SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Methods of Applied Mathematics I
HW4

Yu Cang
018370210001

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1 EXERCISE4.1

Let $f(x)$ be extended as

$$f(x) = \begin{cases} x(\pi - x) x \in [2n\pi, (2n+1)\pi] \\ -x(\pi - x) x \in [-(2n-1)\pi, 2n\pi] \end{cases} \tag{1.1}$$

Then $f(x)$ is both odd and periodic. Thus fouier-sine series can be employed.

$$f(x) = \sum_{n=1}^{\infty} b_n sin(nx) \tag{1.2}$$

Coefficients b_n are calculated by

$$\begin{aligned} b_n &= \frac{2}{\pi} \int_0^{\pi} f(x) sin(nx) dx \\ &= \frac{2}{\pi} \int_0^{\pi} x(\pi - x) sin(nx) dx \\ &= \frac{4[1-(-1)^n]}{n^3 \pi} \quad \text{(Integrate by parts)} \end{aligned} \tag{1.3}$$

Thus

$$f(x) = \sum_{k=0}^{\infty} \frac{8 sin(2k+1)x}{\pi (2k+1)^3} \tag{1.4}$$

Taking $x = \frac{\pi}{2}$ yields

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3} = \frac{\pi^3}{32} \tag{1.5}$$

2 EXERCISE4.2

1. *Proof.* The orthogonal property is justified as

$$\int_0^\pi (\frac{1}{\sqrt{\pi}})^2 dx = \frac{1}{\pi} \int_0^\pi dx = 1 \tag{2.1}$$

$$\int_0^\pi (\sqrt{\frac{2}{\pi}} cos(nx))^2 dx = \frac{2}{\pi} \int_0^\pi cos^2(nx) dx = 1 \tag{2.2}$$

$$\int_0^\pi \frac{1}{\sqrt{\pi}} \sqrt{\frac{2}{\pi}} cos(nx) dx = 0 \tag{2.3}$$

$$\int_0^\pi \sqrt{\frac{2}{\pi}} cos(nx) \sqrt{\frac{2}{\pi}} cos(mx) dx = \frac{2}{\pi} \int_0^\pi cos(nx) cos(mx) dx = 0 \tag{2.4}$$

□

2. *Proof.* □

3.

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