ISTANBUL TECHNICAL UNIVERSITY

BLG354E - Recitation 1

17.04.2024

Question 1

$n=0$

Impulse response of a DT system is given as $h[n] = [0.2, 0.1, 0.5]$

Since there is an arrow here, you should take it as h[0].

$$x(t) = \underbrace{t(u(t) - u(t-30))}_{x_1(t)} + \underbrace{(60-t)(u(t-30) - u(t-60))}_{x_2(t)}$$ is given.

A. Plot the $x(t)$

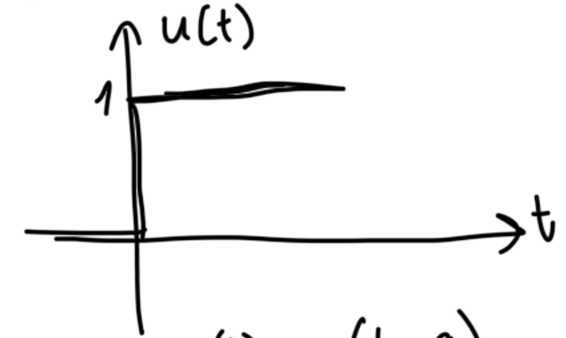
B. Convert $x(t)$ to digital $x[n]$ with $f_s = 0.1$ Hz and plot

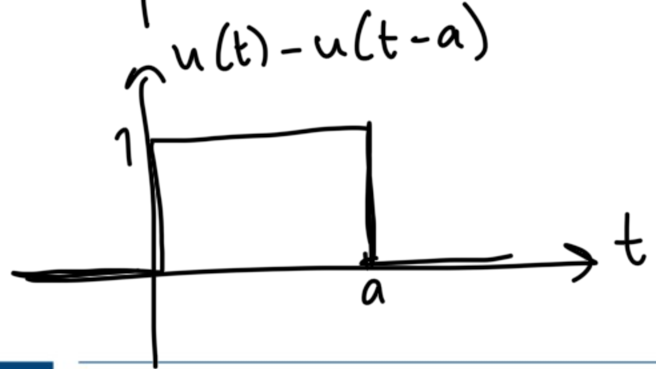
C. Find and sketch the output of the system for the obtained input signal $x[n]$

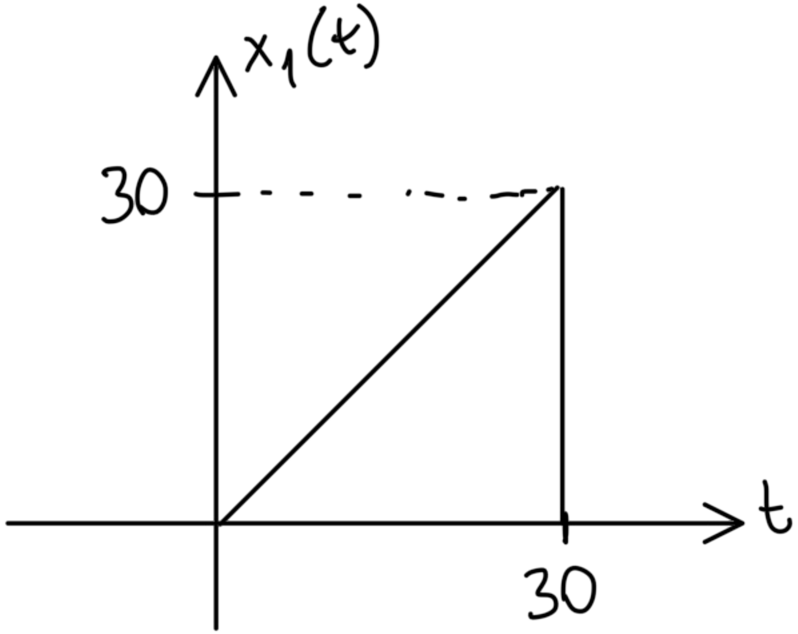
Question 1 - Solution

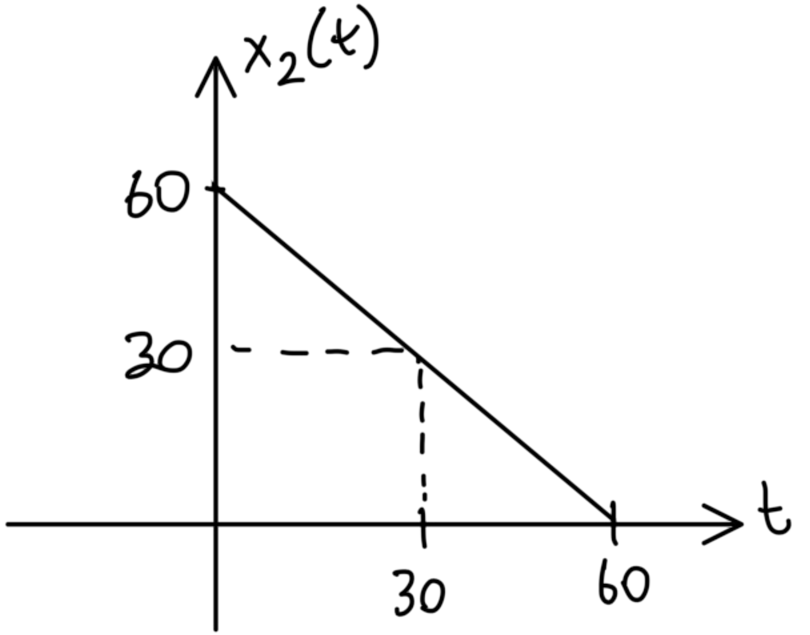
a) $$x(t) = \underbrace{t \cdot \big(u(t) - u(t-30)\big)}_{x_1(t)} + \underbrace{(60-t) \cdot \big(u(t-30) - u(t-60)\big)}_{x_2(t)}$$

Reminder :





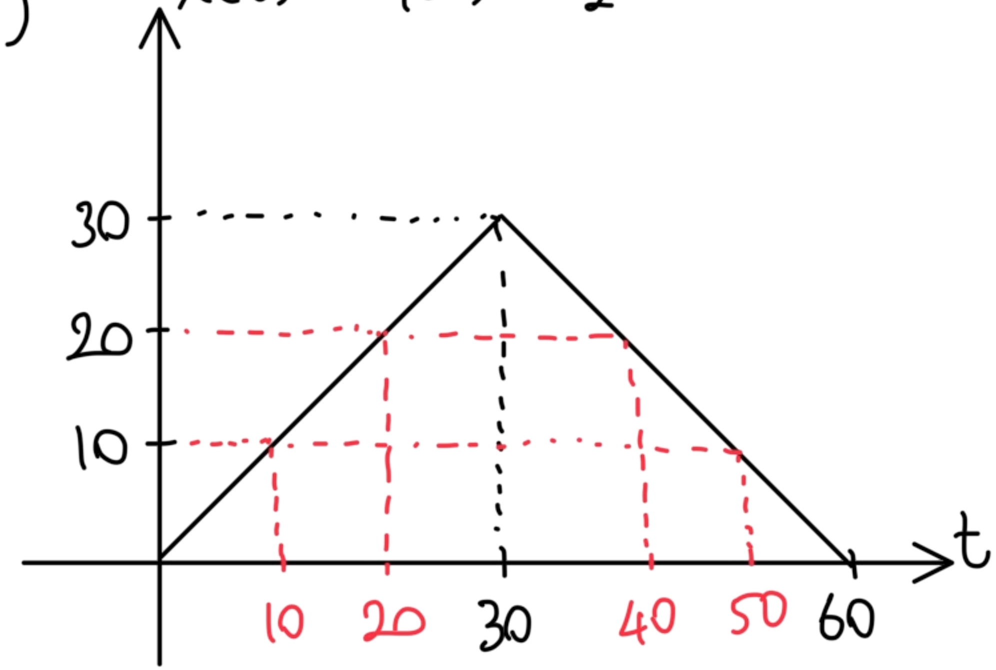




By adding these two graphs we get the final graph.

Question 1 - Solution

a) $x(t) = x_1(t) + x_2(t)$



b) $x[n] = x(t = nT_s)$ is used.

$f_s = 0.1\ \text{Hz} \rightarrow T_s = \frac{1}{f_s} = 10\ \text{sn}$

$x[0] = x(0) = 0$

$x[1] = x(10) = 10$

$x[2] = x(20) = 20$

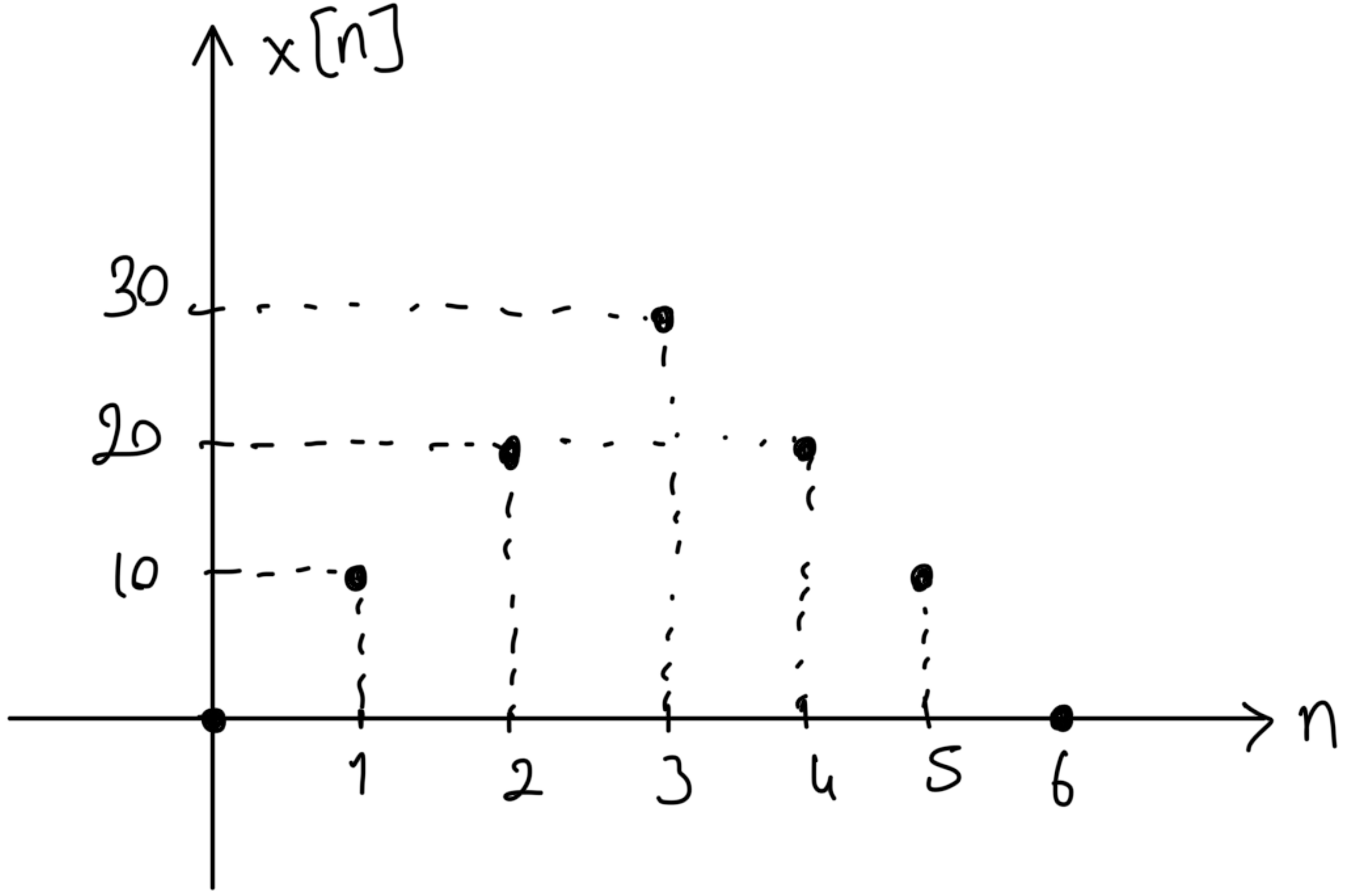
$x[3] = x(30) = 30$

$x[4] = x(40) = 20$

$x[5] = x(50) = 10$

$x[6] = x(60) = 0$

$$x[n] = [0, 10, 20, 30, 20, 10, 0]$$



Question 1 - Solution

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{k=\infty} x[k] \cdot h[n-k]$$

$x[n] = [0, 10, 20, 30, 20, 10, 0]$

$h[n] = [0.2, 0.1, 0.5]$

After filling out the table, you should calculate y[n] for each n values ranging between 0 and 7 (where your data is)

$y[0] = x[0].h[0-0] + x[1].h[0-1] + \dots + x[7].h[0-7]$

$y[1] = x[0].h[1-0] + x[1].h[1-1] + \dots + x[7].h[1-7]$

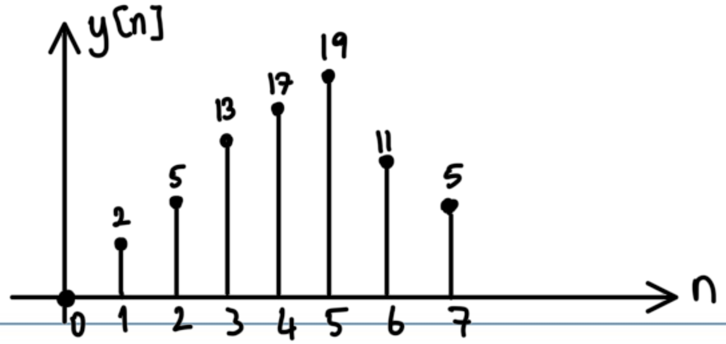
⋮

k	-2	-1	0	1	2	3	4	5	6	7
h[k]			0.2	0.1	0.5					
x[k]			0	10	20	30	20	10	0	
h[-k]	0.5	0.1	0.2							
h[1-k]		0.5	0.1	0.2						
h[2-k]			0.5	0.1	0.2					
h[3-k]				0.5	0.1	0.2				
h[4-k]					0.5	0.1	0.2			
h[5-k]						0.5	0.1	0.2		
h[6-k]							0.5	0.1	0.2	
h[7-k]								0.5	0.1	0.2
y[n]			0	2	5	13	17	19	11	5

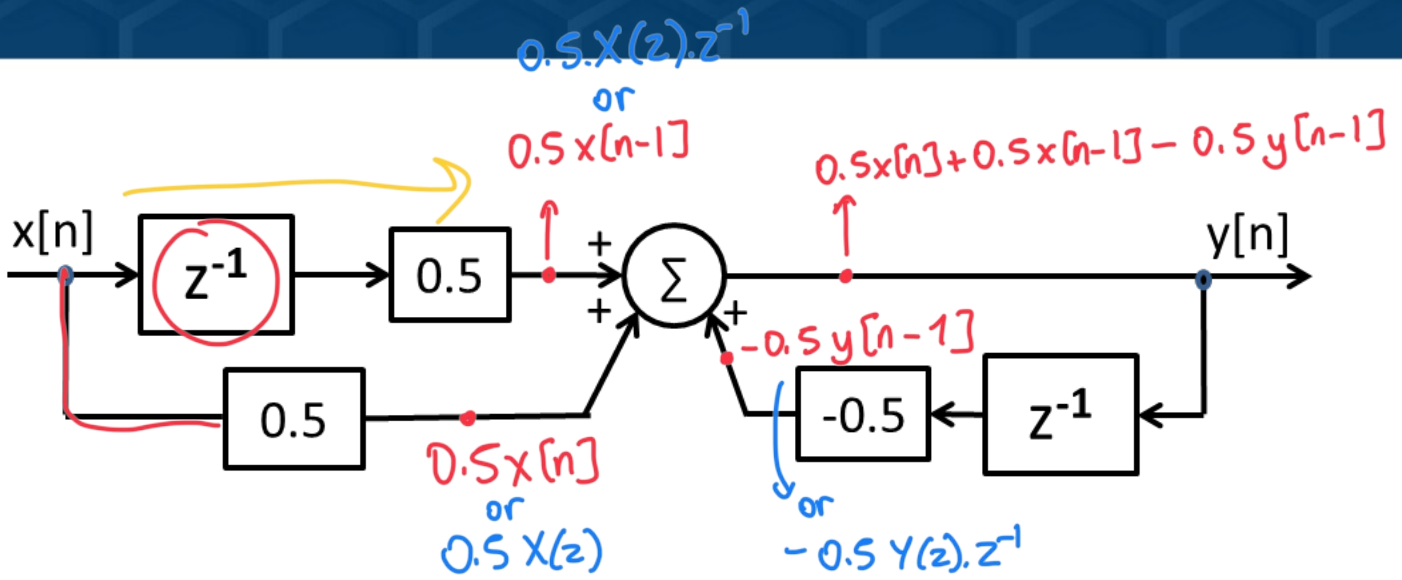
For example:

$y[2] = 20.\ 0{,}2 + 10.\ 0{,}1 + 0.\ 0{,}1$
$= 4 + 1 = 5$

$y[3] = 30.\ 0{,}2 + 20.\ 0{,}1 + 10.\ 0{,}5$
$= 6 + 2 + 5 = 13$



Question 2



Consider the discrete system shown in the figure below where x[n] is the input, y[n] is the output and z^{-1} represents the unit delay.

a) Write the difference equation that relates the output y[n] and the input x[n]

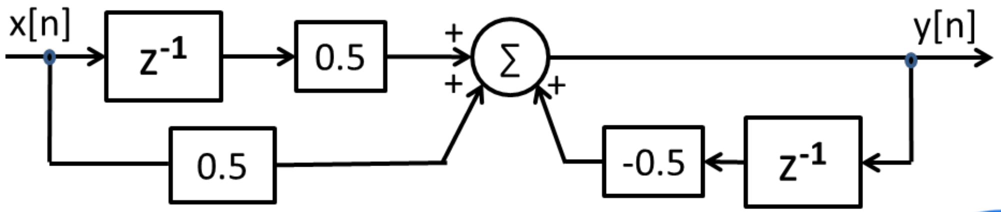
b) Find and draw the system output values for n=0 to 5 if input signal x[n] is defined as:

$$x[n]=n\cdot(u[n-1]-u[n-3])-2n\cdot\delta[n-2]$$

a) $y[n] = 0.5\,x[n] + 0.5\,x[n-1] - 0.5\,y[n-1] = 0.5\left(x[n] + x[n-1] - y[n-1]\right)$

Question 2 - Solution

b)



$$x[n] = n\cdot\left(u[n-1] - u[n-3]\right) - 2n\cdot\delta[n-2]$$

$$3\,(1 - 1)$$

$$2\cdot 1 - 4\cdot 1 = -2$$

$$y[n] = 0.5\left(x[n-1] + x[n] - y[n-1]\right)$$

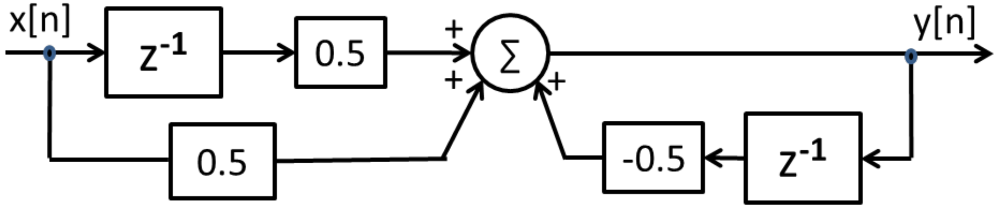
$$1 = 0.5\,(0 + 1 - 0) = 1/2$$

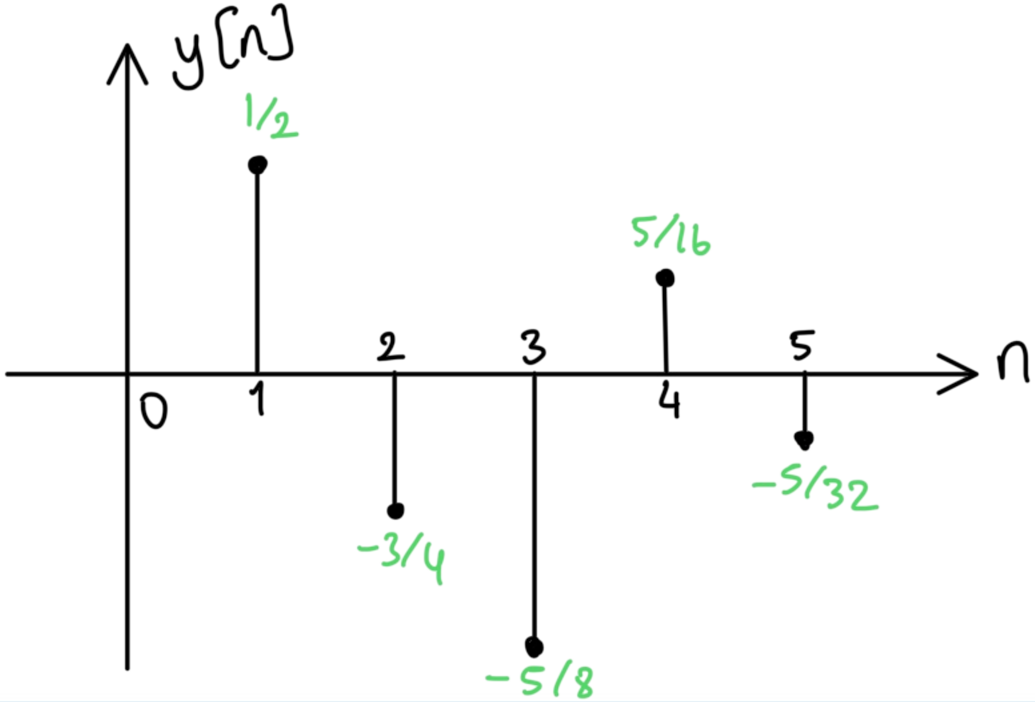
n	x[n]	x[n-1]	y[n]	y[n-1]
0	0	0	0	0
1	1	0	1/2	0
2	−2	1	−3/4	1/2
3	0	−2	−5/8	−3/4
4	0	0	5/16	−5/8
5	0	0	−5/32	5/16

→ calculate each y[n] value and fill out a table like the one above for a clean solution.

Question 2 - Solution

b)



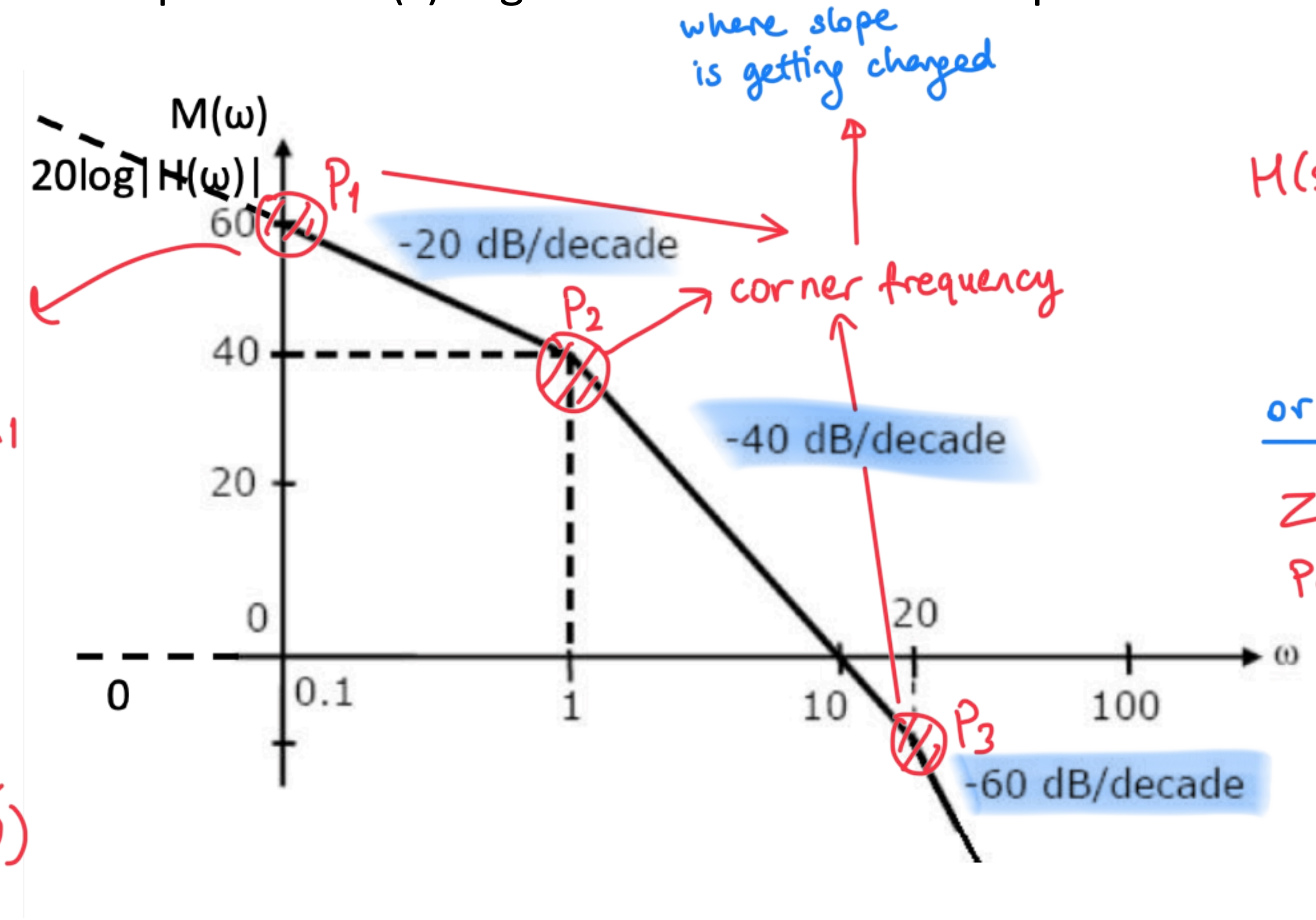


n	x[n]	x[n-1]	y[n]	y[n-1]
0	0	0	0	0
1	1	0	1/2	0
2	-2	1	-3/4	1/2
3	0	-2	-5/8	-3/4
4	0	0	5/16	-5/8
5	0	0	-5/32	5/16

Question 3

Magnitude response of H(s) is given in the below Bode plot. Find the transfer function H(s).

Since the magnitude at $w=0$ is not given, and there is a slope decrease at $w=0.1$ took place, we say that there is a pole at $w=0$ (here 0.1 will be taken as origin therefore we'll say $w=0$)



Standard Transfer Function:

$$H(s) = \frac{K.\left(1+\frac{s}{z_1}\right).\left(1+\frac{s}{z_2}\right)\ldots}{\left(1+\frac{s}{p_1}\right).\left(1+\frac{s}{p_2}\right)\ldots}$$

How to decide whether it is a pole or zero?

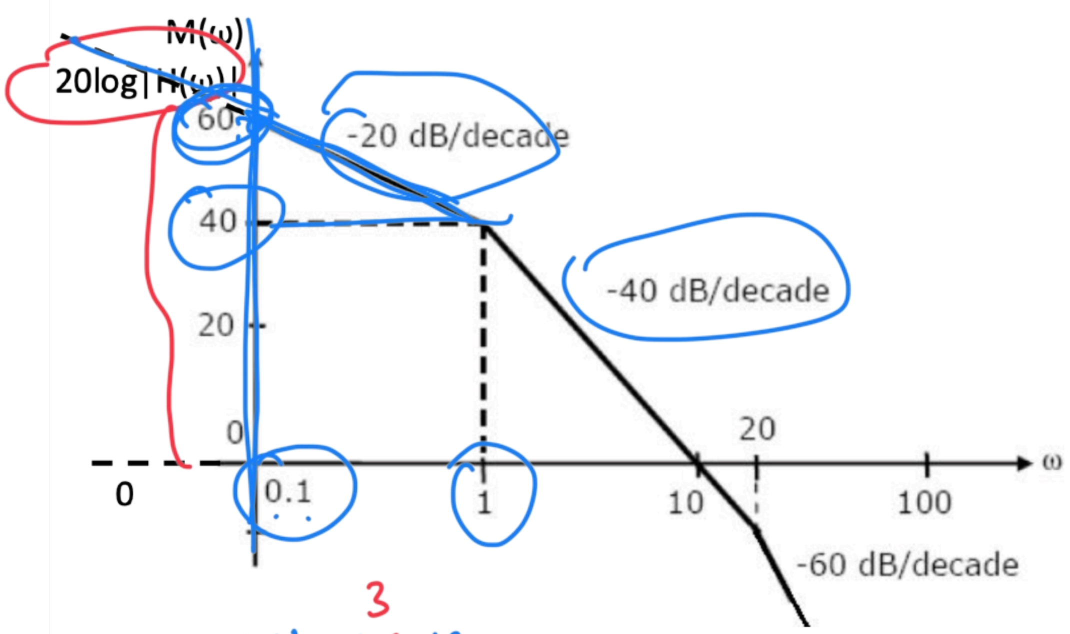
Zero: If slope is increased by 20dB

Pole: If slope is decreased by 20dB

Since the slope is decreased by 20 dB at $w=0$, $w=1$ and $w=20$, we'll say that the system has 3 poles.

$p_1=0$ $\quad p_2=1$ $\quad p_3=20$

Question 3 - Solution



$$H(s) = \frac{K}{s.\left(1+\frac{s}{1}\right).\left(1+\frac{s}{20}\right)}$$

pole at origin

→ s = jw conversion will be made to calculate magnitude

$$|H(jw)| = \left|\frac{K}{jw(1+jw).\left(1+\frac{jw}{20}\right)}\right|_{w=0.1} = 60\ dB$$

$\cancel{20}\ \log|H(w)| = \overset{3}{\cancel{60}}\ dB$

$|H(w)| = 10^3 = 1000$

$$= \frac{K}{w.\left(\sqrt{1+\cancel{w^2}}\right).\left(1+\cancel{\left(\frac{w}{20}\right)^2}\right)} = 10^3 = 1000$$

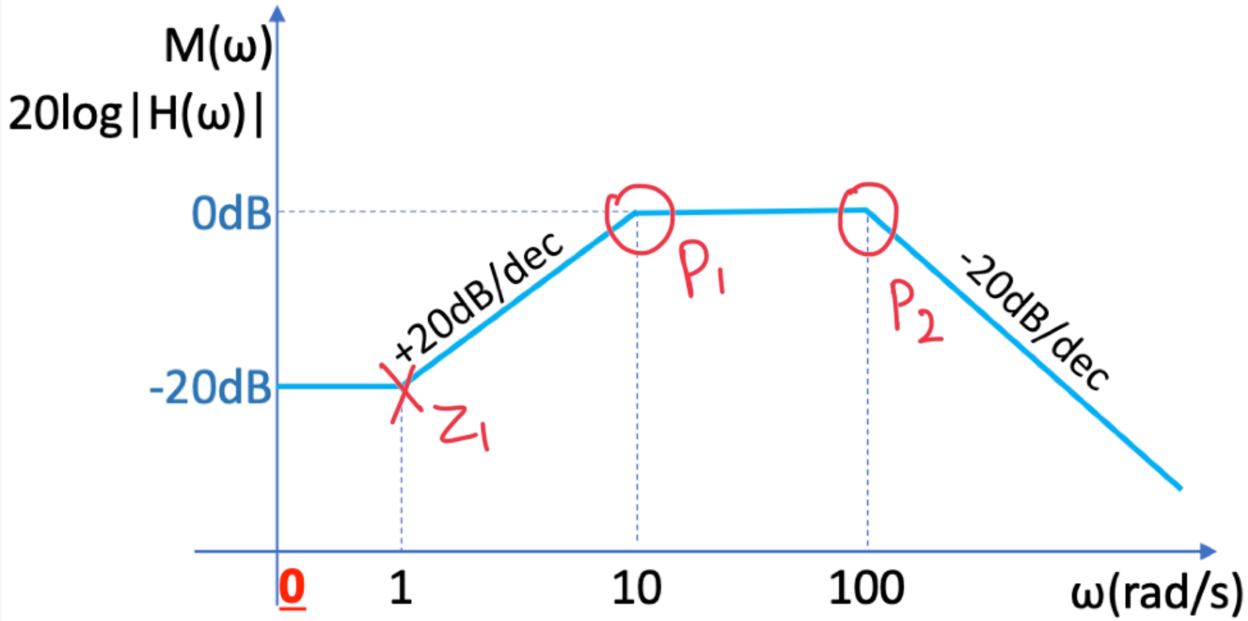
At $w=0.1$, $w^2 \ll 1$ and $\frac{w}{20} \ll 1$ we ignore these.

$$|H(w)| = \frac{K}{\underset{0.1}{w}} = 1000 \rightarrow K = 1000 \cdot 0{,}1 = 100$$

$$\boxed{H(s) = \frac{100}{s.(1+s).(1+0.05s)}}$$

Question 4

Frequency response (magnitude) of H(s) is given in the below Bode plot. Find the transfer function H(s).



<u>Standard Transfer Function:</u>

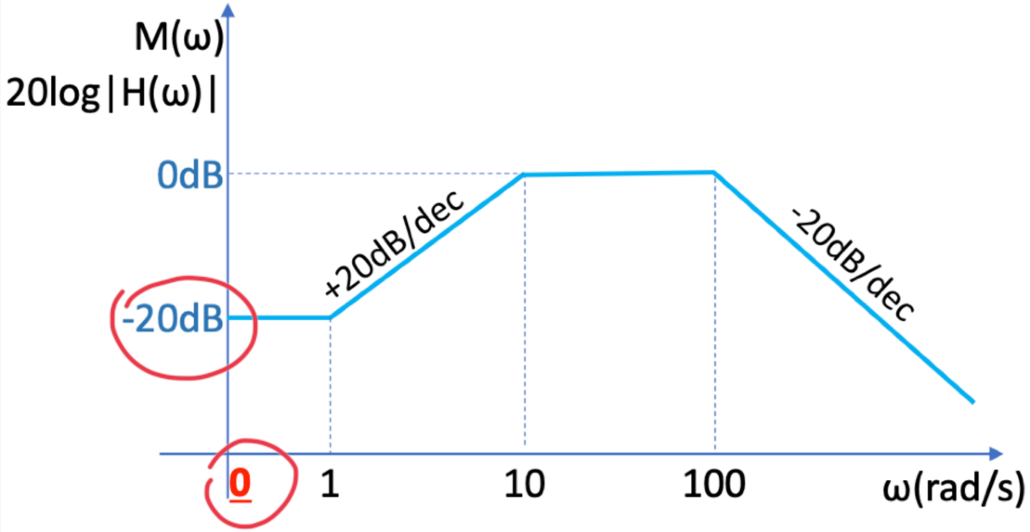
$$H(s)=\frac{K.\left(1+\frac{s}{z_1}\right).\left(1+\frac{s}{z_2}\right)\cdots}{\left(1+\frac{s}{p_1}\right).\left(1+\frac{s}{p_2}\right)\cdots}$$

$z_1=1$, $p_1=10$, $p_2=100$

$$H(s)=\frac{K.\left(1+\frac{s}{1}\right)}{\left(1+\frac{s}{10}\right).\left(1+\frac{s}{100}\right)}$$

$$H(s)=\frac{K.(1+s)}{\frac{1}{10}.(10+s).\frac{1}{100}(100+s)}$$

Question 4 - Solution



$w=0:\ \cancel{20}\log|H(w)| = -\cancel{20}^{1}\,dB$

$|H(w)| = 10^{-1} = 0.1$

$$H(s) = \frac{1000K(1+s)}{(10+s).(100+s)}$$

$$|H(jw)| = \left|\frac{1000K(1+\cancel{jw})}{(10+\cancel{jw}).(100+\cancel{jw})}\right|_{w=0} = -20\,dB$$

0.1

$$= \frac{1\cancel{000}K}{1\cancel{0}.1\cancel{00}} = 0.1 \rightarrow K = 0.1$$

$$\boxed{H(s) = \frac{100(1+s)}{(10+s).(100+s)}}$$

Question 5

Find the Laplace Transform and determine the ROC for the following signals:

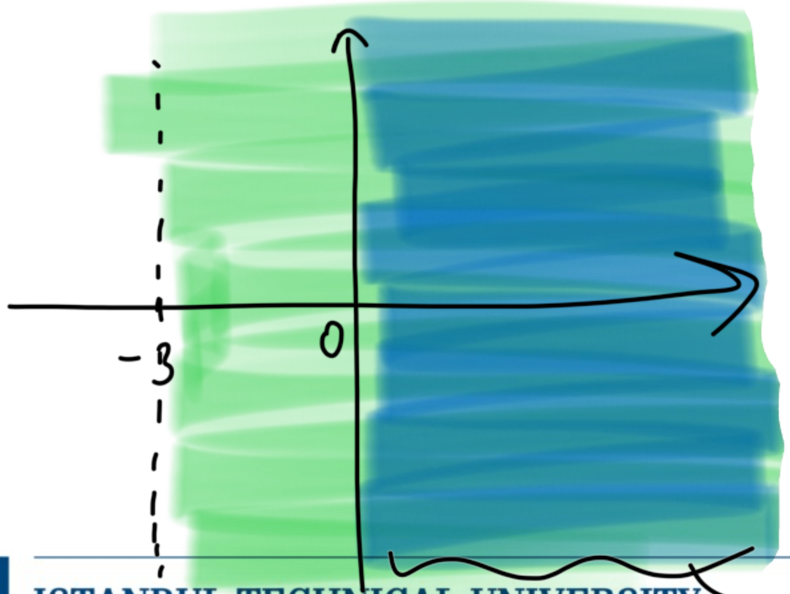
a) $x_1(t) = e^{-2(t-3)}u(t-3)$

b) $x_2(t) = (1-(1-t)e^{-3t})u(t)$ $= u(t) - e^{-3t}u(t) + \boxed{t \cdot e^{-3t} \cdot u(t)}$

c) $x_3(t) = |t|e^{-|t|}$

$$X_2(s) = \frac{1}{s} - \frac{1}{s+3} + \frac{1}{(s+3)^2}$$

$\mathrm{Re}\{s\} > 0$ + $\mathrm{Re}\{s\} > -3$ + $\mathrm{Re}\{s\} > -3$



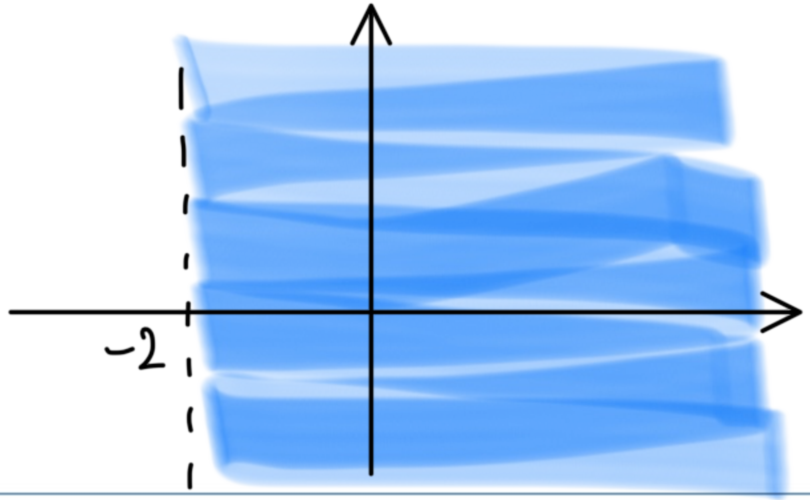
Overall ROC for part b: $\mathrm{Re}\{s\} > 0$

Question 5 - Solution

a) $x_1(t) = e^{-2(t-3)} \cdot u(t-3)$

$$e^{-2t} \cdot u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+2}, \quad Re\{s\} > -2$$

$$e^{-2(t-3)} \cdot u(t-3) \xleftrightarrow{\mathcal{L}} \frac{e^{-3s}}{s+2}, \quad Re\{s\} > -2$$

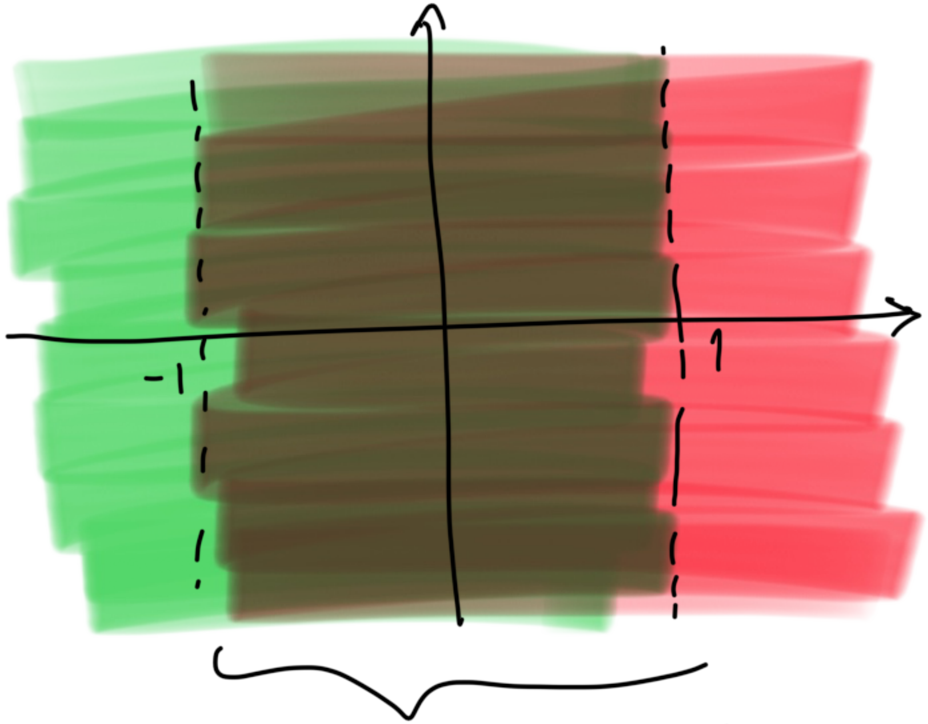


Question 5 - Solution

c) $x_3(t) = |t| e^{-|t|} = t e^{-t} u(t) - t e^{t} u(-t)$

$$X_3(s) = \frac{1}{(s+1)^2} + \frac{1}{(s-1)^2}$$

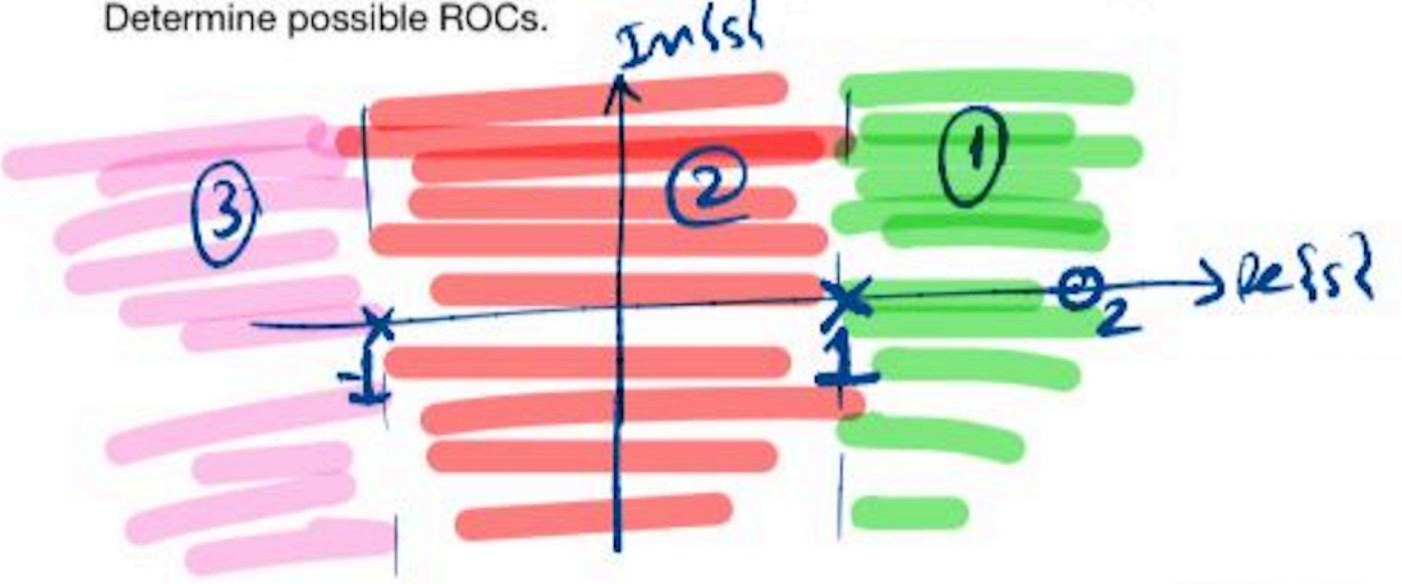
ROC: $Re\{s\} > -1$, $Re\{s\} < 1$



$-1 < Re\{s\} < 1$

Question 5 - Additional Question and Solution

> Pole-zero diagram of an LTI system is given below. Find H(s). Determine possible ROCs.



x : pole
o : zero

1. Re{s} > 1 → Causal, unstable
2. 1 > Re{s} > −1 → Noncausal, stable
3. Re{s} < −1 → Noncausal, unstable

$$H(s) = \frac{s-2}{(s+1)\cdot(s-1)}$$

Question 6

Find the inverse Laplace Transform of

$$X(s) = \frac{2 + 2se^{-2s} + 4e^{-4s}}{s^2 + 4s + 3}, \; Re\{s\} > -1$$

$$X(s) = \underbrace{\frac{2}{s^2+4s+3}}_{X_1(s)} + \underbrace{\frac{2se^{-2s}}{s^2+4s+3}}_{X_2(s).\,e^{-2s}} + \underbrace{\frac{4e^{-4s}}{s^2+4s+3}}_{X_3(s).\,e^{-4s}}$$

$$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$$

$$x(t) = x_1(t) + x_2(t-2) + x_3(t-4)$$

Question 6 - Solution

$$X_1(s) = \frac{2}{(s+1).(s+3)} = \frac{1}{s+1} - \frac{1}{s+3} \longleftrightarrow x_1(t) = u(t).(e^{-t} - e^{-3t})$$

$$X_1(s) = \frac{2s}{(s+1).(s+3)} = \frac{1}{s+1} + \frac{3}{s+3} \longleftrightarrow x_2(t) = u(t).(e^{-t} + 3e^{-3t})$$

$$X_1(s) = \frac{4}{(s+1).(s+3)} = \frac{2}{s+1} - \frac{2}{s+3} \longleftrightarrow x_3(t) = u(t).(2e^{-t} - 2e^{-3t})$$

- From the previous slide $x(t) = x_1(t) + x_2(t-2) + x_3(t-4)$

$$x(t) = u(t).(e^{-t} - e^{-3t}) + u(t-2).(e^{-(t-2)} + 3e^{-3(t-2)}) + u(t-4).(2e^{-(t-4)} - 2e^{-3(t-4)})$$

Additional Question

Determine the discrete-time convolution of x[n] and h[n] for the following two cases:

(a)

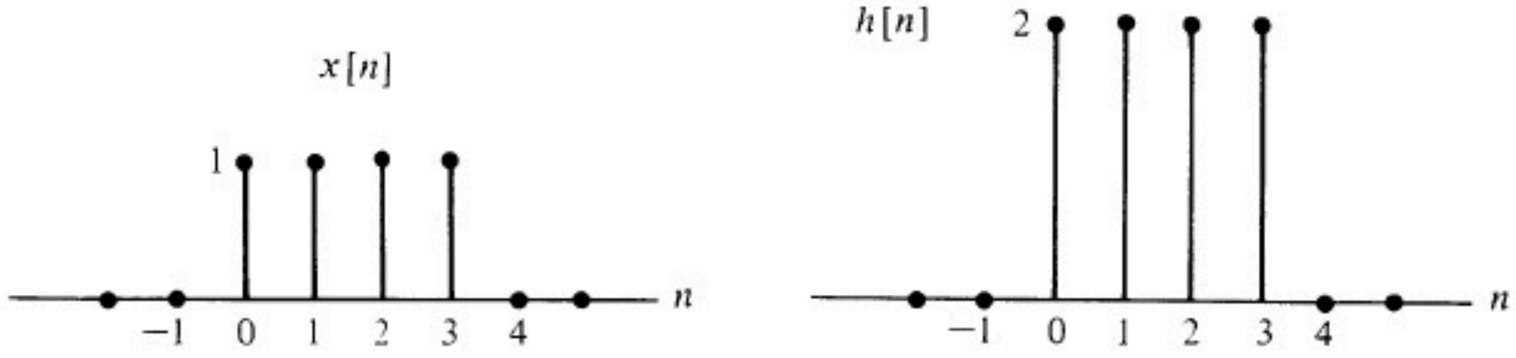


Figure P4.2-1

Additional Question

Determine the discrete-time convolution of x[n] and h[n] for the following two cases:

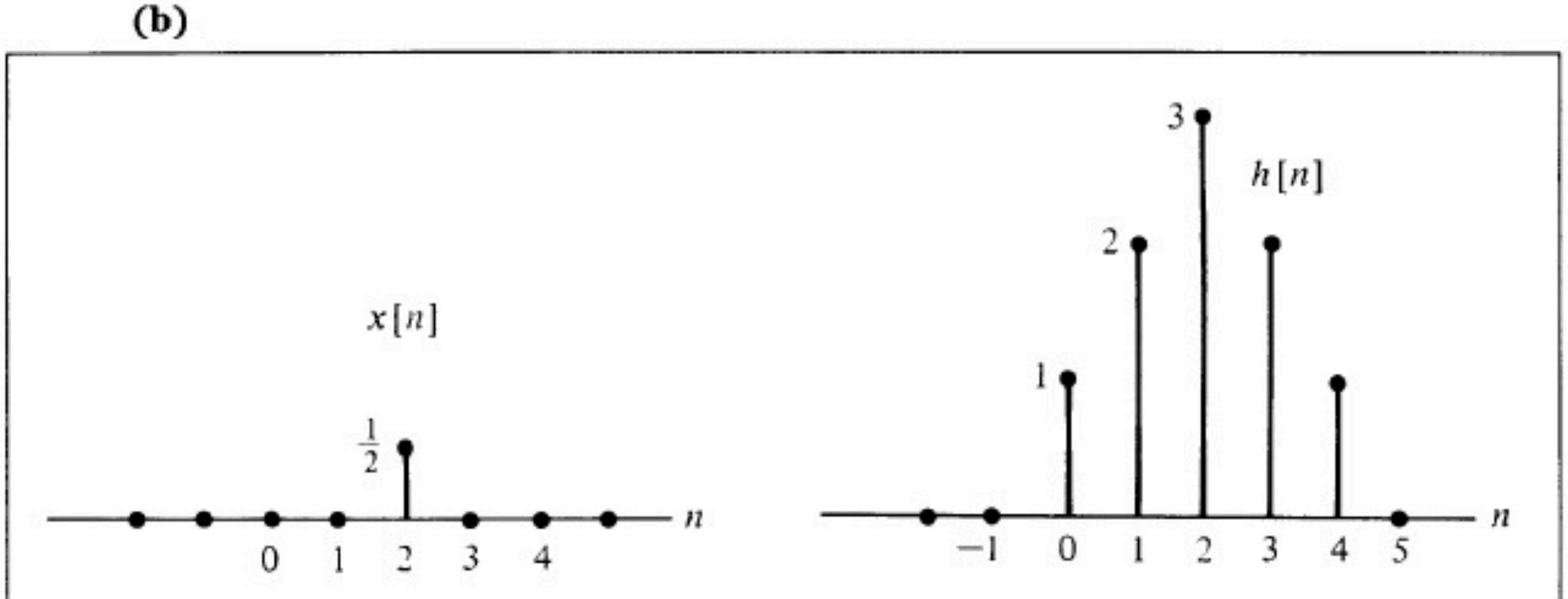


Figure P4.2-2

Additional Question Answer

(a) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-1.

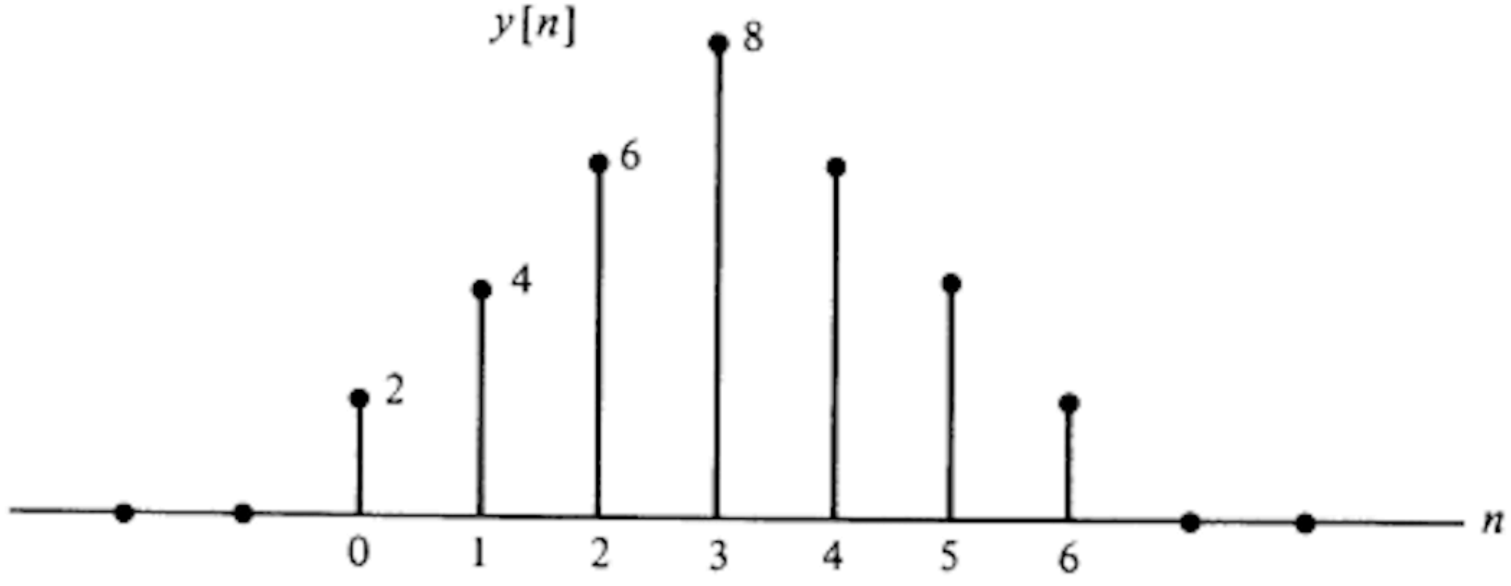


Figure S4.2-1

Additional Question Answer

(b) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure S4.2-2.

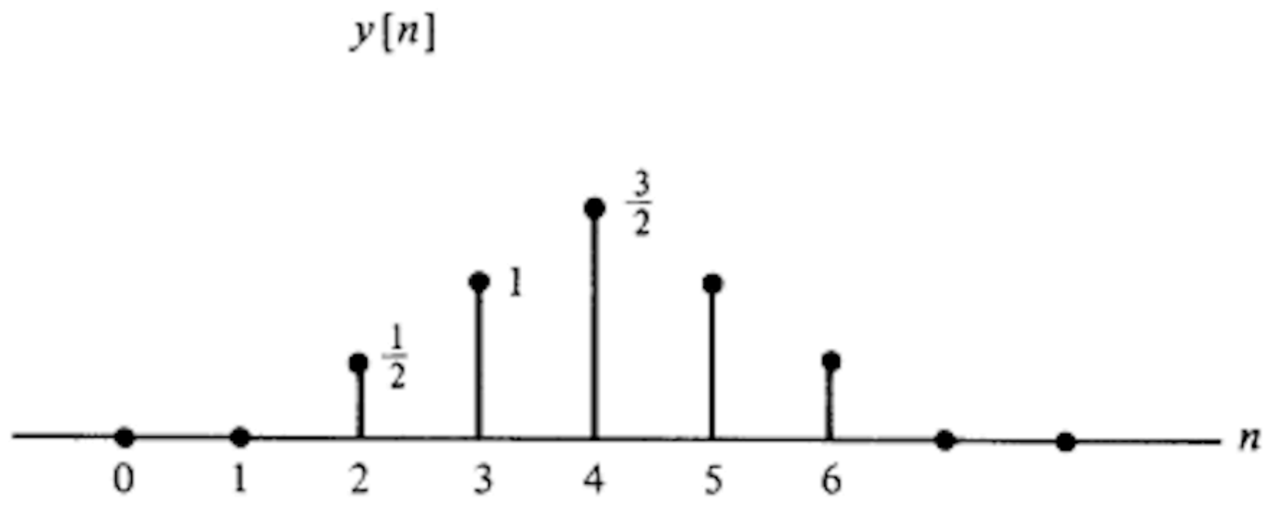


Figure S4.2-2

Notice that $y[n]$ is a shifted and scaled version of $h[n]$.