

Problem 11

Problem 1. $\forall X \in \text{PNat}, L \in \text{NatList}, \text{has}(\text{drop}(L, X), X) = \text{false}$.

Proof. By structural induction on L .

(1) Base case

What to show: $\text{has}(\text{drop}(\text{nil}, x), x) = \text{false}$
 where $x \in \text{PNat}$.

$$\begin{aligned} \text{has}(\underline{\text{drop}(\text{nil}, x)}, x) &\longrightarrow \underline{\text{has}(\text{nil}, x)} && \text{(by drop1)} \\ &\longrightarrow \text{false} && \text{(by has1)} \end{aligned}$$

(2) Induction case

What to show: $\text{has}(\text{drop}(y \mid l, x), x) = \text{false}$
 Induction hypothesis: $\text{has}(\text{drop}(l, x), x) = \text{false}$
 where $x, y \in \text{PNat}$ and $l \in \text{NatList}$.

We use case splitting for our proofs as follows:

Case 1: $y = x$

$$\begin{aligned} \text{has}(\text{drop}(\underline{y} \mid l, x), x) &\longrightarrow \text{has}(\underline{\text{drop}(x \mid l, x)}, x) && \text{(by case splitting)} \\ &\longrightarrow \text{has}(\text{if } \underline{(x = x)} \text{ then } \text{drop}(l, x) \text{ else } (x \mid \text{drop}(l, x)) \text{ fi}, x) && \text{(by drop2)} \\ &\longrightarrow \text{has}(\underline{\text{if } \text{true} \text{ then } \text{drop}(l, x) \text{ else } (x \mid \text{drop}(l, x)) \text{ fi}}, x) && \text{(by equality)} \\ &\longrightarrow \underline{\text{has}(\text{drop}(l, x), x)} && \text{(by if1)} \\ &\longrightarrow \text{false} && \text{(by IH)} \end{aligned}$$

Case 2: $(y = x) = false$

$$\begin{aligned}
\text{has}(\text{drop}(y \mid l, x), x) &\longrightarrow \text{has}(\text{if } \underline{(y = x)} \text{ then } \text{drop}(l, x) \text{ else } (y \mid \text{drop}(l, x)) \text{ fi}, x) \\
&\hspace{15em} (\text{by drop2}) \\
&\longrightarrow \text{has}(\text{if } false \text{ then } \text{drop}(l, x) \text{ else } (y \mid \text{drop}(l, x)) \text{ fi}, x) \\
&\hspace{15em} (\text{by case splitting}) \\
&\longrightarrow \text{has}(y \mid \text{drop}(l, x), x) \hspace{10em} (\text{by if2}) \\
&\longrightarrow \underline{(y = x)} \text{ or } \text{has}(\text{drop}(l, x), x) \hspace{5em} (\text{by has2}) \\
&\longrightarrow \underline{false} \text{ or } \text{has}(\text{drop}(l, x), x) \\
&\hspace{15em} (\text{by case splitting}) \\
&\longrightarrow \text{has}(\text{drop}(l, x), x) \hspace{10em} (\text{by or}) \\
&\longrightarrow false \hspace{15em} (\text{by IH})
\end{aligned}$$

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