Midterm - 2 Solutions

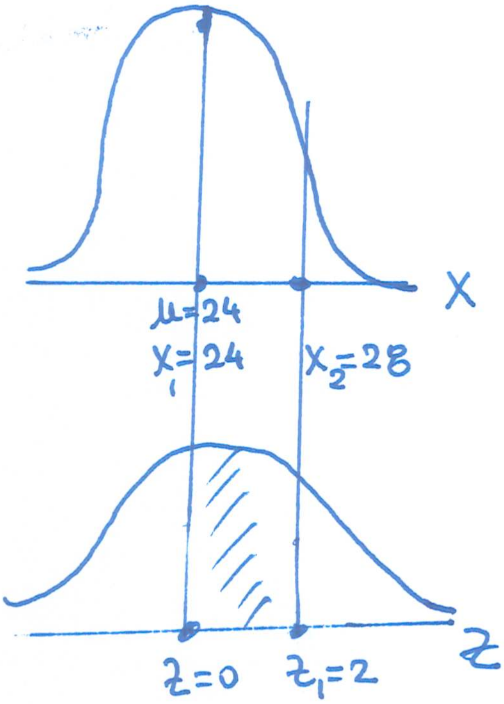
1°. $\mu = 24'$, $\sigma = 2'$ $X \sim N(24, 2)$

a) $P(24 < X < 28)$

$P\left(\frac{24-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{28-\mu}{\sigma}\right)$

$P\left(\frac{24-24}{2} < z < \frac{28-24}{2}\right)$

$P(0 < z < 2) = 0.4772$



b) $P(X > 28)$

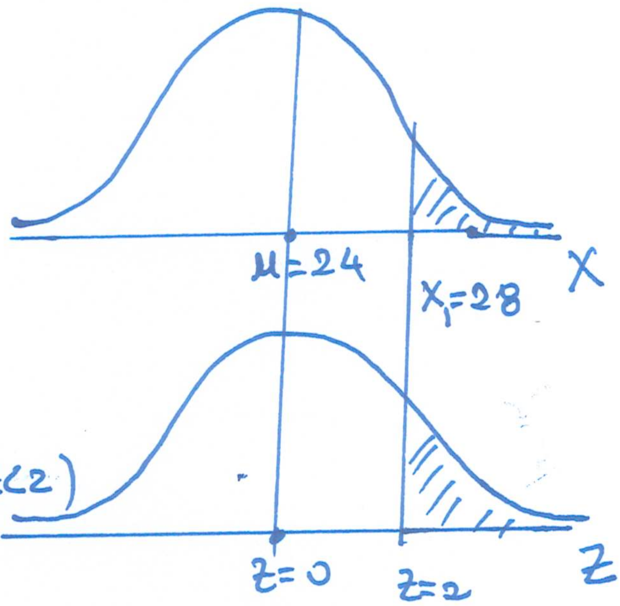
$P\left(\frac{X-\mu}{\sigma} > \frac{28-\mu}{\sigma}\right)$

$P\left(z > \frac{28-24}{2}\right)$

$P(z > 2) = 0.5 - P(0 < z < 2)$

$= 0.5 - 0.4772$

$= 0.0228$



2°. $n = 50$, $\bar{x} = 47000$, $S = 1100$

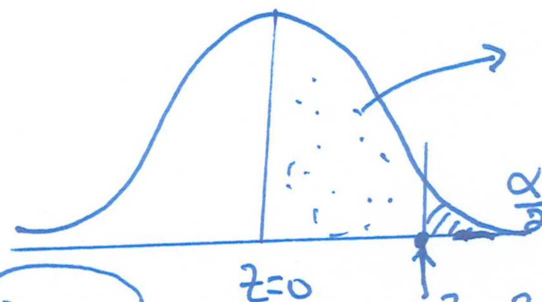
σ is not known

$n = 50 > 30$ so we use Z distribution

$$\bar{x} - Z_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right) < \mu < \bar{x} + Z_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right)$$

99 % C.I

$1 - \alpha = 0.99$, $\alpha = 0.01$, $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$



$\rightarrow 0.5 - 0.005 = 0.4950$

$\frac{\alpha}{2} = 0.005$

$Z = 0$

$P(0 < Z < Z_{\alpha/2}) = 0.4950$

$Z_{\alpha/2} = 2.58$

Z	0.07		0.08
2.5	0.4949	0.4950	0.4951

$Z = 2.57$, $Z = 2.58$, $Z = \frac{(2.57 + 2.58)}{2} = 2.575$

If you take $Z = 2.58$

$$47000 - (2.58)\left(\frac{1100}{\sqrt{50}}\right) < \mu < 47000 + (2.58)\left(\frac{1100}{\sqrt{50}}\right.$$

$$47000 - (2.58)\left(\frac{1100}{7.07}\right) < \mu < 47000 + (2.58)\left(\frac{1100}{7.07}\right)$$

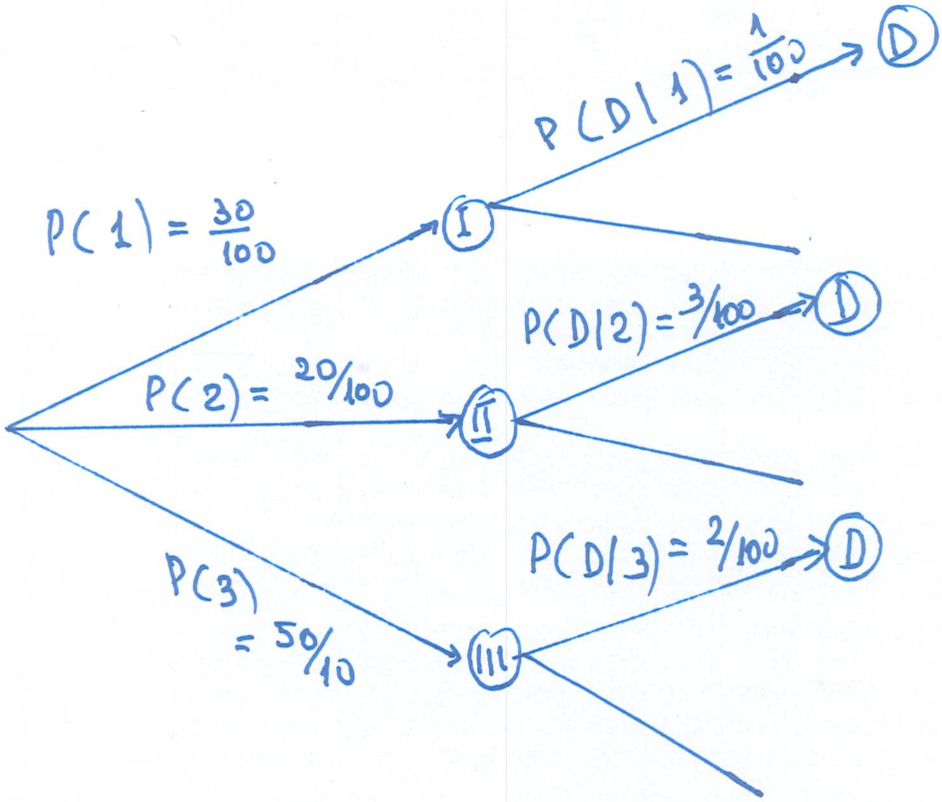
$$47000 - (2.58)(155.58698) < \mu < 47000 + (2.58)(\overset{155.58698}{\cancel{58.69}})$$

$$47000 - 401.414 < \mu < 47000 + 401.414$$

$$46598.596 < \mu < 47401.414$$

$$\boxed{46598.6 < \mu < 47401.4}$$

3°.



$$P(D) = \left(\frac{30}{100}\right)\left(\frac{1}{100}\right) + \left(\frac{20}{100}\right)\left(\frac{3}{100}\right) + \left(\frac{50}{100}\right)\left(\frac{2}{100}\right)$$

$$= \frac{30 + 60 + 100}{10000} = \frac{190}{10000}$$

$$P(1 \mid D) = \frac{\left(\frac{30}{100}\right)\left(\frac{1}{100}\right)}{\left(190/10000\right)} = \frac{30}{190} = \frac{3}{19}$$

$$P(2 \mid D) = \frac{\left(\frac{20}{100}\right)\left(\frac{3}{100}\right)}{\left(190/10000\right)} = \frac{60}{190} = \frac{6}{19}$$

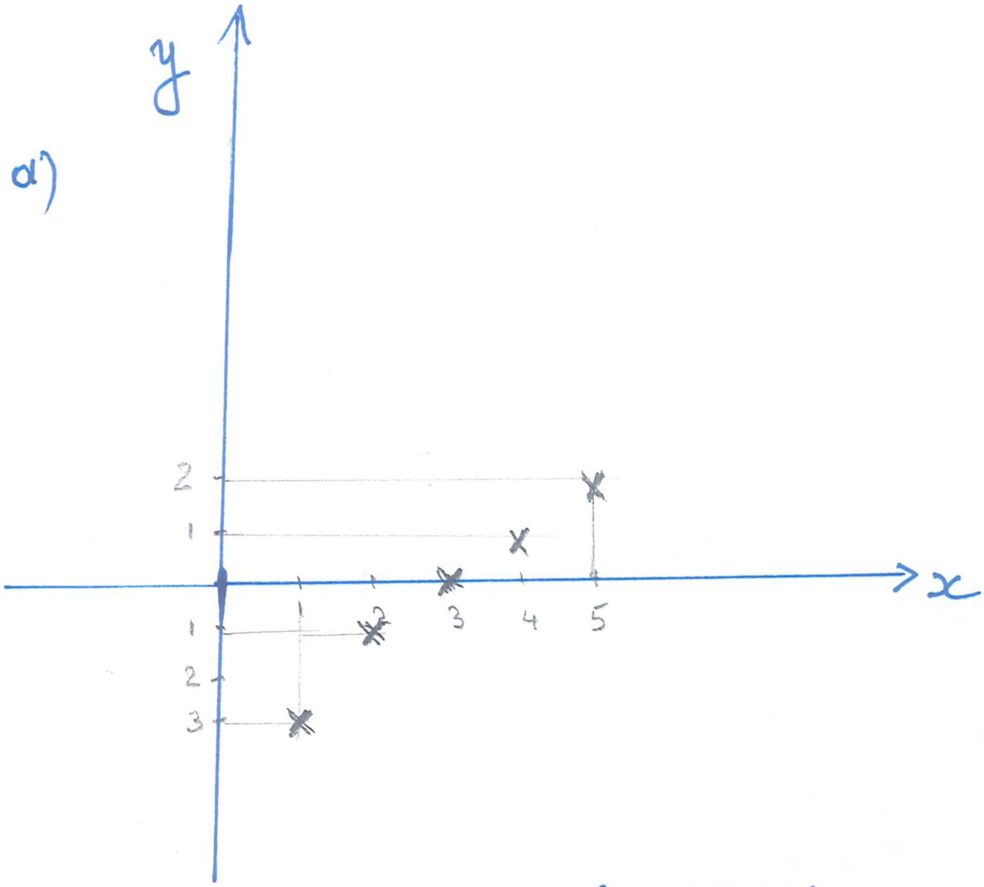
$$P(3 \mid D) = \frac{\left(\frac{50}{100}\right)\left(\frac{2}{100}\right)}{\left(\frac{190}{10000}\right)} = \frac{100}{190} = \frac{10}{19}$$

Plan 3 is most likely used and thus responsible.

4°.

x	y	xy	x^2	y^2
1	-3	-3	1	9
2	-1	-2	4	1
3	0	0	9	0
4	1	4	16	1
5	2	10	25	4
$\Sigma x = 15$	$\Sigma y = -1$	$\Sigma xy = 9$	$\Sigma x^2 = 55$	$\Sigma y^2 = 15$

a)



b)

$$b_1 = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sum x^2 - (\sum x)^2/n} = \frac{9 - \frac{(15)(-1)}{5}}{55 - (15)^2/5}$$

$$b_1 = \frac{9+3}{55-45} = \frac{12}{10} = 1.2$$

$$b_0 = \bar{y} - (b_1)\bar{x}$$

$$= \left(\frac{-1}{5}\right) - (1.2)\left(\frac{15}{5}\right) = \frac{-1}{5} - 3.6 = -0.2 - 3.6$$

$$b_0 = -3.8$$

$$y = b_0 + b_1 x$$

$\boxed{y = -3.6 - 1.2x}$ Regression Equation

Correlation coefficient

$$r = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right]\left[\Sigma y^2 - (\Sigma y)^2/n\right]}}$$

$$r = \frac{(12)}{\sqrt{(10)\left[15 - \frac{(-1)^2}{5}\right]}} = \frac{12}{\sqrt{(10)\left(15 - \frac{1}{5}\right)}}$$

$$= \frac{12}{\sqrt{(10)\left(\frac{75-1}{5}\right)}} = \frac{12}{\sqrt{(10)\left(\frac{74}{5}\right)}}$$

$$= \frac{12}{\sqrt{2(74)}} = \frac{12}{\sqrt{148}} = \frac{12}{12.166} = 0.986$$