Problem 8

Problem 1. $\forall L1, L2 \in \texttt{NatList}, \text{diff}(L1, L2) = \text{diff}(L1, \text{rev}(L2))$

Proof. By structural induction on $L1$.

(1) Base case

What to show: $\text{diff}(nil, l2) = \text{diff}(nil, \text{rev}(l2))$.
where $l2 \in \texttt{NatList}$. Note that $l2$ is a fresh constant[1].

$$\underline{\text{diff}(nil, l2)} \longrightarrow nil \qquad \text{(by diff1)}$$

$$\underline{\text{diff}(nil, \text{rev}(l2))} \longrightarrow nil \qquad \text{(by diff1)}$$

(2) Induction case

What to show: $\text{diff}(x \mid l1, l2) = \text{diff}(x \mid l1, \text{rev}(l2))$
Induction hypothesis: $\text{diff}(l1, l2) = \text{diff}(l1, \text{rev}(l2))$
where $x \in \texttt{PNat}$ and $l1, l2 \in \texttt{NatList}$. Note that $x, l1, l2$ are fresh constants.

$$\underline{\text{diff}(x \mid l1, l2)} \longrightarrow \text{if has}(l2, x) \text{ then } \underline{\text{diff}(l1, l2)} \text{ else } (x \mid \underline{\text{diff}(l1, l2)}) \text{ fi} \qquad \text{(by diff2)}$$

$$\longrightarrow \text{if has}(l2, x) \text{ then diff}(l1, \text{rev}(l2)) \text{ else } (x \mid \text{diff}(l1, \text{rev}(l2))) \text{ fi} \qquad \text{(by IH)}$$

$$\underline{\text{diff}(nil, \text{rev}(l2))} \longrightarrow \text{if } \underline{\text{has}(\text{rev}(l2), x)} \text{ then diff}(l1, \text{rev}(l2)) \text{ else } (x \mid \text{diff}(l1, \text{rev}(l2))) \text{ fi} \qquad \text{(by diff2)}$$

$$\longrightarrow \text{if has}(l2, x) \text{ then diff}(l1, \text{rev}(l2)) \text{ else } (x \mid \text{diff}(l1, \text{rev}(l2))) \text{ fi} \qquad \text{(by Problem 6)}$$

□

[1] A fresh constant of a sort denotes an arbitrary value of the sort, and has never been used before.