

Standard Code Library

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Chapter 1

Algorithms and Datastructures

1.1 High Precision in C

```
#define maxlen 1000

struct HP { int len ,s[ maxlen ] ; };

void PrintHP(HP x){ for(int i=x.len ; i>=1;i--) cout<<x.s[ i ] ; }

void Str2HP(const char *s,HP &x)
{
    x.len=strlen(s);
    for(int i=1;i<=x.len ; i++) x.s[ i ]=s[ x.len-i ]- '0' ;
}

void Int2HP(int inte ,HP &x)
{
    if(inte==0) { x.len=1; x.s[ 1 ]=0; return ; }
    for(x.len=0;inte >0;){ x.s[ ++x.len ]=inte%10; inte /=10; }
}

void Multi(const HP a,const HP b,HP &c)
{
    int i ,j ; c.len=a.len+b.len ;
    for(i=1;i<=c.len ; i++) c.s[ i ]=0;
    for(i=1;i<=a.len ; i++) for(j=1;j<=b.len ; j++) c.s[ i+j-1 ]+=a.s[ i ]*b.s[ j ];
    for(i=1;i<c.len ; i++) { c.s[ i+1 ]+=c.s[ i ]/10; c.s[ i ]%=10; }
    while(c.s[ i ]) { c.s[ i+1 ]=c.s[ i ]/10; c.s[ i ]%=10; i++; }
    while(i>1 && !c.s[ i ]) i--; c.len=i;
}

void Plus(const HP a,const HP b,HP &c)
{
    int i ; c.s[ 1 ]=0;
    for(i=1; i<=a.len || i<=b.len || c.s[ i ] ; i++) {
        if(i<=a.len ) c.s[ i ]+=a.s[ i ];
        if(i<=b.len ) c.s[ i ]+=b.s[ i ];
        c.s[ i+1 ]=c.s[ i ]/10; c.s[ i ]%=10;
    }
    c.len=i-1; if( c.len ==0 ) c.len=1;
}

void Subtract(const HP a,const HP b,HP &c)
{
    for(int i=1,j=0; i<=a.len ; i++) {
        c.s[ i ]=a.s[ i ]-j; if(i<=b.len ) c.s[ i ]-=b.s[ i ];
        if(c.s[ i ]<0){ j=1 ; c.s[ i ]+=10; } else j=0;
    }
}
```

```

c . len=a . len ; while(c . len>1 && !c . s [ c . len ]) c . len --;
}

int HPCompare(const HP x ,const HP y)
{
    if(x . len>y . len ) return 1;
    if(x . len<y . len ) return -1;
    int i=x . len ;
    while(( i>1)&&(x . s [ i]==y . s [ i ])) i--;
    return x . s [ i ]-y . s [ i ];
}

void Divide(const HP a , const HP b , HP &c ,HP &d)
{
    int i ,j ; d . len=1; d . s [ 1]=0;
    for(i=a . len ;i>0;i--) {
        if (!(d . len==1 && d . s [ 1]==0))
            { for(j=d . len ;j>0;j--) d . s [ j+1]=d . s [ j]; ++d . len ; }
        d . s [ 1]=a . s [ i ]; c . s [ i]=0;
        while( ( j=HPCompare(d ,b))>=0 )
            { Subtract(d ,b,d ); c . s [ i]++; if(j==0) break ; }
    }
    c . len=a . len ; while(( c . len>1)&&(c . s [ c . len ]==0)) c . len --;
}

```

1.2 High Precision in C Plus Plus

```

const int maxlen = 10000;

class HP { public:
    int len , s [ maxlen ]; HP() { (*this)=0; };
    HP(int inte) { (*this)=inte; }; HP(const char*str) { (*this)=str; };
    friend ostream& operator<<(ostream &cout ,const HP &x );
    HP operator=(int inte); HP operator=(const char*str );
    HP operator*(const HP &b); HP operator+(const HP &b );
    HP operator-(const HP &b); HP operator/(const HP &b );
    HP operator%(const HP &b); int Compare(const HP &b );
};

ostream& operator<<(ostream &cout ,const HP &x )
{ for(int i=x . len ;i>=1;i--) cout<<x . s [ i ]; return cout; }

HP HP::operator=(const char *str)
{
    len=strlen(str );
    for(int i=1;i<=len ;i++) s [ i]=str [ len-i]-'0';
    return *this ;
}

HP HP::operator=(int inte)
{
    if(inte==0) { len=1; s [ 1]=0; return (*this ); };
    for(len=0;inte >0;){ s [ ++len]=inte %10; inte /=10;};
    return (*this );
}

HP HP::operator*(const HP &b)
{
    int i ,j ; HP c ; c . len=len+b . len ;
    for(i=1;i<=c . len ;i++) c . s [ i]=0;
    for(i=1;i<=len ;i++) for(j=1;j<=b . len ;j++) c . s [ i+j-1]+=s [ i ]*b . s [ j ];
    for(i=1;i<c . len ;i++) { c . s [ i+1]+=c . s [ i ]/10; c . s [ i ]%=10; }
}

```

```

while(c.s[i]) { c.s[i+1]=c.s[i]/10; c.s[i]%=10; i++; }
while(i>1 && !c.s[i]) i--; c.len=i;
return c;
}

HP HP::operator+(const HP &b)
{
    int i; HP c; c.s[1]=0;
    for(i=1; i<=len || i<=b.len || c.s[i] ; i++) {
        if(i<=len) c.s[i]+=s[i];
        if(i<=b.len) c.s[i]+=b.s[i];
        c.s[i+1]=c.s[i]/10; c.s[i]%=10;
    }
    c.len=i-1; if( c.len==0 ) c.len=1;
    return c;
}

HP HP::operator-(const HP &b)
{
    int i,j; HP c;
    for(i=1,j=0; i<=len ; i++) {
        c.s[i]=s[i]-j; if(i<=b.len) c.s[i]-=b.s[i];
        if(c.s[i]<0){ j=1 ; c.s[i]+=10; } else j=0;
    }
    c.len=len; while(c.len>1 && !c.s[c.len]) c.len--;
    return c;
}

int HP::Compare(const HP &y)
{
    if(len>y.len) return 1;
    if(len<y.len) return -1;
    int i=len;
    while((i>1)&&(s[i]==y.s[i])) i--;
    return s[i]-y.s[i];
}

HP HP::operator/(const HP &b)
{
    int i,j; HP d(0),c;
    for(i=len;i>0;i--) {
        if (!(d.len==1 && d.s[1]==0))
            { for(j=d.len;j>0;j--) d.s[j+1]=d.s[j]; ++d.len; }
        d.s[1]=s[i]; c.s[i]=0;
        while( (j=d.Compare(b))>=0 )
            { d=d-b; c.s[i]++; if(j==0) break; }
    }
    c.len=len; while((c.len>1)&&(c.s[c.len]==0)) c.len--;
    return c;
}

HP HP::operator%(const HP &b)
{
    int i,j; HP d(0);
    for(i=len;i>0;i--) {
        if (!(d.len==1 && d.s[1]==0))
            { for(j=d.len;j>0;j--) d.s[j+1]=d.s[j]; ++d.len; }
        d.s[1]=s[i];
        while( (j=d.Compare(b))>=0 ){ d=d-b; if(j==0) break; }
    }
    return d;
}

```

1.3 High Precision Floating-point Number

```
const int fprec = 100; // floating-point precision

HP zero=0;

class FS{public:
    FS(); void SetZero();
    FS(int inte) { (*this)=inte; }
    FS(char *s) { (*this)=s; }
    FS operator=(char *s); FS operator=(int inte);
    FS operator+(FS b); FS operator-(FS b);
    FS operator*(FS b); FS operator/(FS b);
    friend ostream& operator<<(ostream &cout, FS x);
    int sign, prec;
    HP num;
};

void FS::SetZero() { sign=1; num=0; prec=0; }

FS::FS() { SetZero(); }

ostream& operator<<(ostream &cout, FS x)
{
    if(x.sign<0) cout<<"-";
    int i, k, low=1;
    for(i=x.num.len; i>x.prec; i--) cout<<x.num.s[i];
    if(x.num.len<=x.prec) cout<<"0";
    if(x.num.Compare(zero)==0) { cout<<"0"; return cout; }
    k=i;
    while(k>0 && x.num.s[k]==0) k--;
    if(k==0) { cout<<"0"; return cout; }
    cout<<" . ";
    if(x.num.len<x.prec) for(int j=0; j<x.prec-x.num.len; j++) cout<<"0";
    while(x.num.s[low]==0) low++;
    while(i>low) cout<<x.num.s[i-1];
    return cout;
}

FS FS::operator=(int inte)
{
    prec = 0;
    if(inte>=0) { sign = 1; num = inte; }
    else { sign = -1; num = -inte; }
    return (*this);
}

FS FS::operator=(char *s)
{
    int p, i, j, l;
    SetZero();
    if(s[0]=='-') { sign = -1; s++; }
    if(s[0]=='+') { sign = 1; s++; }
    l = strlen(s);
    for(p=0; p<l; p++) if(s[p]=='.') break;
    if(p==l) prec = 0; else prec = l-1-p;
    for(i=l-1, j=0; i>=0; i--) if(s[i]!='.') num.s[++j] = s[i]-'0';
    while(j>1 && num.s[j]==0) --j; num.len = j;
    return (*this);
}
```

```

void LShift(FS &a, int sl)
{
    a.prec+=sl; a.num.len+=sl; int i;
    for(i=a.num.len; i>sl; i--) a.num.s[i]=a.num.s[i-sl];
    while(i>0) a.num.s[i--]=0;
}

void RShift(FS &a, int sl)
{
    a.prec-=sl; a.num.len-=sl; int i;
    for(i=1; i<=a.num.len; i++) a.num.s[i]=a.num.s[i+sl];
}

FS FS::operator+(FS b)
{
    FS c;
    if( prec>b.prec ) LShift(b,prec-b.prec); else
    if( prec<b.prec ) LShift(*this,b.prec-prec);
    if( sign==b.sign ) {
        c.sign=sign; c.prec=prec; c.num=num+b.num;
        if( c.num.Compare(zero)==0 ) c.SetZero();
    } else {
        c.prec=prec;
        if( num.Compare(b.num)==0 ) c.SetZero(); else
        if( num.Compare(b.num)>0 ) { c.sign=sign; c.num=num-b.num; } else
        if( num.Compare(b.num)<0 ) { c.sign=b.sign; c.num=b.num-num; }
        if( c.num.Compare(zero)==0 ) c.SetZero();
    }
    if( c.prec > fprec ) RShift( c, c.prec - fprec );
    return c;
}

FS FS::operator-(FS b)
{
    b.sign = - b.sign;
    FS c = (*this) + b;
    b.sign = - b.sign;
    return c;
}

FS FS::operator*(FS b)
{
    FS c;
    c.sign = sign * b.sign ;
    c.prec = prec + b.prec ;
    c.num = num * b.num ;
    if( c.num.Compare(zero)==0 ) c.SetZero();
    if( c.prec > fprec ) RShift( c, c.prec - fprec );
    return c;
}

FS FS::operator/(FS b) // 355/133 = 3.1415929203539823008849557522124
{
    FS c,d; // c = d / b
    d = (*this); LShift(d, fprec);
    c.sign = d.sign * b.sign ;
    c.prec = d.prec;
    LShift(d, b.prec);
    c.num = d.num / b.num;
    if( c.prec > fprec ) RShift( c, c.prec - fprec );
    return c;
}

```

1.4 Fraction Class

```
int gcd(int a,int b){ if (b==0) return a; return gcd(b,a%b); }
int lcm(int a,int b){ return a/gcd(a,b) * b; }

class Fraction { public:
    int a,b; // ( a/b = numerator/denominator )
    int sign(int x) { return (x>0?1:-1); }
    Fraction():a(0),b(1){}
    Fraction(int x):a(x),b(1){}
    Fraction(int x,int y){
        int m = gcd(abs(x),abs(y));
        a = x/m * sign(y);
        if (a==0) b = 1; else b = abs( y/m );
    }
    int get_denominator() {return b;}
    int get_numerator() {return a;}
    Fraction operator+(const Fraction &f) {
        int m = gcd(b,f.b);
        return Fraction(f.b/m * a + b/m * f.a, b/m * f.b);
    }
    Fraction operator-(const Fraction &f) {
        int m = gcd(b,f.b);
        return Fraction(f.b/m * a - b/m * f.a, b/m * f.b);
    }
    Fraction operator*(const Fraction &f) {
        int m1 = gcd(abs(a),f.b);
        int m2 = gcd(b,abs(f.a));
        return Fraction( (a/m1)*(f.a/m2) , (b/m2)*(f.b/m1) );
    }
    Fraction operator/(const Fraction &f)
    { return (*this)*Fraction(f.b,f.a); }
    friend ostream &operator<<(ostream &out,const Fraction &f) {
        if (f.a==0) cout << 0; else
            if (f.b==1) cout << f.a; else cout << f.a << '/' << f.b;
        return out;
    }
};
```

1.5 Binary Heap

```
#define MAXN 1048576
int n,HeapSize,Heap[MAXN+1];

void HeapUp(int p)
{
    int q=p>>1,a=Heap[p];
    while(q){
        if (a<Heap[q]) Heap[p]=Heap[q]; else break;
        p=q; q=p >> 1;
    }
    Heap[p]=a;
}

void AddToHeap(int a)
{
    Heap[++HeapSize]=a;
    HeapUp(HeapSize);
}
```

```

void HeapDown(int p)
{
    int q=p<<1,a=Heap[p];
    while( q <= HeapSize ) {
        if( q<HeapSize && Heap[q+1]<Heap[q] ) q++;
        if( Heap[q] < a ) Heap[p] = Heap[q]; else break;
        p=q; q=p << 1;
    }
    Heap[p] = a;
}

int GetTopFromHeap()
{
    int TopElement = Heap[1];
    Heap[1] = Heap[HeapSize--];
    HeapDown(1);
    return TopElement;
}

void BuildHeap() // Remember to Let HeapSize = N
{ for(int i=HeapSize;i>0;i--) HeapDown(i); }

```

1.6 Winner Tree

```

const int inf=10000000;
const int maxsize=1048576; // 2^floor(log(n))

int heap[maxsize*2],pos[maxsize*2],n,base;

void Update(int i)
{
    int j=i<<1;
    if(heap[j+1]<heap[j]) j++;
    heap[i]=heap[j]; pos[i]=pos[j];
}

int GetTopFromHeap(int &ps)
{
    int ret=heap[1],p=pos[1];
    ps=p; heap[p]=inf;
    while(p>1) { p>>=1; Update(p); }
    return ret;
}

int main()
{
    int i,j;
    cin >> n;
    for(base=1;base<n;base<<=1) base--;
    for(i=base+1;i<=(base<<1)+1;i++) {
        pos[i]=i;
        if( i<=base+n ) cin >> heap[i]; else heap[i]=inf;
    }
    for(i=base;i>0;i--) Update(i);
    for(i=1;i<=n;i++) cout << GetTopFromHeap(i) << endl;
    return 0;
}

```

1.7 Digital Tree

```
#define maxlen 100
#define maxsize 1000000
#define DataType int

char tree [ maxsize ] ,s [ maxlen ];
int son [ maxsize ] ,bro [ maxsize ] ,num,k ,n;
DataType data [ maxsize ];

DataType find ( const char*s )
{
    int i ,j=0;
    for ( i=0;s [ i ]; i++ ) {
        j=son [ j ];
        while ( j && tree [ j ]!=s [ i ]) j=bro [ j ];
        if ( !j ) return -1;
    }
    return data [ j ];
}

void add ( const char*s ,DataType x )
{
    int i ,j=0,p;
    for ( i=0;s [ i ]; i++ ) {
        p=j; j=son [ j ];
        while ( j && tree [ j ]!=s [ i ]) j=bro [ j ];
        if ( !j ) {
            tree [ ++num ]=s [ i ]; son [ num ]=0;
            bro [ num ]=son [ p ]; son [ p ]=num;
            data [ num ]=-1; j=num;
        }
    }
    data [ j ]=x;
}

void init ()
{ num=0; bro [ num ]=0; son [ num ]=0; data [ 0 ]=-1; }
```

1.8 Segment Tree

```
int cc [ 1 << 22 ],m,n; // memset cc first

void update ( int ii ,int s ,int t ,int ss ,int tt ,bool insert ) {
    if ( ss>tt ) return; int mid (( s+t )/2);
    if ( s==ss && t==tt ) { if ( insert ) cc [ ii ]=t-s+1; else cc [ ii ]=0; return; }
    if ( cc [ ii ]==0 ) if ( !insert ) return; else cc [ ii *2 ]=cc [ ii *2+1 ]=0;
    else if ( cc [ ii ]==t-s+1 ) if ( insert ) return;
        else { cc [ ii *2 ]=mid-s+1; cc [ ii *2+1 ]=t-mid; }
    update ( ii *2 ,s ,mid ,ss ,__min ( mid ,tt ),insert );
    update ( ii *2+1 ,mid+1 ,t ,__max ( mid+1,ss ),tt ,insert );
    cc [ ii ]=cc [ ii *2 ]+cc [ ii *2+1 ];
}

int query ( int ii ,int s ,int t ,int ss ,int tt ) {
    if ( ss>tt ) return 0; int mid (( s+t )/2);
    if ( s==ss && t==tt ) return cc [ ii ];
    if ( cc [ ii ]==0 ) cc [ ii *2 ]=cc [ ii *2+1 ]=0;
    if ( cc [ ii ]==t-s+1 ) { cc [ ii *2 ]=mid-s+1; cc [ ii *2+1 ]=t-mid; }
    return query ( ii *2 ,s ,mid ,ss ,__min ( mid ,tt ))
        +query ( ii *2+1 ,mid+1 ,t ,__max ( mid+1,ss ),tt );
}
```

1.9 Segment Tree in IOI'2001

```
// upper : maximum possible right point of intervals
int upper, tree[maxinterval+1];

void init()
{ upper=0; memset(tree,0,sizeof(tree)); }

void update( int r,int x ) // sum[1..r] +=x
{ while(r<=upper){ tree[r]+=x; r+=(r&(r^(r-1))); } }

int sum( int r ) // return sum[1..r]
{
    int res = 0;
    while ( r>0 ) { res+=tree[r]; r-=(r&(r^(r-1))); }
    return res;
}
```

1.10 Union-Find Set

```
int rank[maxn],pnt[maxn];

void makeset( int x )
{ rank[pnt[x]]=x; }

int find( int x )
{
    int px=x, i ;
    while(px!=pnt[px]) px=pnt[px];
    while(x!=px){ i=pnt[x]; pnt[x]=px; x=i; };
    return px;
}

void merge( int x,int y ) // or just pnt[find(x)]=find(y)
{
    if( rank[x=find(x)]>rank[y=find(y)]) pnt[y]=x;
    else { pnt[x]=y; rank[y]+=(rank[x]==rank[y]); };
}
```

1.11 Quick Sort

```
void quicksort( int b,int e,int a[])
{
    int i=b, j=e, x=a[(b+e)/2];
    do{
        while(a[i]<x) i++;
        while(a[j]>x) j--;
        if( i<=j ) std::swap(a[i++],a[j--]);
    }while(i<j);
    if( i<e) quicksort(i,e,a);
    if( j>b) quicksort(b,j,a);
}
```

1.12 Merge Sort

```
void sort(int b, int e)
{
    if(e-b<=0) return;
    int mid=(b+e)/2, p1=b, p2=mid+1, i=b;
    sort(b, mid); sort(mid+1, e);
    while( p1<=mid || p2<=e )
        if( p2>e || (p1<=mid && a[p1]<=a[p2]) )
            t[i++]=a[p1++]; else t[i++]=a[p2++];
    for( i=b; i<=e; i++) a[i]=t[i];
}
```

1.13 Radix Sort

```
#define base (1<<16)
int n, a[maxn], t[maxn], bucket[base+2];

void RadixSort(int n, int a[], int t[], int bucket[])
{
    int k, i, j;
    for(j=0; j<base; j++) bucket[j]=0;
    for(k=base-1, i=0; i<2; i++, k<<=16){
        for(j=0; j<n; j++) bucket[a[j]&k]++;
        for(j=1; j<base; j++) bucket[j]+=bucket[j-1];
        for(j=n-1; j>=0; j--) t[--bucket[a[j]&k]]=a[j];
        for(j=0; j<n; j++) a[j]=t[j];
    }
}
```

1.14 Select K^{th} Smallest Element

```
int select(int* a, int b, int e, int k)
{
    if( b==e) return a[b];
    int x = a[b+rand()% (e-b+1)], i = b, j = e;
    i--; j++;
    while(i<j) {
        while( a[++i] < x ); while( a[--j] > x );
        if( i<j ) std::swap(a[i], a[j]);
    }
    if(j==e) j--;
    if( k <= i ) return select(a, b, j, k);
    else return select(a, j+1, e, k-i);
}
```

1.15 KMP

```
int fail[maxlen];

void makefail( char *t, int lt )
{
    --t;
    for( int i=1, j=0; i<=lt; i++, j++ ){
        fail[i]=j;
        while(j>0 && t[i]!=t[j]) j=fail[j];
    }
}

// start matching pattern T in S[i..)
// return match pos or longest match length with corresponding pos
```

```

int kmp(char *s, int ls, char *t, int lt, int i, int &longest, int &lp)
{
    longest = lp = 0; --s; --t;
    for (int j=1; i<=ls; i++,j++) {
        while ( j>0 && s[i]!=t[j] ) j=fail[j];
        if ( j>longest ) { longest = j; lp = i-j; }
        if ( j==lt ) return i-lt;
    }
    return -1;
}

```

1.16 Suffix Sort

SuffixSort : input $s[0..n]$, output $id[0..n]$

```

#define nb next      // "new bucket" overlaid on "next"
#define head height // head is never used when computing height
#define rank b       // after SuffixSort, "rank" overlaid on "bucket"

char s[maxn]; int n, id[maxn], height[maxn], b[maxn], next[maxn];

bool cmp(const int &i, const int &j){ return s[i]<s[j]; }

void SuffixSort()
{
    int i, j, k, h;
    for (i=0; i<n; i++) id[i]=i;
    std::sort(id, id+n, cmp);
    for (i=0; i<n; i++)
        if (i==0 || s[id[i]]!=s[id[i-1]]) b[id[i]] = i;
        else b[id[i]]=b[id[i-1]];
    for (h=1; h<n; h<=1)
    {
        for (i=0; i<n; i++) head[i]=next[i]=-1;
        for (i=n-1; i>=0; i--) if (id[i])
        {
            j = id[i]-h; if ( j<0 ) j+=n;
            next[j] = head[b[j]]; head[b[j]] = j;
        }
        j=n-h; next[j] = head[b[j]]; head[b[j]] = j;
        for (i=k=0; i<n; i++) if ( head[i]>=0 )
            for (j=head[i]; j>=0; j=next[j]) id[k++]=j;
        for (i=0; i<n; i++) if ( i>0 && id[i]+h<n && id[i-1]+h<n
            && b[id[i]] == b[id[i-1]] && b[id[i]+h] == b[id[i-1]+h] )
            nb[id[i]] = nb[id[i-1]]; else nb[id[i]] = i;
        for (i=0; i<n; i++) b[i] = nb[i];
    }
}

```

GetHeight : $height[i] = LCP(s[id[i]], s[id[i]-1])$

```

void GetHeight()
{
    int i, j, h; height[0] = 0;
    for (i=0; i<n; i++) rank[id[i]] = i;
    for (h=0, i=0; i<n; i++) if ( rank[i] > 0 )
    {
        j = id[rank[i]-1];
        while ( s[i+h] == s[j+h] ) ++h;
        height[rank[i]] = h;
        if ( h>0 ) --h;
    }
}

```

Chapter 2

Graph Theory and Network Algorithms

2.1 SSSP — Dijkstra + Binary Heap

```
const int inf = 1000000000;

int n,m,num,len,next [maxm],ev [maxm],ew [maxm];
int value [maxn],mk [maxn],nbs [maxn],ps [maxn],heap [maxn];

void update (int r)
{
    int q=ps [r],p=q>>1;
    while (p && value [heap [p]]>value [r]) {
        ps [heap [p]]=q; heap [q]=heap [p];
        q=p; p=q>>1;
    }
    heap [q]=r; ps [r]=q;
}

int getmin ()
{
    int ret=heap [1],p=1,q=2,r=heap [len--];
    while (q<=len) {
        if ( q<len && value [heap [q+1]]<value [heap [q]]) q++;
        if ( value [heap [q]]<value [r] ) {
            ps [heap [q]]=p; heap [p]=heap [q];
            p=q; q=p<<1;
        } else break;
    }
    heap [p]=r; ps [r]=p;
    return ret;
}

void dijkstra (int src,int dst)
{
    int i,j,u,v;
    for (i=1;i<=n;i++) {value [i]=inf; mk [i]=ps [i]=0; };
    value [src]=0; heap [len=1]=src; ps [src]=1;
    while (!mk [dst]) {
        if (len==0) return;
        u=getmin (); mk [u]=1;
        for (j=nbs [u];j;j=next [j]) {
            v=ev [j]; if ( !mk [v] && value [u]+ew [j]<value [v] ) {
                if (ps [v]==0){ heap [++len]=v; ps [v]=len; }
                value [v]=value [u]+ew [j]; update (v);
            }
        }
    }
}
```

```

void readdata()
{
    int i ,u,v,w;
    cin>>n>>m; num=0;
    for ( i=1;i<=n ; i++) nbs[ i ]=0;
    while(m--){
        cin>>u>>v>>w;
        next[ ++num ]=nbs[ u ]; nbs[ u ]=num;
        ev[ num ]=v; ew[ num ]=w;
    }
    dijkstra(1,n); // Minimum Distance saved at value[1..n]
}

```

2.2 SSSP — Bellman Ford + Queue

```

const int maxn = maxm = 1000005
const int inf = 1000000000

int nbs[ maxn ] , next[ maxm ] , value[ maxn ] , open[ maxn ] , open1[ maxn ];
int ev[ maxm ] , ew[ maxm ] , mk[ maxn ] , n ,m, num, cur , tail ;

void BellmanFord( int src )
{
    int i ,j ,k ,l ,t ,u,v,p=0;
    for ( i=1;i<=n ; i++) { value[ i ]=inf; mk[ i ]=0; }
    value[ src ]=tail=0; open[ 0 ]=src ;
    while(++p, tail >=0){
        for ( i=0;i<=tail ; i++) open1[ i ]=open[ i ];
        for ( cur=0,t=tail , tail=-1; cur<=t ; cur++)
            for ( u=open1[ cur ] , i=nbs[ u ]; i ; i=next[ i ] ) {
                v=ev[ i ]; if( value[ u ]+ew[ i ]<value[ v ]){
                    value[ v ]=value[ u ]+ew[ i ];
                    if(mk[ v ]!=p) { open[ ++tail ]=v; mk[ v ]=p; }
                }
            }
        }
    }
}

```

2.3 MST — Kruskal

```

#define maxn 1000005
#define maxm 1000005

int id[ maxm ] , eu[ maxm ] , ev[ maxm ] , ew[ maxm ] , n ,m, pnt[ maxn ];
int cmp( const int &i , const int &j){ return ew[ i ]<ew[ j ]; }
int find( int x){ if(x!=pnt[ x ]) pnt[ x ]=find( pnt[ x ]); return pnt[ x ]; }

int Kruskal()
{
    int ret=0,i ,j ,p;
    for ( i=1;i<=n ; i++) pnt[ i ]=i; // node[1..n]
    for ( i=0;i<m; i++) id[ i ]=i; // ew[0..m-1]
    std::sort( id , id+m,cmp );
    for ( j=-1,i=1; i<n ; i++){
        while( p=id[ ++j ] , find( eu[ p ])==find( ev[ p ]) );
        ret+=ew[ p ]; pnt[ find( ev[ p ])]=find( eu[ p ]);
    }
    return ret;
}

```

2.4 Minimum Directed Spanning Tree

```

int n, g[maxn][maxn], used[maxn], pass[maxn], eg[maxn], more, queue[maxn];

void combine(int id, int& sum) {
    int tot = 0, from, i, j, k;
    for (; id!=0&&!pass[id]; id=eg[id]) { queue[tot++]=id; pass[id]=1; }
    for (from=0; from<tot&&queue[from]!=id; from++);
    if (from==tot) return; more = 1;
    for (i=from; i<tot; i++) {
        sum+=g[eg[queue[i]]][queue[i]];
        if (i!=from) { used[queue[i]]=1;
            for (j = 1; j <= n; j++) if (!used[j])
                if (g[queue[i]][j]<g[id][j]) g[id][j]=g[queue[i]][j];
        }
    }
    for (i=1; i<=n; i++) if (!used[i]&&i!=id) {
        for (j=from; j<tot; j++) { k=queue[j];
            if (g[i][id]>g[i][k]-g[eg[k]][k]) g[i][id]=g[i][k]-g[eg[k]][k];
        }
    }
}

int msdt(int root) { // return the total length of MDST
    int i, j, k, sum = 0;
    memset(used, 0, sizeof(used));
    for (more=1; more;){ more = 0;
        memset(eg, 0, sizeof(eg));
        for (i = 1; i <= n; i++) if (!used[i] && i != root) {
            for (j = 1, k = 0; j <= n; j++) if (!used[j] && i != j)
                if (k == 0 || g[j][i] < g[k][i]) k = j;
            eg[i] = k;
        }
        memset(pass, 0, sizeof(pass));
        for (i=1;i<=n;i++) if (!used[i]&&!pass[i]&&i!=root) combine(i,sum);
    }
    for (i=1; i<=n; i++) if (!used[i] && i!=root) sum+=g[eg[i]][i];
    return sum;
}

```

2.5 Maximum Matching on Bipartite Graph

```

int nx, ny, m, g[MAXN][MAXN], sy[MAXN], cx[MAXN], cy[MAXN];

int path(int u)
{
    for (int v=1;v<=ny;v++) if (g[u][v] && !sy[v]) { sy[v]=1;
        if (!cy[v] || path(cy[v])) { cx[u]=v; cy[v]=u; return 1; }
    } return 0;
}

int MaximumMatch()
{
    int i, ret=0;
    memset(cx,0,sizeof(cx)); memset(cy,0,sizeof(cy));
    for (i=1;i<=nx;i++) if (!cx[i]) { memset(sy,0,sizeof(sy)); ret+=path(i); }
    return ret;
}

```

2.6 Maximum Cost Perfect Matching on Bipartite Graph

```

int cx [ maxn ] , cy [ maxn ] , sx [ maxn ] , sy [ maxn ] , lx [ maxn ] , ly [ maxn ] ;
int nx , ny , match , g [ maxn ] [ maxn ] ;

int path (int u)
{
    sx [ u ] = 1; for (int v = 1; v <= ny; v++) if (g [ u ] [ v ] == lx [ u ] + ly [ v ] && !sy [ v ]) {
        sy [ v ] = 1; if (!cy [ v ] || path (cy [ v ])) { cx [ u ] = v; cy [ v ] = u; return 1; }
    } return 0;
}

void KuhnMunkres()
{
    int i , j , u , min ;
    memset (lx , 0 , sizeof (lx )); memset (ly , 0 , sizeof (ly ));
    memset (cx , 0 , sizeof (cx )); memset (cy , 0 , sizeof (cy ));
    for (i = 1; i <= nx; i++) for (j = 1; j <= ny; j++) if (lx [ i ] < g [ i ] [ j ]) lx [ i ] = g [ i ] [ j ];
    for (match = 0, u = 1; u <= nx; u++) if (!cx [ u ]) {
        memset (sx , 0 , sizeof (sx )); memset (sy , 0 , sizeof (sy ));
        while (!path (u)) {
            min = 0x3fffffff;
            for (i = 1; i <= nx; i++) if (sx [ i ]) for (j = 1; j <= ny; j++) if (!sy [ j ]) {
                if (lx [ i ] + ly [ j ] - g [ i ] [ j ] < min) min = lx [ i ] + ly [ j ] - g [ i ] [ j ];
            } for (i = 1; i <= nx; i++) if (sx [ i ]) { lx [ i ] -= min; sx [ i ] = 0; }
            for (j = 1; j <= ny; j++) if (sy [ j ]) { ly [ j ] += min; sy [ j ] = 0; }
        }
    }
}
}

```

2.7 Maximum Matching on General Graph

```

// total is the maximum cardinality , p[1..n] means a match: i <-> p[i]
int g [ maxn ] [ maxn ] , p [ maxn ] , l [ maxn ] [ 3 ] , n , total , status [ maxn ] , visited [ maxn ];

void solve ()
{
    int i , j , k , pass ;
    memset (p , 0 , sizeof (p ));
    do { i = 0;
        do { if (p [ ++i ]) pass = 0; else {
            memset (l , 0 , sizeof (l ));
            l [ i ] [ 2 ] = 0xff; pass = path (i );
            for (j = 1; j <= n; j++) for (k = 1; k <= n; k++)
                if (g [ j ] [ k ] < 0) g [ j ] [ k ] = -g [ j ] [ k ];
        };
        } while ( i != n && !pass );
        if (pass) total += 2;
    } while ( i != n && total != n );
}

void upgrade (int r)
{
    int j = r , i = l [ r ] [ 1 ];
    for (p [ i ] = j; l [ i ] [ 2 ] < 0xff; ) {
        p [ j ] = i; j = l [ i ] [ 2 ]; i = l [ j ] [ 1 ]; p [ i ] = j;
    } p [ j ] = i;
}

```

```

int path(int r)
{
    int i, j, k, v, t, quit;
    memset(status, 0, sizeof(status)); status[r] = 2;
    do { quit = 1;
        for (i = 1; i <= n; i++) if (status[i] > 1)
            for (j = 1; j <= n; j++) if (g[i][j] > 0 && p[j] != i)
                if (status[j] == 0) {
                    if (p[j] == 0) { l[j][1] = i; upgrade(j); return 1; } else
                    if (p[j] > 0) {
                        g[i][j] = g[j][i] = -1; status[j] = 1;
                        l[j][1] = i; g[j][p[j]] = g[p[j]][j] = -1;
                        l[p[j]][2] = j; status[p[j]] = 2;
                        quit = 0;
                    }
                } else
        if (status[j] > 1 && (status[i] + status[j] < 6)) {
            quit = 0; g[i][j] = g[j][i] = -1;
            memset(visited, 0, sizeof(visited));
            visited[i] = 1; k = i; v = 2;
            while (l[k][v] != 0xff) { k = l[k][v]; v = 3 - v; visited[k] = 1; }
            k = j; v = 2;
            while (!visited[k]) { k = l[k][v]; v = 3 - v; }
            if (status[i] != 3) l[i][1] = j;
            if (status[j] != 3) l[j][1] = i;
            status[i] = status[j] = 3; t = i; v = 2;
            while (t != k) {
                if (status[l[t][v]] != 3) l[l[t][v]][v] = t;
                t = l[t][v]; status[t] = 3; v = 3 - v;
            }
            t = j; v = 2;
            while (t != k) {
                if (status[l[t][v]] != 3) l[l[t][v]][v] = t;
                t = l[t][v]; status[t] = 3; v = 3 - v;
            }
        }
    } while (!quit);
    return 0;
}

```

2.8 Maximum Flow — Ford Fulkson in Matrix

```

// Remember to memset C[maxn][maxn] for a new case
int c[maxn][maxn], f[maxn][maxn], pnt[maxn], open[maxn], d[maxn], mk[maxn];

int maxflow(int n, int s, int t)
{
    int cur, tail, i, j, u, v, flow = 0; memset(f, 0, sizeof(f));
    do { memset(mk, 0, sizeof(mk)); memset(d, 0, sizeof(d));
        open[0] = s; mk[s] = 1; d[s] = 0x3fffffff;
        for (pnt[s] = cur = tail = 0; cur <= tail && !mk[t]; cur++)
            for (u = open[cur], v = 1; v <= n; v++) if (!mk[v] && f[u][v] < c[u][v]) {
                mk[v] = 1; open[++tail] = v; pnt[v] = u;
                if (d[u] < c[u][v] - f[u][v]) d[v] = d[u];
                else d[v] = c[u][v] - f[u][v];
            }
        if (!mk[t]) break; flow += d[t];
        for (u = t; u != s; ) { v = u; u = pnt[v]; f[u][v] += d[t]; f[v][u] -= f[u][v]; }
    } while (d[t] > 0); return flow;
}

```

2.9 Maximum Flow — Ford Fulkson in Link

```

#define maxn 1000
#define maxm 2*maxn*maxn

int c[maxn], f[maxm], ev[maxn], be[maxn], next[maxm], num=0;
int nbs[maxn], pnt[maxn], open[maxn], d[maxn], mk[maxn];

void AddEdge(int u, int v, int cc) // Remember to set nbs[1..n]=num=0
{
    next[++num]=nbs[u]; nbs[u]=num; be[num]=num+1;
    ev[num]=v; c[num]=cc; f[num]=0;
    next[++num]=nbs[v]; nbs[v]=num; be[num]=num-1;
    ev[num]=u; c[num]=0; f[num]=0;
}

int maxflow(int n, int s, int t)
{
    int cur, tail, i, j, u, v, flow=0; // f has been set zero when AddEdge
    do{ memset(mk, 0, sizeof(mk)); memset(d, 0, sizeof(d));
        open[0]=s; mk[s]=1; d[s]=0x3fffffff;
        for(pnt[s]=cur=tail=0; cur<=tail && !mk[t]; cur++)
            for(u=open[cur], j=nbs[u]; j; j=next[j]) { v=ev[j];
                if(!mk[v]&&f[j]<c[j]) {
                    mk[v]=1; open[++tail]=v; pnt[v]=j;
                    if(d[u]<c[j]-f[j]) d[v]=d[u]; else d[v]=c[j]-f[j];
                }
            }
        if(!mk[t]) break; flow+=d[t];
        for(u=t; u!=s; u=ev[be[j]]) { j=pnt[u]; f[j]+=d[t]; f[be[j]]=-f[j]; }
    } while(d[t]>0); return flow;
}

```

2.10 Minimum Cost Maximum Flow in Matrix

```

const int inf=0x3fffffff;

int c [ maxn ] [ maxn ] , f [ maxn ] [ maxn ] , w [ maxn ] [ maxn ] , pnt [ maxn ];
int value [ maxn ] , d [ maxn ] , mk [ maxn ] , open [ maxn ] , oldque [ maxn ];

void mincost (int n, int s, int t, int &flow, int &cost)
{
    int cur , tail , tl , i , j , u , v;
    memset (f,0 , sizeof(f)); flow=0; cost=0;
    do{ memset (d,0 , sizeof(d));
        for ( i=1; i<=n; i++) value [ i ]=inf;
        open [ 0 ]=s; d [ s ]=0x3fffffff; tail=value [ s ]=0;
        while ( tail >=0){
            memset (mk,0 , sizeof(mk));
            memcpy (oldque , open , sizeof(open));
            for ( tl=tail , pnt [ s ]=cur=0, tail=-1; cur<=tl; cur++)
                for ( u=oldque [ cur ] , v=1; v<=n; v++)
                    if ( f [ u ] [ v ]< c [ u ] [ v ] && value [ u ]<inf
                        && value [ u ]+w [ u ] [ v ]<value [ v ] ){
                        if ( !mk [ v ] ) { mk [ v ]=1; open [ ++tail ]=v; };
                        pnt [ v ]=u; value [ v ]=value [ u ]+w [ u ] [ v ];
                        if ( d [ u ]< c [ u ] [ v ]-f [ u ] [ v ] ) d [ v ]=d [ u ];
                        else d [ v ]=c [ u ] [ v ]-f [ u ] [ v ];
                    }
            }
        if ( value [ t ]==inf ) return;
        flow+=d [ t ]; cost+=d [ t ]*value [ t ];
        for ( u=t ; u!=s ; ){
            v=u; u=pnt [ v ]; f [ u ] [ v ]+=d [ t ]; f [ v ] [ u ]=-f [ u ] [ v ];
            if ( f [ u ] [ v ] <0 ) w [ u ] [ v ]=-w [ v ] [ u ]; else
                if ( f [ v ] [ u ] <0 ) w [ v ] [ u ]=-w [ u ] [ v ];
        }
    } while ( d [ t ]>0);
}
}

```

2.11 Minimum Cost Maximum Flow in Link

```

#define maxn 350
#define maxm 100000 // maxm*2
const int inf=0x3fffffff;

int c[maxn], f[maxn], w[maxn], ev[maxn], be[maxn], next[maxn], value[maxn];
int nbs[maxn], pnt[maxn], open[maxn], oldque[maxn], d[maxn], mk[maxn], num=0;

void AddEdge(int u, int v, int cc, int ww) // Remember to set nbs[1..n]=num=0
{
    next[++num]=nbs[u]; nbs[u]=num; be[num]=num+1;
    ev[num]=v; c[num]=cc; f[num]=0; w[num]=ww;
    next[++num]=nbs[v]; nbs[v]=num; be[num]=num-1;
    ev[num]=u; c[num]=0; f[num]=0; w[num]=-ww;
}

void mincost(int n, int s, int t, int &flow, int &cost)
{
    int cur, tail, tl, i, j, u, v;
    memset(f, 0, sizeof(f)); flow=0; cost=0;
    do{ memset(d, 0, sizeof(d));
        for (i=1; i<=n; i++) value[i]=inf;
        open[0]=s; d[s]=0x3fffffff; tail=value[s]=0;
        while (tail>=0){
            memset(mk, 0, sizeof(mk));
            memcpy(oldque, open, sizeof(open));
            for (tl=tail, pnt[s]=cur=0, tail=-1; cur<=tl; cur++)
                for (u=oldque[cur], j=nbs[u]; j; j=next[j]) {
                    v=ev[j];
                    if (f[j]<c[j] && value[u]<inf && value[u]+w[j]<value[v]){
                        if (!mk[v]) { mk[v]=1; open[++tail]=v; };
                        pnt[v]=j; value[v]=value[u]+w[j];
                        if (d[u]<c[j]-f[j]) d[v]=d[u]; else d[v]=c[j]-f[j];
                    }
                }
            if (value[t]==inf) return;
            flow+=d[t]; cost+=d[t]*value[t];
            for (u=t; u!=s; u=ev[be[j]]) { j=pnt[u]; f[j]+=d[t]; f[be[j]]=-f[j]; }
        } while (d[t]>0);
    }
}

```

2.12 Recognizing Chordal Graph

```

int n, m, mk[maxn], degree[maxn], PEO[maxn], g[maxn][maxn];

int Chordal()
{
    memset(mk, 0, sizeof(mk)); memset(degree, 0, sizeof(degree));
    for (int j, k, u, v, i=0; i<n; i++) {
        j=-1; u=-1;
        for (k=0; k<n; k++) if (!mk[k] && (j<0 || degree[k]>degree[j])) j=k;
        mk[j]=1; PEO[i]=j;
        for (k=i-1; k>=0; k--) if (g[j][PEO[k]])
            if (u<0) u=PEO[k]; else if (!g[u][PEO[k]]) return 0;
        for (k=0; k<n; k++) if (!mk[k] && g[j][k]) degree[k]++;
    }
    return 1;
}

```

2.13 DFS — Bridge

```
int n, g[maxn][maxn], mk[maxn], d[maxn], low[maxn];
int color, ti, bridgenum, bridgeu[maxn], bridgev[maxn];

void dfsvisit(int u, int p)
{
    int v, s=0, bBridge=0; low[u]=d[u]=++ti; mk[u]=-color;
    for(v=1; v<=n; v++) if(g[u][v] && v!=p)
        if(mk[v]==0){ dfsvisit(v,u); s++;}
        if(low[v]<low[u]) low[u]=low[v];
        if(low[v]==d[v]){
            bridgeu[bridgenum]=u;
            bridgev[bridgenum+1]=v;
        }
    } else if(d[v]<low[u]) low[u]=d[v];
    mk[u]=color;
}

void dfs()
{
    int i, j, k; memset(mk,0,sizeof(mk));
    color=ti=bridgenum=0;
    for(i=1; i<=n; i++) if(!mk[i]){ ++color; dfsvisit(i,0); }
    cout<<bridgenum<<endl;
}
```

2.14 DFS — Cutvertex

```
int n, g[maxn][maxn], mk[maxn], d[maxn], low[maxn];
int color, ti, cutvertexnum, cutvertexlist[maxn];

void dfsvisit(int u, int p)
{
    int v, s=0, bVertex=0; low[u]=d[u]=++ti; mk[u]=-color;
    for(v=1; v<=n; v++) if(g[u][v] && v!=p)
        if(mk[v]==0){ dfsvisit(v,u); s++;}
        if(low[v]<low[u]) low[u]=low[v];
        if(low[v]>=d[u]) bVertex=1;
    } else if(d[v]<low[u]) low[u]=d[v];
    if((p && bVertex) || (!p && s>1)) cutvertexlist[cutvertexnum++]=u;
    mk[u]=color;
}

void dfs()
{
    int i, j, k; memset(mk,0,sizeof(mk));
    color=ti=cutvertexnum=0;
    for(i=1; i<=n; i++) if(!mk[i]){ ++color; dfsvisit(i,0); }
    cout<<cutvertexnum<<endl;
    for(i=0; i<cutvertexnum; i++) cout<<cutvertexlist[i]<<" ";
    cout<<endl;
}
```

2.15 DFS — Block

```
int n, g[maxn][maxn], mk[maxn], d[maxn], low[maxn], len, que[maxn];
int color, ti, cutvertexnum, cutvertexlist[maxn], blocknum;

void dvsvisit(int u, int p)
{
    int v, s=0, bCutvertex=0; low[u]=d[u]=++ti; mk[u]=-color; que[++len]=u;
    for(v=1; v<=n; v++) if(g[u][v] && v!=p)
        if(mk[v]==0){ dvsvisit(v,u); s++;
            if(low[v]<low[u]) low[u]=low[v];
            if(low[v]>=d[u]){
                while(que[len]!=v) cout<<que[len--]<<" ";
                cout<<que[len--]<<" "<<u<<endl;
                bCutvertex=1; blocknum++;
            }
        } else if(d[v]<low[u]) low[u]=d[v];
    if((p && bCutvertex) || (!p && s>1)) cutvertexlist[cutvertexnum++]=u;
    mk[u]=color;
}

void dfs()
{
    int i, j, k; memset(mk,0,sizeof(mk));
    color=ti=cutvertexnum=blocknum=0;
    for(i=1; i<=n; i++) if(!mk[i]){
        ++color; len=0; dvsvisit(i,0);
        if(len>1 || d[i]==ti){
            while(len>1) cout<<que[len--]<<" ";
            cout<<i<<endl; blocknum++;
        }
    }
    cout<<" Block Number : "<<blocknum<<endl;
    cout<<" Cutvertex Number : "<<cutvertexnum<<endl;
    for(i=0; i<cutvertexnum; i++) cout<<cutvertexlist[i]<<" ";
    cout<<endl<<endl;
}
```

2.16 DFS — Topological Sort

```
int n, mk[maxn], topo[maxn], g[maxn][maxn], ps, topook;

void dfs(int u)
{
    if(mk[u]<0){topook=0; return; } if(mk[u]>0) return; else mk[u]=-1;
    for(int v=1; topook && v<=n; v++) if(g[u][v]) dfs(v);
    topo[ps--]=u; mk[u]=1;
}

void toposort()
{
    int i, j, k; topook=1; ps=n; memset(mk,0,sizeof(mk));
    for(i=1;topook && i<=n; i++) if(!mk[i]) dfs(i);
}

int main()
{
    int i, m, u, v;
    while(cin>>n>>m, n && !cin.fail()){
        memset(g,0,sizeof(g));
        while(m--){ cin>>u>>v; g[u][v]=1; }; toposort();
        for(i=1; i<n; i++) cout<<topo[i]<<" ";
        cout<<topo[n]<<endl;
    }
    return 0;
}
```

2.17 Strongly Connected Component

```
int g[maxn][maxn], n, mk[maxn], list[maxn], num;

void back(int v)
{
    mk[v]=1; cout<<v<<" ";
    for(int u=1; u<=n; u++) if(!mk[u] && g[u][v]) back(u);
}

void dfs(int u)
{
    mk[u]=1;
    for(int v=1; v<=n; v++) if(!mk[v] && g[u][v]) dfs(v);
    list[num--]=u;
}

int main()
{
    int i, j, k, l;
    cin>>n; for(i=1; i<=n; i++) for(j=1; j<=n; j++) cin>>g[i][j];
    memset(mk,0,sizeof(mk)); num=n;
    for(i=1; i<=n; i++) if(!mk[i]) dfs(i);
    memset(mk,0,sizeof(mk));
    for(i=1; i<=n; i++) if(!mk[list[i]]) { back(list[i]); cout<<endl; }
    return 0;
}
```

Chapter 3

Number Theory

3.1 Greatest Common Divisor

```
void gcd(int a, int b, int &d, int &x, int &y)
{
    if (b==0){ d=a; x=1; y=0; return; }
    gcd(b, a%b, d, x, y);
    y -= x * (a/b);
}
```

3.2 Chinese Remainder Theorem

$extended_euclid(a, b) = ax + by$

```
int extended_euclid(int a, int b, int &x, int &y)
{
    if (b==0){ x=1,y=0; return a; } else {
        int res=extended_euclid(b,a%b,x,y);
        int t=x; x=y; y=t-(a/b)*y;
        return res;
    }
}
```

$ax \equiv b \pmod{n}$, $n > 0$

```
void modular_linear_equation_solver(int a, int b, int n)
{
    int d,x,y,e,i;
    d=extended_euclid(a,n,x,y);
    if (b%d!=0) cout<<"No answer!" ; else {
        e=x*(b/d)%n; // x=e is a basic solution
        for (i=0;i<d; i++) cout<<(e+i*(n/d))%n<<endl;
    }
}
```

Given b_i , w_i , $i = 0 \dots len - 1$ which $w_i > 0$, $i = 0 \dots len - 1$ and $(w_i, w_j) = 1$, $i \neq j$
Find an x which satisfies: $x \equiv b_i \pmod{w_i}$, $i = 0 \dots len - 1$

```
int china(int b[], int w[], int len)
{
    int i,d,x,y,x,m,n;
    x=0; n=1; for (i=0;i<len ; i++) n*=w[ i ];
    for (i=0;i<len ; i++){
        m=n/w[ i ];
        d=extended_euclid(w[ i ],m,x,y);
        x=(x+y*m*b[ i ])%n;
    }
    return (n+x%n)%n;
}
```

3.3 Prime Generator

```
#define maxn 10000000
#define maxp 1000000

char mk[maxn];
int prime[maxp], pnum;

void GenPrime(int n)
{
    int i, j, k; pnum = 0; memset(mk, 0, n+1);
    for(i=2, k=4; i<=n; i++, k+=i+i-1) if (!mk[i])
    {
        prime[pnum++] = i;
        if (k<=n) for(j=i+i; j<=n; j+=i) mk[j] = 1;
    }
}
```

3.4 ϕ Generator

$\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$, where p is a prime.
 $\phi(846720) = 193536$

```
int Phi(int n) // O( Sqrt(N) )
{
    int i, j, ret=n;
    for(i=2, j=4; j<=n; i++, j+=i+i-1) if (!(n%i))
    {
        ret = ret / i * (i-1);
        while( !(n%i) ) n/=i;
    }
    if( n>1 ) ret = ret / n * (n-1);
    return ret;
}
```

```
#define maxn 10000000
#define maxp 1000000
```

```
int phi[maxn], prime[maxp], pnum;

void GenPhi(int n) // O( N loglog N )
{
    int i, j, k; pnum = 0;
    memset(phi, 0, (n+1)*sizeof(phi[0]));
    phi[1] = 1;
    for(i=2; i<=n; i++) if (!phi[i])
    {
        prime[pnum++] = i;
        for(j=i; j<=n; j+=i)
        {
            if (!phi[j]) phi[j] = j;
            phi[j] = phi[j] / i * (i-1);
        }
    }
}
```

3.5 Discrete Logarithm

```

#define ll long _int64
inline int mod(int x, int n) {return (x%n+n)%n;}
    //  $ax \equiv 1 \pmod{n}$ 
int Inv(int a, int n)
{
    int d, x, y; Gcd(a, n, d, x, y);
    if (d==1) return mod(x, n); else return -1;
}
    //  $x \equiv a^b \pmod{n}$ ,  $a, b \geq 0$ 
int ModPow(int a, int b, int n)
{
    ll long d(1), i(0); while (b>=((ll long)1<<i)) i++;
    for(--i; i>=0; --i) { d=d*d%n; if (b&(1<<i)) d=d*a%n; }
    return d;
}
    //  $a^x \equiv b \pmod{n}$ ,  $n$  is prime!
int mexp[50000], id[50000];

bool logcmp(const int &a, const int &b) {return mexp[a]<mexp[b];}

int ModLog(int a, int b, int n)
{
    int i, j, m = (int)ceil(sqrt(n)), inv = Inv(ModPow(a, m, n), n);
    for (id[0]=0, mexp[0]=i=1; i<m; i++)
        { id[i]=i; mexp[i] = (mexp[i-1]*(ll long)a)%n; }
    std::stable_sort(id, id+m, logcmp);
    std::sort(mexp, mexp+m);
    for (i=0; i<m; i++) { //  $i*m < n$ 
        j = std::lower_bound(mexp, mexp+m, b)-mexp;
        if (j<m && mexp[j]==b) return i*m+id[j];
        b = (b*(ll long)inv)%n;
    }
    return -1;
}

```

3.6 Square Roots in Z_p

```

#define llong __int64

int ModPow(int a, int b, int n) //  $a^b \bmod n$   $a, b \geq 0$ 
{
    llong d(1), i(0);
    while (b >= ((llong)1 << i)) i++;
    for (--i; i >= 0; --i) { d = d * d % n; if (b & (1 << i)) d = d * a % n; }
    return d;
}

//  $x*x = a \pmod n$  should be a prime and  $\gcd(a, n) == 1$ 
int ModSqrt(int a, int n)
{
    int b, k, i, x;
    if (n == 2) return a % n;
    if (ModPow(a, (n - 1) / 2, n) == 1) {
        if (n % 4 == 3) x = ModPow(a, (n + 1) / 4, n); else {
            for (b = 1; ModPow(b, (n - 1) / 2, n) == 1; b++);
            i = (n - 1) / 2; k = 0; do { i /= 2; k /= 2;
                if ((ModPow(a, i, n) * (llong)ModPow(b, k, n) + 1) % n == 0) k += (n - 1) / 2;
            } while (i % 2 == 0);
            x = (ModPow(a, (i + 1) / 2, n) * (llong)ModPow(b, k / 2, n)) % n;
        }
        if (x * 2 > n) x = n - x;
    }
    return -1;
}

int main()
{
    int a, n, casec, x; cin >> casec;
    while (casec--) {
        cin >> a >> n; x = ModSqrt(a, n);
        if (x < 0) cout << "No root" << endl;
        else if (x * 2 == n) cout << x << endl;
        else cout << x << ' ' << n - x << endl;
    }
    return 0;
}

```

Chapter 4

Algebraic Algorithms

4.1 Linear Equations in Z_2

```
// Gauss Elimination :  $\bigoplus_{0 \leq j < nn} a_{i,j}x_{i,j} = a_{i,nn}$ 
int m, nn, num, list [maxn]; char a [maxn] [maxn];

int reduce()
{
    int i, j, k, r;
    for (i=r=0; i<nn; i++){
        for (j=r; j<m && !a[j][i]; j++); if (j>=m) continue;
        if (j>r) for (k=0; k<=nn; k++) std::swap(a[r][k], a[j][k]);
        for (num=0, k=i; k<=nn; k++) if (a[r][k]) list[num++]=k;
        for (j=0; j<m; j++) if (j!=r && a[j][i])
            for (k=0; k<num; k++) a[j][list[k]] ^= 1;
        ++r;
    }
    for (i=0; i<m; i++)
        if (a[i][nn]){
            for (j=0; j<nn && !a[i][j]; j++);
            if (j==nn) return 0; // else x[j]=a[i][nn]/a[i][j];
        }
    return 1;
}
```

4.2 Linear Equations in Z

```

// Gauss Elimination :  $\sum_{0 \leq j < nn} a_{i,j}x_{i,j} = a_{i,nn}$ 
int m, nn, a[maxn][maxn];

int gcd(int x, int y)
{ if(y==0) return x; else return gcd(y, x%y); }

void yuefen(int b[], int ct)
{
    int i, j=0, k;
    for(i=0; i<ct; i++) if(b[i]) if(j) k=gcd(b[i], k); else {k=b[i]; j=1;}
    if(k!=0) for(i=0; i<ct; i++) b[i]/=k;
}

int reduce() // return 0 means no solution!
{
    int i, j, k, r, tmp;
    for(i=r=0; i<nn; i++){
        for(j=r; j<m && !a[j][i]; j++); if(j>=m) continue;
        if(j>r) for(k=0; k<=nn; k++) std::swap(a[r][k], a[j][k]);
        for(j=0; j<m; j++) if(j!=r && a[j][i]){
            tmp=a[j][i];
            for(k=0; k<=nn; k++) a[j][k]=a[j][k]*a[r][i]-tmp*a[r][k];
            yuefen(a[j], nn+1);
        } ++r;
    }
    for(i=0; i<m; i++) if(a[i][nn]){
        for(j=0; j<nn && !a[i][j]; j++);
        if(j==nn) return 0; // else  $x[j]=a[i][nn]/a[i][j]$ ;
    }
    return 1;
}

```

4.3 Linear Equations in Q

Note: *fraction.h* contains a *Fraction Class* (Section 1.4 on Page 8)

```

#include<fraction.h>

int m, nn; Fraction a[maxn][maxn];
int dcmp(Fraction x){return x.a;}

int reduce()
{
    int i, j, k, r; double tmp;
    for(i=r=0; i<nn; i++){
        for(j=r; j<m && !dcmp(a[j][i]); j++); if(j>=m) continue;
        if(j>r) for(k=0; k<=nn; k++) std::swap(a[r][k], a[j][k]);
        for(j=0; j<m; j++) if(j!=r && dcmp(a[j][i])){
            tmp=a[j][i]/a[r][i];
            for(k=0; k<=nn; k++) a[j][k]=a[j][k]-tmp*a[r][k];
        } ++r;
    }
    for(i=0; i<m; i++) if(dcmp(a[i][nn])){
        for(j=0; j<nn && !dcmp(a[i][j]); j++);
        if(j==nn) return 0; // else  $x[j]=a[i][nn]/a[i][j]$ ;
    }
    return 1;
}

```

4.4 Linear Equations in R

```

const double eps=1e-8;
int m,nn; double a [maxn] [maxn];

int dcmp(double x){ if(x>eps) return 1; if(x<-eps) return -1; return 0; }

int reduce() // r is rank
{
    int i,j,k,r; double tmp;
    for(i=r=0; i<nn; i++){
        for(j=r; j<m && !dcmp(a [j] [i]); j++); if(j>=m) continue;
        if(j>r) for(k=0;k<nn;k++) std :: swap(a [r] [k] ,a [j] [k]);
        for(j=0;j<m; j++) if(j!=r && dcmp(a [j] [i])){
            tmp=a [j] [i]/a [r] [i];
            for(k=0;k<nn;k++) a [j] [k]-=tmp*a [r] [k];
        } ++r;
    }
    for(i=0;i<m; i++) if(dcmp(a [i] [nn])){
        for(j=0;j<nn && !dcmp(a [i] [j]); j++);
        if(j==nn) return 0; // else x[j]=a[i][nn]/a[i][j];
    } return 1;
}

```

4.5 Roots of Polynomial

Find the roots of $f_a(x) = \sum_{i=0}^n a_i x^i$ using *Newton Iterations*, $f_b(x) = f_a(x) \frac{d}{dx}$

```

const double eps=1e-5;
#define genx (rand()%1000)/100.0

int dcmp(double x)
{ if(x>eps) return 1; else if(x<-eps) return -1; else return 0; }

double f(double a[], int n, double x)
{
    double ret=0,xx=1;
    for(int i=0; i<=n; i++) { ret+=a [i]*xx; xx*=x; }
    return ret;
}

double newton(double a[], double b[], int n)
{
    double dy,y,x=genx, lastx=x-1;
    while(y=f(a,n,x), dcmp(lastx-x)){
        lastx=x; dy=f(b,n-1,x);
        if (!dcmp(dy)) x=genx; else x=x-y/dy;
    }
    return x;
}

void solve(double a[], double x[], int n)
{
    int i,j; double b[maxn];
    for(j=n; j>0; j--){
        for(i=0; i<j; i++) b [i]=a [i+1]*(i+1);
        x [j-1]=newton(a,b,j);
        for(b [j]=0, i=j-1; i>=0; i--) b [i]=a [i+1]+b [i+1]*x [j-1];
        for(i=0; i<j; i++) a [i]=b [i];
    }
}

```

4.6 Roots of Cubic and Quartic

$$c_0 + c_1 * x + c_2 * x^2 + c_3 * x^3 + c_4 * x^4 = 0$$

The functions return the number of distinct non-complex roots and put the values into the `s` array.

```

const double pi = acos(-1.0); // 3.14159265358979323846

double cbrt(double x)
{
    if( x> eps ) return pow( x, 1/3.0);
    if( x<-eps ) return -pow( -x, 1/3.0);
    return 0;
}

int SolveQuadric(double c[3], double s[2])
{
    double p, q, d; // normal form:  $x^2 + px + q = 0$ 
    p = c[1]/(2*c[2]); q = c[0]/c[2]; d = p*p-q;
    if( dcmp(d)==0 ) { s[0] = -p; return 1; }
    if( dcmp(d) < 0 ) return 0;
    d = sqrt( d );
    s[0] = -p + d;
    s[1] = -p - d;
    return 2;
}

int SolveCubic(double c[4], double s[3])
{
    int i, num; // normal form:  $x^3 + Ax^2 + Bx + C = 0$ 
    double sub, A, B, C, sqa, p, q, cbp, d;
    A = c[2]/c[3]; B = c[1]/c[3]; C = c[0]/c[3];
    sqa = A * A; //  $x = y - A/3 \Rightarrow x^3 + px + q = 0$ 
    p = 1.0/3 * (-1.0/3 * sqa + B);
    q = 1.0/2 * (2.0/27 * A * sqa - 1.0/3 * A * B + C);
    cbp = p * p * p; // use Cardano's formula
    d = q * q + cbp;
    if( dcmp(d)==0 ) {
        if( dcmp(q)==0 ) { s[0] = 0; num = 1; } // one triple solution
        else { // one single and one double solution
            double u = cbrt( -q );
            s[0] = 2 * u; s[1] = -u; num = 2;
        }
    } else if( dcmp(d)<0 ) { // Casus irreducibilis: three real solutions
        double phi = 1.0/3 * acos(-q / sqrt(-cbp));
        double t = 2 * sqrt(-p);
        s[0] = t * cos(phi);
        s[1] = -t * cos(phi + pi / 3);
        s[2] = -t * cos(phi - pi / 3);
        num = 3;
    } else { /* one real solution */
        d = sqrt(d); double u = cbrt(d-q), v = -cbrt(d+q);
        s[0] = u + v; num = 1;
    }
    /* resubstitute */
    sub = 1.0/3 * A; for( i=0; i<num; ++i ) s[i] -= sub;
    return num;
}

```

```

int SolveQuartic(double c[5], double s[4])
{
    double e[4], z, u, v, sub, A, B, C, d, sqa, p, q, r;
    int i, num; //  $x^4 + Ax^3 + Bx^2 + Cx + D = 0$ 
    A = c[3]/c[4]; B = c[2]/c[4]; C = c[1]/c[4]; d = c[0]/c[4];
    sqa = A * A; //  $x = y - A/4 \Rightarrow x^4 + px^2 + qx + r = 0$ 
    p = -3.0/8 * sqa + B;
    q = 1.0/8 * sqa * A - 1.0/2 * A * B + C;
    r = -3.0/256 * sqa * sqa + 1.0/16 * sqa * B - 1.0/4 * A * C + d;
    if (dcmp(r)==0) { // no absolute term:  $y(y^3 + py + q) = 0$ 
        e[0] = q; e[1] = p; e[2] = 0; e[3] = 1;
        num = SolveCubic(e, s); s[num++] = 0;
    } else { // solve the resolvent cubic ...
        e[0] = 1.0/2 * r * p - 1.0/8 * q * q; e[1] = -r;
        e[2] = -1.0/2 * p; e[3] = 1;
        SolveCubic(e, s);
        z = s[0]; // ... and take the one real solution
        u = z*z-r; v = 2*z-p; // ... to build two quadric equations
        if (dcmp(u)==0) u=0; else if (dcmp(u)>0) u=sqrt(u); else return 0;
        if (dcmp(v)==0) v=0; else if (dcmp(v)>0) v=sqrt(v); else return 0;
        e[0] = z-u; e[1] = dcmp(q)<0 ? -v : v; e[2] = 1;
        num = SolveQuadric(e, s);
        e[0] = z+u; e[1] = dcmp(q)<0 ? v : -v; e[2] = 1;
        num += SolveQuadric(e, s + num);
    }
    sub = 1.0/4*A; for (i=0; i<num; ++i) s[i] -= sub; // resubstitute
    return num;
}

```

4.7 Fast Fourier Transform

```

const double eps=1e-8;
const double pi=acos(-1.0);

#define cp complex<double>

inline int max(int a, int b){ if(a>b) return a; else return b; }
inline int dcmp(double a){ if(a<-eps) return -1; return (a>eps); }

void fft(cp *x, int n, cp *y, int bInv) // y=Wx, w[j, k]=e^ijk
{
    if(n==1) { y[0] = x[0]; return; }
    cp *xeven = new cp[n/2], *xodd = new cp[n/2], w(1,0),
        *yeven = new cp[n/2], *yodd = new cp[n/2], wn; int i;
    if(bInv) wn=cp( cos(-2*pi/n), sin(-2*pi/n) );
    else wn=cp( cos( 2*pi/n), sin( 2*pi/n) );
    for(i=0; i<n/2; i++)
    {
        xeven[i] = x[i*2];
        xodd[i] = x[i*2+1];
    }
    fft(xeven, n/2, yeven, bInv);
    fft(xodd, n/2, yodd, bInv);
    for(i=0; i<n/2; i++)
    {
        y[i] = yeven[i] + w*yodd[i];
        y[i+n/2] = yeven[i] - w*yodd[i];
        w *= wn;
    }
    delete xeven; delete yeven; delete xodd; delete yodd;
}

```

4.8 FFT - Polynomial Multiplication

```

void PolyMulti(double *a, int na, double *b, int nb, double *c, int &nc)
{
    int i, j, n=(na>nb)?na:nb;
    n=1<<((int)ceil(log(2*n)/log(2)-eps));
    cp *x=new cp[n], *ya=new cp[n], *yb=new cp[n], *yc=new cp[n];
    for (i=0; i<n; i++) x[i]=(i<na)?a[i]:0; fft(x, n, ya, 0);
    for (i=0; i<n; i++) x[i]=(i<nb)?b[i]:0; fft(x, n, yb, 0);
    for (i=0; i<n; i++) yc[i]=ya[i]*yb[i]; fft(yc, n, xc, 1);
    for (i=0; i<n; i++) c[i]=x[i].real()/n;
    for (nc=n; nc>0 && dcmp(c[nc-1])==0; nc--);
    delete x; delete ya; delete yb; delete yc;
}

```

4.9 FFT - Convolution

$$r_k = \sum_{i=0}^{n-1} a[i] * b[i - k]$$

```

void Convolution1(int *a, int *b, int *c, int n)
{
    int m, i, j, *rb=new int[n]; rb[0]=b[0];
    for (i=1; i<n; i++) rb[i]=b[n-i];
    PolyMulti1(a, n, rb, n, c, m);
    for (i=0; i<n; i++) c[i]+=c[i+n];
    delete [] rb;
}

\\ N must be power of 2
void Convolution2(int *a, int *b, int *c, int n)
{
    int i, j;
    cp *x=new cp[n], *ya=new cp[n], *yb=new cp[n], *yc=new cp[n];
    x[0]=b[0];
    for (i=1; i<n; i++) x[i]=(i<n)?b[n-i]:0; fft(x, n, yb, 0);
    for (i=0; i<n; i++) x[i]=(i<n)?a[i]:0; fft(x, n, ya, 0);
    for (i=0; i<n; i++) yc[i]=ya[i]*yb[i]; fft(yc, n, xc, 1);
    for (i=0; i<n; i++) c[i]=int(x[i].real()/n+0.5);
    delete x; delete ya; delete yb; delete yc;
}

```

4.10 FFT - Reverse Bits

```

#define for if(0); else for
const double pi = acos(-1.0);
const int MFB = 16;
int **bt = 0;

struct cp { double re, im; };

inline int ReverseBits(int index, int bitnum) {
    int ret = 0;
    for (int i=0; i<bitnum; ++i, index >>= 1)
        ret = (ret << 1) | (index & 1);
    return ret;
}

```

```

void InitFFT() {
    bt = new int *[MFB]; int i, j, length;
    for (i=1, length=2; i<=MFB; ++i, length<<=1) {
        bt[i-1] = new int [length];
        for (j=0; j<length; ++j) bt[i-1][j] = ReverseBits(j, i);
    }
}

inline int FRB(int i, int bitnum) {
    return bitnum <= MFB ? bt[bitnum - 1][i] : ReverseBits(i, bitnum);
}

void FFT(cp *in, cp *out, int n, bool bInv)
{
    int i, j, k, ed, len, bitnum=0; if (!bt) InitFFT();
    while ( !((1<<bitnum)&n) ) bitnum++;
    for (i=0; i<n; ++i) out[FRB(i, bitnum)] = in[i];
    double basicangle = pi * (bInv ? -2 : 2);
    cp a0, a1, a2, a, b;
    for (ed = 1, len = 2; len <= n; len <<= 1) {
        double delta_angle = basicangle / len;
        double sin1 = sin(-delta_angle), sin2 = sin(-delta_angle * 2);
        double cos1 = cos(-delta_angle), cos2 = cos(-delta_angle * 2);
        for (i=0; i<n; i+=len) {
            a1.re=cos1; a1.im=sin1; a2.re=cos2; a2.im=sin2;
            for (j=i, k=0; k<ed; ++j, ++k) {
                a0.re=2*cos1*a1.re-a2.re; a0.im=2*cos1*a1.im-a2.im;
                a2 = a1; a1 = a0; b=out[j+ed];
                a.re = a0.re*b.re - a0.im*b.im;
                a.im = a0.im*b.re + a0.re*b.im;
                out[j+ed].re=out[j].re-a.re;
                out[j+ed].im=out[j].im-a.im;
                out[j].re+=a.re;
                out[j].im+=a.im;
            }
        }
        ed = len;
    }
    if (bInv) for (int i = 0; i < n; ++i) { out[i].re/=n; out[i].im/=n; }
}

// n must be power of 2
void convolution(double *a, double *b, double *r, int n) {
    int i;
    cp *s=new cp[n], *d1=new cp[n], *d2=new cp[n], *y=new cp[n];
    s[0].im=b[0]; s[0].re=0;
    for (i=1; i<n; ++i) s[i].re=b[n-i], s[i].im=0; FFT(s, d2, n, false);
    for (i=0; i<n; ++i) s[i].re=a[i], s[i].im=0; FFT(s, d1, n, false);
    for (i=0; i<n; ++i) {
        y[i].re = d1[i].re*d2[i].re - d1[i].im*d2[i].im;
        y[i].im = d1[i].re*d2[i].im + d1[i].im*d2[i].re;
    }
    FFT(y, s, n, true);
    for (i=0; i<n; ++i) r[i] = s[i].re;
    delete s; delete d1; delete d2; delete y;
}

```

4.11 Linear Programming - Primal Simplex

Primal Simplex Method for solving Linear Programming problem in Standard Form

maximize

$$c_1 x_1 + c_2 x_2 + \cdots + c_n x_n + \text{ans}$$

subject to

$$\begin{aligned} a_{1,1} x_1 + a_{1,2} x_2 + \cdots + a_{1,n} x_n &\leq rhs_1 \\ a_{2,1} x_1 + a_{2,2} x_2 + \cdots + a_{2,n} x_n &\leq rhs_2 \\ &\vdots \\ a_{m,1} x_1 + a_{m,2} x_2 + \cdots + a_{m,n} x_n &\leq rhs_m \end{aligned}$$

```

const double eps = 1e-8;
const double inf = 1e15;

#define OPTIMAL -1
#define UNBOUNDED -2
#define FEASIBLE -3
#define INFEASIBLE -4
#define PIVOT_OK 1

int basic [maxn] ,row [maxm] , col [maxn];
double c0 [maxn];

double dcmp(double x)
{
    if( x > eps ) return 1;
    if( x < -eps ) return -1;
    return 0;
}

int Pivot(int n, int m, double *c, double a [maxn] [ maxn ],
           double *rhs, int &i, int &j)
{
    double min = inf; int k = -1;
    for(j=0; j<=n; j++) if( !basic [ j ] && dcmp(c [ j ]) >0 )
        if( k < 0 || dcmp(c [ j ]-c [ k ]) >0 ) k=j;
    j=k; if( k < 0 ) return OPTIMAL;
    for(k=-1,i=1; i<=m; i++) if( dcmp(a [ i ] [ j ]) >0 )
        if( dcmp(rhs [ i ]/a [ i ] [ j ]-min) < 0 ) { min = rhs [ i ]/a [ i ] [ j ]; k=i; }
    i=k; if( k < 0 ) return UNBOUNDED; else return PIVOT_OK;
}

int PhaseII(int n, int m, double *c, double a [maxn] [ maxn ],
             double *rhs, double &ans, int PivotIndex)
{
    int i,j,k,l; double tmp;
    while(k=Pivot(n,m,c,a,rhs,i,j),k==PIVOT_OK || PivotIndex)
    {
        if( PivotIndex ) { j=0; i=PivotIndex; PivotIndex=0; }
        basic [row [ i ]]=0; col [row [ i ]]=0; basic [ j ]=1; col [ j ]=i; row [ i ]=j;
        tmp=a [ i ] [ j ]; for(k=0;k<=n;k++) a [ i ] [ k ]/=tmp; rhs [ i ]/=tmp;
        for(k=1;k<=m;k++) if(k!=i && dcmp(a [ k ] [ j ]))
        {
            tmp = -a [ k ] [ j ]; for(l=0;l<=n;l++) a [ k ] [ l ]+=tmp*a [ i ] [ l ];
            rhs [ k ] += tmp*rhs [ i ];
        }
        tmp=-c [ j ]; for(l=0;l<=n;l++) c [ l ]+=a [ i ] [ l ]*tmp; ans-=tmp*rhs [ i ];
    }
    return k;
}

```

```

int PhaseI( int n, int m, double *c, double a[maxn][maxn], double *rhs, double &ans)
{
    int i, j, k = -1; double tmp, min = 0, ans0 = 0;
    for( i=1; i<=m; i++) if( dcmp( rhs[ i]-min)<0 ) { min=rhs[ i]; k=i; }
    if( k<0 ) return FEASIBLE;
    for( i=1; i<=m; i++) a[ i][0] = -1;
    for( j=1; j<=n; j++) c0[ j]=0; c0[0] = -1;
    PhaseII(n, m, c0, a, rhs, ans0, k);
    if( dcmp( ans0)<0 ) return INFEASIBLE;
    for( i=1; i<=m; i++) a[ i][0] = 0;
    for( j=1; j<=n; j++) if( dcmp( c[ j]) && basic[ j] )
    {
        tmp = c[ j]; ans += rhs[ col[ j]]*tmp;
        for( i=0; i<=n; i++) c[ i] -= tmp*a[ col[ j]][ i];
    }
    return FEASIBLE;
}

int simplex( int n, int m, double *c, double a[maxn][maxn],
             double *rhs, double &ans, double *x) // standard form
{
    int i, j, k;
    for( i=1; i<=m; i++)
    {
        for( j=n+1; j<=n+m; j++) a[ i][ j]=0;
        a[ i][ n+i] = 1; a[ i][0] = 0;
        row[ i] = n+i; col[ n+i] = i;
    }
    k = PhaseI( n+m, m, c, a, rhs, ans);
    if( k == INFEASIBLE ) return k;
    k = PhaseII(n+m, m, c, a, rhs, ans, 0);
    for( j=0; j<=n+m; j++) x[ j]=0;
    for( i=1; i<=m; i++) x[ row[ i]]=rhs[ i];
    return k;
}

int n, m; double c[maxn], ans, a[maxm][maxn], rhs[maxm], x[maxn];

int main()
{
    ifstream cin("lp.in");
    int i, j;
    while( cin>>n>>m && !cin.fail() )
    {
        for( j=1; j<=n; j++) cin>>c[ j]; cin>>ans; c[0]=0;
        for( i=1; i<=m; i++){ for( j=1; j<=n; j++) cin>>a[ i][ j]; cin>>rhs[ i]; }
        switch( simplex(n, m, c, a, rhs, ans, x) )
        {
            case OPTIMAL :
                printf("OPTIMAL\n%10lf\n", ans);
                for( j=1; j<=n; j++) printf("x[%2d]=%10lf\n", j, x[ j]);
                break;
            case UNBOUNDED :
                printf("UNBOUNDED\n");
                break;
            case INFEASIBLE :
                printf("INFEASIBLE\n");
                break;
        }
        printf("\n");
    }
    return 0;
}

```

Chapter 5

Computational Geometry

5.1 Basic Operations

```
const double eps = 1e-8;
const double pi = acos(-1.0);

struct CPoint{ double x,y; };

double min(double x,double y){ if( x<y ) return x; else return y; }

double max(double x,double y){ if( x>y ) return x; else return y; }

double sqr(double x){ return x*x; }

int dcmp(double x)
{
    if(x<-eps) return -1; else return (x>eps);
}

double cross(CPoint p0,CPoint p1,CPoint p2)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}

double dot(CPoint p0,CPoint p1,CPoint p2)
{
    return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}

double dissqr(CPoint p1,CPoint p2)
{
    return sqr(p1.x-p2.x)+sqr(p1.y-p2.y);
}

double dis(CPoint p1,CPoint p2)
{
    return sqrt(sqr(p1.x-p2.x)+sqr(p1.y-p2.y));
}

int PointEqual(const CPoint &p1,const CPoint &p2)
{
    return dcmp(p1.x-p2.x)==0 && dcmp(p1.y-p2.y)==0;
}
```

5.2 Extended Operations

```

// Crossing Angle of P0P1 -> P0P2, range in (-pi, pi]
double angle(CPoint p0, CPoint p1, CPoint p2)
{
    double cr = cross(p0, p1, p2);
    double dt = dot(p0, p1, p2);
    if(dcmp(cr)==0) cr=0.0;
    if(dcmp(dt)==0) dt=0.0;
    return atan2(cr, dt);
}

int PointOnLine(CPoint p0, CPoint p1, CPoint p2)
{
    return dcmp(cross(p0, p1, p2))==0;
}

int PointOnSegment(CPoint p0, CPoint p1, CPoint p2)
{
    return dcmp(cross(p0, p1, p2))==0 && dcmp(dot(p0, p1, p2))<=0;
}

// 1 = cross; 0 = parallel; -1 = overlap
int LineIntersection(CPoint p1, CPoint p2, CPoint p3, CPoint p4, CPoint &cp)
{
    double u=cross(p1, p2, p3), v=cross(p2, p1, p4);
    if(dcmp(u+v))
    {
        cp.x=(p3.x*v + p4.x*u) / (v+u);
        cp.y=(p3.y*v + p4.y*u) / (v+u);
        return 1;
    }
    if(dcmp(u)) return 0; // else u=v=0;
    if(dcmp(cross(p3, p4, p1))) return 0;
    return -1;
}

int SegmentIntersection(CPoint p1, CPoint p2, CPoint p3, CPoint p4, CPoint &cp)
{
    int ret=LineIntersection(p1, p2, p3, p4, cp);
    if(ret==1) return PointOnSegment(cp, p1, p2) && PointOnSegment(cp, p3, p4);
    if(ret== -1 && (PointOnSegment(p1, p3, p4) || PointOnSegment(p2, p3, p4)
        || PointOnSegment(p3, p1, p2) || PointOnSegment(p4, p1, p2)))
        return -1;
    return 0;
}

int SegmentIntersecTest(CPoint p1, CPoint p2, CPoint p3, CPoint p4)
{
    if(max(p1.x, p2.x) + eps < min(p3.x, p4.x) ||
        max(p3.x, p4.x) + eps < min(p1.x, p2.x) ||
        max(p1.y, p2.y) + eps < min(p3.y, p4.y) ||
        max(p3.y, p4.y) + eps < min(p1.y, p2.y)) return 0;
    int d1=dcmp(cross(p3, p4, p2));
    int d2=dcmp(cross(p3, p4, p1));
    int d3=dcmp(cross(p1, p2, p4));
    int d4=dcmp(cross(p1, p2, p3));
    if(d1*d2==1 || d3*d4 ==1) return 0;
    if(d1==0 && d2==0 && d3==0 && d4==0) return -1;
    return 1;
}

```

```

// 0 = outside;      1 = inside;      2 = boundary
int PointInPolygon (CPoint cp, CPoint p[], int n)
{
    int i, k, d1, d2, wn=0;
    double sum=0;
    p[n]=p[0];
    for (i=0; i<n; i++)
    {
        if ( PointOnSegment (cp, p[i], p[i+1]) ) return 2;
        k = dcmp ( cross (p[i], p[i+1], cp) );
        d1 = dcmp ( p[i+0].y - cp.y );
        d2 = dcmp ( p[i+1].y - cp.y );
        if (k>0 && d1<=0 && d2>0) wn++;
        if (k<0 && d2<=0 && d1>0) wn--;
    }
    return wn!=0;
}

double PointToLine (CPoint p0, CPoint p1, CPoint p2, CPoint &cp)
{
    double d=dis(p1,p2);
    double s = cross(p1,p2,p0)/d;
    cp.x = p0.x + s*(p2.y-p1.y)/d;
    cp.y = p0.y - s*(p2.x-p1.x)/d;
    return s; // **** Signed Magnitude ****
}

void PointProjLine (CPoint p0, CPoint p1, CPoint p2, CPoint &cp)
{
    double t = dot(p1,p2,p0)/dot(p1,p2,p2);
    cp.x = p1.x + t*(p2.x-p1.x);
    cp.y = p1.y + t*(p2.y-p1.y);
}

```

5.3 Convex Hull

Graham Scan, $O(N \log N)$

```

CPoint bp; // for polar sorting

int PolarCmp(const CPoint &p1, const CPoint &p2)
{
    int u=dcmp(cross(bp,p1,p2));
    return u>0 || (u==0 && dcmp(dissqr(bp,p1)-dissqr(bp,p2))<0 );
}

void GrahamScan(CPoint pin[], int n, CPoint ch[], int &m)
{
    int i,j,k,u,v;
    memcpy(ch, pin, n*sizeof(CPoint));
    for (i=k=0; i<n; i++)
    {
        u = dcmp(ch[i].x - ch[k].x);
        v = dcmp(ch[i].y - ch[k].y);
        if (v<0 || (v==0 && u<0)) k=i;
    }
    bp = ch[k];
    std::sort(ch, ch+n, PolarCmp);
    n = std::unique(ch, ch+n, PointEqual)-ch;
    if (n<=1) { m = n; return; }
    if (dcmp(cross(ch[0],ch[1],ch[n-1]))==0)
        { m=2; ch[1]=ch[n-1]; return; }
    ch[n++]=ch[0];
    for (i=1,j=2; j<n; j++)
    {
        while (i>0 && dcmp(cross(ch[i-1],ch[i],ch[j]))<=0) i--;
        ch[++i] = ch[j];
    }
    m=i;
}

void GrahamScanReserved(CPoint pin[], int n, CPoint ch[], int &m)
{
    int i,j,k,u,v;
    memcpy(ch, pin, n*sizeof(CPoint));
    for (i=k=0; i<n; i++)
    {
        u = dcmp(ch[i].x - ch[k].x);
        v = dcmp(ch[i].y - ch[k].y);
        if (v<0 || (v==0 && u<0)) k=i;
    }
    bp = ch[k];
    std::sort(ch, ch+n, PolarCmp);
    n = std::unique(ch, ch+n, PointEqual)-ch;
    if (n>0 && dcmp(cross(ch[0],ch[1],ch[n-1])) )
    {
        for (i=n-1; dcmp(cross(ch[0],ch[n-1],ch[i]))==0; i--);
        std::reverse(ch+i+1, ch+n);
    }
    for (m=0, i=0; i<n; i++)
    {
        while (m>=2 && dcmp(cross(ch[m-2],ch[m-1],ch[i]))<0) m--;
        ch[m++] = ch[i];
    }
}

```

Montone Chain, $O(N \log N)$

```

int VerticalCmp(const CPoint &p1, const CPoint &p2)
{
    return p1.y+eps<p2.y || (p1.y<p2.y+eps && p1.x+eps<p2.x);
}

void MontoneChain(CPoint pin[], int n, CPoint ch[], int &m)
{
    int i, k; CPoint *p = new CPoint[n];
    memcpy(p, pin, n*sizeof(CPoint));
    std::sort(p, p+n, VerticalCmp);
    n = std::unique(p, p+n, PointEqual)-p;
    for (m=i=0; i<n; i++)
    {
        while (m>1 && dcmp(cross(ch[m-2], ch[m-1], p[i]))<=0) m--;
        ch[m++]=p[i];
    }
    k=m;
    for (i=n-2; i>=0; i--)
    {
        while (m>k && dcmp(cross(ch[m-2], ch[m-1], p[i]))<=0) m--;
        ch[m++]=p[i];
    }
    if (n>1) m--;
    delete p;
}

void MontoneChainReserved(CPoint pin[], int n, CPoint ch[], int &m)
{
    int i, k;
    CPoint *p = new CPoint[n]; memcpy(p, pin, n*sizeof(CPoint));
    std::sort(p, p+n, VerticalCmp);
    n = std::unique(p, p+n, PointEqual)-p;
    for (m=i=0; i<n; i++)
    {
        while (m>1 && dcmp(cross(ch[m-2], ch[m-1], p[i]))<0) m--;
        ch[m++]=p[i];
    }
    if (n==m) return;
    k=m;
    for (i=n-2; i>=0; i--)
    {
        while (m>k && dcmp(cross(ch[m-2], ch[m-1], p[i]))<0) m--;
        ch[m++]=p[i];
    }
    if (n>1) m--;
    delete p;
}

```

Javis March, $O(NH)$

```

int ConvexJavisMarchCmp( CPoint p0 , CPoint p1 , CPoint pnew )
{
    int u=dcmp( cross( p0 , p1 , pnew ) );
    return ( u<0 || ( u==0 && dcmp( dissqr( pnew , p0 )-dissqr( p1 , p0 ) )>0 ) );
}

void ConvexJavisMarch( CPoint pin [ ] , int n , CPoint ch [ ] , int &m )
{
    int i , j , k , u , v ;
    char *mk = new char [ n ];
    CPoint *p = new CPoint [ n ];
    memcpy( p , pin , n*sizeof( CPoint ) );
    memset( mk , 0 , n );
    for ( i=k=0 ; i<n ; i++ )
    {
        u=dcmp( p [ i ].x-p [ k ].x );
        v=dcmp( p [ i ].y-p [ k ].y );
        if ( v<0 || ( v==0 && u<0 ) ) k=i ;
    }
    for ( m=0 ; !mk [ k ] ; m++ )
    {
        mk [ k ]=1; ch [ m ]=p [ k ];
        for ( j=k=0 ; j<n ; j++ ) if ( ConvexJavisMarchCmp( ch [ m ] , p [ k ] , p [ j ] ) ) k=j ;
    }
    delete p;
    delete mk;
}

```

5.4 Point Set Diameter

P must be convex in ccw order and no three points on an edge and will be changed after computing it's convex hull

```

double Diameter( CPoint *p , int n )
{
    Convex( p , n , p , n );
    if ( n==1 ) return 0;
    if ( n==2 ) return dis( p [ 0 ] , p [ 1 ] );
    int u , nu , v , nv , k; double ret = 0;
    p [ n ] = p [ 0 ];
    for ( u=0 , v=1 ; u<n ; u=nu )
    {
        nu = u+1;
        while ( 1 )
        {
            nv = ( v+1 )%n;
            k = dcmp( ( p [ nu ].x-p [ u ].x ) * ( p [ nv ].y-p [ v ].y )
                      - ( p [ nv ].x-p [ v ].x ) * ( p [ nu ].y-p [ u ].y ) );
            if ( k<=0 ) break;
            v=nv;
        }
        ret = max( ret , dis( p [ u ] , p [ v ] ) );
        if ( k==0 ) ret = max( ret , dis( p [ u ] , p [ nv ] ) );
    }
    return ret ;
}

```

5.5 Closest Pair

```

#define sqr(z) ((z)*(z))
struct point { double x,y; } pt[maxn]; // [1..n]
int n,o[maxn],on;

int dcmp(double a,double b) {
    if (a - b < 1e-10 && b - a < 1e-10) return 0;
    if (a > b) return 1; return -1;
}

bool cmp(const point& a,const point& b)
{ return dcmp(a.x,b.x) < 0; }

bool cmp2(const int& a,const int& b)
{ return dcmp(pt[a].y,pt[b].y) < 0; }

double dis(point a,point b)
{ return sqrt( sqr(a.x - b.x) + sqr(a.y - b.y) ); }

double min(double a,double b) { return a < b ? a : b; }

double search(int s,int t) {
    int mid = (s + t) / 2,i,j; double ret(1e300);
    if (s >= t) return ret;
    for(i=mid; i>=s && !dcmp(pt[i].x,pt[mid].x); i--) ret=search(s,i);
    for(i=mid; i<=t && !dcmp(pt[i].x,pt[mid].x); i++);
    ret=min(ret,search(i,t)); on=0;
    for(i=mid; i>=s && dcmp(pt[mid].x-pt[i].x,ret)<=0; i--) o[++on]=i;
    for(i=mid+1; i<=t && dcmp(pt[i].x-pt[mid].x,ret)<=0; i++) o[++on]=i;
    std::sort(o+1,o+on+1,cmp2);
    for(i=1; i<=on; i++) for(j=1; j<=10; j++) if(i+j<=on)
        ret = min(ret,dis(pt[o[i]],pt[o[i+j]]));
    return ret;
}

double solve() { std::sort(pt+1,pt+1+n,cmp); return search(1,n); }

```

5.6 Circles

Crossing of $|P - P_0| = r$ and $ax + by + c = 0$

```

int CircleCrossLine_1( CPoint p0, double r,
    double a, double b, double c, CPoint &cp1, CPoint &cp2)
{
    double aa = a * a, bb = b * b, s = aa + bb;
    double d = r*r*s - sqr(a*p0.x+b*p0.y+c);
    if( d+eps<0 ) return 0;
    if( d<eps ) d = 0; else d = sqrt( d );
    double ab = a * b, bd = b * d, ad = a * d;
    double xx = bb * p0.x - ab * p0.y - a * c;
    double yy = aa * p0.y - ab * p0.x - b * c;
    cp2.x = ( xx + bd ) / s; cp2.y = ( yy - ad ) / s;
    cp1.x = ( xx - bd ) / s; cp1.y = ( yy + ad ) / s;
    if( d>eps ) return 2; else return 1;
}

```

Crossing of $|P - P_0| = r$ and $\overrightarrow{P_1P_2}$

```
int CircleCrossLine_2( CPoint p0, double r,
                      CPoint p1, CPoint p2, CPoint &cp1, CPoint &cp2)
{
    double d, d12, dx, dy;
    d = fabs(PointToLine( p0, p1, p2, cp1 ));
    if( dcmp(d-r) >0 ) return 0;
    if( dcmp(d-r)==0 ) { cp2 = cp1; return 1; }
    d = sqrt( r*r - d*d ) / dis( p1, p2 );
    dx = ( p2.x - p1.x ) * d;
    dy = ( p2.y - p1.y ) * d;
    cp2.x = cp1.x + dx; cp2.y = cp1.y + dy;
    cp1.x = cp1.x - dx; cp1.y = cp1.y - dy;
    return 2;
}
```

Crossing of $|P - P_1| = r_1$ and $|P - P_2| = r_2$

```
int CircleCrossCircle_1( CPoint p1, double r1, CPoint p2, double r2,
                        CPoint &cp1, CPoint &cp2 )
{
    double mx = p2.x-p1.x, sx = p2.x+p1.x, mx2 = mx*mx;
    double my = p2.y-p1.y, sy = p2.y+p1.y, my2 = my*my;
    double sq = mx2 + my2, d = -(sq-sqr(r1-r2))*(sq-sqr(r1+r2));
    if( d+eps<0 ) return 0; if( d<eps ) d = 0; else d = sqrt(d);
    double x = mx*( (r1+r2)*(r1-r2) + mx*sx ) + sx*my2;
    double y = my*( (r1+r2)*(r1-r2) + my*sy ) + sy*mx2;
    double dx = mx*d, dy = my*d; sq *= 2;
    cp1.x = ( x - dy ) / sq; cp1.y = ( y + dx ) / sq;
    cp2.x = ( x + dy ) / sq; cp2.y = ( y - dx ) / sq;
    if( d>eps ) return 2; else return 1;
}
```

Crossing of $|P - P_1| = r_1$ and $|P - P_2| = r_2$

```
int CircleCrossCircle_2( CPoint p1, double r1, CPoint p2, double r2,
                        CPoint &cp1, CPoint &cp2 )
{
    double a, b, c; CommonAxis( p1, r1, p2, r2, a, b, c );
    return CircleCrossLine_1( p1, r1, a, b, c, cp1, cp2 );
}
```

Common Axis of $|P - P_1| = r_1$ and $|P - P_2| = r_2$ of the $ax + by + c = 0$ form

```
void CommonAxis( CPoint p1, double r1, CPoint p2, double r2,
                  double &a, double &b, double &c )
{
    double sx = p2.x + p1.x, mx = p2.x - p1.x;
    double sy = p2.y + p1.y, my = p2.y - p1.y;
    a = 2*mx; b = 2*my; c = - sx*mx - sy*my - (r1+r2)*(r1-r2);
}
```

5.7 Largest Empty Convex Polygon

```

#define ABS(x)          ((x)>=0 ? (x) : -(x))
#define CROSS(x1, y1, x2, y2) ((x1)*(y2)-(x2)*(y1))

const double eps = 1e-8;

struct CPoint { int x, y; };

int n; CPoint p[maxn]; double ans;

bool cmp(const CPoint &a, const CPoint &b) {
    int v = CROSS( a.x, a.y, b.x, b.y );
    if ( v>0 ) return true; if ( v<0 ) return false;
    return ( a.x*a.x + a.y*a.y < b.x*b.x + b.y*b.y );
}

CPoint c[maxn]; int nc; double fm[maxn][maxn];

void sweep(int x, int y) {
    int i, j, k, m; double v, best = 0;
    for(nc=i=0; i<n; ++i) if( p[i].y<y || p[i].y==y && p[i].x<x )
        { c[nc].x=p[i].x-x; c[nc++].y=p[i].y-y; }
    if( nc<2 ) return;
    std::sort(c, c + nc, cmp);
    memset(fm, 0, sizeof(fm));
    for( i=1; i<nc; ++i ) {
        j=i-1; while(j>=0 && CROSS(c[i].x, c[i].y, c[j].x, c[j].y)==0) --j;
        int nev = 0, ev[maxn];
        while( j>=0 ) {
            v = CROSS(c[j].x, c[j].y, c[i].x, c[i].y)/2.0; k=j-1;
            while( k>=0 && CROSS( c[j].x-c[i].x, c[j].y-c[i].y,
                c[k].x-c[i].x, c[k].y-c[i].y ) >0 ) --k;
            if( k>=0 ) v += fm[j][k];
            if( v-best>eps ) best = v;
            if( CROSS(c[i].x, c[i].y, c[i-1].x, c[i-1].y) )
                if( v-fm[i][j]>eps ) fm[i][j]=v;
            ev[nev++]=j; j=k;
        }
        if( CROSS(c[i].x, c[i].y, c[i-1].x, c[i-1].y) )
            for(j=nev-2; j>=0; --j) if( fm[i][ev[j+1]]-fm[i][ev[j]]>eps )
                fm[i][ev[j]]=fm[i][ev[j+1]];
    }
    if( best-ans>eps ) ans = best;
}

void main() {
    int t, i; for( scanf("%d", &t); t; --t ) { scanf("%d", &n);
        for(i=0; i<n; ++i) scanf("%d%d", &p[i].x, &p[i].y);
        for(ans=i=0; i<n; ++i) sweep(p[i].x, p[i].y); // main procedure
        printf("%.1lf\n", ans);
    }
}

```

5.8 Triangle Centers

```

// INPUT: (242, 89), (212, 185), (71, 128), OUTPUT: (158.0885, 115.4652)
void Circumcenter(CPoint p0, CPoint p1, CPoint p2, CPoint &cp)
{
    double a1=p1.x-p0.x, b1=p1.y-p0.y, c1=(sqr(a1)+sqr(b1))/2 ;
    double a2=p2.x-p0.x, b2=p2.y-p0.y, c2=(sqr(a2)+sqr(b2))/2 ;
    double d = a1 * b2 - a2 * b1;
    cp.x = p0.x + (c1*b2 - c2*b1) / d;
    cp.y = p0.y + (a1*c2 - a2*c1) / d;
}

// INPUT: (242, 89), (212, 185), (71, 128), OUTPUT: (189.5286, 137.4987)
double Incenter(CPoint A, CPoint B, CPoint C, CPoint &cp)
{
    double s, p, r, a, b, c;
    a = dis(B, C), b = dis(C, A), c = dis(A, B); p = (a + b + c) / 2;
    s = sqrt(p * (p-a) * (p-b) * (p-c)); r = s / p;
    cp.x = (a*A.x + b*B.x + c*C.x) / (a + b + c);
    cp.y = (a*A.y + b*B.y + c*C.y) / (a + b + c);
    return r;
}

// INPUT: (242, 89), (212, 185), (71, 128), OUTPUT: (208.8229, 171.0697)
void Orthocenter(CPoint A, CPoint B, CPoint C, CPoint &cp)
{
    Circumcenter(A, B, C, cp);
    cp.x = A.x + B.x + C.x - 2 * cp.x;
    cp.y = A.y + B.y + C.y - 2 * cp.y;
}

```

Find three numbers r, s, t which make $P = rA + sB + tC$ and $r + s + t = 1$

```

void Parametric( CPoint P, CPoint A, CPoint B, CPoint C,
                  double &r, double &s, double &t )
{
    double d;
    d = cross(A, B, C);
    r = cross(P, B, C) / d;
    s = cross(A, P, C) / d;
    t = cross(A, B, P) / d;
}

void PolygonCentroids(CPoint p[], int n, CPoint &cp)
{
    double sum=0, s=0; cp.x=0; cp.y=0;
    for ( int i=1; i<n-1; i++,sum+=s )
    {
        s= cross(p[0], p[i], p[i+1]);
        cp.x += s*(p[0].x + p[i].x + p[i+1].x );
        cp.y += s*(p[0].y + p[i].y + p[i+1].y );
    }
    cp.x/=sum*3; cp.y/=sum*3;
}

```

5.9 Polyhedron Volume

Remark : All faces are assumed oriented **counterclockwise** from the outside;
Volume6 returns six times the volume of the tetrahedron determined by abc
and the origin d. Volume6 is positive iff d is on the negative side of abc,
where the positive side is determined by the rh-rule. So the volume is positive
if the ccw normal to abc points outside the tetrahedron.

```
struct TPoint { double x, y, z; };
typedef int TFace[3];

double Volume6( TPoint a, TPoint b, TPoint c, TPoint d ) // d = origin
{
    double vol, bdx, bdy, bdz, cdx, cdy, cdz;
    bdx = b.x-d.x; bdy = b.y-d.y; bdz = b.z-d.z;
    cdx = c.x-d.x; cdy = c.y-d.y; cdz = c.z-d.z;
    vol = ( a.z - d.z ) * ( bdx * cdy - bdy * cdx )
        + ( a.y - d.y ) * ( bdz * cdx - bdx * cdz )
        + ( a.x - d.x ) * ( bdy * cdz - bdz * cdy );
    return vol;
}

void main()
{
    int n, F, i, j; double vol;
    TPoint p[maxn]; TFace face[maxn*2-4];
    cin >> n; for(i=0; i<n; i++) cin >> p[i].x >> p[i].y >> p[i].z;
    cin >> F; for(i=0; i<F; i++) for(j=0; j<3; j++) cin >> face[i][j];
    if( F != 2 * n - 4 ) { printf( "Not a simple polyhedron!\n" ); return; }
    for( vol = i = 0; i < F; i++ )
        vol += Volume6( p[face[i][0]], p[face[i][1]], p[face[i][2]], p[0] );
    vol /= 6.0; cout << vol << endl;
}
```

5.10 Planar Graph Contour

```

int x[maxn], y[maxn], g[maxn][maxn], num[maxn], base, n, size, mk[maxn][maxn];
int s[maxn], used[maxn], ans; double angle[maxn];

bool cmp(const int &i, const int&j){ return angle[i] < angle[j]; }

void dfs(int d, int u, int v)
{
    int i, j, w; s[d] = u; used[u]++;
    if( mk[u][v] ) {
        if( d==size ) {
            used[u]--;
            for(j=1; j<=n; j++) if( used[j]>1) break; if(j<=n) return;
            if(j>n) ++ans;
        }
        return;
    }
    mk[u][v]=1;
    for(j=0; j<num[v]; j++) if( g[v][j]==u ) break;
    j = (j+1)%num[v]; w = g[v][j]; dfs(d+1, v, w);
}

void solve()
{
    int i, j, k, l, u, v;
    for(i=1; i<=n; i++) {
        base=i;
        for(j=1; j<=n; j++) angle[j] = atan2(y[j]-y[i], x[j]-x[i]);
        std::sort(g[i], g[i]+num[i], cmp);
    }
    u = 1; memset(mk, 0, sizeof(mk));
    for(i=2; i<=n; i++) if(y[i]<y[u] || (y[i]==y[u] && x[i]<x[u])) u=i;
    for(v=-1, i=0; i<num[u]; i++) {
        j = g[u][i]; if( j==u || j==v ) continue;
        if( v<0 ) { v=j; continue; }
        k = (x[j]-x[u])*(y[v]-y[u]) - (y[j]-y[u])*(x[v]-x[u]);
        if( k<0 ) v=j; else
            if( k==0 ) if( y[j]<y[v] || (y[j]==y[v] && x[j]<x[v]) ) v=j;
    }
    dfs(0, v, u); ans = 0; // outer contour
    for(i=1; i<=n; i++) for(j=0; j<num[i]; j++)
        if( !mk[i][g[i][j]] ) {
            memset(used, 0, sizeof(used));
            dfs(0, i, g[i][j]);
        }
}

int main()
{
    int t, i, j, k, l;
    cin>>t; while(t-->0) {
        cin>>n;
        for(k=0; k<n; k++) {
            cin>>i; cin>>x[i]>>y[i]; cin>>num[i];
            for(j=0; j<num[i]; j++) cin>>g[i][j];
        }
        cin>>size; ans=0; if( size<3) size=3;
        solve(); cout<<ans<<endl;
    } return 0;
}

```

5.11 Rectangles Area

```
struct TSegNode {
    TSegNode(int x, int y):L(x),R(y),Lch(-1),Rch(-1),count(0),len(0){}
    TSegNode():TSegNode(-1,-1);
    int L, R, Lch, Rch, count, len;
};

struct Tevent {
    int L, R, x;
    bool style;
    friend const bool operator< (Tevent a, Tevent b) { return a.x<b.x; }
};

int nlist, list[MAXN*4], total, n, nevent;
TSegNode node[MAXN*4]; Tevent event[MAXN*4];

void CreateTree(int r) {
    if (node[r].R-node[r].L>1) {
        int mid = (node[r].L+node[r].R)>>1;
        node[total] = TSegNode(node[r].L, mid);
        node[r].Lch = total; CreateTree(total++);
        node[total] = TSegNode(mid, node[r].R);
        node[r].Rch = total; CreateTree(total++);
    }
}

void Update(int r, int L, int R, int v) {
    if (L>=node[r].R || R<=node[r].L) return;
    if (L<=node[r].L && R>=node[r].R) {
        node[r].count+=v;
        if (v>0 && v==node[r].count) node[r].len = node[r].R-node[r].L;
        if (v<0 && node[r].count==0) if (node[r].Lch<0) node[r].len = 0;
        else node[r].len = node[node[r].Lch].len + node[node[r].Rch].len;
    } else {
        Update(node[r].Lch, L, R, v); Update(node[r].Rch, L, R, v);
        if (node[r].count==0) node[r].len =
            node[node[r].Lch].len + node[node[r].Rch].len;
    }
}
```

```

int main() {
    int i, j, res, last;
    scanf("%d", &n);
    nevent=0; nlist=0;
    for ( i=0; i<n; ++i ) {
        int lx, ly, ux, uy;
        scanf("%d.%d.%d.%d", &lx, &ly, &ux, &uy);
        if ( lx<ux && ly<uy ) {
            event[ nevent ].x = lx; event[ nevent ].L = ly;
            event[ nevent ].R = uy; event[ nevent++ ].style = true;
            event[ nevent ].x = ux; event[ nevent ].L = ly;
            event[ nevent ].R = uy; event[ nevent++ ].style = false;
        }
        list[ nlist++ ] = ly; list[ nlist++ ] = uy;
    }
    std::sort(event, event+nevent);
    std::sort(list, list+nlist);
    nlist = std::unique(list, list+nlist)-list;
    node[ total=0, total++ ] = TSegNode(0, nlist-1);
    CreateTree( 0 );
    for( i=0; i<total; ++i )
        { node[ i ].L = list[ node[ i ].L ]; node[ i ].R = list[ node[ i ].R ]; }
    res = i = 0;
    while ( i<nevent ) {
        for( last=event[ i ].x; event[ i ].x==last; ++i )
            Update(0, event[ i ].L, event[ i ].R, event[ i ].style ? 1 : -1);
        if ( i < nevent ) res += (event[ i ].x - last) * node[ 0 ].len;
    }
    printf("%d\n", res);
    return 0;
}

```

5.12 Rectangles Perimeter

```

#define ABS(x) ( (x)>=0 ? (x) : -(x) )

struct TSegNode {
    TSegNode(int x, int y):L(x),R(y),Lch(-1),Rch(-1),count(0),len(0){}
    TSegNode():TSegNode(-1,-1);
    int L, R, Lch, Rch, count, len;
};

struct Tevent {
    int L, R, x; bool style;
    friend const bool operator< (Tevent a, Tevent b)
    { if (a.x!=b.x) return a.x<b.x; return (a.style && !b.style); }
};

int n, lx[MAXN], ly[MAXN], ux[MAXN], uy[MAXN], total, nevent, res;
TSegNode node[MAXN*4]; Tevent event[MAXN*4];

void CreateTree(int r) {
    if (node[r].R-node[r].L>1) {
        int mid = (node[r].L+node[r].R)>>1;
        node[total] = TSegNode(node[r].L, mid);
        node[r].Lch = total; CreateTree(total++);
        node[total] = TSegNode(mid, node[r].R);
        node[r].Rch = total; CreateTree(total++);
    }
}

void Update(int r, int L, int R, int v) {
    if (L>=node[r].R || R<=node[r].L) return;
    if (L<=node[r].L && R>=node[r].R) {
        node[r].count+=v;
        if (v>0 && v==node[r].count) node[r].len = node[r].R-node[r].L;
        if (v<0 && node[r].count==0) if (node[r].Lch<0) node[r].len = 0;
        else node[r].len = node[node[r].Lch].len + node[node[r].Rch].len;
    } else {
        Update(node[r].Lch, L, R, v); Update(node[r].Rch, L, R, v);
        if (node[r].count==0) node[r].len =
            node[node[r].Lch].len + node[node[r].Rch].len;
    }
}

```

```

void process() {
    int nlist, list[MAXN*2], last, i, now;
    nevent = 0; nlist = 0;
    for ( i=0; i<n; ++i ) {
        event[nevent].x = lx[i]; event[nevent].L = ly[i];
        event[nevent].R = uy[i]; event[nevent++].style = true;
        event[nevent].x = ux[i]; event[nevent].L = ly[i];
        event[nevent].R = uy[i]; event[nevent++].style = false;
        list[nlist++] = ly[i]; list[nlist+] = uy[i];
    }
    std::sort(event, event+nevent);
    std::sort(list, list+nlist);
    nlist = int(std::unique(list, list+nlist)-list);
    node[total=0, total++] = TSegNode(0, nlist-1);
    CreateTree( 0 );
    for(i=0; i<total; ++i)
        { node[i].L = list[node[i].L]; node[i].R = list[node[i].R]; }
    last = i = 0;
    while( i<nevent ) {
        now = event[i].x;
        while( i<nevent && event[i].x==now && event[i].style )
            { Update(0, event[i].L, event[i].R, 1); ++i; }
        res += ABS(node[0].len-last); last = node[0].len;
        while( i<nevent && event[i].x==now )
            { Update(0, event[i].L, event[i].R, -1); ++i; }
        res += ABS(node[0].len-last); last = node[0].len;
    }
}

int main() {
    int i;
    scanf("%d", &n);
    for ( i=0; i<n; ++i ) scanf("%d_%d_%d_%d", &lx[i], &ly[i], &ux[i], &uy[i]);
    res=0; process();
    for ( i=0; i<n; ++i ) { std::swap(lx[i], ly[i]); std::swap(ux[i], uy[i]); }
    process(); printf("%d\n", res);
    return 0;
}

```

5.13 Smallest Enclosing Circle

$O(N^3)$, compute Convex Hull first! or it will be quite slow!

```

double GetCos( CPoint p0 ,CPoint p1 ,CPoint p2 )
{ return dot(p0,p1,p2)/ dis(p0,p1)/ dis(p0,p2); }

int allin (CPoint p[] ,int n ,int i ,int j ,int k )
{
    for (int l=0; l<n; l++) if (l!=i && l!=j && l!=k) {
        if ( ( cross(p[ i ] ,p[ j ] ,p[ k ])>0 )^( cross(p[ i ] ,p[ j ] ,p[ l ])>0 ) &&
            dcmp( GetCos(p[ k ] ,p[ i ] ,p[ j ]) + GetCos(p[ l ] ,p[ i ] ,p[ j ]) )>0 ) return 0;
        if ( ( cross(p[ j ] ,p[ k ] ,p[ i ])>0 )^( cross(p[ j ] ,p[ k ] ,p[ l ])>0 ) &&
            dcmp( GetCos(p[ i ] ,p[ k ] ,p[ j ]) + GetCos(p[ l ] ,p[ k ] ,p[ j ]) )>0 ) return 0;
        if ( ( cross(p[ i ] ,p[ j ] ,p[ k ])>0 )^( cross(p[ i ] ,p[ l ] ,p[ k ])>0 ) &&
            dcmp( GetCos(p[ j ] ,p[ k ] ,p[ i ]) + GetCos(p[ l ] ,p[ k ] ,p[ i ]) )>0 ) return 0;
    }
    return 1;
}

double SmallestEnclosingCircle (CPoint p[] ,int n ,CPoint &cp )
{
    int i ,j ,k; double di ,cos1 ,cos2 ,co ,si ,r=0;
    if ( n == 1 ) { cp = p[ 0 ]; return 0; }
    if ( n == 2 )
    {
        cp .x = ( p[ 0 ].x + p[ 1 ].x )/2;
        cp .y = ( p[ 0 ].y + p[ 1 ].y )/2;
        return dis(p[ 0 ] ,p[ 1 ])/2;
    }
    for (i=0; i<n; i++) for (j=i+1; j<n; j++)
    {
        di = dis(p[ i ] ,p[ j ]); cos1 = cos2 = -2;
        if ( dcmp(di-r*2)>0 ) r = di/2;
        for (k=0; k<n; k++) if ( k!=i && k!=j )
        {
            co = GetCos(p[ k ] ,p[ i ] ,p[ j ]);
            if ( dcmp(cross(p[ i ] ,p[ j ] ,p[ k ]))>0 )
                { if ( co>cos1 ) cos1=co; }
            else if ( co>cos2 ) cos2=co;
        }
        if ( dcmp(cos1)<=0 && dcmp(cos2)<=0 )
        {
            cp .x = ( p[ i ].x + p[ j ].x )/2;
            cp .y = ( p[ i ].y + p[ j ].y )/2;
            return di/2;
        }
    }
    r = 1e30;
    for (i=0; i<n; i++) for (j=i+1; j<n; j++) {
        di = dis( p[ i ] , p[ j ] );
        for (k=j+1; k<n; k++) {
            co = GetCos( p[ k ] , p[ j ] , p[ i ] );
            si = sqrt(1-sqr(co));
            if ( dcmp( di/si/2-r )<0 && allin(p,n,i,j,k) ) {
                r=di/si/2;
                GetCircleCenter(p[ i ] ,p[ j ] ,p[ k ] ,cp );
            }
        }
    }
    return r;
}

```

5.14 Smallest Enclosing Ball

```
const double eps = 1e-10;

struct point_type { double x, y, z; };

int npoint, nouter;
point_type point[10000], outer[4], res;
double radius, tmp;

inline double dist(point_type p1, point_type p2)
{
    double dx=p1.x-p2.x, dy=p1.y-p2.y, dz=p1.z-p2.z;
    return ( dx*dx + dy*dy + dz*dz );
}

inline double dot(point_type p1, point_type p2)
{
    return p1.x*p2.x + p1.y*p2.y + p1.z*p2.z;
}

void minball(int n)
{
    ball();
    if( nouter<4 )
        for( int i=0; i<n; ++i )
            if( dist(res, point[i])-radius>eps )
            {
                outer[nouter]=point[i];
                ++nouter;
                minball(i);
                --nouter;
                if( i>0 )
                {
                    point_type Tt = point[i];
                    memmove(&point[1], &point[0], sizeof(point_type)*i);
                    point[0]=Tt;
                }
            }
}
```

```

void ball() {
    point_type q[3]; double m[3][3], sol[3], L[3], det; int i, j;
    res.x = res.y = res.z = radius = 0;
    switch ( nouter ) {
        case 1: res=outer[0]; break;
        case 2:
            res.x=(outer[0].x+outer[1].x)/2;
            res.y=(outer[0].y+outer[1].y)/2;
            res.z=(outer[0].z+outer[1].z)/2;
            radius=dist(res, outer[0]);
            break;
        case 3:
            for ( i=0; i<2; ++i ) {
                q[i].x=outer[i+1].x-outer[0].x;
                q[i].y=outer[i+1].y-outer[0].y;
                q[i].z=outer[i+1].z-outer[0].z;
            }
            for ( i=0; i<2; ++i ) for ( j=0; j<2; ++j )
                m[i][j]=dot(q[i], q[j])*2;
            for ( i=0; i<2; ++i ) sol[i]=dot(q[i], q[i]);
            if ( fabs(det=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps ) return;
            L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/det;
            L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/det;

            res.x=outer[0].x+q[0].x*L[0]+q[1].x*L[1];
            res.y=outer[0].y+q[0].y*L[0]+q[1].y*L[1];
            res.z=outer[0].z+q[0].z*L[0]+q[1].z*L[1];
            radius=dist(res, outer[0]);
            break;
        case 4:
            for ( i=0; i<3; ++i ){
                q[i].x=outer[i+1].x-outer[0].x;
                q[i].y=outer[i+1].y-outer[0].y;
                q[i].z=outer[i+1].z-outer[0].z;
                sol[i]=dot(q[i], q[i]);
            }
            for ( i=0; i<3; ++i ) for ( j=0; j<3; ++j )
                m[i][j]=dot(q[i], q[j])*2;
            det = m[0][0]*m[1][1]*m[2][2] + m[0][1]*m[1][2]*m[2][0]
                + m[0][2]*m[2][1]*m[1][0] - m[0][2]*m[1][1]*m[2][0]
                - m[0][1]*m[1][0]*m[2][2] - m[0][0]*m[1][2]*m[2][1];

            if ( fabs(det)<eps ) return;

            for ( j=0; j<3; ++j ){
                for ( i=0; i<3; ++i ) m[i][j]=sol[i];
                L[j]=( m[0][0]*m[1][1]*m[2][2] + m[0][1]*m[1][2]*m[2][0]
                    + m[0][2]*m[2][1]*m[1][0] - m[0][2]*m[1][1]*m[2][0]
                    - m[0][1]*m[1][0]*m[2][2] - m[0][0]*m[1][2]*m[2][1] )
                    / det;
                for ( i=0; i<3; ++i ) m[i][j]=dot(q[i], q[j])*2;
            }
            res=outer[0];
            for ( i=0; i<3; ++i ) {
                res.x += q[i].x * L[i];
                res.y += q[i].y * L[i];
                res.z += q[i].z * L[i];
            }
            radius=dist(res, outer[0]);
    }
}

```

Chapter 6

Classic Problems

6.1 Bernoulli Number Generator

Note: *fraction.h* contains a *Fraction Class* (Section 1.4 on Page 8)

```
#include<fraction.h>

Fraction a[22];
int c[22][22];

int main()
{
    int i, j, k, m;
    c[0][0] = 1;
    for (i=1; i<=21; i++) {
        c[i][0] = 1; c[i][i] = 1;
        for (j=1; j< i; j++) c[i][j] = c[i-1][j] + c[i-1][j-1];
    }
    a[0] = 0;
    while (cin>>k) {
        a[k+1] = Fraction(1, k+1); m = k+1;
        for (i=k; i>=1; i--) {
            a[i] = 0;
            for (j=i+1; j<=k+1; j++)
                if ((j-i+1)%2==0) a[i] = a[i] + a[j]*c[j][j-i+1];
                else a[i] = a[i] - a[j]*c[j][j-i+1];
            a[i] = a[i] * Fraction(1, i);
            m = lcm(m, a[i].get_denominator());
        }
        cout << m << '\n';
        for (i=k+1; i>0; i--) cout << a[i] * m << '\n';
        cout << 0 << endl;
    }
    return 0;
}
```

6.2 Baltic OI'99 Expressions

$f(n, d)$ is the number of trees whose depth is less or equal than d .

```
int f [ maxn ] [ maxd ] , h [ maxn ] [ maxn ] , n , d ;
int main ()
{
    ifstream cin ("input.txt");
    int i , j , k ;
    for (d=1; d<=maxd; d++) {
        memset (h , 0 , sizeof (h));
        for (i=0; i<=d; i++) h [ i ] [ 0 ] = 1;
        for (i=1; i<maxn; i++) for (j=i-d; j<=i; j++)
            if (j >= 0) h [ i ] [ j ] = h [ i - 1 ] [ j ] + h [ i ] [ j - 1 ];
        for (i=1; i<maxn; i++) f [ i ] [ d ] = h [ i ] [ i ];
    }
    while (cin >> n >> d && n) cout << f [ n / 2 ] [ d ] - f [ n / 2 ] [ d - 1 ] << endl;
    return 0;
}
```

6.3 Bead Coloring — Pólya Theory

Use C colors to color L -bead necklace , the non-isomorphic number of the necklaces is :

If L is odd ,

$$f(C, L) = \frac{1}{2L} \left(LC^{\frac{L+1}{2}} + \sum_{K=1}^L C^{(K, L)} \right)$$

If L is even,

$$f(C, L) = \frac{1}{2L} \left(\frac{L}{2} (C^{\frac{L}{2}} + C^{\frac{L}{2}+1}) + \sum_{K=1}^L C^{(K, L)} \right)$$

```
int ans , n , m , mk [ maxn ] , id [ maxn ] , num ;
int main ()
{
    int i , j , k , l , d , u , p [ maxn ];
    while (cin >> n >> m && n && m) {
        for (p [ 0 ] = i = 1; i <= m; i++) p [ i ] = p [ i - 1 ] * n;
        for (ans = num = i = 0; i < m; i++) id [ i ] = i;
        for (l = 0; l < 2; l++) {
            for (i = 0; i < m; i++) {
                memset (mk , 0 , sizeof (mk));
                for (k = j = 0; j < m; j++) if (!mk [ id [ j ] ]) {
                    for (k ++ , u = id [ j ] ; !mk [ u ] ; u = id [ (u + i) % m ]) mk [ u ] = 1;
                    num++;
                    ans += p [ k ];
                }
                std :: reverse ( id , id + m );
            }
            cout << ans / num << endl;
        }
        return 0;
}
```

6.4 Binary Stirling Number

Parity of the Stirling number of the second kind

```
#define int long long
```

```
int calc(int n, int k)
{
    if( k==0 ) if( n==0 ) return 1; else return 0;
    else if( k==1 ) return 1; else
    {
        int p = 0, p2 = 1;
        while( k>p2*2 || n-k/2>p2 ) { p++; p2<<=1; }
        if( k>p2 ) return calc(n-p2,k-p2);
        if( n-k>=p2/2 ) return calc(n-p2/2,k);
        return 0;
    }
}
```

6.5 Box Surface Distance

```
int r,L,H,W,x1,y1,z1,x2,y2,z2;

void turn(int i,int j,int x,int y,int z,int x0,int y0,int L,int W,int H){
    if(z==0){ int R=x*x+y*y; if(R<r) r=R; } else{
        if(i>=0 && i < 2) turn(i+1,j,x0+L+z,y,x0+L-x,x0+L,y0,H,W,L);
        if(j>=0 && j < 2) turn(i,j+1,x,y0+W+z,y0+W-y,x0,y0+W,L,H,W);
        if(i<=0 && i >-2) turn(i-1,j,x0-z,y,x-x0,x0-H,y0,H,W,L);
        if(j<=0 && j >-2) turn(i,j-1,x,y0-z,y-y0,x0,y0-H,L,H,W);
    }
}

int main(){
    while(cin>>L>>W>>H>>x1>>y1>>z1>>x2>>y2>>z2){
        if(z1!=0 && z1!=H) if(y1==0 || y1==W)
            { std::swap(y1,z1); std::swap(y2,z2); std::swap(W,H); } else
            { std::swap(x1,z1); std::swap(x2,z2); std::swap(L,H); }
        if(z1==H) z1=0,z2=H-z2;
        r=0x3fffffff; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
        cout<<r<<endl;
    }
    return 0;
}
```

6.6 Calculate Expression

```
char expr[MAX+1]; int next[MAX], stack[MAX], top;
double calc(int L, int R);

void prefix() {
    int i; top=-1;
    for( i=0; expr[i]; ++i ) {
        next[i]=-1;
        if( expr[i]== '(' ) stack[++top]=i;
        else if( expr[i]== ')' ) next[stack[top--]]=i;
    }
}
```

```

double getnum(int &L) {
    double res = 0;
    if ( expr[L]==‘(’ ) { res=calc(L+1, next[L]-1); L=next[L]+1; }
        else while ( isdigit(expr[L]) ) res=res*10+expr[L++]-‘0’;
    return res;
}

void process(double &a, double b, char op) {
    switch ( op ) {
        case '+': a += b; break;
        case '-': a -= b; break;
        case '*': a *= b; break;
        default : a /= b;
    }
}

double calc(int L, int R) {
    double a, b, c;
    char op1, op2;
    if ( next[L] == R ) return calc(L+1, R-1);
    a = 0; op1 = ( expr[L]==‘-’ ? ‘-’ : ‘+’ );
    L = ( expr[L]==‘+’ || expr[L]==‘-’ ? L+1 : L );
    for ( b = getnum(L); L<R ; ) {
        op2=expr[L++]; c=getnum(L);
        if ( op2==‘+’ || op2==‘-’ || op1==‘*’ || op1==‘/’ ) {
            process(a, b, op1); b=c; op1=op2;
        } else process(b, c, op2);
    }
    process(a, b, op1);
    return a;
}

void main() {
    scanf("%s", expr); prefix();
    printf("%.10lf\n", calc(0, strlen(expr)));
}

```

6.7 Cartesian Tree

```

int lson [maxn], rson [maxn], pnt [maxn], root ,n;

void BuildCartesianTree(int a[], int n)
{
    int i ,j ;
    for ( i=0; i<n; j=i++) {
        pnt [ i]=i-1; lson [ i]=rson [ i]=-1; j=i;
        while ( pnt [ j]>=0 && a[ i]>a[ pnt [ j] ] ) j = pnt [ j ];
        if ( j!=i ) { pnt [ i]=pnt [ j ]; lson [ i]=j; pnt [ j]=i; };
        if ( pnt [ i]>=0) rson [ pnt [ i]]=i;
    }
    for ( i=0;i<n; i++) if (pnt [ i]<0) root=i;
}

```

6.8 Catalan Number Generator

```
\\" Catalán[ 19] = 1767263190 < 2^31
\\" Catalán[ 35] = 3116285494907301262 < 2^63
\\" Catalán[100] = 896519947090131496687170070074100632420837521538745909320

#define maxn 1000
#define maxlen 700
#define maxpnum 400

int prime [maxpnum] , primepos [maxn*2] ,num [maxpnum] ,pnum;

struct HP{ int len ; int s [ maxlen ] ; };

void PrintHP(HP x) { for (int i=x.len ; i>0; i--) cout<<x.s [ i ] ; }

void Multi(HP &x ,int k)
{
    int i ; for (i=1;i<=x.len ; i++) x.s [ i ]*=k;
    x.len +=8; // log (10 ,maxn*2);
    for (i=1;i<=x.len ; i++) { x.s [ i+1 ]+=x.s [ i ]/10; x.s [ i ]%=10; }
    while ( x.len >1 && !x.s [ x.len ] ) x.len --;
}

void Factorize(int x ,int flag)
{
    for (int i=0; prime [ i ]*prime [ i ]<=x ; i++)
        while ( x%prime [ i ]==0 ) { x/=prime [ i ] ; num [ i ]+=flag ; }
    if (x>1) num [ primepos [ x ] ]+=flag ;
}

HP Catalan (int n)
{
    HP x ; memset(&x,0 , sizeof(x)) ; x.len =1; x.s [ 1 ]=1;
    memset(num,0 , sizeof(num)) ; int i ,j ;
    for (i=1;i<=n ; i++) { Factorize (2*n+1-i , 1) ; Factorize (i , -1) ; }
    Factorize (n+1 , -1);
    for (i=0;i<pnum ; i++) while (num [ i ]-->0) Multi (x , prime [ i ] );
    return x;
}

void InitPrimes()
{
    int i ,j ; pnum=0; memset(primepos ,0 , sizeof(primepos));
    for (i=2;i<=maxn*2 ; i++) if (!primepos [ i ] ) {
        primepos [ i ]=pnum; prime [ pnum++ ]=i;
        for (j=i+i ; j<=maxn*2 ; j+=i ) primepos [ j ]=-1;
    }
}

void main()
{
    InitPrimes(); int n;
    while (cin>>n) { PrintHP (Catalan (n)) ; cout<<endl; }
}
```

6.9 Coloring Regular Polygon

Coloring regular n -vertex polygon with m white and $n - m$ black. When $n = 17$ and $m = 8$ OUTPUT : 750

```
int c [ maxn ] [ maxn ] , ans , n , m;

int gcd ( int i , int j ) { if ( j == 0 ) return i ; else return gcd ( j , i % j ) ; }

int main ( )
{
    cin >> n >> m;
    int i , j , k , l , d;
    c [ 0 ] [ 0 ] = 1;
    for ( i = 1; i < maxn; i ++ ) {
        c [ i ] [ 0 ] = 1;
        for ( j = 1; j <= i ; j ++ ) c [ i ] [ j ] = c [ i - 1 ] [ j ] + c [ i - 1 ] [ j - 1 ];
    }
    for ( k = 0; k < m; k ++ ) {
        d = gcd ( m , k );
        if ( n * d % m == 0 ) { l = n * d / m; ans += c [ l - 1 ] [ d - 1 ]; }
    }
    if ( m % 2 == 0 ) {
        if ( n % 2 == 0 ) ans += ( m / 2 ) * c [ n / 2 - 1 ] [ m / 2 - 1 ];
        if ( m == 2 ) ans += ( m / 2 ) * ( n - 1 );
        else {
            for ( i = 2 - n % 2; i <= n - ( m - 2 ); i += 2 )
                ans += ( m / 2 ) * ( i - 1 ) * c [ ( n - i ) / 2 - 1 ] [ ( m - 2 ) / 2 - 1 ];
        } else for ( i = 2 - n % 2; i <= n - ( m - 1 ); i += 2 ) ans += m * c [ ( n - i ) / 2 - 1 ] [ ( m - 1 ) / 2 - 1 ];
    }
    cout << ans / ( 2 * m ) << endl;
    return 0;
}
```

6.10 Counting Inverse Pairs

```
#include<iostream . h>
#include<fstream . h>
#include<algorithm>

#define maxn 10000
int a [ maxn ] , t [ maxn ] , n , ans;

void sort ( int b , int e )
{
    if ( e - b <= 0 ) return ;
    int mid = ( b + e ) / 2 , p1 = b , p2 = mid + 1 , i = b ;
    sort ( b , mid );
    sort ( mid + 1 , e );
    while ( p1 <= mid || p2 <= e )
        if ( p2 > e || ( p1 <= mid && a [ p1 ] <= a [ p2 ] ) ) t [ i ++ ] = a [ p1 ++ ];
        else { t [ i ++ ] = a [ p2 ++ ]; ans += mid - p1 + 1; }
    for ( i = b ; i <= e ; i ++ ) a [ i ] = t [ i ];
}

int main ( )
{
    ifstream cin ( " input . txt" );
    int i , j ;
    while ( cin >> n ) {
        for ( i = 0; i < n ; i ++ ) cin >> a [ i ];
        ans = 0; sort ( 0 , n - 1 ); // Counting Inverse Number
        cout << " Minimum _ exchange _ operations : " << ans << endl;
    }
    return 0;
}
```

6.11 Counting Trees

```
// Rooted {1, 5, 11, 20, 30} => {1, 9, 1842, 12826228, 354426847597 }
// Non-Rooted {1, 3, 10, 25, 30} => {1, 1, 106, 104636890, 14830871802 }
void main()
{
    ifstream cin("input.txt");
    int i, j, n;
    memset(s, 0, sizeof(s)); a[0] = 0; a[1] = 1;
    for (i=1; i<maxn-1; i++)
    {
        a[i+1] = 0;
        for (j=1; j<=i; j++)
        {
            s[i][j] = s[i-j][j] + a[i+1-j];
            a[i+1] += j*a[j]*s[i][j];
        }
        a[i+1] /= i;
    }
    while (cin>>n) // a[n] = Rooted; ans = Non-Rooted
    {
        int ans = a[n];
        for (i=1; 2*i<=n; i++) ans -= a[i] * a[n-i];
        if (n%2==0) ans += (a[n/2]+1) * a[n/2] / 2;
        cout << a[n] << " " << ans << endl;
    }
}
```

6.12 Eight Puzzle Problem

Input: 012345678 123456780 Output: *STEP* = 22

Common Part

```
#define maxlen 10
#define size 362880+1

const int link[9][5]={{ {2,1,3}, {3,0,2,4}, {2,1,5}, {3,0,4,6},
    {4,1,3,5,7}, {3,2,4,8}, {2,3,7}, {3,4,6,8}, {2,5,6} }};

int s[maxlen], p[maxlen], mk[size], open[size], cur, tail;

void encode(int *s, int len, int &x)
{
    int i, j, k, l; for(x=0, i=len-1; i>=0; x+=k*p[i--])
        for(k=s[i], j=i+1; j<len; j++) if(s[j]<s[i]) k--;
}

void decode(int *s, int len, int x)
{
    int i, j, k, l; for(i=len-1; i>=0; i--) { s[i]=x/p[i]; x%=p[i]; }
    for(i=0; i<len; i++) for(j=0; j<i; j++) if(s[j]>=s[i]) s[j]++;
}

void print(int *s, int len)
{
    for(int i=0; i<len; i++)
        cout << s[i];
    cout << endl;
}
```

```

int main()
{
    ifstream cin("input.txt");
    char ch; int i, src, dst;
    for(p[0]=i=1; i<maxlen; i++) p[i]=p[i-1]*i;
    for(i=0; i<9; i++) { cin>>ch; s[i]=ch-'0'; } encode(s, 9, src);
    for(i=0; i<9; i++) { cin>>ch; s[i]=ch-'0'; } encode(s, 9, dst);
    solve(src, dst); cout<<cur<<" " <<tail <<endl;
    return 0;
}

```

Simple Breadth First Search

```

void output(int pos, int num)
{
    if(pos==1) cout<<"Total number of steps = "<<num<<endl;
    else output(mk[open[pos]], num+1);
    decode(s, 9, open[pos]); print(s, 9);
}

void solve(int src, int dst)
{
    int i, j, k, x, l, ps;
    if(src==dst){ cout<<"SRC-DST is the same!"<<endl; return; }
    cur=0; tail=1; open[1]=src; mk[src]=1;
    while(++cur<=tail){
        decode(s, 9, open[cur]); for(ps=0; s[ps]; ps++);
        for(k=1; k<=link[ps][0]; k++) {
            std::swap(s[ps], s[link[ps][k]]); encode(s, 9, x);
            if(!mk[x]){
                mk[x]=cur; open[++tail]=x;
                if(x==dst) { output(tail, 0); return; }
            }
            std::swap(s[ps], s[link[ps][k]]);
        }
    }
    cout<<"No solution!"<<endl;
}

```

Heuristic Breadth First Search

```

int d[size], heap[size], hlen, h[size], dsts[maxlen];
int cmp(const int &i, const int &j){ return h[i]>h[j]; }

void calch(int pos)
{
    int i, j, k; h[pos]=d[pos];
    for(i=0; i<9; i++) if(s[i]!=dsts[i]) h[pos]++;
}

void output(int pos, int num)
{
    if(pos==1) cout<<"Total number of steps = "<<num<<endl;
    else output(mk[open[pos]], num+1);
    decode(s, 9, open[pos]); print(s, 9);
}

```

```

void solve(int src, int dst)
{
    int i, j, k, x, l, ps;
    if(src==dst){ cout<<"SRC-DST is the same!"<<endl; return; }
    tail=1; open[1]=src; mk[src]=1; hlen=1; heap[0]=1; d[1]=0;
    decode(s,9,src); decode(dsts,9,dst); calch(1);
    while(hlen>0){
        cur=heap[0]; std::pop_heap(heap,heap+(hlen--),cmp);
        decode(s,9,open[cur]); for(ps=0; s[ps]; ps++){
            for(k=1; k<=link[ps][0]; k++){
                std::swap(s[ps],s[link[ps][k]]); encode(s,9,x);
                if(!mk[x]){
                    mk[x]=cur; open[++tail]=x; d[tail]=d[cur]+1; calch(tail);
                    heap[hlen++]=tail; std::push_heap(heap,heap+hlen,cmp);
                    if(x==dst) { output(tail,0); return; }
                }
                std::swap(s[ps],s[link[ps][k]]);
            }
        }
    }
    cout<<"No solution!"<<endl;
}

```

Double Breadth First Search

```

int step, di[size];

void out1(int pos)
{
    if(pos>2) out1(mk[open[pos]]); step++;
    decode(s,9,open[pos]); print(s,9);
}

void out2(int pos)
{
    decode(s,9,open[pos]); print(s,9);
    if(pos>2) out2(mk[open[pos]]); step++;
}

void solve(int src, int dst)
{
    int i, j, k, x, l, ps;
    if(src==dst){ cout<<"SRC-DST is the same!"<<endl; return; }
    open[1]=src; mk[src]=1; di[src]=1; cur=0;
    open[2]=dst; mk[dst]=2; di[dst]=2; tail=2;
    while(++cur<=tail){
        decode(s,9,open[cur]); for(ps=0; s[ps]; ps++){
            for(k=1; k<=link[ps][0]; k++){
                std::swap(s[ps],s[link[ps][k]]); encode(s,9,x);
                if(!mk[x]){ mk[x]=cur; open[++tail]=x; di[x]=di[open[cur]]; }
                else if(di[x]!=di[open[cur]]){
                    step=0;
                    if(di[x]==1) { out1(mk[x]); out2(cur); }
                    else { out1(cur); out2(mk[x]); }
                    cout<<" Total number of steps = "<<step<<endl;
                    return;
                }
                std::swap(s[ps],s[link[ps][k]]);
            }
        }
    }
    cout<<"No solution!"<<endl;
}

```

6.13 Extended Honai Tower

```
int P[ML][ML],D[ML][ML],T[ML][ML];  
  
void init()  
{  
    int i,j,x,k,l;  
    for(P[0][0]=l=1;l<ML;l++) {  
        P[0][1] = P[1][0] = 1;  
        for(i=1;i<l;i++) P[i][1-i] = P[i-1][1-i]+P[i][1-i-1];  
    }  
    for(i=0;i<ML;i++) for(k=j=0;j<ML-i && k!=ML;j++)  
        for(x=0;x<P[i][j];x++) { if (k==ML) break; D[i][k++] = 1<<j; }  
    for(i=0;i<ML;i++) T[i][0] = 0;  
    for(j=1;j<ML;j++) for(i=0;i<20;i++) T[i][j] = T[i][j-1]+D[i][j-1];  
}  
  
int main()  
{  
    init();  
    for (int a,b,casec=1; cin>>a>>b && (a||b); casec++)  
        cout << "Case " << casec << ":" << T[b-3][a] << endl;  
    return 0;  
}
```

6.14 High Precision Square Root

```
int x[maxlen],y[maxlen],z[maxlen],bck[maxlen],lx,ly,lz;  
  
int IsSmaller() // is z<=y ?  
{int i=ly; while(i>1 && z[i]==y[i]) i--; return (z[i]<=y[i]); }  
  
void Solve() // y^2=x  
{  
    int i,j,k;  
    lx=(ly+1)/2; ly=lx*2;  
    memset(x,0,sizeof(x)); memset(z,0,sizeof(z));  
    for(i=lx;i>0;i--) {  
        for(j=1;j<10;x[i]=j++) {  
            memcpy(bck,z,sizeof(z));  
            z[2*i-1]++; for(k=i;k<=lx;k++)  
                { z[i-1+k] += 2*x[k]; z[i+k] += z[i-1+k]/10; z[i-1+k] %= 10; }  
            for(k=lx+i;k<=ly;k++) { z[k+1] += z[k]/10; z[k] %= 10; }  
            if (!IsSmaller()) break;  
        };  
        if(j<10) memcpy(z,bck,sizeof(bck));  
    };  
    for(i=lx;i>0;i--) cout<<x[i]; cout<<endl;  
}  
  
int main()  
{  
    char ch,s[maxlen]; int i,j;  
    memset(y,0,sizeof(y));  
    cin>>s; ly=strlen(s);  
    for(i=0;i<ly;i++) y[i+1]=s[ly-1-i]-'0';  
    Solve();  
    return 0;  
}
```

6.15 Largest Empty Rectangle

$O(N^2)$

```

int n,wx,wy,x [ maxn ] ,y [ maxn ] ,id [ maxn ] ;
int xx [ maxn ] ,yy [ maxn ] ,ans ;

bool cmp(const int&i ,const int&j )
{
    return x [ i ] < x [ j ] ;
}

void calc(int i ,int px ,int py)
{
    int ret ,j ,low=0,high=wy ;
    for ( ; i < n ; i ++ ) if ( x [ i ] > px )
    {
        j=(high-low)*(x [ i ] -px); if ( j > ans ) ans=j ;
        if ( y [ i ] < py && y [ i ] >= low ) low = y [ i ] ;
        if ( y [ i ] >= py && y [ i ] <= high ) high = y [ i ] ;
    }
}

int main()
{
    int i ,j ,k ;
    cin >> wx >> wy >> n ; for ( i=0; i < n ; i ++ ) cin >> x [ i ] >> y [ i ] ;
    x [ n ] = y [ n ] = 0; n ++; x [ n ] = wx; y [ n ] = wy; n ++;
    for ( i=0; i < n ; i ++ ) id [ i ] = i ; std :: sort ( id , id +n , cmp );
    for ( i=0; i < n ; i ++ ) { xx [ i ] = x [ id [ i ] ] ; yy [ i ] = y [ id [ i ] ] ; }
    for ( i=0; i < n ; i ++ ) { x [ i ] = xx [ i ] ; y [ i ] = yy [ i ] ; }
    std :: sort ( yy , yy +n ); k=std :: unique ( yy , yy +n ) - yy ;
    ans = 0;
    for ( i=0; i < n ; i ++ ) calc ( i , x [ i ] , y [ i ] );
    for ( j=0; j < k ; j ++ ) calc ( 0 , 0 , yy [ j ] );
    cout << ans << endl ;
    return 0 ;
}

```

$O(D^2)$

```

int x [ maxn ] ,y [ maxn ] ,xlist [ maxn ] ,ylist [ maxn ] ,nx ,ny ,ans ,n ,wx ,wy ;
char g [ maxd ] [ maxd ] ; int u [ maxd ] ,d [ maxd ] ,l [ maxd ] ;

int main()
{
    int i ,j ,px ,py ,up ,down ,tmp ; ans=0;
    cin >> wx >> wy >> n ; for ( i=0; i < n ; i ++ ) cin >> x [ i ] >> y [ i ] ;
    nx=ny=n ; for ( i=0; i < n ; i ++ ) { xlist [ i ] = x [ i ] ; ylist [ i ] = y [ i ] ; }
    xlist [ nx++ ] = ylist [ ny++ ] = 0; xlist [ nx++ ] = wx; ylist [ ny++ ] = wy;
    std :: sort ( xlist , xlist +nx ); nx=std :: unique ( xlist , xlist +nx ) - xlist ;
    std :: sort ( ylist , ylist +ny ); ny=std :: unique ( ylist , ylist +ny ) - ylist ;

    for ( i=0; i < nx ; i ++ ) memset ( g , 0 , n );

    for ( i=0; i < n ; i ++ )
    {
        px = std :: lower_bound ( xlist , xlist +nx , x [ i ] ) - xlist ;
        py = std :: lower_bound ( ylist , ylist +ny , y [ i ] ) - ylist ;
        g [ px ] [ py ] = 1;
    }
}

```

```

for (j=0; j<ny-1; j++)
{
    tmp = wx * ( ylist [ j+1 ] - ylist [ j ] );
    if ( tmp > ans ) ans = tmp;
}
for ( i=1; i<nx; i++)
{
    down=0; up=ny-1;
    for ( j=0; j<ny; j++) if ( i==1 || *(*(g+i-1)+j) )
        { l [ j ]=i-1; d [ j ]=0; down=j; } else
        if ( down > d [ j ] ) d [ j ] = down;
    for ( j=ny-1; j>=0; j--)
    {
        if ( i==1 || *(*(g+i-1)+j) ) { u [ j ]=ny-1; up=j; } else
            if ( up < u [ j ] ) u [ j ] = up;
        tmp = ( xlist [ i ] - xlist [ l [ j ] ] ) * ( ylist [ u [ j ] ] - ylist [ d [ j ] ] );
        if (tmp>ans) ans=tmp;
    }
}
cout<<ans<<endl;
return 0;
}

```

$O(N^2)$

```

int n,wx,wy,id [ maxn ] ,x [ maxn ] ,y [ maxn ] ,ans ,xx [ maxn ] ,yy [ maxn ];

bool xcmp(const int&i,const int &j) { return x [ i ]<x [ j ]; }
bool ycmp(const int&i,const int &j) { return y [ i ]<y [ j ]; }

int main()
{
    int i,j,k,l,tmp,low,high,last;
    cin>>wx>>wy>>n; for (i=0;i<n; i++) cin>>x [ i ]>>y [ i ];
    x [ n ]=y [ n ]=0; n++; x [ n ]=wx; y [ n ]=wy; n++;
    for (i=0;i<n; i++) id [ i ]=i;
    std::sort (id ,id+n,xcmp);
    for (i=0;i<n; i++) { xx [ i ]=x [ id [ i ] ]; yy [ i ]=y [ id [ i ] ]; }
    for (i=0;i<n; i++) { x [ i ]=xx [ i ]; y [ i ]=yy [ i ]; }
    std::sort (id ,id+n,ycmp);
    for (i=0;i<n; i++)
    {
        l=0; last=0;
        for (j=0;j<n; j++) if ( x [ id [ j ] ]<x [ i ] && y [ id [ j ] ]>last )
        {
            if ( y [ id [ j ] ]-last > 1 ) l=y [ id [ j ] ]-last;
            last=y [ id [ j ] ];
        }
        if ( wy-last>l ) l=wy-last;
        if ( l*x [ i ] > ans ) ans =l*x [ i ];
        low=0; high=wy; for (j=i+1;j<n; j++)
        {
            tmp = ( high-low)*(x [ j ]-x [ i ] ) ;
            if ( tmp> ans ) ans=tmp;
            if ( y [ j ]>=y [ i ] && y [ j ]<high ) high = y [ j ];
            if ( y [ j ]<=y [ i ] && y [ j ]>low ) low = y [ j ];
        }
    }
    cout<<ans<<endl;
    return 0;
}

```

6.16 Last Non-Zero Digit of N!

Smart Edition

```
const int ff[10] = {1, 1, 2, 6, 4, 4, 4, 8, 4, 6};

int fact(int n)
{
    int i, x;
    if(n<5) return ff[n];
    x = (ff[n%10] * 6) %10;
    for( i=1; i<=(n/5)%4; i++)
        if( x==6 || x==2) x=(x+10)/2; else x/=2;
    return ( fact(n/5) * x ) % 10;
}
```

High Precision Edition

```
int a[10] = {6,1,2,6,4,4,4,8,4,6};
int b[4] = {1,8,4,2};

void divide(char s[], int &len)
{
    int i;
    char temp[200];
    for(i=0; i<len; i++) temp[i] = s[i]*2; temp[len] = 0;
    for(i=0; i<len; i++) if( temp[i]>9 ) { temp[i]-=10; temp[i+1]++; }
    for(i=0; i<len; i++) s[i] = temp[i+1];
    if( temp[len]==0 ) len--;
}

int fact(char s[])
{
    int resulent=1,power=0,len=strlen(s),i;
    char temp;
    if (len==1&&s[0]== '0') return 1;
    for(i=0;i<len; i++) s[i]-= '0';
    for(i=0;i<len/2; i++){ temp=s[i]; s[i]=s[len-1-i]; s[len-1-i]=temp; }
    while(len){
        resulent=resulent*a[s[0]%10]%10;
        divide(s, len);
        power+=(s[1]*10+s[0])%4;
    }
    resulent=resulent*b[power%4]%10;
    return resulent;
}
```

6.17 Least Common Ancestor

```
int n,h,root; // maxh-1 = h = floor( log ( 2, n-1 ) )
int pnt[maxn][maxh], son[maxn], next[maxn], depth[maxn];
int stack[maxn], mylog[maxn];

int GetParent(int x, int len)
{
    while( len>0 ) {
        x = pnt[x][ mylog[len] ];
        len -= ( 1<<mylog[len] );
    }
    return x;
}
```

```

int LCA(int x, int y) //  $O(\log N)$ 
{
    int nx, ny, px, py, low, mid, high;
    low=0; high = depth[x]<depth[y] ? depth[x] : depth[y];
    px = GetParent(x, depth[x]-high);
    py = GetParent(y, depth[y]-high);
    if( px == py ) return px;
    while( high-low>1)
    {
        mid = mylog[high-low-1];
        nx = pnt[px][mid];
        ny = pnt[py][mid];
        mid = high - (1<<mid);
        if( nx == ny ) low = mid; else { high = mid; px = nx; py = ny; }
    }
    return pnt[px][mylog[high-low]];
}

int LCA_2(int x, int y) //  $O(\log^2 N)$ 
{
    int low, mid, high;
    low = 0; mid = high = depth[x]<depth[y] ? depth[x] : depth[y];
    if( GetParent(x, depth[x]-mid) != GetParent(y, depth[y]-mid) )
    while( low+1<high )
    {
        mid = (low + high) / 2;
        if( GetParent(x, depth[x]-mid) != GetParent(y, depth[y]-mid) )
            high = mid; else low = mid;
    } else low = high;
    return GetParent(x, depth[x]-low);
}

void dfs(int d, int cur)
{
    int i, j; stack[d] = cur; depth[cur] = d;
    for(j=1, i=2; i<=d; j++, i*=2) pnt[cur][j]=stack[d-i];
    for(j=son[cur]; j; j=next[j]) dfs(d+1, j);
}

void main()
{
    int i, j, k, l;
    for(i=0, j=1; j<maxn; i++)
    {
        k = j*2; if( k>maxn ) k = maxn;
        while( j<k ) mylog[j++] = i;
    }
    cin>>n;
    for(i=1; i<=n; i++) {
        son[i] = next[i] = 0;
        for(j=0; j<=h; j++) pnt[i][j] = 0;
    }
    for(i=1; i<n; i++) {
        cin >> j >> k; pnt[j][0] = k;
        next[j]=son[k]; son[k]=j;
    }
    for(i=1; i<=n; i++) if( pnt[i][0]==0 ) { root=i; break; };
    dfs(0, root); // Preprocess Parent Array
    for(cin>>k; k; k--) { cin >> i >> j; cout << LCA(i, j) << endl; }
}

```

6.18 Longest Common Substring

$O(N \log N)$, using Suffix Sort with LCP information

```
int LCS(char *s1, int l1, char *s2, int l2, int &i1, int &i2)
{
    strcpy(s, s1); s[l1] = '$';
    strcpy(s+l1+1, s2); n=l1+l2+1;
    SuffixSort(); GetHeight(); // s[l1]=0;
    int i, j, l=0; i1 = i2 = 0;
    for(i=1; i<n; i++)
    {
        if( height[i]>=l && id[i-1]<l1 && id[i]>l1 )
            { l = height[i]; i1 = id[i-1]; i2 = id[i]-l1-1; }
        if( height[i]>=l && id[i]<l1 && id[i-1]>l1 )
            { l = height[i]; i1 = id[i]; i2 = id[i-1]-l1-1; }
    }
    return l;
}
```

$O(N^2)$, using KMP

```
int LCS(char *s1, int l1, char *s2, int l2, int &ansi, int &ansj)
{
    int i, j, k, l, ans=0; ansi=0; ansj=0;
    for(i=0; i<l1-ans; i++)
    {
        makefail(s1+i, l1-i);
        kmp(s2, l2, s1+i, l1-i, 0, l, j);
        if(l>ans) { ans=l; ansi=i; ansj=j; }
    }
    return ans;
}
```

Example Part

```
char s1[maxlen], s2[maxlen]; int l1, l2;

int main()
{
    ifstream cin("input.txt");
    cin>>s1>>s2; l1=strlen(s1); l2=strlen(s2);
    int i1, i2, i, l = LCS(s1, l1, s2, l2, i1, i2);
    cout<<l<<" "

```

M^{th} Longest Common Substring

```
#define h next // h[i] = Longest Common Substring of s1+0 and s2+i
int mk[maxn]; // already found a common substring = s2[i..mk[i])
struct CAnswer { int pos, len; } ans[maxn];

bool newcmp(const CAnswer &a, const CAnswer &b)
{
    if(a.len != b.len) return a.len>b.len;
    return a.pos<b.pos;
}
```

```

void LCS(char *s1, int l1, char *s2, int l2, int m)
{
    strcpy(s, s1);           s[l1]='$';
    strcpy(s+l1+1, s2);   n=l1+l2+1;
    SuffixSort();           GetHeight(); // s[l1]=0;
    int i, j, k, p, u, v;
    // computing longest common prefix between s1+0 and s2+i
    memset(h, 0, sizeof(h));
    for(i=0; i<n; i++) if( i<n-1 && id[i]<l1 && id[i+1]>l1 ) {
        k=maxlen;
        for(j=i+1; j<n; j++)
            { if( id[j]<l1 ) break; if( height[j]<k) k=height[j]; h[j]=k; }
        i=j-1;
    }
    for(i=n-1; i>0; i--) if( id[i]<l1 && id[i-1]>l1 ) {
        k=maxlen;
        for(j=i-1; j>=0; j--) {
            if( id[j]<l1 ) break; if( height[j+1]<k) k=height[j+1];
            if( k>h[j] ) h[j]=k;
        }
        i=j+1;
    }
    num=0; // Collect Non-Position-Covering Answer
    for(i=0; i<n; i++) {
        if( h[rank[i]]!=0 && ( i==0 || h[rank[i-1]]<=h[rank[i]] ) )
            { k=rank[i]; ans[num].pos=id[k]; ans[num].len=h[k]; num++; }
    }
    std::sort(ans, ans+num, newcmp);
    memset(mk, 0, sizeof(mk));
    for(i=j=0; i<num && j<m; i++) {
        k=rank[ans[i].pos]; // Check Non-Substring-Covering
        if( mk[k]>=h[k] ) continue;
        int ok=1;
        for(u=maxlen, p=k+1; p<n; p++) {
            if( height[p]<u ) u=height[p];
            if( u<h[k] ) break;
            if( mk[p]>=h[k] ) { ok=0; break; }
        }
        if( !ok ) continue;
        for(u=maxlen, p=k-1; p>=0; p--) {
            if( height[p+1]<u ) u=height[p+1];
            if( u<h[k] ) break;
            if( mk[p]>=h[k] ) { ok=0; break; }
        }
        if( !ok ) continue;
        j++; // Check Passed, Set Already Found Substring
        for(v=0; v<h[k]; v++)
            if( mk[rank[id[k]+v]] < h[k]-v ) mk[rank[id[k]+v]] = h[k]-v;
        // LENGTH h[rank[ans[i].pos]]/ POSITION ans[i].pos-l1-1
        char ch = s[ans[i].pos + h[rank[ans[i].pos]]];
        s[ans[i].pos + h[rank[ans[i].pos]]] = 0;
        cout << s+ans[i].pos << endl;
        s[ans[i].pos + h[rank[ans[i].pos]]] = ch;
    }
}

```

6.19 Longest Non Descending Sub Sequence

```
int LNDSS(int a[], int n) // Longest Non-descending Sub Sequence
{
    int i, j, k, *b=new int[n+1], ans=0;
    b[ans]=-0x3f3f3f3f;
    for (i=0; i<n; i++) { // lower_bound for Asending Sub Squence
        j=std::upper_bound(b, b+ans+1, a[i])-b;
        if (j>ans) b[++ans]=a[i]; else if (a[i]<=b[j]) b[j]=a[i];
    }
    delete b; return ans;
}
```

6.20 Join and Disjoin

Note: UnionFind.h contains a Union-Find Set (Section 1.10 on Page 11)

```
#include<unionfind.h>

int Gather(int x, int y)
{
    if (!x && !y) return 0;
    if (!x) return find(y);
    if (!y) return find(x);
    Merge(x, y);
    return find(x);
}

void Join(int x, int y)
{
    int a=Gather(x, y); // x, y never be zero
    int b=Gather(vs[x], vs[y]);
    vs[a]=b; vs[b]=a;
}

void Disjoin(int x, int y)
{
    int a=Gather(x, vs[y]);
    int b=Gather(y, vs[x]);
    vs[a]=b; vs[b]=a;
}
```

6.21 Magic Square

```
#define maxn 1000
int a[ maxn ][ maxn ] ,n;

void build( int n ,int a[ ][ maxn ] ) // No solutions when n=2!
{
    int i ,j ,k ,n2=n*n ,m=n/2 ,m2=m*m;
    for( i=0; i<n; i++ ) for( j=0; j<n; j++ ) a[ i ][ j ]=0;
    if ( n==2 ) return; // No solutions
    if ( n%2==1 )
        for( i=0, j=n/2 ,k=1; k<=n2 ; k++ ) {
            a[ i ][ j ] = k;
            if ( !a[ ( i+n-1)%n ][ ( j+1)%n ] )
                { i=( i+n-1)%n; j=( j+1)%n; } else i=( i+1)%n;
        }
    else if ( n%4==0 )
        for( k=0, i=0; i<n; i++ ) for( j=0; j<n; j++ ) {
            a[ i ][ j ] = ++k;
            if ( i%4==j%4 || i%4+j%4==3 ) a[ i ][ j ] = n2+1-a[ i ][ j ];
        }
    else if ( n%4==2 )
        for( i=0, j=m/2 ,k=0; k<m2 ; k++ ) {
            if ( ( i<=m/2 && !( i==m/2&&j==m/2 ) ) || ( i==m/2+1&&j==m/2 ) ) { // L
                a[ i*2 ][ j*2+1 ]=k*4+1; a[ i*2+1 ][ j*2 ]=k*4+2;
                a[ i*2+1 ][ j*2+1 ]=k*4+3; a[ i*2 ][ j*2 ]=k*4+4;
            } else if ( i>m/2+1 ) { // X
                a[ i*2 ][ j*2 ]=k*4+1; a[ i*2+1 ][ j*2+1 ]=k*4+2;
                a[ i*2+1 ][ j*2 ]=k*4+3; a[ i*2 ][ j*2+1 ]=k*4+4;
            } else { // U
                a[ i*2 ][ j*2 ]=k*4+1; a[ i*2+1 ][ j*2 ]=k*4+2;
                a[ i*2+1 ][ j*2+1 ]=k*4+3; a[ i*2 ][ j*2+1 ]=k*4+4;
            }
            if ( !a[ ( i+m-1)%m*2 ][ ( j+1)%m*2 ] ) i=( i+m-1)%m, j=( j+1)%m;
            else i=( i+1)%m;
        }
    }
int main()
{
    while ( cin>>n ) {
        build( n ,a ); cout<<"Order - "<<n<<" : "<<endl;
        for ( int j ,i=0; i<n; i++ )
            { for ( j=0; j<n; j++ ) cout<<a[ i ][ j ]<<' ' ; cout<<endl; }
    }
    return 0;
}
```

6.22 Optimal Binary Search Tree

```

int n, a [ maxn ] , s [ maxn ] [ maxn ] , h [ maxn ] [ maxn ] , kk [ maxn ] [ maxn ] ;

int solve()
{
    int i , j , k , l;  memset ( h , 0 , sizeof ( h ) );
    for ( i = 1 ; i <= n ; i ++ ) { s [ i ] [ i ] = a [ i ] ; h [ i ] [ i ] = 0 ; kk [ i ] [ i ] = i ;
        for ( j = i + 1 ; j <= n ; j ++ ) s [ i ] [ j ] = s [ i ] [ j - 1 ] + a [ j ] ;
    }
    for ( l = 1 ; l < n ; l ++ ) {
        for ( i = 1 ; i < n ; i ++ ) { j = i + l ; h [ i ] [ j ] = 0 x 0 f f f f f f f f ;
            for ( k = kk [ i ] [ j - 1 ] ; k <= kk [ i + 1 ] [ j ] ; k ++ )
                if ( h [ i ] [ k - 1 ] + h [ k + 1 ] [ j ] - a [ k ] + s [ i ] [ j ] < h [ i ] [ j ] ) {
                    h [ i ] [ j ] = h [ i ] [ k - 1 ] + h [ k + 1 ] [ j ] + s [ i ] [ j ] - a [ k ] ;
                    kk [ i ] [ j ] = k ;
                }
        }
    }
    return h [ 1 ] [ n ];
}

```

6.23 Pack Rectangles — Cut Rectangles

```

struct rect { int x1 , y1 , x2 , y2 ; } r [ maxm ] ;
int mk [ maxm ] ;

int intersect ( rect a , const rect & b , rect out [ 4 ] ) // b cut a
{
    if ( b . x2 <= a . x1 || b . x1 >= a . x2 || b . y2 <= a . y1 || b . y1 >= a . y2 ) return 0 ;
    if ( b . x1 <= a . x1 && b . x2 >= a . x2 && b . y1 <= a . y1 && b . y2 >= a . y2 ) return -1 ;
    rect t ; int nout = 0 ;
    if ( b . x1 > a . x1 ) { t = a ; t . x2 = b . x1 ; a . x1 = b . x1 ; out [ nout ++ ] = t ; }
    if ( b . x2 < a . x2 ) { t = a ; t . x1 = b . x2 ; a . x2 = b . x2 ; out [ nout ++ ] = t ; }
    if ( b . y1 > a . y1 ) { t = a ; t . y2 = b . y1 ; a . y1 = b . y1 ; out [ nout ++ ] = t ; }
    if ( b . y2 < a . y2 ) { t = a ; t . y1 = b . y2 ; a . y2 = b . y2 ; out [ nout ++ ] = t ; }
    return nout ;
}

int main()
{
    rect curr , t [ 4 ] ; int i , j , k , nn , nr , ans , rr , n ;
    cin >> n ; rr = 0 ;
    for ( i = 0 ; i < n ; i ++ ) {
        cin >> curr . x1 >> curr . y1 >> curr . x2 >> curr . y2 ;
        nr = rr ; mk [ rr ] = 1 ; r [ rr ++ ] = curr ;
        for ( j = 0 ; j < nr ; j ++ ) {
            mk [ j ] = 1 ; nn = intersect ( r [ j ] , curr , t ) ; if ( ! nn ) continue ;
            if ( nn < 0 ) mk [ j ] = 0 ; else { r [ j ] = t [ -- nn ] ;
                while ( nn ) { mk [ rr ] = 1 ; r [ rr ++ ] = t [ -- nn ] ; }
            }
        }
        for ( k = j = 0 ; j < rr ; j ++ ) if ( mk [ j ] ) r [ k ++ ] = r [ j ] ; rr = k ;
    }
    for ( ans = i = 0 ; i < rr ; i ++ ) ans += ( r [ i ] . x2 - r [ i ] . x1 ) * ( r [ i ] . y2 - r [ i ] . y1 ) ;
    cout << ans << endl ;
    return 0 ;
}

```

6.24 Pack Rectangles — $O(N^2)$

```
int x1 [maxn] ,y1 [maxn] ,x2 [maxn] ,y2 [maxn] ;
int ylist [maxn*2] ,id [maxn] ,n ,ny ;

bool cmp (const int&i ,const int&j ){ return x1 [i]<x1 [j] ; }

int GetAreaUnion ()
{
    int i ,j ,k ,rx ,l ,ans=0;
    for (ny=0,i=0; i<n; i++) { ylist [ny++]=y1 [i]; ylist [ny++]=y2 [i]; }
    std :: sort ( ylist , ylist+ny); ny=std :: unique ( ylist , ylist+ny)- ylist ;
    for (i=0; i<n; i++) id [i]=i; std :: sort (id ,id+n ,cmp);
    for (j=0; j<ny-1; j++){
        rx = -0x3f3f3f; l=0;
        for (k=0;k<n;k++){ i = id [k];
            if ( y1 [i]<= ylist [j] && y2 [i]>= ylist [j+1] && x2 [i]>rx ) {
                if ( x1 [i]>rx ) l+=x2 [i]-x1 [i]; else l+=x2 [i]-rx ;
                rx = x2 [i];
            }
        }
        ans += l * ( ylist [j+1]- ylist [j] );
    }
    return ans;
}
```

6.25 Parliament

Given $n > 0$, find distinct positive numbers $a_1 + a_2 + \dots + a_k = n$ that maximize $a_1 \cdot a_2 \cdot \dots \cdot a_k$.

```
int main ()
{
    int n ,k ,p ,i ,caseno ;
    for ( cin>>caseno ; caseno --; ) { cin>>n;
        for (p=n , k=2; p>=k; k++) p-=k; k--;
        if (p<=1){ for (i=2;i<k; i++) cout<<i<<" " ; cout<<k+p<<endl; } else
        if (p==k){ for (i=3;i<=k; i++) cout<<i<<" " ; cout<<k+2<<endl; } else
        { for (i=2+(p==k-1);i<=k; i++) if (i!=k-p+1) cout<<i<<" " ;
        cout<<k+1<<endl; }
        if (caseno ) cout<<endl;
    }
    return 0;
}
```

6.26 π Generator

```
int a=10000,b ,c=2800,d ,e ,f [2801] ,g;

void GenPI() {
    for (;b-c ;) f [b++]=a /5;
    for (;d=0,g=c *2; c -=14,printf ("%.4d" ,e+d/a) ,e=d%a)
        for (b=c ; d+=f [b]*a ,f [b]=d%-g ,d=g--,-b; d*=b );
}
```

6.27 Plant Trees — Iteration

```
const int maxlen = 50005;
const int maxn = 50000;

int n, st [ maxlen ] ,a [ maxn ] ,b [ maxn ] ,c [ maxn ] ,up ;

int main(){
    int i ,more ;
    while( cin>>n){
        for( i=0; i<n; i++){
            cin>>a [ i]>>b [ i]>>c [ i ];
            if( ++b [ i]>up ) up=b [ i ];
        }
        memset( st , 0 , sizeof( st ) );
        for( more=1; more; ){
            more = 0;
            for( i=0; i<n; i++) if ( st [ a [ i]]+c [ i]>st [ b [ i ]])
                { st [ b [ i]]=st [ a [ i]]+c [ i ]; more=1; }
            for( i=1; i<=up; i++) {
                if( st [ i-1]+1<st [ i ]){ st [ i-1]=st [ i ]-1; more=1; }
                if( st [ i-1] >st [ i ]){ st [ i ]=st [ i-1]; more=1; }
            }
            for( i=up; i>0; i--){
                if( st [ i ]-1>st [ i-1 ]){ st [ i-1]=st [ i ]-1; more=1; }
                if( st [ i-1]>st [ i ]){ st [ i ]=st [ i-1]; more=1; }
            }
        }
        cout<<st [ up ] << endl;
    }
    return 0;
}
```

6.28 Plant Trees — Segment Tree

```
#define maxn 50000
#define maxup 50006

int nspan ,span [maxn][3] ,up ,tree [maxup];
int iteam [maxup] ,next [maxup] ,num;

int funt_comp( const void *a , const void *b)
{ return ((const int *)b)[0] - ((const int *)a)[0]; }

void add( int r)
{ for ( ; r<=up; r+=r&(r^(r-1))) ++tree [ r ]; }

int sum( int r)
{ int ans = 0; for ( ; r>0; r-=r&(r^(r-1))) ans+=tree [ r ]; return ans; }

void go()
{
    int j ,k ,i ,ans=0; up=0;
    for( i=0; i<nspan; ++i){
        scanf("%d %d %d", &span [ i ][ 0] , &span [ i ][ 1] , &span [ i ][ 2 ] );
        ++span [ i ][ 0]; ++span [ i ][ 1];
        if ( span [ i ][ 1]>up ) up=span [ i ][ 1 ];
    }
    qsort( span , nspan , sizeof( int )*3 , funt_comp );
    for( j=0;j<=up ;j++) next [ j]=j+1;
    next [ up ]=0; memset( tree ,0 ,( up+1)*sizeof( int ) );
    for( i=0; i<nspan ; i++){

```

```

k=sum( span[ i ][ 1 ]) -sum( span[ i ][ 0 ] -1 );
if( k>=span[ i ][ 2 ]) continue; else k=span[ i ][ 2 ] -k ;
j=span[ i ][ 0 ]; if( next[ j-1 ]!=j ) j=next[ j ];
while( k--){
    next[ span[ i ][ 0 ]]=next[ span[ i ][ 0 ] -1 ]=next[ j ];
    ans++; add( j ); j=next[ j ];
}
printf( "%d\n" , ans );
}

int main() {
    while( 1==scanf( "%d" , &nspan ) ) go();
    return 0;
}

```

6.29 Range Maximum Query

$O(N \log N)$ Preprocess, $O(1)$ Query

```

int n,L,q , a[ maxn ] ,h[ maxn ][ maxL ]; // maxL = sqrt{N} + 3

void PreProcess()
{
    int i,j,l;
    for( i=0; i<n; i++ ) h[ i ][ 0 ]=a[ i ];
    for( j=1, l=1; l*2<=n; j++, l*=2 ) for( i=0; i<=n-l*2; i++ )
        h[ i ][ j ] = ( h[ i ][ j-1 ]>h[ i+1 ][ j-1 ] ) ? h[ i ][ j-1 ] : h[ i+1 ][ j-1 ];
}

int Query( int be ,int ed ) // return max{ a[ op .. ed ] }
{
    int j=0, l=1; while( 2*l<=ed-be+1 ) { j++; l*=2; }
    return ( h[ be ][ j ]>h[ ed+1-l ][ j ] ) ? h[ be ][ j ] : h[ ed+1-l ][ j ];
}

```

$O(N)$ Preprocess, $O(\sqrt{N})$ Query

```

int a[ maxn ] ,b[ maxL ] ,n ,L,q;

void PreProcess()
{
    int i,j,up,k; L = ( int )sqrt( n );
    for( i=k=0; i<n; k++ ) {
        up=i+L; if( up>n ) up = n;
        for( j=i+1; j<up; j++ ) if( a[ j ]>a[ i ] ) i=j;
        b[ k ]=i; i=up;
    }
}

int Query( int be ,int ed ) // return max{ a[ op .. ed ] }
{
    int i,up,u,v,k;
    u = be / L; v = ed / L; k = be;
    if( u<v ) {
        k=be; up=(u+1)*L;
        for( i=u+1; i<v ; i++ ) if( a[ b[ i ] ]>a[ k ] ) k = b[ i ];
        for( i=be; i<up; i++ ) if( a[ i ]>a[ k ] ) k = i;
        for( i=v*L; i<=ed; i++ ) if( a[ i ]>a[ k ] ) k = i;
    } else for( i=be; i<=ed; i++ ) if( a[ i ]>a[ k ] ) k = i;
    return k;
}

```

6.30 Travelling Salesman Problem

```
int n,x [maxn] ,y [maxn] ,id [maxn] ;
double g [maxn] [maxn] ;

double dis (int x1,int y1,int x2,int y2)
{ return sqrt ((x1-x2)*(x1-x2)+(y1-y2)*(y1-y2)); }

double solve ()
{
    int i,j,k,l,loop ;
    double cur ,ans=1e30 ;

    for ( i=0;i<n; i++)
        for ( j=0;j<n; j++)
            g [ i ] [ j]=dis (x [ i ] ,y [ i ] ,x [ j ] ,y [ j ] );

    for (k=0;k<n; k++)
    {
        for ( l=0;l<50;l++)
        {
            for ( i=0;i<n; i++) id [ i ]=i ;
            std :: swap ( id [ 0 ] , id [ k ] );
            std :: random_shuffle ( id+1,id+n );

            loop=1;
            while (loop){
                loop=0;
                for ( i=1;i<n; i++)
                    for ( j=i+1;j<n-1; j++)
                        if ( g [ id [ i-1 ] ] [ id [ i ] ] + g [ id [ j ] ] [ id [ j+1 ] ]
                            > g [ id [ i-1 ] ] [ id [ j ] ] + g [ id [ i ] ] [ id [ j+1 ] ] + 1e-8 )
                            {
                                loop=1;
                                std :: reverse ( id+i , id+j+1 );
                            }
            };
            for ( cur=0,i=0; i<n-1; i++)
                cur+=g [ id [ i ] ] [ id [ i+1 ] ];

            if ( cur<ans ) ans=cur ;
        }
    }
    return ans ;
}
```

6.31 Tree Heights

```
#define maxn 5003*4

int n, num, nbs[maxn], next[maxn*2], h[maxn];
int out1[maxn], out2[maxn], son1[maxn], in[maxn], value[maxn];
int son[maxn], pnt[maxn], bro[maxn], weight[maxn], id[maxn];

void solve()
{
    int i, j, k(1), l;
    id[1]=1; weight[1]=in[1]=0;
    for(i=1; i<=k; i++) for(j=son[id[i]]; j; j=bro[j]) id[++k]=j;
    for(i=2; i<=n; i++) weight[i]=1;
    for(k=n; k>0; k--) { i=id[k];
        for(j=son[i]; j; j=bro[j])
            if(out1[j]+weight[j]>=out1[i])
                { out2[i]=out1[i]; out1[i]=out1[j]+weight[j]; son1[i]=j; }
            else if(out1[j]+weight[j]>out2[i]) out2[i]=out1[j]+weight[j];
    }
    for(k=2; k<=n; k++) { i=id[k]; in[i]=0;
        if(in[pnt[i]]>in[i]) in[i]=in[pnt[i]];
        if(i==son1[pnt[i]]) l=out2[pnt[i]]; else l=out1[pnt[i]];
        if(l>in[i]) in[i]=l; in[i]+=weight[i];
    }
}

void dfs(int node)
{
    for(int j, i=nbs[node]; i; i=next[i]){
        j=value[i]; if(j==pnt[node]) continue;
        pnt[j]=node; bro[j]=son[node]; son[node]=j; dfs(j);
    }
}

void out()
{
    int maxh=-1, minh=n+1, i;
    for(i=1; i<=n; i++) {
        if(in[i]<out1[i]) h[i]=out1[i]; else h[i]=in[i];
        if(h[i]>maxh) maxh=h[i]; if(h[i]<minh) minh=h[i];
    }
    cout<<"Best_Roots: ";
    for(i=1; i<=n; i++) if(h[i]==minh) cout<<" " <<i;
    cout<<endl <<"Worst_Roots: ";
    for(i=1; i<=n; i++) if(h[i]==maxh) cout<<" " <<i;
    cout<<endl;
}

int main()
{
    int i, j, k, l;
    while(cin>>n){
        for(i=1; i<=n; i++)
            out1[i]=out2[i]=son1[i]=son[i]=bro[i]=pnt[i]=next[i]=nbs[i]=0;
        for(num=1, i=1; i<=n; i++) { cin>>l; for(k=0; k<l; k++)
            { cin>>j; value[num]=j; next[num]=nbs[i]; nbs[i]=num++; } }
        dfs(1); solve(); out();
    }
    return 0;
}
```

6.32 Minimum Cyclic Presentation

```
int MinimumCyclicPresentation(char *s, int n)
{
    int i, j, x, y, u, v;
    for(x=0, y=1; y<n; y++) if( s[y]<=s[x] )
    {
        i=u=x; j=v=y;
        while( s[i]==s[j] )
        {
            ++u; if( ++i == n ) i=0;
            ++v; if( ++j == n ) j=0;
            if( i==x ) break;
        }
        if( s[i]<=s[j] ) y = v; else
        {
            x = y; if( u>y ) y = u;
        }
    }
    return x;
}
```

6.33 Maximum Clique

```
int list [ maxn ] [ maxn ] , g [ maxn ] [ maxn ] , s [ maxn ] , degree [ maxn ] , behide [ maxn ] ;
int found , n , curmax , curobj ;

void sortdegree ()
{
    for ( int j , k , l , i = 1; i <= n; i ++ ) {
        for ( k = i , j = i + 1; j <= n; j ++ ) if ( degree [ j ] < degree [ k ] ) k = j ;
        if ( k != i ) {
            std :: swap ( degree [ i ] , degree [ k ] );
            for ( l = 1; l <= n; l ++ ) std :: swap ( g [ i ] [ l ] , g [ k ] [ l ] );
            for ( l = 1; l <= n; l ++ ) std :: swap ( g [ l ] [ i ] , g [ l ] [ k ] );
        }
    }
}

void dfs ( int d )
{
    if ( d - 1 > curmax ) { found = 1; return ; }
    int i , j ;
    for ( i = 1; i < list [ d - 1 ] [ 0 ] - curmax + d; i ++ )
        if ( ! found && d + behide [ list [ d - 1 ] [ i ] + 1 ] > curmax &&
            ( list [ d - 1 ] [ 0 ] == i || d + behide [ list [ d - 1 ] [ i + 1 ] ] > curmax ) ) {
            for ( j = i + 1, list [ d ] [ 0 ] = 0; j <= list [ d - 1 ] [ 0 ]; j ++ )
                if ( g [ list [ d - 1 ] [ j ] ] [ list [ d - 1 ] [ i ] ] )
                    list [ d ] [ ++ list [ d ] [ 0 ] ] = list [ d - 1 ] [ j ];
            if ( list [ d ] [ 0 ] == 0 || d + behide [ list [ d ] [ 1 ] ] > curmax ) dfs ( d + 1 );
        }
    }

void solve ()
{
    sortdegree (); behide [ n + 1 ] = 0; behide [ n ] = 1;
    for ( int j , i = n - 1; i > 0; i -- ) {
        curmax = behide [ i + 1 ]; found = list [ 1 ] [ 0 ] = 0;
        for ( j = i + 1; j <= n; j ++ ) if ( g [ j ] [ i ] ) list [ 1 ] [ ++ list [ 1 ] [ 0 ] ] = j ;
        dfs ( 2 ); behide [ i ] = curmax + found ;
    } cout << behide [ 1 ] << endl ;
}

int main ()
{
    int i , j ;
    while ( cin >> n , n ) {
        for ( i = 1; i <= n; i ++ ) for ( j = 1, degree [ i ] = 0; j <= n; j ++ ) {
            cin >> g [ i ] [ j ];
            degree [ i ] += ( g [ i ] [ j ] != 0 );
        } solve ();
    }
    return 0;
}
```

6.34 Maximal Non-Forbidden Submatrix

```

#define forbidden 1

int wx,wy,g [maxn] [maxn] ,h [maxn] ,r [maxn] ,l [maxn] ;

int solve()
{
    int i,j,k,ans, left ,right ;
    ans=0; memset(h,0 ,sizeof(h));
    for (i=0;i<wx; i++) {
        for (j=0;j<wy; j++) if(g [i] [j]!=forbidden) h [j]++; else h [j]=0;
        for (j=0;j<wy; j++) if(h [j] ) {
            if(j==0 || h [j-1]==0) left=j;
            if(i==0 || g [i-1][j]==forbidden) l [j]=left ;
            if(left>l [j] ) l [j]=left ;
        }
        for (j=wy-1;j>=0;j--) if(h [j] ) {
            if(j==wy-1 || h [j+1]==0) right=j;
            if(i==0 || g [i-1][j]==forbidden) r [j]=right ;
            if(right<r [j] ) r [j]=right ;
        }
        for (j=0;j<wy; j++)
            if((r [j]-l [j]+1)*h [j] > ans) ans = (r [j]-l [j]+1)*h [j];
    }
    return ans;
}

```

6.35 Maximum Two Chain Problem

```

typedef struct { int x, y; } point;

int cmp(const void* e1, const void* e2) {
    const point* p1 = (const point*)e1;
    const point* p2 = (const point*)e2;
    if (p1->x != p2->x) return p1->x - p2->x;
    return p1->y - p2->y;
}

int n;
point p[MAX];

void initialize() {
    int i;
    for (scanf("%d", &n), i = 1; i <= n; i++)
        scanf("%d%d", &p [i].x, &p [i].y);

    qsort(&p [1] , n , sizeof(point) , cmp);
    p [0].x = p [0].y = 0;
}

int deg [MAX] = {0} , queue [MAX];
int maxlevel , level [MAX] = {0};
int left [MAX] = {0} , right [MAX] = {0} , mark [MAX] = {0};

```

```

void local_chain() {
    int i, j;
    for (i = 1; i <= n; i++)
        for (j = i + 1; j <= n; j++)
            if (p[i].y <= p[j].y)
                deg[i]++;
}

for (queue[0] = 0, i = 1; i <= n; i++)
    if (deg[i] == 0)
        queue[queue[0]] = i;
for (i = 1, maxlevel = -1; i <= queue[0]; i++)
    for (j = 1; j < queue[i]; j++)
        if (p[j].y <= p[queue[i]].y)
            if (--deg[j] == 0)
                queue[queue[0]] = j, level[j] = level[queue[i]] + 1;
                if (level[j] > maxlevel) maxlevel = level[j];
    }
for (maxlevel++, i = 1; i <= n; i++)
    level[i] = maxlevel - level[i];

for (mark[0] = n + 1, i = 1; i <= n; i++) {
    for (j = 0; j < i; j++)
        if (mark[j] && level[j] == level[i] - 1 && p[j].y <= p[i].y)
            break;
    if (j < i) {
        if (left[level[i]] == 0) left[level[i]] = i, mark[i] = n + 1;
        mark[right[level[i]]] --;
        mark[right[level[i]]] = i++;
    }
}
}

int index[MAX], value[MAX] = {0}, levvalue[MAX];

int index_cmp(const void* e1, const void* e2) {
    return level[*(const int*)e1] - level[*(const int*)e2];
}

void calc_value() {
    int q, i, j, lev;
    for (i = 1; i <= n; i++)
        index[i] = i;
    qsort(index, n, sizeof(int), index_cmp);

    for (q = 1; q <= n; q++) {
        lev = level[i = index[q]];

        if (left[lev] == i && right[lev] == i)
            value[i] = levvalue[lev - 1] + 1;
        else if (left[lev] == i || right[lev] == i)
            value[i] = levvalue[lev - 1] + 2;
        else
            for (j = 0; j < i; j++) {
                if (mark[j]) value[j] = levvalue[level[j]];
                if (p[j].y <= p[i].y && value[j] + level[i] - level[j] + 1 > value[i])
                    value[i] = value[j] + level[i] - level[j] + 1;
            }
        if (value[i] > levvalue[lev])
            levvalue[lev] = value[i];
    }
}

```

```

void put_answer() {
    int i, max = 0;
    for (i = 1; i <= n; i++)
        if (value[i] > max)
            max = value[i];
    printf("%d\n", max);
}

void main() {
    initialize();
    local_chain();
    calc_value();
    put_answer();
}

```

6.36 N Queens Problem

```

int main() {
    int n, i, odd;
    while(cin>>n){
        if(n<4) cout<<"Impossible"; else
        if((n/2)%3!=1){
            cout<<2;
            for(i=4; i<=n; i+=2) cout<<"_"<<i;
            for(i=1; i<=n; i+=2) cout<<"_"<<i;
        } else {
            if(n&1) { n--; odd = 1; } else odd=0;
            cout<<n/2 ;
            for(i=n/2+1; i!=n/2-1; i=(i+2)%n) cout<<"_"<<i+1;
            for(i=(i+n-2)%n; i!=n/2-1; i=(i+n-2)%n) cout<<"_"<<n-i ;
            cout<<"_"<<n-i; if(odd) cout<<"_"<<n+1;
        }
        cout<<endl;
    }
    return 0;
}

```

6.37 de Bruijn Sequence Generator

```

int go[1<<maxn], start, now, n, k, a[(1<<maxn)+maxn], i, ans, caseno;

int main()
{
    ifstream cin("input.txt");
    for(cin>>caseno; caseno--){ cin>>n>>k;
        memset(go, 0, sizeof(go)); memset(a, 0, sizeof(a));
        now=start=(1<<(n-1))-1; i=0;
        do { if(go[now]) { a[i++]=1; now=(now*2+1)&start; }
            else { go[now]=1; a[i++]=0; now=(now*2)&start; }
        } while( now!=start );
        a[i++]=1;
        for(i=0; i<n; i++) a[i+(1<<n)]=a[i];
        for(ans=i=0; i<n; i++) ans=ans*2+a[k+i];
        cout<<ans<<endl;
    }
    return 0;
}

```

6.38 ZOJ 1482 Partition

```
#define maxn 3010

int n, pnt[maxn], rank[maxn];

int find(int x)
{
    if(x!=pnt[x]) pnt[x]=find(pnt[x]);
    return pnt[x];
}

int main()
{
    int i, j, ans(0), x;
    cin >> n;
    memset(pnt, 0, sizeof(pnt));
    for(i=1; i<=n; i++) for(j=1; j<=n; j++){
        cin >> x;
        if(!x){
            if(pnt[j]) pnt[find(j)]=j;
            if(pnt[j-1]) pnt[j-1]=j;
            pnt[j]=j;
        } else { if(pnt[j]==j) ans++; pnt[j] = 0; }
    }
    for(i=1; i<=n; i++) if(pnt[i]==i) ans++;
    cout << ans << endl;
    return 0;
}
```