Problem 16

Problem 1. $\forall L \in \text{NatList}, \text{diff}(\text{rev}(L), L) = nil.$

Proof. By structural induction on L.

(1) Base case

What to show: diff(rev(nil), nil) = nil.

$$\operatorname{diff}(\operatorname{rev}(nil), nil) \longrightarrow \operatorname{diff}(nil, nil)$$
 (by rev1)

$$\longrightarrow nil$$
 (by diff1)

(2) Induction case

What to show: $\operatorname{diff}(\operatorname{rev}(x\mid l), x\mid l) = nil$ Induction hypothesis: $\operatorname{diff}(\operatorname{rev}(l), l) = nil$ where $x \in \operatorname{PNat}$ and $l \in \operatorname{NatList}$.

$$\begin{array}{c} \operatorname{diff}(\underline{\operatorname{rev}(x\mid l)},x\mid l) \longrightarrow \underline{\operatorname{diff}(\operatorname{rev}(l) @ (x\mid l),x\mid l)} & \text{(by rev2)} \\ \longrightarrow \overline{\operatorname{diff}(\operatorname{rev}(l),x\mid l) @ \underline{\operatorname{diff}(x\mid l,x\mid l)}} & \text{(by Problem 9 - Lemma 1)} \\ \longrightarrow \underline{\operatorname{diff}(\operatorname{rev}(l),x\mid l) @ nil} & \text{(by Problem 15)} \\ \longrightarrow \underline{\operatorname{diff}(\operatorname{rev}(l),x\mid l)} & \text{(by Problem 4 - Lemma 2)} \\ \longrightarrow \overline{\operatorname{drop}}(\underline{\operatorname{diff}(\operatorname{rev}(l),l)},x) & \text{(by Problem 14)} \\ \longrightarrow \underline{\operatorname{drop}}(nil,x) & \text{(by IH)} \\ \longrightarrow nil & \text{(by drop1)} \end{array}$$