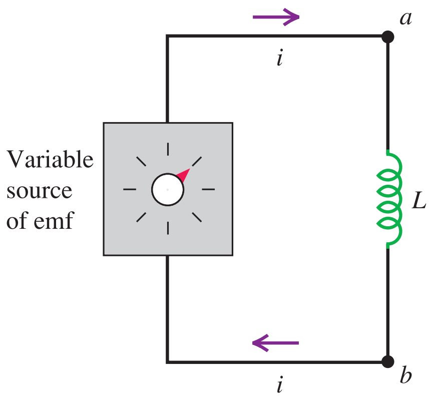
Chp 30: Inductance - (II)

Magnetic field energy



an increasing current i in the inductor causes an emf E between its terminals and a corresponding potential difference V_{ab} between the terminals of the source, with point a at higher potential than point b. Thus the source must be adding energy to the inductor, and the instantaneous power P (rate of transfer of energy into the inductor) is $P = V_{ab}i$.

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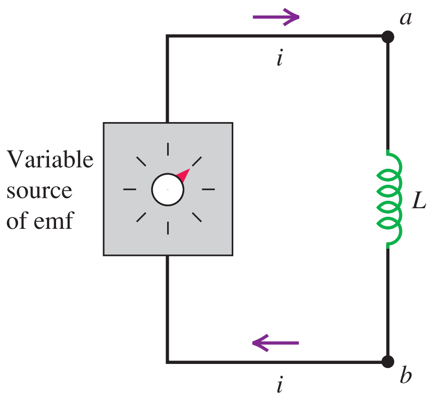

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We can calculate the total energy input U needed to establish a final current I in an inductor with inductance L if the initial current is zero. We assume that the inductor has zero resistance, so no energy is dissipated within the inductor.

$$di/dt > 0 \quad \Rightarrow \quad V_{ab} = L\, di/dt$$

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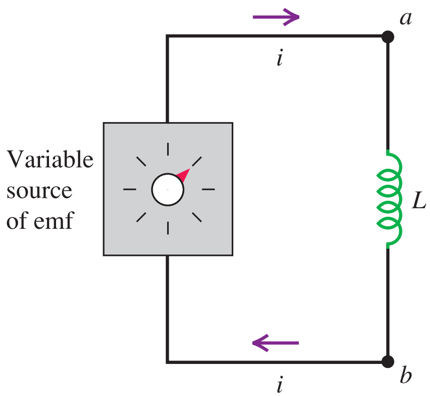

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the rate P at which energy is being delivered to the inductor (equal to the instantaneous power supplied by the external source) is $P = V_{ab}i = L\,i\,di/dt$

The energy dU supplied to the inductor during an infinitesimal time interval dt: $dU = Pdt$

$$\Rightarrow \quad dU = Li\,di$$

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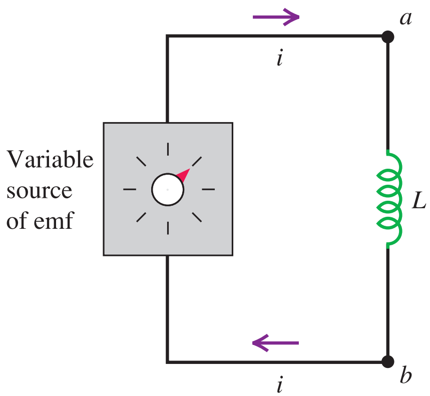

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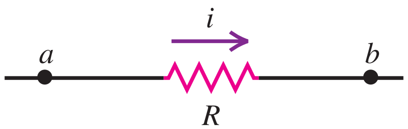
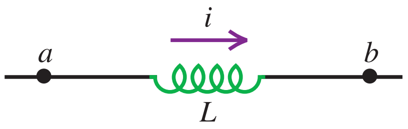
$$\Rightarrow \quad U = L\int_0^I i\, di = \tfrac{1}{2}LI^2 \qquad \text{(energy stored in an inductor)}$$

Resistor with current i: energy is *dissipated*.



Inductor with current i: energy is *stored*.



Magnetic Energy Density

The energy in an inductor is actually stored in the magnetic field within the coil, just as the energy of a capacitor is stored in the electric field between its plates. We can develop relationships for magnetic-field energy analogous to those we obtained for electric-field energy

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The energy *per unit volume,* is *u* = *U/V* ➔

$$u = \frac{U}{2\pi r A} = \tfrac{1}{2}\mu_0 \frac{N^2 I^2}{(2\pi r)^2}$$

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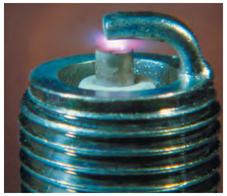
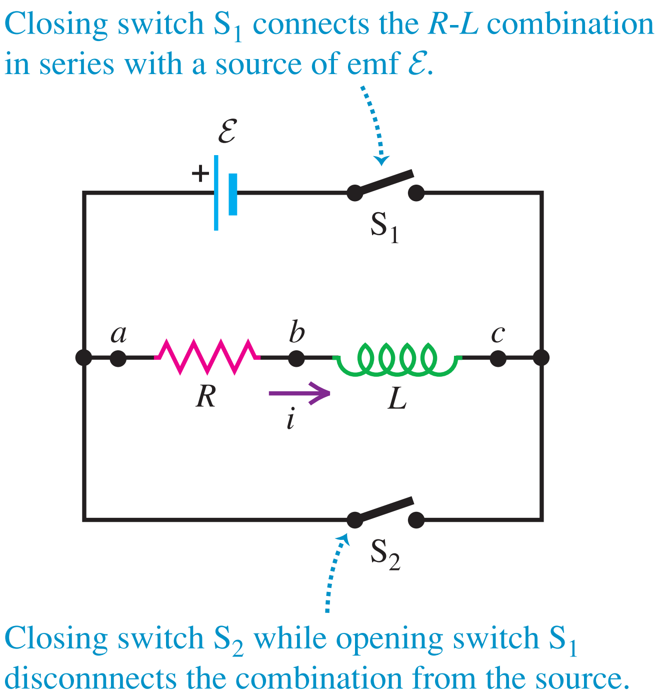


A primary coil of about 250 turns is connected to the car's battery and produces a strong magnetic field. This coil is surrounded by a secondary coil (25,000 turns) When it is time for a spark plug to fire, an emf of tens of thousands of volts is induced in the secondary coil.

The *R-L* circuit



Suppose both switches are open to begin with, and then at some initial time $t = 0$ we close switch S_1. The current cannot change suddenly from zero to some final value, since di/dt and the induced emf in the inductor would both be infinite. Instead, the current begins to grow at a rate that depends only on the value of L in the circuit.

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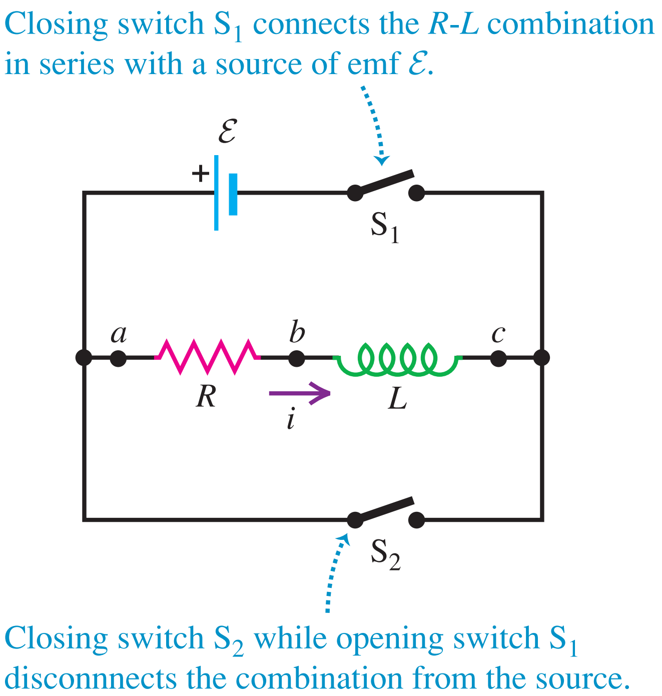

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The potential difference across the resistor and inductor at that time is

$$v_{ab} = iR \qquad v_{bc} = L\frac{di}{dt}$$

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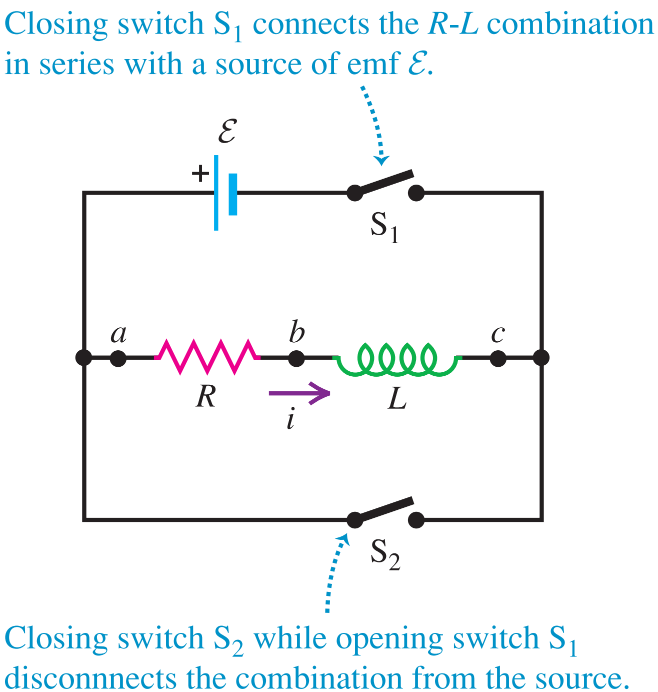

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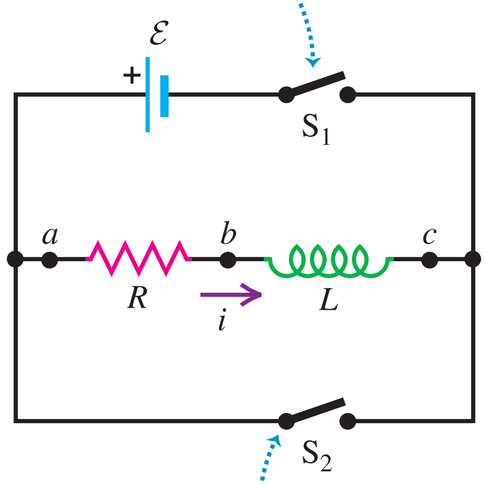

Closing switch S_2 while opening switch S_1 disconnects the combination from the source.

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$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$ ➡ At the instant that switch S_1 is first closed, $i = 0$ ➔ and the potential drop across *R* is zero ➡ $$\left(\frac{di}{dt}\right)_{\text{initial}} = \frac{\mathcal{E}}{L}$$

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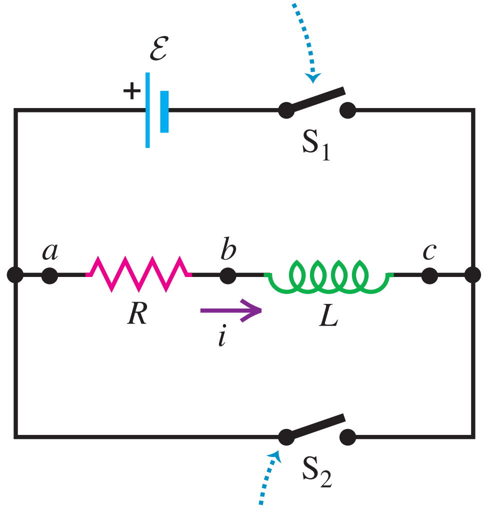

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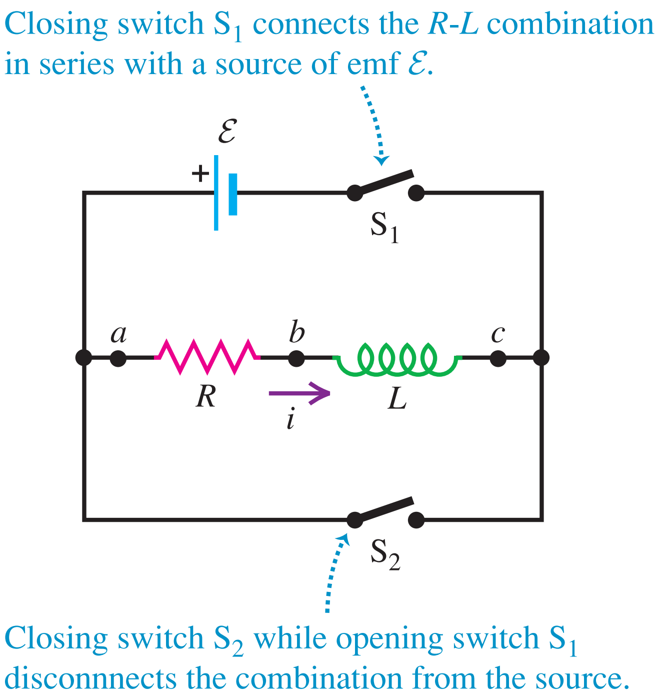
$$\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{\mathcal{E}}{L} - \frac{R}{L}i$$

As the current increases, the term $(R/L)i$ in increases, and the *rate* of increase of becomes smaller and smaller

➔the current is approaching a final, steady-state value *I*.

The R-L circuit



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The potential difference across the resistor and inductor at that time is

$$v_{ab} = iR \qquad v_{bc} = L\frac{di}{dt}$$

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$ ➡

At the instant that switch S_1 is first closed, $i = 0$ ➡ $\left(\frac{di}{dt}\right)_{\text{initial}} = \frac{\mathcal{E}}{L}$

➔ and the potential drop across R is zero

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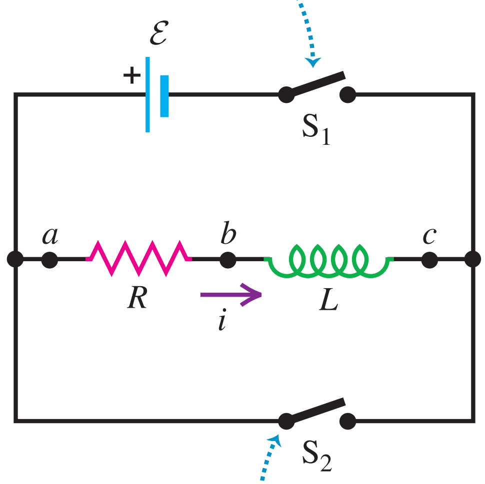
➡ $$\left(\frac{di}{dt}\right)_{\text{final}} = 0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \quad \Leftrightarrow \quad I = \frac{\mathcal{E}}{R}$$

The R-L circuit

Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.



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➔ and the potential drop across R is zero

$$\left(\frac{di}{dt}\right)_{\text{initial}} = \frac{\mathcal{E}}{L}$$

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$$\left(\frac{di}{dt}\right)_{\text{final}} = 0 = \frac{\mathcal{E}}{L} - \frac{R}{L}I \iff I = \frac{\mathcal{E}}{R}$$

$$\frac{di}{i - (\mathcal{E}/R)} = -\frac{R}{L}dt$$

$$\int_0^i \frac{di'}{i' - (\mathcal{E}/R)} = -\int_0^t \frac{R}{L}dt'$$

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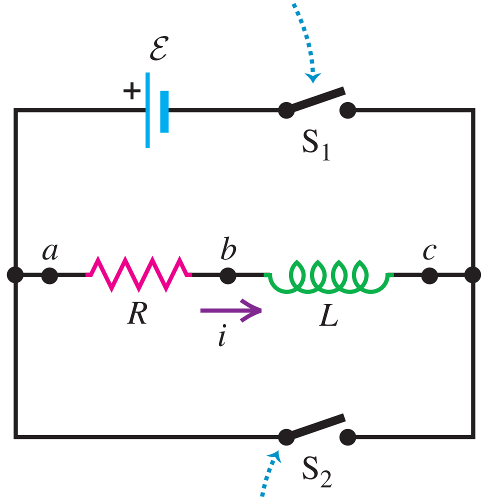

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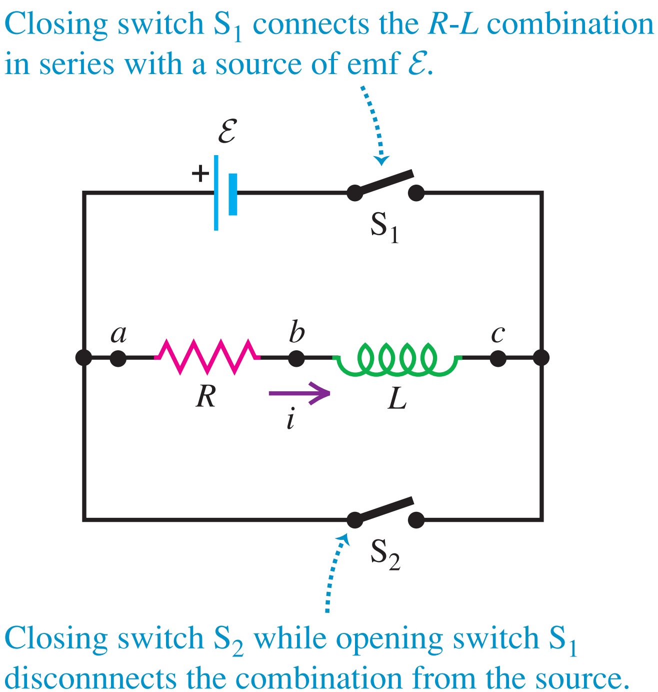
$$\int_0^i \frac{di'}{i' - (\mathcal{E}/R)} = -\int_0^t \frac{R}{L}dt' \quad ➡ \quad \ln\left(\frac{i - (\mathcal{E}/R)}{-\mathcal{E}/R}\right) = -\frac{R}{L}t$$

The *R-L* circuit



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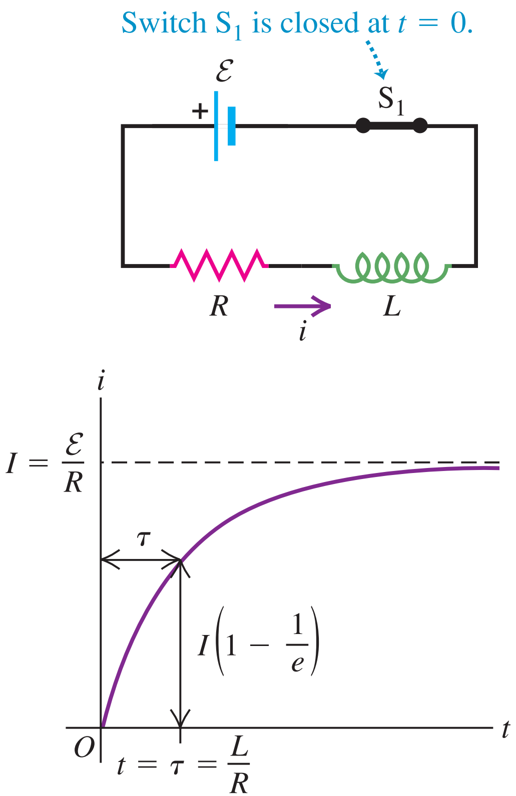

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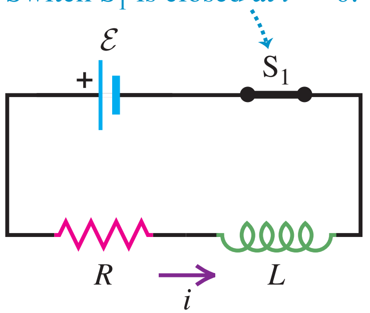
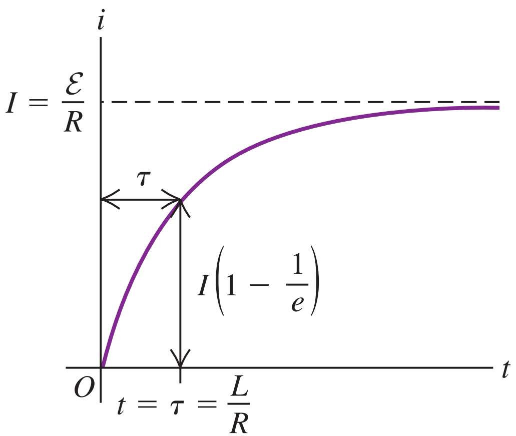
current in an *R-L* circuit with emf

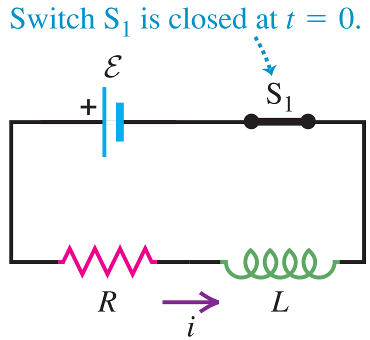


$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$$ current in an *R*-*L* circuit with emf

$$\Rightarrow \quad \frac{di}{dt} = \frac{\mathcal{E}}{L} e^{-(R/L)t}$$

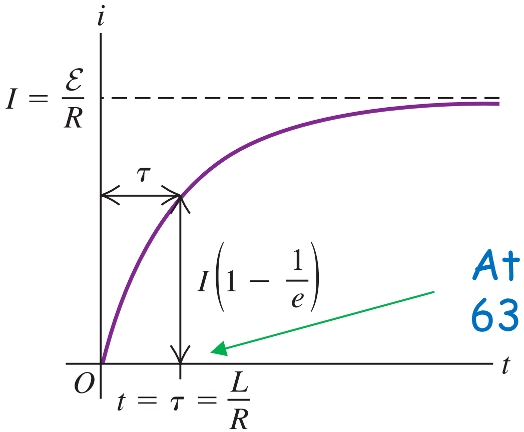


At time $t = 0$, $I = 0$ and $di/dt = \mathcal{E}/L$. As $t \to \infty$, $i \to \mathcal{E}/L$ and $idt \to 0$,



$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$$ current in an R-L circuit with emf

$$\Rightarrow \quad \frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-(R/L)t}$$



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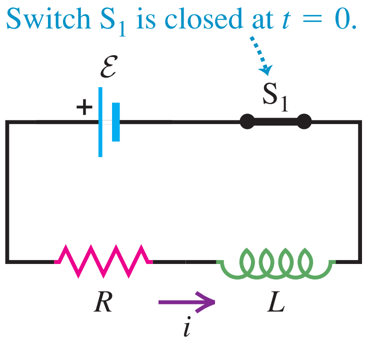
At a time equal to L/R, the current has risen to $(1 - 1/e)$, or about 63%, of its final value



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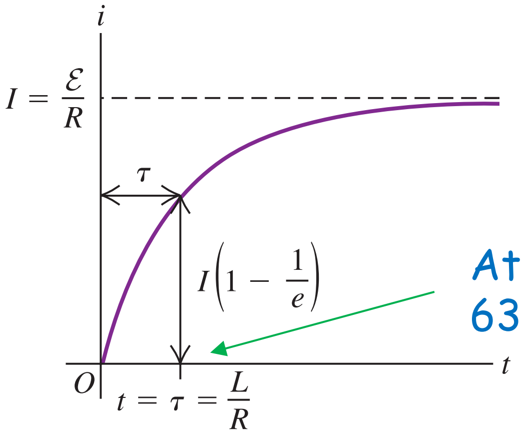

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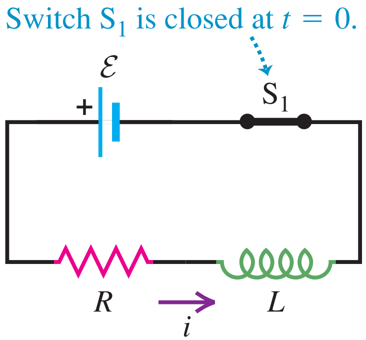
For example, if $R = 100\ \Omega$ and $L = 10$ H $\quad \tau = \frac{L}{R} = \frac{10\text{ H}}{100\ \Omega} = 0.10\text{ s}$



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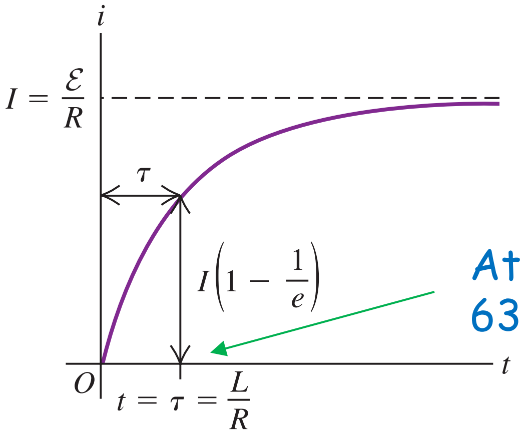

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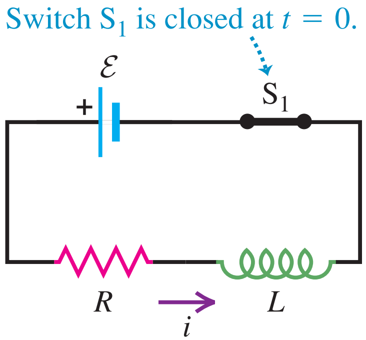
Meaning the current increases to about 63% of its final value in 0.10 s.
(Recall that 1 H = 1 Ω s.) If $L = 0.010$ H, $t = 10^{-4}$ s = 0.10 ms, and the rise is much more rapid.



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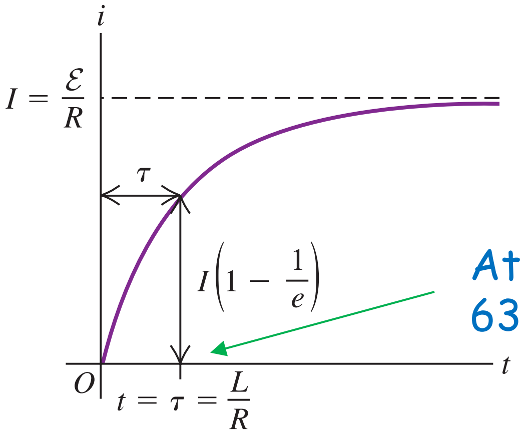

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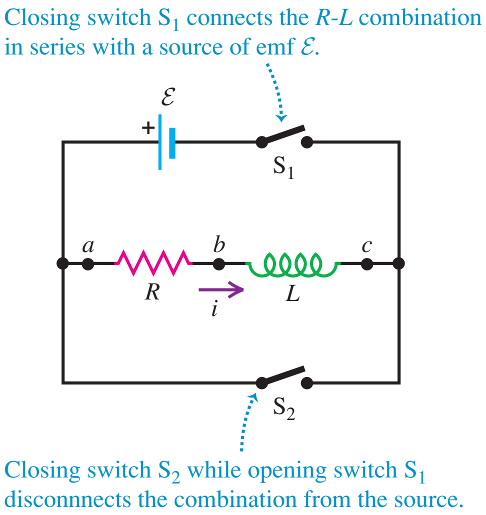
The instantaneous rate at which the source delivers energy to the circuit is $P = \varepsilon i$. The instantaneous rate at which energy is dissipated in the resistor is i^2R, and the rate at which energy is stored in the inductor is $iv_{bc} = Li\,di/dt$

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \quad \Rightarrow \quad \mathcal{E}i = i^2R + Li\frac{di}{dt}$$

Analyzing an R-L circuit



A sensitive electronic device of resistance $R = 175\ \Omega$ is to be connected to a source of emf (of negligible internal resistance) by a switch. The device is designed to operate with a 36-mA current, but to avoid damage to the device, the current can rise to no more than 4.9 mA in the first 58 μs after the switch is closed. An inductor is therefore connected in series with the device, as in figure the switch in question is S_1.

(a) What is the required source emf $\mathcal{E}$? (b)

(b) What is the required inductance L? (c) What is time constant τ?

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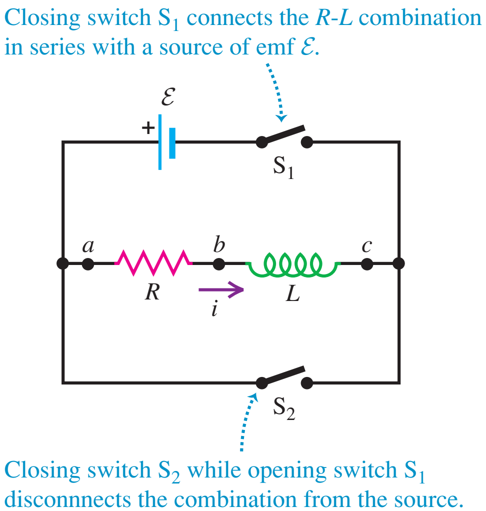

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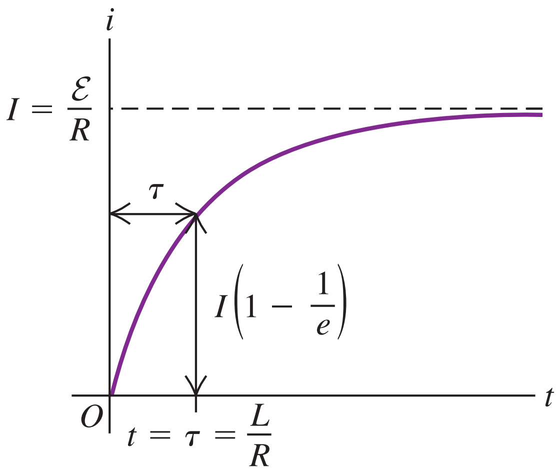

(a) Final current $I = \mathcal{E}/R$ ➔ $\mathcal{E} = IR = (0.036\ \mathrm{A})(175\ \Omega) = 6.3\ \mathrm{V}$

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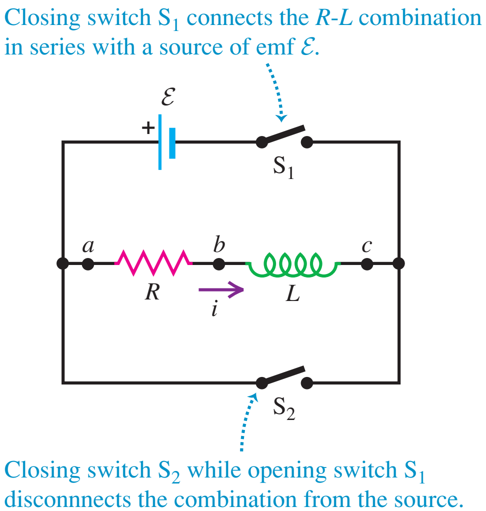
Analyzing an R-L circuit



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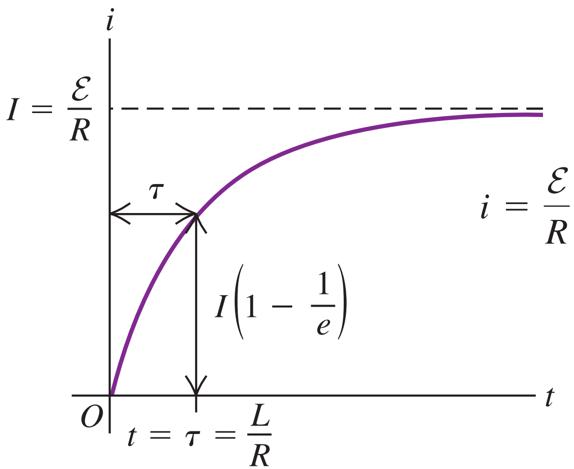

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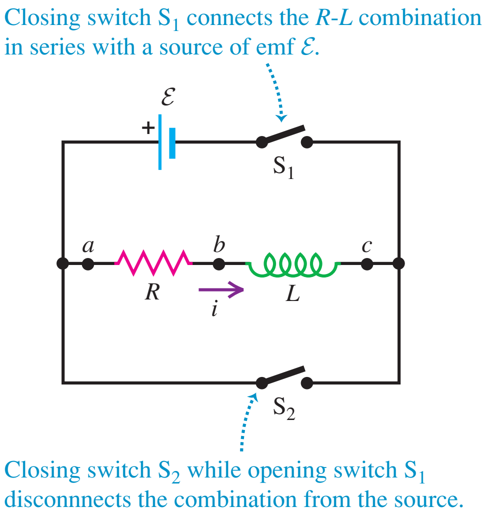
Analyzing an R-L circuit



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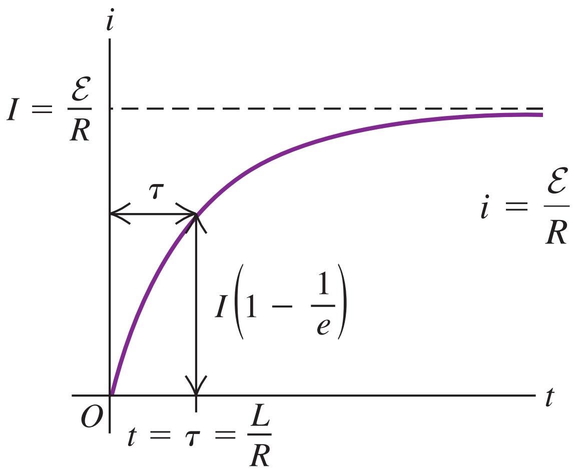

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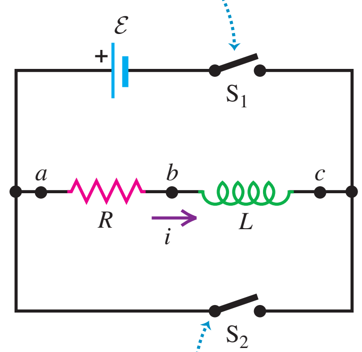
(b) $$L = \frac{-(175\ \Omega)(58 \times 10^{-6}\ \text{s})}{\ln[1 - (4.9 \times 10^{-3}\ \text{A})(175\ \Omega)/(6.3\ \text{V})]} = 69\ \text{mH}$$

Analyzing an R-L circuit

Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.



Closing switch S_2 while opening switch S_1 disconnects the combination from the source.

A sensitive electronic device of resistance $R = 175\ \Omega$ is to be connected to a source of emf (of negligible internal resistance) by a switch. The device is designed to operate with a 36-mA current, but to avoid damage to the device, the current can rise to no more than 4.9 mA in the first 58 μs after the switch is closed. An inductor is therefore connected in series with the device, as in figure the switch in question is S_1.

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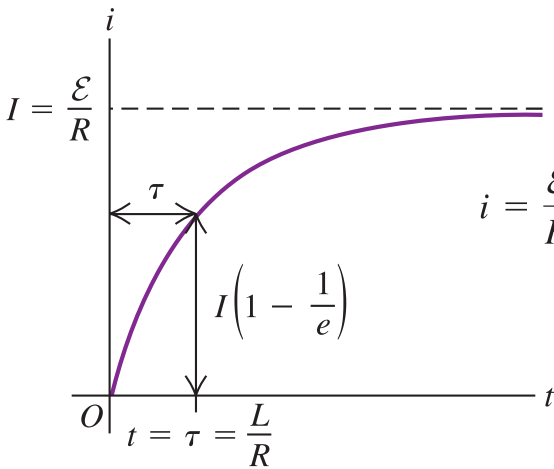

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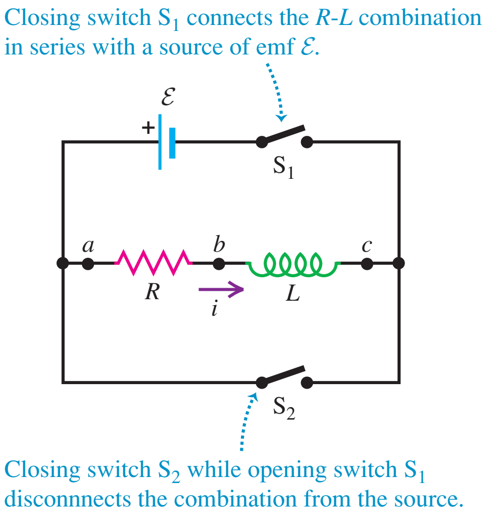
(c)
$$\tau = \frac{L}{R} = \frac{69 \times 10^{-3}\ \text{H}}{175\ \Omega} = 3.9 \times 10^{-4}\ \text{s} = 390\ \mu\text{s}$$

Analyzing an R-L circuit



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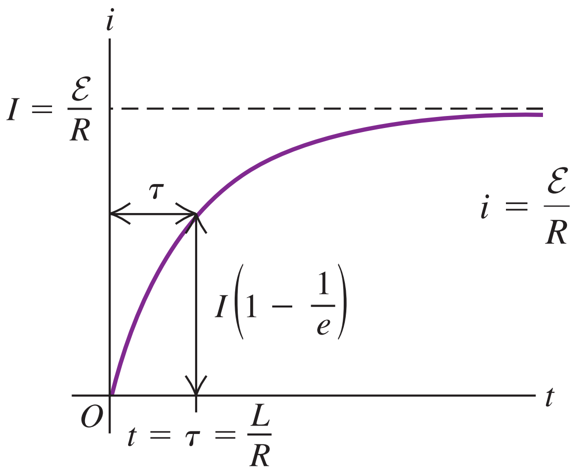

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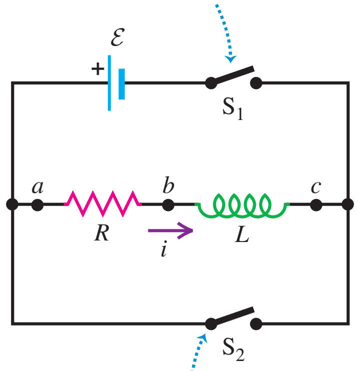
$$\tau = \frac{L}{R} = \frac{69 \times 10^{-3}\ \text{H}}{175\ \Omega} = 3.9 \times 10^{-4}\ \text{s} = 390\ \mu\text{s}$$

Note that 58 μs is much less than the time constant (390 μs). In 58 μs the current builds up from zero to 4.9 mA, a small fraction of its final value of 36 mA; after 390 μs the current equals (1 – 1/*e*) of its final value, or about (0.63)(36 mA) = 23 mA.

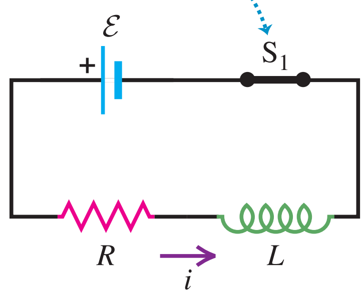
Current Decay in an R-L Circuit

Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.

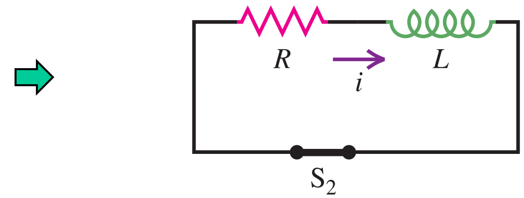


Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Switch S_1 is closed at $t = 0$.

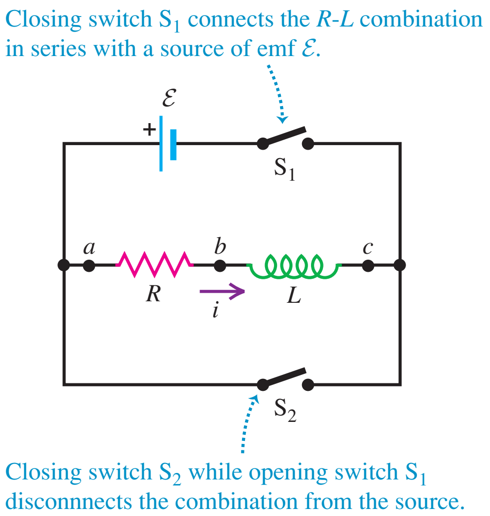


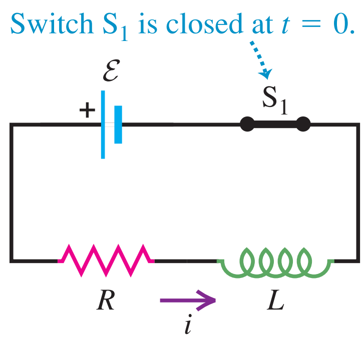
charging



decaying

Current Decay in an R-L Circuit





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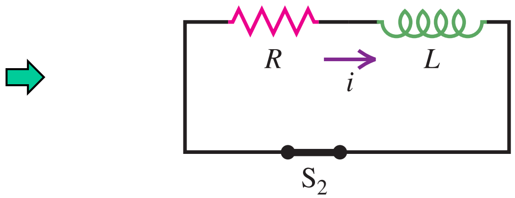

decaying

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

Current Decay in an R-L Circuit

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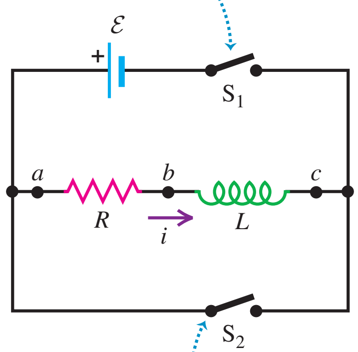
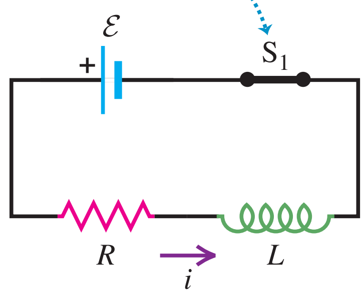


Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Switch S_1 is closed at $t = 0$.



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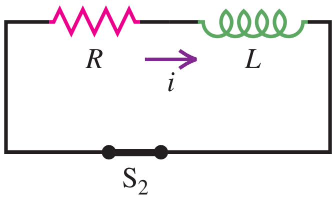

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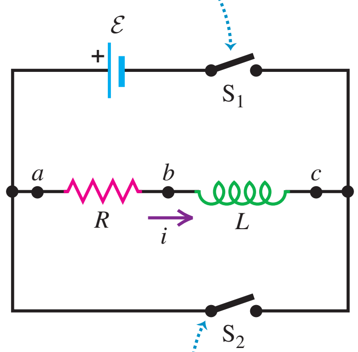

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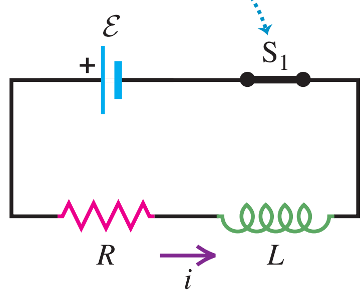

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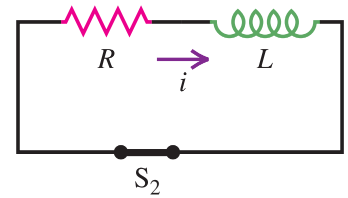
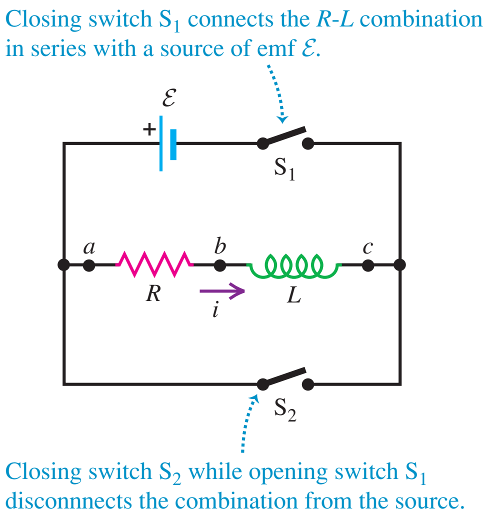


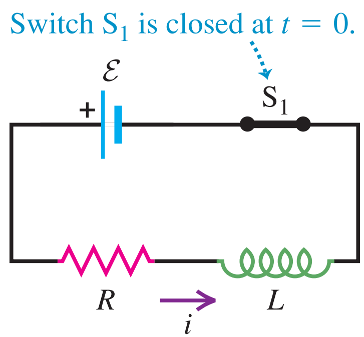
decaying

$$-iR - L\frac{di}{dt} = 0$$

$$i = I_0 e^{-(R/L)t}$$

Current Decay in an R-L Circuit





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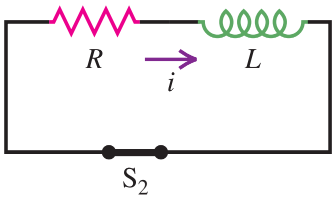

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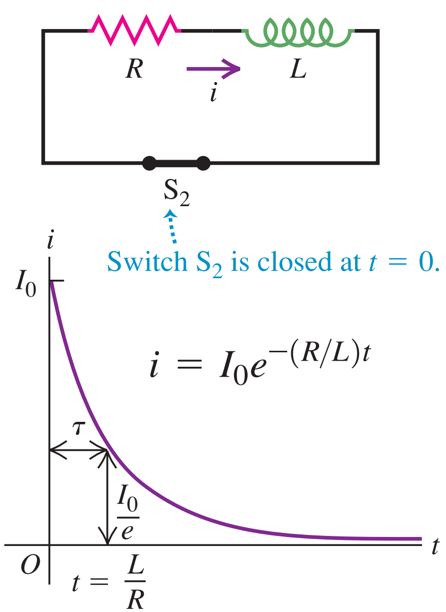
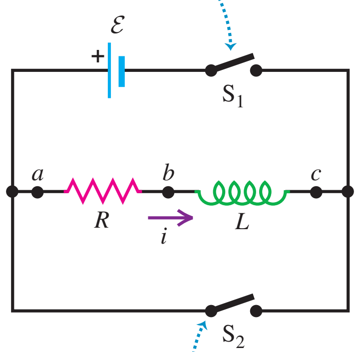


I_0 is the initial current at time $t = 0$

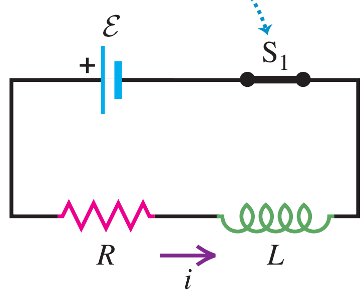
Current Decay in an R-L Circuit

Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.



Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Switch S_1 is closed at $t = 0$.



charging

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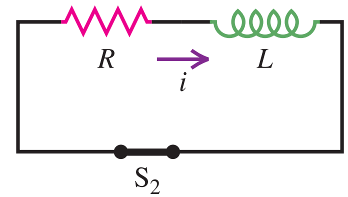

decaying

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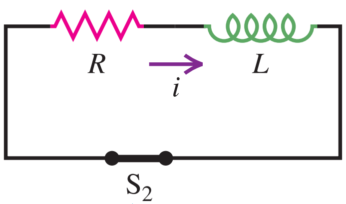
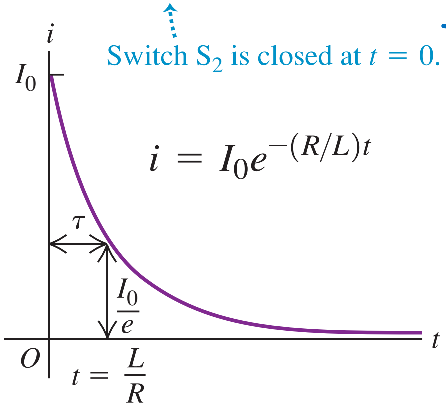


Switch S_2 is closed at $t = 0$.

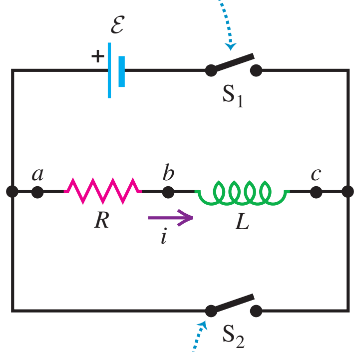


I_0 is the initial current at time $t = 0$

The time constant, $\tau = L/R$ is the time for current to decrease to $1/e$ or about 37%, of its original value

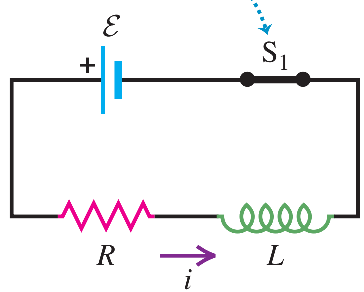
Current Decay in an R-L Circuit

Closing switch S_1 connects the R-L combination in series with a source of emf $\mathcal{E}$.



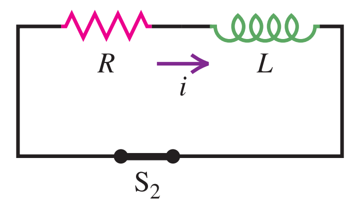
Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Switch S_1 is closed at $t = 0$.



charging

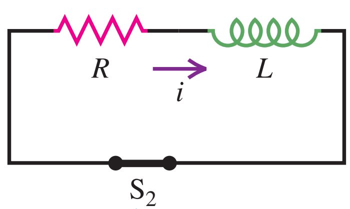
$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$



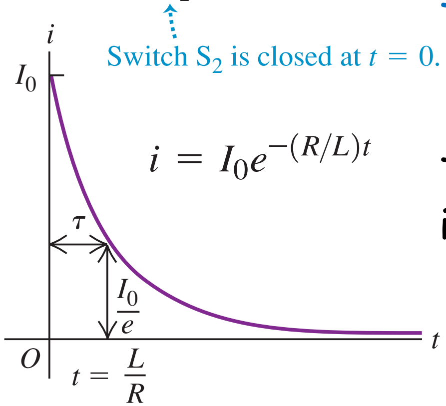
decaying

$$-iR - L\frac{di}{dt} = 0$$

$$i = I_0 e^{-(R/L)t}$$



Switch S_2 is closed at $t = 0$.



I_0 is the initial current at time $t = 0$

The time constant, $\tau = L/R$ is the time for current to decrease to $1/e$ or about 37%, of its original value

The energy that is needed to maintain the current during this decay is provided by the energy stored in the magnetic field of the inductor

$$0 = i^2 R + Li\frac{di}{dt}$$

Current Decay in an R-L Circuit

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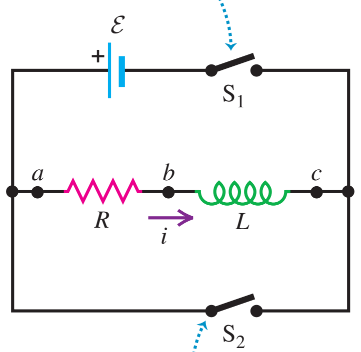
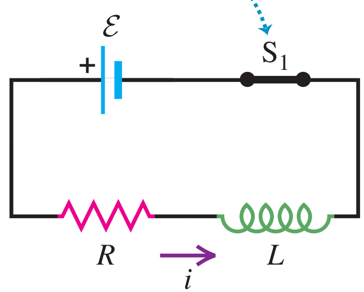


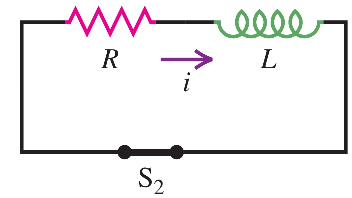
Closing switch S_2 while opening switch S_1 disconnnects the combination from the source.

Switch S_1 is closed at $t = 0$.



charging

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$



decaying

$$-iR - L\frac{di}{dt} = 0$$

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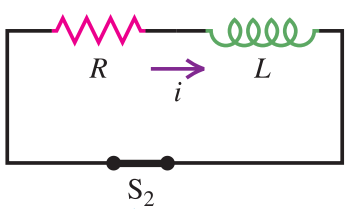
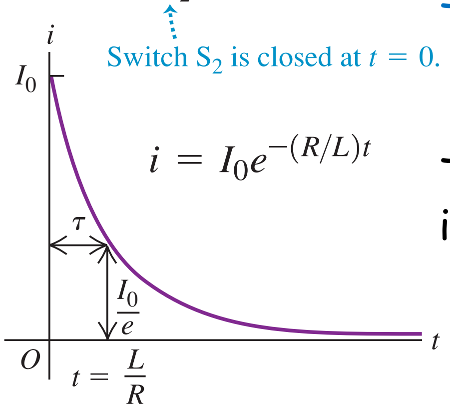


Switch S_2 is closed at $t = 0$.



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the energy stored in the inductor *decreases* at a rate equal to the rate of dissipation of energy i^2R in the resistor.