Chapter 8

LP Modeling Applications: With Computer Analyses in Excel and QM for Windows

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Learning Objectives

Students will be able to:

1. Model a wide variety of medium to large LP problems.
2. Understand major application areas, including marketing, production, labor scheduling, fuel blending, transportation, and finance.
3. Gain experience in solving LP problems with QM for Windows and Excel Solver software.

Chapter Outline

Marketing Applications

- Linear programming models have been used in the advertising field as a decision aid in selecting an effective media mix.
- ***Media selection*** problems can be approached with LP from two perspectives.
  - The objective can be to **maximize** audience exposure or to **minimize** advertising costs.

Marketing Applications

The Win Big Gambling Club

- The table on the following slide presents the number of potential gamblers reached by making use of an advertisement in each of four media.
- It also provides the cost per advertisement placed and the maximum number of ads that can be purchased per week.

Marketing Applications

Media Selection:

Win Big Gambling Club

Medium	Audience Reached Per Ad	Cost Per Ad($)	Maximum Ads Per Week
TV spot (1 minute)	5,000	800	12
Daily newspaper (full page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

Marketing Applications

The Win Big Gambling Club

- The total budget is $8,000.
- The contractual arrangements require that at least 5 radio spots be placed each week.
- Management insists that no more than $1,800 be spent on radio advertising every week.
- Maximize total audience !
- **Let**

X_1 = # of 1-minute TV spots each week

X_2 = # of daily paper ads each week

X_3 = # of 30-second radio spots each week

X_4 = # of 1-minute radio spots each week

Win Big Gambling Club

$$\text{Maximize}: 5000X_1 + 8500X_2 + 2400X_3 + 2800X_4$$

(audience coverage)

Subject to:

$X_1 \leq 12$ (max TV spots/week)

$X_2 \leq 5$ (max newspaper ads/week)

$X_3 \leq 25$ (max 30-sec. radio spots/week)

$X_4 \leq 20$ (max 1-min. radio spots/week)

$800X_1 + 925X_2 + 290X_3 + 380X_4 \leq 8000$

(Weekly ad budget)

$X_3 + X_4 \geq 5$ (min radio spots/week)

$290X_3 + 380X_4 \leq 1800$ (max radio expense)

Win Big Gambling Club - QM for Windows

Screen shot for problem in QM for Windows

Linear Programming

File Edit View Module Tables Window Help

Print Screen Step Edit Data

Instruction: This cell can not be changed.

Objective

- (•) Maximize
- ( ) Minimize

Linear Programming Results

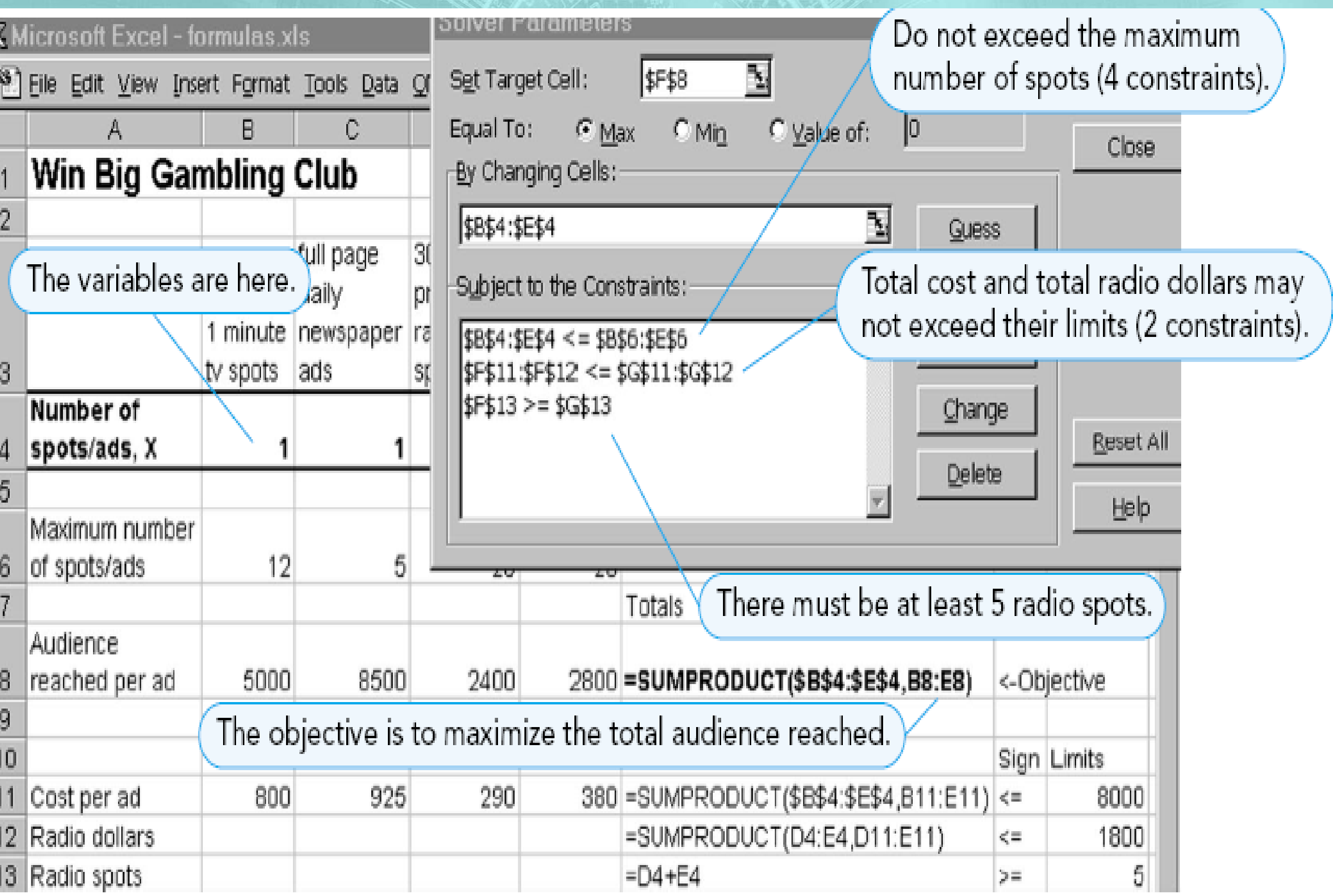
Win Big Gambling Club Solution

	X1	X2	X3	X4		RHS	Dual
Maximize	5,000.	8,500.	2,400.	2,800.			
Constraint 1	1.	0.	0.	0.	<=	12.	0.
Constraint 2	0.	1.	0.	0.	<=	5.	2,718.75
Constraint 3	0.	0.	1.	0.	<=	25.	0.
Constraint 4	0.	0.	0.	1.	<=	20.	0.
Constraint 5	800.	925.	290.	380.	<=	8,000.	6.25
Constraint 6	0.	0.	1.	1.	>=	5.	0.
Constraint 7	0.	0.	290.	380.	<=	1,800.	2.0259
Solution->	1.9688	5.	6.2069	0.		67,240.3	

Ranging Iterations

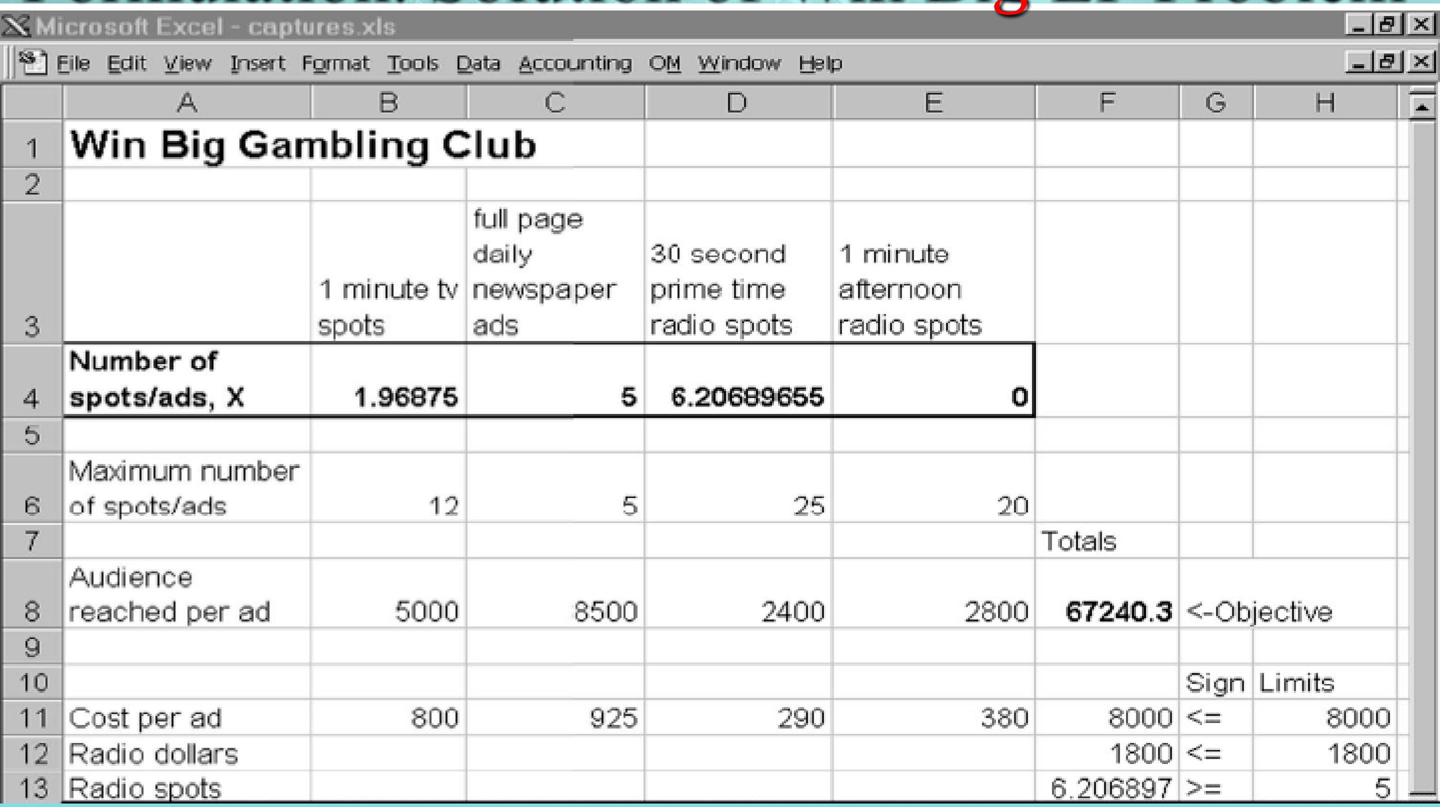
Win Big Gambling Club – Solver in Excel

Screen shot for Excel Formulation of Win Big's LP Problem using Solver



Win Big Gambling Club – Solver in Excel

Screen shot for Output from Excel Formulation. Solution of Win Big LP Problem



	A	B	C	D	E	F	G	H
1	**Win Big Gambling Club**							
2								
3		1 minute tv spots	full page daily newspaper ads	30 second prime time radio spots	1 minute afternoon radio spots			
4	**Number of spots/ads, X**	**1.96875**	**5**	**6.20689655**	**0**			
5								
6	Maximum number of spots/ads	12	5	25	20			
7						Totals		
8	Audience reached per ad	5000	8500	2400	2800	**67240.3**	<-Objective	
9								
10							Sign	Limits
11	Cost per ad	800	925	290	380	8000	<=	8000
12	Radio dollars					1800	<=	1800
13	Radio spots					6.206897	>=	5

$\mathbf{X_1} = 1.97$	TV spots
$\mathbf{X_2} = 5$	newspaper ads
$\mathbf{X_3} = 6.2$	30-second radio spots
$\mathbf{X_4} = 0$	1-minute radio spots

Win Big Gambling Club – Solver in Excel

- The solution gives an audience exposure of 67,240 contacts.
- Because X_1 and X_3 are fractional, Win Big would probably round them to 2 and 6, respectively.
- Situations that require all integer solutions are discussed in detail in Chapter 11.

Marketing Applications

- Linear programming has also been applied to ***marketing research*** problems and the area of consumer research.
- The next example illustrates how statistical pollsters can reach strategy decisions with LP.

Marketing Applications

Management Sciences Associates (MSA)

- The MSA determines that it must fulfill several requirements in order to draw statistically valid conclusions:

1. Survey at least 2,300 U.S. households.
2. Survey at least 1,000 households whose heads are 30 years of age or younger.
3. Survey at least 600 households whose heads are between 31 and 50 years of age.
4. Ensure that at least 15% of those surveyed live in a state that borders on Mexico.
5. Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a state that borders on Mexico.
6. MSA decides that all surveys should be conducted in person.

Marketing Applications

Management Sciences Associates (MSA)

- MSA estimates that the costs of reaching people in each age and region category are as follows:

	COST PER PERSON SURVEYED ($)		
REGION	AGE ≤ 30	AGE 31–50	AGE ≥ 51
State bordering Mexico	$7.50	$6.80	$5.50
State not bordering Mexico	$6.90	$7.25	$6.10

MSA’s goal is to meet the sampling requirements at the least possible cost.

	<=30	31-50	>=51
State bordering Mexico	X1	X2	X3
State not bordering Mexico	X4	X5	X6

Let:

X_1 = # of 30 or younger & in a border state

X_2 = # of 31-50 & in a border state

X_3 = # 51 or older & in a border state

X_4 = # 30 or younger & *not* in a border state

X_5 = # of 31-50 & *not* in a border state

X_6 = # 51 or older & *not* in a border state

Marketing Applications

Management Sciences Associates (MSA)

Objective function:

minimize total interview costs =

$$\$7.5X_1 + \$6.8X_2 + \$5.5X_3 + \$6.9X_4 + \$7.25X_5 + \$6.1X_6$$

subject to:

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 2{,}300$ (total households)

$X_1 + X_4 \geq 1{,}000$ (households 30 or younger)

$X_2 + X_5 \geq 600$ (households 31-50 in age)

$X_1 + X_2 + X_3 \geq 0.15(X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$ (border states)

$X_3 \leq 0.2(X_3 + X_6)$ (limit on age group 51+ in border state)

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$

Marketing Applications

Management Sciences Associates (MSA)

- The computer solution to MSA's problem costs $15,166 and is presented in the table and screen shot on the next slide, which illustrates the input and output from QM for Windows.
- Note that the variables in the last two constraints are moved to the left-hand side of the inequality leaving a zero as the right-hand-side value.

Marketing Applications

Management Sciences Associates (MSA)

Computer solution for the 6 variables

REGION	AGE ≤ 30	AGE 31–50	AGE ≥ 51
State bordering Mexico	0	600	140
State not bordering Mexico	1,000	0	560

Input and Output from QM for Windows

QM for Windows - C:\Prentice\Data\Renstair\Mansc393.lin

Objective

- Maximize
- Minimize (selected)

Linear Programming Results

Management Science Associates Solution

	X1	X2	X3	X4	X5	X6		RHS	Dual
Minimize	7.5	6.8	5.5	6.9	7.25	6.1			
Constraint 1	1.	1.	1.	1.	1.	1.	>=	2,300.	-5.98
Constraint 2	1.	0.	0.	1.	0.	0.	>=	1,000.	-0.92
Constraint 3	0.	1.	0.	0.	1.	0.	>=	600.	-0.82
Constraint 4	0.85	0.85	0.85	-0.15	-0.15	-0.15	>=	0.	0.
Constraint 5	0.	0.	0.8	0.	0.	-0.2	<=	0.	0.6
Solution->	0.	600.	140.	1,000.	0.	559.9999		15,166.	

Manufacturing Applications

Production Mix

- A fertile field for the use of LP is in planning for the optimal mix of products to manufacture.
- A company must meet a myriad of constraints, ranging from financial concerns to sales demand to material contracts to union labor demands.
- Its primary goal is to generate the largest profit possible.

Manufacturing Applications

Fifth Avenue Industries

Four varieties of ties produced:

1. one is an expensive, all-silk tie,
2. one is an all-polyester tie, and
3. two are blends of polyester and cotton.

The table on the following slide illustrates the cost and availability (per monthly production planning period) of the three materials used in the production process

Manufacturing Applications

Fifth Avenue Industries

Monthly cost & availability of material

MATERIAL	COST PER YARD ($)	MATERIAL AVAILABLE PER MONTH (YARDS)
Silk	21	800
Polyester	6	3,000
Cotton	9	1,600

The table on the next slide summarizes the

- contract demand for each of the four styles of ties,
- the selling price per tie, and
- the fabric requirements of each variety.

Manufacturing Applications

Fifth Avenue Industries

Data:

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MONTHLY CONTRACT MINIMUM	MONTHLY DEMAND	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL REQUIREMENTS
All silk	6.70	6,000	7,000	0.125	100% silk
All polyester	3.55	10,000	14,000	0.08	100% polyester
Poly–cotton blend 1	4.31	13,000	16,000	0.10	50% polyester–50% cotton
Poly–cotton blend 2	4.81	6,000	8,500	0.10	30% polyester–70% cotton

Fifth Avenue's goal is to maximize its monthly profit. It must decide upon a policy for product mix.

Let

X1 = # of all-silk ties produced per month

X2 = # polyester ties

X3 = # of blend 1 poly-cotton ties

X4 = # of blend 2 poly-cotton ties

Manufacturing Applications

Fifth Avenue Industries

Calculate profit for each tie:

***Profit* = Sales price**
– Cost per yard X Yards per tie

Silk ties = \$6.70 – \$21 x 0.125 = \$4.08

Polyester = \$3.55 – \$6 x 0.08 = \$3.07

Poly-blend 1 = \$4.31 – (\$6 x 0.05 + \$9 x 0.05) = \$3.56

Poly-blend 2 = \$4.81 – (\$6 x 0.03 + \$9 x 0.07) = \$4.00

Manufacturing Applications

Fifth Avenue Industries

Objective function:

maximize profit =

$$\$4.08X_1 + \$3.07X_2 + \$3.56X_3 + \$4.00X_4$$

Subject to:

$$
\begin{aligned}
0.125X_1 &\le 800 \\
0.08X_2 + 0.05X_3 + 0.03X_4 &\le 3{,}000 \\
0.05X_3 + 0.07X_4 &\le 1{,}600 \\
X_1 &\ge 6{,}000 \\
X_1 &\le 7{,}000 \\
X_2 &\ge 10{,}000 \\
X_2 &\le 14{,}000 \\
X_3 &\ge 13{,}000 \\
X_3 &\le 16{,}000 \\
X_4 &\ge 6{,}000 \\
X_4 &\le 8{,}500 \\
X_1, X_2, X_3, X_4 &\ge 0
\end{aligned}
$$

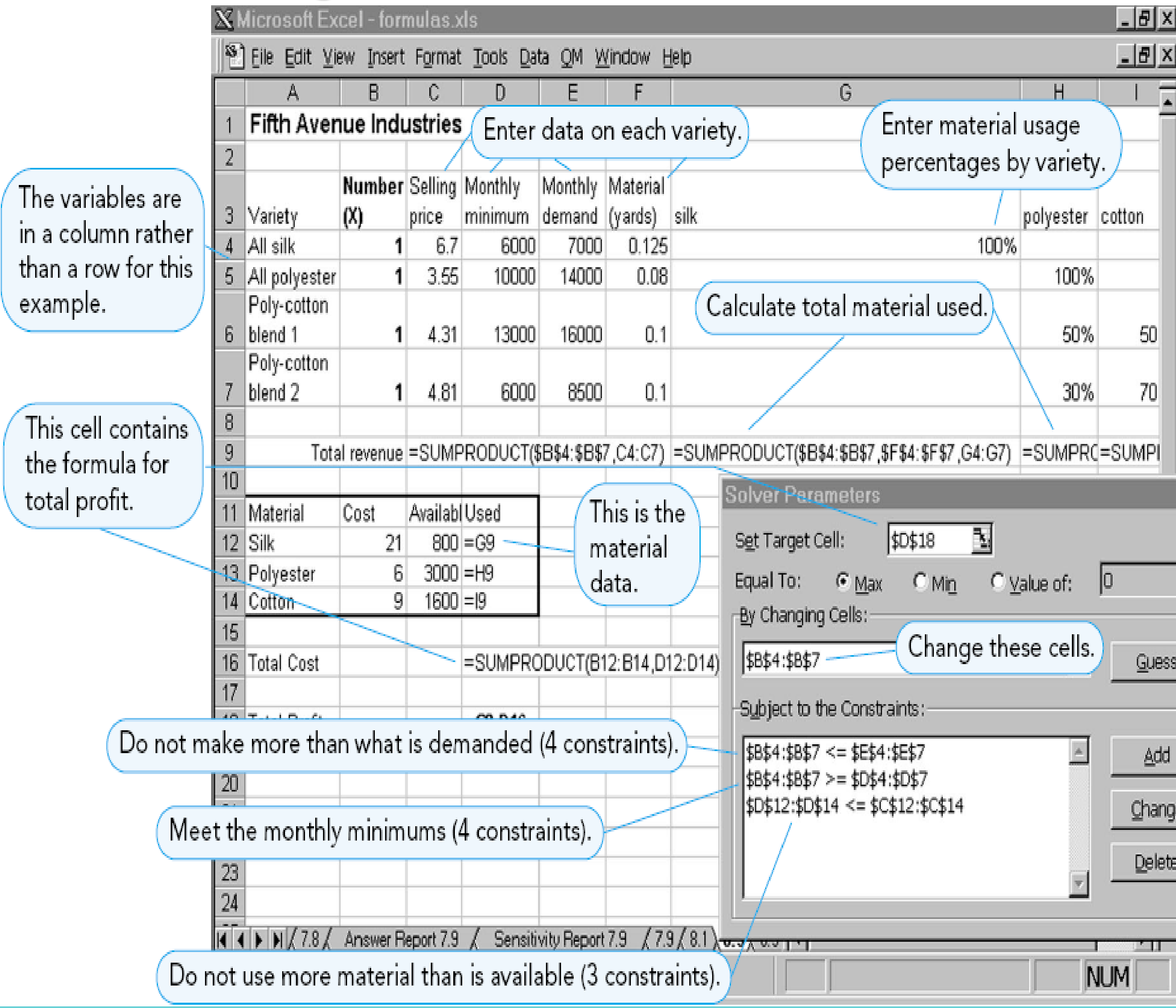
Manufacturing Applications
Fifth Avenue

- Using Solver in Excel, the computer-generated solution is to produce
  1. 6,400 all-silk ties each month;
  2. 14,000 all-polyester ties;
  3. 16,000 poly–cotton blend 1 ties; and
  4. 8,500 poly–cotton blend 2 ties.

- This produces a profit of $160,020 per production period.
- See Programs 8.3A and 8.3B on the next page for details.

Manufacturing Applications Fifth Avenue

Formulation for Fifth Avenue Industries LP Problem Using Solver



Manufacturing Applications
Fifth Avenue

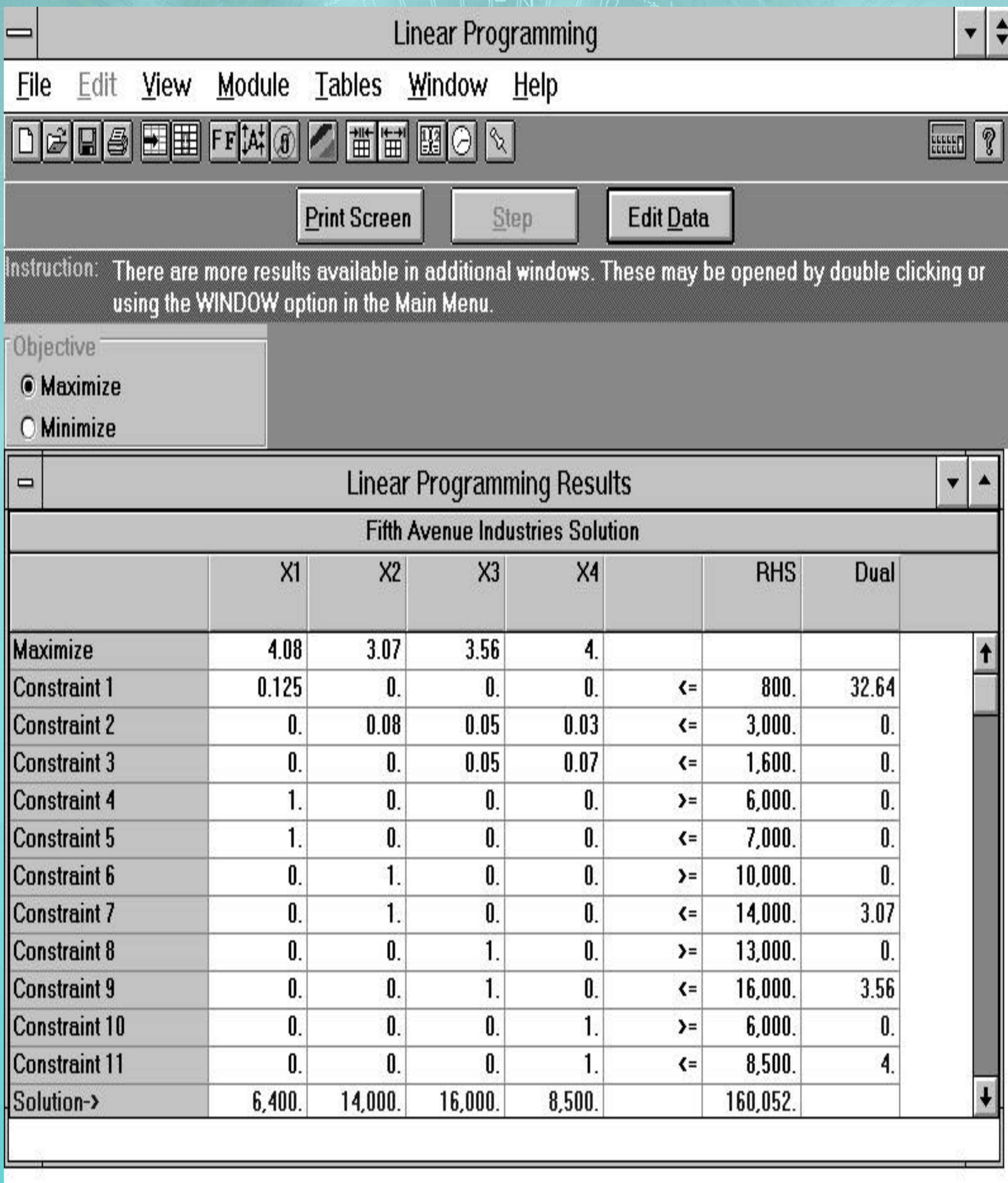
Excel Output for Solution to the LP Problem

Microsoft Excel - captures.xls

File Edit View Insert Format Tools Data Accounting OM Window Help

	A	B	C	D	E	F	G	H	I
1	**Fifth Avenue Industries**								
2									
3	Variety	**Number (X)**	Selling price	Monthly minimum	Monthly demand	Material (yards)	silk	polyester	cotton
4	All silk	**6400**	6.7	6000	7000	0.125	100%		
5	All polyester	**14000**	3.55	10000	14000	0.08		100%	
6	Poly-cotton blend 1	**16000**	4.31	13000	16000	0.1		50%	50%
7	Poly-cotton blend 2	**8500**	4.81	6000	8500	0.1		30%	70%
8									
9		Total revenue	202425				800	2175	1395
10									
11	Material	Cost	Available	Used					
12	Silk	21	800	800					
13	Polyester	6	3000	2175					
14	Cotton	9	1600	1395					
15									
16	Total Cost			42405					
17									
18	Total Profit			**160020**					

Fifth Avenue QM for Windows



	X1	X2	X3	X4		RHS	Dual
Maximize	4.08	3.07	3.56	4.			
Constraint 1	0.125	0.	0.	0.	<=	800.	32.64
Constraint 2	0.	0.08	0.05	0.03	<=	3,000.	0.
Constraint 3	0.	0.	0.05	0.07	<=	1,600.	0.
Constraint 4	1.	0.	0.	0.	>=	6,000.	0.
Constraint 5	1.	0.	0.	0.	<=	7,000.	0.
Constraint 6	0.	1.	0.	0.	>=	10,000.	0.
Constraint 7	0.	1.	0.	0.	<=	14,000.	3.07
Constraint 8	0.	0.	1.	0.	>=	13,000.	0.
Constraint 9	0.	0.	1.	0.	<=	16,000.	3.56
Constraint 10	0.	0.	0.	1.	>=	6,000.	0.
Constraint 11	0.	0.	0.	1.	<=	8,500.	4.
Solution->	6,400.	14,000.	16,000.	8,500.		160,052.	

Manufacturing Applications

Production Scheduling

- This type of problem resembles the product mix model for each period in the future.
- The objective is either to maximize profit or to minimize the total cost (production plus inventory) of carrying out the task.
- Production scheduling is well suited to solution by LP because it is a problem that must be solved on a regular basis.
- When the objective function and constraints for a firm are established, the inputs can easily be changed each month to provide an updated schedule.

Manufacturing Applications

Production Scheduling

- Setting a low-cost production schedule over a period of weeks or months is a difficult and important management problem.
- The production manager has to consider many factors:
  - labor capacity,
  - inventory and storage costs,
  - space limitations,
  - product demand, and
  - labor relations.
- Because most companies produce more than one product, the scheduling process is often quite complex.

Employee Scheduling Applications

Assignment Problems

- Assignment problems involve finding the most efficient assignment of
  - people to jobs,
  - machines to tasks,
  - police cars to city sectors,
  - salespeople to territories, and so on.
- The objective might be
  - to minimize travel times or costs or
  - to maximize assignment effectiveness.
- We can assign people to jobs using LP or use a special assignment algorithm discussed in Chapter 10.

Employee Scheduling Applications

Assignment Problems

- Assignment problems are unique because
  - they have a coefficient of 0 or 1 associated with each variable in the LP constraints;
  - the right-hand side of each constraint is always equal to 1.
- The use of LP in solving assignment problems, as shown in the example that follows, yields solutions of either 0 or 1 for each variable in the formulation.

Assignment Problem
Ivan and Ivan Law Firm Example

- The law firm maintains a large staff of young attorneys.
- Ivan, concerned with the effective utilization of its personnel resources, seeks some objective means of making lawyer-to-client assignments.
- Seeking to maximize the overall effectiveness of the new client assignments, Ivan draws up the table on the following slide, in which the estimated effectiveness (on a scale of 1 to 9) of each lawyer on each new case is rated.

Assignment Problem
Ivan and Ivan Law Firm Example

Ivan's Effectiveness Ratings

	CLIENT'S CASE			
LAWYER	DIVORCE	CORPORATE MERGER	EMBEZZLEMENT	EXHIBITIONISM
Adams	6	2	8	5
Brooks	9	3	5	8
Carter	4	8	3	4
Darwin	6	7	6	4

To solve using LP, double-subscripted variables are used.

Let

$$X_{ij} = \begin{cases} 1 \text{ if attorney } i \text{ if assigned to case } j \\ 0 \text{ otherwise} \end{cases}$$

where

i = 1,2,3,4 for Adams, Brooks, Carter, and Darwin, respectively

j = 1,2,3,4 for divorce, merger, embezzlement, and exhibitionism, respectively

Assignment Problem
Ivan and Ivan Law Firm Example

The LP formulation follows:

$$\text{maximize effectiveness} = 6X_{11} + 2X_{12} + 8X_{13} + 5X_{14} + 9X_{21} + 3X_{22} + 5X_{23} + 8X_{24} + 4X_{31} + 8X_{32} + 3X_{33} + 4X_{34} + 6X_{41} + 7X_{42} + 6X_{43} + 4X_{44}$$

Coefficients correspond to effectiveness table

Darwin has a rating of 6 in Divorce

Subject to:

$X_{11} + X_{21} + X_{31} + X_{41} = 1$ (divorce case)

$X_{12} + X_{22} + X_{32} + X_{42} = 1$ (merger)

$X_{13} + X_{23} + X_{33} + X_{43} = 1$ (embezzlement)

$X_{14} + X_{24} + X_{34} + X_{44} = 1$ (exhibitionism)

$X_{11} + X_{12} + X_{13} + X_{14} = 1$ (Adams)

$X_{21} + X_{22} + X_{23} + X_{24} = 1$ (Brooks)

$X_{31} + X_{32} + X_{33} + X_{34} = 1$ (Carter)

$X_{41} + X_{42} + X_{43} + X_{44} = 1$ (Darwin)

Assignment Problem
Ivan and Ivan Law Firm Example

The law firm's problem can be solved in QM for Windows and Solver in Excel. There is a total effectiveness rating of 30 by letting $X_{13} = 1$, $X_{24} = 1$, $X_{32} = 1$, and $X_{41} = 1$. All other variables are equal to zero.

QM for Windows Solution Output

QM for Windows - C:\Prentice\Data\RenderStair7\Ivan.LIN

Objective
- (•) Maximize
- ( ) Minimize

Linear Programming Results

Ivan and Ivan Solution

	x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	x41	x42	x43	x44		RHS	Dual
Maximize	6.	2.	8.	5.	9.	3.	5.	8.	4.	8.	3.	4.	6.	7.	6.	4.			
Divorce	1.	1.	1.	1.													=	1.	5.
Merger					1.	1.	1.	1.									=	1.	8.
Embezzlement									1.	1.	1.	1.					=	1.	4.
Exhibitionism													1.	1.	1.	1.	=	1.	5.
Adams	1.				1.				1.				1.				=	1.	1.
Brooks		1.				1.				1.				1.			=	1.	4.
Carter			1.				1.				1.				1.		=	1.	3.
Darwin				1.				1.				1.				1.	=	1.	0.
Solution->	0.	0.	1.	0.	0.	0.	0.	1.	0.	1.	0.	0.	1.	0.	0.	0.		30.	

Employee Scheduling Applications

Labor Planning

- Labor planning problems address staffing needs over a specific time period.
- They are especially useful when managers have some flexibility in assigning workers to jobs that require overlapping or interchangeable talents.
- Large banks frequently use LP to tackle their labor scheduling.

Alternate Optimal Solutions

- Alternate optimal solutions are common in many LP problems.
- The sequence in which you enter the constraints into QM for Windows can affect the solution found.
- No matter the final variable values, every optimal solution will have the same objective function value.

Financial Applications

Portfolio Selection

- The selection of specific investments from a wide variety of alternatives is a problem frequently encountered by
  - managers of banks,
  - mutual funds,
  - investment services, and
  - insurance companies.
- The manager's overall objective is usually to maximize expected return on investment
  - given a set of legal, policy, or risk restraints.

Financial Applications
Portfolio Selection

International City Trust (ICT)

- ICT has $5 million available for immediate investment and wishes to do two things:

  (1) maximize the interest earned on the investments made over the next six months, and

  (2) satisfy the diversification requirements as set by the board of directors.

Financial Applications

Portfolio Selection

International City Trust (ICT)

The specifics of the investment possibilities are as follows:

INVESTMENT	INTEREST EARNED (%)	MAXIMUM INVESTMENT ($ MILLIONS)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stocks	19	1.5
Construction loans	15	1.8

Also, the board specifies that

- at least 55% of the funds invested must be in gold stocks and construction loans, and that
- no less than 15% be invested in trade credit.

Financial Applications

Portfolio Selection

International City Trust (ICT)

Formulate ICT's decision as an LP problem.

Let

X_1 = dollars invested in trade credit

X_2 = dollars invested in corporate bonds

X_3 = dollars invested in gold stocks

X_4 = dollars invested in construction loans

The objective function formulation is presented on the next slide.

Financial Applications

Portfolio Selection

International City Trust (ICT)

Objective:

maximize dollars of interest earned =

$0.07X_1 + 0.11X_2 + 0.19X_3 + 0.15X_4$

subject to:

$$
\begin{array}{lllll}
X_1 & & & & \leq 1{,}000{,}000 \\
 & X_2 & & & < 2{,}500{,}000 \\
 & & X_3 & & < 1{,}500{,}000 \\
 & & & X_4 & < 1{,}800{,}000 \\
 & & X_3 + & X_4 & > 0.55(X_1+X_2+X_3+X_4) \\
X_1 & & & & > 0.15(X_1+X_2+X_3+X_4) \\
X_1 + & X_2 + & X_3 + & X_4 & < 5{,}000{,}000 \\
 & & X_1, X_2, X_3, X_4 & & > 0
\end{array}
$$

SOLUTION:

Maximum interest is earned by making X_1 = \$750,000, X_2 = \$950,000, X_3 = \$1,500,000, and X_4 = \$1,800,000.

Total interest earned = \$712,000

Transportation Applications

Shipping Problem

- Transporting goods from several origins to several destinations efficiently is called the "transportation problem."
- It can be solved with LP or with a special algorithm introduced in Chapter 10.
- The transportation or shipping problem involves determining the amount of goods or items to be transported from a number of origins to a number of destinations.

Transportation Applications

Shipping Problem

- The objective usually is to minimize total shipping costs or distances.
- Constraints in this type of problem deal with capacities at each origin and requirements at each destination.
- The transportation problem is a very specific case of LP; a special algorithm has been developed to solve it.

Transportation Applications

Shipping Problem

The Top Speed Bicycle Co.

- The firm has final assembly plants in two cities in which labor costs are low,
  - New Orleans and Omaha.
- Its three major warehouses are located near the large market areas of
  - New York, Chicago, and Los Angeles.
- The sales requirements and factory capacities for each city are as follows:

CITY	DEMAND	CAPACITY
New York	10,000	NA
Chicago	8,000	NA
Los Angeles	15,000	NA
New Orleans	NA	20,000
Omaha	NA	15,000

Transportation Applications

Shipping Problem

The Top Speed Bicycle Co.

- The cost of shipping one bicycle from each factory to each warehouse differs, and these unit shipping costs are as follows:

To \ From	New Orleans	Omaha
New York	$2	$3
Chicago	$3	$1
Los Angeles	$5	$4

- The company wishes to develop a shipping schedule that will minimize its total annual transportation costs.

Transportation Applications

Shipping Problem

The Top Speed Bicycle Co.

Let

X_{11} = bicycles shipped from New Orleans to New York

X_{12} = bicycles shipped from New Orleans to Chicago

X_{13} = bicycles shipped from New Orleans to Los Angeles

X_{21} = bicycles shipped from Omaha to New York

X_{22} = bicycles shipped from Omaha to Chicago

X_{23} = bicycles shipped from Omaha to Los Angeles

- In a transportation problem, there will be one constraint for each demand source and one constraint for each supply destination.

Transportation Applications
Shipping Problem

The Top Speed Bicycle Co.

Objective:

Minimize total shipping costs =

$$2X_{11} + 3X_{12} + 5X_{13} + 3X_{21} + 1X_{22} + 4X_{23}$$

Subject to:

$$X_{11} + X_{21} = 10{,}000$$
$$X_{12} + X_{22} = 8{,}000$$
$$X_{13} + X_{23} = 15{,}000$$
$$X_{11} + X_{12} + X_{13} \leq 10{,}000$$
$$X_{21} + X_{22} + X_{23} \leq 15{,}000$$
$$\text{All variables} \geq 0$$

Transportation Applications

Shipping Problem

The Top Speed Bicycle Co.

- Using Solver in Excel, the computer-generated solution to Top Speed's problem is shown in the following table and in the screen shot on the next slide.
- The total shipping cost is $96,000.

From / **To**	New Orleans	Omaha
New York	10,000	0
Chicago	0	8,000
Los Angeles	8,000	7,000

Transportation Applications

Shipping Problem

The Top Speed Bicycle Co.

Solution Output From Solver in Excel:

Microsoft Excel - captures.xls

File Edit View Insert Format Tools Data Accounting OM Window Help

Top Speed Bicycle Company

Transportation

Enter the transportation costs, supplies and demands in the shaded area. Then go to TOOLS, SOLVER, SOLVE on the menu bar at the top.
If SOLVER is not a menu option in the Tools menu then go to TOOLS, ADD-INS.

Data

COSTS	New York	Chicago	Los Angel	Supply
New Orleans	2	3	5	20000
Omaha	3	1	4	15000
Demand	10000	8000	15000	33000 \ 35000

Shipments

Shipments	New York	Chicago	Los Angel	Row Total
New Orleans	**10000**	**0**	**8000**	18000
Omaha	**0**	**8000**	**7000**	15000
Column Total	10000	8000	15000	33000 \ 33000

Total Cost **96000**

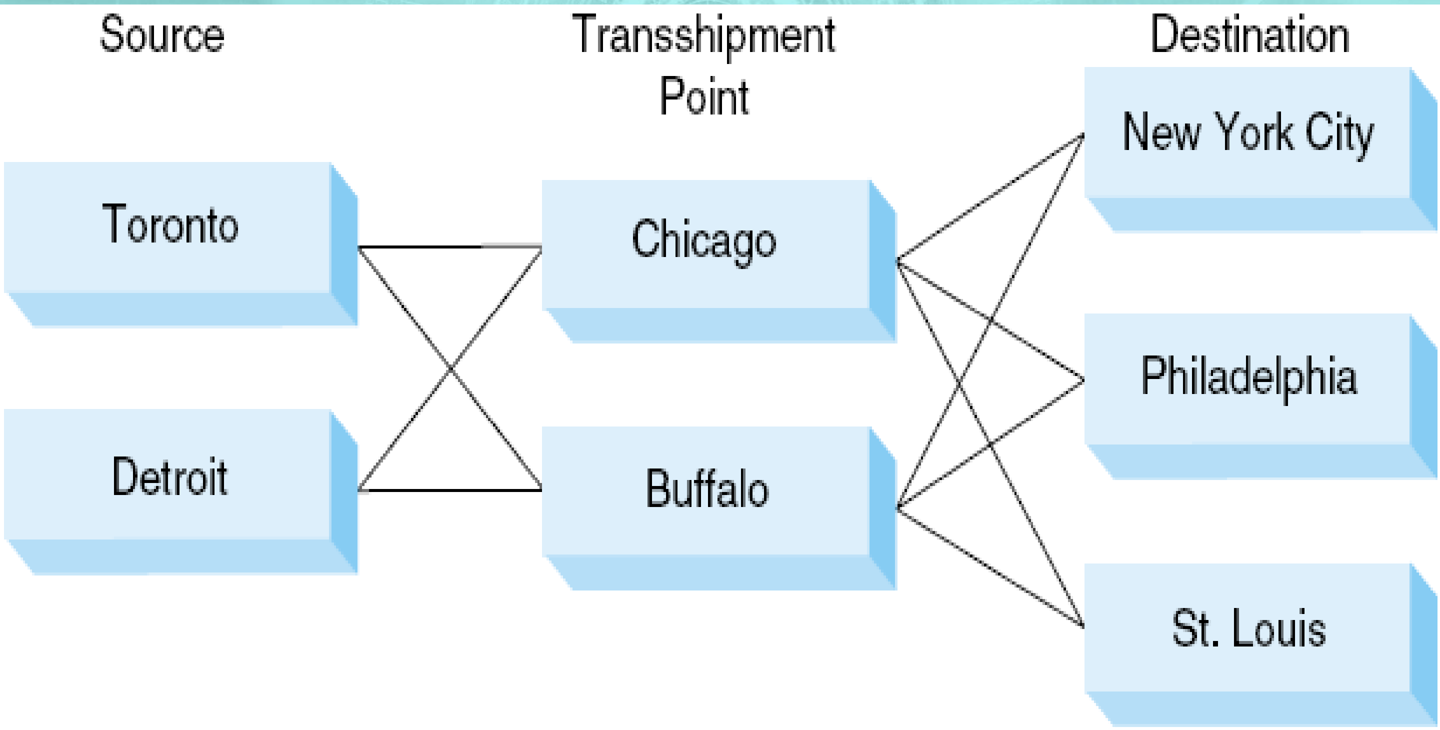
Transshipment Applications

- The transportation problem is actually a special case of the transshipment problem.
- If items are being transported from the source through an intermediate point (called a ***transshipment point***) before reaching a final destination, then the problem is called a ***transshipment problem.***
- For example, a company might be manufacturing a product at several factories to be shipped to a set of regional distribution centers. From these centers the items are shipped to retail outlets that are the final destinations.

Transshipment Applications
Distribution Centers

Frosty Machines

The figure below illustrates the basic network representation of this example:



Transshipment Applications
Distribution Centers

Frosty Machines

- The shipping costs vary, seen in the following table.
- Forecasted demands for New York, Philadelphia, and St. Louis are also seen in this table, as are the available supplies.

FROM	TO: CHICAGO	BUFFALO	NEW YORK CITY	PHILADELPHIA	ST. LOUIS	SUPPLY
Toronto	$4	$7	—	—	—	800
Detroit	$5	$7	—	—	—	700
Chicago	—	—	$6	$4	$5	—
Buffalo	—	—	$2	$3	$4	—
Demand	—	—	450	350	300	

Transshipment Applications

Distribution Centers

Frosty Machines

- The goal is to minimize the transportation costs associated with shipping sufficient supply to meet the demands at the three destinations while not exceeding the supply of each factory.
- Therefore, we have supply and demand constraints similar to the transportation problem.
- Since there are no units being produced in Chicago or Buffalo, anything shipped from these transshipment points must have arrived from either Toronto or Detroit.
- Therefore, Chicago and Buffalo will each have a constraint indicating this.

Transshipment Applications
Distribution Centers

Frosty Machines

- Statement of the Problem:

Minimize cost, subject to:

1. The number of units shipped from Toronto is not more than 800.
2. The number of units shipped from Detroit is not more than 700.
3. The number of units shipped to New York is 450.
4. The number of units shipped to Philadelphia is 350.
5. The number of units shipped to St. Louis is 300.
6. The number of units shipped out of Chicago is equal to the number of units shipped into Chicago.
7. The number of units shipped out of Buffalo is equal to the number of units shipped into Buffalo.

Ingredient Blending Applications
Diet Problems

- The *diet problem*, one of the earliest applications of LP, was originally used by hospitals to determine the most economical diet for patients.
- Known in agricultural applications as the *feed mix problem*, the diet problem involves specifying a food or feed ingredient combination that satisfies stated nutritional requirements at a minimum cost level.

Ingredient Blending Applications

Diet Problems

The Whole Food Nutrition Center

The cost of each bulk grain and the protein, riboflavin, phosphorus, and magnesium units per pound of each are shown below:

GRAIN	COST PER POUND (CENTS)	PROTEIN (UNITS/LB)	RIBOFLAVIN (UNITS/LB)	PHOSPHORUS (UNITS/LB)	MAGNESIUM (UNITS/LB)
A	33	22	16	8	5
B	47	28	14	7	0
C	38	21	25	9	6

- The minimum adult daily requirement (called the U.S. Recommended Daily Allowance, or USRDA)
  - for protein is 3 units;
  - for riboflavin, 2 units;
  - for phosphorus, 1 unit; and
  - for magnesium, 0.425 unit.
- Whole Food wants to select the blend of grains that will meet the USRDA at a minimum cost.

Ingredient Blending Applications
Diet Problems

The Whole Food Nutrition Center

Let

X_A = pounds of grain A in one serving
X_B = pounds of grain B in one serving
X_C = pounds of grain C in one serving

Objective function:

minimize total cost of mixing a serving =

$$\$0.33X_A + \$0.47X_B + \$0.38X_C$$

subject to:

$22X_A + 28X_B + 21X_C \geq 3$	(protein units)
$16X_A + 14X_B + 25X_C \geq 2$	(riboflavin units)
$8X_A + 7X_B + 9X_C \geq 1$	(phosphorus units)
$5X_A + 0X_B + 6X_C \geq 0.425$	(magnesium units)
$X_A + X_B + X_C = 0.125$	(total is 0.125 lbs.)

$$X_A, X_B, X_C \geq 0$$

Ingredient Blending Applications
Diet Problems

The Whole Food Nutrition Center

- The solution to this problem requires mixing together
  1. 0.025 lb of grain A,
  2. 0.050 lb of grain B, and
  3. 0.050 lb of grain C.
- Another way of stating the solution is in terms of the proportion of the 2-ounce serving of each grain, namely,
  1. 0.4 ounce of grain A,
  2. 0.8 ounce of grain B, and
  3. 0.8 ounce of grain C in each serving.
- The cost per serving is $0.05.
- The next slide illustrates this solution using the QM for Windows software package.

Ingredient Blending Applications

Diet Problems

QM for Windows - C:\Prentice\Data\RenderStair7\Whole417.lin

Objective

- Maximize
- Minimize

Linear Programming Results

Whole Food Nutrition Center Solution

	X1	X2	X3		RHS	Dual
Minimize	0.33	0.47	0.38			
Constraint 1	22.	28.	21.	>=	3.	-0.038
Constraint 2	16.	14.	25.	>=	2.	0.
Constraint 3	8.	7.	9.	>=	1.	-0.088
Constraint 4	5.	0.	6.	>=	0.425	0.
Constraint 5	1.	1.	1.	=	0.125	1.21
Solution->	0.025	0.05	0.05		0.05	

Ingredient Blending Applications

Ingredient Mix or Blending Problems

- Diet and feed mix problems are actually special cases of a more general class of LP problems known as ***ingredient mix*** **or** ***blending problems***.
- Blending problems arise when a decision must be made regarding the blending of two or more resources to produce one or more products.
- Resources, in this case, contain one or more essential ingredients that must be blended so that each final product contains specific percentages of each ingredient.

Post Office Example

(Winston)

A PO requires different numbers of employees on different days of the week. Union rules state each employee must work 5 consecutive days and then receive two days off. Find the minimum number of employees needed.

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Staff Needed	17	13	15	19	14	16	11

The decision variables are xi (# of employees starting on day i)

$$\min z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

$$x_1 + x_4 + x_5 + x_6 + x_7 \geq 17$$

$$x_1 + x_2 + x_5 + x_6 + x_7 \geq 13$$

$$x_1 + x_2 + x_3 + x_6 + x_7 \geq 15$$

$$x_1 + x_2 + x_3 + x_4 + x_7 \geq 19$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 14$$

$$+ x_2 + x_3 + x_4 + x_5 + x_6 \geq 16$$

$$+ x_3 + x_4 + x_5 + x_6 + x_7 \geq 11$$

Sailco Example

- Sailco must determine how many sailboats to produce in the next 4 quarters. The demand is known to be 40, 60, 75, and 25 boats. Sailco must meet its demands. At the beginning of the 1st quarter Sailco starts with 10 boats in inventory. Sailco can produce up to 40 boats with regular time labor at $400 per boat, or additional boats at $450 with overtime labor. Boats made in a quarter can be used to meet that quarter's demand or held in inventory for the next quarter at an extra cost of $20.00 per boat

Minimize the total cost!

The decision variables are for $t = 1,2,3,4$

xt = # of boats in quarter t built in regular time

yt = # of boats in quarter t built in overtime

it = # of boats in inventory at the end of the quarter t

dt = demand in quarter t

We are given that

$xt \leq 40, \forall t$

$it = it\text{-}1 + xt + yt - dt, \forall t.$

Demand is met iff $it \geq 0, \forall t$

$xt, yt \geq 0, \quad \forall t$

$$Min\ z = 400\ (x1 + x2 + x3 + x4) + 450\ (y1 + y2 + y3 + y4) + 20\ (i1 + i2 + i3 + i4)$$

Customer Service Level Example

CSL services computers. Its demand (hours) for the time of skilled technicians in the next 5 months is

t	Jan	Feb	Mar	Apr	May
dt	6000	7000	8000	9500	11000

It starts with 50 skilled technicians at the beginning of January. Each technician can work 160 hrs/month. To train a new technician they must be supervised for 50 hrs by an experienced technician for a period of one month time. Each experienced technician is paid $2K/mth and a trainee is paid $1K/mth. Each month 5% of the skilled technicians leave. CSL needs to meet demand and minimize costs.

Let

xt = # to be trained in month t

yt = # experienced tech. start at t-th month

dt = demand during the month t

Then, we must minimize the total cost.

min $z = 2000\ (y1 + .. + y5) + 1000\ (x1 + .. + x5)$

subject to

$160yt - 50xt \geq dt$ for $t = 1, \ldots, 5$

$y1 = 50$

$yt = .95yt\text{-}1 + xt\text{-}1$ for $t = 2,3,4,5$

$xt, yt \geq 0$