Q1. From the momentum equation of the fluctuating velocity, derive the following two-point correlation equation:

$$
\begin{aligned}
&D^{(i)}R_{ij} + D^{(j)}R_{ij} = \frac{\partial R_{ij}}{\partial t} + U_k(\vec{x},t)\frac{\partial R_{ij}}{\partial x_k} + [U_k(\vec{x}^{(1)},t^{(1)}) - U_k(\vec{x},t)]\frac{\partial R_{ij}}{\partial r_k} \\
&+ R_{kj}\frac{\partial U_i(\vec{x},t)}{\partial x_k} + R_{ik}\frac{\partial U_j(\vec{x}^{(1)},t^{(1)})}{\partial x_k^{(1)}} + \frac{\partial R_{(ik)j}}{\partial x_k} - \frac{\partial}{\partial r_k}(R_{(ik)j} - R_{i(jk)}) \\
&+ \frac{1}{\rho}\frac{\partial \overline{p'u'_j}}{\partial x_i} - \frac{1}{\rho}\frac{\partial \overline{p'u'_j}}{\partial r_i} + \frac{1}{\rho}\frac{\partial \overline{p'u'_i}}{\partial r_j} - \nu[\frac{\partial^2 R_{ij}}{\partial x_k \partial x_k} - 2\frac{\partial^2 R_{ij}}{\partial x_k \partial r_k} + 2\frac{\partial^2 R_{ij}}{\partial r_k \partial r_k}] \\
&- \overline{f'_i u'_j} - \overline{f'_j u'_i},
\end{aligned}
\tag{1}
$$

where $D^{(i)}R_{ij} \equiv \overline{u'_j(\vec{x}^{(1)},t^{(1)})N_i\{\vec{x},t\}}$ and $D^{(j)}R_{ij} \equiv \overline{u'_i(\vec{x},t)N_j\{\vec{x}^{(1)},t^{(1)}\}}$, in which $N_i\{\vec{x},t\}$ is the momentum equation of $u'_i(\vec{x},t)$.

Q2. For the pure shear turbulence case, derive the kinetic energy equation of each velocity component, i.e. $\overline{u_1^2}$, $\overline{u_2^2}$ and $\overline{u_3^2}$. Estimate the magnitude of each term.