Vv556 Methods of Applied Mathematics I
Linear Operators

Date Due: 12:10 PM, Thursday, the 20th of September 2018

This assignment has a total of **(15 Marks)**.

Exercise 1.1
Let

$$U = \{x \in \mathbb{R}^4 : x_1 + x_2 + x_3 = 0,\ x_1 + 3x_2 = x_4\}, \qquad V = \{x \in \mathbb{R}^4 : x_1 = x_4\},$$
$$U + V := \{x \in \mathbb{R}^4 : x = u + v,\ u \in U,\ v \in V\}.$$

Find $\dim U$, $\dim V$ and $\dim U + V$.
(3 Marks)

Exercise 1.2
Calculate the pointwise limit, if it exists, for each of the following function sequences $\{f_n\}_{n\in\mathbb{N}}$ on the given domain and decide whether the convergence is uniform.

i) $f_n(x) = \sqrt[n]{x}, \quad \operatorname{dom} f_n = [0,1],$
(2 Marks)

ii) $f_n(x) = \dfrac{nx}{1+n+x}, \quad \operatorname{dom} f_n = [0,\infty),$
(2 Marks)

iii) $f_n(x) = \sqrt{1/n + x} - \sqrt{x}, \quad \operatorname{dom} f_n = (0,\infty),$
(2 Marks)

Exercise 1.3
For $p \in \mathbb{N} \setminus \{0\}$ we define the ℓ^p-spaces of real sequences by

$$\ell^p := \left\{ (a_n) : \mathbb{N} \to \mathbb{R} : \sum_{n=0}^{\infty} |a_n|^p < \infty \right\}$$

i) Prove that $\ell^p \subset \ell^q$ for $p < q$.
(2 Marks)

ii) Find a sequence (a_n) such that $(a_n) \in \ell^p$ for all $p > 1$ but $(a_n) \notin \ell^1$.
(2 Marks)

iii) Find[1] a sequence (a_n) of real numbers such that $\lim_{n\to\infty} a_n = 0$ but $(a_n) \notin \ell^p$ for all $p \in [1,\infty)$.
(2 Marks)

[1] *Kreyszig*, Section 1.2, question 4