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Methods of Applied Mathematics I
HW5

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1 EXERCISE5.1

Proof. Suppose $\dim(U) = N$ and $\dim(V) = M$. Since all the norm are equivalent in finite-dimension spaces, the linear operator L can be represented in matrix form as $\{l_{ji}\}$. Then

$$\begin{aligned} ||Lu|| &= \sqrt{\sum_{j=1}^{M} \Big| \sum_{i=1}^{N} l_{ji} u_i \Big|^2} \\ &\leq \sqrt{\sum_{j=1}^{M} \Big[\sum_{i=1}^{N} l_{ji}^2 \sum_{i=1}^{N} u_i^2 \Big]} \quad \text{(Cauchy-Schwartz)} \\ &= \sqrt{\sum_{j=1}^{M} ||u||^2 \sum_{i=1}^{N} l_{ji}^2} = ||u|| \sqrt{\sum_{j=1}^{M} \sum_{i=1}^{N} l_{ji}^2} \end{aligned} \tag{1.1}$$

Hence, L is bounded. □

2 EXERCISE5.2

Proof. Denote $w = \alpha u + \beta v$, then

$$(Tw)(x) = xw(x) = x(\alpha u(x) + \beta v(x)) = \alpha x u(x) + \beta x v(x) = \alpha (Tu)(x) + \beta (Tv)(x) \tag{2.1}$$

Hence, T is linear.
Further

$$||T|| = \sup_{u\in\mathbb{C}[0,1]} \frac{||Tu||}{||u||} = \sup_{u\in\mathbb{C}[0,1]} \frac{\sup_{x\in[0,1]}|xu(x)|}{\sup_{x\in[0,1]}|u(x)|} = \sup_{u\in\mathbb{C}[0,1]} \frac{\sup_{x\in[0,1]}|u(x)|}{\sup_{x\in[0,1]}|u(x)|} = 1 \tag{2.2}$$

□

3 Exercise5.3