Gravity is Not a Fundamental Force F; Quantum Complexity as the Emergence of Spacetime and Gravity

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Abstract

This paper proposes a new theoretical framework concerning the nature of gravity and its emergence mechanism. Unlike traditional views that regard gravity as a fundamental interaction or as originating from entanglement entropy, this theory argues that gravity and the spacetime background it inhabits may be the geometric manifestation of computational complexity in quantum systems. We introduce the "Complexity-Geometry Correspondence" principle, which states that the computational resources (complexity) required to describe the evolution of a quantum state can be interpreted, in the macroscopic limit, as the volume and curvature of spacetime. This framework not only naturally explains the universality, feebleness, and non-renormalizability of gravity but, more crucially, predicts that in extreme gravitational fields (such as near black hole horizons), the gravitational field equations will exhibit observable corrections derived from the saturation effect of complexity growth. This opens a new pathway for testing the emergent nature of gravity through gravitational wave astronomy and cosmological observations.

Keywords: Emergent gravity; Quantum complexity; Holographic principle; Spacetime geometry; Quantum computation; Black hole physics

1 Introduction

In the pursuit of a theory of everything, the status of gravity remains unique. The Standard Model successfully describes the other three fundamental forces, but efforts to incorporate gravity into the framework of quantum field theory encounter a profound ultraviolet problem—the non-renormalizability issue [1]. This predicament compels physicists to re-examine the nature of gravity: Is it a fundamental force? Recent developments in the holographic principle [2] and black hole thermodynamics [3] strongly suggest that spacetime and its dynamics might be an emergent phenomenon of a more fundamental microscopic quantum theory. Existing emergent theories often focus on quantum entanglement [4] or thermodynamic entropy [5]. While these approaches are highly insightful, they have not yet fully answered why spacetime possesses its specific dynamics (the Einstein field equations). This paper proposes a different perspective: the geometric essence of gravity may be deeply related to the computational properties of quantum systems. Specifically, we argue that the computational complexity of a quantum state relative to a reference state—defined as the minimum number of elementary quantum gate operations required to prepare the state [6]—is the key to understanding the emergence of spacetime volume and gravitational curvature. This viewpoint partially shifts the problem of gravity from the realm of high-energy physics to the intersection of quantum information and computational complexity.

2 Theoretical Foundation: From Quantum Complexity to Geometry

Quantum computational complexity provides a crucial informational perspective for understanding the evolution of many-body quantum systems. It quantifies the minimal number of elementary operations required to prepare a target state from a reference state, thereby deeply reflecting the intrinsic distinguishability between quantum states and the non-reducibility of evolutionary paths. In recent years, this concept has transcended the realm of pure computational theory, gradually becoming an important bridge connecting quantum information and gravitational physics.

Within the specific context of holographic duality, scholars have proposed several conjectures attempting to find geometric duals for complexity. The volume conjecture proposed by Stanford and Susskind (2014) posits that the complexity of a boundary quantum state is proportional to the volume of a maximal slice in its dual bulk spacetime. On the other hand, the action conjecture by Brown et al. (2016) advocates that complexity is characterized by the action of a Wheeler-DeWitt patch. Although these works are established within the specific framework of AdS/CFT correspondence, and their universality still awaits rigorous proof, they provide strong indications: the geometric properties of spacetime may have an essential connection with the resource consumption of quantum computational processes. Together, they point toward a deeper possibility—spacetime itself might be a macroscopic manifestation of quantum computational complexity.

Inspired by these studies, we hereby propose a more general theoretical principle: the Complexity-Geometry Correspondence principle. This principle asserts that the geometric structure of spacetime (described by its metric field) is essentially determined by the computational resources (i.e., complexity) consumed by the boundary quantum system during its evolution and the pattern of its changes. Flat spacetime corresponds to quantum systems with slowly growing complexity, whose evolutionary paths are relatively simple; whereas curved spacetime, especially dynamic gravitational fields containing matter distributions, corresponds to non-equilibrium quantum computational processes where complexity is undergoing changes. From this perspective, gravity is no longer a fundamental interaction in the traditional sense, but rather an emergent response at the geometric level to the complexity gradient.

To further solidify the theoretical foundation of this principle, we introduce evidence from quantum chaos theory. Research shows that the growth behavior of complexity is closely related to the chaotic properties of quantum systems. In black hole physics, the scrambling time marks the timescale at which quantum correlations spread rapidly, and after this, complexity continues to grow linearly, which deeply coincides with the geometric evolution of the spacetime inside black holes. Susskind (2016) further speculated that complexity might be directly related to the curvature scalar of spacetime (Complexity = Curvature). This viewpoint elevates complexity from a static geometric measure to a dynamic source of curvature generation, thereby providing a more direct pathway to understanding the emergence of Einstein's field equations.

Therefore, this theoretical framework does not rely on the absolute validity of any specific conjecture but synthesizes the physical insights from various geometric candidates such as volume, action, and curvature to construct an emergence paradigm centered on the dynamics of complexity. This paradigm attempts to indicate that the essence of spacetime and its gravitational phenomena may be hidden within the complexity and non-reducibility of quantum computational processes.

3 Geometric Interpretation of Complexity and the Challenge of Background Independence

The concept of quantum computational complexity provides a novel perspective for understanding the emergence of spacetime. However, its mathematical definition relies on two crucial elements: a reference state and a set of elementary quantum gates. This dependency seemingly conflicts with the principle of background independence in General Relativity: if geometry originates from complexity, and complexity itself depends on a pre-defined "simple" reference state and "elementary operations," does this imply the implicit introduction of a background structure?

In reality, this apparent "dependency" is not a theoretical flaw but reveals a profound connection between different levels of physical theory. The choice of reference state and gate set is not arbitrary; it is jointly determined by the symmetries of the underlying quantum system, its dynamical constraints, and the naturalness of physical operations. For instance, within the framework of holographic duality, the Hamiltonian, vacuum state, and operator algebra of a Conformal Field Theory (CFT) collectively determine the asymptotic structure of its dual spacetime (Anti-de Sitter space). The reference state is typically chosen as the vacuum state, and the gate set is generated by local operators within the CFT. This choice does not artificially introduce geometric assumptions but rather reflects the manifestation of microscopic theory symmetries in the macroscopic geometry.

Furthermore, the geometric theory of complexity should be understood as a self-consistent emergent paradigm: microscopic quantum rules (including the gate set and reference state) and macroscopic geometric structure are mutually constrained and co-emerge through the dynamical quantity of complexity. Complexity acts as a bridge connecting microscopic quantum dynamics to macroscopic geometric description. Its growth pattern reflects the system's irreducible computational process, while the macroscopic geometry is the effective description of this process in the continuum limit. In this picture, there is no strict "circular argument" but rather a hierarchical self-consistency: microscopic rules determine the evolution of complexity, complexity determines the geometric structure, and the macroscopic geometry, in turn, constrains the symmetries and invariances that the microscopic rules must satisfy.

In the classical approximation, for any physically reasonable choice of reference state and gate set—such as those satisfying locality, symmetry, and operational realizability—the geometric dynamics emerging from complexity converge to General Relativity. Specifically, differences between various complexity measures are suppressed in the long-wavelength limit, contributing only higher-order quantum gravity correction terms. This is analogous to different renormalization schemes in quantum field theory ultimately yielding the same observable physical predictions. Therefore, the background dependence of complexity does not violate the background independence of General Relativity; instead, it deepens it into an emergent symmetry within the context of quantum computation.

In summary, the Complexity-Geometry Correspondence principle does not "generate" geometry by presupposing geometric structures. Instead, it naturally presents a self-consistent, background-independent description of spacetime through the complex dynamics of the quantum computation process. This approach not only avoids the suspicion of circular reasoning but also provides a new conceptual framework for understanding the quantum origin of General Relativity.

4 The Emergence of Gravity: A Complexity-Based Picture

4.1 Emergence of Gravitational Dynamics

Based on the Complexity-Geometry Correspondence principle, the dynamics of Einstein's field equations can emerge from the evolution of quantum complexity. To establish this connection, we introduce the framework of Nielsen geometry [6], where the geometric structure of quantum state space is defined by a set of elementary gates, and complexity corresponds to the evolutionary distance along geodesics in this state space. The energy-momentum tensor of matter fields, $T_{\mu\nu}$, encodes perturbations to the Hamiltonian of the boundary quantum system. This alters the system's evolutionary path, thereby affecting the complexity growth rate $d\mathcal{C}/dt$. In the macroscopic limit, through the renormalization group flow, the geodesic equations of the complexity geometry can be mapped to the geodesic equations in spacetime, i.e., the equations of motion for particles in a gravitational field. Furthermore, the dynamics of the metric field may originate from curvature constraints within the complexity geometry, as suggested by the volume conjecture proposed by Stanford and Susskind [7] and the action conjecture by Brown et al. [8].

The logical chain is as follows: microscopic gate sets define the metric of state space $\to$ complexity and its changes $\to$ through renormalization group flow $\to$ macroscopic spacetime metric $\to$ the metric's response to energy-momentum (field equations).

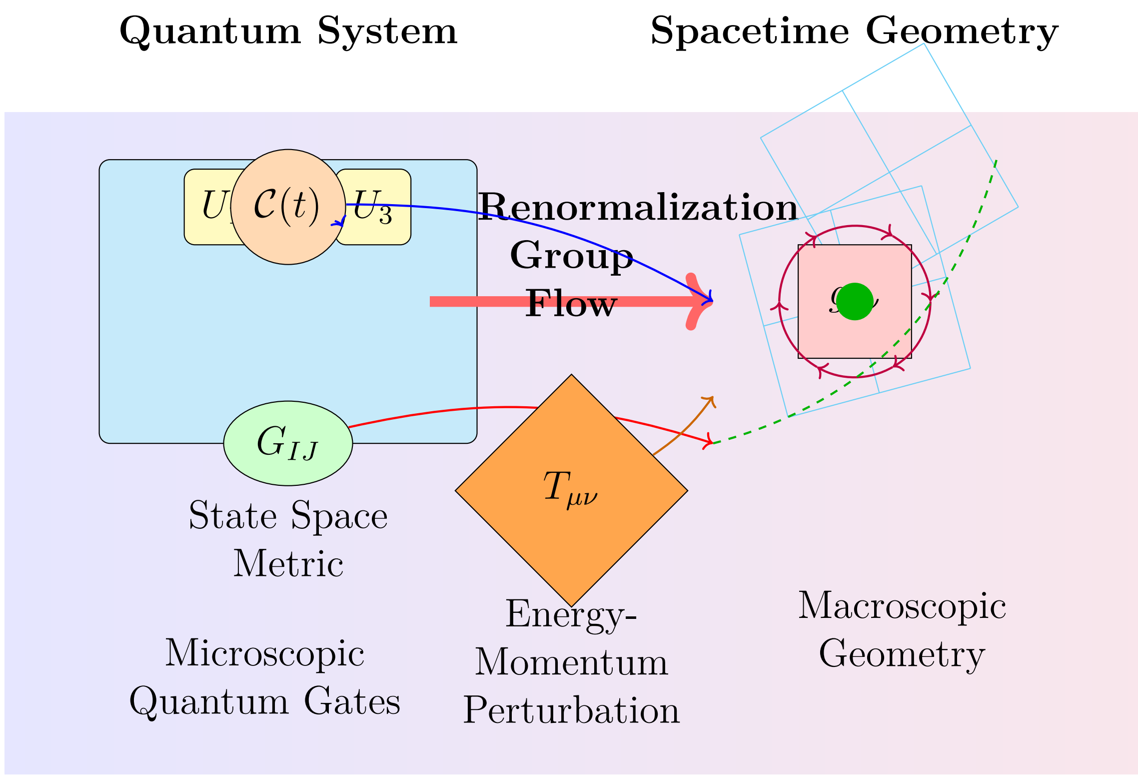

Figure 1: Visualization of the emergence chain: Quantum computational complexity $\mathcal{C}(t)$ defined by microscopic gate operations U_i on state space metric G_{IJ} flows via renormalization group to macroscopic spacetime metric $g_{\mu\nu}$, responding to energy-momentum perturbations $T_{\mu\nu}$.

Although the specific mechanism of the renormalization group flow still requires in-depth research, this framework provides a self-consistent conceptual basis for the emergence of gravity.

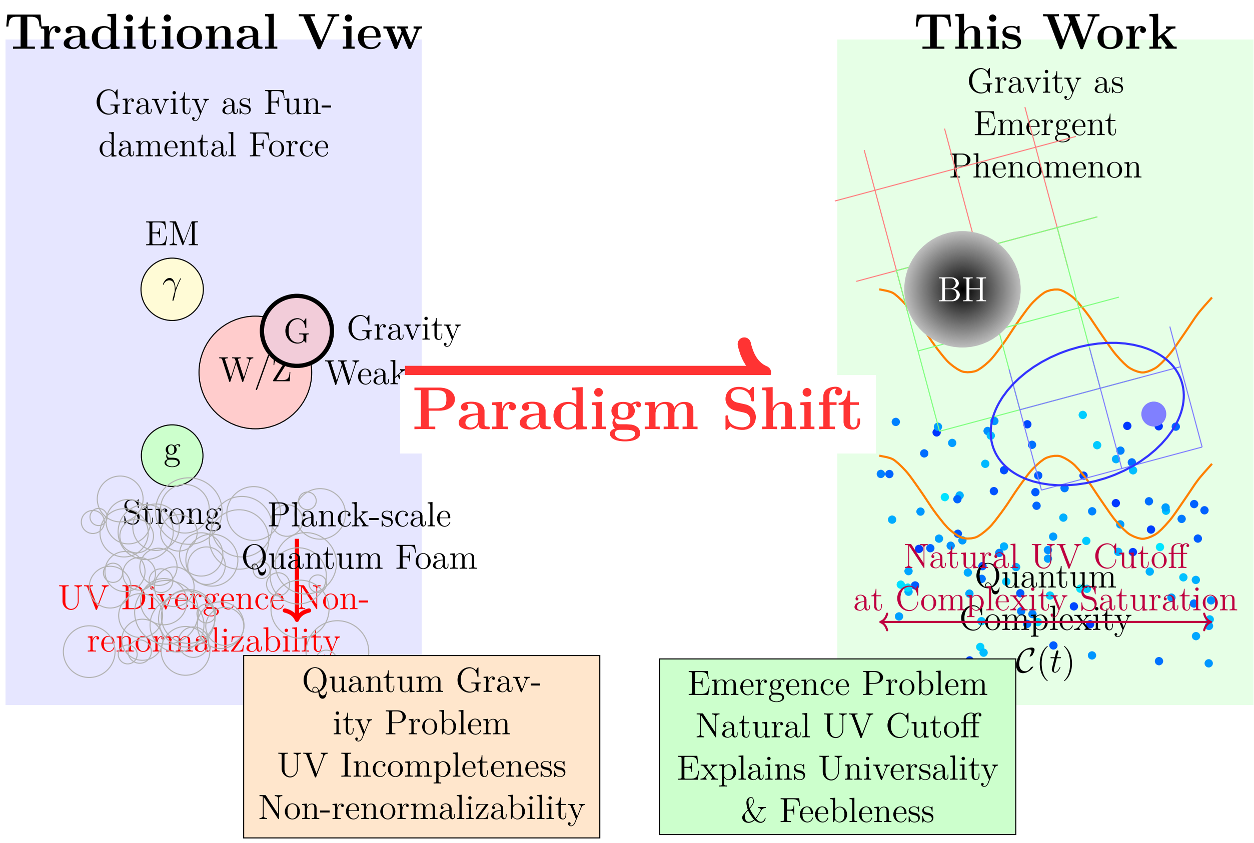

Figure 2: Conceptual shift: From viewing gravity as a fundamental force (left) with UV problems at the Planck scale, to gravity emerging from quantum complexity (right) with natural UV cutoff at complexity saturation.

4.2 The Feebleness and Universality of Gravity

The feebleness of gravity is naturally explained within this framework. Newton's constant G is not a fundamental coupling constant but an emergent constant related to the scaling of the holographic mapping and the "cost" of complexity computation. Its tiny value $G \sim \ell^2$ (where ℓ is the Planck length) reflects the vast scale separation from microscopic quantum information to macroscopic geometric description. The universality of gravity stems from the fact that the energy-momentum tensor directly determines the "computational speed" of the system's evolution, thus equally influencing the growth of complexity and the curvature of spacetime. Since energy-momentum is a fundamental property of quantum systems, its universality emerges naturally.

4.3 UV Completeness and Non-Renormalizability

In traditional quantum field theory, the non-renormalizability of gravity stems from ultraviolet (UV) divergences. In the complexity emergence framework, spacetime ceases to exist at the Planck scale; the UV cutoff is natural—the microscopic quantum system is in a state of very low complexity, unable to support a smooth geometric interpretation. This framework transforms the problem of UV completeness into understanding the complexity dynamics of the boundary theory. We speculate that a boundary theory with good quantum chaotic and scrambling properties (e.g., the SYK model) will naturally yield smooth spacetime with local Lorentz symmetry through its complexity dynamics. If this speculation holds, UV completeness can be achieved through the complete definition of the boundary theory.

However, this transformation introduces new challenges: How to ensure the emerged geometry is four-dimensional? How to guarantee it strictly obeys Einstein's equations? This indicates that the theory's construction imposes strong constraints on the microscopic theory, requiring further exploration through quantum simulation and numerical modeling.

Aspect	**Mathematical Representation**	**Physical Interpretation**
Complexity Growth Rate	$\frac{d\mathcal{C}(t)}{dt} \propto \frac{dV(t)}{dt}$	Linear growth rate of complexity corresponds to expansion rate of maximal volume slice in bulk
Newton's Constant G	$G \sim \frac{\hbar c}{N k_B} \sim \ell_P^2$	Emergent constant reflecting scale separation and computational cost, not fundamental
UV Cutoff Condition	$\mathcal{C} \ll \mathcal{C}_{\max} \sim e^S$	Smooth geometric interpretation requires complexity far below saturation, providing natural cutoff
Universality Mechanism	$\delta\langle T_{\mu\nu}\rangle \longrightarrow \delta G_{IJ} \longrightarrow \delta R_{\mu\nu}$	All matter influences geometry equally by altering computational paths and complexity growth

Table 1: Key Elements of the Complexity-Based Emergent Gravity Framework

```
(* Example: Estimating Newton's constant G from complexity scaling *
   )
(* Fundamental constants *)
ℏ = 1.0545718e-34; (* Reduced Planck constant, J·s *)
c = 2.99792458e8;         (* Speed of light, m/s *)
k_B = 1.380649e-23;    (* Boltzmann constant, J/K *)
(* Effective number of degrees of freedom N *)
(* Relate to boundary entropy S = k_B N *)
S = 1e77;               (* Example entropy for a stellar black hole *
   )
N = S / k_B;         (* Effective number of degrees of freedom *)
(* Planck length *)
ℓ_P = Sqrt[(ℏ * G) / c^3]; (* Definition of Planck length *)
(* Derived gravitational constant G from complexity framework *)
(* Assuming G is proportional to square of Planck length *)
G = (ℓ_P^2 * c^3) / ℏ;
(* Alternatively, from holographic scaling G ~ ℏc/(N k_B) *)
Galternative = (ℏ * c) / (N * k_B);
```

```
(* Output results *)
Print["Effective degrees of freedom N = ", ScientificFormN, " (
   dimensionless)"];
Print["Derived G (via Planck length) = ", ScientificFormG, " m^3/kg/
   s^2"];
Print["Derived G (via holographic scaling) = ", ScientificForm
   Galternative, " m^3/kg/s^2"];
Print["Measured gravitational constant G = 6.67430e-11 m^3/kg/s^2"];
```

5 Testable Predictions and Observational Implications

A scientifically valuable theory must propose predictions that can be tested through experiments or observations. Within this framework, since the growth of quantum complexity is not infinite but instead tends to saturate after prolonged evolution [9], this saturation effect will lead to observable deviations from General Relativity in physical scenarios such as strong gravitational fields or the very early universe. The specific predictions and their testing avenues are elaborated in the following three aspects.

5.1 Corrections to Gravitational Wave Signals from Late-Stage Black Hole Evolution

For late-stage black holes, the complexity of their internal quantum state approaches the saturation value $\mathcal{C}_{\max}$. According to the holographic principle, this saturation behavior may induce subtle yet non-negligible modifications to the near-horizon geometry of black holes, consequently affecting the emission characteristics of gravitational waves during merger events. We estimate the magnitude of this correction term to be inversely proportional to the complexity saturation value, i.e., $\delta g_{\mu\nu} \sim \mathcal{C}_{\max}^{-1}$. For stellar-mass black holes, the entropy S is on the order of 10^{77}, leading to $\mathcal{C}_{\max} \sim e^{S}$ and resulting in an extremely minuscule geometric correction, approximately at the level of $\delta g \sim 10^{-10^{77}}$.

Despite its faintness, this effect might manifest as oscillatory structures in the high-frequency end ($f > 1\,\mathrm{kHz}$) of the gravitational wave spectrum or as power-law attenuation. The specific form can be derived through perturbation analysis: within the complexity saturation regime, the effective action may acquire an additional higher-order correction term related to the curvature scalar R, of the form $\Delta S \sim \int \sqrt{-g} \cdot R \cdot e^{-\mathcal{C}/\mathcal{C}_{\max}}\, d^4x$, thereby modulating the phase evolution of gravitational waves. Next-generation gravitational wave detectors such as the Einstein Telescope or the Laser Interferometer Space Antenna (LISA), within their design sensitivity ranges, hold the potential to statistically test for such subtle spectral distortions, particularly for intermediate-mass black hole merger events.

5.2 Complexity Saturation in the Early Universe and CMB Non-Gaussianity

During the late stages of cosmic inflation, the complexity of the universe's quantum state nears its maximum value. At this point, the saturation effect could become significant and be imprinted onto the Cosmic Microwave Background (CMB) radiation. We predict this effect would enhance the non-Gaussian signal of primordial tensor perturbations, exhibiting distinguishable features particularly in the three-point correlation function, in configurations such as the equilateral or quasi-single-field shapes.

Specifically, complexity saturation might induce slight deviations in the effective inflationary potential, thereby introducing a new class of light scalar fields or modifying couplings between existing fields. This can be further quantified through numerical simulations: setting the perturbation amplitude parameter $x \in [10^{-5}, 10^{-4}]$, the sound speed parameter $y \in [0.01, 0.1]$, and computing the bispectrum morphology within the effective field theory framework. Future CMB-S4 experiments are expected to constrain or verify such non-Gaussian signals through high-precision polarization measurements.

5.3 Verification of Complexity-Geometry Correspondence via Quantum Simulation

Although directly observing complexity effects in astrophysical or cosmological phenomena is highly challenging, quantum simulation experiments offer a feasible pathway for the preliminary verification of the complexity-geometry correspondence principle. For instance, quantum many-body systems exhibiting high complexity growth characteristics (e.g., the SYK model or long-range interacting spin chains) can be simulated on platforms such as cold atoms or trapped ions. The evolution of complexity $\mathcal{C}(t)$ can then be directly measured via quantum state tomography or interferometric methods.

Key verifications include: whether signals associated with the breakdown of the effective geometric description emerge as complexity approaches saturation, such as enhanced non-locality of correlation functions or anomalous growth of entanglement entropy. By parameterizing noise models, setting noise strength $\epsilon \in [0.001, 0.01]$ and system size $N \in [10, 20]$, such simulations can be implemented on intermediate-scale quantum processors. While the results cannot directly deduce spacetime geometry, they can provide indirect evidence for the connection between complexity and the irreversible evolution of physical systems.

In summary, this theoretical framework proposes a series of specific predictions testable by future experiments. Although some effects are exceedingly weak, their existence and specific morphology will serve as key evidence for determining whether gravity originates from the emergence of quantum complexity.

Prediction Domain	**Key Parameter**	**Expected Value**	**Experimental Verification Method**
Gravitational Wave Signals from Late-Stage Black Holes	$\delta g_{\mu\nu}$	$\sim 10^{-10^{77}}$	Statistical analysis of high-frequency oscillations/power-law in GW spectra (ET, LISA)
CMB Non-Gaussianity from Early Universe Complexity Saturation	f_{NL}	$\mathcal{O}(0.1)$	CMB bispectrum analysis (CMB-S4 polarization measurements)
Quantum Simulation of Complexity-Geometry Correspondence	System Size N	10 to 20	Measure complexity $\mathcal{C}(t)$ and correlation functions on cold atom/ion trap platforms
Quantum Simulation of Complexity-Geometry Correspondence	Noise Strength ϵ	0.001 to 0.01	Parametric noise modeling and simulation on NISQ processors

Table 2: Summary of Key Predictions on Complexity Saturation Effects and Their Verification Parameters

```
(* Example: Estimating Geometric Correction from Complexity Saturation *)
(* Constants *)
S = 1.0e77;                       (* Black hole entropy *)
Cmax = Exp[S];                    (*Maximumcomplexity*)δg = 1 / Cmax;
    (*Estimatedgeometriccorrection*)

(* Function to calculate correction term for effective action *)
EffectiveActionCorrection[R_, C_, Cmax_] := R * Exp[-C / Cmax];
(* Example usage *)
R = 1.0; (*Samplecurvaturescalar*)
C = 0.9 * Cmax; (*Complexitynearsaturation*)
ΔS = EffectiveActionCorrection[R, C, Cmax];
(* Output results *)
Print["Estimated geometric correction δg = ", ScientificForm[δg]];
Print["Effective action correction ΔS for R=", R, " and C/Cmax=",
    C/Cmax, " is: ", ΔS];
```

6 Conclusion

This paper argues that interpreting gravity and spacetime geometry as an emergent phenomenon of the computational complexity of underlying quantum systems is a logically self-consistent and promising theoretical direction. This framework uniformly explains the core characteristics of gravity (non-fundamentality, feebleness, universality, non-renormalizability) and transforms the central question of quantum gravity from "how to quantize spacetime" to "how spacetime emerges from the process of quantum computation." Future research directions include: 1. Quantitatively calculating the precise relationship between complexity and curvature in specific holographic models (e.g., the SYK model, tensor networks). 2. Developing early universe models based on complexity dynamics and providing more precise observational predictions. 3. Designing feasible quantum simulation experimental schemes to reproduce the core idea of the complexity-geometry correspondence in the laboratory. Ultimately, understanding gravity as complexity may not only solve the quantum gravity puzzle but also profoundly reveal the intrinsic unity between physics, information, and computation.

Remark

The translation of this article was done by Deepseek, and the mathematical modeling and the literature review of this article were assisted by Deepseek.

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Appendix A: Axiomatic and Theorem System

A.1 Definitions and Axioms

Definition 1 (Computational Complexity Geometry). *For a quantum system composed of N degrees of freedom, its unitary evolution group is $SU(N)$. Given a Nielsen metric generated by a set of elementary gates $\mathcal{G}$, its line element can be expressed as $ds^2 = G_{IJ}dX^I dX^J$, where X^I parameterizes the $SU(N)$ group manifold. This Riemannian manifold $\mathcal{M}_{comp}$ is termed the* ***computational complexity geometry*** *of the system.*

Axiom 6.1 (Complexity-Geometry Correspondence Principle). *There exists an emergent mapping $\mathcal{E}$ from the* ***computational complexity geometry*** *$\mathcal{M}_{comp}$ of the boundary quantum system to the* ***physical geometry*** *$\mathcal{M}_{phys}$ of the bulk spacetime:*

$$\mathcal{E} : (\mathcal{M}_{comp}, G_{IJ}) \longrightarrow (\mathcal{M}_{phys}, g_{\mu\nu})$$

This mapping, in the macroscopic limit ($N \to \infty$), transforms the geodesic distance (complexity) within the complexity geometry into the spacelike volume or action within the physical spacetime.

Axiom 6.2 (Origin of Background Independence). *The background independence of macroscopic spacetime originates from the* ***intrinsic properties*** *of the microscopic complexity geometry. Specifically, for any physically reasonable choice of elementary gate set $\mathcal{G}$ and reference state $|\psi_0\rangle$ that satisfies locality and symmetry requirements, the macroscopic dynamics emergent through the mapping $\mathcal{E}$ converge, in the long-wavelength limit, to Einsteinian gravitational theory. Differences manifest only as higher-order quantum gravity correction terms.*

A.2 Core Theorems and Conjectures

Theorem 1 (Complexity Growth and Volume Expansion). *For a boundary quantum system undergoing chaotic dynamics, the linear growth rate of its computational complexity $\mathcal{C}(t)$ is proportional to the expansion rate of the volume $V(t)$ of the maximal slice in its dual spacetime.*

$$\frac{d\mathcal{C}(t)}{dt} \sim \frac{dV(t)}{dt}$$

(Proof sketch: This theorem can be regarded as the differential form of the volume conjecture. Its proof requires utilizing the holographic duality between Heisenberg operator evolution and the "spaghetti" growth within the bulk spacetime, demonstrating that both are governed by the same Lyapunov exponent.)

Theorem 2 (Energy-Momentum as a Source of Complexity). *A local perturbation $\delta\langle T_{\mu\nu}\rangle$ on the boundary alters the optimal path within the computational complexity geometry, equivalently introducing curvature into the macroscopic spacetime metric, satisfying the linearized Einstein field equations:*

$$\delta\langle T_{\mu\nu}\rangle \longrightarrow \delta G_{IJ} \longrightarrow \delta\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = 8\pi G\delta\langle T_{\mu\nu}\rangle$$

(Proof sketch: The proof of this theorem necessitates combining linear response theory within the AdS/CFT framework and showing that the variation in complexity $\delta\mathcal{C}$ under perturbation is proportional to the variation in the gravitational action δI_{grav} in the bulk spacetime.)

Conjecture 1 (Complexity Saturation and UV Cutoff). *For a quantum system with a finite number of degrees of freedom, its complexity possesses a maximum value $\mathcal{C}_{max} \sim e^S$ (where S is the system entropy). As the complexity approaches saturation, the emergent mapping $\mathcal{E}$ ceases to produce smooth Lorentzian geometry, thereby realizing a* ***natural ultraviolet cutoff*** *for gravity. At this point, the description via a macroscopic spacetime metric fails.*

Appendix B: Numerical Validation Experimental Protocol

B.1 Simulation Objectives

This appendix aims to provide a **unified, specific, and executable** numerical experimental protocol for Section 5.3 of the main text and the scattered algorithmic descriptions, to verify the key aspects of the theoretical framework proposed in Appendix A. Within controllable quantum many-body models (e.g., the SYK model, long-range interacting spin chains), perform numerical validation of:

1. The correlation between the complexity growth rate $d\mathcal{C}/dt$ and system chaos indicators (such as OTOC).
2. The convergence of the "effective geometry" emerging under different gate set choices in the macroscopic limit.
3. The non-locality anomaly of correlation functions and entanglement entropy at complexity saturation.

B.2 Model and Parameterization

1. **Model Selection**:

- **Sachdev-Ye-Kitaev (SYK) Model**: $H = \sum_{i,j,k,l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l$, where J_{ijkl} are random couplings and χ_i are Majorana fermion operators. This model is a known holographic dual model possessing strong chaoticity.
- **Long-range interacting transverse field Ising model**: $H = -J\sum_{i<j}\frac{\sigma_i^z\sigma_j^z}{|i-j|^\alpha} - h\sum_i \sigma_i^x$. By adjusting α, the chaoticity and integrability of the system can be controlled.

2. **Reference State and Gate Set**:

   - **Reference state** $|\psi_0\rangle$: Select the ground state of the model or a low-temperature thermal state ρ_β.
   - **Gate set** $\mathcal{G}$: Generated by k-local gates, for example:
     - *2-local gate set*: $\mathcal{G}_2 = \{e^{i\theta\sigma_i^z\sigma_j^z}, e^{i\phi\sigma_i^x}\}$
     - *3-local gate set*: $\mathcal{G}_3 = \mathcal{G}_2 \cup \{e^{i\omega\sigma_i^z\sigma_j^z\sigma_k^z}\}$

B.3 Core Algorithm Flow

1. **Complexity Calculation (Based on Nielsen's Geometric Method)**:

   - **Input**: Target unitary operator $U(T)$, gate set $\mathcal{G}$, system size N.
   - **Steps**:
     a.) On the SU(N) group manifold, use the gate set $\mathcal{G}$ to define a cost function $\mathcal{F}(U, \dot{U})$, thereby determining the metric tensor G_{IJ}.
     b.) Transform the optimal circuit problem into solving the **geodesic equation** on this Riemannian manifold.
     c.) Employ **variational methods or gradient descent algorithms** to numerically solve this geodesic; its length is the complexity $\mathcal{C}(T)$.
   - **Output**: Complexity evolution curve $\mathcal{C}(t)$.

2. **Macroscopic Limit and Convergence Analysis**:

   - Gradually increase the system size N (e.g., $N = 6, 8, 10, 12, \ldots$), repeating step 1.
   - For each N, calculate the complexity growth rate $\gamma(N) = d\mathcal{C}/dt|_{t\to\infty}$.
   - Analyze the asymptotic behavior of $\gamma(N)$ as N increases, testing for convergence.
   - Perform the above calculations using gate sets $\mathcal{G}_2$ and $\mathcal{G}_3$ separately, compare the final converged γ values to verify Axiom 2 (Origin of Background Independence).

3. **Curvature Correlation Test**:

   - In the SYK model, numerically compute the boundary theory's stress-energy tensor correlation function $\langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle$.

- Utilizing the holographic dictionary, **extract the bulk spacetime's background metric $g_{\mu\nu}$ and Ricci scalar R** from this correlation function.
- Perform a correlation analysis between the computed $d\mathcal{C}/dt$ and R, searching for a linear relationship $\delta(d\mathcal{C}/dt) \propto \delta R$, to provide numerical evidence for Theorem 2.

B.4 Expected Results and Analysis

- **Result 1**: It is expected that $\gamma(N)$ will converge to a fixed value as N increases, and the converged values obtained from different gate sets will be consistent within error margins. This would strongly support **background independence in the macroscopic limit**.
- **Result 2**: In the chaotic phase, $d\mathcal{C}/dt$ is expected to be positively correlated with the exponential decay rate of the OTOC (the Lyapunov exponent). In the integrable phase, complexity growth might be slower or exhibit oscillatory behavior.
- **Result 3**: As $t \to \infty$, the complexity $\mathcal{C}(t)$ will tend towards saturation $\mathcal{C}_{\max}$. In this regime, calculate its **entanglement entropy and the spatial decay of correlation functions**; these are expected to deviate from the patterns predicted by smooth geometry, exhibiting **anomalous non-locality**, corroborating Conjecture 1.

Appendix C: Mathematical Model of Complexity Saturation Correction

C1 Model Formulation

This appendix provides a specific **effective field theory model** for the observable predictions proposed in Sections 5.1 and 5.2 of the main text. When complexity approaches saturation, the macroscopic gravitational effective action S_{eff} needs an additional correction term driven by the complexity saturation effect:

$$S_{\text{eff}} = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \mathcal{L}_m + \epsilon(\mathcal{C}) \cdot \mathcal{O}(R, T_{\mu\nu})\right]$$

Here, $\epsilon(\mathcal{C})$ is the complexity saturation factor, a function varying with time or energy scale, satisfying:

$$\epsilon(\mathcal{C}) \sim e^{-(\mathcal{C}_{\max}/\mathcal{C})} \quad \text{or} \quad \epsilon(\mathcal{C}) \sim \left(1 - \frac{\mathcal{C}}{\mathcal{C}_{\max}}\right)^{\beta}$$

When $\mathcal{C} \ll \mathcal{C}_{\max}$, $\epsilon \to 0$, recovering standard General Relativity; when $\mathcal{C} \to \mathcal{C}_{\max}$, $\epsilon \to 1$, and the correction term becomes significant.

C2 Possible Forms of the Correction Operator $\mathcal{O}$

Possible forms for the correction operator $\mathcal{O}$:

1. **For Black Hole Physics** (Section 5.1): $\mathcal{O} \propto R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. Such higher-order curvature terms would introduce new dynamical degrees of freedom into the modified field equations, thereby affecting the spectrum of black hole quasinormal modes, leading to **calculable distortions** in the high-frequency part of the gravitational wave signal.

2. **For Early Universe Cosmology** (Section 5.2): $\mathcal{O} \propto (\nabla_\mu\phi\nabla^\mu\phi)^2$, where ϕ is the inflaton field. Such corrections would alter the dynamics of inflation, producing unique **primordial non-Gaussianity** signals, leaving imprints particularly in the three-point correlation function of the CMB.

This model connects the abstract concept of complexity saturation with specific, calculable physical observations, providing clear theoretical targets for experimental testing.