# Chp 29: Electromagnetic Induction - (II)

# Lenz's Law

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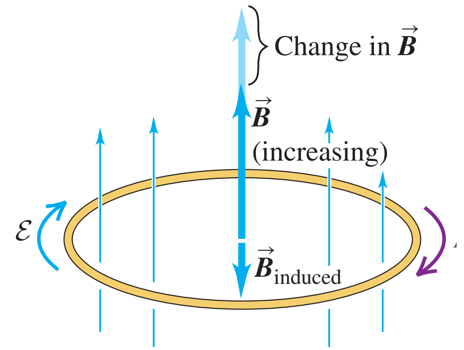
Lenz's Law : **The direction of any magnetic induction effect is such as to oppose the cause of the effect.**



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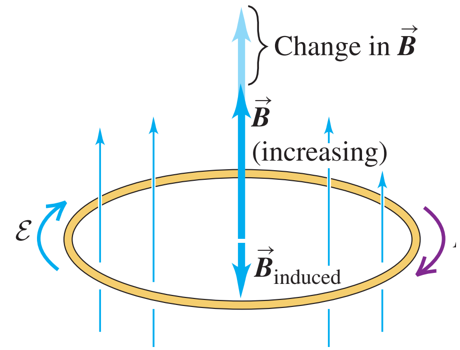
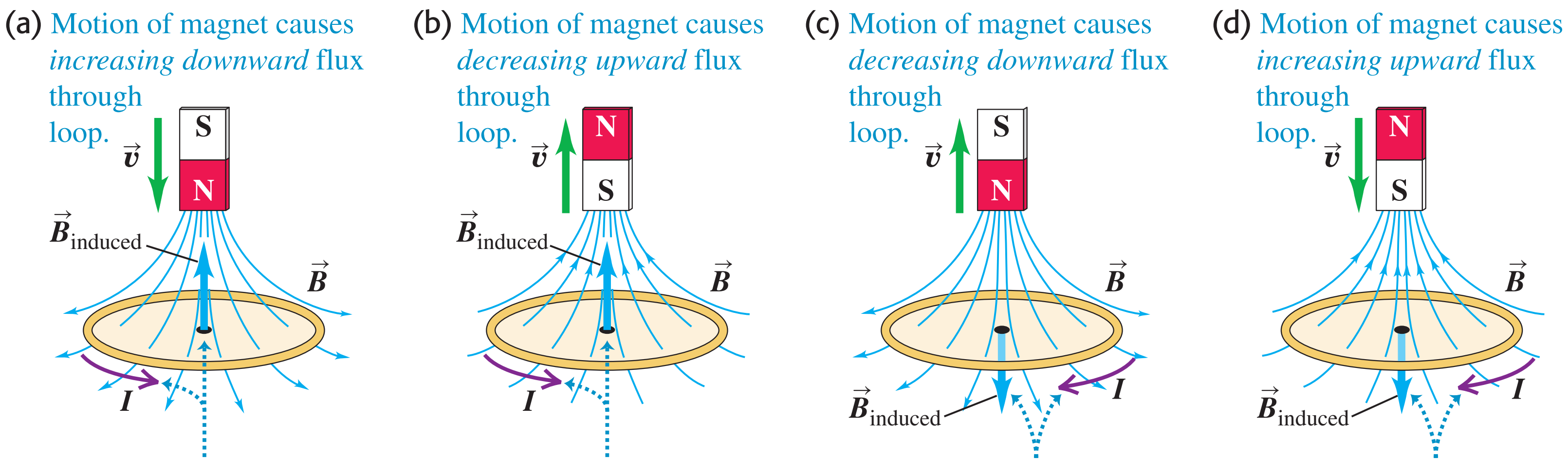
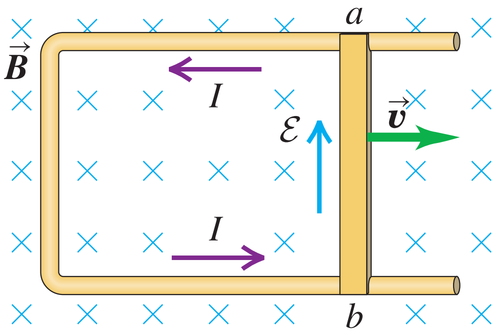



The induced magnetic field is *upward* to oppose the flux change. To produce this induced field, the induced current must be *counterclockwise* as seen from above the loop.

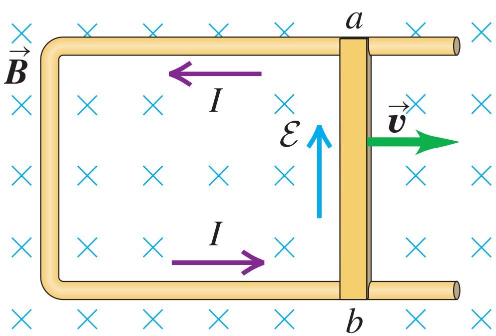
The induced magnetic field is *downward* to oppose the flux change. To produce this induced field, the induced current must be *clockwise* as seen from above the loop.

# Motional electromotive force

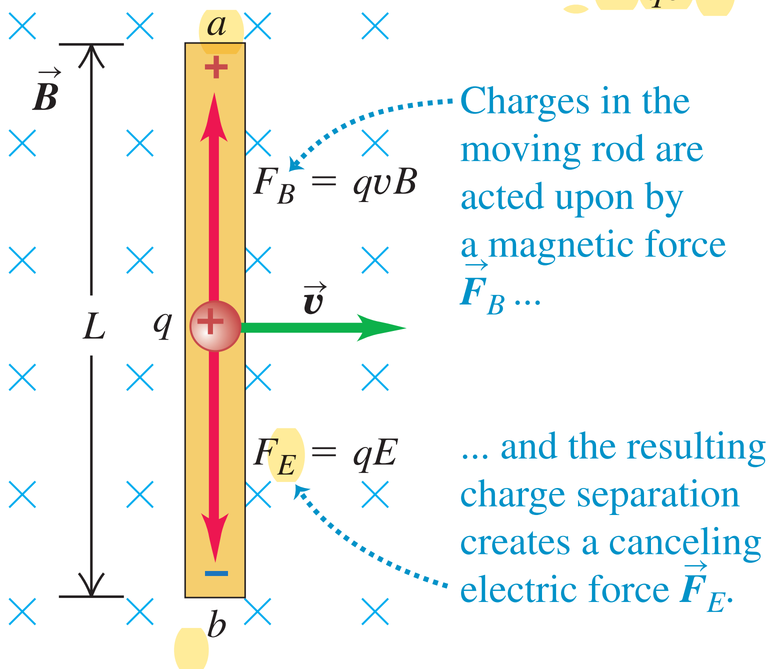


# Motional electromotive force

$\vec{F} = q\vec{v} \times \vec{B}$




This magnetic force causes the free charges in the rod to move, creating an excess of positive charge at the upper end *a* and negative charge at the lower end *b*. This in turn creates an electric field ***E*** within the rod, in the direction from *a* toward *b* (opposite to the magnetic force).
➔ $qE = qvB$ and the charges are in equilibrium.



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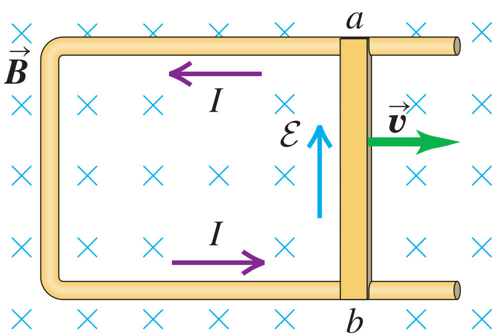


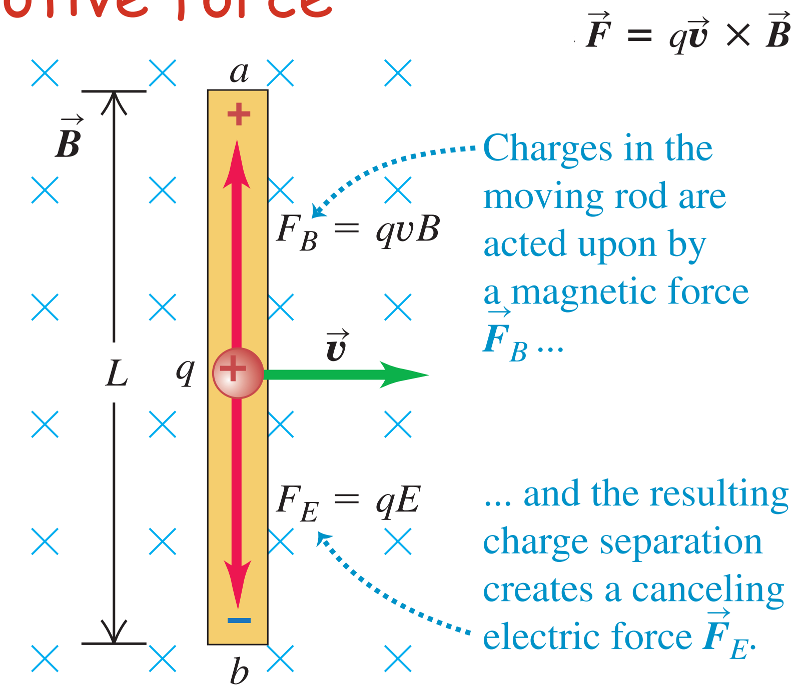


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The magnitude of the potential difference $V_{ab} = V_a - V_b$ is equal to the electric- field magnitude *E* multiplied by the length *L* of the rod. From the above discussion, $E = vB$

$$V_{ab} = EL = vBL$$

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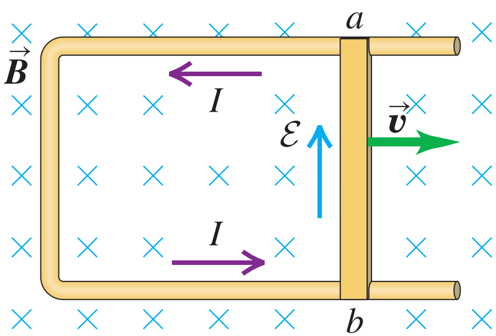


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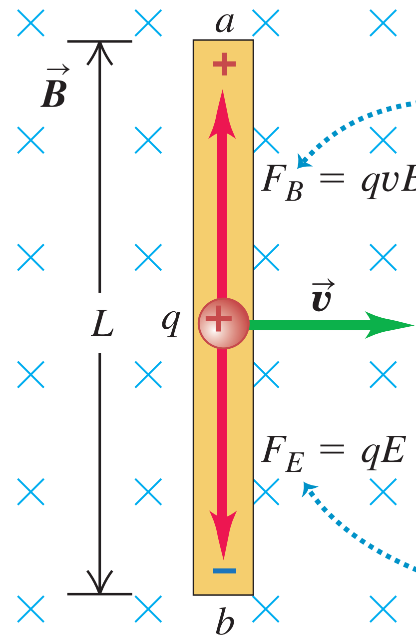


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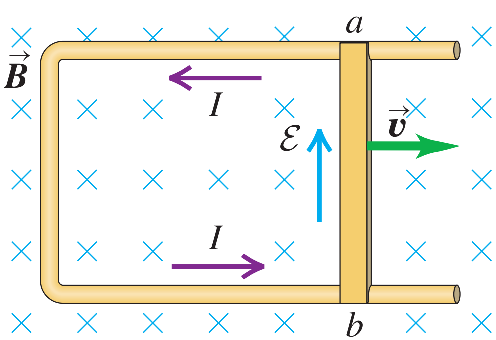


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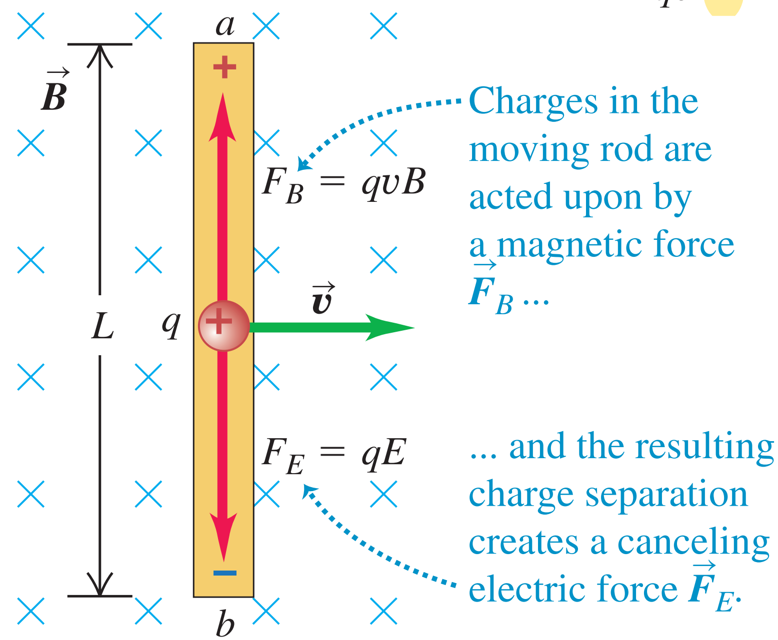


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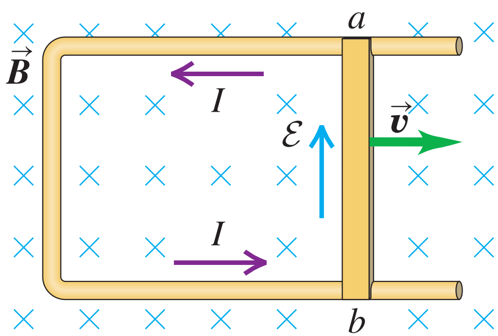


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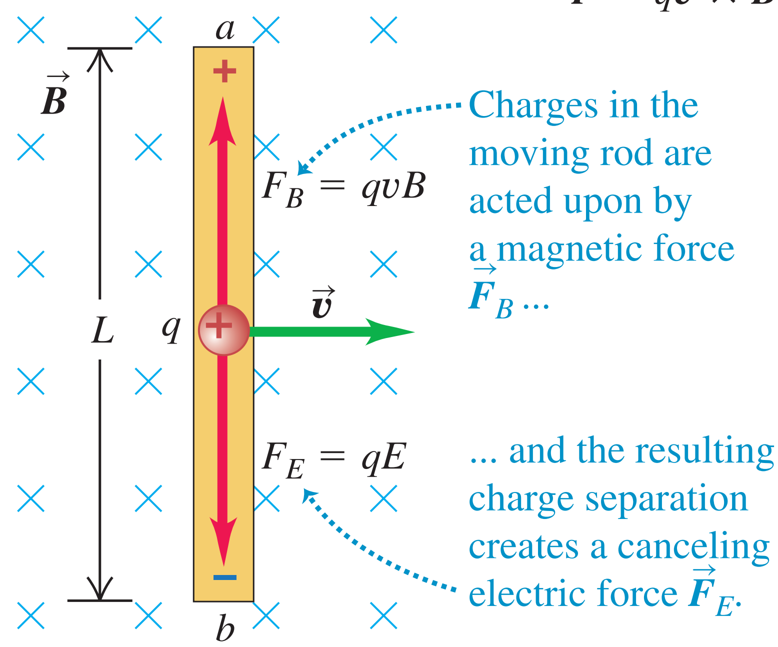


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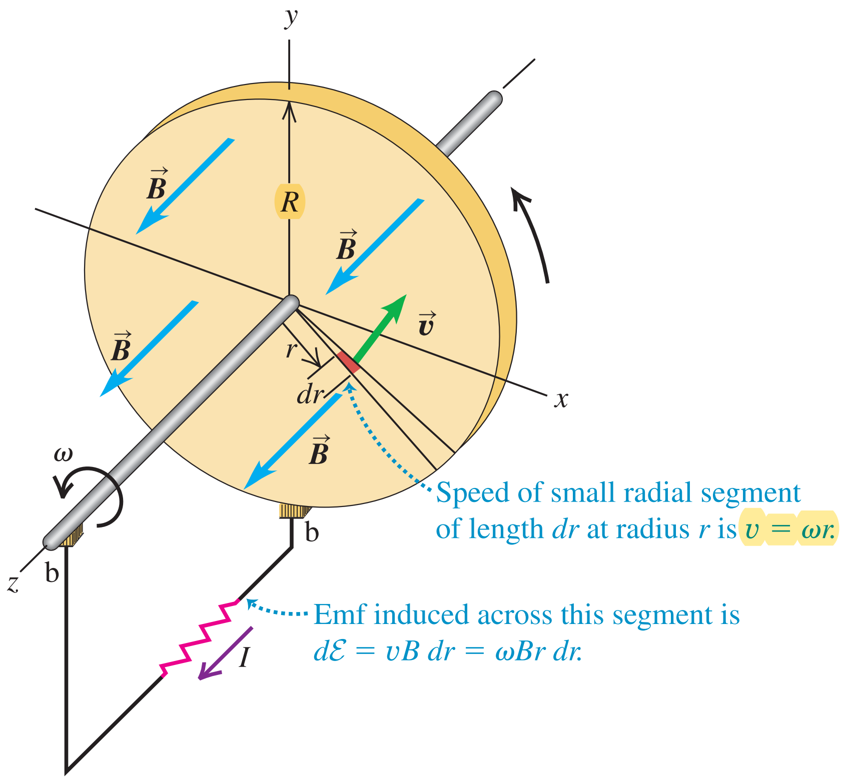
Only for moving conductors!

$$\mathcal{E} = -d\Phi_B/dt$$ (stationary conductors: Faraday's law)

# The Faraday disk dynamo

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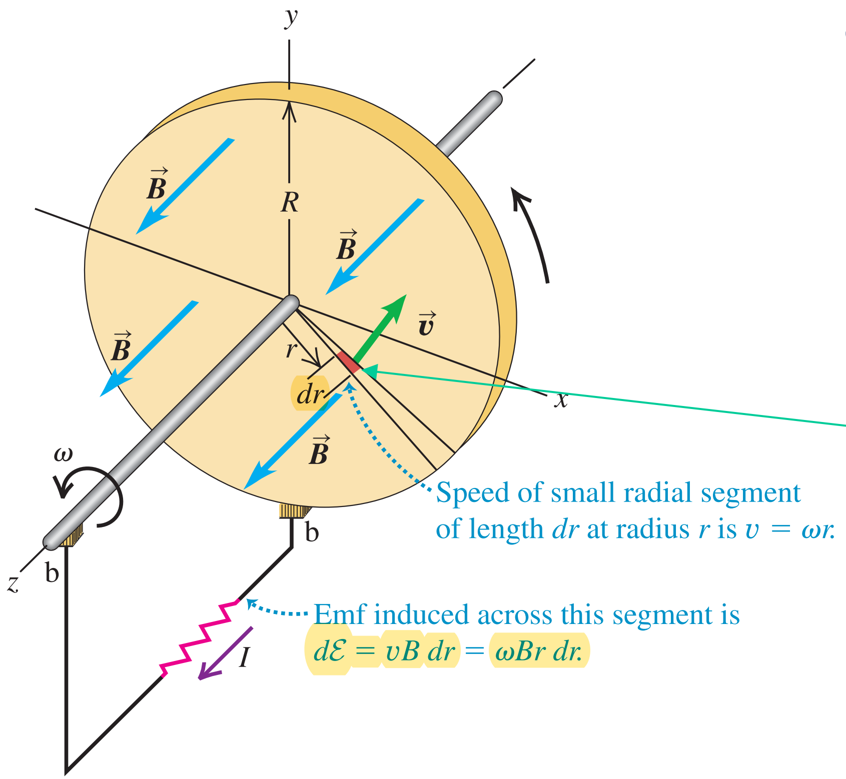


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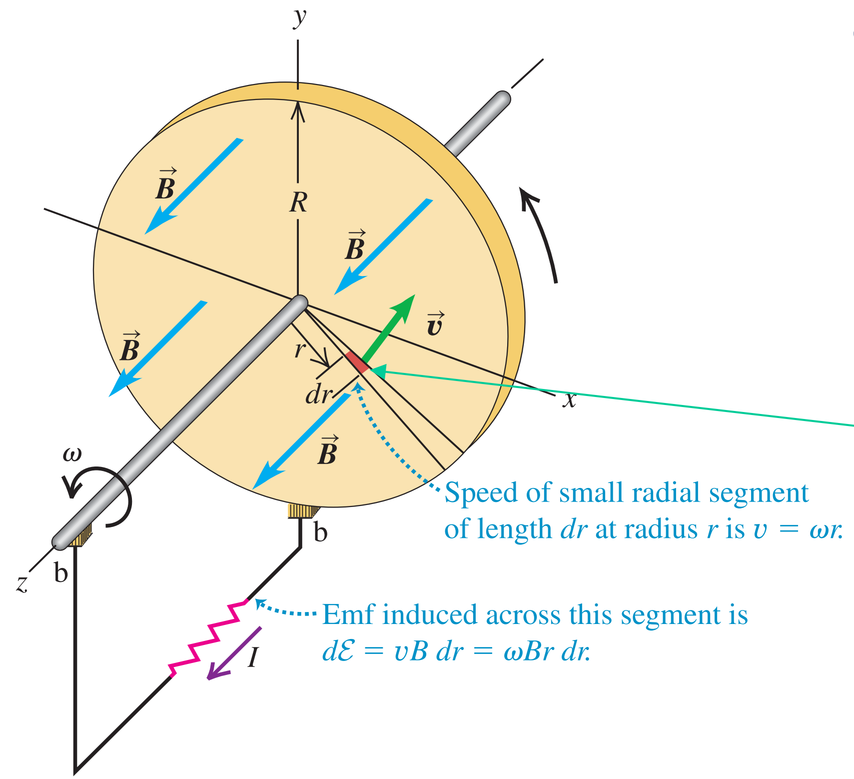


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$$\Rightarrow \quad d\mathcal{E} = \omega B r\, dr$$

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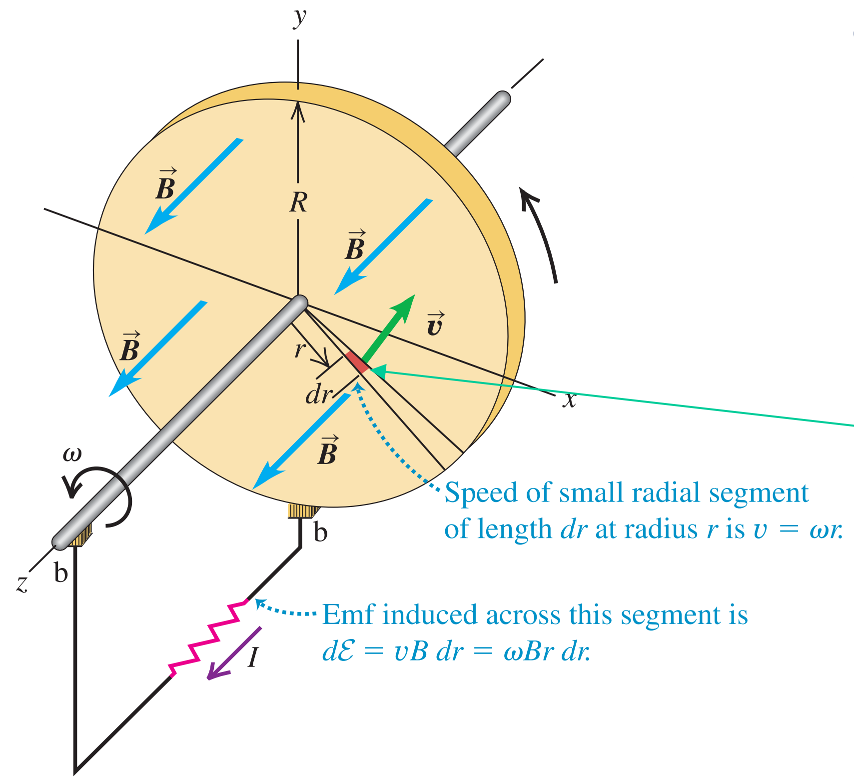


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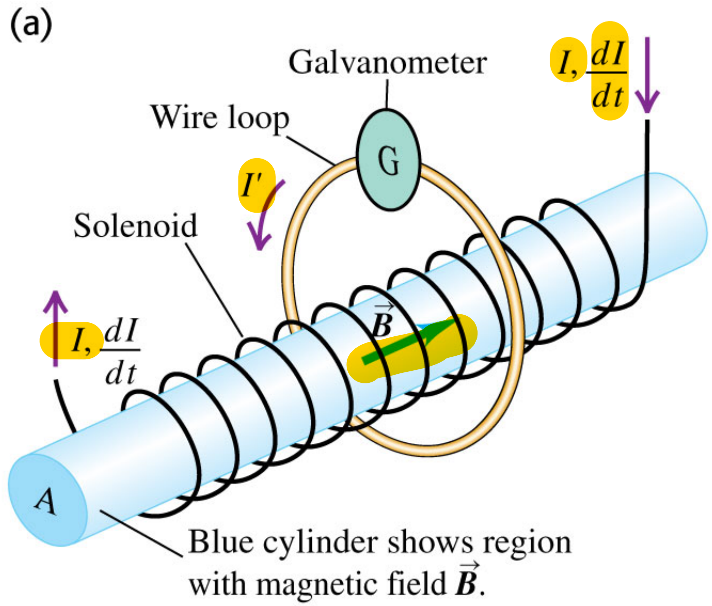
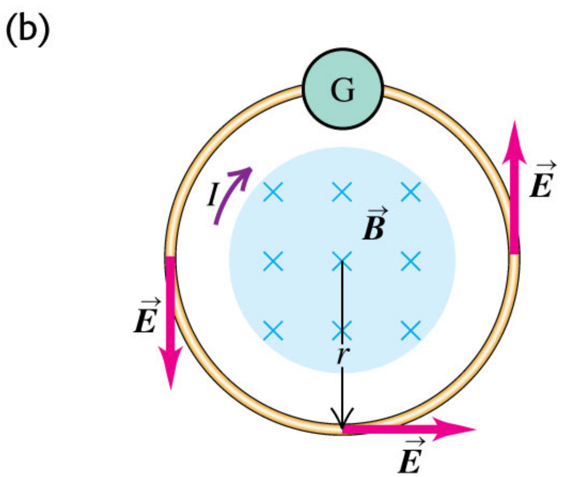
# Induced electric fields




When a conductor moves in a magnetic field, we can understand the induced emf on the basis of magnetic forces on charges in the conductor.

But an induced emf also occurs when there is a changing flux through a stationary conductor. What is it that pushes the charges around the circuit in this type of situation?

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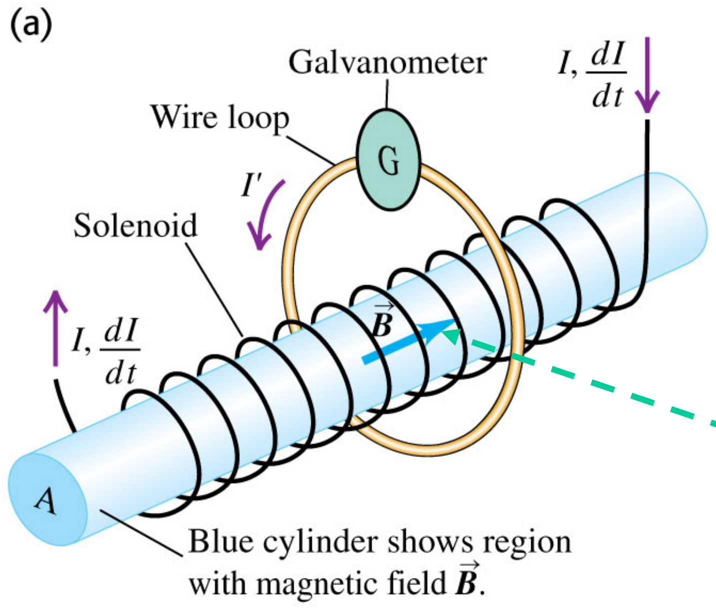


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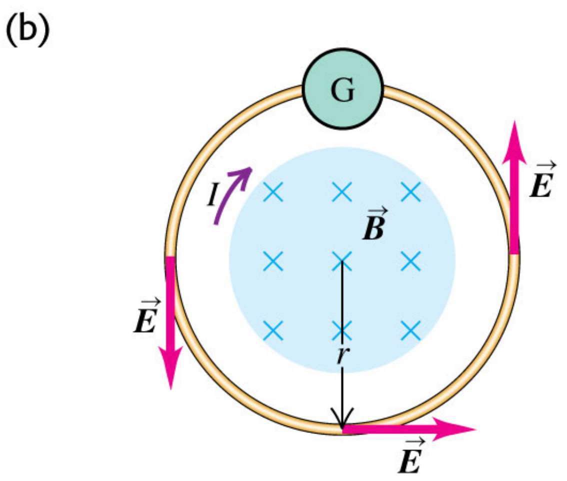

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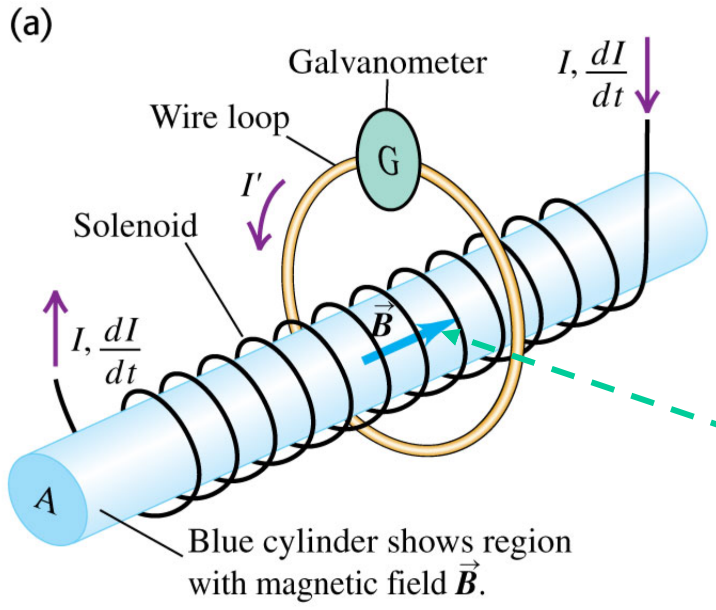


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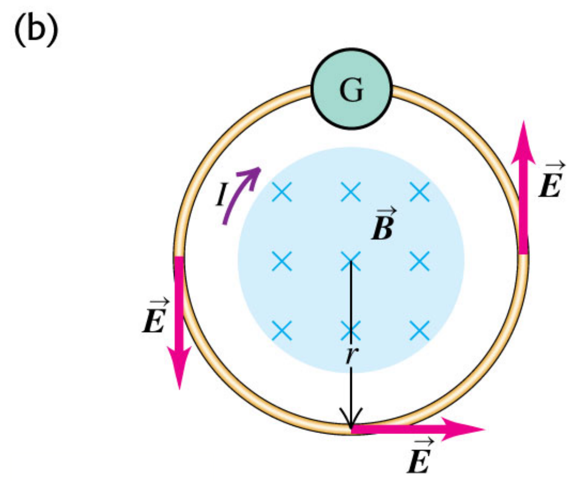


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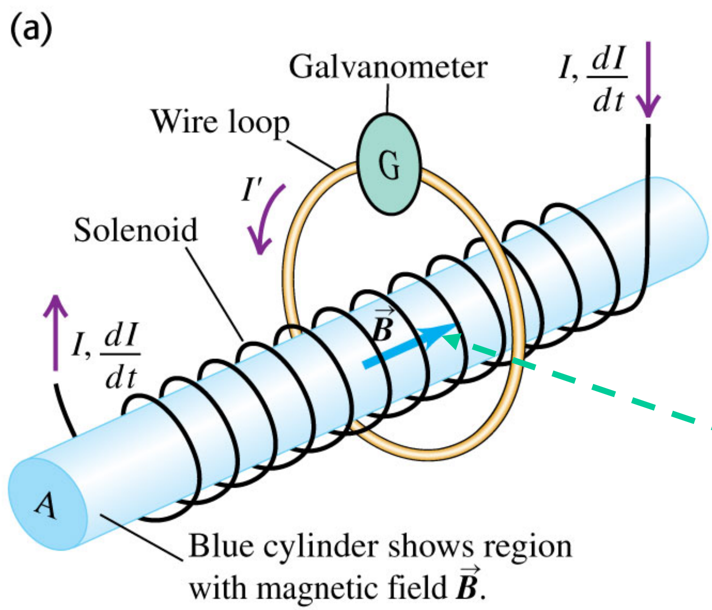


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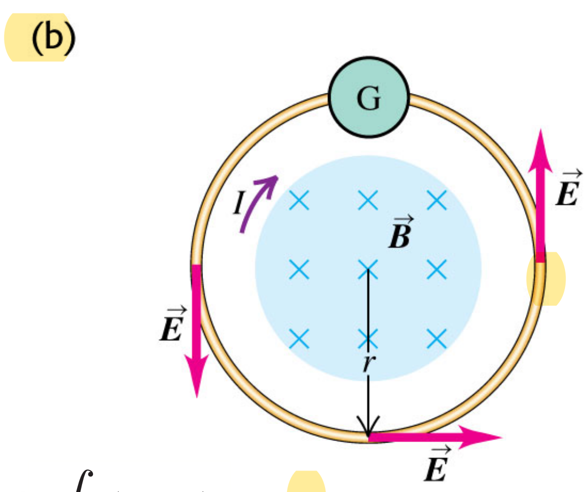


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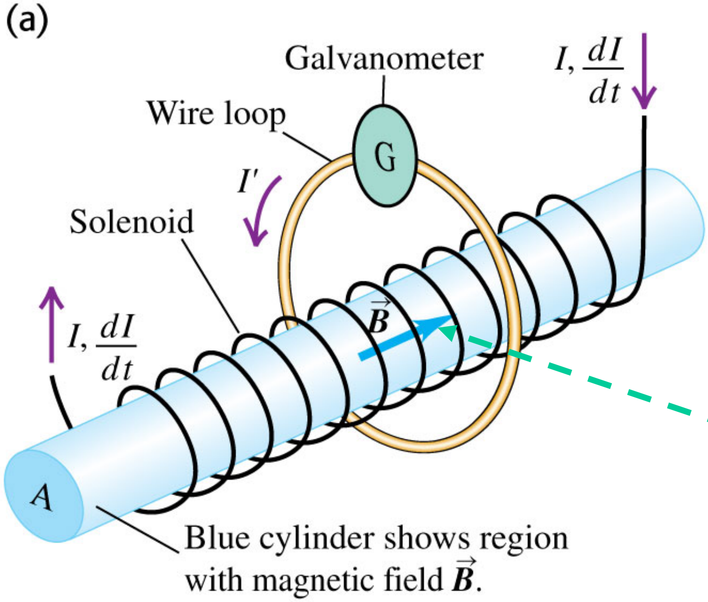
But what *force* makes the charges move around the wire loop? It can't be a magnetic force because the loop isn't even *in* a magnetic field → **induced electric field** in the conductor *caused by the changing magnetic flux.*

# Induced electric fields

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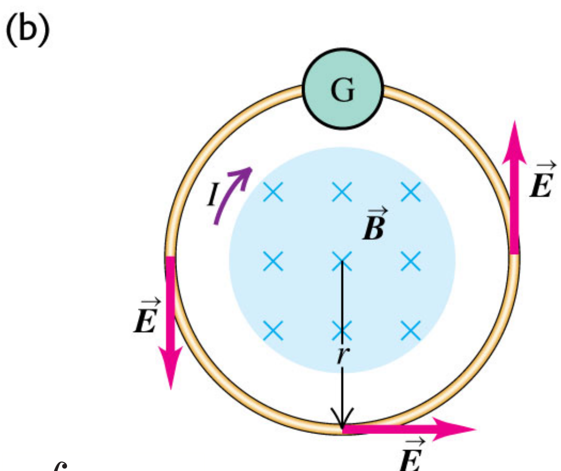


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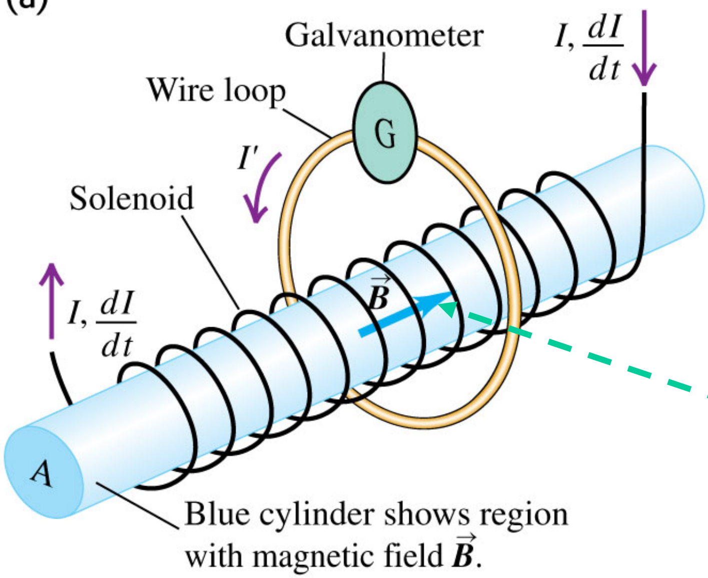


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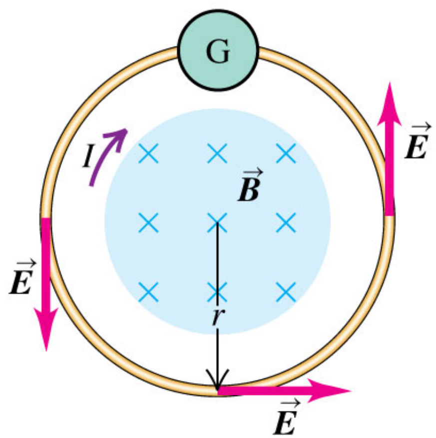


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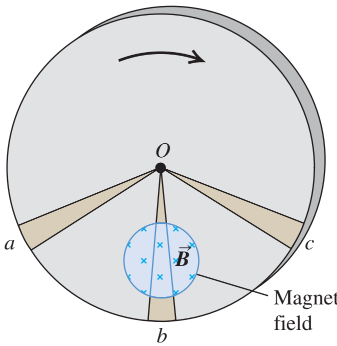
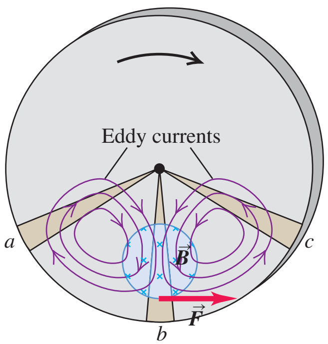
# Eddy Currents

(a) Metal disk rotating through a magnetic field




(b) Resulting eddy currents and braking force

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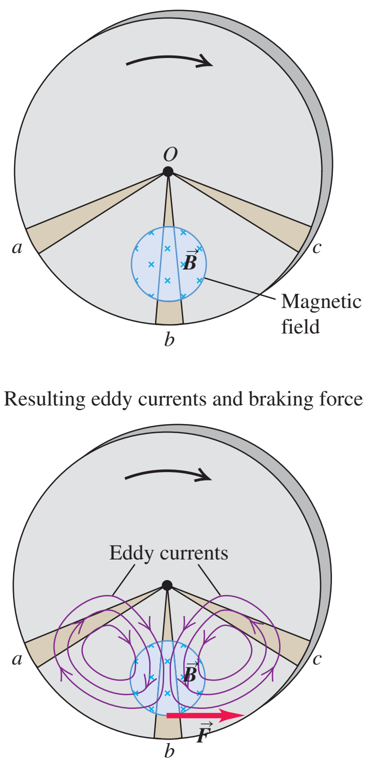


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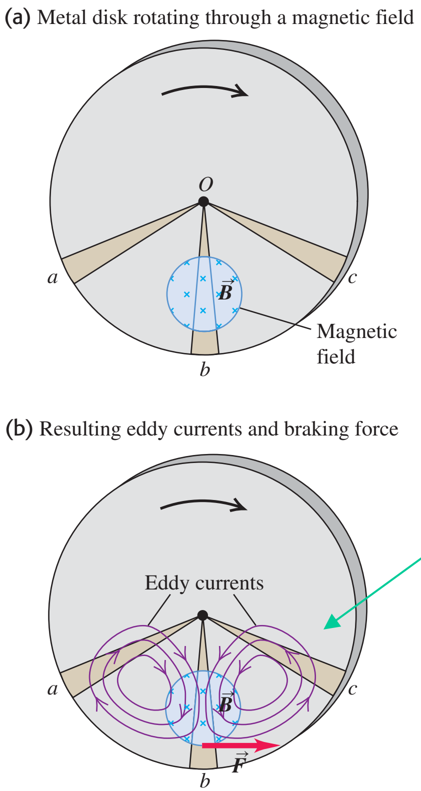


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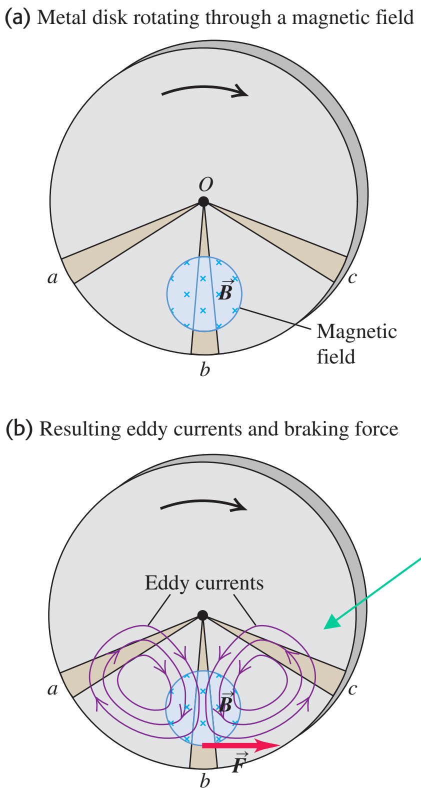


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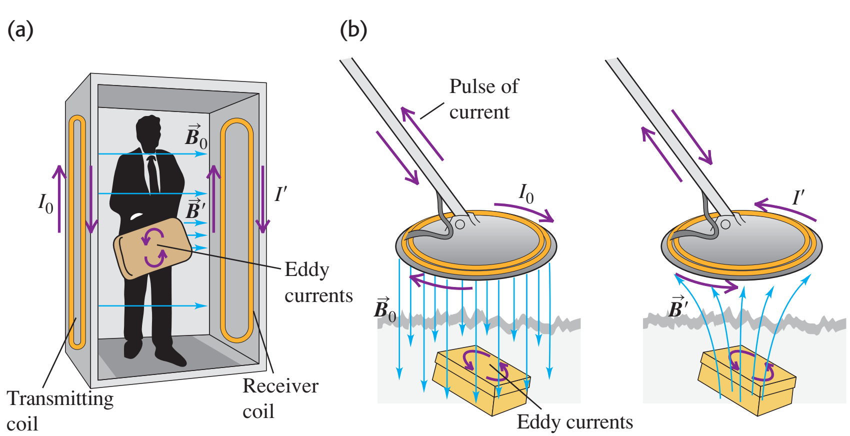
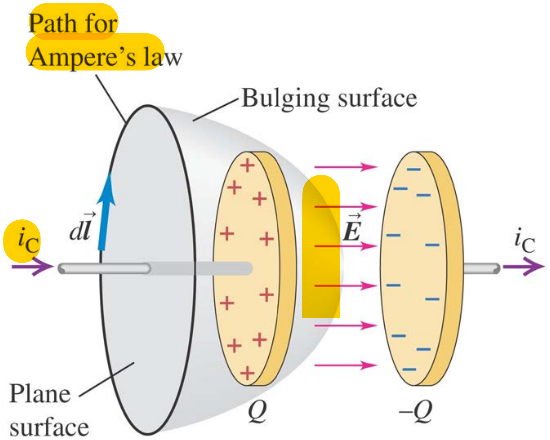
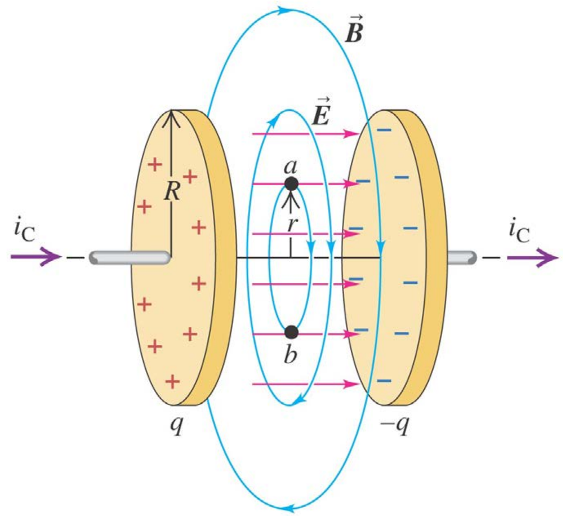



A metal detector at an airport security checkpoint generates an alternating magnetic field $B$. This induces eddy currents in a conducting object carried through the detector. The eddy currents in turn produce an alternating magnetic field $B$, and this field induces a current in the detector's receiver coil. (b) Portable metal detectors work on the same principle.

# Displacement current




Ampere's Law (incomplete):

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

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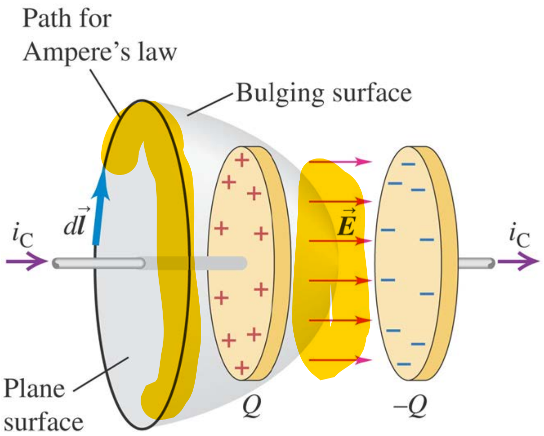


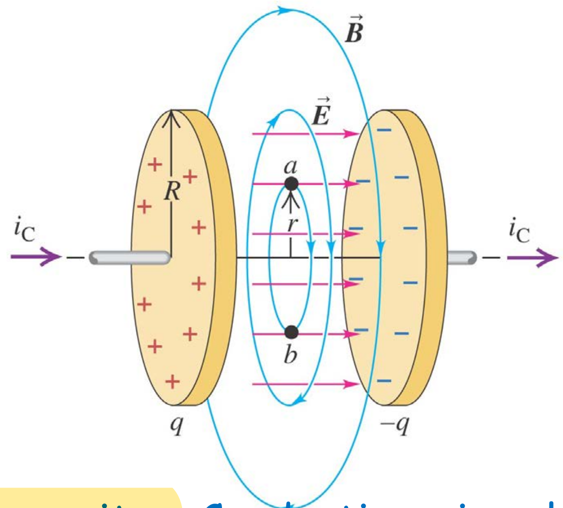


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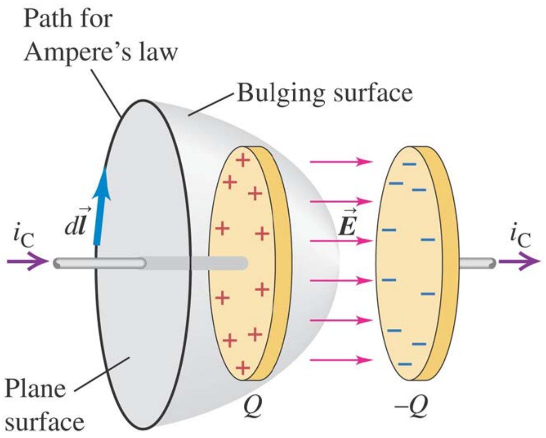


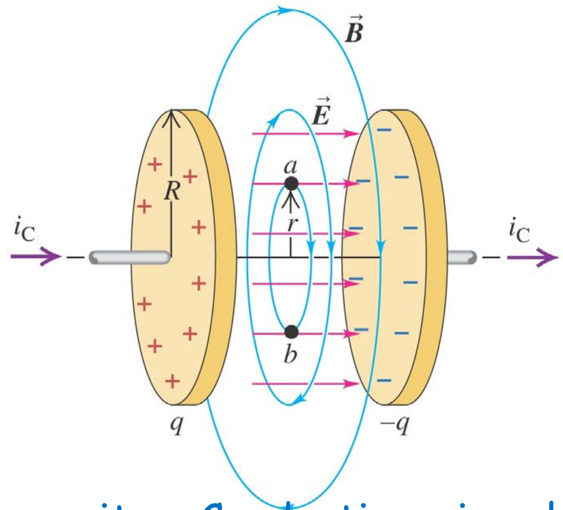


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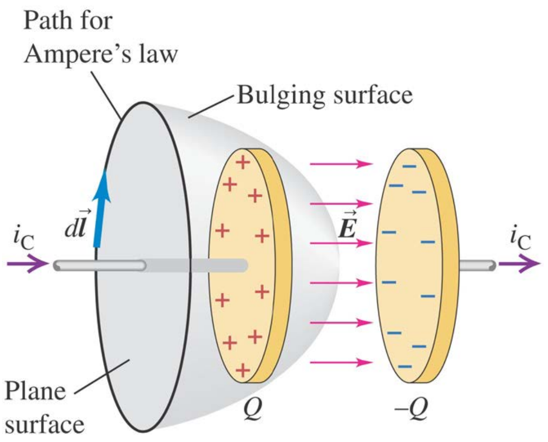


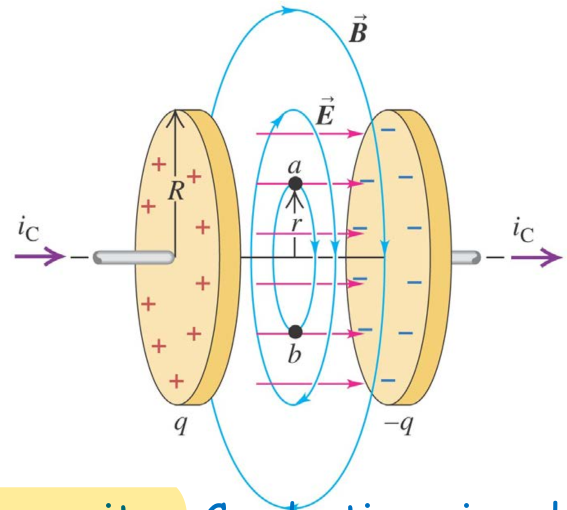


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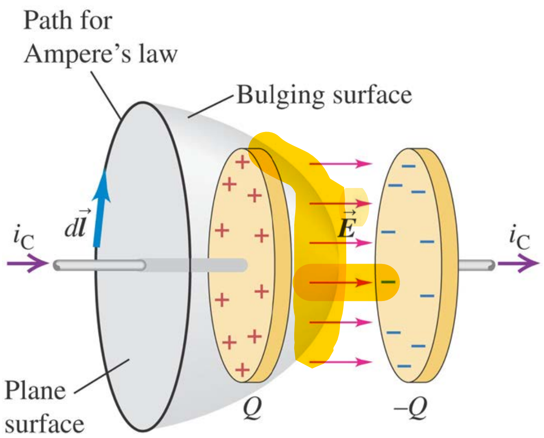


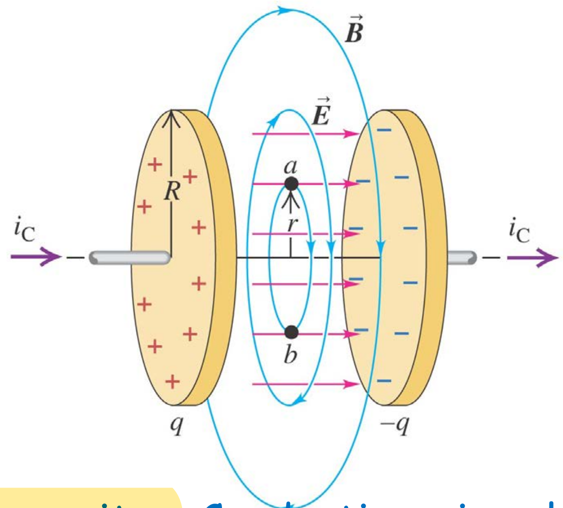


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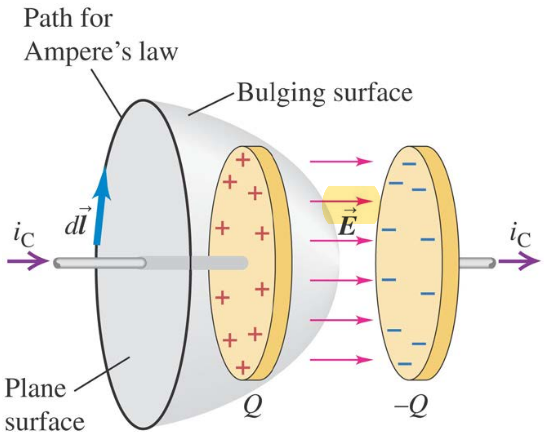


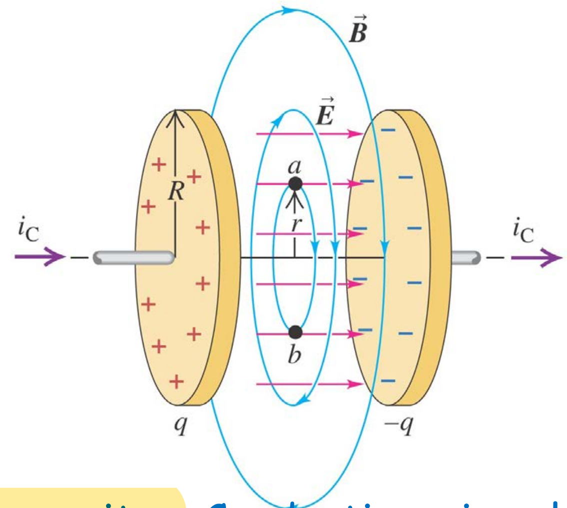


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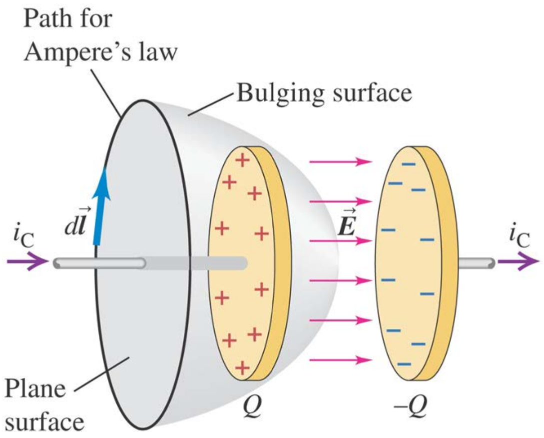


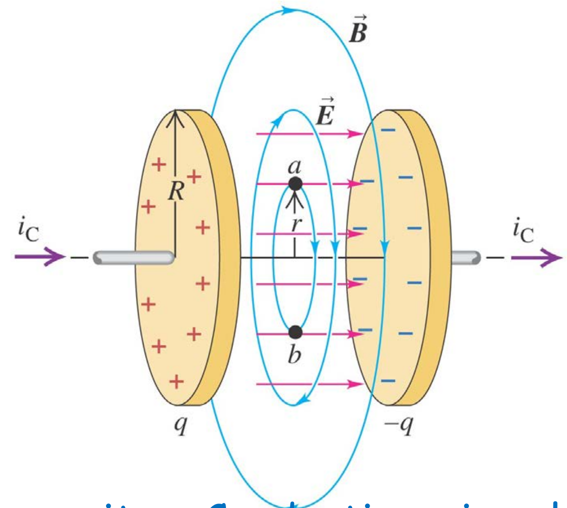


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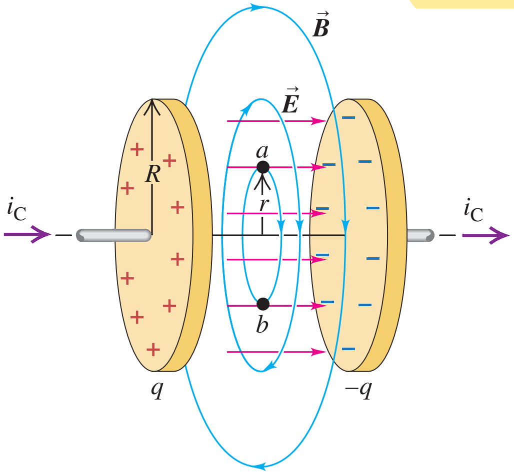
Generalized Ampere's Law

# The Reality of Displacement Current




The fictitious current $i_D$ was invented in 1865 by the Scottish physicist James Clerk Maxwell (1831–1879), who called it displacement current. There is a corresponding *displacement current density* $j_D = i_D / A$. using $\Phi_E = EA$

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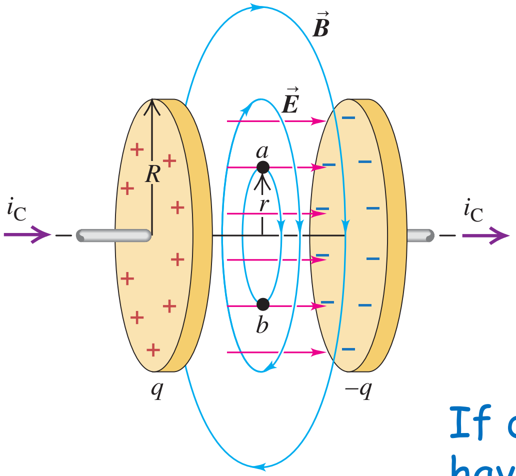


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If displacement current really plays the role in Ampere's law that we have claimed, then there ought to be a magnetic field in the region between the plates while the capacitor is charging. We can use our generalized Ampere's law, including displacement current, to predict what this field should be.

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r B = \mu_0 \frac{r^2}{R^2} i_C \quad \Rightarrow \quad B = \frac{\mu_0}{2\pi} \frac{r}{R^2} i_C$$

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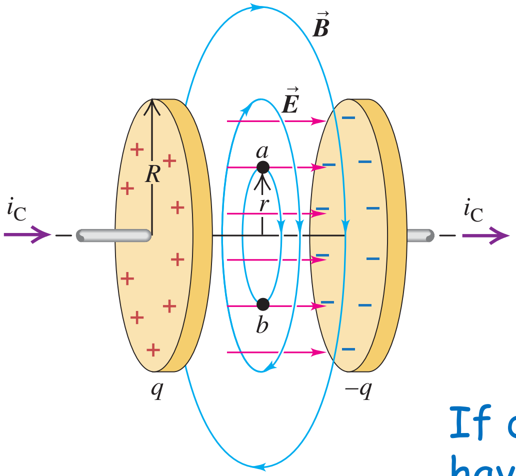


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When we *measure* the magnetic field in this region, we find that it really is there and that it behaves just as generalized Ampere law predicts. This confirms directly the role of displacement current as a source of magnetic field. It is now established beyond reasonable doubt that displacement current, far from being just an artifice, is a fundamental fact of nature. Maxwell's discovery was the bold step of an extraordinary genius.

# Maxwell's Equations of Electromagnetism

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$$\Phi_E = \int \vec{E} \cdot d\vec{A} \qquad \Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

In empty space $i_C = 0$; $Q_{encl} = 0$

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