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Turbulence
HW1

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1 EXERCISE1

Let

$$\Phi = \oiiint_{\Omega} \phi dv \tag{1.1}$$

where ϕ is the passive scalar, and Ω is the control volume.
Then the change of Φ is due to the gradiant diffusion through boundary of Ω, which can be written as

$$\frac{D}{Dt}\Phi = \oiint_{\partial\Omega} \Gamma\nabla\phi d\vec{S} \tag{1.2}$$

where Γ indicates the diffusivity, with unit of m^2/s.
The material derivative of Φ in Euler field is given as follows using RTT

$$\frac{D}{Dt}\Phi = \oiiint_{\Omega} \frac{\partial\phi}{\partial t} dv + \oiint_{\partial\Omega} \phi\vec{U}\cdot d\vec{S} \tag{1.3}$$

When field of ϕ is assumed to be smooth enough, the divergence theorem can be applied, which yields

$$\oiiint_{\Omega} \frac{\partial\phi}{\partial t} dv + \oiiint_{\Omega} \nabla\cdot(\phi\vec{U}) dv = \oiiint_{\Omega} \nabla\cdot(\Gamma\nabla\phi) dv \tag{1.4}$$

namely

$$\frac{\partial\phi}{\partial t} + \nabla\cdot(\phi\vec{U}) = \nabla\cdot(\Gamma\nabla\phi) \tag{1.5}$$

The equation can be further simplified when the flow is assumed to be both incompressible and constant-property.

$$\frac{\partial \phi}{\partial t} + \vec{U} \cdot \nabla \phi = \Gamma \nabla^2 \phi \tag{1.6}$$

which can be written in material derivative format as

$$\frac{D}{Dt}\phi = \Gamma \nabla^2 \phi \tag{1.7}$$

2 EXERCISE2

Let X and Y be 2 sequences of random numbers, the correlation coefficient of these 2 random variables is calculated as

$$\rho_{XY} = \frac{cov(X,Y)}{\sqrt{E[X^2]E[Y^2]}} \tag{2.1}$$

the covariance of X and Y is calculated as

$$cov(X,Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y] \tag{2.2}$$

With $E[XY], E[X], E[Y], E[X^2], E[Y^2]$ provided, the correlation coefficient can be therefore calculated. And numerical experiment has been done with 500 times trial.

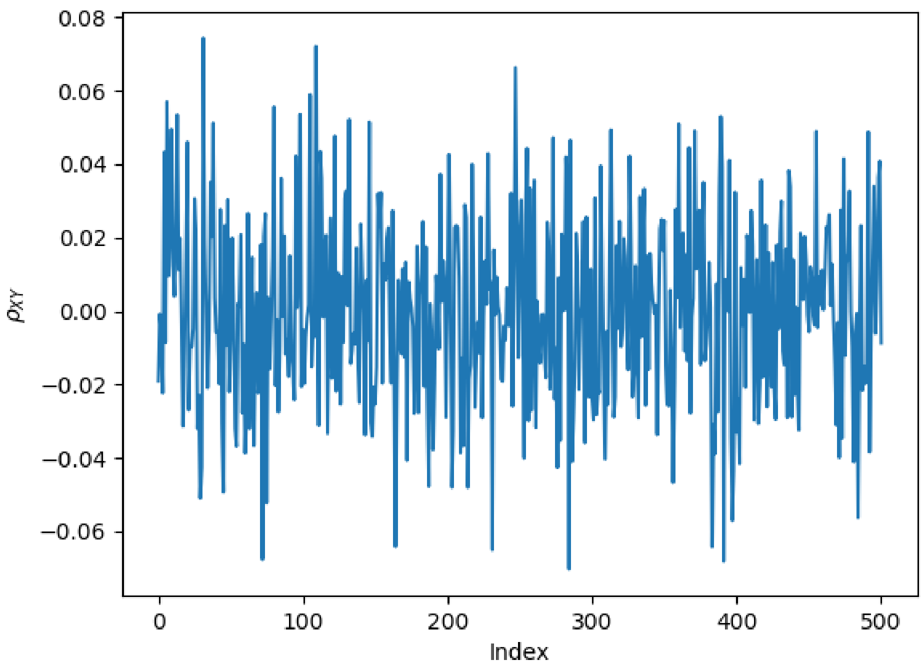

Figure 2.1: Covariance of random numbers

3 Exercise3

Assume the angular speed of the motion is $\omega = 4\pi rad/s$, with radius $r = \frac{d}{2} = \frac{l}{2} = 5cm$, the mean velocity can be calculated as

$$u = \frac{1}{2}\omega r = 0.1\pi m/s \tag{3.1}$$

Thus, the energy input rate is estimated as

$$\epsilon \sim \frac{u^3}{l} \approxeq 9.8696 \times 10^{-1}(m^2 \cdot s^{-3}) \tag{3.2}$$

With the kinematic viscosity of water taken as $\nu = 1.006 \times 10^{-6}(m^2 \cdot s^{-1})$, the Kolmogorov scale is estimated as

$$\eta \sim (\frac{\nu^3}{\epsilon})^{\frac{1}{4}} \approxeq 3.187 \times 10^{-2} mm \tag{3.3}$$

4 Exercise4

Load the give txt datafile, denote the input 1D data as u, then perform FFT on u yields u^*. The energy spectrum is calculated as $E(k) = \frac{1}{2}u^*(k)^2$ for each u^*. The log plot of the energy spectrum is provided.

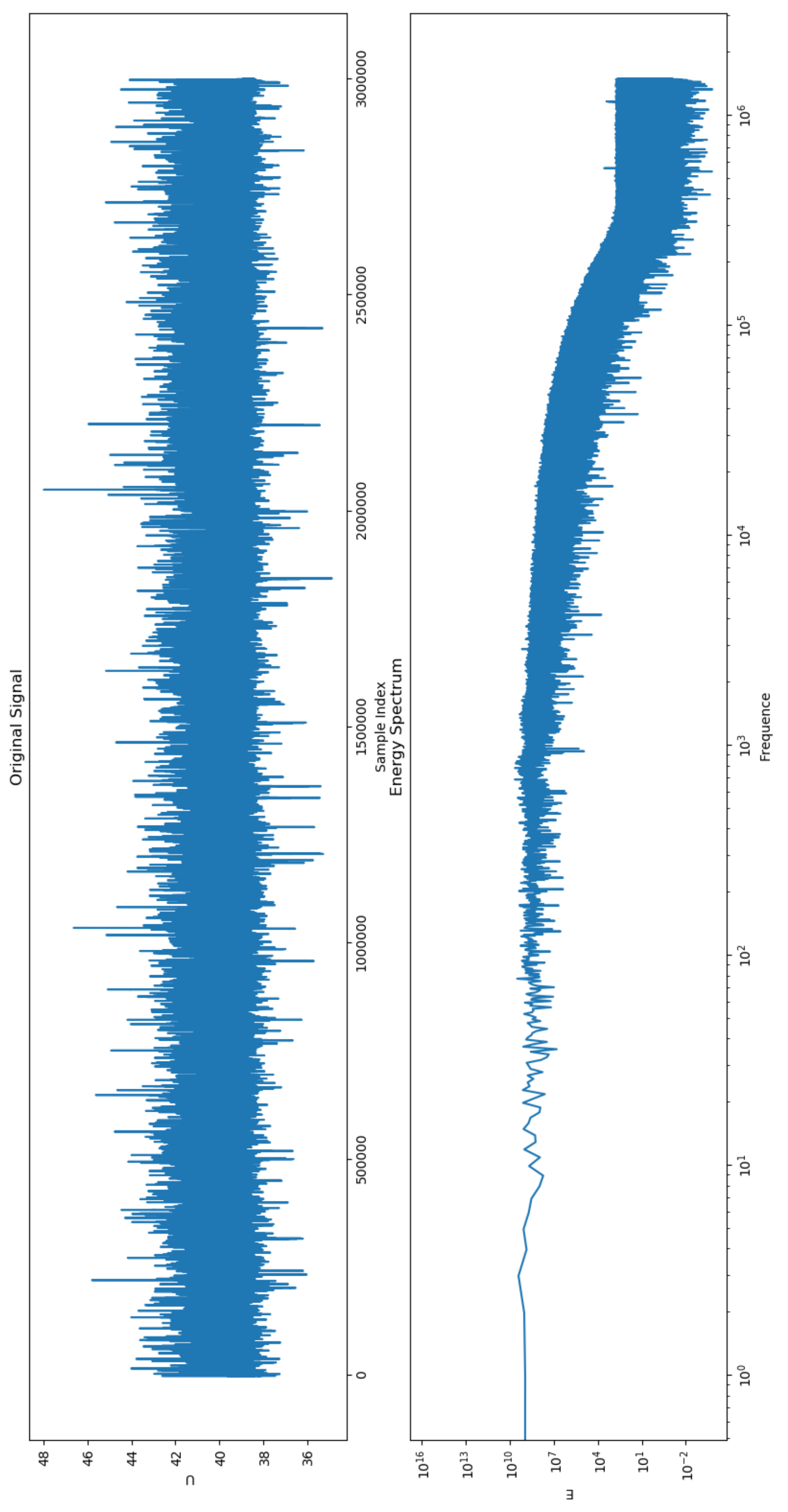

Figure 4.1: Energy Spectrum