1/ a) What is the probability that a randomly selected student from this class is a female.

Solution:

Let F denote the event of a randomly selected student is female

$$P(F) = \frac{number\ of\ female\ students}{total\ numbr\ of\ students} = \frac{550}{1000} = 0.55$$

b) If a selected student form this class is female, then what is the probability that she is from Major A.

Solution:

Let F denote the event of a randomly selected student is female and A denote the event of a randomly selected student is from Major A.

$$P(A|F) = \frac{P(A \cap F)}{P(F)} = \frac{250/100}{0.55} = 0.45$$

MATH 2240 EXAMPLES (DISCRETE PROBABILITY DISTRIBUTIONS)

2/ 1. The number of e-mail messages received per hour has the following distribution:

x = number of messages	1	2	3	4	5
$f(x)$	0.15	0.15	0.30	0.20	0.20

a. Determine the mean and standard deviation of the number of messages received per hour.

Solution:

$$\mu = \sum_{i=1} x_i P(x_i) = 1 \times 0.15 + 2 \times 0.15 + 3 \times 0.30 + 4 \times 0.20 + 5 \times 0.20 = 3.15$$

$$\sum_{i=1} x_i^2 P(x_i) = 1^2 \times 0.15 + 2^2 \times 0.15 + 3^2 \times 0.30 + 4^2 \times 0.20 + 5^2 \times 0.20 = 11.65$$

$$\sigma^2 = \sum_{i=1}^{5} x^2 P(x_i) - \mu^2 = 11.65 - 3.15^2 = 1.73$$

$$\sigma = \sqrt{1.73} = 1.32$$

3°. $\mu = E(X) = 12$

$\mu = np = 12$

$Var(X) = \sigma^2 = 3$

$\sigma^2 = npq$

$npq = 3$

$12q = 3$

$q = \dfrac{3}{12} = \dfrac{1}{4}$

$p + q = 1$

$p = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\therefore p = \dfrac{3}{4} \ , \ q = \dfrac{1}{4}$

$npq = 3$

$n\left(\dfrac{3}{4}\right)\left(\dfrac{1}{4}\right) = 3$

$n = \dfrac{3 \times 16}{3} = 16$

4°. $\lambda = 2$ per week , $\lambda_1 = 2 * \lambda = 2 \times 2 = 4$ per fortnight

$$P(X = x) = \dfrac{e^{-\lambda} (\lambda)^x}{x!} \ , \ x = 0, 1, \dots$$

$P(X < 3) = P(0) + P(1) + P(2)$

$= \dfrac{e^{-4}(4)^0}{0!} + \dfrac{e^{-4}(4)^1}{1!} + \dfrac{e^{-4}(4)^2}{2!}$

From the table , $= 0.0183 + 0.0733 + 0.1465 = 0.2381$

5°.   (A ∪ B) ∩ (C ∪ D)

P((A ∪ B) ∩ (C ∪ D))

[P(A) + P(B) − P(A ∩ B)] ∩ [P(C) + P(D) − P(C ∩ D)]

[0.8 + 0.6 − 0.48] ∩ [(0.7) + (0.8) − (0.7)(0.8)]

(0.92)(0.94)

0.8648

OR

[1 − P(Ā) P(B̄)] [1 − P(C̄) P(D̄)]

[1 − (0.2)(0.4)] [1 − (0.2)(0.2)]

(0.92)(0.94)

0.8648