SHANGHAI JIAO TONG UNIVERSITY, UM-SJTU JOINT INSTITUTE

Methods of Applied Mathematics I
HW3

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1 EXERCISE3.1

Proof. Let $\phi(x) = (x^2-1)^n$, then

$$\phi(\pm 1) = \phi^{(1)}(\pm 1) = \cdots = \phi^{(n-1)}(\pm 1) = 0 \tag{1.1}$$

The L^2 norm of $P_n(x)$ is

$$\begin{aligned}
||P_n||_2 &= \sqrt{\int_{-1}^{1} P_n^2(x)dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 \int_{-1}^{1} [\phi^{(n)}(x)]^2 dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (-1)^n \int_{-1}^{1} \phi^{(2n)}(x)\phi(x)dx} \quad \text{(Integrate by parts)} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (-1)^n (2n)! \int_{-1}^{1} (x^2-1)^n dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (-1)^n (2n)! \cdot 2 \int_{0}^{\frac{\pi}{2}} (sin^2(x)-1)^n cos(x)dx} \\
&= \sqrt{(\frac{1}{2^n n!})^2 (2n)! \cdot 2 \int_{0}^{\frac{\pi}{2}} cos^{2n+1}(x)dx}
\end{aligned} \tag{1.2}$$

Since

$$\int_0^{\frac{\pi}{2}} cos^{2n+1}(x)dx = \frac{2 \times 4 \times \cdots \times (2n)}{1 \times 3 \times \cdots \times (2n+1)} \tag{1.3}$$

substitute it into the norm equation of P_n yields

$$||P_n||_2 = \sqrt{(\frac{1}{2^n n!})^2 (2n)! \cdot 2 \cdot \frac{2 \times 4 \times \cdots \times (2n)}{1 \times 3 \times \cdots \times (2n+1)}} = \sqrt{\frac{2}{2n+1}} \tag{1.4}$$

$\square$

2 Exercise3.2

3 Exercise3.3