

Problem 2

Problem 1. $\forall X \in \text{PNat}, \text{rev}(\text{mkl1}(X)) = \text{mkl2}(X)$.

Proof. By structural induction on X .

(1) Base case

What to show: $\text{rev}(\text{mkl1}(0)) = \text{mkl2}(0)$.

$$\begin{aligned}
 \text{rev}(\underline{\text{mkl1}(0)}) &\longrightarrow \underline{\text{rev}(\text{nil})} && \text{(by mkl1-1)} \\
 &\longrightarrow \text{nil} && \text{(by rev1)} \\
 \underline{\text{mkl2}(0)} &\longrightarrow \underline{\text{smkl2}(0, \text{nil})} && \text{(by mkl2)} \\
 &\longrightarrow \text{nil} && \text{(by smkl2-1)}
 \end{aligned}$$

(2) Induction case

What to show: $\text{rev}(\text{mkl1}(\text{s}(x))) = \text{mkl2}(\text{s}(x))$

Induction hypothesis: $\text{rev}(\text{mkl1}(x)) = \text{mkl2}(x)$

where $x \in \text{PNat}$.

$$\begin{aligned}
 \text{rev}(\underline{\text{mkl1}(\text{s}(x))}) &\longrightarrow \underline{\text{rev}(\text{s}(x) \mid \text{mkl1}(x))} && \text{(by mkl1-2)} \\
 &\longrightarrow \underline{\text{rev}(\text{mkl1}(x)) \text{ @ } (\text{s}(x) \mid \text{nil})} && \text{(by rev2)} \\
 &\longrightarrow \underline{\text{mkl2}(x) \text{ @ } (\text{s}(x) \mid \text{nil})} && \text{(by IH)} \\
 &\longrightarrow \underline{\text{smkl2}(x, \text{nil}) \text{ @ } (\text{s}(x) \mid \text{nil})} && \text{(by mkl2)} \\
 \underline{\text{mkl2}(\text{s}(x))} &\longrightarrow \underline{\text{smkl2}(\text{s}(x), \text{nil})} && \text{(by mkl2)} \\
 &\longrightarrow \underline{\text{smkl2}(x, \text{s}(x) \mid \text{nil})} && \text{(by smkl2-2)} \\
 &\longrightarrow \text{smkl2}(x, \text{nil}) \text{ @ } (\text{s}(x) \mid \text{nil}) && \text{(by Lemma 1)}
 \end{aligned}$$

□

Lemma 1. $\forall X \in \text{PNat}, L \in \text{NatList}, \text{smkl2}(X, L) = \text{smkl2}(X, \text{nil}) \text{ @ } L$.

Proof. By structural induction on X .

(1) Base case

What to show: $\text{smkl2}(0, l) = \text{smkl2}(0, \text{nil}) @ l$
where $l \in \text{NatList}$.

$$\begin{aligned} \text{smkl2}(0, l) &\longrightarrow l && \text{(by smkl2-1)} \\ \text{smkl2}(0, \text{nil}) @ l &\longrightarrow \text{nil} @ l && \text{(by smkl2-1)} \\ &\longrightarrow l && \text{(by @1)} \end{aligned}$$

(2) Induction case

What to show: $\text{smkl2}(s(x), l) = \text{smkl2}(s(x), \text{nil}) @ l$
Induction hypothesis: $\text{smkl2}(x, L) = \text{smkl2}(x, \text{nil}) @ L$
where $x \in \text{PNat}$ and $l, L \in \text{NatList}$.

$$\begin{aligned} \text{smkl2}(s(x), l) &\longrightarrow \text{smkl2}(x, s(x) \mid l) && \text{(by smkl2-2)} \\ &\longrightarrow \text{smkl2}(x, \text{nil}) @ (s(x) \mid l) && \text{(by IH)} \\ \text{smkl2}(s(x), \text{nil}) @ l &\longrightarrow \text{smkl2}(x, s(x) \mid \text{nil}) @ l && \text{(by smkl2-2)} \\ &\longrightarrow (\text{smkl2}(x, \text{nil}) @ (s(x) \mid \text{nil})) @ l && \text{(by IH)} \\ &\longrightarrow \text{smkl2}(x, \text{nil}) @ ((s(x) \mid \text{nil}) @ l) && \text{(by assoc@)} \\ &\longrightarrow \text{smkl2}(x, \text{nil}) @ (s(x) \mid (\text{nil} @ l)) && \text{(by @2)} \\ &\longrightarrow \text{smkl2}(x, \text{nil}) @ (s(x) \mid l) && \text{(by @1)} \end{aligned}$$

□