

Methods of Applied Mathematics I

HW1

Yu Cang
018370210001

September 20, 2018

1 EXERCISE 1.1

1.
$$\dim U = 2 \tag{1.1}$$

As there are 4 components within x , and 2 constraints.

2.
$$\dim U = 3 \tag{1.2}$$

As there are 4 components within x , and 1 constraint.

3.
$$\begin{aligned} \dim U + V &= \dim U + \dim V - \dim U \cap V \\ &= 2 + 3 - 2 \\ &= 3 \end{aligned} \tag{1.3}$$

The dimension of $U \cap V$ can be observed from

$$\begin{aligned} U \cap V &= \{x \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 = 0, x_1 + 3x_2 = x_4, x_1 = x_4\} \\ &= \{x \in \mathbb{R}^4 \mid x_2 = 0, x_1 + x_3 = 0\} \end{aligned} \tag{1.4}$$

2 EXERCISE1.2

1. The pointwise limit is given as follows

$$f = \begin{cases} 0, & x = 0 \\ 1, & x \in (0, 1] \end{cases} \quad (2.1)$$

It's of uniform convergence as

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sup_{x \in [0, 1]} |f_n(x) - f(x)| \\ &= \begin{cases} 0, & x = 0 \\ \lim_{x \rightarrow 0, n \rightarrow \infty} |x^{\frac{1}{n}} - 1|, & x \in (0, 1] \end{cases} \\ &= 0 \end{aligned} \quad (2.2)$$

2. The pointwise limit is given as

$$f(x) = x \quad (2.3)$$

It's not of uniform convergence.

3. The pointwise limit is given as

$$f(x) = 0 \quad (2.4)$$

It's of uniform convergence as

$$\begin{aligned} & \lim_{n \rightarrow \infty} \sup_{x \in (0, \infty)} |f_n(x) - f(x)| \\ &= \lim_{n \rightarrow \infty} \sup_{x \in (0, \infty)} \left| \sqrt{\frac{1}{n} + x} - \sqrt{x} \right| \\ &= \lim_{n \rightarrow \infty} \sup_{x \in (0, \infty)} \left| \frac{1}{n(\sqrt{\frac{1}{n} + x} + \sqrt{x})} \right| \\ &= \lim_{n \rightarrow \infty, x \rightarrow 0} \frac{1}{n(\sqrt{\frac{1}{n} + x} + \sqrt{x})} \\ &= \lim_{n \rightarrow \infty, x \rightarrow 0} \frac{1}{2n\sqrt{x}} = 0 \end{aligned} \quad (2.5)$$

3 EXERCISE1.3

1. Not figured out...
2. For example

$$a_n = \frac{1}{n} \quad (3.1)$$

3. Consider a sequence constructed as follows

$$\begin{aligned}
 x_1 &= \frac{1}{1} & (1^1 = 1 \text{ item}) \\
 x_2 = \dots = x_5 &= \frac{1}{2} & (2^2 = 4 \text{ items}) \\
 x_6 = \dots = x_{32} &= \frac{1}{3} & (3^3 = 27 \text{ items}) \\
 &\dots
 \end{aligned} \tag{3.2}$$

This sequence certainly converges to 0. But not in L^p for any $p \geq 1$ as

$$\sum_{n=1}^{\infty} = 1 + \frac{2^2}{2^p} + \frac{3^3}{3^p} + \dots + \frac{n^n}{n^p} + \dots \tag{3.3}$$

for any $n > p$, $\frac{n^n}{n^p} > 1$.