Chp 29: Electromagnetic Induction - (I)

Goals for Chapter 29

- To examine experimental evidence that a changing magnetic field induces an emf
- To learn how Faraday's law relates the induced emf to the change in flux
- To determine the direction of an induced emf
- To calculate the emf induced by a moving conductor
- To learn how a changing magnetic flux generates an electric field
- To study the four fundamental equations that describe electricity and magnetism

Introduction

- How is a credit card reader related to magnetism?
- Energy conversion makes use of electromagnetic induction.
- Faraday's law and Lenz's law tell us about induced currents.
- Maxwell's equations describe the behavior of electric and magnetic fields in *any* situation.

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for the vast majority of electric devices that are used in industry and in the home (including any device that you plug into a wall socket), the source of emf is *not* a battery but an electric generating station. Such a station produces electric energy by converting other forms of energy: gravitational potential energy at a hydroelectric plant, chemical energy in a coal- or oil-fired plant, nuclear energy at a nuclear plant.
But how is this energy conversion done?

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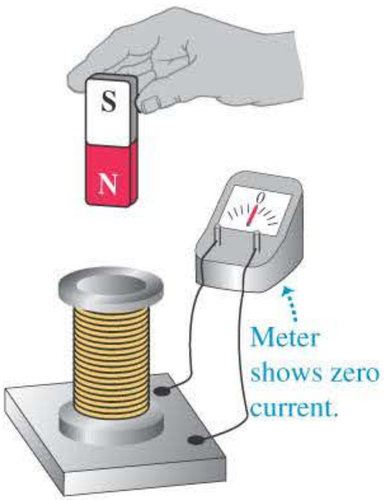
The answer is a phenomenon known as *electromagnetic induction:*
If the magnetic flux through a circuit changes, an *emf* and a current are induced in the circuit.

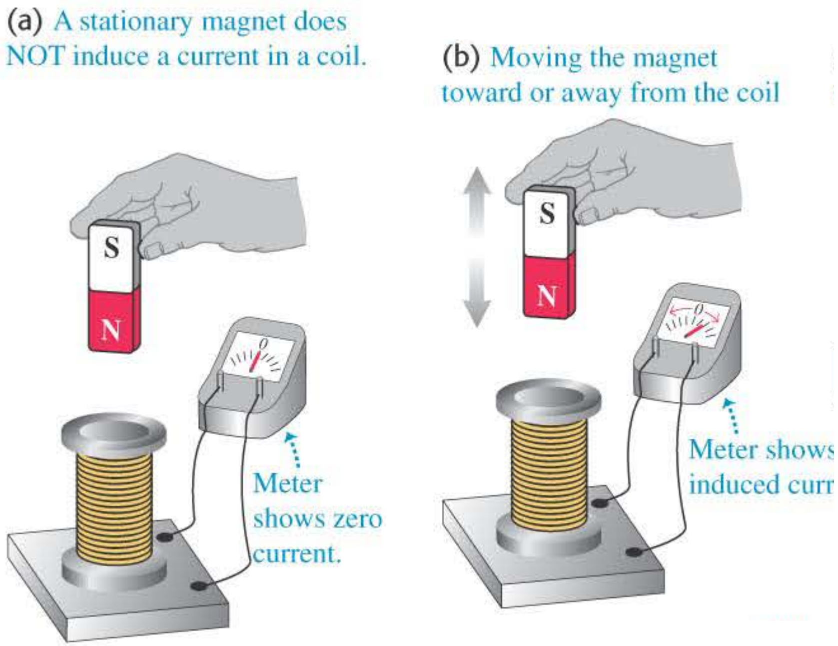
←→ Faraday's Law

Induced current

(a) A stationary magnet does NOT induce a current in a coil.



Induced current



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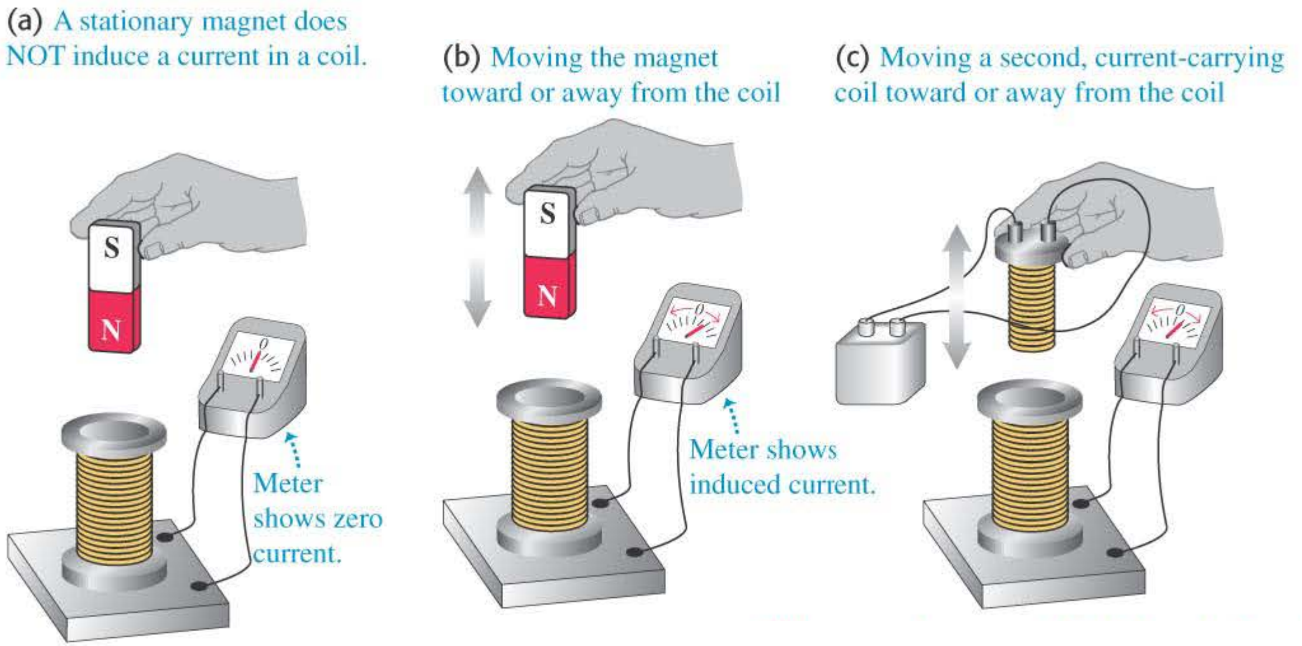

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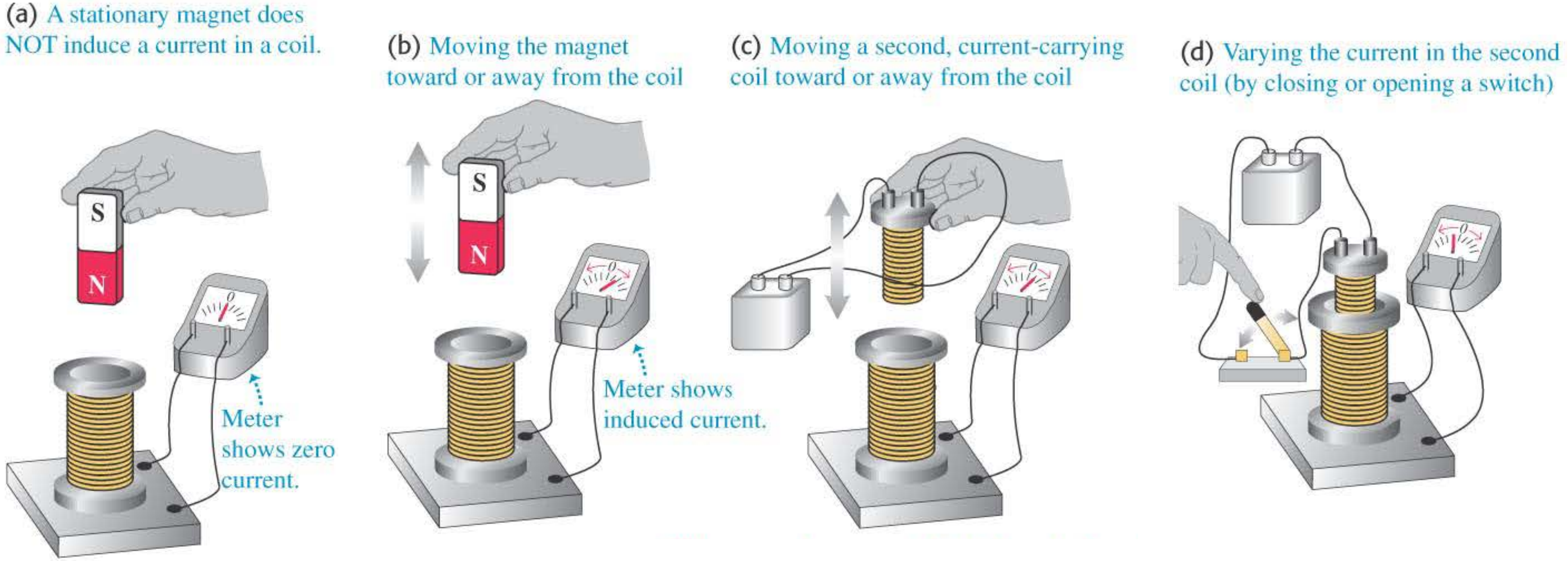

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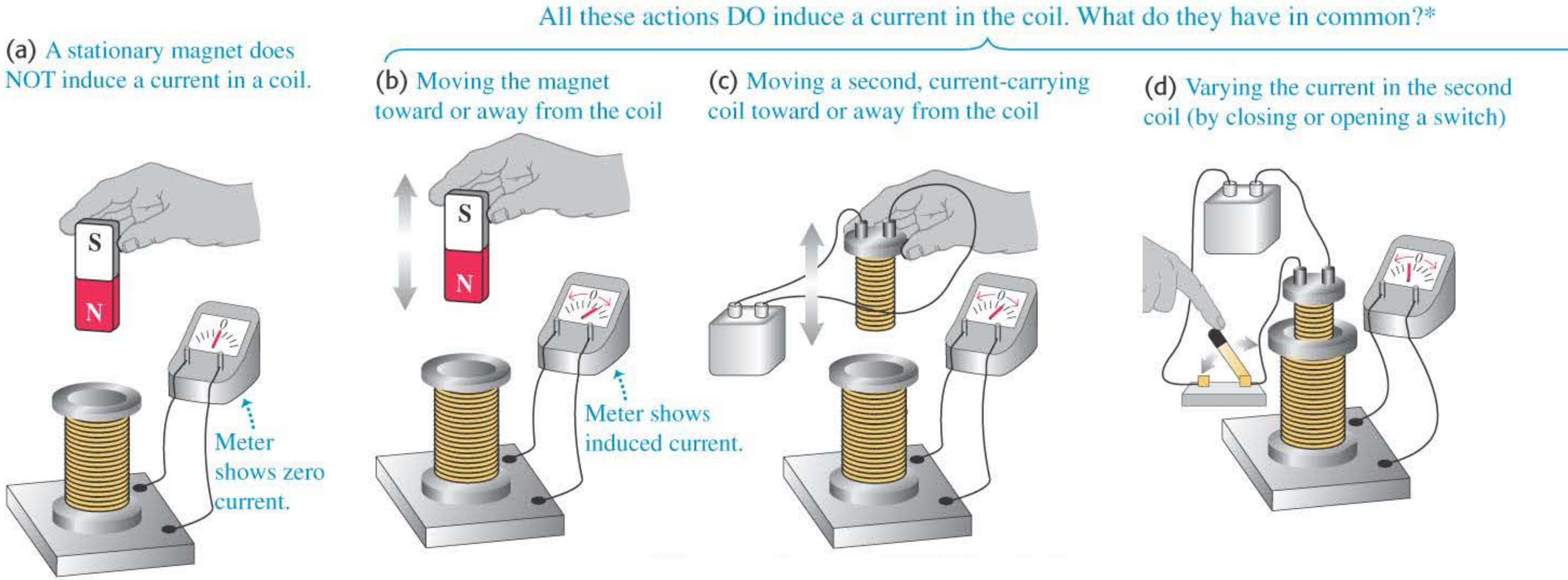

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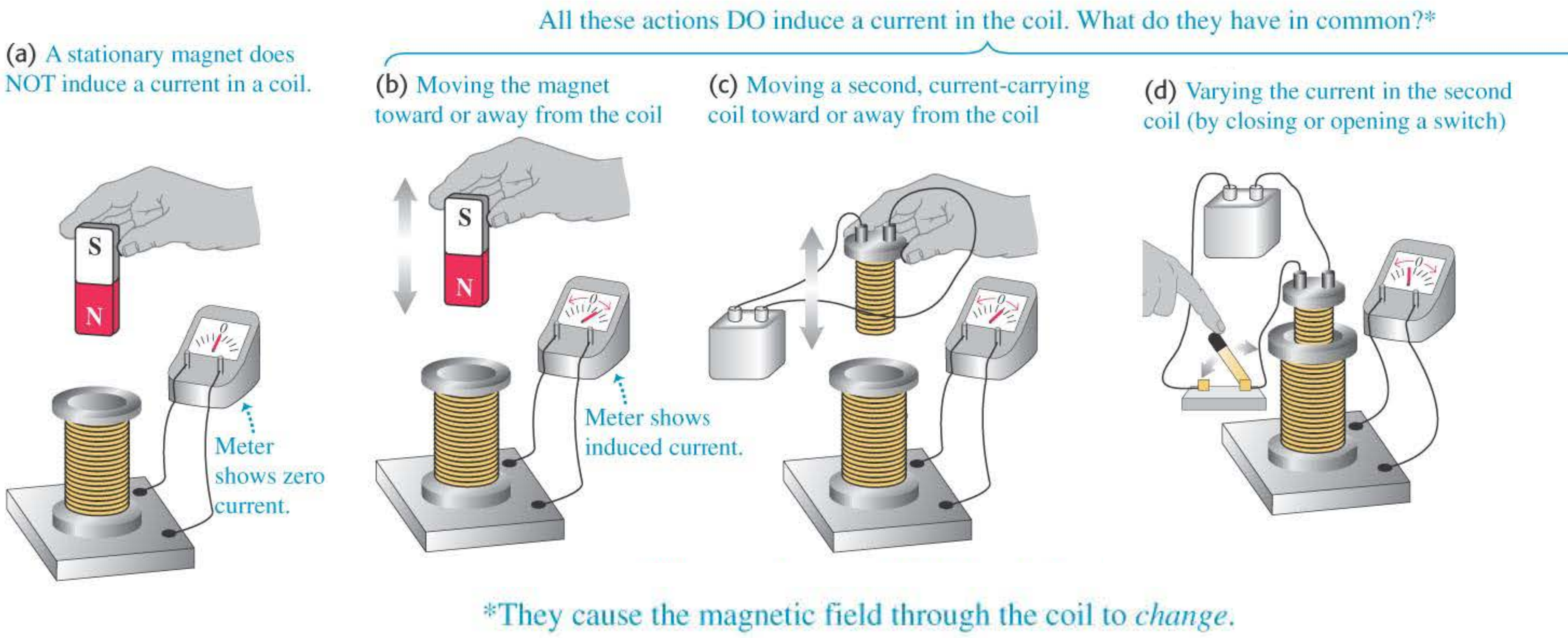
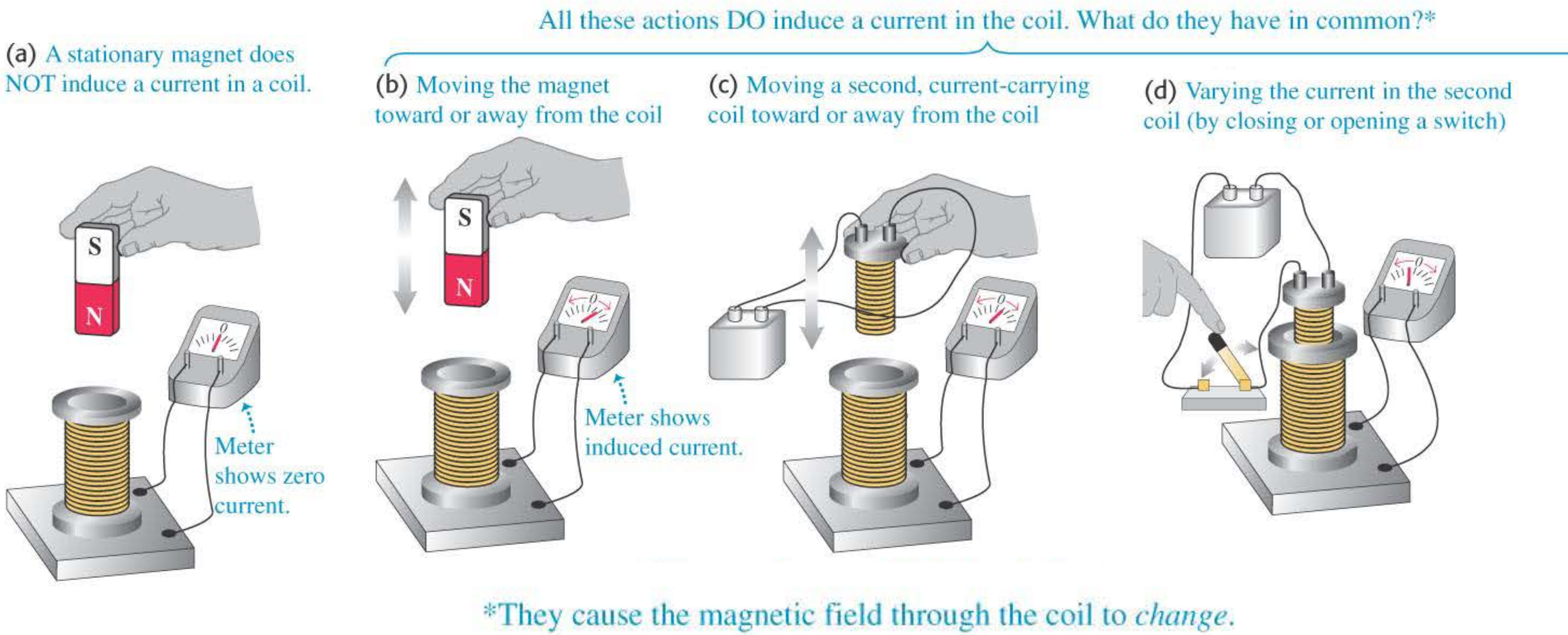
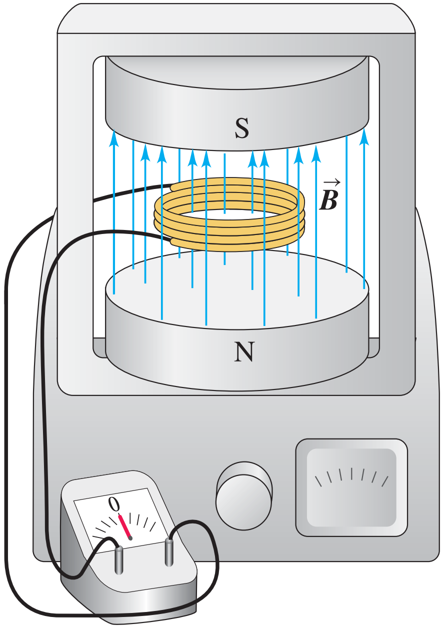


Induced current



- A changing magnetic flux causes an *induced current*
- The *induced emf* is the corresponding emf causing the current.

Induced *emf*



A coil in a magnetic field.
When the ***B*** field is constant and the shape, location, and orientation of the coil do not change, no current is induced in the coil.

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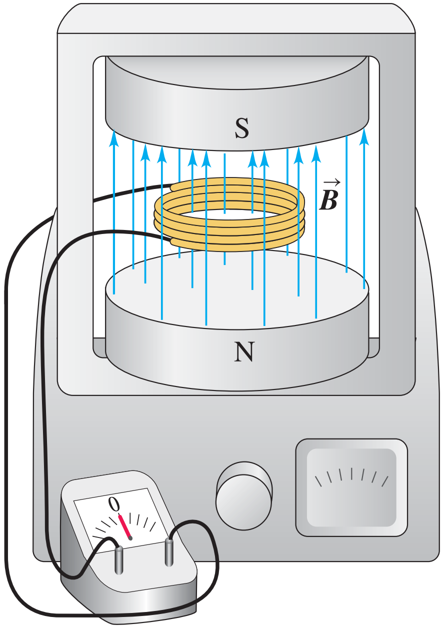

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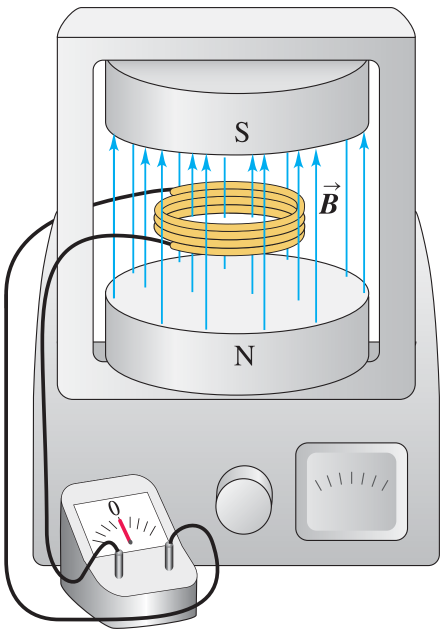

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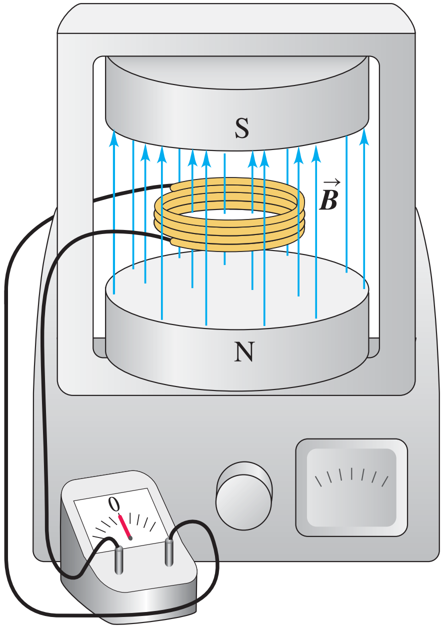

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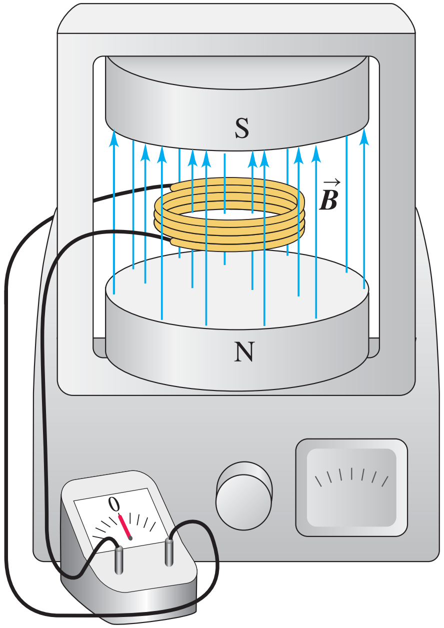

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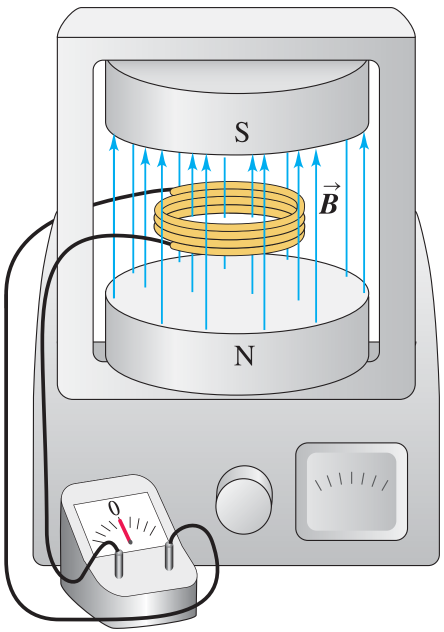

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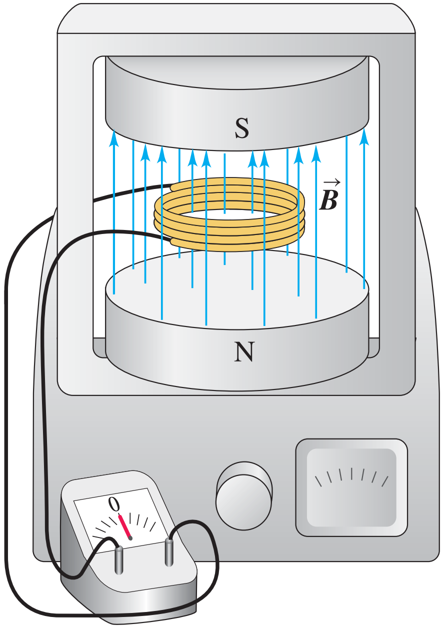
6. If we jerk the coil out of the magnetic field, →current during the motion, in the same direction as when we decreased the area.
7. decrease the number of turns in the coil →current during the unwinding, in the same direction as when we decreased the area.

Induced *emf*



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8. When the magnet is turned off, there is a momentary current (direction opposite when it was turned on).
9. The faster we carry out any of these changes, the greater the current.

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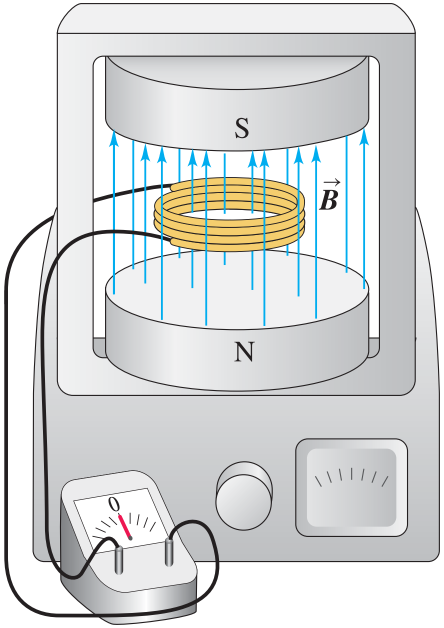

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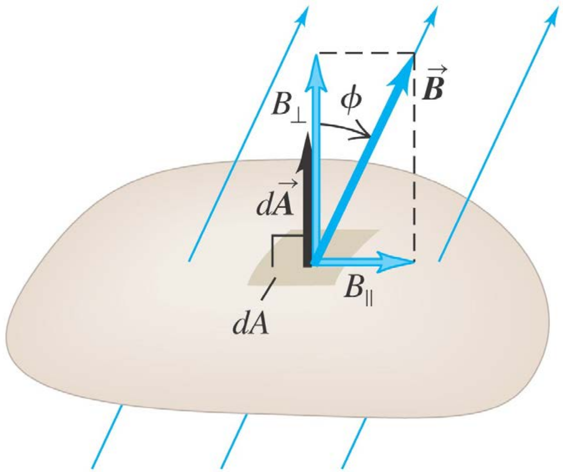
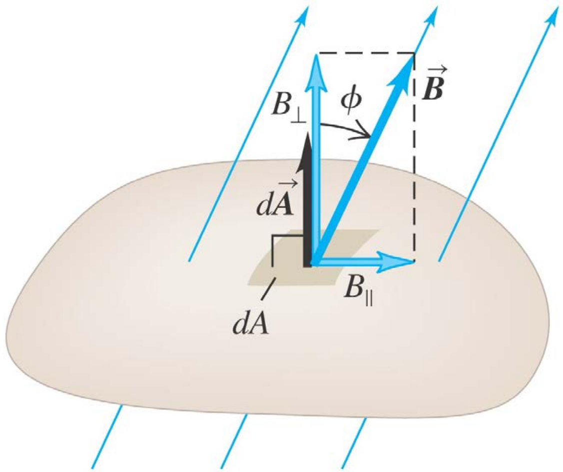
10. different material and different resistance → current in each case is inversely proportional to the total circuit resistance. This shows that the induced emfs that are causing the current do not depend on the material of the coil but only on its shape and the magnetic field.

Faraday's law



Magnetic flux through element of area $d\vec{A}$:
$$d\Phi_B = \vec{B} \cdot d\vec{A} = B_{\perp}\, dA = B\, dA \cos\phi$$

Faraday's law



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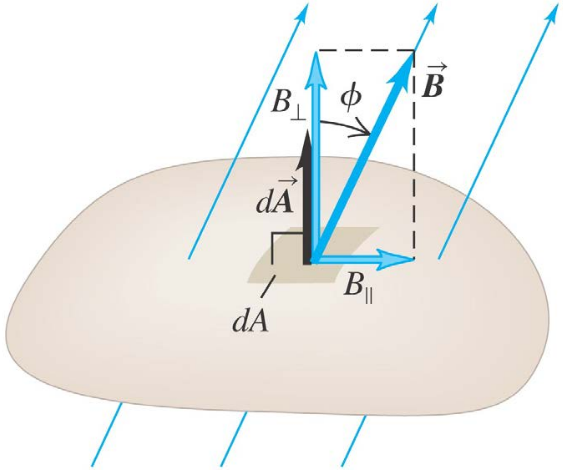

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- *Faraday's law*: The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop,

$$\xi = -d\Phi_B/dt$$

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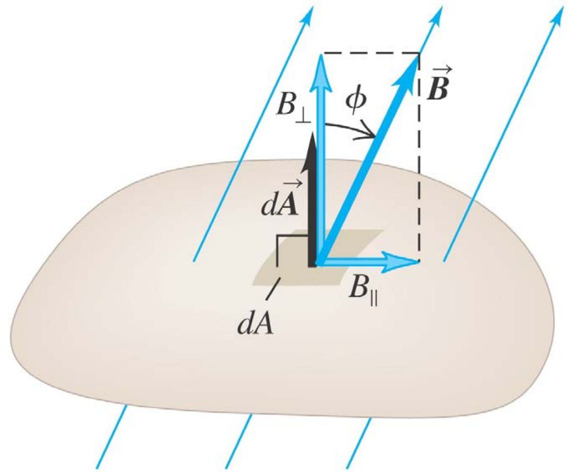

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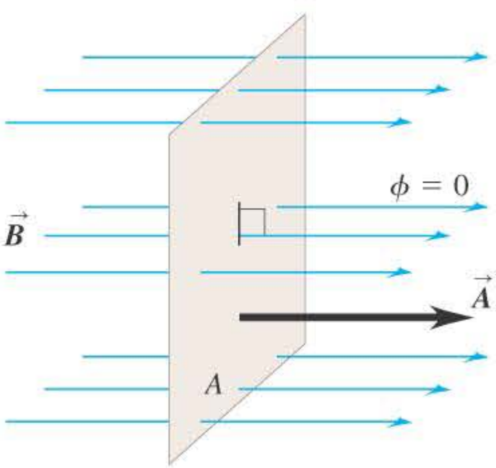
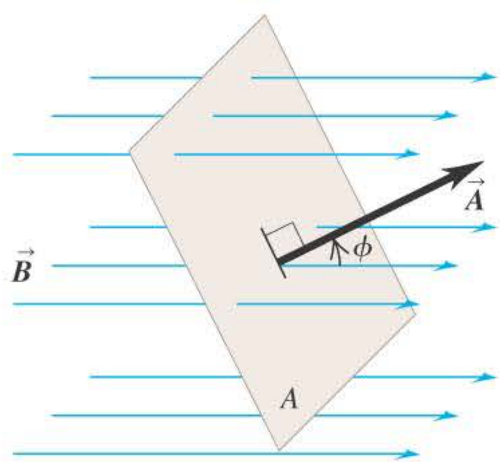
$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\, dA \cos\phi$$

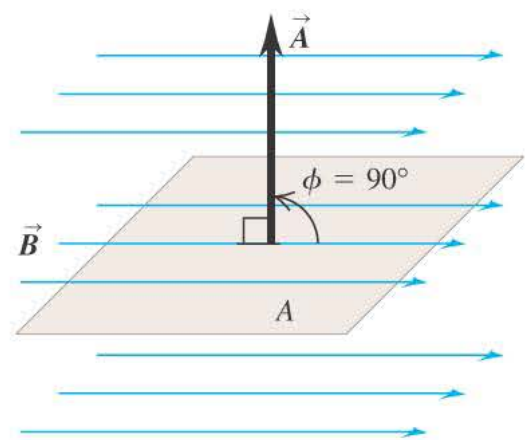
Surface is face-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.



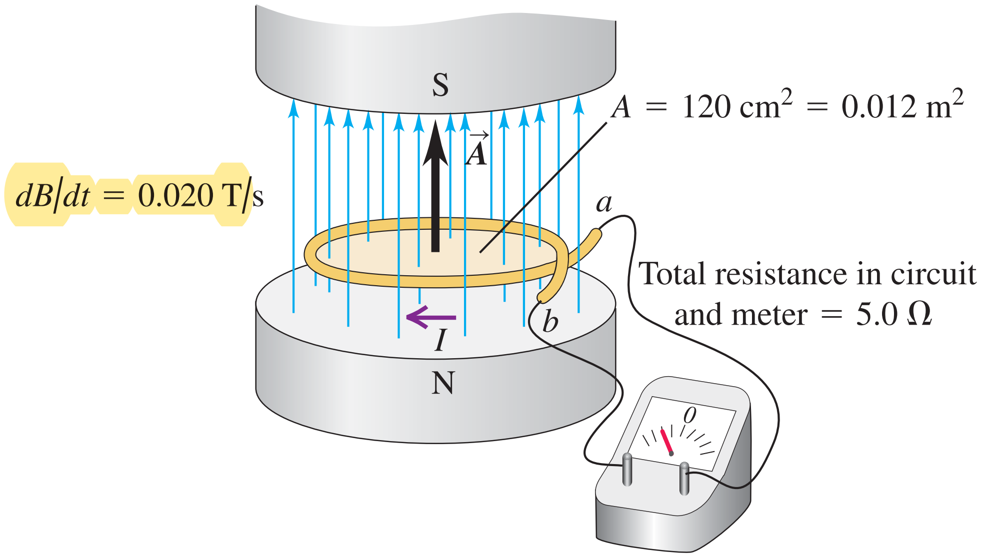
Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{B}$ and $\vec{A}$ is ϕ.
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos\phi$.



Surface is edge-on to magnetic field:
- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 90°$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA \cos 90° = 0$.

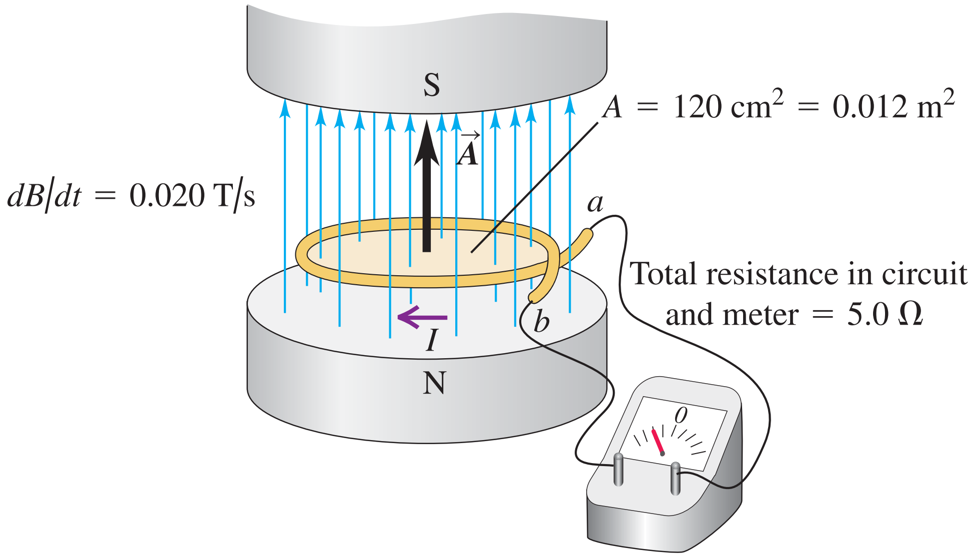


Example: Emf and current induced in a loop



The magnetic field between the poles of the electromagnet is uniform at any time, but its magnitude is increasing at the rate of 0.020 T/s. The area of the conducting loop in the field is 120 cm^2, and the total circuit resistance, including the meter is 5 **Ω**

Example: Emf and current induced in a loop



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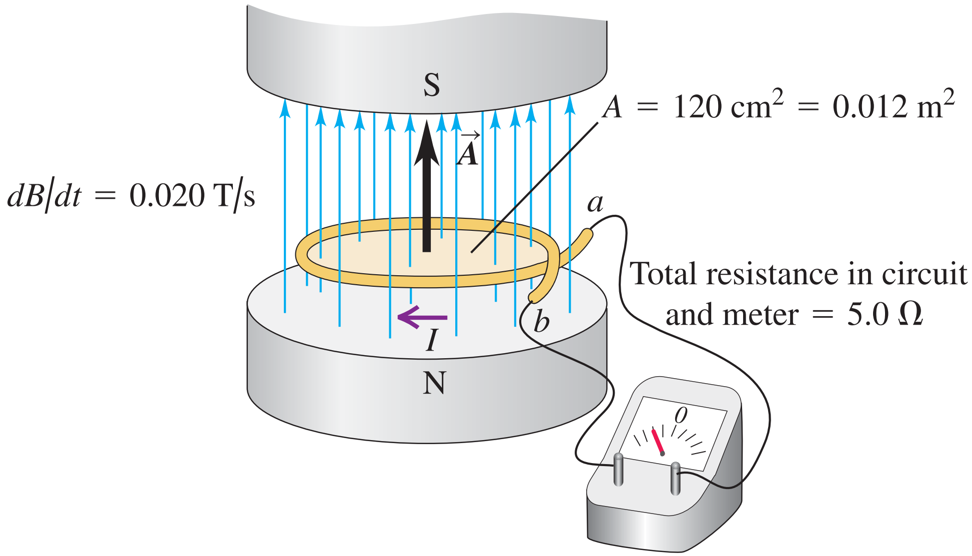

The magnetic field between the poles of the electromagnet is uniform at any time, but its magnitude is increasing at the rate of 0.020 T/s. The area of the conducting loop in the field is 120 cm^2, and the total circuit resistance, including the meter is 5 **Ω**

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 0 = BA \Rightarrow \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{dB}{dt}A = (0.020\ \text{T/s})(0.012\ \text{m}^2) = 2.4 \times 10^{-4}\ \text{V} = 0.24\ \text{mV}$$

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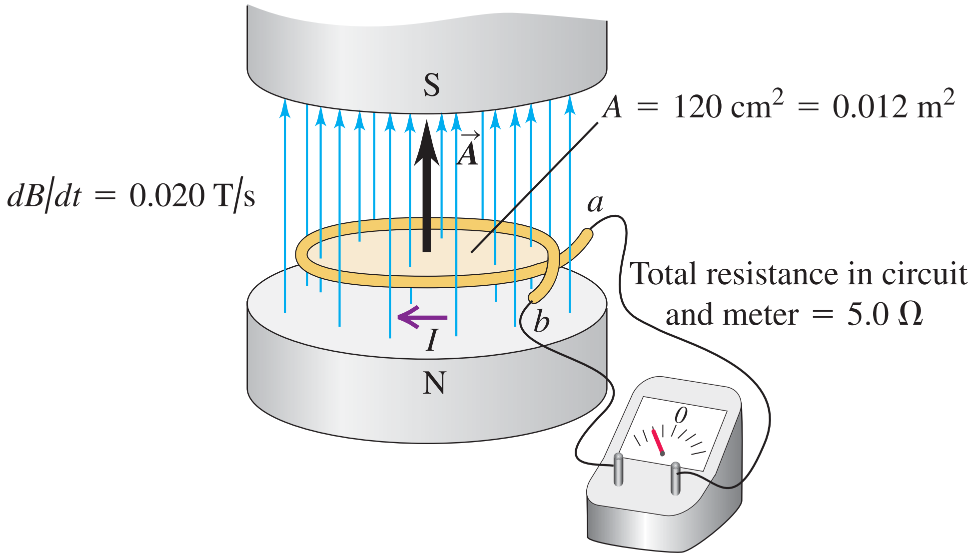

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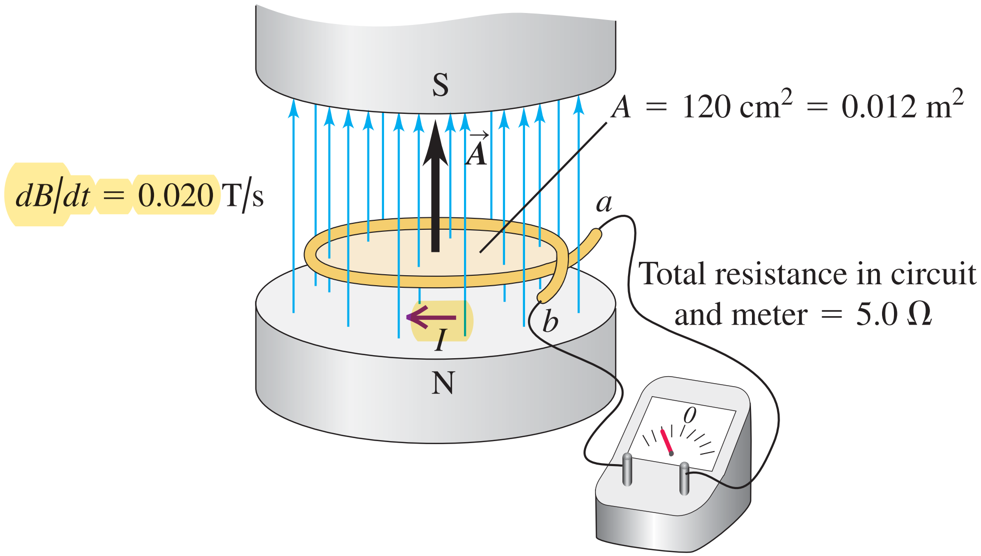

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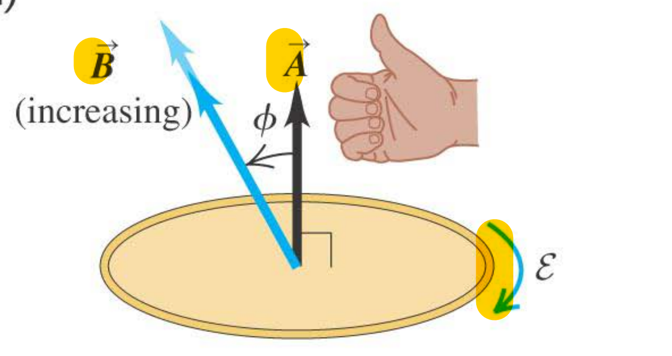
Direction of Induced emf

1. Define a positive direction for the vector area ***A***.
2. From the directions of ***A*** and the magnetic field ***B***, determine the sign of the magnetic flux Φ_B and its rate of change $d\Phi_B/dt$.
3. Determine the sign of the induced emf or current. If the flux is increasing, so $d\Phi_B/dt$ is positive, then the induced emf or current is negative; if the flux is decreasing, $d\Phi_B/dt$ is negative and the induced emf or current is positive.
4. Finally, determine the direction of the induced emf or current using your right hand. Curl the fingers of your right hand around the ***A*** vector. If the induced emf or current in the circuit is *positive*, it is in the same direction as your curled fingers; if the induced emf or current is *negative*, it is in the opposite direction.

Direction of the induced emf

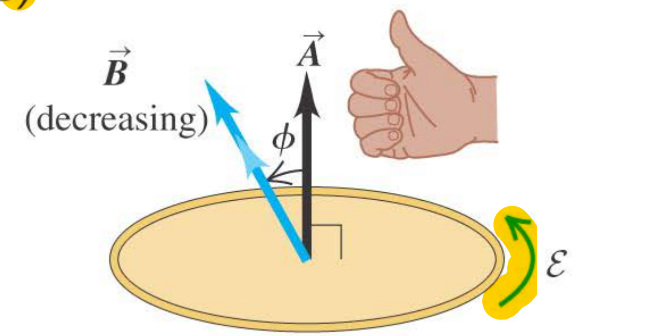
$$\xi = -d\Phi_B/dt$$

(a)



- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

(b)



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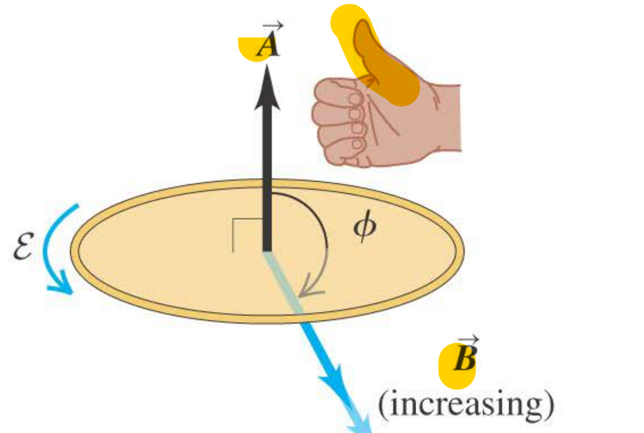
(c)



- Flux is negative ($\Phi_B < 0$) ...
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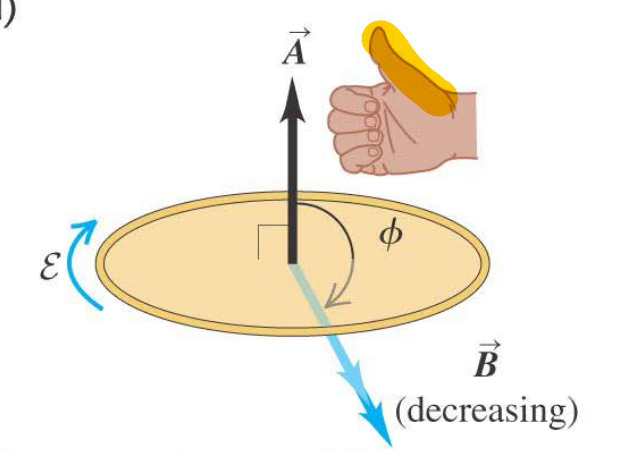
(d)



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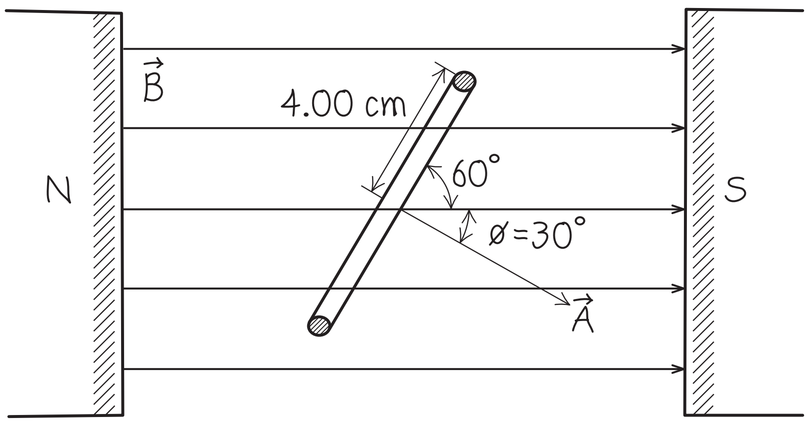
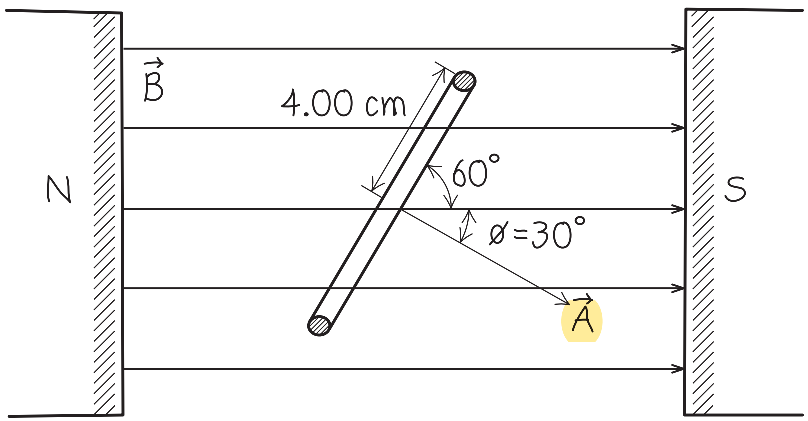
Example: Magnitude and direction of an induced emf



A 500-loop circular wire coil with radius 4.00 cm is placed between the poles of a large electromagnet. The magnetic field is uniform and makes an angle of 60° with the plane of the coil; it **decreases at 0.200 T/s**. What are the magnitude and direction of the induced emf?

Example: Magnitude and direction of an induced emf



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$\phi = 30^0$

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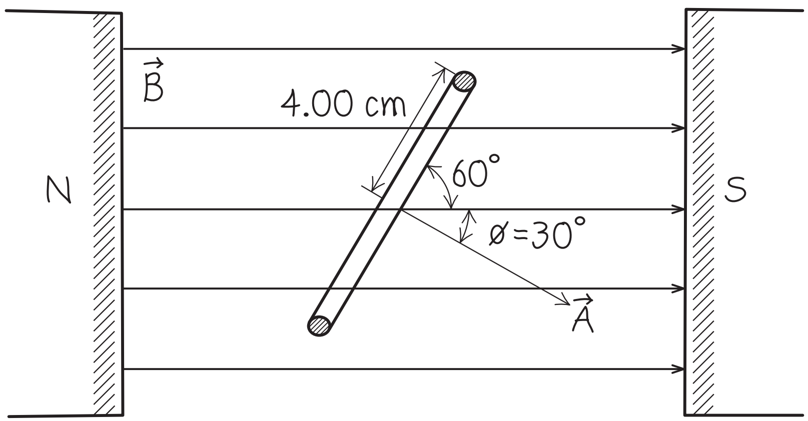

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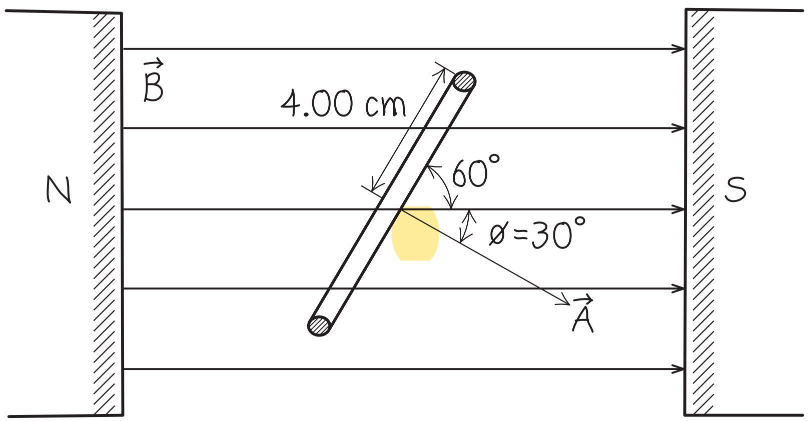
$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = N\frac{dB}{dt}A\cos\phi$$ = 500 (0.2 T/s) π (0.04 m^2) Cos 30^0 = 0.435 V

Example: Magnitude and direction of an induced emf



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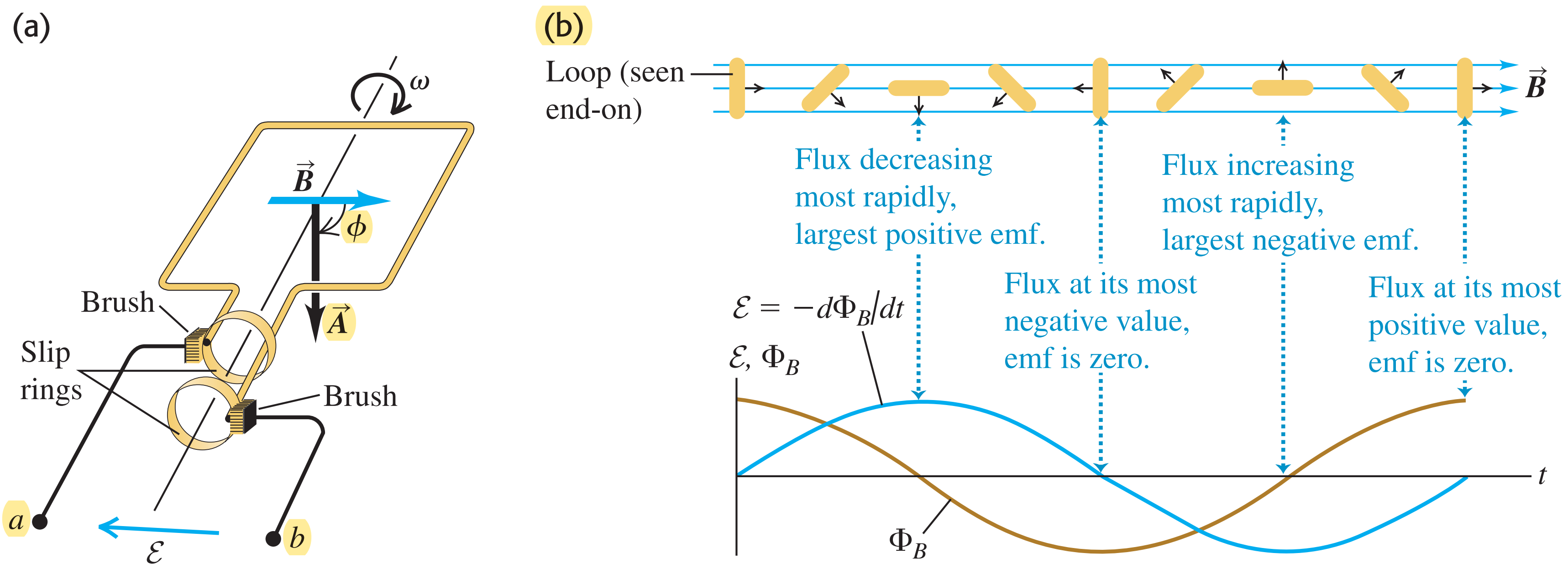
Flux rate negative (decreases)

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = N\frac{dB}{dt}A\cos\phi$$ = 500 (0.2 T/s) π (0.04 m^2) Cos 30^0 = 0.435 V

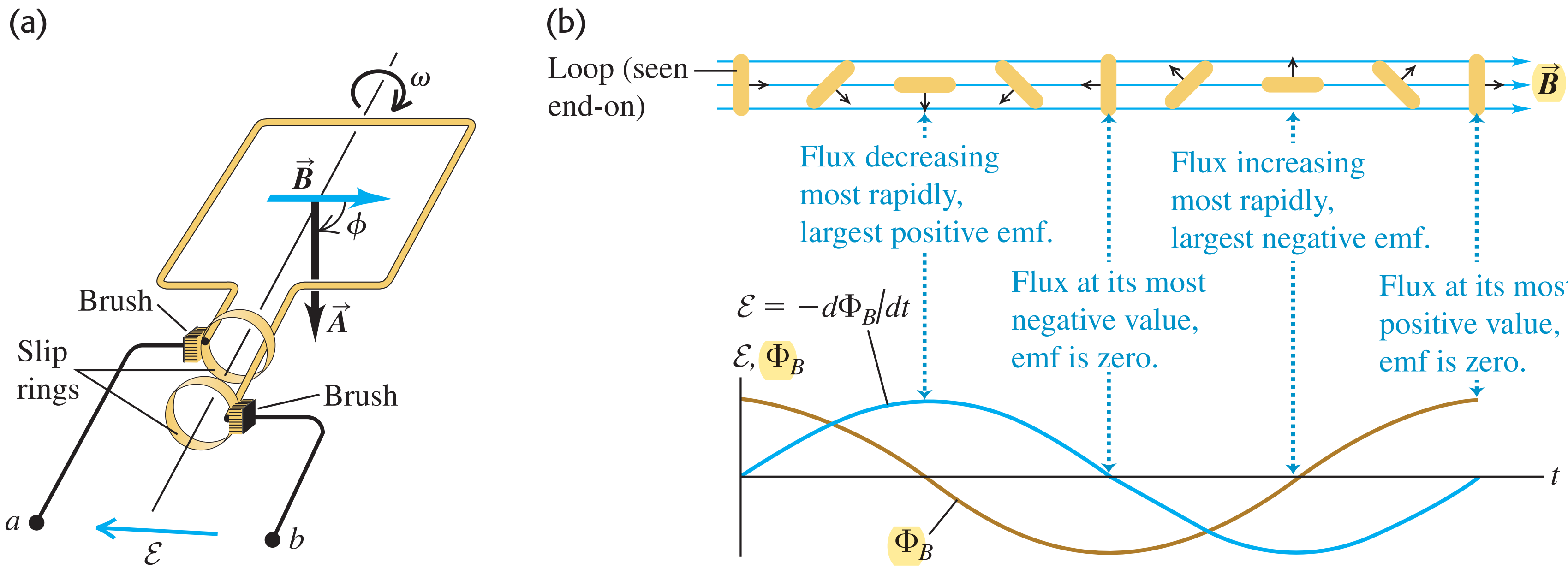
The positive answer means that when you point your right thumb in the direction of the area vector ***A*** (30° below the magnetic field ***B***), the positive direction for E is in the direction of the curled fingers of your right hand. If you viewed the coil from the left in and looked in the direction of ***A***, the emf would be clockwise.

A simple alternator



The magnetic field B is uniform and constant.($t = 0$, $\phi = 0$). Determine the induced emf.

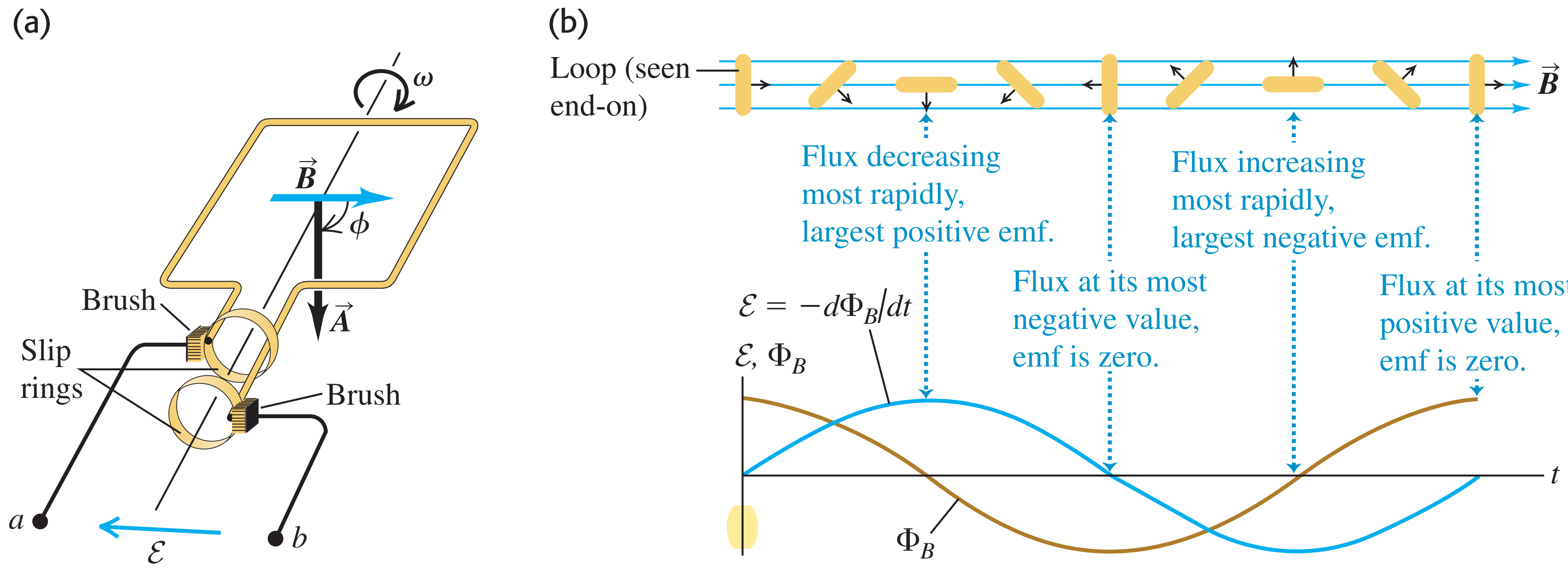
A simple alternator



A conducting loop rotates in a uniform and constant magnetic field, producing an emf. Connections from each end of the loop to the external circuit are made by means of that end's slip ring. The system is shown at the time when the angle $\phi = \omega t = 90°$.

At time $t = 0$, $\phi = 0$. Determine the induced emf.

A simple alternator



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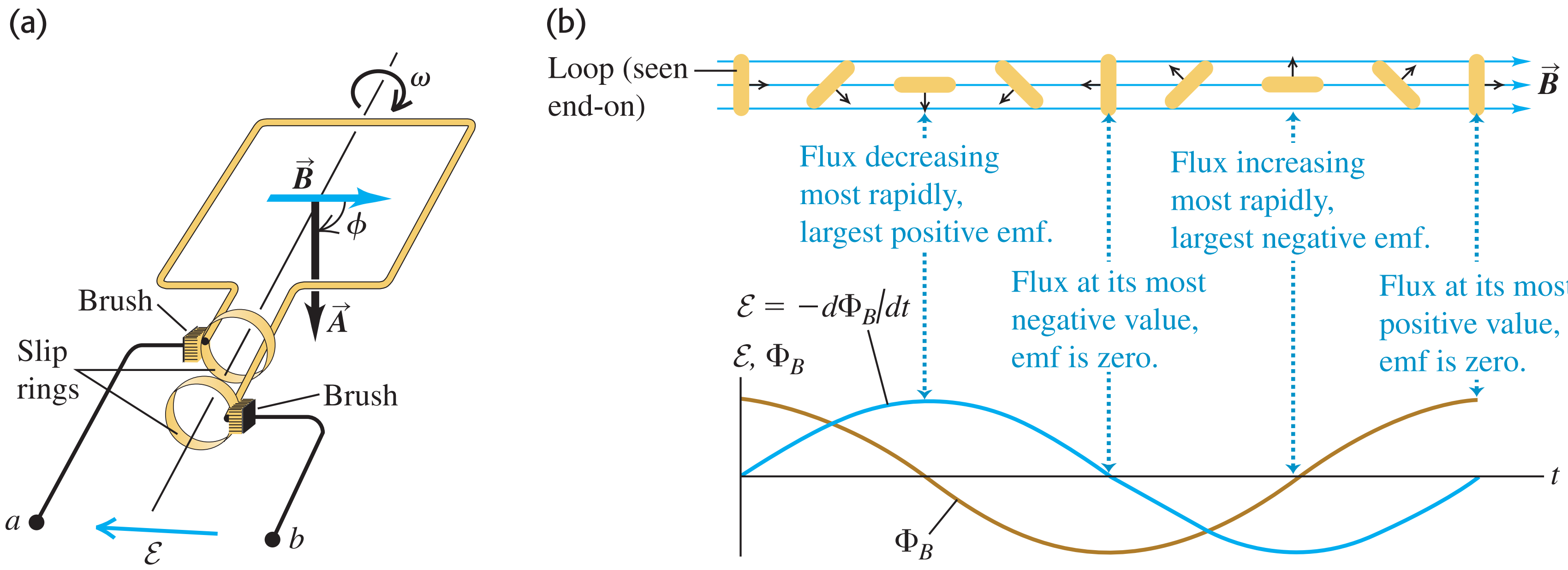
At time $t = 0$, $\phi = 0$. Determine the induced emf.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\omega t) = \omega BA\sin\omega t$$

constant magnetic field

Area is changing

A simple alternator



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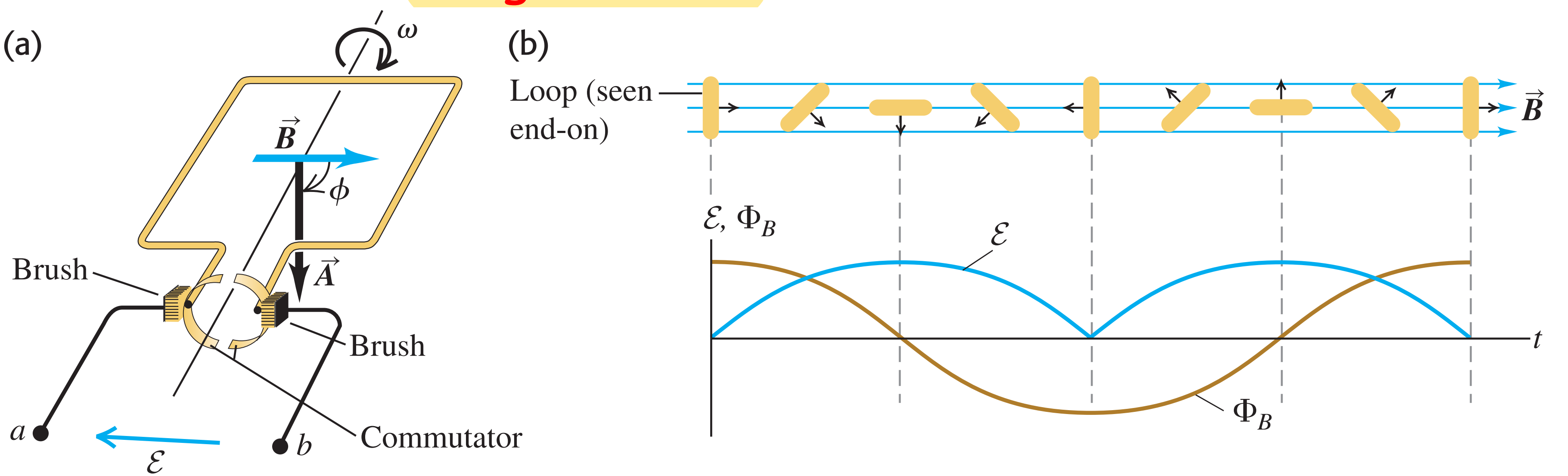
constant magnetic field

Area is changing

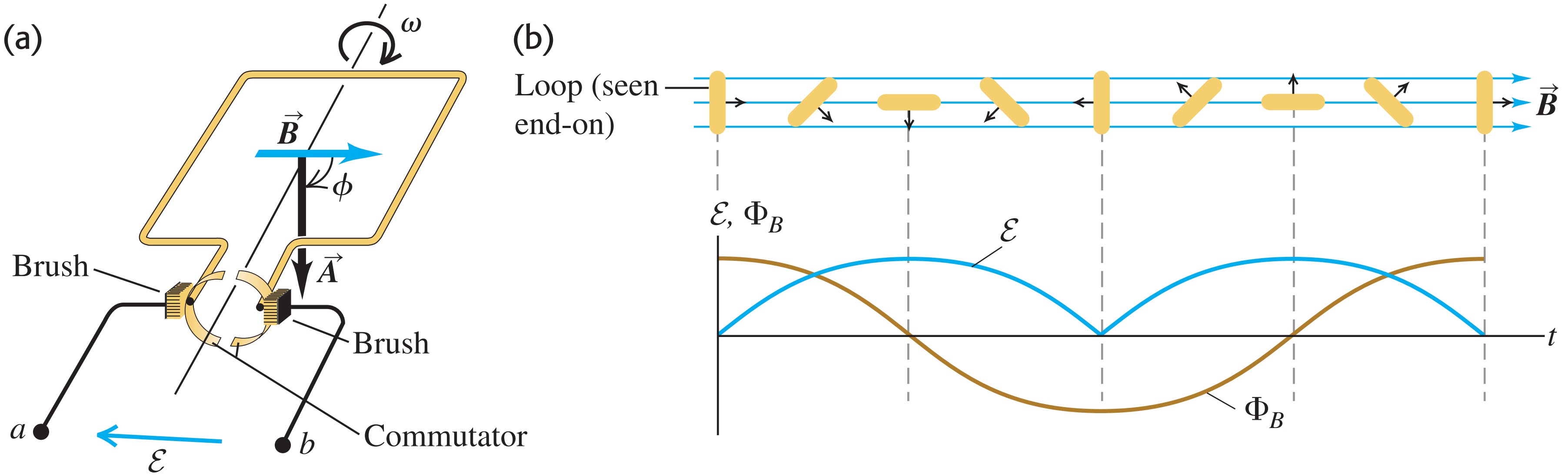
Since the emf varies sinusoidally, the current that results in the circuit is an *alternating* current that also varies sinusoidally in magnitude and direction.

A DC generator and back emf in a motor



Schematic diagram of a dc generator, using a split-ring commutator.
The ring halves are attached to the loop and rotate with it.

A DC generator and back emf in a motor

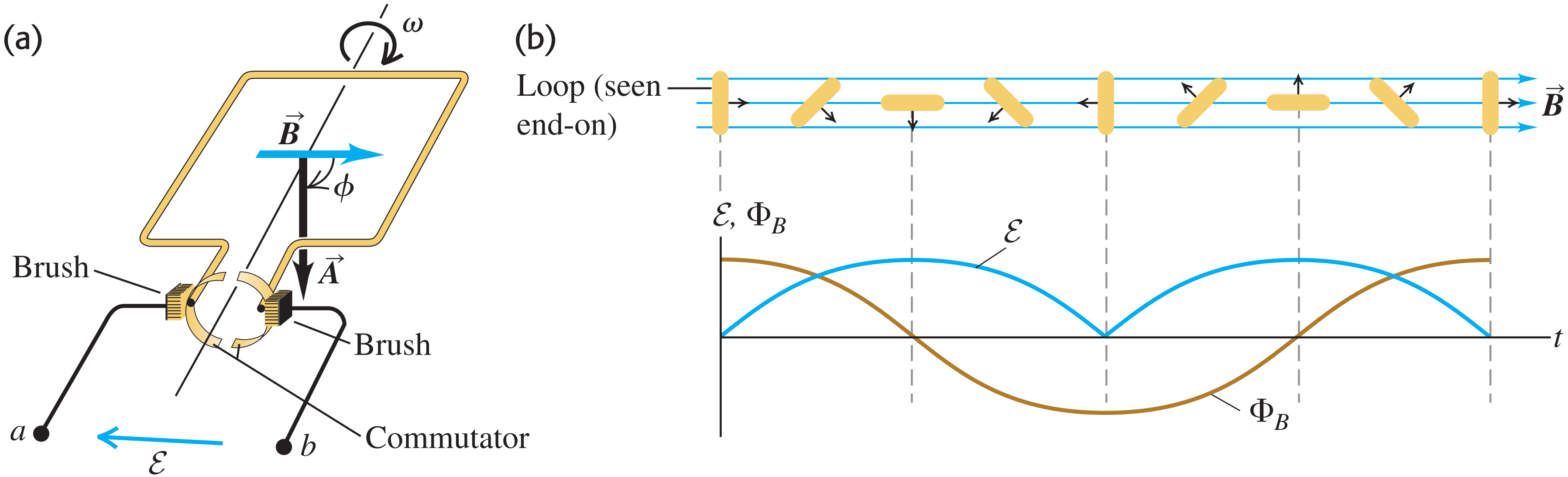


Schematic diagram of a dc generator, using a split-ring commutator.
The ring halves are attached to the loop and rotate with it. *Commutator*, reverses the connections to the external circuit at angular positions at which the emf reverses.

N loops ➔ $|\mathcal{E}| = N\omega BA|\sin\omega t|$

Half cycle : $t = 0$ to $t = T/2 = \pi/\omega$

A DC generator and back emf in a motor



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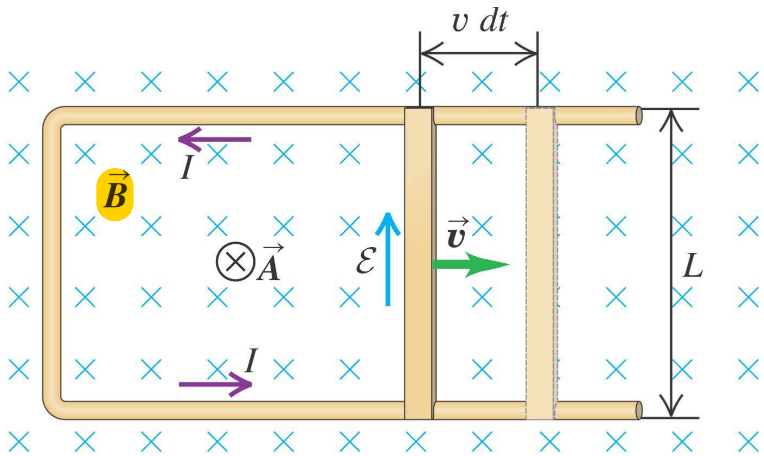
Half cycle : $t = 0$ to $t = T/2 = \pi/\omega$ ➔ $$(|\sin\omega t|)_{av} = \frac{\int_0^{\pi/\omega} \sin\omega t\, dt}{\pi/\omega} = \frac{2}{\pi}$$

➡ $$\mathcal{E}_{av} = \frac{2N\omega BA}{\pi}$$

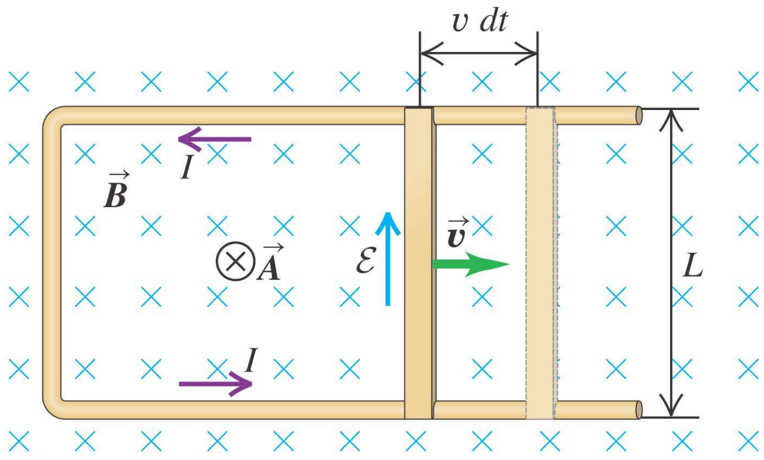
$$1\ \text{V} = 1\ \text{Wb/s} = 1\ \text{T}\cdot\text{m}^2/\text{s}$$

Slidewire generator



a U-shaped conductor in a uniform magnetic field $\boldsymbol{B}$ perpendicular to the plane of the figure and directed *into* the page. We lay a metal rod (the "slidewire") with length L across the two arms of the conductor, forming a circuit, and move it to the right with constant velocity $\boldsymbol{v}$.

Slidewire generator



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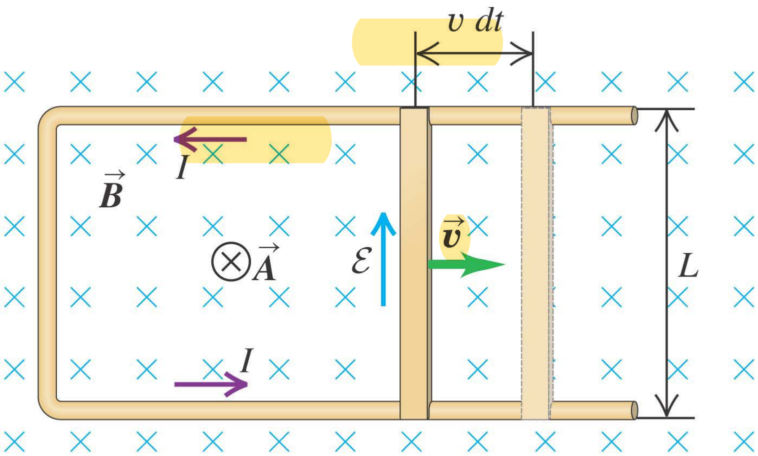
$\Phi_B = BA$ ⇨ $\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt}$

Slidewire generator



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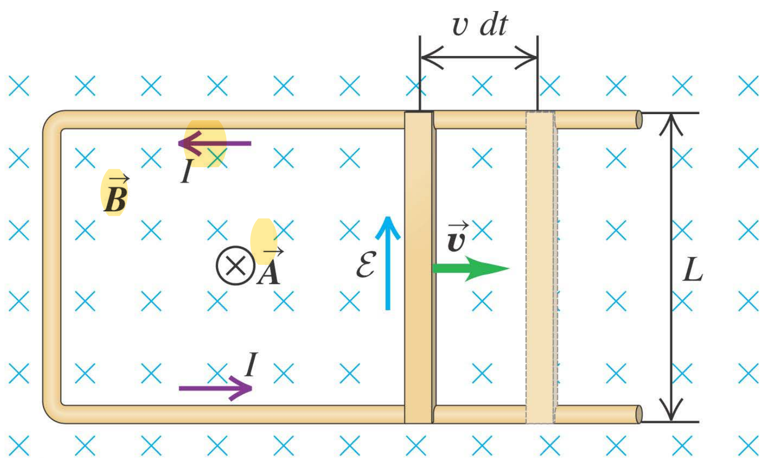
$$dA = Lv\,dt \quad \Rightarrow \quad \mathcal{E} = -B\frac{Lv\,dt}{dt} = -BLv$$

Slidewire generator



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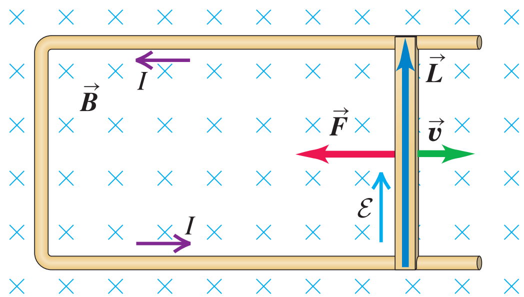
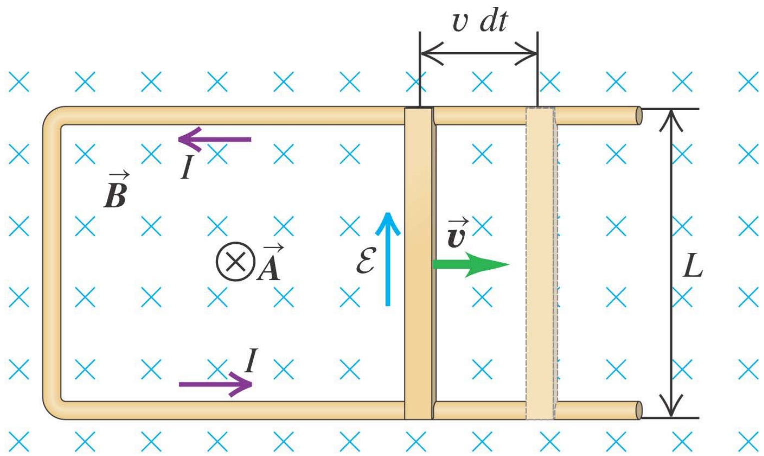
Work and power in the slidewire generator



$$\vec{F} = I\vec{L} \times \vec{B}$$

due to the induced current

Slidewire generator



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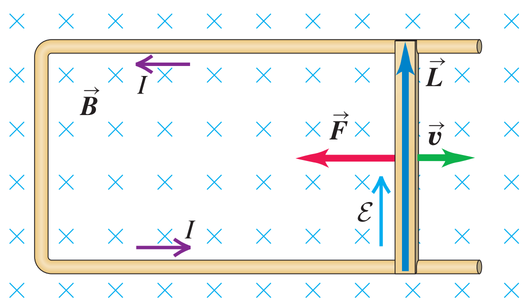
Work and power in the slidewire generator

$$\mathcal{E} = -BLv \qquad I = |\mathcal{E}|/R \Rightarrow I = Blv/R$$



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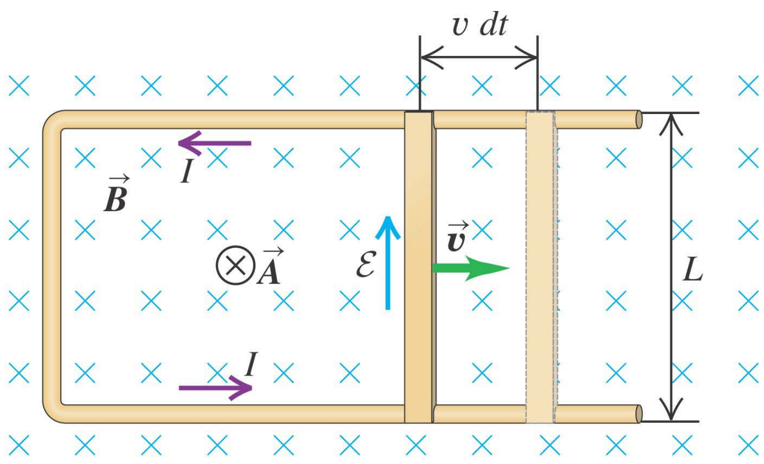

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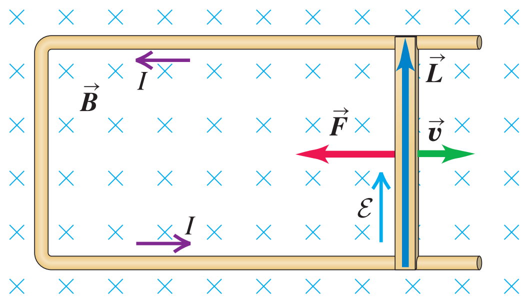


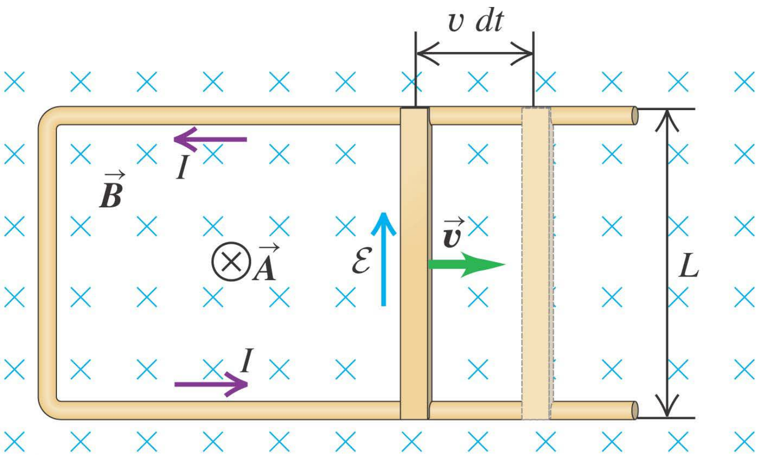
$$\mathcal{E} = -BLv \qquad I = |\mathcal{E}|/R \quad \Rightarrow \quad I = Blv/R$$

$$\Rightarrow \quad P_{\text{dissipated}} = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

due to the induced current

Slidewire generator



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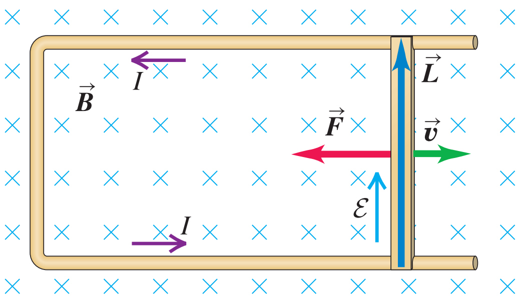

$$\Rightarrow P_{\text{dissipated}} = I^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

$$F = ILB = \frac{BLv}{R}LB = \frac{B^2L^2v}{R}$$

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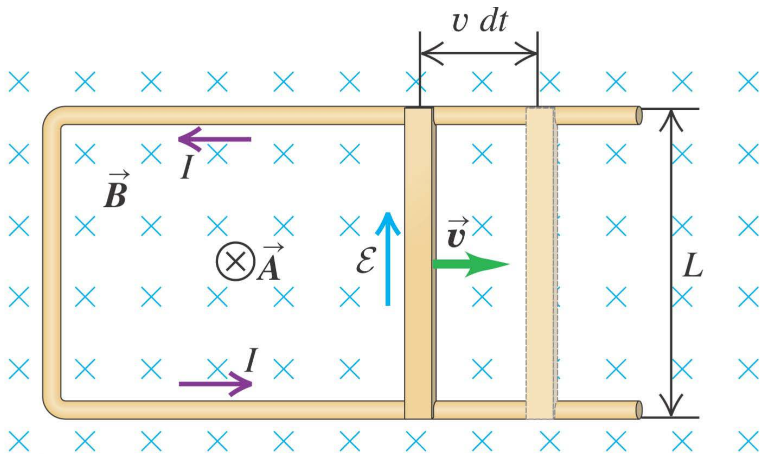
due to the induced current

Slidewire generator



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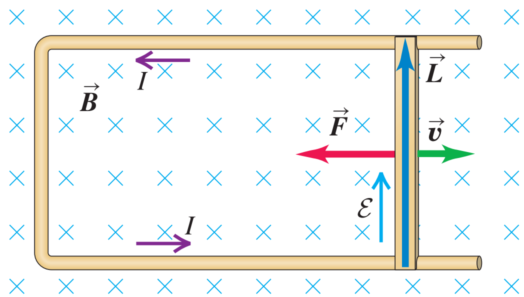
Work and power in the slidewire generator



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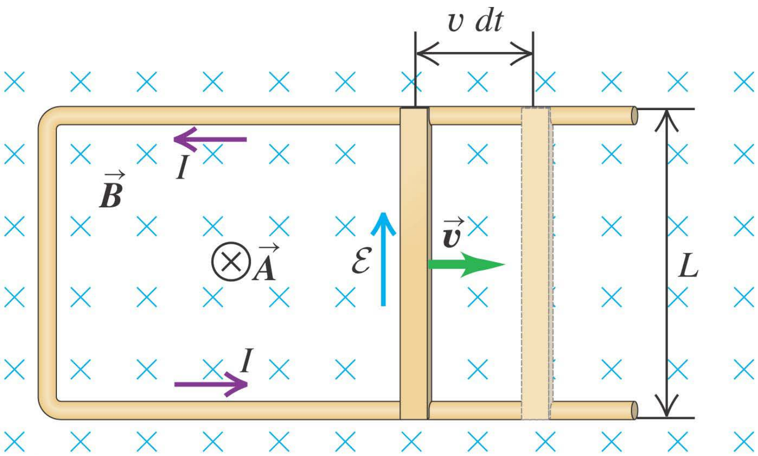
$$\vec{F} = I\vec{L} \times \vec{B}$$

due to the induced current

$$\Rightarrow \quad P_{\text{applied}} = Fv = \frac{B^2L^2v^2}{R}$$

Slidewire generator



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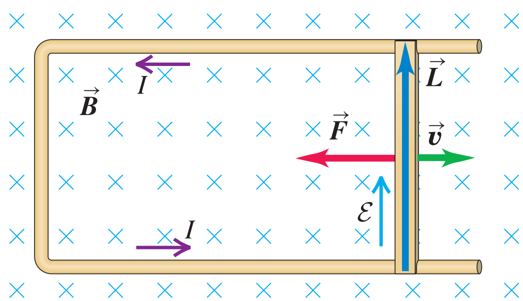
Work and power in the slidewire generator



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$$\vec{F} = I\vec{L} \times \vec{B}$$

due to the induced current

$$\Rightarrow \quad P_{\text{applied}} = Fv = \frac{B^2L^2v^2}{R}$$

The rate at which work is done is exactly *equal* to the rate at which energy is dissipated in the resistance.