SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Methods of Applied Mathematics I
HW2

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1 EXERCIS2.1

1. *Proof.*

$$\begin{aligned}
RHS &= \frac{1}{4}(||x+y||^2 - ||x-y||^2) \\
&= \frac{1}{4}(< x+y, x+y > - < x-y, x-y >) \\
&= \frac{1}{4}(< x+y, x > + < x+y, y > - < x-y, x > - < x-y, -y >) \\
&= \frac{1}{4}(\overline{< x, x+y >} + \overline{< y, x+y >} - \overline{< x, x-y >} + \overline{< y, x-y >}) \\
&= \frac{1}{4}(\overline{< x, x >} + \overline{< x, y >} + \overline{< y, x >} + \overline{< y, y >} - \overline{< x, x >} + \overline{< x, y >} + \overline{< y, x >} - \overline{< y, y >}) \\
&= \frac{1}{2}(\overline{< x, y >} + \overline{< y, x >}) \\
&= \frac{1}{2}(< x, y > + < y, x >) \\
&= < x, y > = LHS
\end{aligned} \tag{1.1}$$

The last line is valid as the inner-product is defined on real space s.t. $< x, y > = < y, x >$. □

2. *Proof.* As have been proved aboved

$$\begin{aligned}&\frac{1}{4}(||x+y||^2-||x-y||^2)\\=&\frac{1}{2}(<x,y>+<y,x>)\end{aligned}\tag{1.2}$$

Also, replace y with iy yields

$$\begin{aligned}&\frac{i}{4}(||x+iy||^2-||x-iy||^2)\\=&\frac{i}{2}(<x,iy>+<iy,x>)\\=&\frac{i}{2}(i<x,y>+\bar{i}<y,x>)\\=&\frac{1}{2}(-<x,y>+<y,x>)\end{aligned}\tag{1.3}$$

Thus

$$\begin{aligned}RHS&=\frac{1}{4}(||x+y||^2-||x-y||^2)-\frac{i}{4}(||x+iy||^2-||x-iy||^2)\\&=\frac{1}{2}(<x,y>+<y,x>)-\frac{1}{2}(-<x,y>+<y,x>)\\&=<x,y>=LHS\end{aligned}\tag{1.4}$$

□

3. *Proof.*

$$\begin{aligned}LHS&=||x+y||^2+||x-y||^2\\&=<x+y,x+y>+<x-y,x-y>\\&=<x+y,x>+<x+y,y>+<x-y,x>-<x-y,y>\\&=\overline{<x,x+y>}+\overline{<y,x+y>}+\overline{<x,x-y>}-\overline{<y,x-y>}\\&=\overline{<x,x>}+\overline{<x,y>}+\overline{<y,x>}+\overline{<y,y>}+\overline{<x,x>}-\overline{<x,y>}-\overline{<y,x>}+\overline{<y,y>}\\&=2(<x,x>+<y,y>)\\&=2(||x||^2+||y||^2)=RHS\end{aligned}\tag{1.5}$$

□

4.

2 Exercise2.2

1.