Intrapersonal Utility Comparisons as Interpersonal Utility Comparisons: Welfare and Robustness in Behavioral Policy Problems with Normative Ambiguity*

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Abstract

We consider the optimal policy problem of a benevolent planner, who is uncertain about an individual's true preferences because of inconsistencies in revealed preferences across behavioral frames. We adapt theories of expected utility maximization and ambiguity aversion to characterize the planner's objective, which results in welfarist criteria similar to social welfare functions, with intrapersonal frames replacing interpersonal types. Under paternalistic risk aversion or ambiguity aversion, a policy is less desirable to the planner, holding all else fixed, when it leads to more disagreement about welfare from revealed preferences. We map some examples of behavioral models into our framework and describe how this notion of robustness plays out in applied settings.

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There's always a reality in what you are doing
Sometimes it's so hard to see which one is the true one
–Gene Clark

1 Introduction

A large body of research in economics considers interpersonal comparisons of utility. In the real world, Pareto improvements are rare, so in evaluating policy reforms we must confront the problem of how to trade off gains to winners and losses to losers. It is an understatement to say that the problem remains unresolved among economists, let alone philosophers, but a common practical approach is the use of social welfare functions like

$$w(x(\theta)) = \sum_{\theta \in \Theta} \psi(\theta) u(x(\theta), \theta). \tag{1}$$

Here $\theta \in \Theta$ captures interpersonal heterogeneity, $x(\theta)$ is an option allocated to type θ, which must be privately optimal under incentive compatibility. In taking this approach to data, we usually think of $u(x, \theta)$ as being identified by revealed preference from the choices of type θ, while $\psi(\theta)$ reflects a judgment on the part of the observer or social planner about the social value of increasing type θ's utility. This clarifies the role of the economist in optimal policy analysis, where interpersonal comparisons are involved: we can identify $u(.)$ by revealed preference, and we can describe how the results, combined with normative judgments about how to weigh utilities in $\psi(.)$ map to social welfare.

A need for normative judgments that cannot be resolved by revealed preference alone also arises in behavioral economics, but here the problem involves different potential views of welfare for a given individual [see e.g. Bernheim and Rangel, 2009]. In this paper, we develop a theoretical approach to the analysis of individual welfare in the presence of behavioral frictions that parallels the conventional approach to the analysis of social welfare with individual heterogeneity. Suppose we reinterpret $\psi(\theta)$ above as a judgment about what utility function the planner should use to evaluate individual welfare, and θ is a *frame*, which captures intrapersonal uncertainty over welfare due to inconsistencies in revealed preferences. We should modify incentive compatibility to require that the individual's choices maximize their utility in the specific frame in which the individual chooses [Rees-Jones and Taubinsky, 2018; Danz et al., 2022]. Our paper is examines the foundations and practical implications of the resulting welfare criterion and related alternative criteria.

The core assumptions of our framework are as follows: an individual has *normative preferences* that describe what they should choose in any situation. Holding the frame fixed, the individual's revealed preferences are rational, i.e. all inconsistencies are explained by framing effects. Normative preferences must coincide with revealed preferences in some frame, called the *normative frame*. With this core setup, we show that the welfare criterion of Bernheim [2009], which we label BR-dominance, is formally admissible and normative preferences must be constant over the frame (i.e. frames affect biases not true welfare, by definition).

When the normative frame is known to the planner, the benevolent planner's optimal policy problem takes a form studied in much prior literature in which *true preferences* are known to the planner or point-identified by empirical data [e.g. O'Donoghue and Rabin, 2006; Mullainathan et al., 2012; Allcott and Taubinsky, 2015; Allcott et al., 2019]. In contrast, we are interested optimal policy problems in which true preference are unknown, which we formulate in terms of uncertainty about which frame is normative. We provide three sets of conditions under which familiar forms of welfare criteria from interpersonal problems represent the benevolent planner's objective. In the first, we endow the planner's preferences over which option the individual consumes with rationality (completeness and transitivity), continuity, and respect for BR-dominance. Noting the similarity of BR-dominance and Pareto dominance, we adapt an argument from Kaplow and Shavell [2001]. So long as there is a good the individual always prefers in strictly larger amounts, the information in revealed preferences in each frame must be sufficient for the planner's objective, i.e. welfare must take what Kaplow and Shavell [2001] call a "welfarist" form.

This first result does not impose structure on how the planner trades off welfare across potentially normative frames, which requires a stronger notion of cardinal comparability of welfare across frames [Debreu, 1959; Sen, 1986; Wakker and Zank, 1999]. We propose two more structured approaches to such traadeoffs. In one approach, we assume the planner has a unique prior about which frame is normative, and they trade off welfare under an independence condition like that of Von Neumann and Morgenstern [1953], which generates yields a utilitarian form of the planner's objective like equation (1). In the other approach, the normative frame is *ambiguous* rather than *probabilistically uncertain*, i.e. the planner does not have a unique prior about which frame is normative but there is a set of priors they find plausible [Knight, 1921; Ellsberg, 1961]. Here, following Gilboa and Schmeidler [1989], we assume the planner's objective satisfies independence over certain outcomes but when faced with unresolvable uncertainty, the planner prefers to hedge. We find that with these conditions, the planner adopts a maxmin expected welfare criterion over plausible priors.

We then turn to some more practical considerations about how we apply our intrapersonal welfare criteria to policy problems. We first examine the usefulness of money-metric equivalent variation, which is often used to cardinalize welfare in applied settings in prior work. The key questions for us involve whether equivalent variation is sufficiently well-behaved and comparable across frames to be a useful input into our welfare objectives. We provide a set of conditions under which we can use equivalent variation to represent frame-dependent preferences, and stronger conditions under which equivalent variation from some suitably chosen counterfactual situation exhibits ordinal level comparability with the cardinal utility function in our planner's objective. This insight prompts a broader consideration of comparability with many parallels to the literature on social welfare functions. We do not claim to solve the comparability problem, but argue that fundamentally, deciding how to compare welfare across frames is the same problem as deciding how to compare welfare across interpersonal types [as debated in Harsanyi, 1955; Sen, 1976; Weymark, 1991].

Next, we examine the perturbation approach to optimal policy analysis under our normative objectives. Compared to prior approaches where true preferences are known, first-order welfare effects of marginal policy reforms are different in two main ways. The first is that we should replace the reduced-form determinants of welfare we find when normative preferences are known with their expected values under unknown normative preferences. These expected values are insufficient to fully characterize the first-order welfare effect of a marginal policy reform when one of two robustness concepts is at play. Under probabilistic uncertainty, the planner prefers to reduce the variance of realized welfare across normative frames, if they are risk averse over a given welfare metric like money-metric equivalent variation – we clarify below what we mean by paternalistic risk aversion over a given welfare metric. Second, under ambiguity, welfare effects are expressed as expectations not over a unique prior distribution but over the worst-case scenario, i.e. the distribution that yields minimal welfare over all plausible distributions.

We provide a range of examples illustrating how prior work maps into our framework. Much prior work can be thought of in terms of the question of whether some behavioral phenomenon reflects a bias or a strange preference [e.g. Goldin and Reck, 2022; Reck and Seibold, 2023; Lockwood et al., 2023]. We capture this idea with a two-frame example in our model. We propose a modification of intertemporal selves models with present focus [Laibson, 1997; Laibson et al., 1998] and derive conditions under which we can also think of welfare under present focus in terms of biases versus strange preferences. We also present an example in which it is ambiguous whether some feature of the decision-making environment is actually a frame.

Our paper is related to prior work in social choice theory, normative decision theory, and behavioral economics, especially behavioral welfare economics. We discuss this relationship throughout the exposition below. Relative to prior work in behavioral welfare economics [reviewed in Bernheim and Taubinsky, 2018], the novelty of our approach primarily lies in the development of robustness criteria for setting policy in the presence of uncertainty or ambiguity in true preferences. These criteria resolve the incompleteness of the welfare criteria proposed by Bernheim and Rangel [2009] using principled reasoning drawn from normative decision theory, which is important given that incompleteness makes criteria like BR-dominance uninformative for a wide range of behavioral policy problems [Benkert and Netzer, 2016]. We illustrate how these robustness criteria play out in three settings: defaults, reference dependence, and corrective taxation given an unknown internality. We show that the first two of these share some common elements in that setting the default or reference point at the *intrinsic optimum* – the choice the individual would make if they did not care about costs of opting out of the defualt or gain-loss utility relative to the reference point – is a robust optimum in both models. Setting an extreme default or reference point, meanwhile, is attractive when the behavioral friction is viewed as a bias, but we show that such extreme potential optima are *not robust*. For corrective taxation, we show that compared to the case where we ignore uncertainty and set the corrective tax rate at the (expected) marginal internality [Mullainathan et al., 2012; Allcott and Taubinsky, 2015], the robust optimal corrective tax is shaded toward the marginal internality according to the worst-case-scenario view of welfare.

Outline. In Section 2 we develop our normative criteria and their axiomatic foundations. Section 3 discusses cardinal comparability, including money-metric welfare metrics. Section 4 describes the first-order welfare effects of perturbations to policy. Section 5 introduces some examples from prior literature and maps them into our framework. We develop characterizations of optimal policy that apply our robustness concepts in Section 6.

2 General Model

2.1 Individual Choices and Welfare

A choice situation is fully characterized by (σ, θ^D), where $\sigma \in \Sigma$ captures variation in the option set and $\theta^D \in \Theta$ is a decision-making frame drawn from a finite set Θ. For a given σ, the set of options available is $X(\sigma) \in \mathcal{X}$. Options are denoted $x = (x_1, ..., x_N)$. We assume $\mathcal{X} \subseteq \mathbb{R}^N$ and that $\mathcal{X}$ is convex. The choice function is $x : \Sigma \times \Theta \to \mathcal{X}$ such that $x(\sigma, \theta^D) \in X(\sigma)$.

Assumption 1. ***Normative Preferences Exist.*** *The individual's normative choice – the choice the individual should make – maximizes a complete and transitive binary relation $\succeq_*$ defined on $\mathcal{X} \times \Theta$.*

Because it describes the preferences the individual should maximize in any situation, the normative preference is in principle defined over both options x and frames θ^D, but subsequent assumptions impose that the normative preference is constant over frames, so we will mainly think of $\succeq^*$ as a relation on $\mathcal{X}$. If we think in terms of a normative choice rule describing what the individual should choose, Assumption 1 requires that the normative choice rule satisfies the Generalized Axiom of Revealed Preference (GARP). For $\succeq^*$ and other preference relations introduced below, we use standard notation for e.g. indifference, strict preference, and reversed preference – $\sim^*$, $\succ^*$, and $\preceq^*$, respectively.

Assumption 2. ***Frame-Dependent Rational Preferences.*** *For all $\theta^D \in \Theta$, there is a complete and transitive preference relation $\succeq_{\theta^D}$ on $\mathcal{X}$, such that for any (σ, θ^D), for any $x \in X(\sigma)$, $x(\sigma, \theta^D) \succeq_{\theta^D} x$.*

The behavioral characterization of this assumption is that holding the frame θ^D fixed, the choice function $x(\sigma, \theta^D)$ satisfies GARP.

Intuitively, this assumption allows revealed preferences to be inconsistent across frames but consistent within frames. The main limitation of the assumption is that we exclude models featuring limited attention/consideration, where variation in the choice set rather than a frame generates inconsistencies.[1] We discuss the limitations of this assumption and how it relates to prior work below.

Now we introduce the normative assumption that enables revealed preference analysis, by relating preferences individuals should maximize to those they do maximize.

Assumption 3. ***Revealed Preference Coincidence.*** *There exists $\theta^* \in \Theta$ such that for any $x, x' \in \mathcal{X}$ and any $\theta^D \in \Theta$,*

$$x \succeq_{\theta^*} x' \iff (x, \theta^D) \succeq_* (x', \theta^D).$$

We label such a θ^* a *normative frame*. The normative behavioral characterization of this assumption is that in any environment (σ, θ^D), choices in the normative frame reveal what the individual should choose.

[1] In this sense, our choices with frames model is more similar to that of Salant and Rubinstein [2008], who label our Assumption 2 "Salient Consideration," than to the setup of Bernheim and Rangel [2009].

These initial assumptions have two straightforward but useful implications. As we assume the implication of Revealed Preference Coincidence (henceforth RP-Coincidence) holds for any decision-making frame (θ^D), an obvious requirement of RP-Coincidence is that the option the individual should choose in any given situation cannot depend nontrivially on the frame.

Lemma 1. ***Frame Exclusion.*** *Under Assumptions 1-3, for any $x, x' \in \mathcal{X}$ and any $\theta, \theta' \in \Theta$,*

$$(x, \theta) \succeq_* (x', \theta) \implies (x, \theta') \succeq_* (x', \theta').$$

Proof. $(x, \theta) \succeq_* (x', \theta) \implies x \succeq_{\theta^*} x'$ by Assumption 3 (" $\impliedby$ "). Then, $x \succeq_{\theta^*} x' \implies (x, \theta') \succeq_* (x', \theta')$ by Assumption 3 (" $\implies$ "). ■

We may therefore think of the normative preference $\succeq_*$ as a preference relation over options alone. Going forward, we suppress the frame θ when describing normative preferences. Assumptions 1-3 also admit the welfare criterion proposed by Bernheim and Rangel [2009]:

Lemma 2. ***BR-Dominance.*** *Under Assumptions 1-3, given any $x, x' \in \mathcal{X}$,*

$$\forall \theta \in \Theta,\ x \succeq_\theta x' \implies x \succeq_* x'. \tag{2}$$

Proof. This follows straightforwardly from Assumption 3. Suppose that $\forall \theta \in \Theta$, $x \succeq_\theta x'$. Then for the normative frame $\theta^* \in \Theta$ whose existence is assured by RP-Coincidence, it must be that $x \succeq_{\theta^*} x'$, and then RP-Coincidence implies $x \succeq_* x'$. ■

Next, we introduce a standard continuity assumption on frame-dependent preferences.

Assumption 4. ***Continuity.*** *For any $x_0 \in \mathcal{X}$ and any $\theta^D \in \Theta$, the sets $\{x \in \mathcal{X} : x \succeq_{\theta^D} x_0\}$ and $\{x \in \mathcal{X} : x \preceq_{\theta^D} x_0\}$ are closed.*

Under Assumptions 2 and 4, we obtain an ordinal utility function denoted $u(x, \theta^D)$, which represents the frame-dependent preferences $\succeq_{\theta^D}$.

Finally, we assume there is one good that the individual prefers to consume in strictly increasing amounts regardless of the frame. This allows us to employ an argument based on Kaplow and Shavell [2001] below.

Assumption 5. *There is some good x_n such that for every θ, $\succeq_\theta$ is strictly monotonic in x_n.*

2.2 Relationship to Prior Literature

Assumption 2 is a restriction on the model of Bernheim and Rangel [2009] requiring that all inconsistencies in choice are driven by framing effects. Relatedly, without Assumption 2, the dominance criterion in Lemma 2 is not equivalent to Bernheim and Rangel's proposed welfare criterion. This restriction imposes some useful discipline on welfare. To see why, consider that Assumption 2 rules out limited attention, where inconsistencies can arise across menus within a frame because the individual does not always consider all the available options. Masatlioglu et al. [2012] demonstrate that Bernheim and Rangel's welfare criterion is not generally admissible in models of limited attention: it could be the case that in all choice situations where two options x and x' are available, the individual might only pay attention to x and choose it even though they prefer x'. As a result, Assumption 2 fails. This in turn causes the more general version of RP-coincidence to fail, because there may be no situation in which the individual expresses a revealed a preference for x' over x. Ruling out limited attention by Assumption 2 frees us from this problem, and the other two assumptions clarify when BR-dominance is generally admissible.

Could we relax Assumption 2 and/or integrate limited attention into our framework? One route would be to follow Bernheim and Rangel [2009] and restrict inference to a "welfare-relevant" subset of situations, which only includes situations in which the individual understands/considers all their options. We could then relax Assumption 2 to hold within the welfare-relevant subset. But our objective is to analyze judgments within the model, so we do not wish to require judgments overturning revealed preferences ex ante. A more attractive solution would be to introduce an "attention cost" that rationalizes what would otherwise appear to be inconsistencies due to limited attention. Whether to respect the attention cost or to consider welfare under an alternative, perfect attention frame would then align with the type of question we take up in our "Biases Versus Strange Preferences" example [see also Goldin and Reck, 2022; Bronchetti et al., 2023]. However, to formalize such an approach to welfare analysis, the attention cost would need to depend on the menu $X(\sigma)$. We might capture this by simply allowing $u(x, \theta)$ to also depend flexibly on σ. However, we are not confident about how the resulting state-dependence in utility would interact with some other complications we introduce below, so we defer this to future work.

Setting Assumption 2 aside, we argue that Assumption 3, RP-Coincidence, is implicitly or explicitly assumed in all literature from behavioral welfare economics that seeks to use choices to inform welfare. For example, in Chetty et al. [2009], RP-coincidence is imposed by their assumption that when a tax is salient, the individual chooses optimally. Other papers in a similar modelling tradition require an assumption that rules out behavioral biases other than the bias that is the focus of the model, implying that RP-coincidence obtains in a frame in which the bias of interest is fully alleviated. More broadly, Lemma 2 shows that RP-coincidence formally justifies the welfare criterion proposed by Bernheim and Rangel [2009].

In reflecting on the model and some examples below, readers might naturally wonder: can we freely introduce normative frames into models whenever we want? What if it is impossible to directly observe behavior in such a frame? Here the answers must come by reasoning about welfare within a model. For example, the results in Masatlioglu et al. [2012] allow for the separate identification of preferences and attention in their model, which pins down choices in a perfect attention frame without assuming that we observe the individual making choices in this frame. In other words, given some assumptions in their model, choice data contain sufficient information to extrapolate to behavior in a potentially normative, perfect-attention frame. In more applied settings, we find similar extrapolations that rely on stronger assumptions to accommodate even more limited choice data [e.g. Goldin and Reck, 2020; Allcott et al., 2019]. However, there may be models of decision-making where it makes no sense to assume a normative decision-making frame exists, or where introducing a normative frame into the model requires ad hoc assumptions that amount to inventing a normative preference out of whole cloth. In such models, revealed-preference analysis seems ill-advised.

Together with Assumption 1, RP-Coincidence also formalizes the notion that the frame θ should be an aspect of the choice situation that matters for choices but not for welfare (Lemma 1). Bernheim and Rangel verbally resist making the assumption that a normative preference exists, but they impose the Frame Exclusion condition from Lemma 1 informally. Without a notion a notion of well-being rooted in the concept of a normative preference ($\succeq_*$), we have difficulty formalizing the premise that frames should be features of choice situations that "have no direct bearing on well-being, but instead impact biases" [Bernheim and Taubinsky, 2018].

Does assuming a normative preference exists contradict theories of choice commonly adopted in psychology? Psychological theories of choice suggest that preferences are constructed endogenously at the moment of choice [see e.g. Lichtenstein and Slovic, 2006]. This is one reason we might resist the assumption that normative preferences exist [see also Bernheim, 2016; Bernheim and Taubinsky, 2018]. Assuming θ^* is known to an observer or social planner, as in much prior work focused on understanding "true preferences," is clearly vulnerable to this critique. Assuming θ^* *exists but is unknown*, however, allows for a deeper analysis of normative ambiguity and judgments, which in turn helps us understand potential sources of disagreement about whether and how we might resolve normative ambiguity. Moreover, the possibility that the normative frame might be fundamentally unknowable rather than just uncertain in a probabilistic sense motivates our consideration of ambiguity below in the tradition of Knight [1921]. This perspective allows us to derive principled solutions to the "incomplete ordering" problem of Bernheim and Rangel's welfare criteria: in situations where BR-dominance does not apply, their framework provides no normative guidance. Among others, Benkert and Netzer [2016] argue that this type of incompleteness limits our ability to infer optimal policies from choices using BR-dominance alone, rather like Pareto efficiency for interpersonal problems.

2.3 Welfare and Optimal Policy Problems

Notation. We now introduce some notation that helps us think about policy variation and welfare. One of the components of $\sigma = (P, \tilde{\sigma})$ is a policy $P \in \mathcal{P} \subseteq \mathbb{R}^{N_P}$ affecting the individual's option set; the other components $\tilde{\sigma}$ are suppressed where they are obviously held fixed. The set of feasible policies $\mathcal{P}$ is compact and convex. The frame may also depend on policy, when this is dependence is relevant for the matter at hand we write $\theta^D(P)$, but otherwise we suppress this input.

2.3.1 Known Normative Frame

We begin, for instructive purposes, with the case where the normative frame θ^* is known, i.e. there is no normative uncertainty/ambiguity in the model. A benevolent planner's objective is to choose the policy $P \in \mathcal{P}$ that the individual would choose for themself according to $\succeq_*$. That is, we should characterize the policy that is optimal subject to the constraint that the individual will choose $x = x(P, \theta^D)$ – the Behavioral Incentive Compatibility (BIC) constraint [Rees-Jones and Taubinsky, 2018; Danz et al., 2022].[2]

RP-Coincidence implies that a benevolent planner should adopt as their the welfare function the utility function that represents $\succeq_{\theta^*}$. The planner's problem under known θ^* is therefore

$$\max_{P \in \mathcal{P}} u(x, \theta^*) \qquad (3)$$
$$\text{subject to } x = x(P, \theta^D(P)). \quad \text{(BIC)}$$

Equivalently, the planners' objective is to maximize the following indirect utility function:

$$W^*(P; \theta^*) \equiv u(x(P, \theta^D(P)), \theta^*).$$

Normative indirect utility W^* is parameterized by θ^* and represents normative preferences over choice situations.

Many policy problems in the literature on behavioral public economics take the form above, where θ^* is implicitly or explicitly assumed to be known [see Bernheim and Taubinsky, 2018, for a review]. From all this work, we have reduced-form characterizations of the welfare effects of policy variation for known θ^* [e.g. Mullainathan et al., 2012; Allcott and Taubinsky, 2015], and structural characterizations of optimal policies for a variety of more specific structural models [e.g. O'Donoghue and Rabin, 2006].

2.3.2 Unknown Normative Frame

Now we wish to characterize a benevolent planner's objective when the normative preference is unknown. Our policy problem is re-written as

$$\max_{P \in \mathcal{P}} w(x) \qquad (4)$$
$$\text{subject to } x = x(P, \theta^D(P)). \quad \text{(BIC)}$$

Unlike problem (3), here we do not impose that $w(x)$ coincides with utility under a particular normative frame. The indirect utility function under BIC is now denoted $W(P) \equiv w(x(P, \theta^D(P)))$.

What structure should we impose on the planner's objective? The planner's preferences over which option the individual consumes are denoted by a relation $\succeq_w$ on $\mathcal{X}$.[3] In writing (4), we are already imposing that $\succeq_w$ has a representation $w : \mathbb{R}^N \to \mathbb{R}$. We should ensure this representation exists. In classical policy problems and behavioral problems where the normative frame is unknown, we say that a planner is *benevolent* if

$$x \succeq_* x' \implies w(x) \geq w(x'). \qquad (5)$$

This is insufficient to fully characterize $w(x)$ when the normative frame is unknown, but property (5) does imply that a benevolent planner should respect BR-dominance. We therefore

[2]Incorporating an additional constraint like the government budget constraint is straightforward – one can think of this as imposing structure on the set of feasible policies $\mathcal{P}$.

[3]Note we are using the implication of Lemma 1 in positing that $w(x)/\succeq_w$ is independent of the frame.

begin with the following structure on $\succsim_w$:

Assumption 6. ***Basic Structure on Planner's Preferences***

Assumption 6.1. ***Rationality.*** *$\succeq_w$ is complete and transitive.*

Assumption 6.2. ***Continuity.*** *For any $x \in \mathcal{X}$, the sets $\{x' \in \mathcal{X} : x' \succeq_w x\}$ and $\{x' \in \mathcal{X} : x' \preceq_w x\}$ are closed.*

Assumption 6.3. ***Weak BR-dominance.*** *For any $x, x' \in \mathcal{X}$, if $x \succeq_\theta x'$ for every θ, then $x \succeq_w x'$.*

Assumptions 1 and 3 justify BR-dominance per Lemma 2; we now impose this directly via Assumption 6.3. How does the planner make policy decisions when BR-dominance does not apply and the planner does not have perfect knowledge of which choices are normative? Turning to the parallel with social welfare, adapting an argument from Kaplow and Shavell [2001], we find that our assumptions so far require that $w(x)$ takes what they call a "welfarist" form [see also Sher, 2023].[4]

Proposition 1. *Maintain Assumptions 2, 4 and 5. Assumption 6 holds if and only if for any representation of ordinal preferences $u(x, \theta^*)$, there is a function $\mathcal{W} : \mathbb{R}^{|\Theta|} \to \mathbb{R}$ such that the planner's preferences are represented by*

$$w(x) = \mathcal{W}\left(\{u(x, \theta^*)\}_{\theta^* \in \Theta}\right), \tag{6}$$

and $\mathcal{W}$ is weakly increasing in every argument.

Proof. This argument is due to Kaplow and Shavell [2001]. We observe that $\succ_w$ does not have the proposed representation if and only if there are two options x, x' such that for every θ, $x \sim_\theta x'$ but $w(x) \neq w(x')$. Toward a contradiction, suppose we find two such x, x'; without loss of generality $x \succ_w x'$. Starting from x', construct x'' by increasing the good x_n from Assumption 5 by a small amount $\delta > 0$. By continuity (6.2), if δ is sufficiently small we must have $x \succ_w x''$. But for every θ, $x'' \succ_\theta x' \sim_\theta x$, so BR-dominance requires $x'' \succsim_w x$. This establishes sufficiency of our assumptions for representation (6); necessity is easily verified. ∎

2.4 Probabilistic Uncertainty versus Ambiguity

How does the planner trade off welfare across different potentially normative frames? The challenge we must confront to go further, as with the theory of social welfare functions [see e.g. Sen, 1986], is comparability, in our case comparability of utility across normative frames.

Without some approach to comparability, we have difficulty thinking through policy problems in which there are tradeoffs between potential realizations of individual welfare depending on the normative frame. We have rich economic theory of optimality in the presence of tradeoffs that seems useful for analyzing such policy problems, but in order to use this theory we require a notion of cardinality [Debreu, 1959]. We introduce conditions that require a utility function

[4] There is some disagreement about whether the concept of a "welfarist" criterion requires additional conditions like the independence assumption introduced below; the way Kaplow and Shavell use the term it does not, but see also the comment by Fleurbaey et al. [2003]. This is a purely semantic issue for our purposes.

that is fully comparable (in level and unit) across normative frames below, and we return to the conceptual question of how we might approach comparability in intrapersonal problems in Section 3 and in our examples.

Given any practical resolution to the comparability question, we confront a related modelling choice. Our objective is to apply a normative theory of decision-making under uncertainty to uncertainty about the normative frame. We have many approaches from the rich literature on the topic. A unifying analysis of many diferent approaches by Wakker and Zank [1999] offers the insight that in any model we might use, the key structure will indeed be how the planner trades off realized welfare across potentially normative frames. In our view, the most important distinction between potential approaches is whether we can think of the planner as having well-formed beliefs about the probability that each frame is normative (or equivalently about the weights they ought to attach to each view of welfare). We therefore seek to develop two approaches: one in which the planner does have such beliefs and one in which they do not. For the first approach, we adapt classical expected utility theory from Von Neumann and Morgenstern [1953], and for the other we adapt the model of ambiguity aversion from Gilboa and Schmeidler [1989]. Approaches in which beliefs/weights emerge as a consequence of assumptions about more abstract primitives are available [e.g. Savage, 1954], but we use simpler assumptions that will be familiar to a broad audience.

An assumption, implicit or explicit, in canonical models of decision-making under uncertainty is that exactly one state is the true state. In our model where the state corresponds to the normative frame, this is explicitly assured under Assumptions 1 and 3, so we re-introduce these assumptions for the remaining results.

2.4.1 Probabilistic Uncertainty

Primitives. Here, we assume the planner weighs views of welfare according to a distribution over Θ, denoted $\psi \in \Delta(\Theta)$. We interpret $\psi(\theta)$ as the probability that the frame θ is normative according the planner's information. For example, in the case where θ^* is known, only one view of welfare receives positive weight so ψ is degenerate. We assume here that ψ is unique, which makes the next result a characterization of welfare under *probabilistic uncertainty*. In the next section, we relax this assumption and consider ambiguity.

Adapting classical Expected Utility Theory to this setting requires us to conceive of counterfactuals that describe situations in which the planner has different beliefs about the normative frame. We introduce the notion of an intrapersonal lottery to capture this. The primitive components of such a lottery are an option $x \in \mathcal{X}$, the state space Θ, and a distribution $\psi \in \Delta(\Theta)$. The outcomes of a lottery entail consuming a particular x in a particular state θ. We conceive of a lottery $L(x, \psi)$ in terms of a vector of probabilities $\left(\psi(\theta_1), ..., \psi(\theta_{|\Theta|})\right)$ and a vector of outcomes $\left((x, \theta_1), ..., (x, \theta_{|\Theta|})\right)$. Compound lotteries entail mixtures of probabilities: for $p \in [0, 1]$ and two distributions ψ_1, ψ_2 we describe these using the notation

$$pL_1(x) + (1-p)L_2(x) = L(x, p\psi_1 + (1-p)\psi_2),$$

where $L_n(x) = L(x, \psi_n)$.

We abuse notation slightly by denoting the planner's preferences over lotteries by $\succeq_w$. Now, the planner's preferences are defined not only over what option the individual consumes but also over the planner's beliefs: $\succeq_w$ is a binary relation on the set of lotteries $\mathcal{L}$. We strengthen Assumption 6 as follows:

Assumption 7. ***Expected Utility Assumptions Over Intrapersonal Lotteries.***

Assumption 7.1. ***Rationality.*** *$\succsim_w$ is complete and transitive on $\mathcal{L}$.*

Assumption 7.2. ***Continuity.*** *For any $L \in \mathcal{L}$, the sets $\{L' \in \mathcal{L} : L' \succsim_w L\}$ and $\{L' \in \mathcal{L} : L' \precsim_w L\}$ are closed.*

Assumption 7.3. ***Strong BR-Dominance.*** *For any ψ, x, if $x \succsim_\theta x'$ for every θ, then $L(x,\psi) \succsim_w L(x',\psi)$. If, additionally, there exists $\theta \in \Theta$ such that $x \succ_\theta x'$ and $\psi(\theta) > 0$, then $L(x,\psi) \succ_w L(x',\psi)$.*

Assumption 7.4. ***Independence.*** *For any x, any $L_1(x), L_2(x), L_3(x) \in \mathcal{L}$, and any $p \in [0,1]$*

$$L_1(x) \succsim_w L_2(x) \implies pL_1(x) + (1-p)L_3(x) \succsim_w pL_2(x) + (1-p)L_3(x). \tag{7}$$

Proposition 2. *Maintain Assumptions 1-5. Then Assumption 7 holds if and only if there is a function $u : \mathcal{X} \times \Theta \to \mathbb{R}$ such that $u(x,\theta)$ represents individual preferences $\succeq_\theta$ for every θ, and the planner's preferences $\succeq_w$ are represented by*

$$w(x;\psi) = \sum_{\theta^* \in \Theta} \psi(\theta^*) u(x,\theta). \tag{8}$$

Moreover, u is continuous and unique up to positive affine transformation.

Proof. Assumptions 7.1, 7.2, and 7.4 are the axioms of classical expected utility over the outcomes (x,θ). We therefore obtain a payoff function $u : \mathcal{X} \times \Theta \to \mathbb{R}$ such that the planner's preferences take the expected utility form $w(x,\psi) = \sum_{\theta^* \in \Theta} \psi(\theta^*) u(x,\theta)$. That $u(x,\theta)$ must be a representation of $\succeq_\theta$ follows from Proposition 1; the strong form of BR-dominance is required to rule out the degenerate case where u is constant over x. This establishes sufficiency of Assumption 7 for the desired representation of $\succeq_w$; necessity is easily verified. ∎

That some fully comparable utility function must exist under Assumption 7, especially the independence assumption 7.4, does not shed much light on what representation of frame-dependent preferences we ought to assume is fully comparable in any given setting or model. In thinking through this issue, we find useful a Corollary to Proposition 2, in which we consider an alternative representation.

Definition. We say that two utility functions $u(x,\theta)$ and $v(x,\theta)$ exhibit *ordinal level comparability* if for any (x,θ) and (x',θ'),

$$u(x,\theta) \geq u(x',\theta') \iff v(x,\theta) \geq v(x',\theta').$$

Corollary 2.1. *Maintain Assumptions 1-5 and 7 and consider a utility function $u(x,\theta)$ that gives the representation in Proposition 2.1. For any function $v(x,\theta)$ that exhibits ordinal level comparability with $u(x,\theta)$, there is a transformation $\omega : \mathbb{R} \to \mathbb{R}$ such that*

$$w(x;\psi) = \sum_{\theta^* \in \Theta} \psi(\theta^*)\omega(v(x,\theta)). \tag{9}$$

Moreover, ω is strictly increasing, continuous, and unique up to positive affine transformation.

The function ω converts representation $v(x,\theta)$ into the cardinal units the planner uses to conduct welfare comparisons across frames. Below, we introduce some structure under which money-metric equivalent variation provides a representation of individual preferences that exhibits ordinal level comparability with cardinal utility, in which case ω should account for any non-linearity of the individual's preference for money. To understand how such welfare metrics relate to the planner's objective in our model, it is instructive to impose some smoothness the transformation ω from Corollary 2.1.

Corollary 2.2. ***Variance Representation*** *Assume the function ω from representation (9) is twice differentiable. Then, the planner's objective can be approximated by a mean-variance representation as follows.*

$$w(x,\psi) \approx \omega\Big(E_\psi[v(x,\theta)]\Big) + \frac{\omega''\Big(E_\psi[v(x,\theta)]\Big)}{2} \cdot Var_\psi\Big[v(x,\theta)\Big] \tag{10}$$

where $E_\psi[v(x,\theta)] = \sum_{\theta\in\Theta}\psi(\theta)v(x,\theta)$ and $Var_\psi\Big[v(x,\theta)\Big] = \sum_{\theta\in\Theta}\psi(\theta)\Big[v(x,\theta) - E_\psi[v(x,\theta)]\Big]^2$.

Proof. For ease of notation shorten $v = v(x,\theta)$, a random variable with respect to θ for given x, and $\overline{v} = E_\psi[v(x,\theta)]$, a deterministic number for given x, ψ. Using a Taylor Expansion of ω around $\overline{v}$ we find

$$\begin{aligned}
&\omega(v) \approx \omega(\overline{v}) + \omega'(\overline{v}) \cdot (v - \overline{v}) + \omega''(\overline{v}) \cdot (v-\overline{v})^2 \\
\implies\ &\mathbb{E}_\psi\big[\omega(v)\big] \approx \underbrace{\omega(\overline{v})}_{\text{Fixed Number}} + \omega'(\overline{v}) \cdot \underbrace{\mathbb{E}_\psi[v-\overline{v}]}_{=0} + \frac{\omega''(\overline{v})}{2} \cdot \mathbb{E}_\psi[v-\overline{v}]^2
\end{aligned}$$

The result follows, as $w(x,\psi) = \mathbb{E}_\psi\big[\omega(v(x,\theta))\big]$. ■

We say that the planner's preferences exhibit *paternalistic risk aversion over v* if $\omega'' < 0$ and *paternalistic risk neutrality over v* if $\omega'' = 0$. Corollary 2.2 motivates our first notion of robustness, which arises under paternalistic risk aversion over a welfare metric v under probabilistic uncertainty. Under paternalistic risk neutrality over v, the planner's objective coincides with mean welfare according to the welfare metric v, i.e. $E_\psi[v(x,\theta)]$. But under paternalistic risk aversion over v, the variance of the welfare metric v across normative frames begins to matter (up to second-order approximation), and in particular welfare is decreasing in this variance. When, according to the welfare metric v, there is more disagreement in revealed preferences across frames about welfare under some policy P_0 compared to an alternative P_1, and mean welfare is similar between the two, paternalistic risk aversion over v suggests that P_0 is less desirable than P_1. Unlike the second notion of robustness introduced below, this notion of robustness is specific to the welfare metric we have in mind. Moreover, Proposition 2 tells us that under our assumptions, there will always be some measure of welfare $u(x, \theta)$ over which the planner's preferences exhibit paternalistic risk neutrality.

We say that the planner's preferences exhibit *scale invariance over v* if ω is a homogeneous function. In this case, the variance formulation has a simple and intuitive functional form. Scale invariance implies the following functional form for ω, for a parameter η:

$$\omega(v) = \begin{cases} \frac{v^{1-\eta}}{1-\eta}, & \eta \neq 1 \\ \log(v) & \eta = 1. \end{cases} \tag{11}$$

Paternalistic risk aversion over v further implies $\eta > 0$. Employing the Taylor Approximation from Corrollary 2.2 with the form in (11), we find

$$w \approx \frac{\overline{v}^{1-\eta}}{1-\eta} - \frac{\eta\overline{v}^{-\eta-1}}{2} \sum_{\theta^*} \psi(\theta^*)[v - \overline{v}]^2 \tag{12}$$

$$= \overline{v}^{1-\eta} \left\{ \frac{1}{1-\eta} - \frac{\eta}{2} \sum_{\theta^*} \psi(\theta^*) \left[\frac{v-\overline{v}}{\overline{v}}\right]^2 \right\}. \tag{13}$$

2.4.2 Ambiguity

A potential objection to the approach to behavioral welfare analysis implied by our results in the previous section is that the planner may not have well-formed beliefs over normative frames. Here we adapt the well-known model of ambiguity aversion due to Gilboa and Schmeidler [1989] – which in turn was motivated by the seminal contributions of Knight [1921] and Ellsberg [1961] – to propose a welfare criterion when the planner does not have well-formed beliefs (i.e. a unique prior) about normative frames.

Primitives. Rather than representing the planner's beliefs by a unique distribution $\psi \in \Delta(\Theta)$, suppose instead that the planner is endowed with a *closed and convex set of beliefs* about normative frames $\Psi^* \subseteq \Delta(\Theta)$, which we interpret as a set of possible distributions the planner finds plausible.

We say that a lottery $L(x, \psi)$ is *constant over u* if for the given x, $u(x, \theta) = u(x, \theta')$ for any θ, θ', i.e. if it generates a constant payoff for every normative frame. Abandoning Assumption 7.4, we introduce conditions on the planner's preferences drawn from Gilboa and Schmeidler [1989].

Assumption 8. ***Ambiguity Aversion Assumptions.***

Assumption 8.1. ***Rationality.*** *$\succsim_w$ is complete and transitive on $\mathcal{L}$.*

Assumption 8.2. ***Continuity.*** *For any $L \in \mathcal{L}$, the sets $\{L' \in \mathcal{L} : L' \succsim_w L\}$ and $\{L' \in \mathcal{L} : L' \precsim_w L\}$ are closed.*

Assumption 8.3. ***Certainty Independence*** *There is a representation $u(x, \theta)$ such that for any x, any* *pair $L_1(x), L_2(x) \in \mathcal{L}$, any lottery $L_3^c(x)$ that is constant over u, and any $p \in (0,1)$,*

$$L_1(x) \succsim_w L_2(x) \implies pL_1(x) + (1-p)L_3^c(x) \succsim_w pL_2(x) + (1-p)L_3^c(x).$$

Assumption 8.4. ***Weak BR-dominance*** *For any $x, x' \in \mathcal{X}$ and any $\psi \in \Delta(\Theta)$, if $x \succsim_\theta x'$ for every θ, then $L(x,\psi) \succsim_w L(x',\psi)$.*

Assumption 8.5. ***Uncertainty Aversion.*** *For any x, any pair $L_1(x), L_2(x)$, and $p \in (0,1)$,*

$$L_1(x) \sim_w L_2(x) \implies pL_1(x) + (1-p)L_2(x) \succsim_w L_1(x).$$

Assumption 8.6. ***Non-degeneracy*** *There exists $L, L' \in \mathcal{L}$ such that $L \succ_w L'$.*

The six conditions we present here correspond to the six axioms of Gilboa and Schmeidler [1989], except we replace their weak monotonicity assumption with weak BR-dominance and our definition of a constant lottery involves constant a constant payoff $u(x,\theta)$ across frames θ, which is the analogue in our model of a "constant act" in their framework. Relative to Assumption 7, Assumption 8.3 weakens Assumption 7.4 so that it only holds when we mix a given pair of lotteries with a constant lottery. Assumption 8.6 rules out the degenerate case where the planner is indifferent across all policies/options.

Crucially, Assumption 8.5 implies that when the planner is weighing welfare under ambiguity over two plausible distributions $\psi_1, \psi_2 \in \Psi^*$, the planner prefers to hedge in setting policy.

Proposition 3. ***MaxMin Welfare Under Ambiguity Aversion.*** *Maintain Assumptions 1-4. Assumption 8 holds if and only if there is a function $u : \mathcal{X} \times \Theta \to \mathbb{R}$ such that $u(x,\theta)$ represents $\succsim_\theta$ for every θ and the planner's preferences $\succsim_w$ are represented by*

$$w(x) = \min_{\psi \in \Psi^*} \left\{ \sum_{\theta^*} \psi(\theta^*) u(x, \theta^*)) \right\}. \tag{14}$$

Proof. Take the representation of individual preferences whose existence is implied by 8.3 and denote this $\tilde{u}$. Observe that BR-dominance implies Gilboa and Schmiedler's weak monotonicity condition over realizations of $\tilde{u}(x,\theta)$ for this representation. Theorem 1 of Gilboa and Schmeidler [1989] then implies there is a strictly increasing transformation $\omega(\tilde{u})$ such that the planner's preferences are represented by $w(x) = \min_{\psi \in \Psi^*} \{\sum_{\theta^*} \psi(\theta^*)\omega(\tilde{u}(x,\theta^*))\}$. The result follows, as $u \equiv \omega(\tilde{u})$ is also a representation of individual preferences by construction. ■

Forms of Ambiguity. Gilboa and Schmeidler [1989] defer the structure of the set Ψ^* to applications, apart from the requirement that Ψ^* is closed and convex [for a useful discussion, see Hansen and Sargent, 2001]. We envision two potential approaches. The first would be to define a subset of the set of frames $\Theta^* \subseteq \Theta$, and let $\Psi^* = \Delta(\Theta^*)$. This approach is very similar in spirit to the concept of a "welfare relevant domain" in Bernheim and Rangel [2009], and seems suitable when the planner cannot form a prior over Θ^* because they regard normative preferences as fundamentally ambiguous. The second approach is drawn from the literature on robust control [e.g. Hansen and Sargent, 2008]: the planner begins with a specific distribution ψ that represents their best guess about the true distribution, and accounts for ambiguity in a neighborhood of this distribution. In this case, Ψ^* should be a ball of distributions around ψ whose radius is determined by a tolerance parameter $\kappa \geq 0$: $\Psi^* = B(\psi, \kappa) \equiv \{\psi' \in \Delta(\Theta) \text{ s.t. } ||\psi' - \psi|| \leq \kappa\}$. This approach seems more applicable in the case where the planner uses a statistical model like the "counterfactual normative consumer" approach to identify normative welfare but nevertheless faces some model uncertainty.

These restrictions on Ψ^* intersect at a global robustness criterion, which also coincides with the objective in a limiting case of representation (9). Formally, we define the *global max-min* criterion as the one implied by equation (14) for $\Psi^* = \Delta(\Theta)$. The global max-min criterion is analogous to Rawlsian social welfare.

Corollary 3.1. ***Intersection of Various Objectives at Global Max-Min***

- *If $\Psi^* = B(\kappa, \psi)$, for any ψ, the planner's objective in* (14) *coincides with the global max-min criterion for $\kappa > 1$.*
- *If $\Psi^* = \Delta(\Theta^*)$, the planner's objective in* (14) *coincides with the global max-min criterion for $\Theta^* = \Theta$.*
- *Given a welfare metric v under scale invariance over v for the parameter η, the planner's objective under probabilistic uncertainty – Equation* (9) *with the functional form restriction in* (11) *– approaches the global max-min criterion as $\eta \to \infty$.*[5]

Proof. The first two claims are obvious from equation (14). The third has a well-known analogue in the nesting of Rawlsian welfare functions in the family of generalized utilitarian welfare functions taking the form in equation (11). ■

3 Approaches to Comparability

Here, we discuss comparisons of welfare across normative frames. As comparability is famously controversial in the theory of social welfare functions, we do not expect to propose an approach to comparability that will be universally acceptable to all readers, nor one that is applicable in all settings. The broader point of our paper is that the same difficulties we wrestle with in writing down a social welfare function also arise in behavioral policy problems. Instead, we examine one common approach and examine its usefulness for intrapersonal problems, and then we broaden our perspective and consider the parallel to debates about comparability involving interpersonal welfare.

3.1 Money Metric Welfare

Many practical characterizations of welfare in prior literature on behavioral welfare economics, and public economics more generally, employ money-metric equivalent variation. In this section, we explore the relationship between money-metric welfare measures and the welfarist criteria we developed above.

[5]The interpersonal analogue of this is a well-known result about Rawlsian social welfare functions; for a discussion of the relationship between this result and the notion of robustness for interpersonal problems, refer to Lockwood et al. [2021].

We begin with some notation and assumptions that discipline equivalent variation. We introduce a new feature of the choice environment denoted $Z \in \mathbb{R}$; choice environments are now described using $\sigma = (P, Z, \tilde{\sigma})$ and the decision frame θ^D.

Assumption 9. ***Ordinal Equivalent Variation Admissibility.***

- ***9.1. Strict Monotonicity in Money.*** *For any two Z, Z' any two θ^D, θ, and any $P, \tilde{\sigma}$,*

$$Z > Z' \iff x(P, Z, \tilde{\sigma}, \theta^D) \succ_\theta x(P, Z', \tilde{\sigma}, \theta^D).$$

- ***9.2. Continuity.*** *For any $(P, Z, \tilde{\sigma})$ and any two $\theta^D, \theta \in \Theta$, the sets $\{Z' : x(P, Z', \tilde{\sigma}, \theta^D) \succsim_\theta x(P, Z, \tilde{\sigma}, \theta^D)\}$ and $\{Z' : x(P, Z', \tilde{\sigma}, \theta^D) \precsim_\theta x(P, Z, \tilde{\sigma}, \theta^D)\}$ are closed.*

- ***9.3. Equalizability.*** *For any $x \in \mathcal{X}$, any $P, \theta^D, \tilde{\sigma}$ and any θ^*, the sets $\{Z' : x(P, Z', \tilde{\sigma}, \theta^D) \succ_{\theta^*} x\}$ and $\{Z' : x(P, Z', \tilde{\sigma}, \theta^D) \prec_{\theta^*} x\}$ are non-empty.*

Combined with RP-Coincidence, Assumption 9.1 ensures that giving the individual more Z always improves welfare in the normative frame. One reason Assumption 9.1 might fail is if, for example, in an "addicted" frame θ^D, the individual spends all their money on an addictive substance that is not a "good" but a "bad" from the perspective of the normative frame θ^*. Assumption 9.2 implies that welfare is continuous in Z in every frame. Assumption 9.3 ensures that all changes to welfare driven by variation in choices can be fully offset by money, which is obviously key for the existence of equivalent variation. All this is assumed regardless of which θ^* is normative.

Together, these assumptions discipline *equivalent variation* in any potentially normative frame θ, which is defined as $\zeta(x, \theta; P_0, Z_0 \theta_0^D)$ such that for a given *baseline* (P_0, Z_0, θ_0^D),

$$x \sim_\theta x(P_0, Z_0 + \zeta, \theta_0^D). \tag{15}$$

Lemma 3. ***Existence and uniqueness of EV****. Under Assumptions 2, 4 and 9, for any x, any θ^* and any (P_0, Z_0, θ_0^D), equivalent variation ζ exists and is unique. Moreover, $\zeta(x, \theta; P_0, Z_0, \theta_0^D)$ represents the individual's ordinal preferences $\succeq_\theta$.*

Proof. Suppose first that $x \succ_{\theta^*} x(P_0, Z_0, \theta_0^D)$, i.e. $u(x, \theta^*) > u(x(P_0, Z_0, \theta_0^D), \theta^*)$. Assumption 9.1 ensures that $u(x(P_0, Z_0 + \zeta, \theta_0^D), \theta^*)$ is a strictly increasing function in ζ; Assumption 9.2 ensures this function is continuous. Assumption 9.3 implies that there is some $\hat{\zeta}$ such that $u(x(P_0, Z_0 + \hat{\zeta}, \theta_0^D), \theta^*) > u(x, \theta^*)$. The result follows from the Intermediate Value Theorem – note that $u(x, \theta^*)$ is continuous by Assumption 4. The same logic applies where $x \prec_{\theta^*} x(P_0, Z_0, \theta_0^D)$, and in the case of indifference, $\zeta = 0$.

Having established existence and uniqueness, that $\zeta(x, \theta^*)$ represents $u(x, \theta^*)$ follows from

$$\begin{aligned} x \succeq_{\theta^*} x' &\iff x(P_0, Z_0 + \zeta(x; \theta^*), \theta_0^D) \succeq_{\theta^*} x(P_0, Z_0 + \zeta(x'; \theta^*), \theta_0^D) \text{ by definition \& transitivity} \\ &\iff \zeta(x; \theta^*) \geq \zeta(x'; \theta^*) \text{ by Assumption 9.1.} \end{aligned}$$

■

Lemma 3 implies that for a given baseline, $\zeta(x, \theta; P_0, Z_0, \theta_0^D)$ is a unique representation of revealed preferences under θ, $u(x, \theta)$.

Combining the representation result in Lemma 3 with the idea in Proposition 1, we find that introducing Assumption 9 yields the following:

Proposition 4. ***Planner's Preferences and Equivalent Variation.*** *Under Assumptions 2, 4, 6, and 9, for any baseline P_0, Z_0, θ_0^D, there is a function $\mathcal{W}_\zeta : \mathbb{R}^{|\Theta|} \to \mathbb{R}$ such that the planners preferences are represented by $w(x) = \mathcal{W}_\zeta \left(\{\zeta(x; \theta^*, P_0, Z_0, \theta_0^D)\}_{\theta^* \in \Theta} \right)$.*

Proof. The result follows the exact same logic as the proof of Proposition 1, but we use small amounts of Z to construct BR-dominant options rather than small amounts of the good described by Assumption 5 (which is no longer required). ∎

Proposition 4 suggests that provided that equivalent variation is well-behaved per Assumption 9, the model will allow us to use equivalent variation welfare metrics to describe the welfare effects of local policy perturbations. Practical applications of such approaches in interpersonal problems often leave the marginal value of a dollar under a given type/frame θ, $\frac{\partial \mathcal{W}_z}{\partial \zeta_\theta}$, unspecified [e.g. Hendren and Sprung-Keyser, 2020]. To go beyond local perturbations in employing equivalent variation, we must impose structure on this aspect of the planner's objective. With the structure we impose under probabilistic uncertainty (Assumption 7), we observe that the application of Corollary 2.1 to a representation rooted in equivalent variation requires ordinal level comparability between normative, cardinal utility $u(x, \theta)$ and $\zeta(x, \theta; P_0, Z_0, \theta_0^D)$. Our next result characterizes when ordinal level comparability obtains for some baseline situation. To economize on notation we express the option the individual chooses in a given baseline as $x_0 \equiv x(P_0, Z_0, \theta_0^D)$.

Assumption 10. ***Cardinal Equivalent Variation Admissibility.*** *Let $u(x, \theta)$ be a cardinal utility function from the representation in Proposition 2 or 3. There is a baseline situation (P_0, Z_0, θ_0^D) under which the following conditions hold:*

Assumption 10.1. ***Baseline Indifference.*** *For any θ, θ',*

$$u(x_0, \theta) = u(x_0, \theta').$$

Assumption 10.2. ***Comparable Value of Money At Baseline.*** *For any θ, θ' and any $\zeta, \zeta' \in \mathbb{R}$,*

$$u(x(P_0, Z_0 + \zeta, \theta_0^D), \theta) - u(x_0, \theta) \geq u(x(P_0, Z_0 + \zeta', \theta_0^D), \theta') - u(x_0, \theta') \iff \zeta \geq \zeta'.$$

Lemma 4. ***Ordinal Level Comparability of Equivalent Variation.*** *Maintain Assumptions 1-4 and 9. Assumption 10 holds if and only if for some baseline (P_0, Z_0, θ_0^D),*

$$u(x, \theta) \geq u(x', \theta') \iff \zeta(x, \theta; P_0, Z_0, \theta_0^D) \geq \zeta(x', \theta'; P_0, Z_0, \theta_0^D).$$

Proof. First we prove that level comparability implies Assumption 10. Assuming ordinal level comparability, Assumption 10.1 follows from the observation that $\zeta(x_0, \theta) = \zeta(x_0, \theta') = 0$

by construction, so then ordinal level comparability implies $u(x_0, \theta) = u(x_0, \theta')$. The second condition then follows from Assumption 9.1 (strict monotonicity over money).

Second we prove that Assumption 10 implies ordinal level comparability. Take any x, x', θ, θ'. Using Assumption 10.1 we have

$$u(x, \theta) \geq u(x', \theta') \iff u(x, \theta) - u(x_0, \theta) \geq u(x', \theta') - u(x_0, \theta').$$

Using the definition of equivalent variation, suppressing the baseline input, we have

$$\begin{aligned} & u(x, \theta) - u(x_0, \theta) \geq u(x', \theta') - u(x_0, \theta') \\ \iff & u(x(P_0, Z_0 + \zeta(x, \theta)), \theta) - u(x_0, \theta) \geq u(x(P_0, Z_0 + \zeta(x', \theta')), \theta') - u(x_0, \theta'). \end{aligned}$$

Now using Assumption 10.2, we find

$$\begin{aligned} & u(x(P_0, Z_0 + \zeta(x, \theta)), \theta) - u(x_0, \theta) \geq u(x(P_0, Z_0 + \zeta(x', \theta')), \theta') - u(x_0, \theta') \\ \iff & \zeta(x, \theta) \geq \zeta(x, \theta'). \end{aligned}$$

■

Putting the previous two Lemmas together with Corollary 2.1, we have a proof of the following proposition:

Proposition 5. *Under Assumptions 1-4, 7, 9 and 10, there is a function $\omega_\zeta : \mathbb{R} \to \mathbb{R}$ and a baseline situation (P_0, Z_0, θ_0^D) such that the planner's preferences are represented by*

$$w(x, \psi) = \sum_{\theta \in \Theta} \psi(\theta) \omega_\zeta(\zeta(x, \theta; P_0, Z_0, \theta_0^D)). \tag{16}$$

Under Assumptions 1-4, 8, 9 and 10, there is a function $\omega_\zeta : \mathbb{R} \to \mathbb{R}$ and a baseline situation (P_0, Z_0, θ_0^D) such that the planner's preferences are represented by

$$w(x, \psi) = \min_{\psi \in \Psi^*} \sum_{\theta \in \Theta} \psi(\theta) \omega_\zeta(\zeta(x, \theta; P_0, Z_0, \theta_0^D)). \tag{17}$$

Bernheim and Rangel [2009] derive bounds on equivalent variation from BR-dominance alone. Proposition 5 provides stronger conditions under which the lower bound identified by their approach is in fact a sufficient statistic for optimal policy: if $\Psi^* = \Delta(\Theta)$, maximizing (17) is equivalent to maximizing the minimum of equivalent variation for a suitably chosen baseline (satisfying Assumption 10).

Happiness-Metric Equivalent Variation? Assumptions 9 and 10 describes the key properties of the variable Z in our approach to welfare analysis. These assumptions do not require that Z be money. Adopting assumptions so that monetary transfers have the properties of good Z would be essential for estimating money-metric equivalent variation in an empirical application. But the variable Z could correspond to something in the real world other than money, such as a mental state.

3.2 Relationship to Interpersonal Comparability

The primary advantage of the money-metric utility concept is that we have tools identifying money-metric utility with empirical data (under some assumptions). Even if we are willing to make enough assumptions that ensure ζ is a well-defined and comparable measure of intrapersonal welfare, the estimation of money-metric willingnesses-to-pay in potential normative frames does not on its own resolve the difficulty with comparability: we also need to consider changes in money map to cardinal welfare, i.e. the function ω_ζ.

What structure might we impose the function in ω_ζ? If we endow the individual with Von Neumann-Morgenstern preferences over intrapersonal lotteries, then we could show that the individual's preferences over lotteries will also have an expected utility representation like (8), and if Assumptions 9 and 10 obtain, we could then formulate their Bernoulli utility function as a transformation of the monetary equivalent payoff in any situation, $\mu(\zeta)$. Then respecting BR-dominance would require $\omega = \mu$, individual risk aversion would imply $\omega'' < 0$, and more fundamentally we would be required to use the intrapersonal utilitarian criterion from Proposition 2 rather than the ambiguity averse criterion from Proposition 3. This line of reasoning echoes a classic argument for utilitarianism due to Harsanyi [1955], and the intuition behind it resembles a "veil of ignorance" thought experiment for intrapersonal problems. Related arguments not relying on notions of objective probabilities/lotteries are found in d'Aspremont and Gevers [1977] and Maskin [1978].

However, endowing individuals with vNM preferences over intrapersonal lotteries and using this as the basis for welfare analysis could be controversial, as argued for interpersonal problems in Sen [1976] and Weymark [1991]. Our ambiguity averse criterion runs parallel to Rawls' [1971] alternative approach to interpersonal welfare, and the formal conditions under which we derive a maxmin criterion are related to those proposed by Hammond [1976].

There are two differences between intrapersonal problems and interpersonal analogues we find noteworthy. First, at least in some behavioral models, we find ordinal level comparability a little bit more plausible, because there is a set of situations where the revealed preferences of the selves agree about the value of money. If we assume that the level of utility is then comparable across frames in these situations, then if we use one of these situations as our baseline, we obtain Assumption 10.1. For money-metric equivalent variation (or another welfare measure with similar properties), the comparability problem is then simplified to specifying the cardinal units for comparisons. However, this insight is model-specific: it seems applicable in some of the examples below but not all of them. Second, even if we do adopt the utilitarian perspective and wish to specify structure on ω_ζ, assuming individuals are endowed with vNM preferences in order to resolve the comparability question following Harsanyi [1955] is likely to be controversial for intrapersonal problems, because in some behavioral models, departures from Expected Utility Theory are core to the behavioral phenomenon of interest. If we wish to entertain that these departures reflect normative preferences rather than biases (e.g. Example 1.3 below), the rational for an approach like Harsanyi's fails.

Leveraging insights about specific classes of models from behavioral decision theory might reveal more and better ways to approach this problem. For example, Ellis and Masatlioglu

[2022] present a model in which an individual evaluates options by partitioning the menu space into categories on the basis of a particular option in the space – they call this option the reference point but it could also be interpreted as a default or status quo – and then uses a different utility function to evaluate options within each category. Theorem 2 of their paper presents conditions under which cardinal comparisons of differences in utility *across categories* are well-defined, which implies intrapersonal welfare comparisons of interest to us will be well-behaved. The results above involving ordinal level comparability simplifying the comparability problem applies to this class of models, if we are willing to assume the level of utility can be assumed to be the same across all categories when the individual chooses the reference point.

4 Perturbations

In this section, we explore a perturbation approach to evaluating policy reforms in our framework. We consider one-dimensional policy variation, supposing $\mathcal{P} = \mathbb{R}$. We do not compare policy perturbations in more than one direction, but under some regularity conditions, doing so is feasible [Golosov et al., 2014]. We assume all the derivatives necessary to apply the perturbation approach exist.

4.1 General Derivation With Unstructured Welfare Weights

We begin with the least restrictive representation of the planner's objective from Proposition 1: $w(x) = \mathcal{W}(\{u(x,\theta)\}_\theta)$, where $u(x,\theta)$ can be any representation of frame-dependent ordinal preferences. We also suppose equivalent variation representations of preferences are well-behaved so that we can think in terms of willingnesses to pay (Assumption 9); this is common in prior work employing the perturbation approach but not essential. As above, express the indirect utility function under a given normative frame θ^*, imposing BIC, as

$$W^*(P, Z(P), \theta^D; \theta^*) = u(x(P, Z(P), \theta^D; \theta^*).$$

Assuming all the necessary differentiability requirements are met in some status quo situation (P, Z, θ^D), for any policy perturbation dP, which could also affect Z but which does not affect θ^D, we have:

$$dW(P, Z(P), \theta^D) = \sum_{\theta^* \in \Theta} \frac{\partial \mathcal{W}(\{W^*(\cdot;\theta^*)\}_{\theta^*})}{\partial W^*(\cdot;\theta^*)} \frac{\partial W^*(P, Z, \theta^D; \theta^*)}{\partial Z} d\zeta(x, \theta^*; P, Z, \theta^D). \tag{18}$$

where $d\zeta(x, \theta^*; P, Z, \theta^D)$ is the equivalent variation of dP given a normative frame θ^* and the baseline (P, Z, θ^D). In other words, the $d\zeta$ term is the money-metric willingness to pay for the marginal reform when the normative frame is θ^*. Using the implicit function theorem for a marginal reform and definition (15), we have:

$$d\zeta = \zeta(x(P + dP, Z + dZ, \theta^D), \theta^*, P, Z, \theta^D) = \frac{\frac{\partial W^*}{\partial P} dP + \frac{\partial W^*}{\partial Z} \frac{\partial Z}{\partial P} dP}{\frac{\partial W^*}{\partial Z}}. \tag{19}$$

The product $\frac{\partial \mathcal{W}}{\partial W^*(\cdot;\theta^*)}\frac{\partial W^*(\cdot;\theta^*)}{\partial Z}$ from equation (18) resembles a welfare weight like those used often in contemporary optimal tax theory for interpersonal problems [e.g. Saez, 2001]. Essentially all we know about this term with the structure of Proposition 1 is that this weight is weakly positive and defined up to a multiplicative constant. The additional structure introduced in subsequent results above imposes further structure on the welfare weights. For example under Assumption 7/Proposition 2, we may impose $\frac{\partial \mathcal{W}}{\partial W^*(\cdot;\theta^*)} = \psi(\theta^*)$. The second part of the weight is the marginal value of a dollar under frame θ. If we further impose that $u = \zeta$ as in Proposition 4, the second part of the welfare weight becomes $\frac{\partial W^*_\zeta(\cdot;\theta^*)}{\partial Z} = 1$ by construction and evaluating whether the derivative in equation (18) is positive is analogous to a Kaldor-Hicks criterion (using equivalent rather than compensating variation). But this requires Assumption 10 obtains for the status quo as the baseline. One can derive a similar characterization to (18) using equivalent variation from an arbitrarily chosen baseline rather than the status quo, in which case $\frac{\partial W^*_\zeta(\cdot;\theta^*)}{\partial Z} = 1$ is not assured when we evaluate welfare at the status quo. In this sense the differential value of a dollar accross frames is accounted for when we specify an admissible baseline per Assumption 10; going from a local to a global optimalty condition rooted in equivalent variation further requires accounting for potential nonlinearity in the value of money (ω_ζ).

Technical Aside. We do not apply equations (18) and (19) to marginal policy reforms that vary the decision-making frame θ^D because the set of frames is finite, but we note that in our examples, variation in the frame is equivalent to variation in continuous preference parameters, which allows us to use approximations like the one implied by (19) to evaluate first-order welfare effects when the decision-making frame is also affected by policy.

4.2 Applying the Envelope Theorem with Fully Specified Objectives

We now use the envelope theorem to further characterize $\frac{\partial W}{\partial P}$. Doing so requires a slight modification of our setup. Suppose we can conceive of every option $\tilde{x} \in \mathcal{X}$ as a component the individual can choose and a fixed feature like a default: $\tilde{x} = (x, P)$. Here, we assume there is a bijection such that every value of the fixed feature corresponds to a value of the policy P, so we might as well denote the fixed feature by P. Given a cardinal utility function now expressed as $u(x, P, \theta)$, we introduce the notion of *disagreements* about welfare between the decision frame θ^D and any other (potentially normative) frame θ.

$$V(x, \theta) = u(x, P, \theta^D) - u(x, P, \theta).$$

Expressing the planner's welfare as $W(P, Z, \theta^D) = w(x(P, Z, \theta^D), P)$ where $w(x, P)$ is the objective of the form introduced in the representation results above. We present three characterizations, one leveraging Proposition 2, where we think of u as a utility function that is fully comparable (Paternalistic Risk Neutrality), one based on Corollary 2.1/Equation (16), where we think of u as a money-metric utility function that is ordinally level comparable to cardinal utility with diminishing utility over money $\omega''_\zeta < 0$ (Paternalistic Risk Aversion), and one rooted in Proposition 3 (Ambiguity Aversion) given a fully comparable utility function.

4.2.1 Under Risk Neutrality

We begin with the planner's objective under probabilistic uncertainty about the normative frame and risk-neutrality. Applying the envelope theorem of Milgrom and Segal [2002] under θ^D, we find[6]

$$
\begin{aligned}
\frac{\partial W(P,Z,\theta^D)}{\partial P} = &\underbrace{E_\psi\left[\frac{\partial u(x(P,Z,\theta^D),P,\theta)}{\partial P}\right]}_{\text{Direct Effect}} \\
&- \underbrace{\frac{\partial x(P,Z,\theta^D)}{\partial P}}_{\text{Beh. Resp.}} \cdot \underbrace{(1-\psi(\theta^D))E_\psi\left[\frac{\partial V(x(P,Z,\theta^D),\theta)}{\partial x}\middle|\,\theta \neq \theta^D\right]}_{\text{Marginal Internality}}
\end{aligned}
\tag{20}
$$

To clarify notation, in equation (20), the partial derivative $\frac{\partial u(x(P,Z,\theta^D),P)}{\partial P}$ is the derivative of $u(x,P,\theta^D)$ with respect to its second argument, where the first argument is evaluated at $x(P,Z,\theta^D)$.

In a model in which normative preferences are known – i.e. a model with a singular alternative view of welfare θ^A, as in Example 1, and weight $\psi(\theta^A) = 1$ – this derivation matches the reduced-form characterization of welfare in Mullainathan et al. [2012].[7] Here, we extend this characterization to accommodate uncertainty about true preferences. We replace the direct effect under some certain view of welfare with the expected direct effect under an uncertain view, and we evaluate the behavioral effect by multiplying the behavioral response by an expected marginal internality rather than a known marginal internality. This derivation focuses our analysis of normative ambiguity on specific first-order questions: how do views of welfare differ in their implied direct effects and marginal internalities? Answering these specific questions in our examples builds intuition.

How do disagreements about welfare shape the effects in (20)? We can see the answer for the internality term in equation (20). For the direct effect it is useful (if obvious) to observe that

$$
\begin{aligned}
E_\psi\left[\frac{\partial u(x(P,Z,\theta^D),P,\theta)}{\partial P}\right] = &\frac{\partial u(x(P,Z,\theta^D),P,\theta^D)}{dP} \\
&- [1-\psi(\theta^D)]E_\psi\left[\frac{\partial V(x(P,Z,\theta^D),P,\theta)}{\partial P}\middle|\,\theta \neq \theta^D\right].
\end{aligned}
\tag{21}
$$

4.2.2 Under Paternalistic Risk Aversion

How do disagreements in money-metric welfare matter for policy evaluation? Assuming ordinal level comparability of equivalent variation $\zeta(x,P,\theta)$ (suppressing the baseline parameters) and diminishing marginal utility of money, we find an intuitive representation that leverages the mean-variance characterization from Corrollary 2.2. Because $\zeta(x,\theta^D)$ does not depend on the normative frame, the variance of welfare equals the variance of the disagreement with θ^D.

[6] Note that P is one-dimensional by assumption here. When x is multidimensional, the second term of this expression should be regarded as a dot product of the vectors $\frac{\partial x}{\partial P}$ and $\frac{\partial V}{\partial x}$.

[7] With a known normative frame it is also not necessary to account for potential differences in the value of a dollar across frames in characterizing when a local perturbation improves welfare, so we could freely use any equivalent variation representation of preferences in the application of the perturbation approach [see also Allcott and Taubinsky, 2015].

We express disagreements here as V_ζ to remind the reader these are in the units of ζ, not u.

$$Var_\psi\left[\zeta(x,P,\theta)\right] = Var_\psi\left[V_\zeta(x,P,\theta)\right]$$

Denoting mean indirect utility at (P,Z,θ^D) by $\overline{W}_\zeta(P,Z,\theta^D;\psi) = E_\psi[\zeta(x(P,Z,\theta^D),P)]$, we find that up to second-order approximation of ω_ζ,

$$\frac{dW(P,Z(P),\theta^D)}{dP} \approx \omega'(\overline{W}_\zeta)\frac{d\overline{W}_\zeta}{dP} + \frac{\omega''(\overline{W}_\zeta)}{2} \cdot \frac{dVar_\psi\left[V_\zeta(x(P,Z,\theta^D),P,\theta)\right]}{dP}. \tag{22}$$

By construction, equations (20) and (21) above characterize the effect of dP on mean welfare, $\frac{d\overline{W}_\zeta}{dP}$ in the first term. The second term then captures how disagreements matter when the planner values robustness (in the sense of being risk averse). The characterization is intuitive (and arguably obvious from Corrollary 2.2): given $\omega''_\zeta < 0$ a policy reform that increases the variance of disagreements about money-metric welfare is less desirable, holding the effect on expected welfare $\overline{W}_\zeta$ fixed.

Remark: Setting Aside Money Metrics. The above characterization holds for any welfare metric under ordinal level comparability and paternalistic risk aversion, but we stated it in terms of money-metric utility to relate to emphasize the relationship with prior work. We do not engage directly with the money-metric welfare concept for the remainder of the paper. We typically assume, for better or worse, that we can write down a utility function that is comparable across frames. We do consider the variance of a given utility representation over frames and typically interpret this under paternalistic risk aversion. This can be interpreted in terms of money-metric utility, or more broadly as any utility representation rooted in Corollary 2.1.

4.2.3 Under Paternalistic Ambiguity Aversion

Now we turn to the ambiguity averse objective from Proposition 3. We let

$$\psi^*(\theta,P) \equiv \arg\min_{\psi\in\Psi^*} E_\psi[W(P,Z,\theta^D;\theta)].$$

Following Hansen and Sargent [2008], we develop intuition by thinking of ψ^* as being chosen by an "evil agent" who minimizes welfare given the planner's choice of policy.

When ψ^* is differentiable in P, we find

$$\frac{dw}{dP} = \frac{d\overline{W}_{\psi^*}(P,Z,\theta^D)}{dP} \tag{23}$$

This welfare effect is the same as $\frac{d\overline{W}}{dP}$ above (direct effects and behavoiral effects multiplied by marginal internalities) but mean welfare is evaluated over the welfare-minimizing distribution ψ^*. Re-optimization by the evil agent ($\partial\psi^*/\partial P$) does not have a first-order welfare effect as a consequence of the envelope theorem where it applies. However, we note that in some examples below, ψ^* is discontinuous in P in relevant ways, so that the envelope theorem does not apply. Understanding the importance of such discontinuities requires a more global perspective rather than a local perturbation approach.

5 Examples

We next illustrate via examples how prior work on behavioral welfare economics fits within our framework.

5.1 Example 1: Biases Versus Strange Preferences

Let us introduce a running example in which the key intrapersonal question is whether some behavioral phenomenon arises due to a bias or a normative preference. Suppose the decision-making frame is some fixed frame θ^D and there is just one alternative frame denoted θ^A. Our representation of welfare from equation (9) becomes

$$w(x) = \psi(\theta^D)u(x,\theta^D) + [1-\psi(\theta^D)]u(x,\theta^A). \tag{24}$$

With just one view of welfare we suppress θ when we express disagreements: $V(x) = u(x,\theta^D) - u(x,\theta^A)$. To see how we relate our framework to prior work, it is instructive to re-write $w(x)$ using the definition of $V(x)$:

$$w(x) = u(x,\theta^D) - [1-\psi(\theta^D)]V(x) \tag{25}$$

$$= u(x,\theta^A) + \psi(\theta^D)V(x). \tag{26}$$

$$u(x,\theta^D) = u(x,\theta^A) + V(x). \tag{27}$$

Prior work on behavioral frictions often uses a formulation like equation (26), where we think of $V(x)$ as the "behavioral" component of preferences, while decision utility takes a form like equation (27). The behavioral component of preferences $V(x)$ can be interpreted as a deviation from classical forms of preferences that may or may not be due to a bias. When $\psi(\theta^D) = 1$, for instance, the planner is certain that V is a non-standard but normative preference rather than a bias. When $\psi(\theta^D) = 0$, the planner is certain that V reflects a bias.

Next we refine this example by considering specific behavioral frictions from prior literature.

Example 1.1. Defaults. We now denote elements of $\mathcal{X}$ by (x,d), where the first element is a choice object and the second is the default, a fixed feature as in the previous section. In any situation $\sigma = (d,\tilde{\sigma})$, both of these are drawn from a set of available options: $X(\tilde{\sigma})$. To nest the fixed cost model of default effects in Example 1 we specify

$$V(x,d) = -1\{x \neq d\}\gamma. \tag{28}$$

where γ is the fixed cost of choosing some option other than the default [see e.g. Carroll et al., 2009; Bernheim et al., 2015]. The fixed opt-out cost structure matches key empirical aspects of default effects, e.g. an increase in the default rate of contribution to pension plans increases the number of individuals not contributing to the plan.[8] Goldin and Reck [2022] shows that

[8] This empirical pattern is observed in widely varied contexts [Choi et al., 2006; Haggag and Paci, 2014; Altmann et al., 2013; Brown et al., 2013] but not everywhere [Brot-Goldberg et al., 2023]. The complexity and opacity of the Medicare Part D plans studied in Brot-Goldberg et al. [2023] suggests that another important factor might be individuals' understanding of the options they could get upon opting out. One could employ our overall normative approach to evaluate defaults while allowing for this possibility, but this is not nested by Example 1.1.

the fixed cost structure nests a wide variety of different mechanisms by which default effects influence behavior, but depending on the mechanisms and our normative interpretation of them, the fixed cost may or may not reflect a real normative cost.

The alternative view of welfare implied by (28) is the utility function individuals maximize when they opt out of the default and make an active choice. Drawing parallels between this example and the next, we might label the utility function $u(x, \theta^A)$ "intrinsic utility." When $\psi(\theta^D) = 1$, we effectively assume that γ represents a welfare-relevant cost; when $\psi(\theta^D) = 0$, γ reflects a bias.[9]

In treating default adherence as a "biases versus strange preferences" question, we do not allow active choosers to make mistakes. This restriction is relaxed in the more general version of the model in Goldin and Reck [2022], but doing so obviously requires introducing more frames than the two we posit in this example. An analogous limitation to Example 1 applies in general: in models of biases versus strange preferences, the modeller picks one behavioral factor to consider as a bias or a strange preference, and assumes away deviations from individual welfare maximization due to any other behavioral factor.

Example 1.2. Refewence Dependence. Reference dependence is the subject of a rich theoretical and empirical literature [Kahneman and Tversky, 1979; Tversky and Kahneman, 1991; Kőszegi and Rabin, 2006; Crawford and Meng, 2011; Thakral and Tô, 2021], including many policy-relevant applications [DellaVigna et al., 2017; Rees-Jones, 2018; Seibold, 2021]. A lack of consensus about whether to regard this phenomenon as a bias or a preference has hindered our ability to evaluate policy in these settings [O'Donoghue and Sprenger, 2018]. Reck and Seibold [2023] consider a model, nested by Example 1, in which the behavioral component of preferences $V(.)$ is a reference-dependent payoff featuring loss aversion.

We use a similar setup to the previous example, but replace the default d with a reference point $r \in X(\tilde{\sigma})$. When researchers model reference-dependent choice, they introduce a utility function over x with classical properties labelled "intrinsic utility" or "consumption utility" [e.g. Kőszegi and Rabin, 2006], which is additively separable from a gain-loss payoff over (x, r). We can nest this in our biases versus strange preferences setup if we posit a naturally occurring frame in which the individual makes choices based on the sum of intrinsic and gain-loss utility, and an alternative frame in which the indiviudal makes choices based on intrinsic utility alone.[10] Following Kőszegi and Rabin [2006], we assume intrinsic utility is additively separable, so that we may write $u(x, r, \theta^A) = \sum_{i=1}^N u_i(x_i)$. For parameters $\Lambda_i > 0, \beta \in (0, 1]$, we specify a gain-loss payoff of the form

$$V(x, r) = -\sum_{i=1}^{N} 1\{x_i \leq r_i\} \Lambda_i \left[u_i(r_i) - u_i(x_i)\right]^\beta, \tag{29}$$

The individual only incurs a payoff along some dimension if they incur a loss, $x_i \leq r_i$. The parameter Λ_i governs the strength of loss aversion along dimension i, while the parameter β

[9]When a real cost is inflated above its true value, for instance due to present bias, we capture this possibility by $0 < \psi(\theta^D) < 1$. Convexifying the possible views of welfare in this way also effectively captures views of welfare according to which fixed costs are partially but not fully normative, e.g. models of present bias.

[10]We borrow the term "naturally occuring frame" from Bernheim et al. [2015].

governs diminishing sensitivity. We separately consider the case without diminishing sensitivity ($\beta = 1$) and with it ($\beta < 1$) below.

This is a similar form to that proposed by Kőszegi and Rabin [2006] – gains and losses are evaluated over "utils" rather than units of each good – except that 1) we disregard gain domain payoffs where $x_i > r_i$ along some dimension, and 2) we allow the extent of loss aversion Λ_i to vary across dimensions i rather than being fixed. These choices are motivated by a more detailed analysis of forms of gain-loss utility in Reck and Seibold [2023]. Including a gain-domain payoff whose strength is governed an additional parameter, usually denoted by η [Tversky and Kahneman, 1991], would not change the results of interest to us provided that η_i is not too strong along any given dimension i, in a sense formalized in [Reck and Seibold, 2023], Appendix B6.

Example 1.1 and Example 1.2 under $\beta = 1$ are both cases of the "Affine Categorical Thinking Model" from Ellis and Masatlioglu [2022]. The salience model of Bordalo et al. [2012] is also an Affine Categorical Thinking Model. One could adapt the approach we develop here to analyze welfare in this salience model, or another Affine Categorical Thinking Model. Not all such models can be nested within Example 1, but our overall approach is applicable to these models because they feature a family of intrapersonally comparable utility functions by definition. Our next example does not fall within this class of models.

Example 1.3. Probability Re-Weighting. Starting with Kahneman and Tversky [1979], researchers have modelled deviations from expected utility theory due to the reweighting of objective probabilities [see also Prelec, 1998; Abdellaoui, 2000; Chateauneuf et al., 2007]. In a recent welfare analysis of state-run lotteries, Lockwood et al. [2023] present a model in which the main behavioral factor is probability-reweighting and it is ambiguous whether re-weighting reflects a bias or a normative preference. In particular, individuals' revealed preferences – identified empirically using demand responses to changes in lottery prizes – suggest their utility function puts much more weight on the jackpot payoff than any other payoff, and moreover more weight than expected utility would require, given the extremely low probability of winning a jackpot. This finding suggests a particular form of probability re-weighting, and the main question they confront for welfare analysis is whether this jackpot payoff effect reflects a bias or a normative preference.

To nest their model in Example 1, we think of each component of $x = (x_1, ..., x_N)$ as the payoff that is realized for each realization of an uncertain state variable. Objective probability of each realized state is $\pi = (\pi_1, ..., \pi_N)$; this is the main aspect of the situation σ that is relevant for the behavioral friction. Individuals reweight each objective probability according to a function $f(\pi)$. Individuals are endowed with a Bernoulli utility function $\mu(x_n)$. Utility in the fixed decision-making frame is $u(x, \pi, \theta^D) = \sum_n f(\pi_n)\mu(x_n)$.

When $f(\pi) = \pi$ everywhere, we have classical expected utility maximization. If we view the independence axiom as normative, then normative utility should coincide with expected utility. For an alternative frame θ^A in which the individual's choices respect the independence axiom, we have $u(x, \pi, \theta^A) = \sum_n \pi_n\mu(x_n)$. The disagreement between these two views of welfare is then, by our definition,

$$V(x,\pi) = \sum_n [\pi_n - f(\pi_n)]\mu(x_n). \tag{30}$$

Now with our framework, we can think of a planner who is uncertain about whether the excess weight on the jackpot payoff (and the resulting under-weighting of payoffs in other states) is normative. Lockwood et al. [2023] model the extent to which re-weighting reflects a bias with a parameter that is isomorphic to $\psi(\theta^A)$ here.

5.2 Example 2: Present Focus

Our next example is motivated by prior work on present focus and the notion of intertemporal selves [e.g. Laibson et al., 1998; Caliendo and Findley, 2019]. The options are lifetime consumption plans: $\mathcal{X} = \mathbb{R}^T_+$, where T is the number of time periods. An option from the set is denoted $x = (x_1, ..., x_T)$. The frame in this model is the vantage point from which individuals evaluate a consumption plan. We characterize individuals' preferences under commitment, i.e. we think of the individual selecting a full consumption plan in each period. We assume individuals are quasi-hyperbolic discounters as in Laibson [1997]. We also assume there is a period 0 in which the individual is entirely forward-looking, i.e. they do not consume or receive flow utility. For two parameters $\beta > 0$, $\delta \leq 1$, a flow utility function $\mu(x_t)$, and a vantage point $\tau = 0, ..., T$ we specify:

$$u(x,\tau) = \mathbb{1}\{\tau > 0\}\delta^\tau \mu(x_\tau) + \beta \sum_{t \neq \tau} \delta^t \mu(x_t). \tag{31}$$

Note that with this formulation, we endow the period τ self with preferences over the prior selves' consumption; this is unconventional and we discuss the rationale for this modelling choice below. A commonly adopted approach to welfare analysis in models like this is to respect the revealed preferences of the period 0 self, sometimes called the "long-run view." The period 0 self is a classical exponential discounter, and in fact we find that a planner's welfarist objective based on formulation (31) has a representation along similar lines to the Biases versus Strange Preferences example.

$$u(x,\tau) = \beta \sum_{t=1}^{T} \delta^t \mu(x_t) + \mathbb{1}\{\tau > 0\}(1-\beta)\delta^\tau \mu(x_\tau). \tag{32}$$

The planner's welfare function takes the following form:

$$\begin{aligned} w(x) &= \sum_\tau \psi(\tau) u(x,\tau) \\ &= \beta \sum_{t=1}^{T} \delta^t \mu(x_t) + \sum_{\tau=0}^{T} \psi(\tau)\mathbb{1}\{\tau > 0\}(1-\beta)\delta^\tau \mu(x_\tau) \\ &= u(x,0) + (1-\psi(0)) \sum_{\tau=1}^{T} \psi(\tau|\tau > 0)[u(x,\tau) - u(x,0)] \end{aligned} \tag{33}$$

If we adopt the intuitive restriction that the normative weight $\psi(\tau)$ is constant for $\tau > 0$ – i.e. that the planner does not want to put differential weight on the present-focus payoff of any particular present-focused self – the above expression becomes:

$$w(x) = u(x,0) + \psi^D \sum_{\tau=1}^{T} [u(x,\tau) - u(x,0)] \tag{34}$$

$$= \beta \sum_{t=1}^{T} \delta^t \mu(x_t) + \psi^D (1-\beta) \sum_{\tau=1}^{T} \delta^\tau \mu(x_\tau), \tag{35}$$

where $\psi^D = \frac{1-\psi(0)}{T}$ is the mean welfare weight on present-focused selves. Because the extent of present focus β and the normative weight $\psi(\tau)$ are both constant over $\tau > 0$ with this structure, we find an intuitive interpretation of welfare that is very similar to Example 1: ψ^D captures the extent to which the planner respects the modification of classical payoffs that is due to present focus. This is reflected in the above expression by the fact that counterfactual values of β, as would be introduced by a notion of "normative present focus," and variation in ψ^D affect $w(x)$ in equivalent ways.[11]

Remark on Intertemporal Selves Preferences Formulation. Holding x_s fixed for $s < \tau$, the above generates the same choices as the conventional β-δ representation of preferences, i.e. $\tilde{u}(x_{t \geq \tau}, \tau) = \mathbb{1}\{\tau > 0\}\mu(x_\tau) + \beta \sum_{t=\tau+1}^{T} \delta^{t-\tau} \mu(x_t)$. The formulation differs from most prior work on intertemporal selves in that the period τ self is endowed with classically discounted preferences over consumption in periods $t < \tau$. This approach appears to fix multiple related issues with the intertemporal selves model identified by Bernheim and Rangel [2009], at the cost of being, admittedly, philosophically confusing.

What does it mean for the period τ self, who cannot go back in time to choose a different amount of consumption, to have preferences over past consumption? Loosely speaking, we address this question by assuming that the period-τ self agrees with most of their prior selves about intertemporal consumption tradeoffs, so that endowing this self with preferences over past consumption does not generate any new choice inconsistencies relative to those we find for observable, forward-looking choices. More formally, we assume that for any pair $\tau > 0$, $\tau' > 0$, if we consider two consumption plans x, x' such that $x_\tau = x'_\tau$ and $x_{\tau'} = x'_{\tau'}$ then we have $u(x,\tau) \geq u(x',\tau) \iff u(x,\tau') \geq u(x',\tau')$. Behaviorally, the period-τ and period-τ' selves make the same choices when we hold consumption in τ and τ' fixed in the menu. This approach works well for the β-δ model but appears to be less well-suited to more generic models of non-classical discounting.

Welfare with this formulation accords with BR-Dominance. The setup is "rectangular" in that for any frame τ individuals have frame-dependent rational preferences over the entire option space; Bernheim and Rangel show that a lack of rectangularity in the naive application of "intertemporal-self Pareto optimality" leads to conceptual problems: if all selves care only about current and future consumption, allocating all resources to the last-period self will al-

[11] If the normative present focus is β^*, $\psi^D = 0$ is equivalent to $\beta^* = 1$, and $\psi^D = 1$ is equivalent to $\beta^* = \beta$. If $\beta < 1$, each $\psi^D \in (0,1)$ gives the same utility function some $\beta^* \in (\beta, 1)$. If $\beta > 1$, the same is true for some $\beta^* \in (1, \beta)$. If $\beta = 1$, all inconsistencies/disagreements vanish and welfare is invariant to ψ^D.

ways be an intertemporal-self Pareto optimum, but revealed preference does not suggest that the individual robustly prefers options that defer all consumption to the final period. We address this problem by adopting formulation (31) and considering choices under commitment.

Why focus on revealed preferences under commitment to define welfare? Doing so brings our welfare analysis of intertemporal selves models in line with the way we usually think about welfare in interpersonal games. In an interpersonal game, each player maximizes preferences that depend on what other players do, but a player's deep structural preferences can be thought of as coming from revealed preferences over the actions of that player *and other players*. For example, in order to say that the Defect-Defect equilibrium of the Prisoner's Dilemma is Pareto inefficient we need to be able to evaluate a player's welfare over variation in the actions of another player. The only way to root a comparison of Defect-Defect to Cooperate-Cooperate in revealed preference is to conceive of a player making choices over counterfactual scenarios in which they control the action of the other player. Commitment serves the same purpose here. Likewise, players in conventional games can have deep structural preferences over the actions of prior movers even when they cannot practically go back in time and change another player's actions, which suggests another rationale for endowing the period t self with primitive preferences over the consumption of prior selves.

Finally, we remark on comparability of welfare across intertemporal selves. The units of utility here are essentially pinned down by $\mu(x)$, which is cardinal (so that we can evaluate intertemporal tradeoffs) [Montiel Olea and Strzalecki, 2014]. It is a normative assumption to suppose that the units of μ are the correct cardinal units for welfare analysis, but adopting this, comparisons of changes in utility across τ with the formulation above are well-defined. For $\tau > 0$, level comparability also does not seem to be a problem: evaluating equation (32), we find that a constant consumption growth path generates the same level of utility for any $\tau > 0$. For $\tau = 0$ versus $\tau > 0$, examining equation (32), we find that the conventional level normalization, $\mu(0)_\tau = 0$ for every τ, implies that the present-focused self with $\beta < 1$ will always have a higher utility level than the period 0 self due to the present-focused payoff in the second term of equation (32). Formally, if $\mu_\tau(x)$ is weakly increasing in x,

$$\forall \tau > 0, \mu_\tau(0) = 0 \implies \forall x \geq 0, \min_{\psi \in \Delta(\Theta)} \psi(\tau) u(x, \tau) = u(x, 0). \tag{36}$$

In words, the implication is that the globally ambiguity averse planner adopts the long run view of welfare. Whether this is a novel rationale for the long-run view or an artefact of our setup is debatable. If it is deemed a problem, the solution appears to require specifying a nonzero level of constant consumption at which we impose that the level of utility is equal between $\tau = 0$ and $\tau > 0$ selves.

5.3 Example 3: Is a Feature of the Environment a Frame?

Note that in Example 1.1, the default cannot be a frame by construction; the same is true of the reference point in Example 1.2.[12] Readers might find this confusing: if we treat default

[12] To see this, suppose that θ^A is the normative frame, where opt-out costs are purely a bias. Then, if $X = \mathbb{R}$, $d = 2 \implies 2 \succ 1$ but $d = 1 \implies 2 \prec 1$, which violates Lemma 1.

adherence as a normative preference then by definition the default should not be a frame, but if we do not, then it could be one and we might think of choices made given each default as coming from distinct frames rather than a unitary, naturally occurring frame. In the present example, there is a component of the situation σ, labelled d and drawn from a finite set D,[13] and the frame has two components: $\theta = (\theta_1, \theta_2)$. The first component, $\theta_1 \in \{0, 1\}$ indicates whether the second component, $\theta_2 \in D$, can really be viewed as a frame ($\theta_1 = 1 \implies \theta_2/d$ is a frame), i.e. whether we obtain the frame exclusion condition from Lemma 1.

We express the utility function as $u(x, d, \theta_1, \theta_2)$ and we make two restrictions to capture our intuition. When $\theta_1 = 0$, saying feature d is not a frame requires that $u(x, d, 0, \theta_2)$ must be constant over θ_2, which we capture with a utility function $u_0(x, d) \equiv u(x, d, 0, \theta_2)$ for any θ_2. If $\theta_1 = 1$, feature d is a frame so frame exclusion requires $u(x, d, 1, \theta_2)$ to be constant over d, which we capture with a function $u_1(x, \theta_2) = u(x, d, 1, \theta_2)$. Denoting disagreements between the $\theta_1 = 0$ and $\theta_1 = 1$ cases by $V(x, d, \theta_2) = u_0(x, d) - u_1(x, \theta_2)$ and letting $\psi^0 = \sum_{\theta_2} \psi(0, \theta_2)$ be the total weight on $\theta_1 = 0$, we derive an identity similar to equation (25):

$$w(x) = u_0(x, d) - (1 - \psi^0) \sum_{\theta^2 \in D} \frac{\psi(1, \theta_2)}{1 - \psi^0} V(x, d, \theta_2). \tag{37}$$

The weight ψ_0 is similar to $\psi(\theta^D)$ in Example 1. With this setup, we confront more ambiguity than in the biases versus strange preferences case from Example 1. For instance, if the planner knew with certainty that the effect of d on choices reflects a bias, then this resolves all ambiguity in Example 1 ($\psi(\theta^D) = 0$), but if analogously $\psi_0 = 0$ in (37), substantial ambiguity in welfare remains due to choice inconsistencies as d varies. We note that the most models of default effects considered in Bernheim et al. [2015] are nested in Example 1.1, but the anchoring model they consider resembles Example 3 under $\psi_0 = 0$. Understanding the difference between these examples clarifies why the results using the anchoring model lead to much more ambiguous welfare effects.

We do not engage deeply with models like Example 3 in the remainder of this paper, but this is done in the interest of providing simple illustrations of our robustness concept rather than our thinking that the approach applied by Example 1 is superior to the one implied by Example 3 for any particular behavioral phenomenon.

5.4 Discussion: Identifying Normative Weights.

Much of the prior literature we draw from for these examples contains analysis that attempts to resolve the uncertainty about normative preferences that is primitive in our model. Regarding Example 1.3, for instance, Lockwood et al. [2023] seek to identify the normative weight on probability-reweighting ($\psi(\theta^A)$) with additional data analysis. In this paper, we do not formally analyze the identification of normative weights ψ using approaches like the "counterfactual normative consumer" these authors use. We do notice similarities between prior approaches to intrapersonal welfare analysis like this and attempts to pin down interpersonal welfare weights using additional normative assumptions and data.

[13] We require a finite set of frames so D must be finite to nest this example in our general model, but we view this as a technical issue and do not expect much to change e.g. when D is a continuum.

Conceptually, we can think of strategies for resolving normative ambiguity in terms of assumptions under which the appropriate weighting function ϕ can be inferred from information observable to the planner. The simplest approach is to assume the answer from some normative principle, e.g. by aggregating welfare using inverse-consumption weights in interpersonal problems. Such a treatment of intrapersonal welfare is found, for example, in the assumption by O'Donoghue and Rabin [2006] that the "long-run view" of welfare is normative, which is derived essentially from the philosophical principle that inter-temporal preferences should be time consistent – Bernheim [2009] provides a contrary perspective [see also Caliendo and Findley, 2019]. A less restrictive version of this approach is the "counterfactual normative consumer" approach [Allcott and Taubinsky, 2015; Goldin and Reck, 2020; Allcott et al., 2019], in which the choices of "debiased" individuals or experts are assumed to pin down the normative objective and the planner has sufficient information to account for all normatively relevant differences in preferences between experts and non-experts via extrapolation.

Second, researchers use revealed preference methods that echo the "veil of ignorance" thought experiment. In the ideal version of this experiment, an individual, being aware of framing effects but not subject to them (being aware of interpersonal inequality but having not been assigned a type), makes choices that require them to trade off welfare under various frames (types). For example, Saez and Stantcheva [2016] and Capozza and Srinivasan [2023] experimentally estimate welfare weights by having participants make choices to reveal their willingness-to-pay to transfer income from person A to person B, varying the incomes of A and B. Relatedly, Landais and Spinnewijn [2021] estimate people's willingness-to-pays to transfer income between themselves in different states of the world (employment vs unemployment). In Allcott and Kessler [2019], the authors implement a meta-choice approach by eliciting the willingness to pay to be nudged and interpreting this elicitation as if individuals know how to evaluate the potentially biased choices they will make after getting the nudge or not.

Lastly, many recent studies use implemented policies such as tax schedules [Hendren, 2020; Lockwood and Weinzierl, 2016] and transfer policies [Hendren and Sprung-Keyser, 2020] to reveal social welfare weights. The idea is to use the chosen policy to reverse-engineer the weights which would have meant that policy was optimal. This is an interesting approach which could be applied to our setting. One simple example is that if a social-planner sets a "penalty-default" in the case of Example 1.1. (defaults), it is likely that the revealed intrapersonal welfare weight likely sets $\psi(\theta^D) \approx 0$. More ambitiously, we can imagine inferring policymakers' normative judgments about present focus from the design of illiquid/mandated savings policies, which is similar to the central exercise in Beshears et al. [2020].

We do not engage with the question of what might be the best way to resolve the uncertainty captured by ψ in this paper. The methods developed in behavioral welfare economics for approaching this problem seem informative, but every approach requires untestable normative assumptions. As such, we view welfare analysis in our framework – how variation in normative judgments affects optimal policy and our notions of robustness – as a useful complement to tools that help us identify what the appropriate normative objective might be. In other words, if there is even a little room for doubt about the validity of any of the approaches summarized above, our approach, which gives us a way to assess the importance of such doubts for optimal policymaking, should prove useful.

6 Robust Optimality

In this section, we explore how the robustness concepts we developed above play out in some of our examples.

6.1 Robustness and Perturbations

We focus on risk aversion and ambiguity aversion in Example 1; understanding welfare under risk neutrality is straightforward from prior literature and the results above. Beginning with probabilistic uncertainty, with just two frames θ^D and θ^A, recalling there we denote disagreements using $V(x) = u(x, \theta^D) - u(x, \theta^A)$, we find that the variance term from Corollary 2.2 simplifies to

$$Var_\psi(V) = \psi(\theta^D)(1 - \psi(\theta^D))V(x, P)^2 \tag{38}$$

Evaluating the second term term from equation (22), we find:

$$\frac{\omega''(\overline{W})}{2} \cdot \frac{dVar_\psi\left[V(x(P, Z, \theta^D), P)\right]}{dP} = \omega''(\overline{W})Var_\psi(V) * \frac{1}{V}\frac{dV}{dP} \tag{39}$$

where V and dV/dP are evaluated at $(x(P, Z, \theta^D), P)$. This is a reduced-form expression that carries some intuition. Note that the last term resembles a semi-elasticity; this term is positive when V moves away from zero following a marginal change in P. The importance of disagreements for policy evaluation depends on 1) how concave is the planner's welfare function over our measure of welfare, 2) how much disagreement is there in the status quo, and 3) the change in the magnitude of disagreement generated by the reform.

The character of $\frac{1}{V}\frac{dV}{dP}$ depends on more specific features of the model. Let us illustrate this in Example 1.1. To obtain differentiability we introduce some unobserved heterogeneity (conventional uncertainty about the individual's type) so that instead of $V = -1\{x \neq d\}\gamma$, we have $V = -Pr[x \neq d]\gamma$.[14] The right-hand side of (39) becomes

$$\omega''(\overline{W}) \underbrace{\psi(\theta^D)(1 - \psi(\theta^D))Pr[x \neq d]^2\gamma^2}_{Var_\psi(V)} \left\{ \frac{1}{Pr[x \neq d]} \frac{\partial Pr[x \neq d]}{\partial d} \right\} \tag{40}$$

The last term in this expression is the semi-elasticity of opt-outs with respect to a change in the default [see also Brot-Goldberg et al., 2023]. A reform of the default rule that increases opt-outs will be less desirable when the planner values robustness, to an extent governed by the other terms in the expression. In Example 1.2, the analogous semi-elasticity term is a weighted semi-elasticity of losses across various dimensions, where the weights depend on the relative strength of loss aversion in each dimension.

Under ambiguity aversion, the evil agent selects the $\psi \in \Psi^*$ that puts maximal weight on the frame in which welfare is lowest: when $V < 0$, ψ^* places maximal weight on θ^D and

[14] We acknowledge this is informally construed in the interest of avoiding extra notation. We continue to assume that γ is uniform for simplicity, so the unobserved heterogeneity should involve other preference parameters. See Goldin and Reck [2022] for a more thorough treatment of the question of interpersonal heterogeneity in this setting.

where $V > 0$, the evil agent places maximal weight on θ^A. Because $V < 0$ everywhere in Examples 1.1 and 1.2, the evil agent always places maximal weight on θ^D in these models. By similar reasoning to the probabilistic uncertainty case, this will make policies where opt-outs are frequent less desirable in Example 1.1, and it will make policies where losses relative to the reference point are larger less desirable in Example 1.2.

6.2 Robustness in General

Now we turn to more global robust optimality concepts. We begin by defining three notions of optimality:

Definition. A policy P^* is a *ψ-optimum* for $\psi \in \Delta(\Theta)$ if $P^* \in \arg\max_{P \in \mathcal{P}} E_\psi[u(x(P), \theta)]$.

Definition. For a given set of distributions $\Psi^* \subseteq \Delta(\Theta)$, a policy P^* is a *robust optimum* if

$$P^* \in \arg\max_{P \in \mathcal{P}} \min_{\psi' \in \Psi^*} E_{\psi'}[u(x(P), \theta)].$$

Definition. A policy P^* is a *globally robust optimum* if it is a ψ-optimum for all $\psi \in \Delta(\Theta)$.

Obviously, a globally robust optimum will also be a robust optimum for any $\Psi^* \subseteq \Delta(\Theta)$. Global robustness also has a straightforward relationship to BR-dominance:

Lemma 5. ***BR-Optimality and Robustness.*** *A policy $P^* \in \mathcal{P}$ is a globally robust optimum if and only if for every $P' \in \mathcal{P}$, for every $\theta \in \Theta$, $x(P^*) \succeq_\theta x(P')$.*

Proof. Suppose $x(P^*)$ BR-dominates any other $x(P')$. Then global optimality of P^* follows from the monotonicity of expected welfare. For the other direction, suppose P^* does not BR-dominate some P', i.e. there is some θ' strictly better off under P' than P^*. Let $\psi(\theta) = 1\{\theta = \theta'\}$. As P^* is not a ψ-optimum for this ψ, it cannot be globally optimal. ■

Partial Characterization of Robust Optimality. Our next result provides a sufficient condition for a policy that is a ψ-candidate optimum for $\psi \in \Psi^*$ to also be the robust optimum. Note that this is not a full characterization as we do not obtain necessity; the condition nevertheless builds intuition and proves useful in applications.

Proposition 6. ***Sufficient Condition for a ψ-Optimum to be a Robust Optimum.*** *Let $P^* \in \mathcal{P}$ be a $\psi-$optimum for some $\psi \in \Delta(\theta)$. Then, for any $\Psi^* \subseteq \Delta(\Theta)$ such that $\psi \in \Psi^*$, P^* is a robust optimum if*

$$P^* \in \arg\min_{P \in \mathcal{P}} \max_{\psi' \in \Psi^*} \sum_{\theta \neq \theta^D} \left(\psi'(\theta) - \psi(\theta)\right) \cdot V\left(x(P, Z, \theta^D), \theta\right) \tag{41}$$

Proof. By supposition,

$$
\begin{aligned}
P^* &\in \arg\min_{P\in\mathcal{P}} \max_{\psi'\in\Psi^*} \sum_{\theta\neq\theta^D} \left(\psi'(\theta)-\psi(\theta)\right)\cdot V\left(x(P,Z,\theta^D),\theta\right) \\
&= \arg\min_{P\in\mathcal{P}} \left\{ \sum_{\theta\neq\theta^D} \psi(\theta)\cdot V\left(x(\theta^D,P),\theta,P\right) - \min_{\psi'\in\Psi^*} \sum_{\theta\neq\theta^D} \psi'(\theta)\cdot V\left(x(\theta^D,P),\theta,P\right) \right\} \\
&= \arg\min_{P\in\mathcal{P}} \Bigg\{ u(x(P,Z,\theta^D)),\theta^D) - \sum_{\theta\neq\theta^D} \psi(\theta)\cdot V\left(x(P,Z,\theta^D),\theta\right) \\
&\qquad - \min_{\psi'\in\Psi^*} \left[u(x(P,Z,\theta^D),\theta^D) - \sum_{\theta\neq\theta^D} \psi'(\theta)\cdot V\left(x(P,Z,\theta^D),\theta\right) \right] \Bigg\} \\
&= \arg\min_{P\in\mathcal{P}} \left\{ W(P,Z,\theta^D;\psi) - \min_{\psi'\in\Psi^*} W(P,Z,\theta^D;\psi') \right\} \\
&\iff \forall P\in\mathcal{P}, W(P^*,Z,\theta^D;\psi) - \min_{\psi'\in\Psi^*} W(P^*,Z,\theta^D;\psi') \leq W(P,Z,\theta^D;\psi) - \min_{\psi'\in\Psi^*} W(P,Z,\theta^D;\psi')
\end{aligned}
$$

However, $W(P^*,Z,\theta^D;\psi) \geq W(P,Z,\theta^D;\psi)$ as P^* is a $\psi-$optimum. We therefore obtain

$$
\min_{\psi'\in\Psi^*} W(P^*,Z,\theta^D;\psi') - \min_{\psi'\in\Psi^*} W(P,Z,\theta^D;\psi') \geq W(P^*,Z,\theta^D;\psi) - W(P,Z,\theta^D;\psi) \geq 0 \qquad (42)
$$

So P^* is a robust optimum. ■

The condition from Proposition 6 for a given ψ-optimum to be robust is more likely to be met when disagreements about welfare evaluated at that policy are not too large and the set Ψ^* over which the planner evaluates robustness is a relatively close neighborhood around the relevant distribution ψ.

6.3 Robustness in Applications

We now turn to characterizing robust optimal policies within our examples.

6.3.1 Optimal Defaults

We begin with the optimal defaults problem studied in Carroll et al. [2009]; Bernheim et al. [2015]; Chesterley [2017]; Goldin and Reck [2022], and others. In the model we introduced in Example 1.1, the *intrinsic optimum* $x^* \equiv \arg\max_x u(x,\theta^A)$ is implicitly assumed to be known to the social planner.[15] We begin there, and then consider the case where the intrinsic optimum is not known, which which makes the planner's normative objective equivalent to aggregate welfare in Bernheim et al. [2015] and social welfare in Goldin and Reck [2022]. Aggregation over potential intrinsic optima is interpreted in these papers as arising due to unobservable interpersonal heterogeneity rather than intrapersonal concerns.

Our illustrations of this model are simulations built on the assumption that the choice variable is one-dimensional, $x \in \mathbb{R}$. We suppose utility is approximately quadratic: $u(x,\theta^A) =$

[15] The intrinsic optimum x^* is called the "ideal option" in earlier work on defaults [Bernheim et al., 2015; Goldin and Reck, 2022] and the analogous option is called the "intrinsic optimum" in the reference dependence literature [Kőszegi and Rabin, 2006; Reck and Seibold, 2023]. In both cases, x^* does not depend on the default/reference point by construction; it obviously does depend on other aspects of the choice situation encoded in σ, e.g. prices. We suppress dependence of x^* on σ for convenience. We suppose x^* is unique for simplicity.

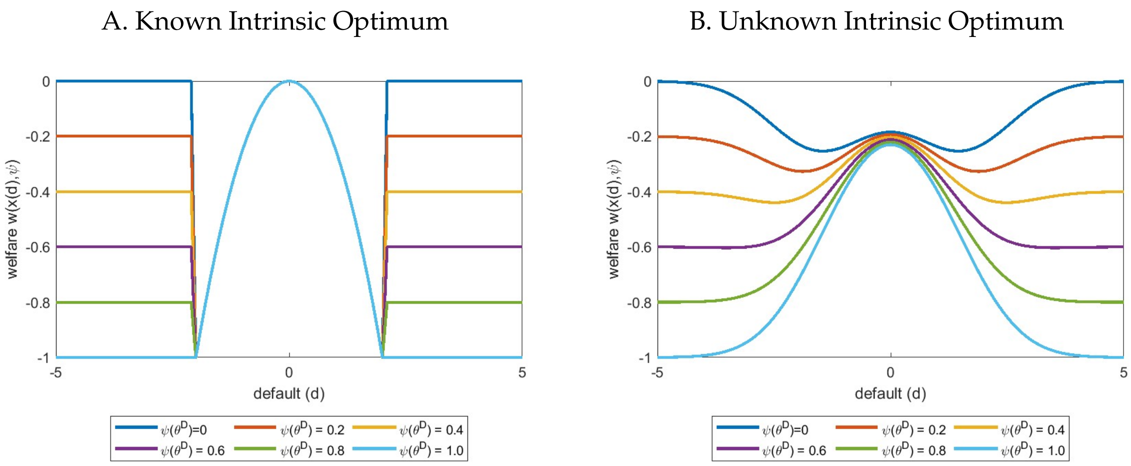
$-\frac{\alpha}{2}(x-x^*)^2$ for a known(/homogeneous) parameter $\alpha > 0$ and the intrinsic optimum x^*. For simplicity, we further assume the opt-out cost γ is known and x^* is either known to equal 0 or it follows a Gaussian distribution centered around 0. This is sufficient structure to pin down the shape of the welfare function, which we illustrate in Figure 1.

Figure 1A depicts welfare as a function of the default (under BIC), given a known intrinsic optimum. We plot welfare for varying weights on the frame θ^D, $\psi(\theta^D)$ from 0 to 1. Applying our definitions of optimality, we observe the following, which turn out to be true in full generality, i.e. without any of the restrictions introduced in the previous paragraph:

- The ψ-optimal defaults are the intrinsic optimum x^* and any default under which the individual chooses actively.[16]
- The intrinsic optimum x^* is the unique globally robust optimum.
- The active choice optima is robust if and only if there is no ambiguity (Ψ^* is singleton) and $\psi(\theta^D) = 0$.

The fact that we find a globally robust optimum in this case clearly depends on the assumption that x^* is known, i.e. that the policymaker can set as the default the option that the individual would choose if they opt out. In this case, setting that option as the default is ensured to give the individual the best possible option and avoid any potentially normative opt-out cost. Relaxing the assumption that the planner knows x^*, we find the following characterization of robustness in the quadratic/Gaussian case:

Figure 1: Illustration of the Optimal Default



Proposition 7. ***Robust Optimal Defaults when the Intrinsic Optimum is Unknown***

- *The ψ-optima are the expected intrinsic optimum and the most extreme default possible in the positive or negative direction (henceforth the* extremum default*).*

[16] Formally, the set of ψ-optima is $\{d | d = x^* \text{ or } d < \underline{x} \text{ or } d > \overline{x}\}$, where $\underline{x}$ and $\overline{x}$ are the thresholds for active choice. These thresholds are equal to -2 and +2 in the illustration in Figure 1A.

- *None of the ψ-optima are globally robust.*
- *If the expected intrinsic optimum is ψ-optimal for some ψ in the interior of Ψ^*, the expected intrinsic optimum is the unique robust optimum.*
- *If the expected intrinsic optimum is not ψ-optimal for any $\psi \in \Psi^*$, the extremum default is the unique robust optimum.*
- *In the knife-edge case where the expected intrinsic optimum is ψ-optimal for some ψ on the boundary of Ψ^* but not the interior, both the expected intrinsic optimum and the extremum default are robust optima.*

Corollary 7.1. ***Robust Control and the Optimal Default.*** *Suppose $\Psi^* = B(\kappa, \psi)$ for some $\kappa > 0$ and some $\psi \in \Delta(\Theta)$. If the extremum defualt is ψ-optimal, there is a threshold $\overline{\kappa}$ such that*

- *the extremum default is the unique robust optimum for $\kappa < \overline{\kappa}$, but*
- *the expected intrinsic optimum is the unique robust optimum for $\kappa > \overline{\kappa}$.*[17]

If the expected intrinsic optimum is ψ-optimal, the expected intrinsic optimum is the robust optimum.

The logic of the proof is illustrated by Figure 1B; see Appendix A for a formal argument. When the intrinsic optimum is unknown, the default that maximizes welfare depends on the normative judgment about whether and to what extent the opt-out cost implied by revealed preferences, γ, is normative. The welfare-maximizing default is the expected intrinsic optimum when $\psi(\theta^D)$ is sufficiently large, while the extremum default maximizes welfare when $\psi(\theta^D)$ is sufficiently small. As such, a global robustness criterion like Bernheim and Rangel [2009] is inapplicable, provided that sufficiently extreme defaults (i.e. those where sufficiently many individuals make active choices) are feasible. Even so, however, the above characterization reveals that there is still a sense in which the setting the expected intrinsic optimum as the default (i.e. minimizing opt-outs) is a more robust policy recommendation than an extremum default. As we can see in Figure 1B, the expected intrinsic optimum remains a local optimum as we vary normative weights, while the active choice policy becomes strictly worse when we put more weight on the possibility that opt-out costs are normative. The intuition that this makes the the extremum default a less robust optimum appears in Goldin and Reck [2022]; here we find that operationalizing robustness allows us to formalize that intuition. Our proof of this result leverages the simplifying structure of our illustrative simulations, but the result should generalize to some degree. For less restrictive treatments of the optimal defaults problem, refer to Goldin and Reck [2022]; Bernheim et al. [2015]; Bernheim and Gastell [2021].

6.3.2 (Unconstrained) Optimal Reference Points

In this example, we employ a two dimensional version of Example 1.2 and some additional simplifying structure to derive a reduced-form representation of the planner's normative objective with some interesting commonalities to the previous example.

[17] In the knife-edge case $\kappa = \overline{\kappa}$, both the extremum default and the expected intrinsic optimum are κ-ψ robust.

We suppose options are two-dimensional $x = (x_1, x_2)$, and that intrinsic utility is quasi-linear with x_2 the numeraire. The individual faces a budget constraint for given prices and income (components of σ), with price of p_2 normalized to 1 and $p_1 = p$. The form of gain-loss utility follows from equation (29).

$$u(x_1, x_2, \theta^A) = \log(x_1) + x_2.$$

$$px_1 + x_2 = Z.$$

$$V(x_1, x_2, r_1, r_2) = -1\{x_1 \le r_1\}\Lambda_1[\log(r_1) - \log(x_1)]^\beta - 1\{x_2 \le r_2\}\Lambda_2[r_2 - x_2]^\beta \tag{43}$$

We are interested in whether and when the planner might wish to induce the individual to use a different reference point. Evidence from the lab and the field suggests that policy reforms can indeed *shift reference points to some extent* [e.g. Homonoff, 2018; Rees-Jones, 2018; Seibold, 2021]. However, the full policy space $\mathcal{P}$ is difficult to characterize in applied settings where reference dependence appears to matter, because there is no consensus about how to model the formation of reference points. Here, we consider an environment with fixed prices and incomes, and we suppose that the planner can set the reference point at any point on the budget constraint: $\mathcal{P} = \{(r_x, r_y) | pr_x + r_y = Z\}$.[18] This confers a likely unrealistic amount of power on the planner to shape the individual's reference point, but with this approach we nevertheless find a thought-provoking characterization of robustness. To see why, first note that the model admits a reduced-form representation for welfare in terms of a single dimension of choice, x_1, that has some common features with the previous example. Assuming an interior solution, for fixed p and Z, we can re-write intrinsic utility as:

$$u(x_1, \theta^A) = \log(x_1) + Z - px_1.$$

$$V(x_1, r_1) = \begin{cases} -\Lambda_1[\log(r_1) - \log(x_1)]^\beta, & x_1 \le r_1 \\ -\Lambda_2[px_1 - pr_1]^\beta, & x_1 > r_1. \end{cases} \tag{44}$$

The *intrinsic optimum* is here characterized by $x_1^* = \frac{1}{p}$; for numeric illustration here we simply set $p = 0.1 \implies x_1^* = 10$.[19] To simulate welfare in the model, we suppose $\Lambda_1 = \Lambda_2 = 0.5$, we set $Z = 10$. We express welfare in equivalent variation units relative to any baseline where $x_2 > r_2$, and further normalize this as a share of income.[20]

Figure 2 plots welfare as a function of the reference point for x_1, where the reference point for good 2 is then pinned down by the budget constraint. In the first panel, we rule out diminishing sensitivity ($\beta = 1$), and in the second we include it, supposing $\beta = 0.5$. Without diminishing sensitivity, we find that the intrinsic optimum is the globally robust optimum, as in the defaults model under known x^*, and in this case it is also the unique ψ-candidate optimum for any ψ. That the optimal reference point is the intrinsic optimum when $\psi(\theta^D) = 1$

[18] If we relax the restriction that (r_1, r_2) must lie on the budget constraint, the globally robust optimum is for the planner to set the lowest reference point possible along each dimension; see Reck and Seibold [2023] Appendix B for further discussion.

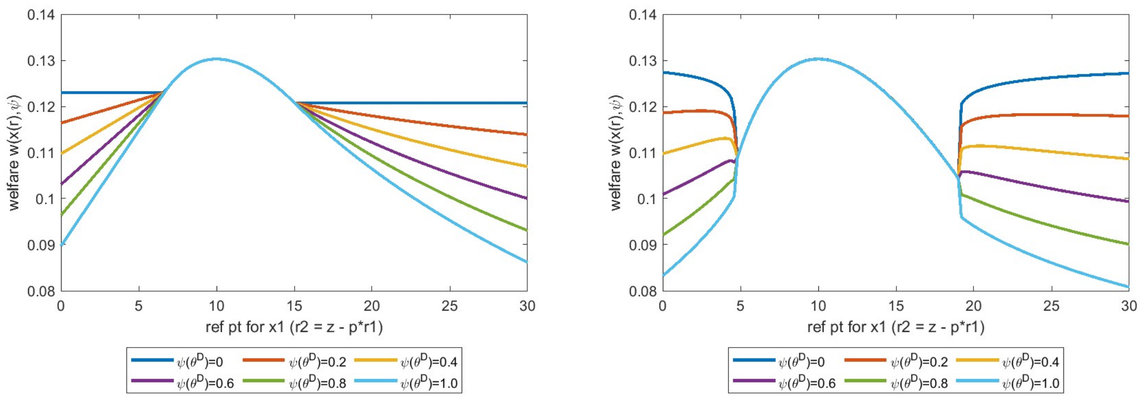
[19] Varying prices is a straightforward extension building on our work in the next subsection. Introducing price variation requires specifying how such variation affects the reference point.

[20] In other words, we plot $\tilde{w}(x, \psi) = \frac{E_\psi[u(x,\theta)] - Z}{Z}$. This serves to make the y-axis more interpretable but it is not essential for the main point here.

Figure 2: Illustration of Optimal Reference Points

A. Without Diminishing Sensitivity ($\beta = 1$)

B. With Diminishing Sensitivty ($\beta = 0.5$)



is key to the Preferred Personal Equilibrium concept of Kőszegi and Rabin [2007]. In a non-stochastic environment, selecting a Preferred Personal Equilibrium from the set of Personal Equilibria is equivalent to solving our planner's problem under $\psi(\theta^D) = 1$. In fact, we learn here that the planner – or an individual setting a reference point to maximize the welfare off their future, reference-dependent self as in Fudenberg and Levine [2006] – would also want to set the intrinsic optimum as the reference point for any $\psi(\theta^D)$.

We observe that welfare behaves differently in three domains in both panels of Figure 2A. To understand why, first observe that when the reference point is on the budget constraint, the individual can either consume the reference point itself, a bundle with $x_1 < r_1$ and $x_2 > r_2$ (a loss over good 1) or a budle with $x_1 > r_1$ and $x_2 < r_2$ (a loss over good 2). When the reference point falls around the intrinsic optimum of $x_1^* = 10$, the individual chooses the reference point, so $V(x, r) = 0$ because there are no gains or losses, and their welfare is peaked around 10 because this is the intrinsic optimum. At a very high reference point for good 1, the individual chooses to consume some $x_1 > x_1^*$ to reduce their losses over good 1 due to Loss Aversion. Similarly at a very low reference point for good 1, the individual consumes more x_2 to reduce losses in x_2 and therefore consumes less of good 1 than x_1^*. Without diminishing sensitivity x_1 is constant over r in the latter two cases, so under $\psi(\theta^D) = 0$, welfare is flat. When $\psi(\theta^D) > 0$, changing the reference point has direct welfare effects, by increasing the size losses, and consequently, welfare falls further as r moves to further extremes and the losses grow.

With diminishing sensitivity, welfare unsurprisingly behaves similarly in the domain where $x = r$ but we find non-monotonicity at extreme reference points for small $\psi(\theta^D)$. The intuition here is similar to active choices above: under diminishing sensitivity, as losses grow to an extreme, the individual stops trying to avoid losses. When the losses themselves receive little welfare weight (($\psi(\theta^D)$ is near zero), this is desirable, because the planner is judging that the individual *should* stop trying to avoid losses. However, when loss aversion is viewed more as a normative preference ($\psi(\theta^D)$ is near 1) the direct negative welfare effect of imposing extreme losses on the individual makes extreme reference points highly undesirable. Like extremum defaults, reference points that generate extreme losses are desirable under $\psi(\theta^D) = 0$ but this desirability is *not robust*. Based on what we found in Figure 1, it is obvious that if we introduced some uncertainty about the intrinsic optimum, we could even get an extreme reference point to be ψ-optimal for some sufficiently small $\psi(\theta^D)$, but this will tend not to be robust just as in Proposition 7.

That our notion of robustness plays out very similarly in the defaults and reference points models (compare Figure 1B and Figure 2B) can be understood as an implication of Proposition 6. In both examples, setting the default or reference point at the (expected) intrinsic optimum minimizes disagreements about welfare across frames.

6.3.3 Corrective Taxation

Let us consider optimal corrective taxation in the biases versus strange preferences example. Suppose for simplicity that we are in the quasi-linear environment from the previous example, and introduce a nonlinear tax on good 1 according to a tax schedule $T(x_1)$, which is fully incident on consumers – i.e. prices p are invariant to T. Utility under the alternative/classical preferences frame θ^A is

$$u(x_1, \theta^A) = \mu(x_1) + Z - px_1 - T(x_1) + R$$

where the sub-utility function $\mu(x_1)$ is twice differentiable, increasing, and concave. The variable R is rebated revenue from the corrective tax, which is determined by the simple government budget constraint $R = T(x)$. Suppose further that the tax is unrelated to the behavioral friction, so $V(x)$ is invariant to T; this rules out misperception of tax incentives.[21]

Expressing the disagreement $V(x) = u(x, \theta^D) - u(x, \theta^A)$ as a function of x_1 (leveraging the budget constraint as in the previous example), we write

$$w(x) = u(x_1, \theta^A) + \psi(\theta^D)V(x_1)$$

Following the same logic as Mullainathan et al. [2012], the ψ-optimal corrective tax is

$$T^*(x; \psi) = [1 - \psi(\theta^D)]V(x) + C, \tag{45}$$

where C is a constant pinned down by the government budget constraint. This can be understood by taking a derivative with respect to x_1. Where T^* is differentiable with respect to x_1, we find

$$\frac{\partial T(x_1; \psi)}{\partial x_1} = [1 - \psi(\theta^D)]\frac{dV(x_1)}{dx_1},$$

equating the marginal tax rate with the expected marginal internality (see equation 20). With this tax schedule, the individual's choice of x_1 optimizes the planner's expectation for their

[21]This is a common assumption in prior work on corrective taxation, but there are many proposed theories of tax misperception in which the assumption is obviously violated. Integrating the theory of corrective taxes for internalities with the theory of tax misperceptions is beyond the scope of this paper.

welfare over x_1:

$$\begin{aligned} u(x_1, \theta^D) &= \mu(x_1) + Z - px_1 - T(x_1) + R + V(x_1) \\ &= \mu(x_1) + Z - px_1 + \psi(\theta^D)V(x_1) + R \\ &= u(x_1, \theta^A) + \psi(\theta^D)V(x_1) = w(x_1) \end{aligned} \tag{46}$$

For a given amount of x_1 chosen by the individual (under BIC in the frame θ^D), we note that welfare is increasing in $\psi(\theta^D)$ if $V(x_1) > 0$ and decreasing if $V(x_1) < 0$. These observations lead straightforwardly to the following characterization of the robust optimal corrective marginal tax rate:

Proposition 8. *Let $\underline{\psi} \equiv \min_{\psi \in \Psi^*} \psi(\theta^D)$, and let $\overline{\psi} \equiv \max_{\psi \in \Psi^*} \psi(\theta^D)$. The robust optimal marginal tax rate given Ψ^* is*

$$\frac{dT^*(x_1)}{dx_1} = \begin{cases} [1 - \underline{\psi}]\frac{dV(x_1)}{dx_1} & V(x_1) > 0 \\ [1 - \overline{\psi}]\frac{dV(x_1)}{dx_1} & V(x_1) < 0 \\ 0 & V(x_1) = 0. \end{cases} \tag{47}$$

The intuition is as follows: the ambiguity averse planner wishes to set a tax rate that is optimal in the worst case scenario for true preferences. When $V(x_1) > 0$ at some chosen x_1, by construction $u(x_1, \theta^D) > u(x_1, \theta^A)$, so the worst-case scenario places maximal weight on the "biases" case θ^A and minimal weight on the "strange preferences" case θ^D. When $V(x_1) < 0$, the opposite is true and the worst-case scenario places maximal weight on the strange preferences view. While we do not examine in detail the joint optimality of a two-dimensional policy involving e.g. a default and a corrective tax, we observe that at a robust optimum (e.g. opt-out minimization for defaults), $V(x_1)$ will be small or zero (e.g. depending on whether x^* is known). This in turn suggests a small or zero corrective tax.

7 Conclusion

The core argument of our paper is that a primary obstacle to the development of behavioral welfare economics – the question of how to do welfare analysis when we get conflicting information from revealed preferences – is the intrapersonal analogue of a much older and more familiar problem: interpersonal comparisons of utility. We exploit the parallel between interpersonal and intrapersonal problems to develop criteria for the welfarist evaluation of policy in the presence of uncertainty about an individuals' normative preferences, and we explored what insights the resulting criteria might provide, in general and in the context of specific examples.

Showing that welfare in intrapersonal and intrapersonal problems can be modelled very similarly could be interpreted optimistically or pessimistically, depending on one's views about how economists typically approach interpersonal comparisons. From a pragmatic perspective, our results give applied researchers the tools to conduct welfare analysis when they wish to respect some revealed preferences but are unsure how to resolve ambiguity in revealed preferences. Specifically, we provide applied researchers the conceptual tools to separate empirical quantities that are informative for policy (e.g. what is the magnitude of potential internalities, how does behavior respond to a policy reform, etc) from normative judgments about how exactly these quantities map to an optimal policy. We require some potentially objectionable assumptions like those surrounding comparability, but after having made these assumptions we can ask how disagreements about normative judgments and ambiguity map to disagreements or ambiguity in the optimal policy. While such an approach is far from universally accepted in interpersonal problems, it has become popular in recent decades because it allows economists to inform optimal policy using empirical data, without taking a stand on difficult normative questions like the value of equity.

We identify a few opportunities for future work. From a theoretical perspective, a few generalizations of our results are available, upon overcoming some technical challenges. One could formalize the derivation of a normative criteria without endowing the planner with primitive beliefs, e.g. using the approach of Savage [1954] rather than Von Neumann and Morgenstern [1953]. One could allow for a continuum of frames, which might clarify to some degree the relationship of our approach to the "counterfactual normative consumer" approach. More ambitiously, it might be possible to extend our framework to accommodate limited attention, as discussed in Section 2.2. One could also derive more and better ways to think about comparability using behavioral decision theory, as suggested by some of the results in Ellis and Masatlioglu [2022]. From a more applied perspective, our work on how our notions of robustness play out in examples barely scratches the surface of what is possible. Future work, potentially including future revisions of this paper, should consider what more we can learn by applying these notions of optimality and robustness in a broader array of settings.

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A Proofs

Proposition 7.

Proof. In the defaults case, the policy parameter is 1-d $\sigma = d$, the default option. As discussed previously, this is usually thought of as an example of Bias vs Strange Preferences - where under θ^D, the as-if cost implied by behaviour is normative, and under θ^A it is a pure bias. $\psi(\theta^D)$, which we abbreviate to just $\psi = \mathbb{P}[\theta = \theta^D]$.

$$u(x, \theta^D, d) = u(x) - \gamma \cdot \mathbb{1}\{x \neq d\} \tag{48}$$
$$u(x, \theta^A, d) = u(x) \tag{49}$$

Therefore, $V(x, d) = -\gamma \cdot \mathbb{1}\{x \neq d\}$ and welfare $W(d) = u\big(x(d)\big) - \psi^D \cdot \gamma \cdot \mathbb{1}\{x(d) \neq d\}$.

As a simple example, let $u(x) = -\frac{\alpha}{2}(x - x^*)^2$ where x^* is unknown. $x, x^* \in X$, the choice set which is $X \subset \mathbb{R}$ and defaults at the max and min of X force the consumer to choose actively. $\psi \in [0, 1]$.

1. First, show that the expected intrinsic optimum d_{min} default is a $\kappa - \psi$ robust optimum for any ψ making d_{min} a candidate optimum. This $\psi \approx 1$. So, $B(\kappa, \psi) = [\psi - \kappa, min(\psi + \kappa, 1)]$. Recall $W(d, \psi') = -\frac{\alpha}{2}(x(d) - x^*)^2 - \psi' \cdot \gamma \cdot \mathbb{1}\{x(d) \neq d\}$. Since $\gamma > 0$...

$$\arg \min_{\psi' \in B(\kappa, \psi)} W(d_{min}, \psi') = min(\psi + \kappa, 1) \tag{50}$$

The evil agent wants to make the opt-out cost as large as possible so chooses ψ' as large as possible. Therefore, the $\kappa - \psi$ robust optimum is defined by...

$$d^* = \arg\max_d -\frac{\alpha}{2}(x(d) - x^*)^2 - min(\psi + \kappa, 1) \cdot \gamma \cdot \mathbb{1}\{x(d) \neq d\} \tag{51}$$

Since d_{min} is a candidate optimum for ψ, it is also a candidate optimum for $\psi' > \psi$ since under those judgements the opt-out cost is strictly more likely to be normative - suggesting that minimizing opt-outs will be better. Therefore, d_{min} is a $\kappa - \psi$ robust optimum for any κ.

2. Now show that the penalty default is only a $\kappa - \psi$ robust optimum for small κ. Let ψ be the judgement which makes the penalty default a candidate optimum ($\psi \approx 0$). Then, $B(\kappa, \psi) = [max(0, \psi - \kappa), \psi + \kappa]$. Similarly to the minimizing opt-outs example, the evil agent wants to maximise ψ' and so sets...

$$\arg \min_{\psi' \in B(\kappa,\psi)} W(d_{pen}, \psi') = \psi + \kappa \tag{52}$$

By definition, $x(d_{pen}) = x^*$ and the individual opts-out for sure, therefore...

$$\min_{\psi' \in B(\kappa,\psi)} W(d_{pen}, \psi') = 0 - \gamma(\psi + \kappa) \tag{53}$$

Consider an alternative policy $\bar{d} = \mathbb{E}[x^*]$, i.e. the minimizing opt-out default. From before, we know that...

$$\min_{\psi' \in B(\kappa,\psi)} W(\bar{d}, \psi') = \underbrace{-\frac{\alpha}{2}(\mathbb{E}[x^*] - x^*)^2}_{=-\Lambda \text{ fixed w.r.t. } \kappa} - (\psi + \kappa) \cdot \gamma \cdot \underbrace{\mathbb{P}\{x(\bar{d}) \neq \bar{d}\}}_{=p \text{ small}} \tag{54}$$

Therefore, $\bar{d} \succ d_{pen}$ if

$$\begin{aligned} & -\Lambda - (\psi + \kappa) \cdot \gamma \cdot p > -\gamma(\psi + \kappa) \\ \iff & \gamma \cdot (\psi + \kappa) \cdot (1 - p) > \Lambda \\ & \iff \kappa > \frac{\Lambda}{\gamma \cdot (1 - p)} - \psi \triangleq \bar{\kappa} \end{aligned}$$

where $\bar{\kappa}$ is most likely > 0 given $\psi \approx 0$. I.e. d_{pen} is only a $\kappa - \psi$ robust optimum for at most $\kappa < \bar{\kappa}$. Importantly, note that $\bar{\kappa}$ is **decreasing** in $\gamma = V(x(d_{pen}), d_{pen})$.

■