

## Problem 11

**Problem 1.**  $\forall X \in \mathbf{PNat}, \forall L \in \mathbf{NatList}, \text{has}(\text{drop}(L, X), X) = \text{false}$ .

*Proof.* By structural induction on  $L$ .

**(1) Base case**

What to show:  $\text{has}(\text{drop}(\text{nil}, x), x) = \text{false}$   
 where  $x \in \mathbf{PNat}$ .

$$\begin{aligned} \text{has}(\underline{\text{drop}(\text{nil}, x)}, x) &\longrightarrow \underline{\text{has}(\text{nil}, x)} && \text{(by drop1)} \\ &\longrightarrow \text{false} && \text{(by has1)} \end{aligned}$$

**(2) Induction case**

What to show:  $\text{has}(\text{drop}(y \mid l, x), x) = \text{false}$   
 Induction hypothesis:  $\text{has}(\text{drop}(l, x), x) = \text{false}$   
 where  $x, y \in \mathbf{PNat}$  and  $l \in \mathbf{NatList}$ .

We use case splitting for our proofs as follows:

**Case 1:**  $y = x$

$$\begin{aligned} \text{has}(\text{drop}(\underline{y} \mid l, x), x) &\longrightarrow \text{has}(\underline{\text{drop}(x \mid l, x)}, x) && \text{(by case splitting)} \\ &\longrightarrow \text{has}(\text{if } \underline{(x = x)} \text{ then } \text{drop}(l, x) \text{ else } (x \mid \text{drop}(l, x)) \text{ fi}, x) && \text{(by drop2)} \\ &\longrightarrow \text{has}(\underline{\text{if } \text{true} \text{ then } \text{drop}(l, x) \text{ else } (x \mid \text{drop}(l, x)) \text{ fi}}, x) && \text{(by equality)} \\ &\longrightarrow \underline{\text{has}(\text{drop}(l, x), x)} && \text{(by if1)} \\ &\longrightarrow \text{false} && \text{(by IH)} \end{aligned}$$

**Case 2:**  $(y = x) = false$

$$\begin{aligned} \text{has}(\text{drop}(y \mid l, x), x) &\longrightarrow \text{has}(\text{if } \underline{(y = x)} \text{ then } \text{drop}(l, x) \text{ else } (y \mid \text{drop}(l, x)) \text{ fi}, x) \\ &\hspace{15em} (\text{by drop2}) \\ &\longrightarrow \text{has}(\text{if } false \text{ then } \text{drop}(l, x) \text{ else } (y \mid \text{drop}(l, x)) \text{ fi}, x) \\ &\hspace{15em} (\text{by case splitting}) \\ &\longrightarrow \text{has}(y \mid \text{drop}(l, x), x) \hspace{10em} (\text{by if2}) \\ &\longrightarrow \underline{(y = x)} \text{ or } \text{has}(\text{drop}(l, x), x) \hspace{10em} (\text{by has2}) \\ &\longrightarrow \underline{false} \text{ or } \text{has}(\text{drop}(l, x), x) \\ &\hspace{15em} (\text{by case splitting}) \\ &\longrightarrow \text{has}(\text{drop}(l, x), x) \hspace{10em} (\text{by or}) \\ &\longrightarrow false \hspace{15em} (\text{by IH}) \end{aligned}$$

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