Yaşar University

Department of Mathematics

MATH 2240 Probability and Statistics for Engineers

Quiz-1, 02.11.2023

1. (30) The average GMAT scores for the top 30 ranked graduate business schools are listed below.

718	651	686	690	645	680	676	703	662	669
705	660	651	689	700	645	652	688	703	637
691	695	642	670	681	703	641	674	690	636

   a) Construct the following table:

Class Boundaries	Class Limits	Frequency	Cumulative Frequency	Cumulative Relative Frequency
629.5–644.5	630–644			
644.5–659.5	645–659			
659.5–674.5	660–674			
674.5–689.5	675–689			
689.5–704.5	690–704			
704.5–719.5	705–719			

   b) Draw a histogram and a frequency polygon for the grouped data.

   c) Draw the OGIVE curve.

2. (25) Shown here are the numbers of stores in the 10 tallest buildings in İzmir.

<div align="center">

14   18   28   2   14   10   8   16   8   12

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    a) Calculate the sample variance S^2.

    b) Draw the Box-Whisker diagram and interpret the result.

3. (20) When two dice were rolled, it is known that the sum was an even number. Find the probability that the sum was 8.

4. (25) We are given three identical urns as follows:

Urn A contains 3 Red 5 White marbles. Urn B contains 2 Red 1 White. Urn C contains 2 Red 3 White marbles. An urn is selected at random, and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn B?

$$\bar{x} = \frac{\sum x_i}{n} \qquad\qquad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$Q_1 = \frac{n+1}{4}, \qquad Q_2 = 2\left(\frac{n+1}{4}\right), \qquad Q_3 = 3\left(\frac{n+1}{4}\right)$$

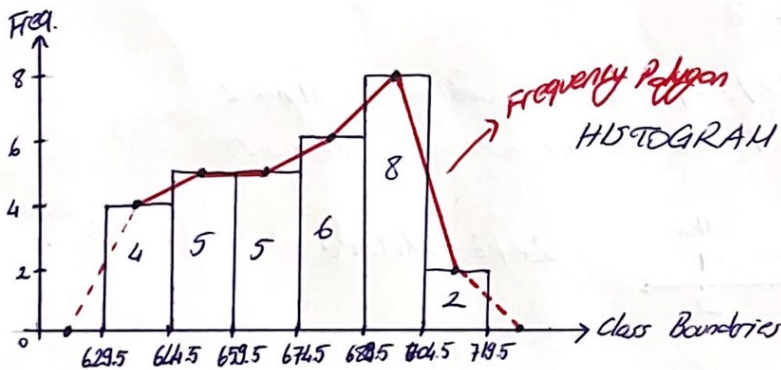$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$
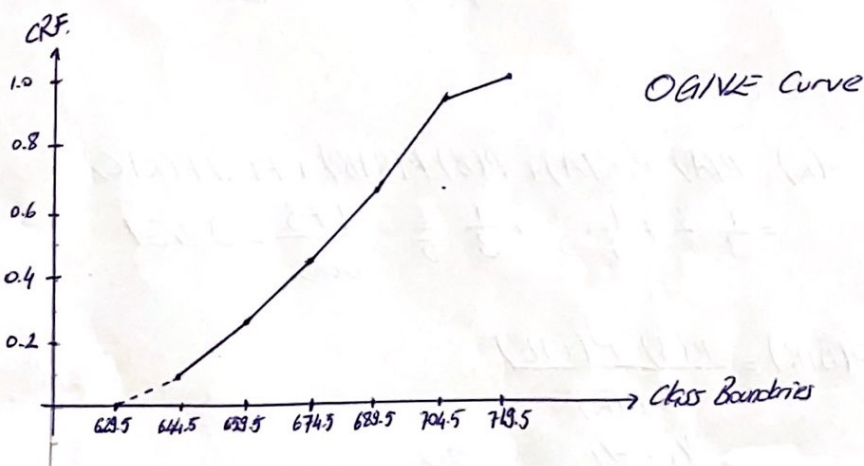
MATH 2240

Quiz 1 — Answer Key

(1)a)

Class Boundaries	Class Limits	Frequency	Cum Freq	C.R.F.
629.5 − 644.5	630 − 644	4	4	4/30 = 0.133
644.5 − 659.5	645 − 659	5	9	9/30 = 0.300
659.5 − 674.5	660 − 674	5	14	14/30 = 0.467
674.5 − 689.5	675 − 689	6	20	20/30 = 0.667
689.5 − 704.5	690 − 704	8	28	28/30 = 0.933
704.5 − 719.5	705 − 719	+ 2	30	30/30 = 1.000
		Total = 30		

b)

Frequency Polygon

HISTOGRAM

c)

OGIVE Curve

— 1 —

② a) $\bar{X} = \sum_{i=1}^{n} \frac{x_i}{n} = \dfrac{14 + 18 + 28 + \cdots + 16 + 8 + 12}{10} = \dfrac{130}{10} = 13$

$S^2 = \sum_{i=1}^{n} \dfrac{(x_i - \bar{X})^2}{n-1} = \dfrac{(14-13)^2 + (18-13)^2 + \cdots + (8-13)^2 + (12-13)^2}{10-1} = \dfrac{442}{9} = 49.11$

b) Ordered Data

¹	²	³	⁴	⁵	⁶	⁷	⁸	⁹	¹⁰
2	8	8	10	12	14	14	16	18	28

Locations $\quad Q_{1L} = \dfrac{10+1}{4} = 2.75 \qquad Q_{2L} = 2\left(\dfrac{10+1}{4}\right) = 5.5 \qquad Q_{3L} = 3\left(\dfrac{10+1}{4}\right) = 8.25$
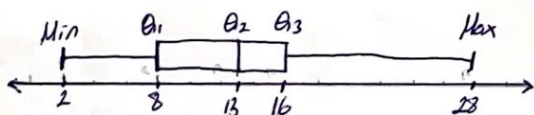
$Q_1 = 8 \qquad\qquad Q_2 = \dfrac{12+14}{2} = 13 \qquad\qquad Q_3 = 16$

$IQR = Q_3 - Q_1 = 16 - 8 = 8$

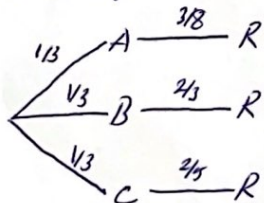$Q_1 - 1.5\, IQR = 8 - 1.5(8) = -4 \quad$ no min outlier $\qquad$ Min = 2

$Q_3 - 1.5\, IQR = 16 + 1.5(8) = 28 \quad$ no max outlier $\qquad$ Max = 28

Min $\quad Q_1 \quad\quad Q_2 \; Q_3 \quad\quad$ Max $\qquad$ Left-Skewed

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Min     Q₁      Q₂ Q₃        Max
 |——————[——|——]—————|
 2       8   13 16         28
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③ A: sum was an even number
B: sum was 8 $\qquad\qquad P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{5/36}{18/36} = \dfrac{5}{18}$

④ Tree Diagram

```
        3/8
   1/3 ⟋A ——— R
      ⟋
 ⟨  1/3  B ——²/³—— R
      ⟍
   1/3 ⟍C ——²/⁵—— R
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$P(R) = P(A)\cdot P(R|A) + P(B)\,P(R|B) + P(C)\,P(R|C)$

$\quad = \dfrac{1}{3}\cdot\dfrac{3}{8} + \dfrac{1}{3}\cdot\dfrac{2}{3} + \dfrac{1}{3}\cdot\dfrac{2}{5} = \dfrac{173}{360} = 0.481$

$P(B|R) = \dfrac{P(B)\cdot P(R|B)}{P(R)}$

$\qquad = \dfrac{\frac{1}{3}\cdot\frac{2}{3}}{173/360} = \dfrac{80}{173} = 0.462$