BLG202E

HW3

MUSTAFA CAN ÇALIŞKAN

150200097

Q1)

(Q1) Using Rayleight Quotient, we can find the dominant eigen-value which is belong to eigenvector that have found using power method.

$$\lambda = \frac{Ax \cdot X}{X \cdot X} \rightarrow \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} h \\ 1 \end{bmatrix} = \begin{bmatrix} 5h+2 \\ 4h+7 \end{bmatrix}$$

$$(5h+2, 4h+7) \cdot (h, 1) = 5h^2 + 2h + 4h + 7 = 5h^2 + 6h + 7$$

$$(h, 1) \cdot (h, 1) = h^2 + 1$$

$$\boxed{\lambda = \frac{5h^2 + 6h + 7}{h^2 + 1}}$$

Q2)

Q2)

a)

i	x_i	$f[x_i]$	$f[x_{i-1}, x_i]$	...		
0	0	$\boxed{1}$	$\frac{9-1}{1-0} = \boxed{8}$	$\frac{14-8}{2-0} = \boxed{3}$	$\frac{7-3}{4-0} = \boxed{1}$	$\frac{1-1}{6-0} = \boxed{0}$
1	1	9	$\frac{23-9}{2-1} = 14$	$\frac{35-14}{4-1} = 7$	$\frac{12-7}{6-1} = 1$	
2	2	23	$\frac{93-23}{4-2} = 35$	$\frac{83-35}{6-2} = 12$		
3	4	93	$\frac{259-93}{6-4} = 83$			
4	6	259				

c) The diagonal entries yield the coefficients.

$$f(x) = 1 + 8(x-0) + 3(x-0)(x-1) + 1(x-0)(x-1)(x-2) + 0(x-0)(x-1)(x-2)(x-4)$$

$$= 1 + 8x + 3x^2 - 3x + x^3 - 2x^2 - x^2 + 2x$$

$$= x^3 + 7x + 1 \rightarrow \text{Newton Interpolation Polynomial.}$$

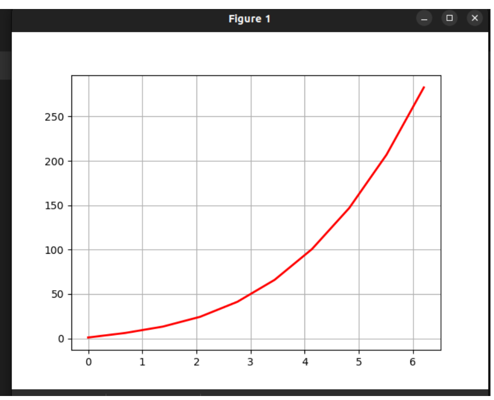
$$f(4.2) = (4.2)^3 + 7(4.2) + 1$$

$$= 104,488$$

PROBLEMS OUTPUT TERMINAL

TERMINAL

```
canetizen@mcc:~$ /bin/python3 /home/canetizen/q2.py
Enter x values in the format x0, x1, x2,...: 0,1,2,4,6
Enter y values in the format y0, y1, y2,...: 1,9,23,93,259
Enter an a value to determine y at x = a: 4.2
esting
Using Divided Difference method, y at x = 4.2 is: 104.48800000000003
```



Q3)

(Q_3)

a.i)

	$\theta_0(x) = 1$	$\theta_1(x) = x$	$\theta_2(x) = x^2$
$x = -1.2$	1	-1.2	1.44
$x = 0.3$	1	0.3	0.09
$x = 1.1$	1	1.1	1.21

$$\underbrace{\begin{bmatrix} 1 & -1.2 & 1.44 \\ 1 & 0.3 & 0.09 \\ 1 & 1.1 & 1.21 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c \\ b \\ a \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -5.76 \\ -5.61 \\ -3.69 \end{bmatrix}}_{b}$$

$Ax = b$

$x = A^{-1}b = \begin{bmatrix} -6 \\ 1 \\ 1 \end{bmatrix} \longrightarrow$ The polynomial is : $\boxed{x^2 + x - 6}$

Q3)

a.ii)

$$L_0(x) = \frac{x-x_1}{x_0-x_1}\cdot\frac{x-x_2}{x_0-x_2} = \frac{(x-0.3)(x-1.1)}{(-1.2-0.3)(-1.2-1.1)}$$

$$L_1(x) = \frac{(x-x_0)}{x_1-x_0}\cdot\frac{x-x_2}{x_1-x_2} = \frac{(x+1.2)(x-1.1)}{(0.3+1.2)(0.3-1.1)}$$

$$L_2(x) = \frac{(x-x_0)}{x_2-x_0}\cdot\frac{x-x_1}{x_2-x_1} = \frac{(x+1.2)(x-0.3)}{(1.1+1.2)(1.1-0.3)}$$

$$g(x) = L_0(x).f(x_0) + L_1(x).f(x_1) + L_2(x).f(x_2)$$

$$= \frac{(x-0.3)(x-1.1)}{3.45}.(-5.76) + \frac{(x+1.2)(x-1.1)}{-1.2}(-5.61)$$

$$+ \frac{(x+1.2)(x-0.3)}{1.84}(-3.69)$$

$$= -1.67(x-0.3)(x-1.1) + 4.68(x+1.2)(x-1.1) - 2(x+1.2)(x-0.3)$$

$$\sim \boxed{1.01x^2 + 1.006x - 6.0087}$$

```
22    plt.plot(x, lagrange(x), color='red', lw=2)
```

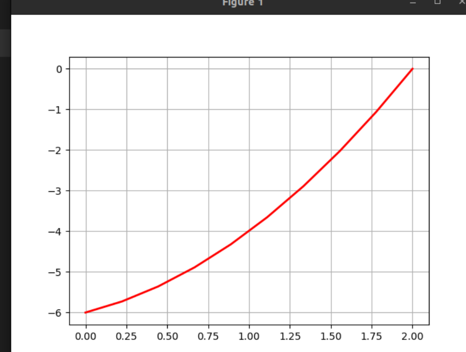
PROBLEMS OUTPUT TERMINAL

TERMINAL

```
canetizen@mcc:~$ /bin/python3 /home/canetizen/q3.py
Enter x values in the format x0, x1, x2,...: -1.2,0.3,1.1
Enter y values in the format y0, y1, y2,...: -5.76,-5.61,-3.69
Enter an a value to determine y at x = a: 0

Using Lagrange Interpolation method, y at x = 0.0 is: -5.999999999999999
```

Figure 1



$Q_3)$

c) From a.i, $f(x) = x^2 + x - 6$

$\boxed{f(0) = -6}$

Q4)

Q4)

a) $x_0 = 0.0 \quad f[x_0] = a$

$x_1 = 0.4 \quad f[x_1] = b$

$x_2 = 0.7 \quad f[x_2] = 6$

$$f[x_0, x_1] = \frac{b-a}{0.4-0} = c \longrightarrow \frac{3-a}{0.4} = 5 \to a = 1$$

$$f[x_1, x_2] = \frac{6-b}{0.7-0.4} = 10 \to \boxed{b = 3}$$

$$f[x_0, x_1, x_2] = \frac{10-c}{0.7-0} = \frac{50}{7} \to \boxed{c = 5}$$

$$\boxed{\begin{aligned} f[x_0] &= 1 \\ f[x_1] &= 3 \\ f[x_0, x_1] &= 5 \end{aligned}}$$

b)

i	x_i	$f[x_i]$	$f[x_{i-1}, x_i]$	...
0	0	$\boxed{1}$		
			$\frac{3-1}{0.4-0} = \boxed{5}$	
1	0.4	3		$\frac{10-5}{0.7-0.4} = \boxed{\frac{50}{7}}$
			$\frac{6-3}{0.7-0.4} = 10$	
2	0.7	6		

The diagonal entries yield the coefficients.

$$f(x) = 1 + 5(x-0) + \frac{50}{7}(x-0).(x-0.4) = \boxed{\frac{50}{7}x^2 + \frac{15}{7}x + 1}$$ (in order of x_0, x_1, x_2)

Q4)

c)

i	x_i	$f[x_i]$	$f[x_{i-1}, x_i]$	...
2	0.7	$\boxed{6}$		
			$\frac{3-6}{0.4-0.7} = \boxed{10}$	
1	0.4	3		$\frac{5-10}{0-0.7} = \boxed{\frac{50}{7}}$
			$\frac{1-3}{0-0.4} = 5$	
0	0	1		

The diagonal entries yield the coefficients.

$$f(x) = 6 + 10(x-0.7) + \frac{50}{7}(x-0.7)(x-0.4) = \boxed{\frac{50x^2}{7} + \frac{15x}{7} + 1}$$ (in order of x_2, x_1, x_0)

Q4)

d)

$$g(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + \left(\frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} \right)(x - x_0)(x - x_1)$$

$$h(x) = f(x_2) + \frac{f(x_1) - f(x_2)}{x_1 - x_2}(x - x_2) + \left(\frac{\frac{f(x_0) - f(x_1)}{x_0 - x_1} - \frac{f(x_1) - f(x_2)}{x_1 - x_2}}{x_0 - x_2} \right)(x - x_2)(x - x_1)$$

$$\left.\begin{aligned} g(x_0) &= h(x_0) \\ g(x_1) &= h(x_1) \\ g(x_2) &= h(x_2) \end{aligned}\right\}$$

$g(x)$ and $h(x)$ are the <u>second</u> order polynomials, and they have the same <u>y</u> values at the same <u>x</u> values for three points. Therefore the polynomials are the <u>same</u>.