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BLG202E

HW1

Q1)

a)

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$$1621 = 810 \cdot 2 + 1$$
$$810 = 405 \cdot 2 + 0$$
$$405 = 202 \cdot 2 + 1$$
$$202 = 101 \cdot 2 + 0$$
$$101 = 50 \cdot 2 + 1$$
$$50 = 25 \cdot 2 + 0$$
$$25 = 12 \cdot 2 + 1$$
$$12 = 6 \cdot 2 + 0$$
$$6 = 3 \cdot 2 + 0$$
$$3 = 1 \cdot 2 + 1$$
$$1 = 0 \cdot 2 + 1$$
$$0 = 0 \cdot 2 + 0$$

Remainders from bottom to top show binary representation.

$$(1621)_{10} = (11001010101)_2$$

b)

b) $\frac{443}{2048} = \underline{\underline{0}},\ldots$ so the non fractional part of the binary representation must be 0. $(0 = 0.2 + \underline{\underline{0}})$

$$\frac{443}{2048} \cdot 2 = \frac{443}{1024} = \underline{\underline{0}},\ldots$$

$$\frac{443}{1024} \cdot 2 = \frac{443}{512} = \underline{\underline{0}},\ldots$$

$$\frac{443}{512} \cdot 2 = \frac{443}{256} = \underline{\underline{1}},\ldots$$

$$\left(\frac{443}{256} - 1\right) \cdot 2 = \frac{187}{128} = \underline{\underline{1}},\ldots$$

Integral parts from top to bottom show binary representation of fractional part.

$$\frac{443}{512} \cdot 2 = \frac{443}{256} = \underline{\underline{1}}, \ldots$$ fractional part.

$$\left(\frac{443}{256} - 1\right) \cdot 2 = \frac{187}{128} = \underline{\underline{1}}, \ldots$$

$$\left(\frac{187}{128} - 1\right) \cdot 2 = \frac{59}{64} = \underline{\underline{0}}, \ldots$$

$$\frac{59}{64} \cdot 2 = \frac{59}{32} = \underline{\underline{1}}, \ldots$$

$$\left(\frac{59}{32} - 1\right) \cdot 2 = \frac{27}{16} = \underline{\underline{1}}, \ldots$$

$$\left(\frac{27}{16} - 1\right) \cdot 2 = \frac{11}{8} = \underline{\underline{1}}, \ldots$$

$$\left(\frac{11}{8} - 1\right) \cdot 2 = \frac{3}{4} = \underline{\underline{0}}, \ldots$$

$$\frac{3}{4} \cdot 2 = \frac{3}{2} = \underline{\underline{1}}, \ldots$$

$$\left(\frac{3}{2} - 1\right) \cdot 2 = 1 = \underline{\underline{1}}, 0$$

$$0 \cdot 2 = 0$$

Therefore,

$$\left(\frac{448}{2048}\right)_{10} = (0.00110111011)_2$$

Q2)

Execution results:

```
PS C:\Users\mcanc> & C:/Users/mcanc/AppData/Local/Programs/Python/Python311/python.exe c:/U
sers/mcanc/OneDrive/Desktop/q2.py
Enter a rational number in either x.yzt or x/y format: -443/2048
The binary representation of given rational number is:
-0.00110111011
PS C:\Users\mcanc>
```

```
PS C:\Users\mcanc> & C:/Users/mcanc/AppData/Local/Programs/Python/Python311/python.exe c:/U
sers/mcanc/OneDrive/Desktop/q2.py
Enter a rational number in either x.yzt or x/y format: 11.25
The binary representation of given rational number is:
1011.01
PS C:\Users\mcanc>
```

Q3)

Bisection method:

initial interval $= [a, b] = [\sqrt[3]{8}, \sqrt[3]{27}]$

$\sqrt[3]{8} < \sqrt[3]{17} < \sqrt[3]{27}$

Let f(x) = x^3 - 17
This comparison implies f(5/2) * f(27^(1/3)) < 0
(int value theorem)

(1) $\frac{a+b}{2} = \frac{5}{2} \rightarrow \frac{5}{2} < \sqrt[3]{17} \rightarrow$ new interval $= [\frac{5}{2}, \sqrt[3]{27}]$

$\frac{5}{2} < \sqrt[3]{17} < \sqrt[3]{27}$

This comparison implies f(5/2) * f(11/4) < 0
(int value theorem)

(2) $\frac{a+b}{2} = \frac{11}{4} \rightarrow \frac{11}{4} > \sqrt[3]{17} \rightarrow$ new interval. $= [\frac{5}{2}, \frac{11}{4}]$

$\frac{5}{2} < \sqrt[3]{17} < \frac{11}{4}$

(3) $\frac{a+b}{2} = \frac{21}{8} \rightarrow \frac{21}{8} > \sqrt[3]{17} \rightarrow$ new interval $= [\frac{5}{2}, \frac{21}{8}]$

$\frac{5}{2} < \sqrt[3]{17} < \frac{21}{8}$

(4) $\frac{a+b}{2} = \frac{41}{16} \rightarrow \frac{41}{16} < \sqrt[3]{17} \rightarrow$ new interval $= [\frac{41}{16}, \frac{21}{8}]$

$\frac{41}{16} < \sqrt[3]{17} < \frac{21}{8}$

(5) $\frac{a+b}{2} = \frac{83}{32} \rightarrow \frac{83}{32} > \sqrt[3]{17} \rightarrow$ new interval $= [\frac{41}{16}, \frac{83}{32}]$

$\vdots$

$$\frac{41}{16} < \sqrt[3]{17} < \frac{83}{32}$$

(6) $x_6 = \frac{a+b}{2} = \boxed{\frac{165}{64}}$ → Best guess after 6 iteration.

$$\text{error} \Rightarrow |x^* - x_n| \leq \frac{b-a}{2} \cdot 2^{-n}$$

$$= |x^* - \tfrac{165}{64}| \leq \tfrac{1}{2} \cdot 2^{-6}$$

↳ obtained from initial interval.

$$= |x^* - \tfrac{165}{64}| \leq 2^{-7}$$

$$\boxed{\text{max. error} = 2^{-7} = 0.0078125}$$

absolute error = |17^(1/3) - 165/64| = 0.00684340934

Q4)

Execution results:

```
PS C:\Users\mcanc> & C:/Users/mcanc/AppData/Local/Programs/Python/Python311/python.exe c:/Users/mcanc/O
neDrive/Desktop/q4.py
Enter a: 243
Enter e: 0.01
The approximation is:
3.003387451171875
Absolute Error:
0.003387451171875
PS C:\Users\mcanc> 
```

```
PS C:\Users\mcanc> & C:/Users/mcanc/AppData/Local/Programs/Python/Python311/python.exe c:/Users/mcanc/O
neDrive/Desktop/q4.py
Enter a: 3125
Enter e: 0.5
The approximation is:
4.9591064453125
Absolute Error:
0.04089355468750089
PS C:\Users\mcanc> 
```