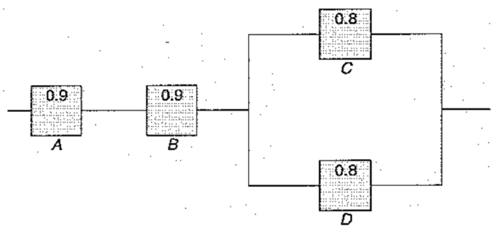
Ex1. An electrical system consists of four components as illustrated in the figure. The system works if components A and B work and either of the components C or D works. The reliability (probability of working) of each component is also shown in the figure. Find the probability that the entire system works. Assume that the four components work independently.



$$P(\text{system works}) = P(A \underset{\text{and}}{\cap} B \underset{\text{and}}{\cap} (C \underset{\text{or}}{\cup} D))$$

$$= P(A) \cdot P(B) \cdot P(C \cup D)$$

$$= P(A) \cdot P(B) \cdot [P(C) + P(D) - P(C \cap D)]$$

$$= P(A) \cdot P(B) \cdot [P(C) + P(D) - P(C) \cdot P(D)]$$

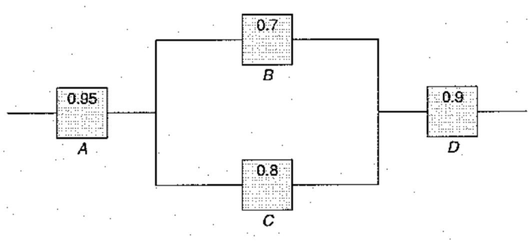
$$= (0.9)(0.9) \cdot [(0.8) + (0.8) - (0.8)(0.8)]$$

$$= (0.81) \cdot (0.96) = 0.7776$$

1

Ex2. Suppose the diagram of an electrical system is as given in the figure. What is the probability that the system works?

Assume the components fail independently.



$$P(\text{system works}) = P(A \underset{\text{and}}{\cap} (B \underset{\text{or}}{\cup} C) \underset{\text{and}}{\cap} D)$$

$$= P(A) \cdot P(B \cup C) \cdot P(D)$$

$$= P(A) \cdot [P(B) + P(C) - P(B \cap C)] \cdot P(D)$$

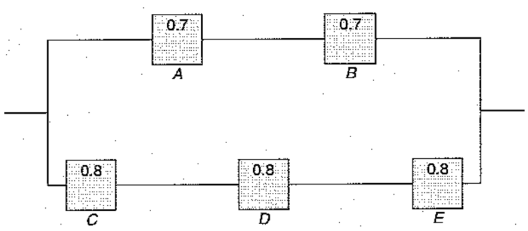
$$= P(A) \cdot [P(B) + P(C) - P(B) \cdot P(C)] \cdot P(D)$$

$$= (0.95) \cdot [(0.7) + (0.8) - (0.7) \cdot (0.8)] \cdot (0.9)$$

$$= (0.95) \cdot (0.94) \cdot (0.9) = 0.8037$$

2

Ex3. A circuit system is given in the figure. What is the probability that the system works? Assume the components fail independently.



$P(\text{system works}) = P((A\cap B)\cup(C\cap D\cap E))$

$= P(A\cap B) + P(C\cap D\cap E) - P((A\cap B)\cap(C\cap D\cap E))$

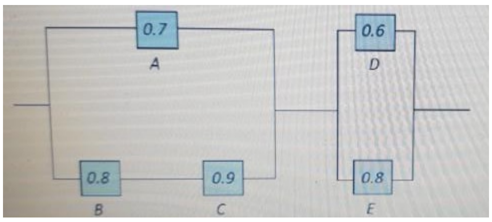
$= [P(A)\cdot P(B)] + [P(C)\cdot P(D)\cdot P(E)] - [P(A\cap B)\cdot P(C\cap D\cap E)]$

$= [(0.7)\cdot(0.7)] + [(0.8)\cdot(0.8)\cdot(0.8)] - [(0.7)^2\cdot(0.8)^3]$

$= 0.49 + 0.512 - 0.25088 = 0.75112$

3

Ex4. A circuit system is given in the figure. What is the probability that the system works? Assume the components fail independently.



$P(\text{system works}) = P[(A \underset{or}{\cup} (B \underset{and}{\cap} C)) \underset{and}{\cap} (D \underset{or}{\cup} E)]$

$= P[A\cup(B\cap C)]\cdot P(D\cup E)$

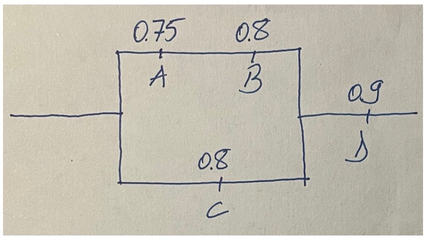
$= [P(A) + \underbrace{P(B\cap C)} - \underbrace{P(A\cap B\cap C)}]\cdot[P(D) + P(E) - \underbrace{P(D\cap E)}]$

$= [(0.7) + (0.8)\cdot(0.9) - (0.7)(0.8)(0.9)]\cdot[(0.6) + (0.8) - (0.6)\cdot(0.8)]$

$= (0.916)\cdot(0.92) = 0.84272$

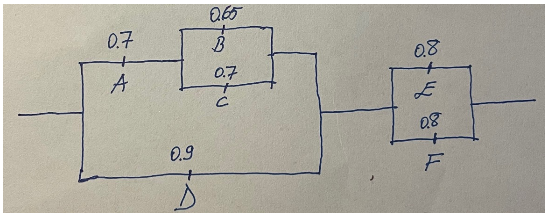
4

Ex5. A circuit system is given in the figure. What is the probability that the system works?
Assume the components fail independently.



$$P(\text{system works}) = P[((A\cap B)\cup C)\cap D]$$
$$= P[(A\cap B)\cup C]\cdot P(D) =$$
$$= [P(A\cap B) + P(C) - P(A\cap B\cap C)]\cdot P(D)$$
$$= [P(A)\cdot P(B) + P(C) - P(A)P(B)\cdot P(C)]\cdot P(D)$$
$$= [(0.75)(0.8) + (0.8) - (0.75)(0.8)(0.8)]\cdot(0.9) = 0.828$$

Ex6. A circuit system is given in the figure. What is the probability that the system works?
Assume the components fail independently.



$$P(\text{system works}) = P[[(A\cap(C\cup B))\cup D]\cap(E\cup F)]$$
$$= P[(A\cap(B\cup C))\cup D]\cdot P(E\cup F)$$
$$= [P(A\cap(B\cup C)) + P(D) - P(A\cap(B\cup C)\cap D)]\cdot P(E\cup F)$$
$$= [P(A)\cdot P(B\cup C) + P(D) - P(A)\cdot P(B\cup C)P(D)]\cdot P(E\cup F)$$
$$\text{if} \quad P(B\cup C) = P(B)+P(C)-P(B\cap C) = (0.65)+(0.7)-(0.65)(0.7) = 0.895$$
$$\text{if} \quad P(E\cup F) = P(E)+P(F)-P(E\cap F) = (0.8)+(0.8)-(0.8)\cdot(0.8) = 0.96$$
$$= [(0.7)\cdot(0.895) + (0.9) - (0.7)\cdot(0.895)\cdot(0.9)]\cdot(0.96)$$
$$= (0.96265)\cdot(0.96) = 0.924144$$