Problem 8

Problem 1. $\forall L1, L2 \in \texttt{NatList}, \mathrm{diff}(L1, L2) = \mathrm{diff}(L1, \mathrm{rev}(L2))$

Proof. By structural induction on $L1$.

(1) Base case

What to show: $\mathrm{diff}(nil, l2) = \mathrm{diff}(nil, \mathrm{rev}(l2))$.
where $l2 \in \texttt{NatList}$.

$$\underline{\mathrm{diff}(nil, l2)} \longrightarrow nil \qquad \text{(by diff1)}$$
$$\underline{\mathrm{diff}(nil, \mathrm{rev}(l2))} \longrightarrow nil \qquad \text{(by diff1)}$$

(2) Induction case

What to show: $\mathrm{diff}(x \mid l1, l2) = \mathrm{diff}(x \mid l1, \mathrm{rev}(l2))$
Induction hypothesis: $\mathrm{diff}(l1, l2) = \mathrm{diff}(l1, \mathrm{rev}(l2))$
where $x \in \texttt{PNat}$ and $l1, l2 \in \texttt{NatList}$.

$$\begin{aligned}
\underline{\mathrm{diff}(x \mid l1, l2)} &\longrightarrow \text{if } \mathrm{has}(l2, x) \text{ then } \underline{\mathrm{diff}(l1, l2)} \text{ else } (x \mid \underline{\mathrm{diff}(l1, l2)}) \text{ fi} \\
&\qquad \text{(by diff2)} \\
&\longrightarrow \text{if } \mathrm{has}(l2, x) \text{ then } \mathrm{diff}(l1, \mathrm{rev}(l2)) \text{ else } (x \mid \mathrm{diff}(l1, \mathrm{rev}(l2))) \text{ fi} \\
&\qquad \text{(by IH)} \\
\underline{\mathrm{diff}(nil, \mathrm{rev}(l2))} &\longrightarrow \text{if } \underline{\mathrm{has}(\mathrm{rev}(l2), x)} \text{ then } \mathrm{diff}(l1, \mathrm{rev}(l2)) \text{ else } (x \mid \mathrm{diff}(l1, \mathrm{rev}(l2))) \text{ fi} \\
&\qquad \text{(by diff2)} \\
&\longrightarrow \text{if } \mathrm{has}(l2, x) \text{ then } \mathrm{diff}(l1, \mathrm{rev}(l2)) \text{ else } (x \mid \mathrm{diff}(l1, \mathrm{rev}(l2))) \text{ fi} \\
&\qquad \text{(by Problem 6)}
\end{aligned}$$

$\square$