SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Methods of Applied Mathematics I
HW3

Yu Cang
018370210001

October 6, 2018

1 EXERCISE3.1

Proof. Let $\phi(x) = (x^2 - 1)^n$, then

$$\phi(\pm 1) = \phi^{(1)}(\pm 1) = \cdots = \phi^{(n-1)}(\pm 1) = 0 \tag{1.1}$$

The L^2 norm of $P_n(x)$ is

$$\begin{aligned}
||P_n||_2 &= \sqrt{\int_{-1}^{1} P_n^2(x)dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 \int_{-1}^{1} [\phi^{(n)}(x)]^2 dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (-1)^n \int_{-1}^{1} \phi^{(2n)}(x)\phi(x) dx} \quad \text{(Integrate by parts)} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (-1)^n (2n)! \int_{-1}^{1} (x^2 - 1)^n dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (-1)^n (2n)! \cdot 2 \int_{0}^{\frac{\pi}{2}} (sin^2(x) - 1)^n cos(x) dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (2n)! \cdot 2 \int_{0}^{\frac{\pi}{2}} cos^{2n+1}(x) dx}
\end{aligned} \tag{1.2}$$

Since

$$\int_0^{\frac{\pi}{2}} cos^{2n+1}(x)dx = \frac{2\times 4\times\cdots\times(2n)}{1\times 3\times\cdots\times(2n+1)} \tag{1.3}$$

substitute it into the norm equation of P_n yields

$$||P_n||_2 = \sqrt{(\frac{1}{2^n n!})^2(2n)!\cdot 2\cdot\frac{2\times 4\times\cdots\times(2n)}{1\times 3\times\cdots\times(2n+1)}} = \sqrt{\frac{2}{2n+1}} \tag{1.4}$$

□

2 EXERCISE3.2

1. Consider the projection of $f(x)$ onto $P_0(x)$, $P_1(x)$ and $P_2(x)$ correspondingly

$$(f,P_0) = \int_{-1}^{1} e^x dx = e - \frac{1}{e} \tag{2.1}$$

$$(f,P_1) = \int_{-1}^{1} xe^x dx = \frac{2}{e} \tag{2.2}$$

$$(f,P_2) = \int_{-1}^{1} \frac{3x^2-1}{2} e^x dx = e - \frac{7}{e} \tag{2.3}$$

As $(P_i,P_j) = 0$ when $i \neq j$, $p(x)$ can be written as linear combination of P_i

$$p(x) = a_0P_0(x) + a_1P_1(x) + a_2P_2(x) \tag{2.4}$$

with coefficient a_i given as

$$a_i = \frac{(f,P_i)}{(P_i,P_i)} = \frac{2i+1}{2}(f,P_i) \tag{2.5}$$

Finally

$$p(x) = 1.1752P_0(x) + 1.1036P_1(x) + 0.3578P_2(x) \tag{2.6}$$

2. The plot(Fig 2.1) is provided for comparsion.

   The distinction is quite obvious and Legendre polynomials does better than taylor series expansion at zero point.

3 EXERCISE3.3

1.

2.

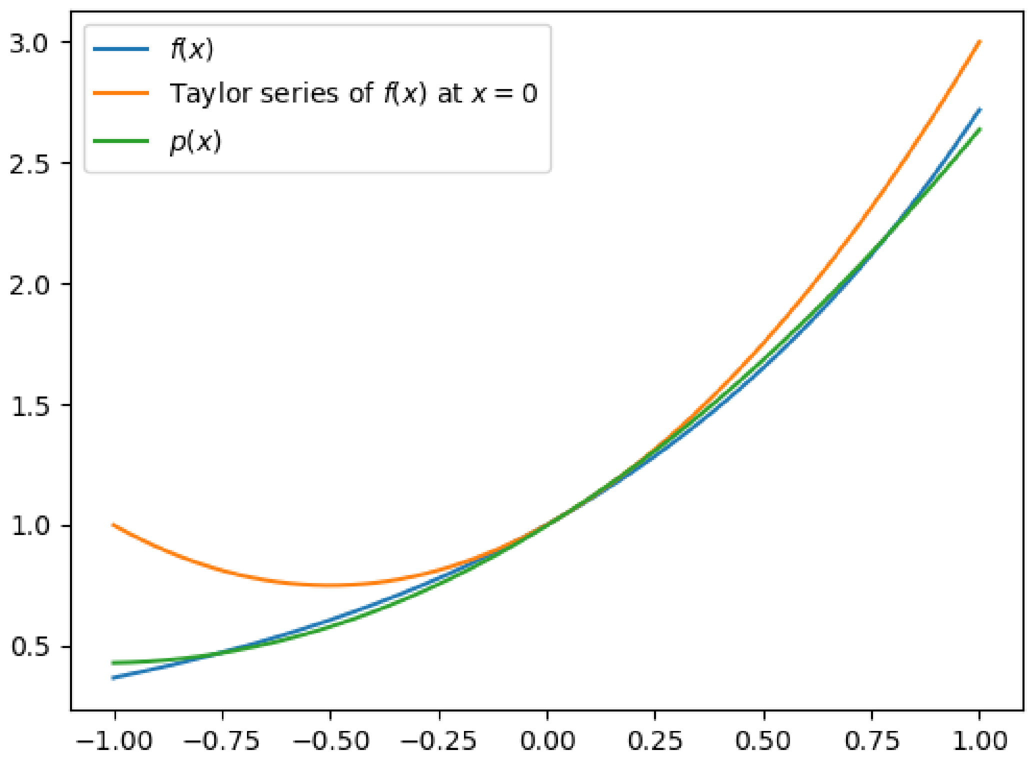

Figure 2.1: Comparsion of different approximation for e^x