Portfolio designer

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In this project, I analyzed the price of the End of Day for eight US Stock Prices from 2013 to 2017. I developed a code to estimate the mean return of each stock and designed a portfolio which minimizes the expected risk calculating the expected return at the end of the year.

INTRODUCTION

Modern portfolio theory proposed by Harry Markowitz awared with the Nobel Prize in Economics in 1990. This method looks for obtain the best way to distribute the investments in a market with different possible inversions with different volatility each. The main problem is that some investments although showing a good average return, they could be also a bad investment while it shows a high volatility (stochastic fluctuations) since the expected returns can vary suddenly.

Through this method, portfolios giving a fixed mean return, an investor chooses the portfolio with smallest variance of the return. The model is subjective both in its choice of optimality criterion but also in its dependence on the investors beliefs about he means of the returns of the various available assets as well as the covariances between those returns.

I. THEORY

Consider a model evolving for one time period from time 0 to time 1. We first assume that there are s assets, $i = 1, \cdots, s$ for which the prices at time 0 are given by the vector $\mathbf{S_0} = (S_{1,0}, \cdots, S_{s,0})^\dagger$ which is just a constant vector in $\mathbb{R}^s$. The prices at time 1 are given by the random vector $\mathbf{S_1} = (S_{1,1}, \cdots, S_{s,01})^\dagger$ with the value of $\mathbf{S_1}$ not observed until time 1. An investor constructs a portfolio at time 0 by choosing a vector $\mathbf{x} = (x_1, \cdots x_s)^\dagger$ where xi is the proportion of his time-0 wealth that he invests in asset i, so that $\sum_{i=1}^{s} x_i = 1$. We allow the possibility that $x_i < 0$ which is known as being short in asset i, or having a short position in asset i; this involves borrowing an amount $|x_i|$ of asset i at time 0 which must be repaid at time 1. By contrast, holding a positive amount of an asset is referred to as being long in the asset, or having a long position in the asset.

Let $\mathbf{R} = (R_1, \cdots, R_s)^\dagger$ be the random vector representing the rates of return on the assets, so that $R_i = \frac{S_{i,1}}{S_{i,0}}$. When w represents the initial wealth of an investor who forms the portfolio determined by $\mathbf{x}$ at time 0, then his wealth at time 1 is the random variable

$$W = \left(\sum_{i=1}^{s} x_i R_i \right) w \tag{1}$$

The rate of return on his portfolio $\mathbf{x}$ is given by $W/w = \mathbf{x}^\dagger \mathbf{R}$.

We assume that $\mathbf{R}$ is a random vector with mean vector $r = \mathbb{E}(\mathbf{R})$, where $\mathbf{r} = (r_1, \cdots, r_s)$ with $\mathbb{E}(R_i) = r_i$, and covariance matrix:

$$V = \mathrm{Cov}(R) = \mathbb{E}(\mathbf{R} - \mathbf{r})(\mathbf{R} - \mathbf{r})^\dagger \tag{2}$$

The approach that we will adopt is to assume that for some fixed mean rate of return $\rho = \mathbb{E}(x)$ the investor seeks to minimize the variance of the return over portfolios $\mathbf{x}$. The variance is given by

$$\sigma^2 = \mathrm{Var}(x^\dagger R) = \mathbb{E}[x^\dagger (\mathbf{R} - \mathbf{r})(\mathbf{R} - \mathbf{r})^\dagger x] = xVx^\dagger \tag{3}$$

The problem for the investor then reduces to solving the quadratic programming problem: minimize

$$\frac{1}{2} xVx^\dagger \tag{4}$$

subject to:

$$\begin{aligned} x^\dagger e &= 1 \\ x^\dagger r &= \rho \end{aligned} \tag{5}$$

with $e = (1, \cdots, 1)^\dagger$. The solution. x_g which minimizes the variance is given by:

$$x_g = \frac{1}{\alpha} V^{-1} e \tag{6}$$

with $\alpha = e^\dagger V^{-1} e$

II. ANALYZING STOCK PRICES

Consider an investor trying to design their portfolio based on the behavior of nine assets:

- Home Depot
- JP Morgan
- Disney
- Golman Sachs

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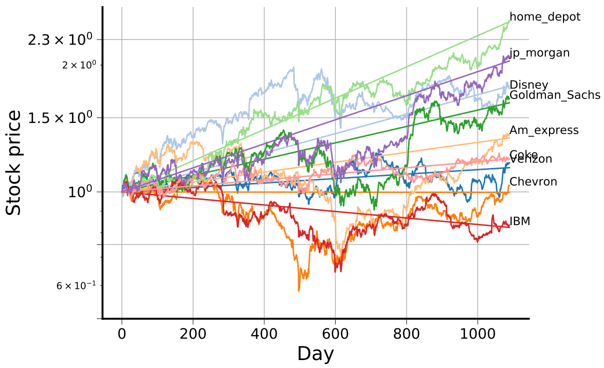

FIG. 1. Fluctuations on stock price for nine companies over the period (2013-2014). These data were fitted to the mean variation (straight lines).

- American Express
- Coca Cola
- Verizon
- Chevron IBM

I obtained and analyzed the data available in QUANDL about the end of the day stock prices of these companies. How these prices change along the time relative to the initial price (set equal to 1 at time $t = 0$) are shown in FIG 1.

Mean growth rate during one day μ_i was calculed taking five points in the data. The current stock price and four prices: two for the two later days and two future days. These five points were fitted to an exponential function $s_j(t) \approx s_j(0)\exp(\mu_j t)$. Taking these μ_j for all the days except in the extremes, the daily increments are estimated. At a given time (j), we have a vector of return rates μ_i, $i \in \{1,\ldots,s\}$ related to R_i in (1) by:

$$\mathbf{R} = (R_1,\ldots,R_s) = (\exp(\mu_i),\ldots,\exp(\mu_s)) \tag{7}$$

if we assume that each μ_i is a random variable with mean $\overline{\mu}_i = \mathrm{E}(\mu_j)$ with j the time instant. The fluctuations over $\overline{\mu}_i$ are estimated from the fluctuations along the trend line ignoring the time correlation of the signal. As seen in Table 1, the intensity of these fluctuations can be measured by the coefficient of variation C_i

$$C_i = \frac{\sigma_i}{\overline{\mu}_i} \tag{8}$$

where σ_i is the variation of the daily returns for the asset i over the mean value $\exp\overline{\mu_i}$:

$$\sigma_i = \sqrt{\mathrm{var}(\exp\mu_i)} \tag{9}$$

Finally, I obtained the best portfolio x_g minimizing the variance of the returns over all the possible portfolios applying the formula (6). The recommended portfolio x_i to each asset i and the observed properties of these assets are shown in Table 1.

TABLE I. Observed properties of the studied assets

Asset	$\overline{\mu_i}$	$\sigma_i/\overline{\mu_i}$	x_i
Verizon	4.0 % a year	0.38% daily	15.0%
Disney	21.0 % a year	0.39% daily	11.0%
Chevron	0.0 % a year	0.44% daily	-6.0%
American Express	10.0 % a year	0.44% daily	4.0%
Goldman$sachs$	17.0 % a year	0.45% daily	-3.0%
Home Depot	36.0 % a year	0.36% daily	10.0%
IBM	-6.0 % a year	0.39% daily	7.0%
Coca Cola	6.0 % a year	0.28% daily	48.0%
JP Morgan	27.0 % a year	0.43% daily	13.0%

With this portfolio, the expected return is 12% at the end of the year.

III. DISCUSSION

In this project, I designed a python-based script that studied the stock price evolution of nine companies along three years. The script estimates the mean return rate and the fluctuations over this trend for each asset.

The main approach done in the data analysis is that fluctuations in the stock return rate have no-correlation along the time. As can be seen in Figure 2, this correlation decays to zero approximately after seven days:

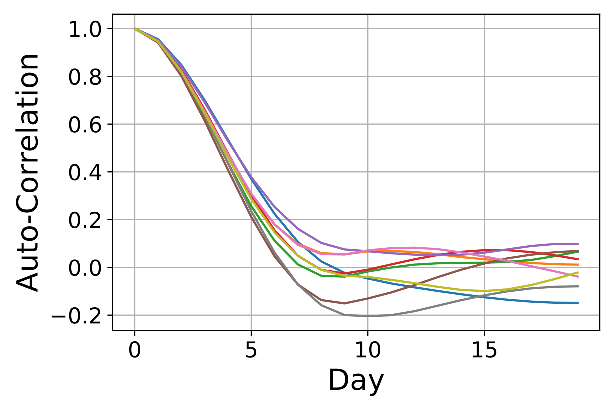

FIG. 2. Autocorrelation in the stock return with different periods.

Based on these statistical properties, I designed the portfolio minimizing the expected variability. As main result, depicted in Table 1, the main investment (48%) must be done in the Coca-Cola asset. This, mainly because this asset shows the lowest variability ($\approx 0.28\% daily$) and a positive return along the year. The companies showing relatively high volatility like Goldman Sachs (0.45% daily) are high punished advising negative investment on that asset.