Exam 1

Intro to Robotics

Name:

SHOW WORK FOR ALL QUESTIONS FOR FULL CREDIT!

1

Given $v_1 = \begin{bmatrix} 0 \\ 2 \\ 2.24 \end{bmatrix}$ $v_2 = \begin{bmatrix} 4 \\ 0 \\ -3 \end{bmatrix}$

1. Compute the cross product of $v_1 \times v_2$

2. Compute the angle between v_1 and v_2. Hint: $2.24^2 \approx 5$ and arccos is valid for all real numbers between -1 and 1.

3. Use your answers from 1 and 2 to reason that the matrix $\begin{bmatrix} v_1 & v_2 & n \end{bmatrix}$ is non-singular. n is the normal vector computed in question 1.

2

Given frames: Frame {A} = universe,

Frame {B} = $\{{}^A_B R(30°) = \begin{bmatrix} .87 & -.5 \\ .5 & .87 \end{bmatrix}, {}^A P_{Borg} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}\}$,

Frame {C} = $\{{}^B_C R(90°) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, {}^B P_{Corg} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}\}$

Given points: ${}^A P_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$, ${}^B P_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, ${}^C P_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

1. Find ${}^B P_1$
2. Find ${}^A P_3$

3

Given frames: Frame $\{A\}$ = universe,

Frame $\{B\} = \{{}^A_BR = \begin{bmatrix} .5 & -.15 & .85 \\ .5 & .85 & -.15 \\ -.71 & .5 & .5 \end{bmatrix}, {}^AP_{Borg} = \begin{bmatrix} 2 \\ .7 \\ 1 \end{bmatrix}\}$

1. Find the eulerian angles for ${}^A_BR_{Z'Y'X'}$.

2. Given ${}^BP_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ what is ${}^AP_2$?

3. When does gimbal lock occur? and when solving this question did you encounter gimbal lock ?

4

1. rotate the 2d vector $P_4 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ by $30°$ using complex rotations. Hint: no need for calculator if you look at question 2.
2. Express $R_{Z'X'Y'}(90°, 0°, 45°)$ in matrix form. Pay close attention to the subscript.

5 formulas

1. $\begin{bmatrix} {}^A_B R^T & -({}^A_B R^{T\,A} P_{Borg}) & \\ 0 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} c\alpha c\beta & s\gamma s\beta c\alpha - c\gamma s\alpha & c\gamma s\beta c\alpha + s\gamma s\alpha \\ s\alpha c\beta & s\gamma s\beta s\alpha + c\gamma c\alpha & c\gamma s\beta s\alpha - s\gamma c\alpha \\ -s\beta & s\gamma c\beta & c\gamma c\beta \end{bmatrix}$

3. $cos(45^\circ) = sin(45^\circ) = .71$