SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Turbulence
HW1

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1 EXERCISE1

Let

$$\Phi = \oiiint_{\Omega} \phi dv \tag{1.1}$$

where ϕ is the passive scalar, and Ω is the control volume.
Then the change of Φ is due to the gradiant diffusion through boundary of Ω, which can be written as

$$\frac{D}{Dt}\Phi = \oiint_{\partial\Omega} \Gamma\nabla\phi d\vec{S} \tag{1.2}$$

where Γ indicates the diffusivity, with unit of m^2/s.
The material derivative of Φ in Euler field is given as follows using RTT

$$\frac{D}{Dt}\Phi = \oiiint_{\Omega} \frac{\partial\phi}{\partial t} dv + \oiint_{\partial\Omega} \phi\vec{U}\cdot d\vec{S} \tag{1.3}$$

When field of ϕ is assumed to be smooth enough, the divergence theorem can be applied, which yields

$$\oiiint_{\Omega} \frac{\partial\phi}{\partial t} dv + \oiiint_{\Omega} \nabla\cdot(\phi\vec{U}) dv = \oiiint_{\Omega} \nabla\cdot(\Gamma\nabla\phi) dv \tag{1.4}$$

namely

$$\frac{\partial\phi}{\partial t} + \nabla\cdot(\phi\vec{U}) = \nabla\cdot(\Gamma\nabla\phi) \tag{1.5}$$

The equation can be further simplified when the flow is assumed to be both incompressible and constant-property.

$$\frac{\partial \phi}{\partial t} + \vec{U} \cdot \nabla \phi = \Gamma \nabla^2 \phi \tag{1.6}$$

which can be written in material derivative format as

$$\frac{D\phi}{Dt} = \Gamma \nabla^2 \phi \tag{1.7}$$

2 EXERCISE2

3 EXERCISE3

4 EXERCISE4