# Vv556 Methods of Applied Mathematics I

## Linear Operators

## Assignment 5

**Date Due: 12:10 PM, Thursday, the 25th of October 2018**

This assignment has a total of **(10 Marks)**.

**Exercise 5.1**

Let $(U, \|\cdot\|_U)$ and $(V, \|\cdot\|_V)$ be finite-dimensional vector spaces and $L\colon U \to V$ a linear map. Show that $L$ is bounded.
**(4 Marks)**

**Exercise 5.2**

Define the multiplication operator

$$T\colon C([0,1]) \to C([0,1]), \qquad (Tu)(x) = x \cdot u(x), \quad x \in [0,1].$$

Show that $T$ is linear and bounded. Find $\|T\|$ and prove your result.
**(4 Marks)**

**Exercise 5.3**

We denote the vector space of continuously differentiable functions on the interval $[a,b]$ by $C^1([1,b])$. Show that

$$\|u\| = \sup_{x\in[a,b]} |u(x)| + \sup_{x\in[a,b]} |u'(x)|$$

defines a norm on $C^1([1,b])$. Is the map

$$T\colon C^1([0,1]) \to C([0,1]), \qquad u \mapsto \frac{du}{dx}$$

continuous if $C([0,1])$ is endowed with the $\|\cdot\|_\infty$ norm? Prove your statement or provide a counterexample.
**(2 Marks)**