

# Turbulence

## HW1

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### 1 EXERCISE 1

Let

$$\Phi = \iiint_{\Omega} \phi dv \quad (1.1)$$

where  $\phi$  is the passive scalar, and  $\Omega$  is the control volume.

Then the change of  $\Phi$  is due to the gradient diffusion through boundary of  $\Omega$ , which can be written as

$$\frac{D}{Dt} \Phi = \iint_{\partial\Omega} \Gamma \nabla \phi d\vec{S} \quad (1.2)$$

where  $\Gamma$  indicates the diffusivity, with unit of  $m^2/s$ .

The material derivative of  $\Phi$  in Euler field is given as follows using RTT

$$\frac{D}{Dt} \Phi = \iiint_{\Omega} \frac{\partial \phi}{\partial t} dv + \iint_{\partial\Omega} \phi \vec{U} \cdot d\vec{S} \quad (1.3)$$

When field of  $\phi$  is assumed to be smooth enough, the divergence theorem can be applied, which yields

$$\iiint_{\Omega} \frac{\partial \phi}{\partial t} dv + \iiint_{\Omega} \nabla \cdot (\phi \vec{U}) dv = \iiint_{\Omega} \nabla \cdot (\Gamma \nabla \phi) dv \quad (1.4)$$

namely

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \vec{U}) = \nabla \cdot (\Gamma \nabla \phi) \quad (1.5)$$

The equation can be further simplified when the flow is assumed to be both incompressible and constant-property.

$$\frac{\partial \phi}{\partial t} + \vec{U} \cdot \nabla \phi = \Gamma \nabla^2 \phi \quad (1.6)$$

which can be written in material derivative format as

$$\frac{D}{Dt} \phi = \Gamma \nabla^2 \phi \quad (1.7)$$

## 2 EXERCISE2

Let  $X$  and  $Y$  be 2 sequences of random numbers, the correlation coefficient of these 2 random variables is calculated as

$$\rho_{XY} = \frac{cov(X, Y)}{\sqrt{E[X^2]E[Y^2]}} \quad (2.1)$$

the covariance of  $X$  and  $Y$  is calculated as

$$cov(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y] \quad (2.2)$$

With  $E[XY], E[X], E[Y], E[X^2], E[Y^2]$  provided, the correlation coefficient can be therefore calculated. And numerical experiment has been done with 500 times trial.

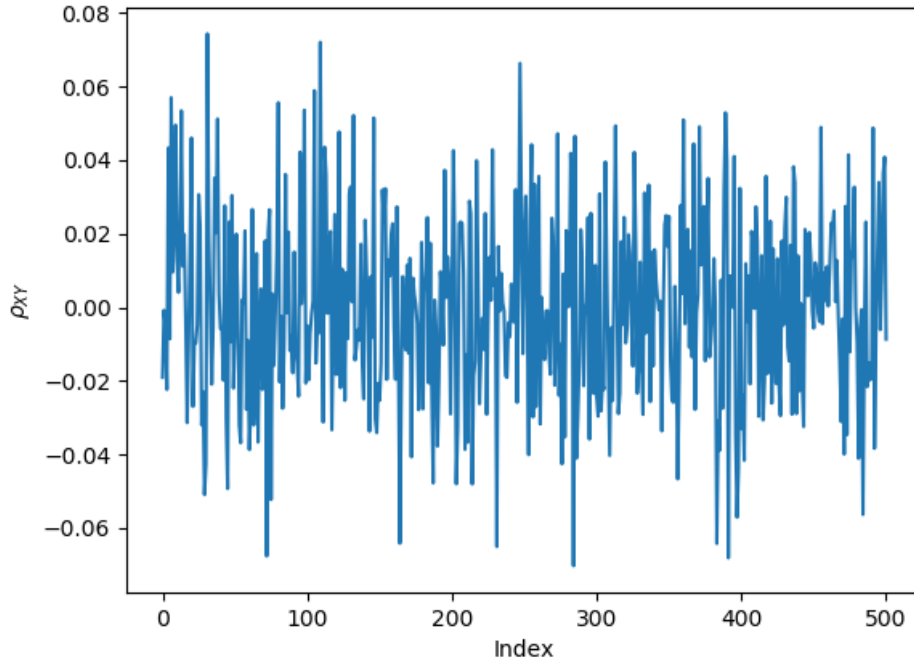


Figure 2.1: Covariance of random numbers

3 EXERCISE3

4 EXERCISE4