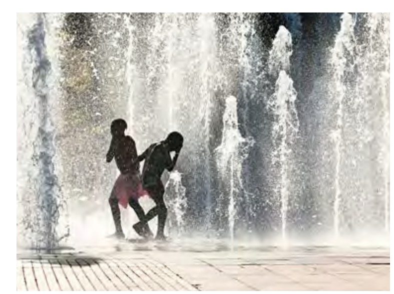

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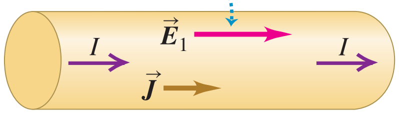

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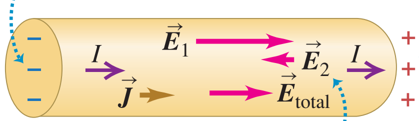

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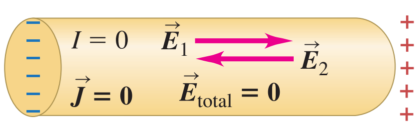

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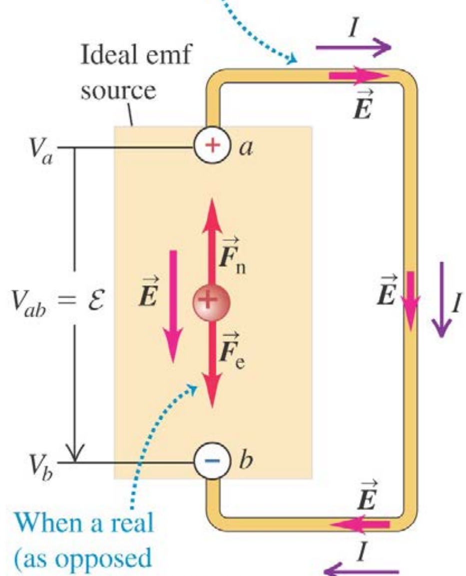

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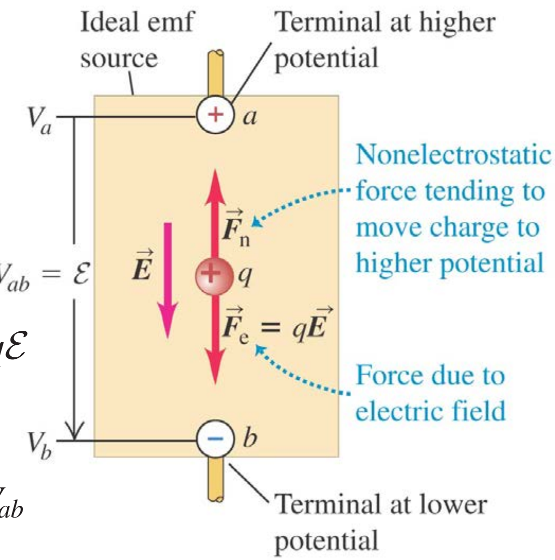


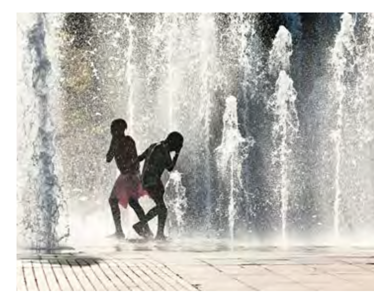
$$W_n = q\mathcal{E}$$

$$q\mathcal{E} = qV_{ab}$$

When the emf source is not part of a closed circuit, $F_n = F_e$ and there is no net motion of charge between the terminals.

Unit of emf (ε) is the same as potential (1 V = 1 J/C)

A battery with an *emf* of 1.5 V does 1.5 J of work on every coulomb of charge that pass through it



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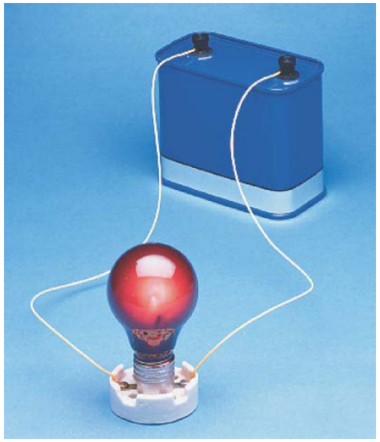
Internal resistance



Real sources of emf actually contain some *internal resistance r*.

The *terminal voltage* of an emf source is $V_{ab} = \mathcal{E} - Ir$.

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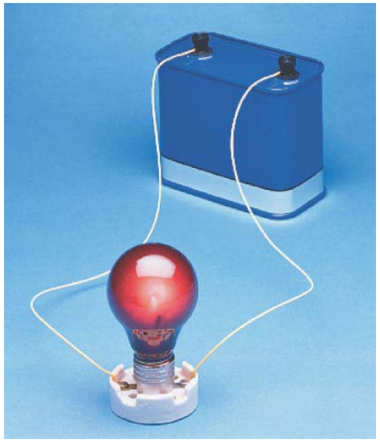

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The terminal voltage of the 12-V battery shown at the right is less than 12 V when it is connected to the light bulb.

The potential V_{ab}, called the **terminal voltage**, is less than the emf $\mathcal{E}$ because of the term Ir representing the potential drop across the internal resistance r.

$$\Rightarrow \quad \mathcal{E} - Ir = IR \qquad I = \frac{\mathcal{E}}{R + r}$$

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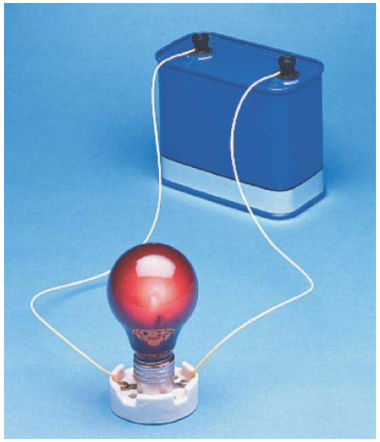

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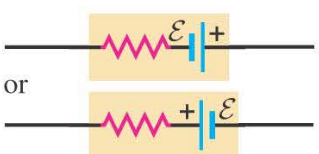
Symbols for circuit diagrams

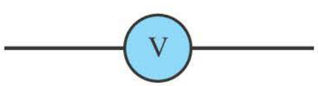
Conductor with negligible resistance

Resistor

Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)



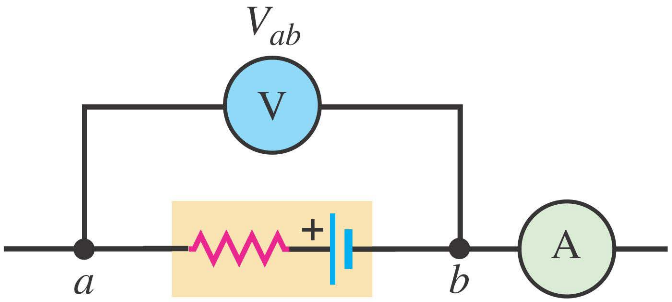
Source of emf with internal resistance r (r can be placed on either side)



Voltmeter (measures potential difference between its terminals)

Ammeter (measures current through it)

A source in an open circuit



$r = 2\ \Omega, \mathcal{E} = 12$ V

There is *zero* current because there is no complete circuit. Hence the ammeter reads $I = 0$. Because there is no current through the battery, there is no potential difference across its internal resistance. The potential difference V_{ab} across the battery terminals is equal to the emf. So the voltmeter reads $V_{ab} = \mathcal{E} = 12$ V.

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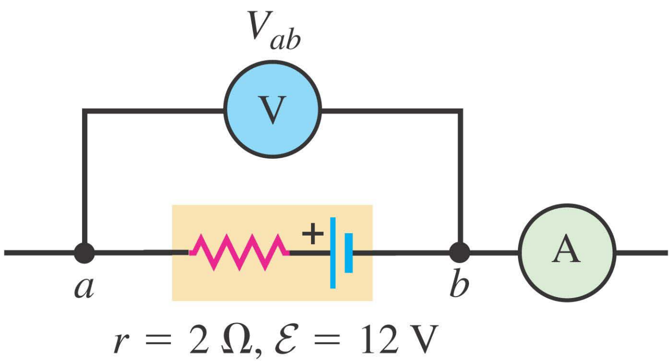

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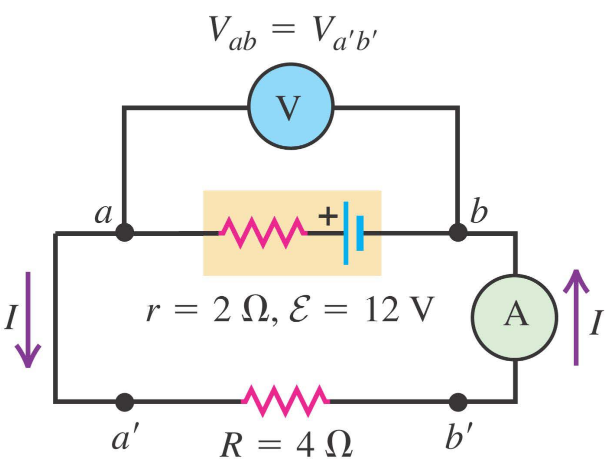
Source in a complete circuit

The ideal ammeter has zero resistance, so the total resistance external to the source is $R = 4\ \Omega$. The current through the circuit $aa'bb'$ is then

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\ \text{V}}{4\ \Omega + 2\ \Omega} = 2\ \text{A}$$



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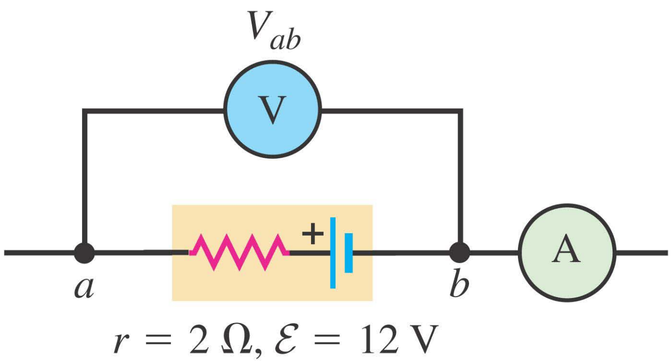

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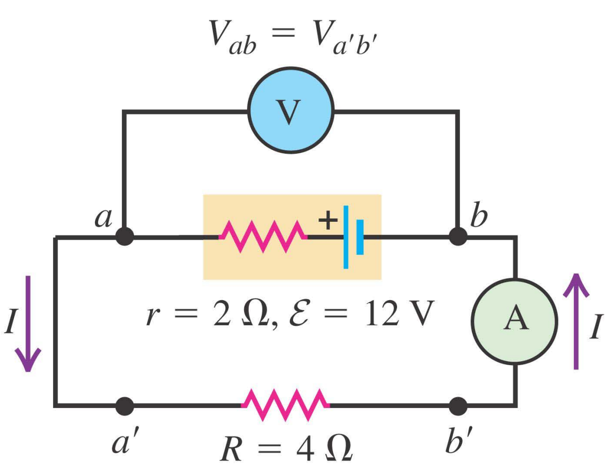

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There is no potential difference between points a and a' or between points b and b'; that is, $V_{ab} = V_{a'b'}$:

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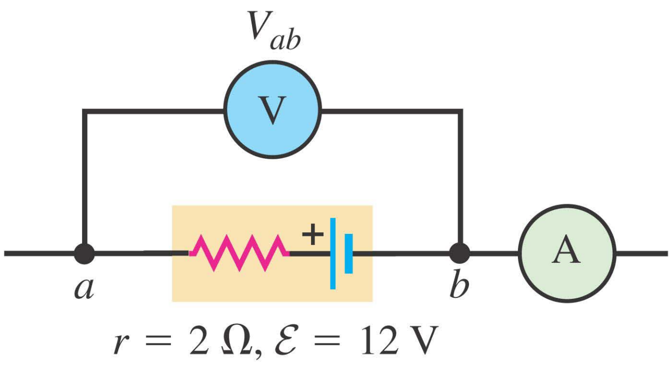

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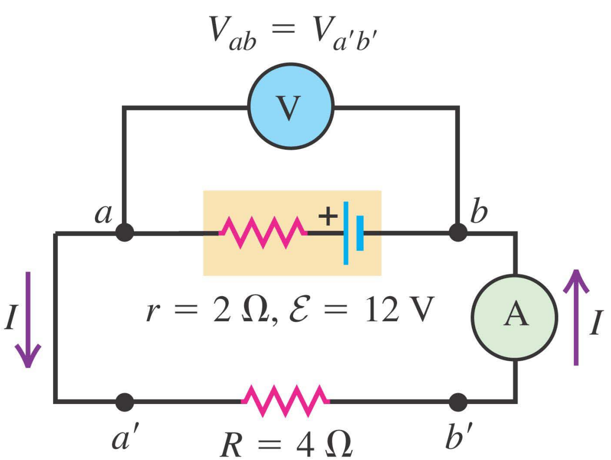

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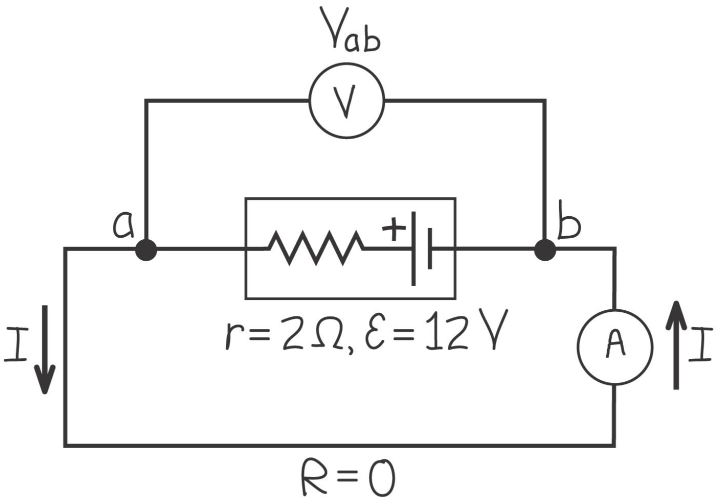
A source with a short circuit



There is now a zero-resistance path between points *a* and *b*, through the lower loop, so the potential difference between these points must be zero:

$V_{ab} = IR = I(0) = 0.$

$$V_{ab} = \mathcal{E} - Ir = 0 \qquad I = \frac{\mathcal{E}}{r} = \frac{12\ \text{V}}{2\ \Omega} = 6\ \text{A}$$

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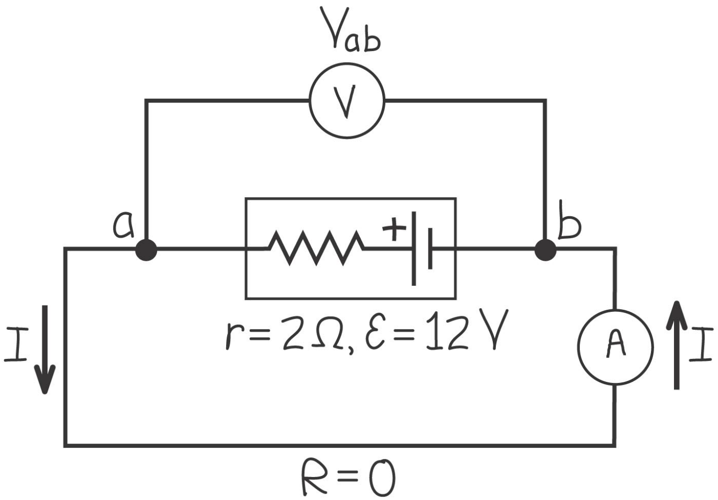

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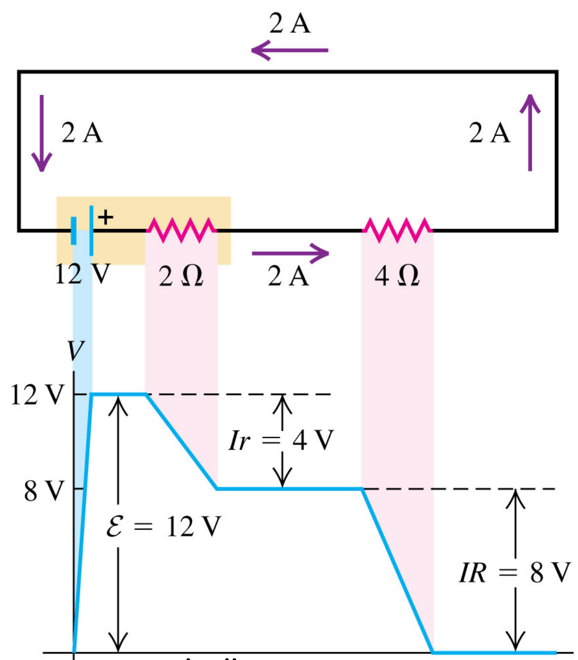
Potential changes around a circuit



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A potential gain of $\mathcal{E}$ is associated with the *emf*, and potential drops of Ir and IR

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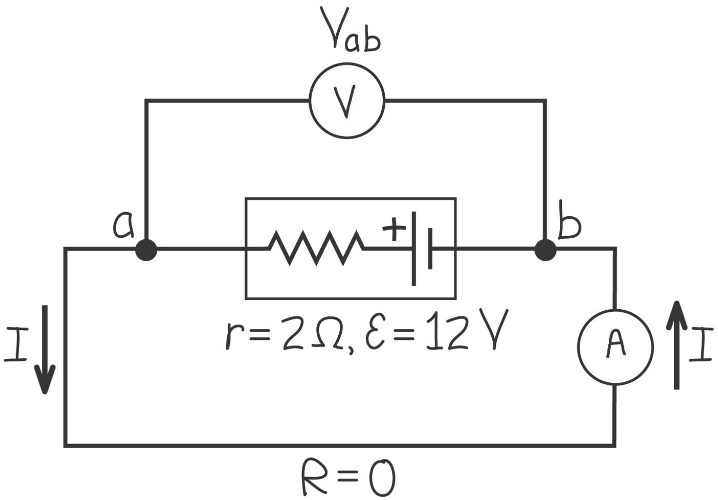

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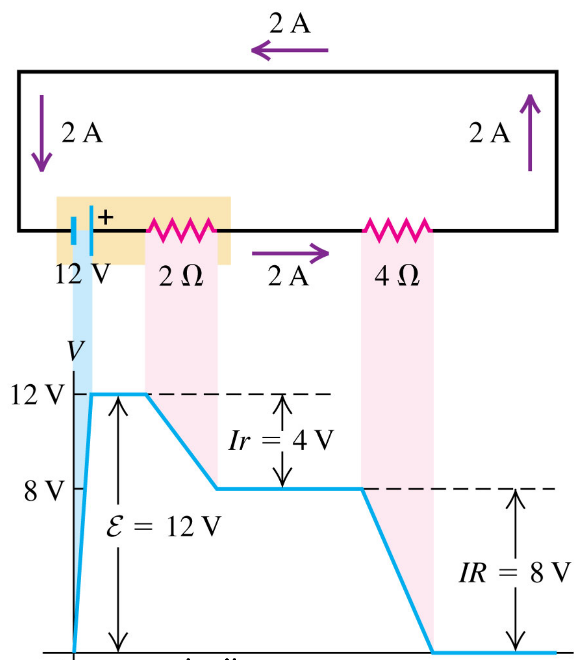

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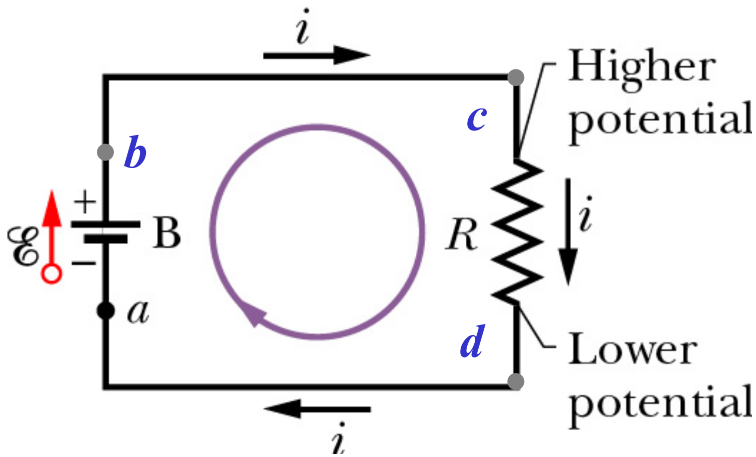
A potential gain of $\mathcal{E}$ is associated with the *emf*, and potential drops of *Ir* and *IR*

The principal difference between a fresh flashlight battery and an old one is NOT in the *emf*, which decreases only slightly with use, but in the internal resistance (r), which may increase from less than an ohm when the battery is fresh to as much as 1000 Ω or more after long use.

Example: CW or CCW around a single-loop circuit

Assume current direction as shown



- **Traverse clockwise from a:**

$$\Delta V_{ba} = V_b - V_a = +\varepsilon$$
$$\Delta V_{cb} = 0$$
$$\Delta V_{dc} = V_d - V_c = -iR$$
$$\Delta V_{ad} = 0$$

$$\sum_{\text{closed loop}} \Delta V = 0 = \varepsilon + 0 - iR + 0$$

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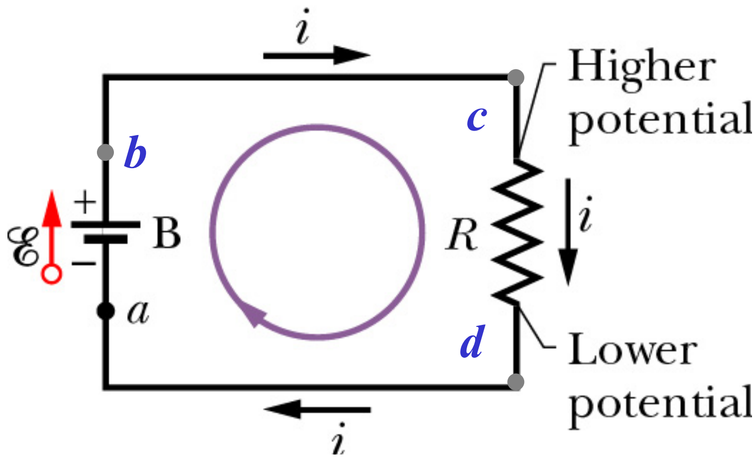

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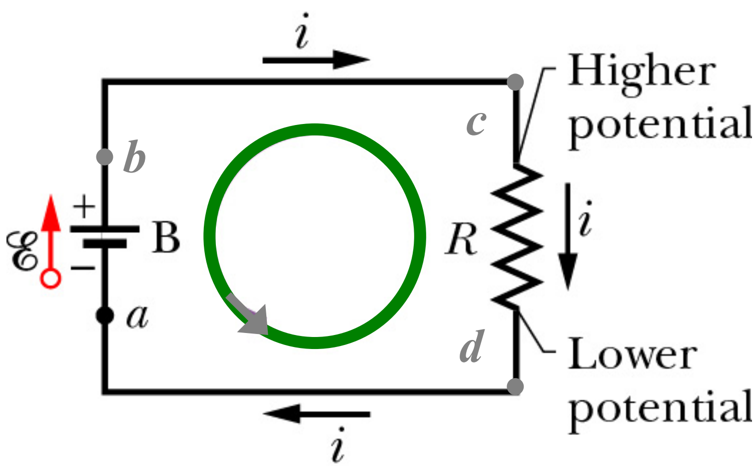


- **Traverse counterclockwise from a:**

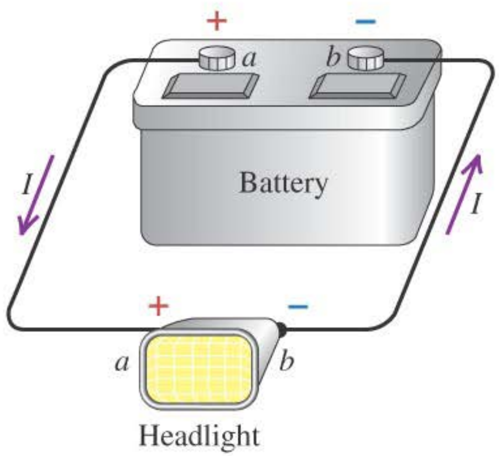
$$\Delta V_{da} = 0$$
$$\Delta V_{cd} = V_c - V_d = +iR$$
$$\Delta V_{bc} = 0$$
$$\Delta V_{ab} = V_a - V_b = -\varepsilon$$

$$\sum_{\text{closed loop}} \Delta V = 0 = 0 + iR + 0 - \varepsilon$$

$$\boxed{\begin{gathered} iR - \varepsilon = 0 \\ i = \frac{\varepsilon}{R} \end{gathered}}$$

Same result

Energy and power in electric circuits



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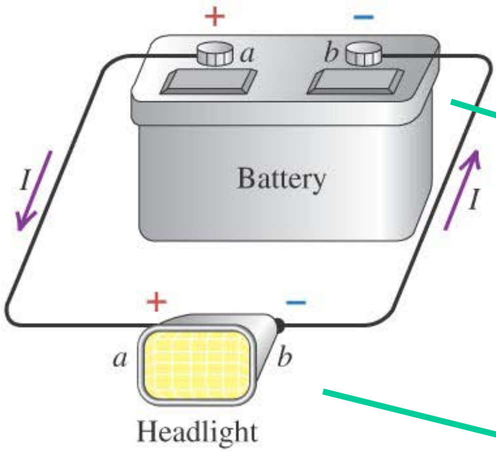

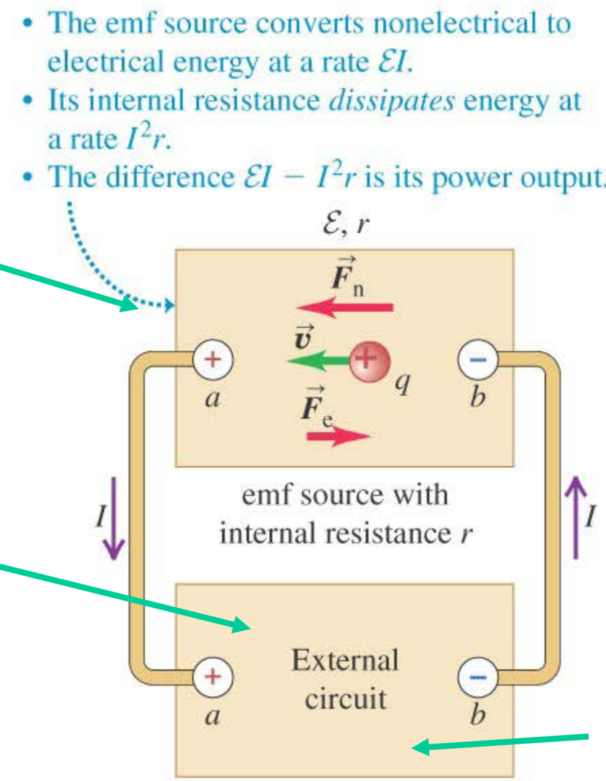

Chemical energy converted to electrical energy

Internal resistance R

The power input to the circuit element between a and b is

$$P = (V_a - V_b)I = V_{ab}I$$

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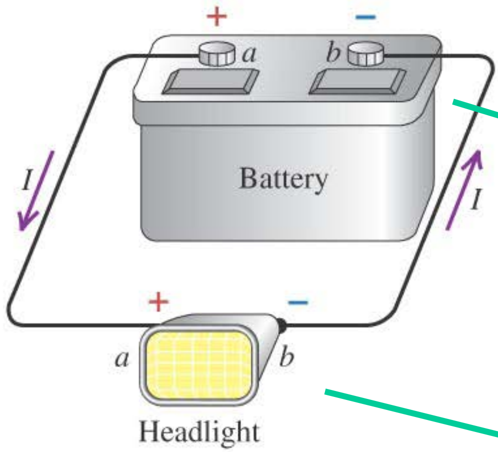
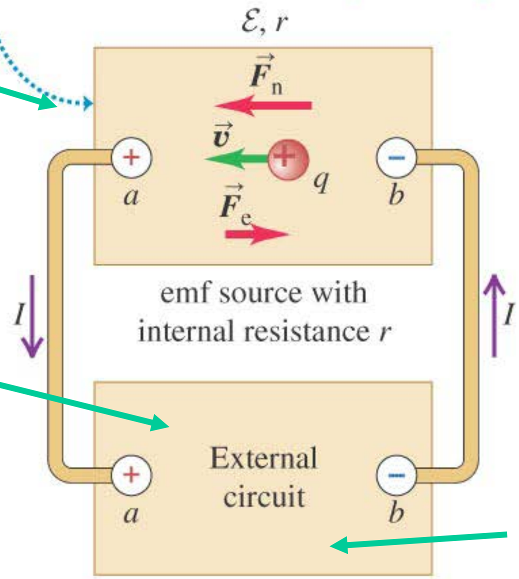


- The emf source converts nonelectrical to electrical energy at a rate $\mathcal{E}I$.
- Its internal resistance *dissipates* energy at a rate I^2r.
- The difference $\mathcal{E}I - I^2r$ is its power output.



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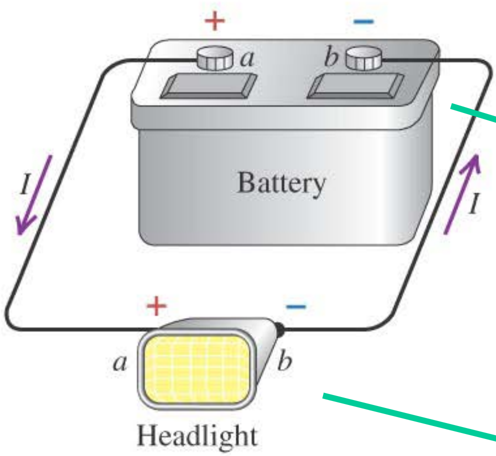
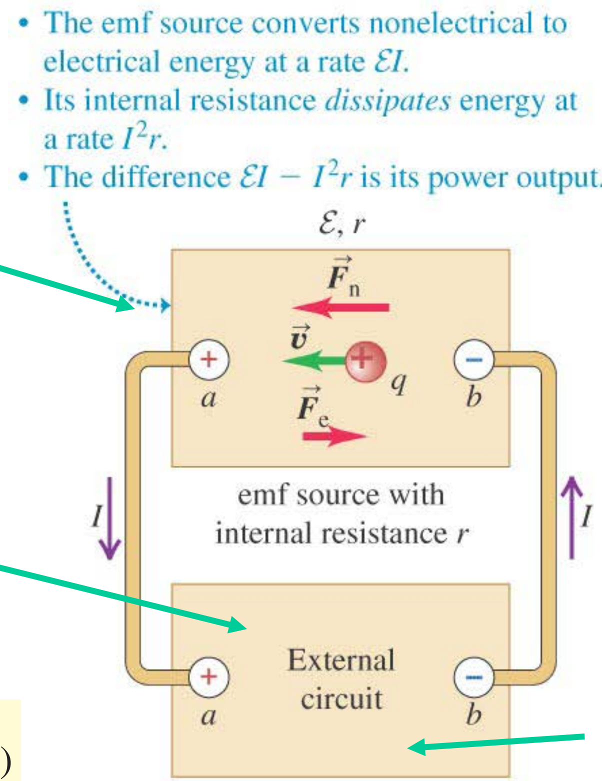
$$P = V_{ab}I$$ (rate at which energy is delivered to or extracted from a circuit element)

1 W = 1 J/s = (1 J/C) x (1 C/s)

Energy and power in electric circuits





Chemical energy converted to electrical energy

Internal resistance R

$$V_{ab} = IR$$

$$P = V_{ab}I = I^2R = \frac{{V_{ab}}^2}{R} \quad \text{(power delivered to a resistor)}$$

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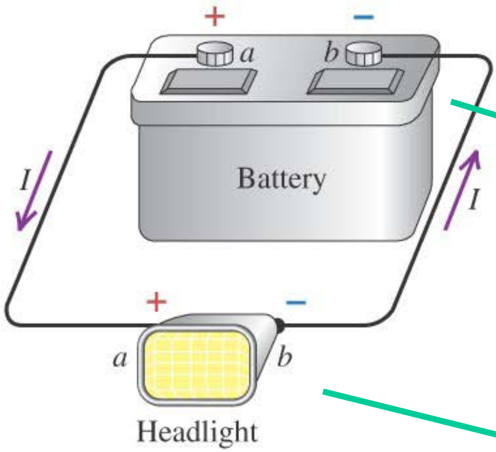

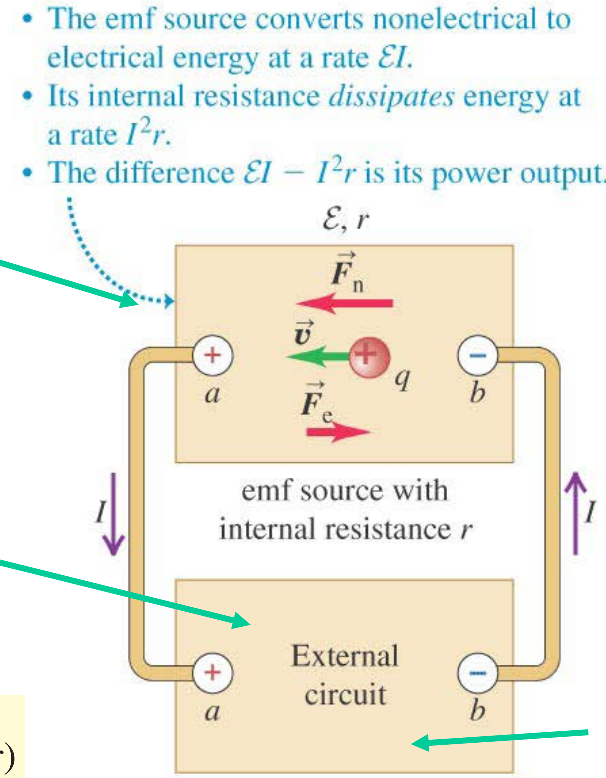

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Power Output of a Source

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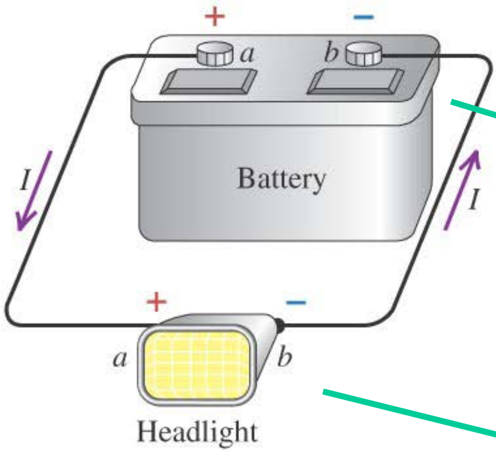
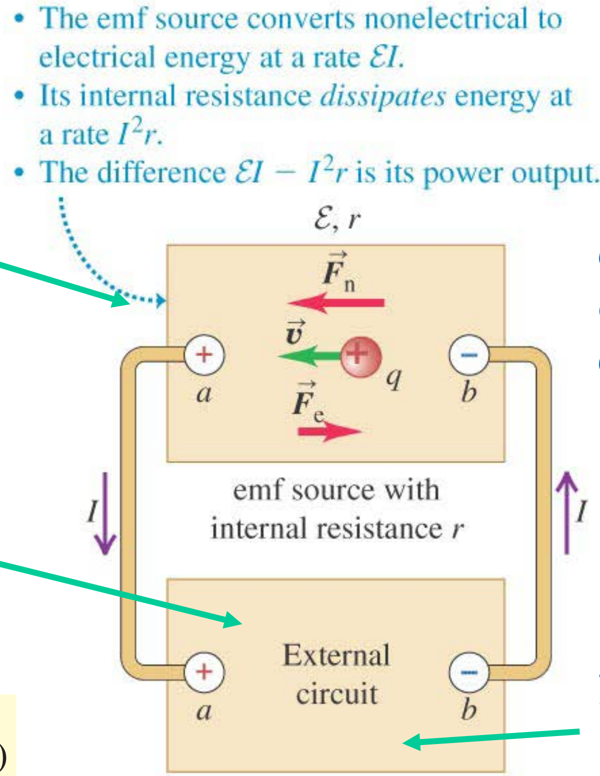
Energy and power in electric circuits





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I^2r = Rate of dissipation of the source

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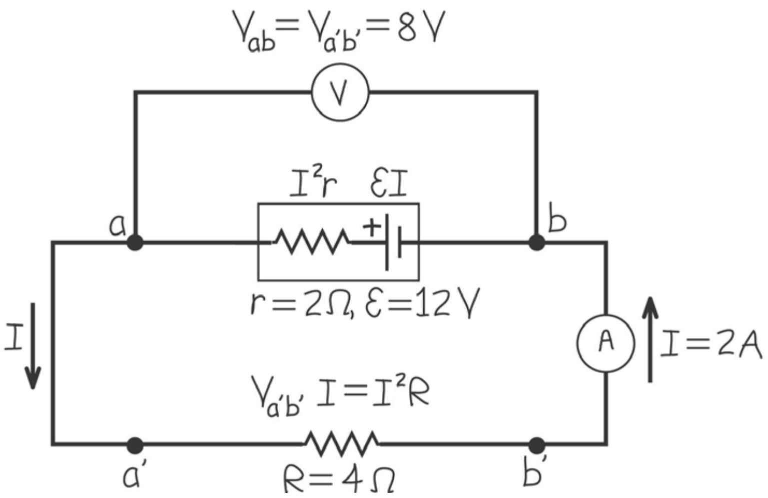

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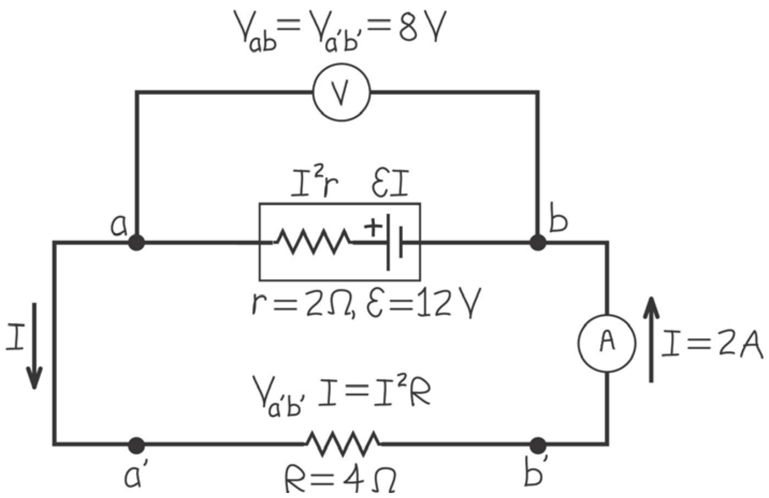

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$$\mathcal{E}I = (12\ \text{V})(2\ \text{A}) = 24\ \text{W}$$

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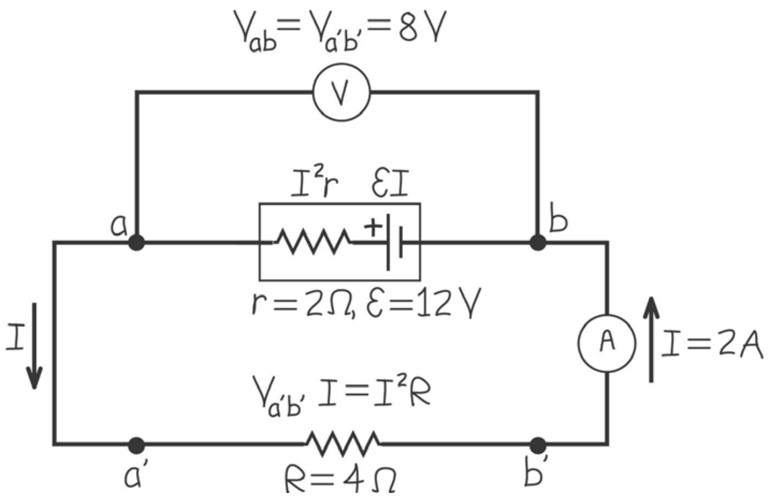

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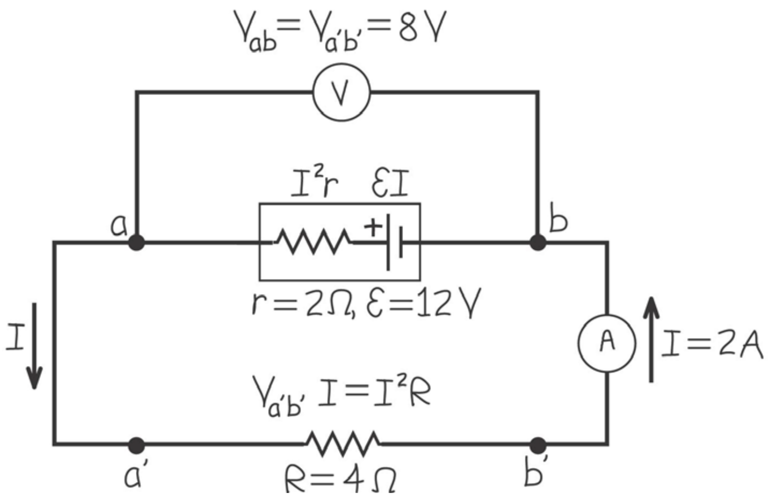

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$\mathcal{E}I - I^2r = 16\ \mathrm{W}$

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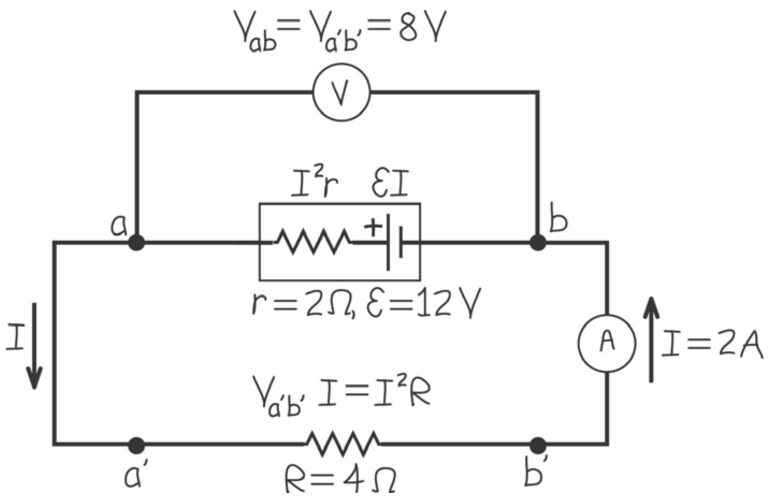

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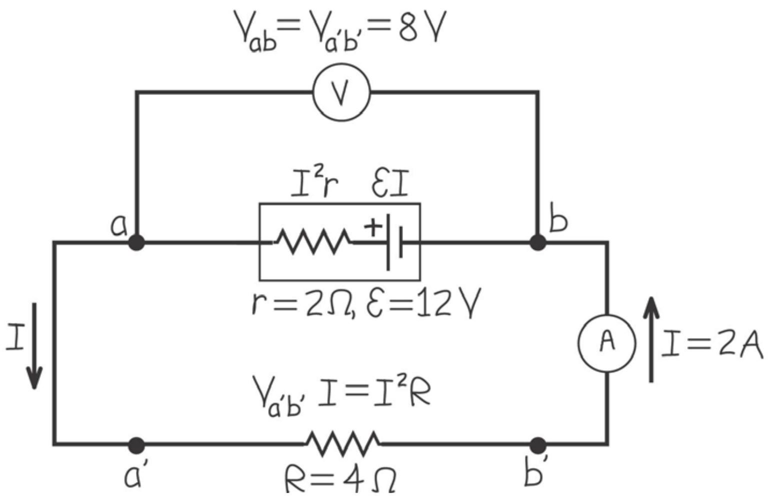

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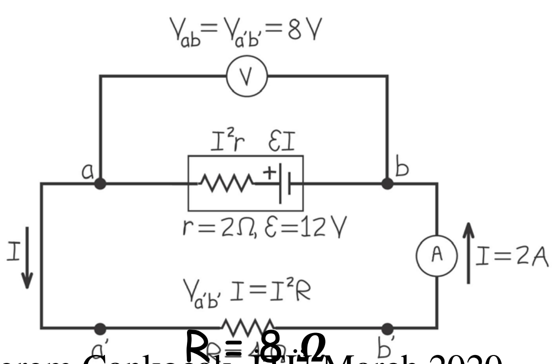
Increasing the resistance (4 Ω → Ω 8)



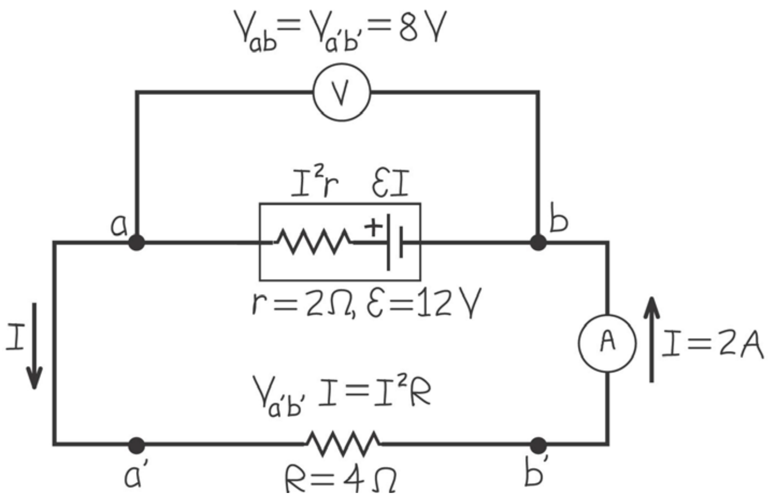
$$I = \frac{\mathcal{E}}{R + r} = \frac{12\ \text{V}}{8\ \Omega + 2\ \Omega} = 1.2\ \text{A}$$

Current decreases

$$V_{ab} = IR = (1.2\ \text{A})(8\ \Omega) = 9.6\ \text{V}$$

Potential diff. increases

Power input and output



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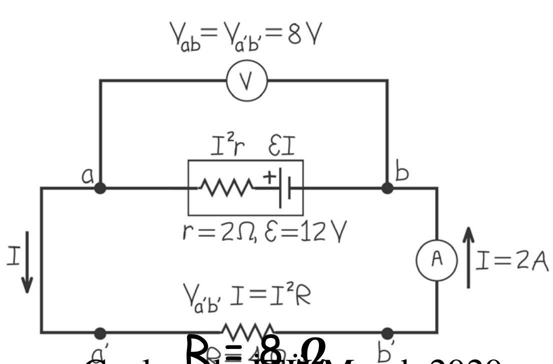

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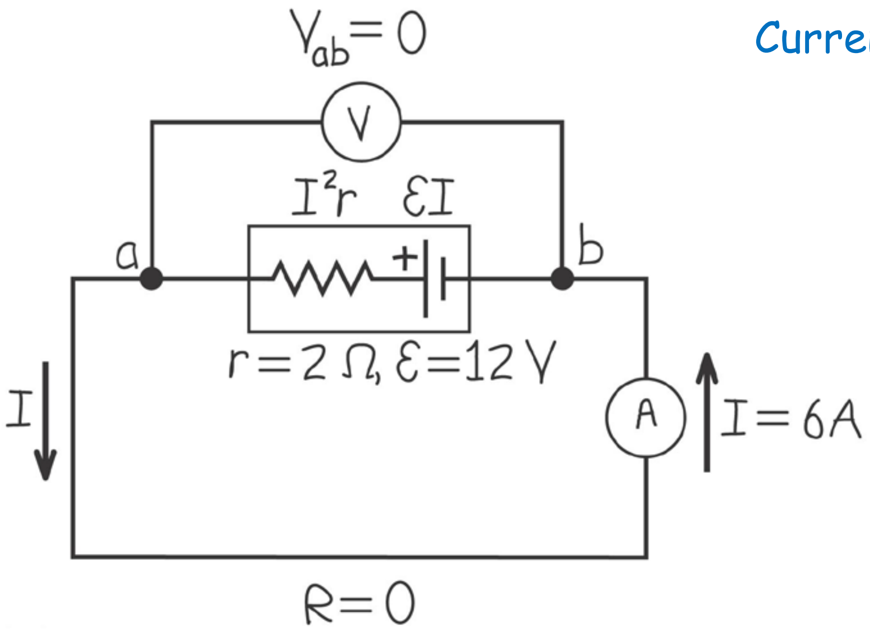
$$V_{ab} = IR = (1.2\ \text{A})(8\ \Omega) = 9.6\ \text{V}$$ Potential diff. increases

$$P = I^2R = (1.2\ \text{A})^2(8\ \Omega) = 12\ \text{W}$$

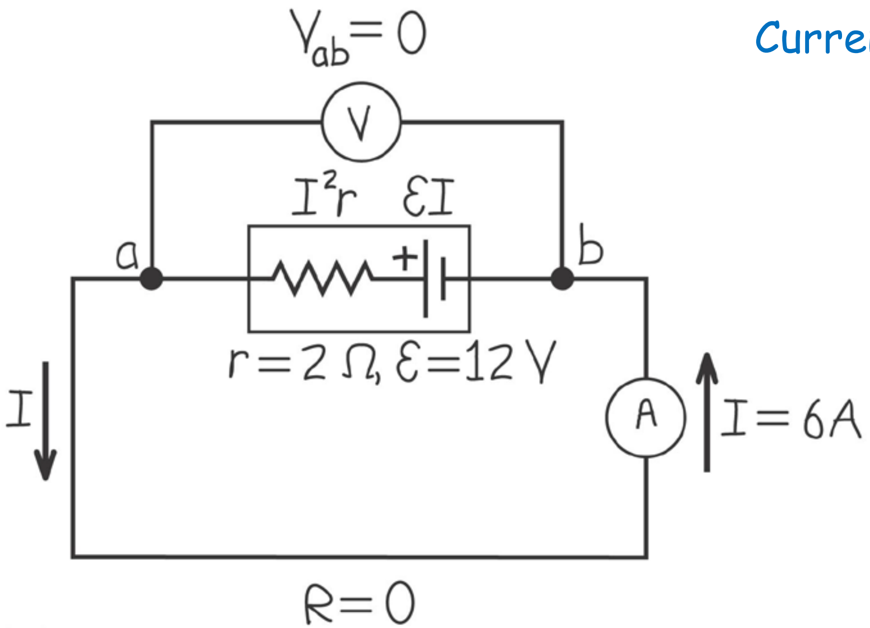
Power dissipated in R decreases (50 W light bulb has a greater resistance than a 100 W light bulb)

Power in a short circuit



Current I is ε/r = 6 A

Power in a short circuit



Current I is $\mathcal{E}/r$ = 6 A

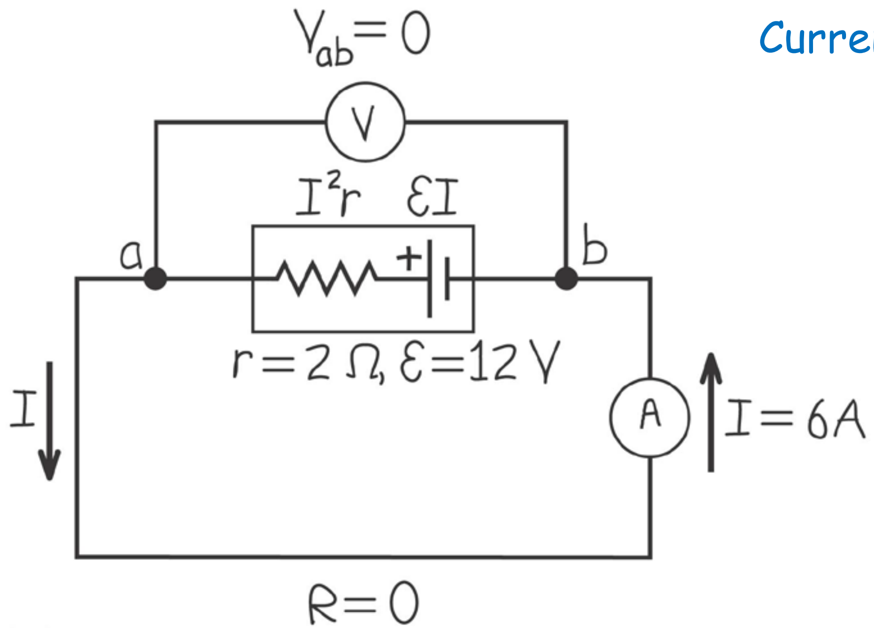
the rate of energy conversion (chemical to electrical) in the battery is then

$$\mathcal{E}I = (12\ \mathrm{V})(6\ \mathrm{A}) = 72\ \mathrm{W}$$

the rate of dissipation of energy in the battery is

$$I^2r = (6\ \mathrm{A})^2(2\ \Omega) = 72\ \mathrm{W}$$

Power in a short circuit



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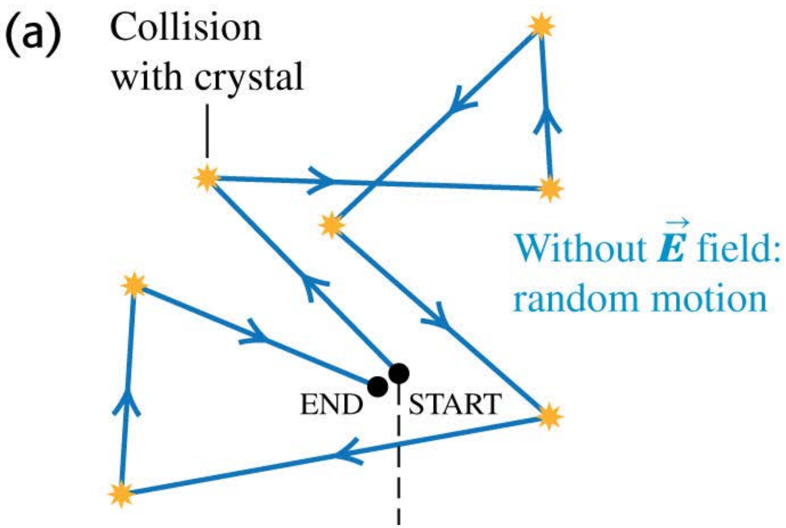
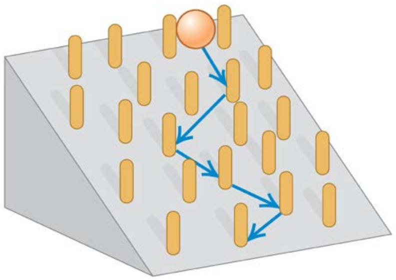
The net power is $\mathcal{E}I - I^2r = 0$

Same result from $P = V_{ab}I = 0$ (since $V_{ab} = 0$)

Theory of metallic conduction



Average speed of random motion is 10^6 m/s



If there is no electric field, the electrons move in straight lines between collisions, the directions of their velocities are random, and on average net displacement is ZERO

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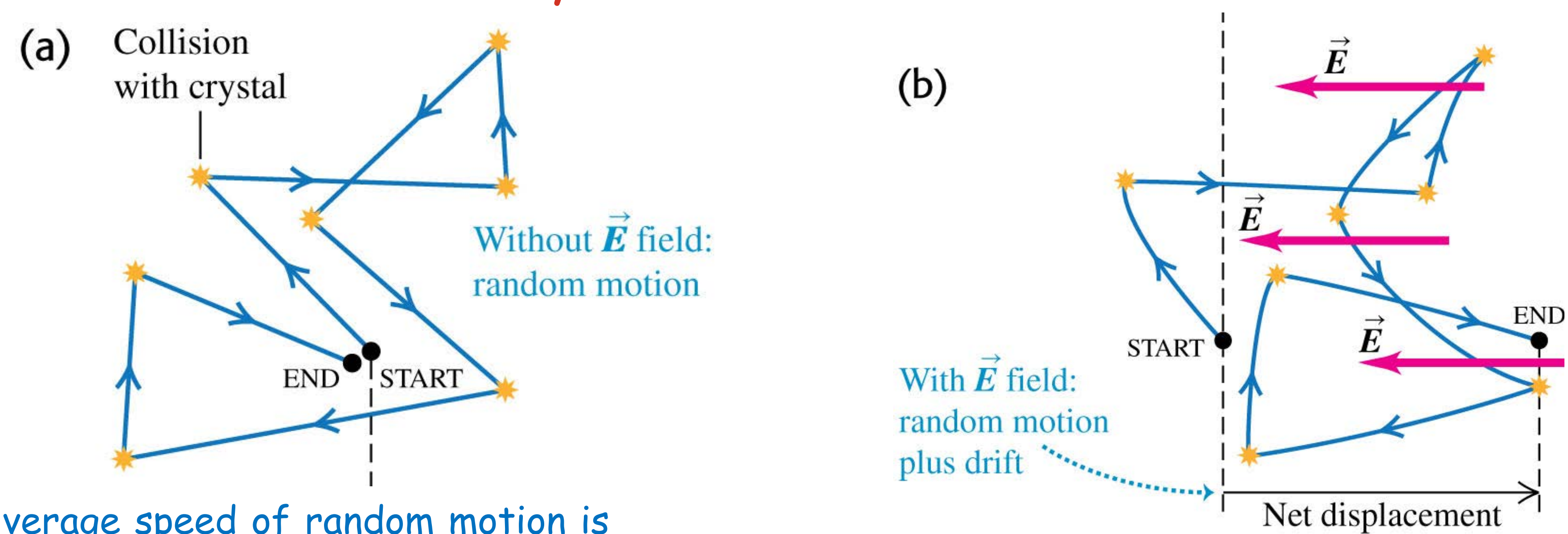
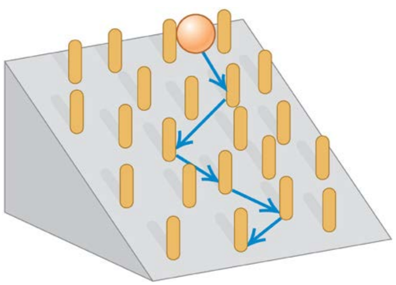


Average speed of random motion is 10^6 m/s

Average drift speed is 10^{-4} m/s



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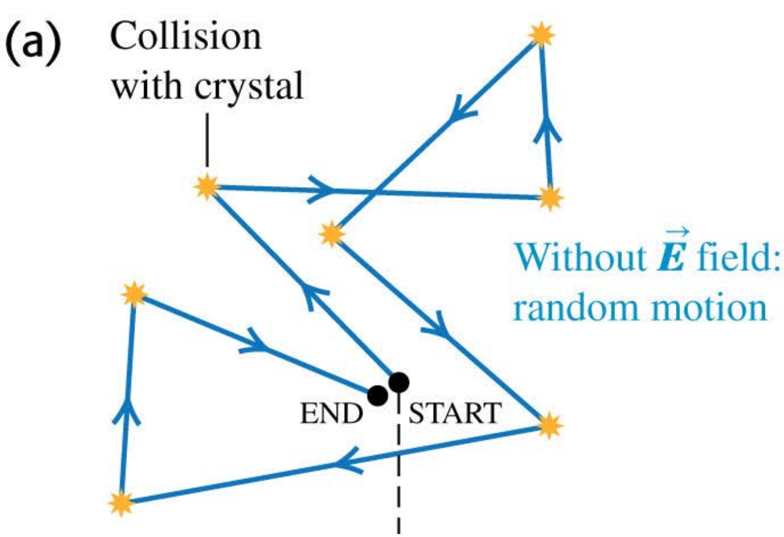

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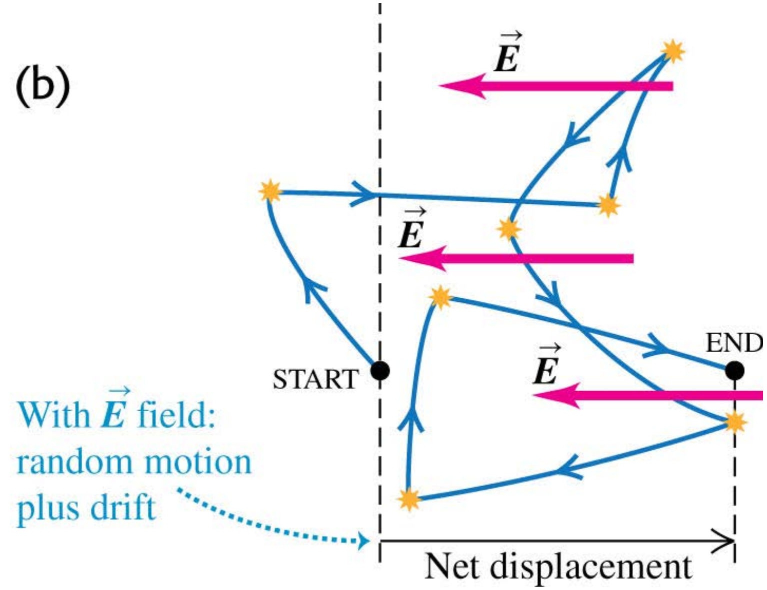

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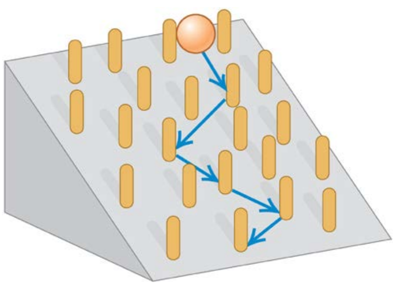

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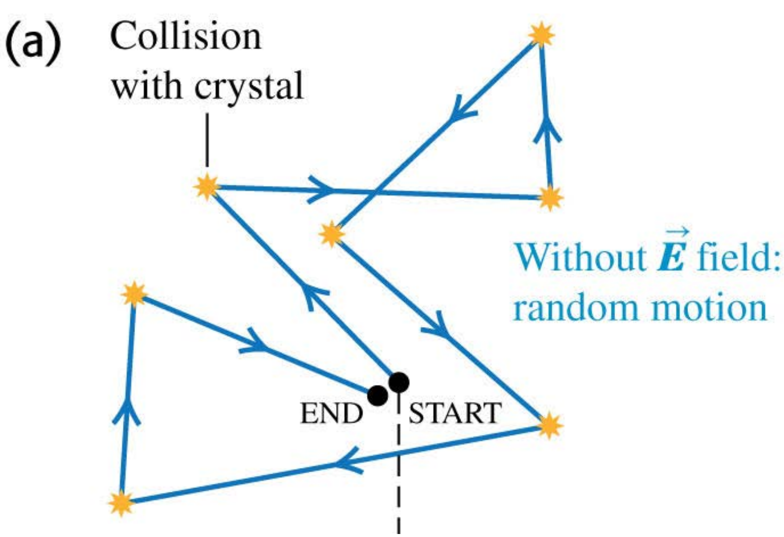

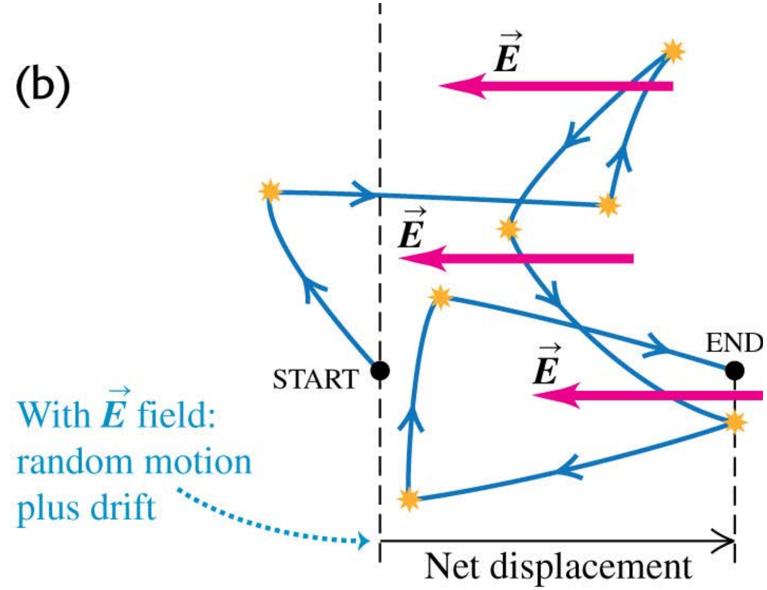

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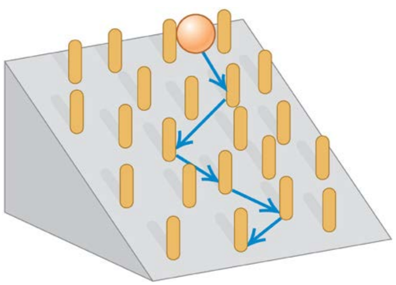

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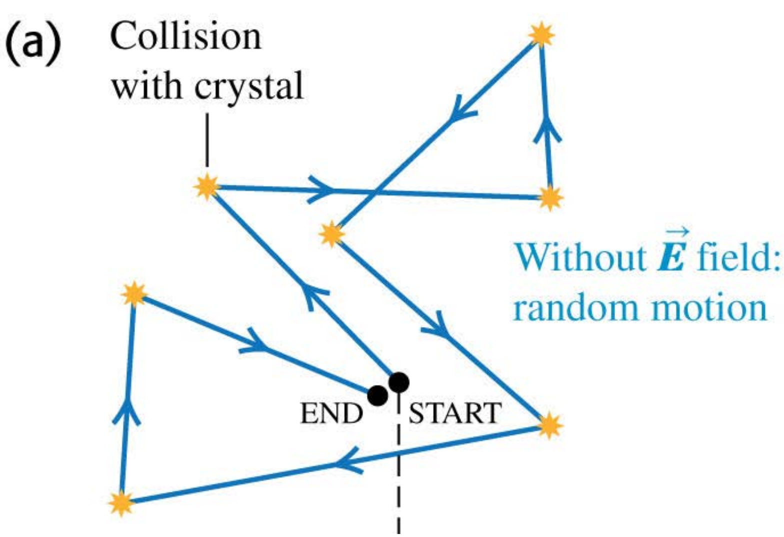

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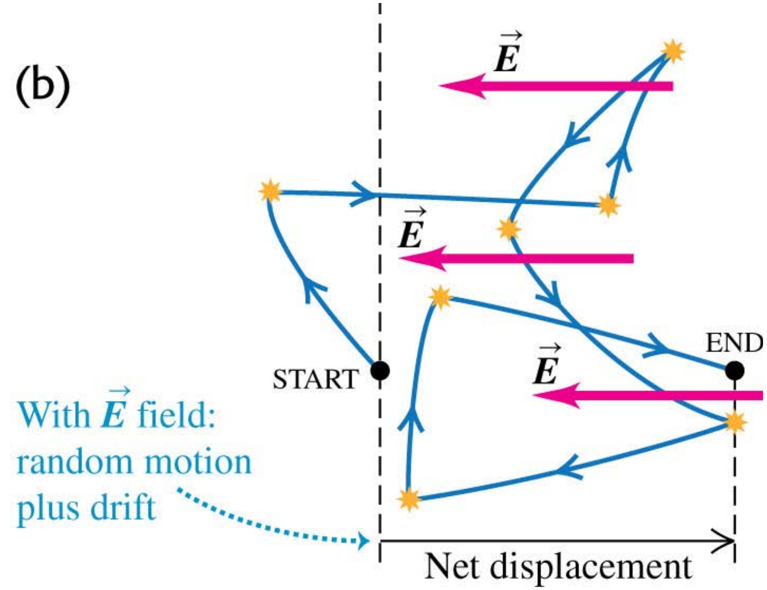

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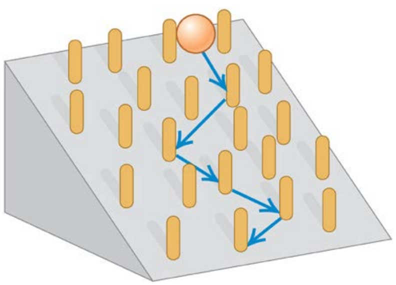

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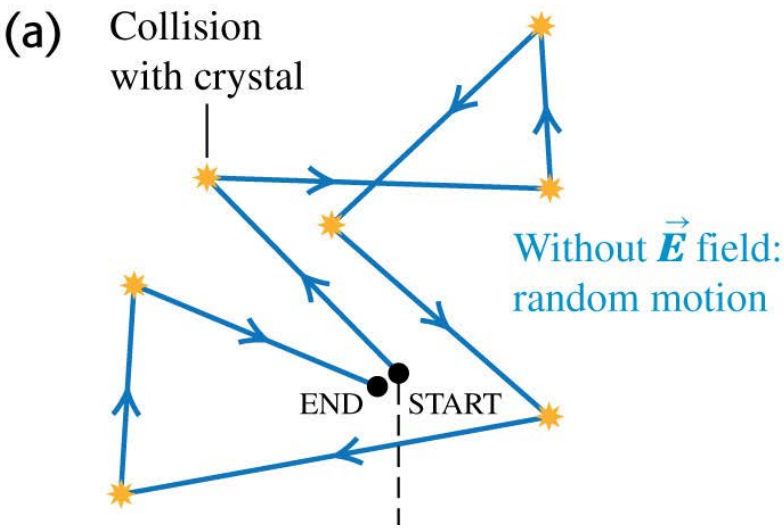
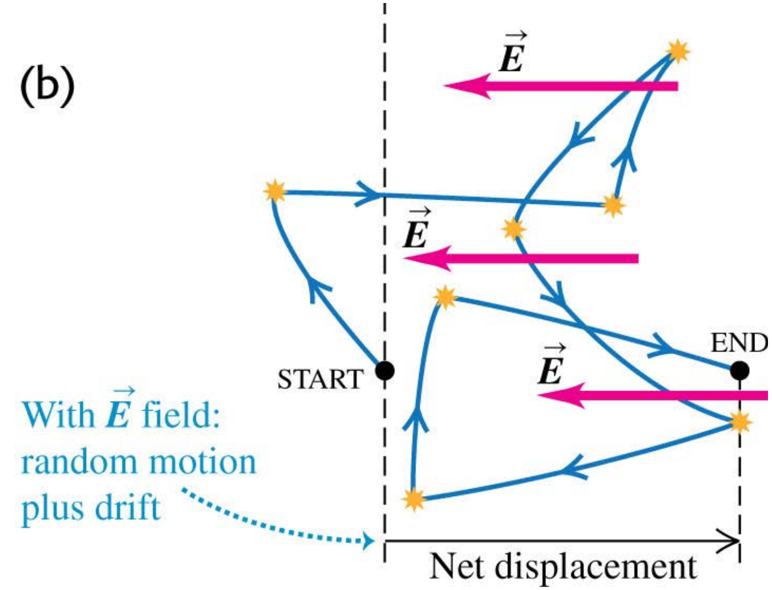
$$\vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m}\vec{E} \qquad \rho = \frac{m}{ne^2\tau}$$

Theory of metallic conduction





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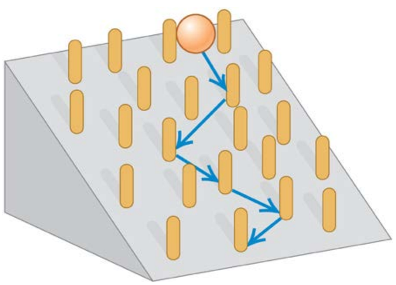

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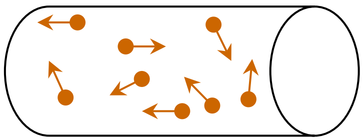
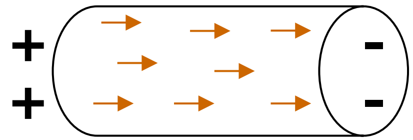
If n and τ are independent of E, then the resistivity is independent of E and the conducting material obeys Ohm's law.

What makes current flow?

conductivity $\sigma = 1/\rho$.



APPLIED FIELD = 0
Random motion
flow left = flow right



APPLIED FIELD NOT ZERO
Moving charges collide with fixed ions
and flow with **drift** velocity

$$\mathbf{J} = \sigma \mathbf{E}$$

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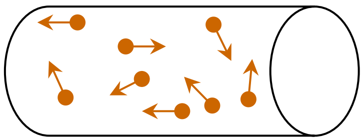

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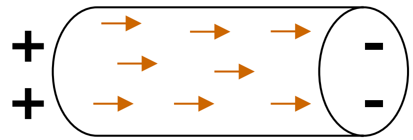

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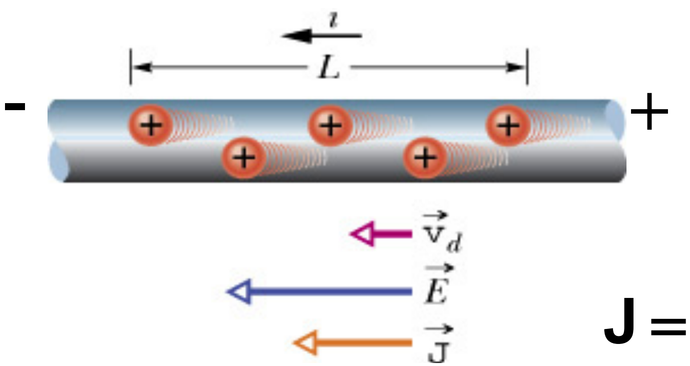

- $n \equiv$ density of charge carriers Units : # /volume
- nv_D = # of charge carriers crossing unit area per unit time

$$\mathbf{J} = \mathbf{q n v_D} = \text{net charge crossing area A per unit time}$$