Random Variables

Random Variable

- Represents a numerical value associated with each outcome of a probability distribution.
- Denoted by **x**
- Examples
  - x = Number of sales calls a salesperson makes in one day.
  - x = Hours spent on sales calls in one day.

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Discrete Probability Distributions

- Lists each possible value the random variable can assume, together with its probability.
- Must satisfy the following conditions:
  - The probability of each value of the discrete random variable is between 0 and 1, inclusive.
    - $0 \leq P(x) \leq 1$
  - The sum of all the probabilities is 1.
    - $\Sigma P(x) = 1$
- Because probabilities represent relative frequencies, a discrete probability distribution can be graphed with a relative frequency histogram.

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During the summer months, a rental agency keeps track of the number of chain saws it rents each day during a period of 90 days. The number of saws rented per day is represented by the variable X. The results are shown in the table. Calculate the probability P(X) for each X and construct a probability distribution and graph for the data.

X	**Nb. of days**
0	45
1	30
2	15
Total	**90**

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Mean, Variance and Standard Deviation

Mean of a discrete probability distribution

$\mu = \Sigma\, xP(x)$

- Each value of x is multiplied by its corresponding probability and the products are added.

Variance of a discrete probability distribution

$$\sigma^2 = \Sigma(x-\mu)^2 P(x) \qquad \sigma^2 = \sum X^2 P(X) - \mu^2$$

Standard deviation of a discrete probability distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x-\mu)^2 P(x)}$$

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Five balls numbered 0, 2, 4, 6, and 8 are placed in a bag. After the balls are mixed, one is selected, its number is noted, and then it is replaced. If this experiment is repeated many times, find the mean, variance and standard variance of the numbers on the balls.

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The probability that 0, 1, 2, 3, or 4 people will be placed on hold when they call a radio talk show is shown in the probability distribution table. The radio has four phone lines. When all lines are full, a busy signal is heard. Find the mean and standard deviation.

X	0	1	2	3	4
P(X)	0.18	0.34	0.23	0.21	0.04

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Expected Value

Expected value of a discrete random variable

- Equal to the mean of the random variable.

$E(x) = \mu = \Sigma\, xP(x)$

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Ex1. A ski resort loses $70,000 per season when it does not snow very much (less than 75 inches) and makes $250,000 profit when it does snow a lot (at least 75 inches). The probability of its snowing at least 75 inches is 40%. Find the expectation for the profit.

Ex2. At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. You buy one ticket. Find the expected value and interpret its meaning.

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Binomial Experiments

1. The experiment is repeated for a fixed number of trials, where each trial is independent of other trials.
2. There are only two possible outcomes of interest for each trial. The outcomes can be classified as a success (S) or as a failure (F).
3. The probability of a success, P(S), is the same for each trial.
4. The random variable x counts the number of successful trials.

Notation for Binomial Experiments

Symbol	*Description*
n	The number of times a trial is repeated
p	The probability of success in a single trial
q	The probability of failure in a single trial ($q = 1 - p$)
x	The random variable represents a count of the number of successes in n trials: $x = 0, 1, 2, 3, \ldots, n$.

Binomial Probability Formula

- The probability of exactly x successes in n trials is

$$P(x) = {}_nC_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

- n = number of trials
- p = probability of success
- q = 1 – p probability of failure
- x = number of successes in n trials
- Note: number of failures is n – x

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Ex1. A survey found that one out of five Americans say he or she has visited a doctor in any given month. If 10 people are selected at random, find the probability that exactly 3 have visited a doctor last month.

Ex2. A survey from Teenage Research Unlimited, found that 30% of teenage consumers receive their money from part-time jobs. If 5 teenagers are selected at random, find the probability that at least 3 of them will have part-time jobs.

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Mean, Variance, and Standard Deviation of a Binomial Distribution

- **Mean: $\mu = np$**
- **Variance: $\sigma^2 = npq$**
- **Standard Deviation:** $\sigma = \sqrt{npq}$

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Ex1. A die is rolled 480 times. Find the mean, variance, and standard deviation of the number of 2 that will be rolled.

Ex2. In Pittsburgh, Pennsylvania, about 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days during the month of June.

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Multinomial Probability Formula

- If X consists of events E_1, E_2, … , E_k, which have corresponding probabilities p_1, p_2, … , p_k occurring, and X_1 is the number of times E_1 will occur, X_2 is the number of times E_2 will occur, etc., then the probability that X will occur is

$$P(X) = \frac{n!}{X_1!\,X_2!\ \dots X_k!} p_1^{x_1} p_2^{x_2} \dots p_k^{x_k}$$

- where $X_1 + X_2 + \dots + X_k = n$
- and $p_1 + p_2 + p_k = 1$

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Ex1. In a large city, 50% of the people choose a movie, 30% choose a dinner, and %20 choose shopping as a leisure activity. If a sample of 5 people is randomly selected, find the probability that 3 are planning to go to a movie, 1 to a dinner, and 1 to a shopping mall.

Ex2. A box contains 4 white balls, 3 red balls, and 3 blue balls. A ball is selected at random, and its color is written down. It is replaced each time. Find the probability that if five balls are selected, 2 are white, 2 are red, and 1 is blue.

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Poisson Distribution

- A discrete probability distribution
- Satisfies the following conditions
  1. The experiment consists of counting the number of times an event, x, occurs in a given interval. The interval can be an interval of time, area, or volume.
  2. The probability of the event occurring is the same for each interval.
  3. The number of occurrences in one interval is independent of the number of occurrences in other intervals.

- The probability of exactly x occurrences in an interval is

$$P(X) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where e ≈ 2.71818 and
λ is the mean number of occurrences

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Ex1. If there are 200 typographical errors randomly distributed in a 500-page manuscript, find the probability that a given page contains exactly three errors.

Ex2. The mean number of accidents per month at a certain intersection is 3. What is the probability that in any given month four accidents will occur at this intersection?

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Ex3. If approximately 2% of the people in a room of 200 are left-handed, find the probability that exactly five people there are left-handed.

Geometric Distribution

- A discrete probability distribution.
- Satisfies the following conditions
  1. A trial is repeated until a success occurs.
  2. The repeated trials are independent of each other.
  3. The probability of success p is constant for each trial.
  4. The random variable x represents the number of the trial in which the first success occurs.

- The probability that the first success will occur on trial x is $P(x) = p(q)^{x-1}$
  - where $q = 1 - p$.

Ex1. Basketball player LeBron James makes a free throw shot about 74% of the time. Find the probability that the first free throw shot LeBron makes occurs on the third or fourth attempt.

Hypergeometric Distribution

- Given a population with only two types of objects (females and males, defective and non-defective, success or failure, etc.), such that there are *a* items of one kind and *b* items of another kind and *a+b* equals to the total population.
- The probability of selecting without replacement a sample size of *n* with *X* items of type *a* and *n-X* items of type *b* is

$$P(X) = \frac{{}^{a}C_{x}\,{}^{b}C_{n\,-\,x}}{{}^{a+b}C_{n}}$$

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Ex1. Ten people apply for a job as assistant manager of a restaurant. Five have completed college and five have not. If the manager selects 3 applicants at random, find the probability that all 3 are college graduates.

Ex2. A recent study found that 4 out of 9 houses were underinsured. If 5 houses are selected from the 9 houses, find the probability that exactly 2 are underinsured.

Ex3. A lot of 12 compressor tanks is checked to see whether there are any defective tanks. Three tanks are checked for leaks. If one or more of the three is defective, the lot is rejected. Find the probability that the lot will be rejected.

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Negative Binomial Distribution

- A discrete probability distribution.
- Satisfies the following conditions
    1. A trial is repeated until the specific number of success occurs.
    2. The repeated trials are independent of each other.
    3. The probability of success p is constant for each trial.
    4. The random variable x represents the trial number in which the r^{th} success occurs.
    5. The first (x–1) trials must result in (r–1) successes.
    6. The x^{th} trial must be a success
- The probability that the r^{th} success occurs on the x^{th} trial is $P(X=x) = {}^{x-1}C_{r-1}\, p^r\, (1-p)^{x-r}$

Ex1. A coin is tossed repeatedly until Head come up for the 6^{th} time. What is the probability that this happens on the 15^{th} toss?

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Summary

Discrete Probability Distributions			
	Formula	**Mean**	**Variance**
Binomial	$P(X=x) = {}^{n}C_x\, p^x\, (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = npq$
Multinomial	$P(X) = \frac{n!}{X_1!\,X_2!\,\ldots Xk!} {p_1}^{x_1} {p_2}^{x_2} \ldots {p_k}^{x_k}$		
Poisson	$P(X) = \frac{\lambda^x e^{-\lambda}}{x!}$	$\mu = \lambda$	$\sigma^2 = \lambda$
Geometric	$P(x) = p(q)^{x-1}$	$\mu = \frac{1}{p}$	$\sigma^2 = \frac{1-p}{p^2}$
Hypergeometric	$P(X) = \frac{{}^{a}C_x\, {}^{b}C_{n-x}}{{}^{a+b}C_n}$	$\mu = np$	$\sigma^2 = \left(\frac{N-n}{N-1}\right) np(1-p)$
Negative Binomial	$P(X=x) = {}^{x-1}C_{r-1}\, p^r\, (1-p)^{x-r}$	$\mu = \frac{r}{p}$	$\sigma^2 = \frac{r(1-p)}{p^2}$

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