SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Turbulence
HW2

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1 EXERCISE1

Since the structure function and two-point correlation function are defined as

$$N_i\{\vec{x},t\} = \frac{\partial u_i'}{\partial t} + \overline{u_k}\frac{\partial u_i'}{\partial x_k} + u_k'\frac{\partial \overline{u_i}}{x_k} + \frac{\partial (u_i' u_k')}{\partial x_k} - \frac{\partial \overline{u_i' u_k'}}{\partial x_k} + \frac{1}{\rho}\frac{\partial p'}{\partial x_i} - \nu \frac{\partial^2 u_i'}{\partial x_k \partial x_k} - f_i' = 0 \tag{1.1}$$

and

$$R_{ij} = \overline{u_i'(\vec{x},t)u_j'(\vec{x}+\vec{r},t+\tau)} \tag{1.2}$$

Thus, for $D^{(i)}\{R_{ij}\} = \overline{u_j'(\vec{x}+\vec{r},t+\tau)N_i\{\vec{x},t\}}$, components in the expansion are calculated as

$$\begin{aligned}
\overline{u_j'(\vec{x}+\vec{r},t+\tau)\frac{\partial u_i'(\vec{x},t)}{\partial t}} &= \overline{\frac{\partial (u_i'(\vec{x},t)u_j'(\vec{x}+\vec{r},t+\tau))}{\partial t}} - \overline{u_i'(\vec{x},t)\frac{\partial u_j'(\vec{x}+\vec{r},t+\tau)}{\partial t}} \\
&= \frac{\partial \overline{u_i'(\vec{x},t)u_j'(\vec{x}+\vec{r},t+\tau)}}{\partial t} - \overline{u_i'(\vec{x},t)\frac{\partial u_j'(\vec{x}+\vec{r},t+\tau)}{\partial (t+\tau)}} \\
&= \frac{\partial R_{ij}}{\partial t} - \overline{u_i'(\vec{x},t)\frac{\partial u_j'(\vec{x}+\vec{r},t+\tau)}{\partial \tau}} \\
&= \frac{\partial R_{ij}}{\partial t} - \frac{\partial \overline{u_i'(\vec{x},t)u_j'(\vec{x}+\vec{r},t+\tau)}}{\partial \tau} \\
&= \frac{\partial R_{ij}}{\partial t} - \frac{\partial R_{ij}}{\partial \tau}
\end{aligned} \tag{1.3}$$

$$
\begin{aligned}
\overline{u'_j(\vec{x}+\vec{r},t+\tau)\overline{u_k}\frac{\partial u'_i}{\partial x_k}} &= \overline{u_k(\vec{x},t)}\,\overline{u'_j(\vec{x}+\vec{r},t+\tau)\frac{\partial u'_i(\vec{x},t)}{\partial x_k}} \\
&= \overline{u_k(\vec{x},t)}\left[\frac{\partial\overline{u'_i(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial x_k} - \overline{u'_i(\vec{x},t)\frac{\partial u'_j(\vec{x}+\vec{r},t+\tau)}{\partial(x_k+r_k)}}\right] \\
&= \overline{u_k(\vec{x},t)}\left[\frac{\partial R_{ij}}{\partial x_k} - \overline{u'_i(\vec{x},t)\frac{\partial u'_j(\vec{x}+\vec{r},t+\tau)}{\partial r_k}}\right] \\
&= \overline{u_k(\vec{x},t)}\left[\frac{\partial R_{ij}}{\partial x_k} - \frac{\partial\overline{u'_i(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial r_k}\right] \\
&= \overline{u_k(\vec{x},t)}\left[\frac{\partial R_{ij}}{\partial x_k} - \frac{\partial R_{ij}}{\partial r_k}\right]
\end{aligned}
\tag{1.4}
$$

$$
\overline{u'_j(\vec{x}+\vec{r},t+\tau)u'_k(\vec{x},t)\frac{\partial\overline{u_i(\vec{x},t)}}{\partial x_k}} = \overline{u'_k(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}\frac{\partial\overline{u_i(\vec{x},t)}}{\partial x_k} = R_{kj}\frac{\partial\overline{u_i(\vec{x},t)}}{\partial x_k}
\tag{1.5}
$$

$$
\begin{aligned}
\overline{u'_j(\vec{x}+\vec{r},t+\tau)\frac{\partial p'(\vec{x},t)}{\partial x_i}} &= \frac{\overline{\partial p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial x_i} - \overline{p'(\vec{x},t)\frac{\partial u'_j(\vec{x}+\vec{r},t+\tau)}{\partial x_i}} \\
&= \frac{\partial\overline{p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial x_i} - \overline{p'(\vec{x},t)\frac{\partial u'_j(\vec{x}+\vec{r},t+\tau)}{\partial(x_i+r_i)}} \\
&= \frac{\partial\overline{p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial x_i} - \overline{p'(\vec{x},t)\frac{\partial u'_j(\vec{x}+\vec{r},t+\tau)}{\partial r_i}} \\
&= \frac{\partial\overline{p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial x_i} - \frac{\overline{\partial p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial r_i} \\
&= \frac{\partial\overline{p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial x_i} - \frac{\partial\overline{p'(\vec{x},t)u'_j(\vec{x}+\vec{r},t+\tau)}}{\partial r_i}
\end{aligned}
\tag{1.6}
$$

$$
\overline{u'_j(\vec{x}+\vec{r},t+\tau)\frac{\partial^2 u'_i(\vec{x},t)}{\partial x_k\partial x_k}} =
\tag{1.7}
$$

2 Exercise 2