Vv556 Methods of Applied Mathematics I

Linear Operators

Assignment 3

Date Due: 12:10 PM, Thursday, the 11th of October 2018

This assignment has a total of **(10 Marks)**.

Exercise 3.1

Let P_n be the nth Legendre polynomial. Show that

$$\|P_n\|_{L^2} = \sqrt{\frac{2}{2n+1}}.$$

(2 Marks)

Exercise 3.2

The first three Legendre polynomials are

$$P_0(x) = 1, \qquad P_1(x) = x, \qquad P_2(x) = \frac{1}{2}(3x^2 - 1).$$

i) Use these polynomials to find an approximation $p(x)$ to the function $f\colon [-1,1] \to \mathbb{R}$, $f(x) = e^x$.
(2 Marks)

ii) Plot in one graph the functions e^x, $p(x)$ found above and the Taylor series at $x_0 = 0$, $e^x \approx 1 + x + x^2/2$. Comment on the quality of the approximation.
(2 Marks)

Exercise 3.3

In this exercise we use the scalar product

$$\langle u, v\rangle := \int_a^b \overline{u(x)} v(x)\, dx$$

on $C([a,b])$ for any interval $[a,b] \subset \mathbb{R}$

i) Show that the family of functions defined on the interval $[-1,1]$ and given by

$$\left\{\frac{1}{\sqrt{2}}, \cos(\pi n x), \sin(\pi n x)\right\}_{n=1}^{\infty}$$

is an orthonormal system.
(2 Marks)

ii) Show that if $\{e_n\}$ is an orthonormal system in $C([-1,1])$, then $\{\widetilde{e}_n\}$ defined by

$$\widetilde{e}_n(x) = \sqrt{\frac{2}{b-a}} \cdot e_n\left(\frac{2}{b-a}\left(x - \frac{b+a}{2}\right)\right)$$

is an orthonormal system in $C([a,b])$.
(2 Marks)