SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Introduction to Numerical Analysis
HW3

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1 Cantor's Set

1. *Proof.* As C_i is closed and C is the intersection of these closed sets, C is closed.
   It's clear that C is bounded. Thus, by Heine-Borel theorem, C is compact. □

2. *Proof.* Suppose $x \in C_m$, $y \in C_n$ and $x < y$. There are 2^m subsets in C_m and the length of each subset is $\frac{1}{3^m}$. Also, there are 2^n subsets in C_n and the length of each subset is $\frac{1}{3^n}$.
   As $C \subset C_m \cap C_n$, suppose $m \leq n$, there must exist a subset in C_n s.t. $x \in C_n$.
   If x and y lie in the same subset of C_n, with further division of the subset, there will be a gap between x and y, thus, there exists an element z lies in the gap and satisfies $x < z < y$.
   If x and y lie in different subsets of C_n, denoted as $C_{n,i}$ and $C_{n,j}$,it's obvious that such a z exists in the gap between $C_{n,i}$ and $C_{n,j}$ and satisfies $x < z < y$. □

3. a) 0

   b) *Proof.* As there are 2^n subsets in C_n and the length of each subset is $\frac{1}{3^n}$, thus the Lebesgue measure of C_n is the sum of the Lebesgue measure of each closed subset. Thus $\lambda(C_n) = (\frac{2}{3})^n$.
   As $C = \bigcap\limits_{n=1}^{\infty} C_n$, then $\lambda(C) \leq \lambda(C_n) = (\frac{2}{3})^n$, thus $\lambda(C) = 0$. □

4. a) *Proof.* As the end points of each subset in C_n is not removed in any subdivision, thus C is not empty. □

b)

$$x = \sum_{i=1}^{\infty} \frac{a_i}{3^i}, \text{ with } a_i \in \{0, 2\} \tag{1.1}$$

c) Consider the $i-th$ digit in elemnet s_i, it will be possible to construct such an element that the $i-th$ digit is different from that in s_i, thus s is not included in the original list.

d) *Proof.* Suppose C is countable, express each $x \in C$ in ternary form, then each digit in x is either 0 or 2. Thus, the choice of each digit appears to be binary. Consider the $i-th$ digit in elemnet x_i, it will be possible to construct such an element t, whose $i-th$ digit is different from that in x_i(complementary). Thus t is not included in the original list, which implies the assumption fails. □

5. Althouth C is uncountable, but the measure of its complement is 1, which is illustrated below, thus the measure of C is 0.

$$\begin{aligned}\lambda(C^c) &= \frac{1}{3} + 2 * \frac{1}{9} + ... \\ &= \frac{1}{3} \sum_{i=0}^{\infty} (\frac{2}{3})^n \\ &= \frac{1}{3} \lim_{n \to \infty} \frac{1-(2/3)^n}{1-(2/3)} \\ &= 1\end{aligned} \tag{1.2}$$

2 Cantor's Function

1.
2.
3.
4.

3 Taylor's theorem

1.
2.

4 Convergence of rationals to irrationals

1.
2.
3.