Homework 1

Intro to Robotics

Due Date: March 6th, 2023

MAKE SURE TO SHOW WORK FOR ALL QUESTIONS, DO BY HAND!!

1 Matrix addition, scalar multiplication and transpose

$$5 * \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$

What is the transpose of Matrix D ?

$$D = \begin{bmatrix} -1 & 9 & 17 \\ 44 & -122 & 8 \\ 24 & 0 & 1 \end{bmatrix}$$

2 Matrix multiplication

$$\begin{bmatrix} 7 & 0 & 8 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 9 & 6 \\ 1 & 5 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 2 \\ 9 & 6 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 7 & 0 & 8 \\ 2 & 4 & 3 \end{bmatrix} =$$

Explain why the matrix multiplication below is not possible.

$$\begin{bmatrix} 7 & 0 & 8 \\ 2 & 4 & 3 \\ 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} 7 & 0 \\ 2 & 4 \end{bmatrix} =$$

3 Inverse and RREF

What is the inverse matrix of A?

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 0 & 7 \\ 2 & 3 & 1 \end{bmatrix}$$

Explain why the inverse of B does not exist?

$$B = \begin{bmatrix} 1 & -2 & 1 \\ 3 & 9 & 3 \\ -9 & -27 & -9 \end{bmatrix}$$

4 Determinant

What is the determinant of C? is C singular ?

$$C = \begin{bmatrix} 3 & 7 & 1 \\ 1 & -4 & 6 \\ 8 & 8 & 8 \end{bmatrix}$$

5 Cross product and normal vector

Compute the cross product of $v_1 \times v_2$

$$v_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \; v_2 = \begin{bmatrix} 8 \\ -4 \\ 3 \end{bmatrix}$$

Let n be the normal vector. Draw v_1, v_2, and n with respect to each other. Show that the matrix M is non-singular without computing the determinant.

$M = \begin{bmatrix} v_1 v_2 n \end{bmatrix}$

6 Dot product

Solve for the angle between v_1 and v_2

$$v_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \; v_2 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$$

7 Frames, translations and rotations

Given frames: Frame {A} = universe,

Frame {B} = $\{{}^A_B R = \begin{bmatrix} cos(135) & -sin(135) \\ sin(135) & cos(135) \end{bmatrix}, {}^A P_{Borg} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}\}$,

Frame {C} = $\{{}^B_C R = \begin{bmatrix} cos(-30) & -sin(-30) \\ sin(-30) & cos(-30) \end{bmatrix}, {}^B P_{Corg} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}\}$,

Frame {D} = $\{{}^A_D R = \begin{bmatrix} cos(0) & -sin(0) \\ sin(0) & cos(0) \end{bmatrix}, {}^A P_{Dorg} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}\}$,

Frame {E} = $\{{}^A_E R = \begin{bmatrix} cos(60) & -sin(60) \\ sin(60) & cos(60) \end{bmatrix}, {}^A P_{Eorg} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$,

Frame {F} = $\{{}^A_F R = \begin{bmatrix} cos(45) & -sin(45) \\ sin(45) & cos(45) \end{bmatrix}, {}^A P_{Forg} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}\}$,

Frame {G} = $\{{}^F_G R = \begin{bmatrix} cos(90) & -sin(90) \\ sin(90) & cos(90) \end{bmatrix}, {}^F P_{Gorg} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}\}$

Given points: ${}^A P_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, ${}^B P_2 = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$, ${}^C P_3 = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$, ${}^D P_4 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, ${}^E P_5 = \begin{bmatrix} .7 \\ .7 \end{bmatrix}$, ${}^F P_6 = \begin{bmatrix} -3.14 \\ 2.718 \end{bmatrix}$, ${}^G P_7 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Questions:

1. Draw the frames and points given above.
2. Compute ${}^A_C T$ and ${}^A_G T$.
3. Find ${}^D P_1$ and ${}^A P_4$.
4. Find ${}^E P_1$ and ${}^A P_5$.
5. Find ${}^B P_1$ and ${}^A P_2$.
6. Find ${}^F P_1$ and ${}^A P_6$.
7. Find ${}^F P_2$ and ${}^B P_6$.
8. Find ${}^D P_5$ and ${}^E P_4$.
9. Find ${}^C P_7$ and ${}^G P_3$.

A. Apply a rotation of $-80°$ and translate by vector $v = \begin{bmatrix} -6 \\ 7 \end{bmatrix}$ to ${}^A P_1$.

B. Apply a rotation of $130°$ and translate by vector $v = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ to ${}^C P_7$.

8 Programming question 1

In python use numpy, opencv, or code your own functions to verify your answers are correct for parts 1-6. Or you can use c++ with eigen(https://eigen.tuxfamily.org/index.php?title=Main_Page) or install opencv in c++, to verify your answers for parts 1-6. **Submit as .py or .cpp file. I should be able to run your code and get the answers for parts 1-6.**

9 Programming question 2

Create a class in python or c++ to represent a 2D frame object. Make sure to include at least parent frame, child frame, origin, and orientation.
1. Write a function for multiplying frames. This should return a new frame object.

2. Write a function that takes as input a point, rotation, and translation, then returns a new point that has been rotated and translated.

3. Write a function that takes from_frame, to_frame, and a point in from_frame as input, and returns the point in to_frame.

4. Use your class to verify your answers from part 7.
submit the .py or .cpp files you created