Problem 16

Problem 1. $\forall L \in \texttt{NatList}, \mathrm{diff}(\mathrm{rev}(L), L) = nil$.

Proof. By structural induction on L.

(1) Base case

What to show: $\mathrm{diff}(\mathrm{rev}(nil), nil) = nil$.

$$\begin{aligned} \mathrm{diff}(\underline{\mathrm{rev}(nil)}, nil) &\longrightarrow \underline{\mathrm{diff}(nil, nil)} && \text{(by rev1)} \\ &\longrightarrow nil && \text{(by diff1)} \end{aligned}$$

(2) Induction case

What to show: $\mathrm{diff}(\mathrm{rev}(x \mid l), x \mid l) = nil$

Induction hypothesis: $\mathrm{diff}(\mathrm{rev}(l), l) = nil$

where $x \in \texttt{PNat}$ and $l \in \texttt{NatList}$. Note that x, l are fresh constants[1].

$$\begin{aligned} \mathrm{diff}(\underline{\mathrm{rev}(x \mid l)}, x \mid l) &\longrightarrow \underline{\mathrm{diff}(\mathrm{rev}(l) \mathbin{@} (x \mid l), x \mid l)} && \text{(by rev2)} \\ &\longrightarrow \mathrm{diff}(\mathrm{rev}(l), x \mid l) \mathbin{@} \underline{\mathrm{diff}(x \mid l, x \mid l)} \\ & && \text{(by Problem 9 - Lemma 1)} \\ &\longrightarrow \underline{\mathrm{diff}(\mathrm{rev}(l), x \mid l) \mathbin{@} nil} && \text{(by Problem 15)} \\ &\longrightarrow \underline{\mathrm{diff}(\mathrm{rev}(l), x \mid l)} && \text{(by Problem 4 - Lemma 2)} \\ &\longrightarrow \mathrm{drop}(\underline{\mathrm{diff}(\mathrm{rev}(l), l)}, x) && \text{(by Problem 14)} \\ &\longrightarrow \underline{\mathrm{drop}(nil, x)} && \text{(by IH)} \\ &\longrightarrow nil && \text{(by drop1)} \end{aligned}$$

□

[1] A fresh constant of a sort denotes an arbitrary value of the sort, and has never been used before.