Vv556 Methods of Applied Mathematics I

Linear Operators

Assignment 10

Date Due: 12:10 PM, Thursday, the 6th of December 2018

This assignment has a total of **(12 Marks)**.

Exercise 10.1
Consider the Sturm-Liouville problem

$$-u'' = \lambda u \quad \text{on } (0,1), \qquad u(0) = 0, \qquad u(1) + u'(1) = 0.$$

i) Show directly that $L = -\frac{d^2}{dx^2}$ is positive definite on

$$U = \{u \in C^2([a,b])\colon u(0) = u(1) + u'(1) = 0\} \subset L^2([0,1]).$$

(2 Marks)

ii) Deduce that the eigenvalues of L are strictly positive and show that they satisfy

$$\sqrt{\lambda} = -\tan(\sqrt{\lambda}).$$

(2 Marks)

iii) Use a computer to obtain a numerical value for the two lowest eigenvalues λ_1 and λ_2.
(2 Marks)

iv) Use the Rayleigh-Ritz method to estimate the lowest eigenvalue by taking

$$V_1 = \mathcal{P}_2 \cap U,$$

where $\mathcal{P}_n$ denotes the space of polynomials of degree not more than n.
(2 Marks)

v) Improve your estimate by taking

$$V_2 = \mathcal{P}_3 \cap U$$

You are encouraged to use a computer to assist in calculating integrals and performing the minimization.
(2 Marks)

vi) By setting

$$V_3 = V_1^{\perp} \cap U \cap \mathcal{P}_3,$$

find an estimate for the second eigenvalue.
(2 Marks)