SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Introduction to Numerical Analysis HW7

Yu Cang
018370210001

July 11, 2018

1 QUESTION 1

(a) For example

$$y = tan(x), \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2}) \tag{1.1}$$

It's differentiable over $(-\frac{\pi}{2}, \frac{\pi}{2})$, and the derivative is

$$y' = \frac{1}{cos^2(x)} \tag{1.2}$$

It's obvious that $y' \to \infty$ when $x \to \frac{\pi}{2}$.

(b) *Proof.* Denote $g(x)$ over $[x_1, x_2]$ as

$$g(x) = f(x) - f(x_1) - \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1) \tag{1.3}$$

where $a < x_1 < x_2 < b$. Then, $g(x_1) = 0$ and $g(x_2) = 0$.

Thus, from Rolle's theorem, there exists $\xi \in (x_1, x_2)$ s.t.

$$g'(\xi) = 0 \tag{1.4}$$

namely

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = f'(\xi) \tag{1.5}$$

Since f' is bounded, then $|f'(\xi)| \leq M$. Thus

$$|f(x_2) - f(x_1)| \leq M|x_2 - x_1| \tag{1.6}$$

which means that $f(x)$ is Lipschitz continous. □

(c) For example

$$y = \frac{1}{x}, \quad x \in (0,1) \tag{1.7}$$

y is obviously differentiable and its derivative is

$$y' = -\frac{1}{x^2}, \quad x \in (0,1) \tag{1.8}$$

Suppose there exist a constant $c > 0$ s.t.

$$|y_2 - y_1| \leq c|x_2 - x_1| \tag{1.9}$$

is valid for all $0 < x_1 < x_2 < 1$. Let $y(b) - y(a) = c(b-a)$, where $a < b$then

$$c = \frac{1}{ab} \tag{1.10}$$

Take the mid-point of a, b, then

$$\frac{y(a) - y(\frac{b+a}{2})}{\frac{b+a}{2} - a} = \frac{1}{a(\frac{b+a}{2})} > \frac{1}{ab} = c \tag{1.11}$$

Thus, the assumption fails, which means that y is not Lipschitz continous.

(d) For example

$$y = |x|, \quad x \in (-1,1) \tag{1.12}$$

It's Lipschitz continous as for any $-1 < x_1 < x_2 < 1$

$$\frac{|y(x_2) - y(x_1)|}{|x_2 - x_1|} \leq 1 \tag{1.13}$$

but it is not differentiable at $x = 0$.

2 Question 2

3 Question 3