

## Problem 3

**Problem 1.**  $\forall X \in \mathbf{PNat}, \text{fact}(X) = \text{fold}*(\text{mkl2}(X))$ .

*Proof.* By direct proof.

What to show:  $\text{fact}(x) = \text{fold}*(\text{mkl2}(x))$

where  $x \in \mathbf{PNat}$ .

$$\begin{aligned}
 \underline{\text{fact}(x)} &\longrightarrow \underline{\text{fold}*(\text{mkl1}(x))} && \text{(by Lemma 1)} \\
 &\longrightarrow \underline{\text{fold}*(\text{rev}(\text{mkl1}(x)))} && \text{(by Problem 1)} \\
 &\longrightarrow \underline{\text{fold}*(\text{mkl2}(x))} && \text{(by Problem 2)}
 \end{aligned}$$

□

**Lemma 1.**  $\forall X \in \mathbf{PNat}, \text{fact}(X) = \text{fold}*(\text{mkl1}(X))$ .

*Proof.* By structural induction on  $X$ .

**(1) Base case**

What to show:  $\text{fact}(0) = \text{fold}*(\text{mkl1}(0))$ .

$$\begin{aligned}
 \underline{\text{fact}(0)} &\longrightarrow \underline{s(0)} && \text{(by fact1)} \\
 \text{fold}*(\underline{\text{mkl1}(0)}) &\longrightarrow \underline{\text{fold}*(\text{nil})} && \text{(by mkl1-1)} \\
 &\longrightarrow \underline{s(0)} && \text{(by fold*-1)}
 \end{aligned}$$

**(2) Induction case**

What to show:  $\text{fact}(s(x)) = \text{fold}*(\text{mkl1}(s(x)))$

Induction hypothesis:  $\text{fact}(x) = \text{fold}*(\text{mkl1}(x))$

where  $x \in \mathbf{PNat}$ .

$$\begin{aligned}
 \underline{\text{fact}(s(x))} &\longrightarrow \underline{s(x) * \text{fact}(x)} && \text{(by fact2)} \\
 &\longrightarrow \underline{(x * \underline{\text{fact}(x)}) + \underline{\text{fact}(x)}} && \text{(by *2)} \\
 &\longrightarrow \underline{(x * \text{fold}*(\text{mkl1}(x))) + \text{fold}*(\text{mkl1}(x))} && \text{(by IH)} \\
 \text{fold}*(\underline{\text{mkl1}(s(x))}) &\longrightarrow \underline{\text{fold}*(s(x) \mid \text{mkl1}(x))} && \text{(by mkl1-2)} \\
 &\longrightarrow \underline{s(x) * \text{fold}*(\text{mkl1}(x))} && \text{(by fold*-2)} \\
 &\longrightarrow \underline{(x * \text{fold}*(\text{mkl1}(x))) + \text{fold}*(\text{mkl1}(x))} && \text{(by *2)}
 \end{aligned}$$

□