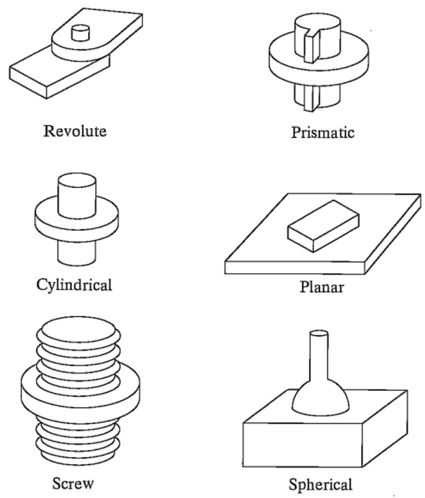
Kinematics

1 Introduction

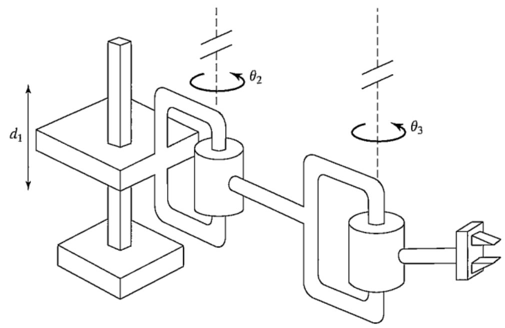
Kinematics is the science of motion that treats the subject without regard to the forces that cause it. Within the science of kinematics, one studies the position, the velocity, the acceleration, and all higher order derivatives of the position variables(with respect to time or any other variable(s)).

Links and joints are the fundamental components of a robot. **Joints** represent movement on one rotational axis(x, y or z as rotational axis), or a one parameter translation(motion on x,y, or z axis)(one degree of freedom per joint). To fully represent an object in 3d space, we need six parameters; three for rotation (α, β, γ) and three for translation((x, y, z)-position of joint) (any more is considered redundant), because of this most robots are constructed using six joints to be able to fully express their position and orientation in 3d. **Links** are the static(unchanging) parts of a robot that connect joints to each other. Given n joints you will have n-1 links; think of links as edges and joints as nodes in a graph.

Examples of joints:
prismatic, spherical, planar(translation with one degree of freedom)
revolute, screw, cylindrical(rotational with one degree of freedom)



2 Conventions for link-frame assignment

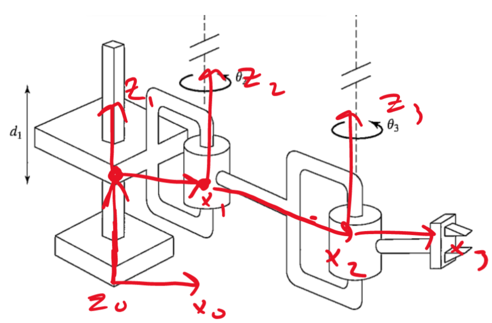


Algorithm:
1. Identify the joint axes and imagine (or draw) infinite lines along them. For steps 2 through 5 below, consider two of these neighboring lines (at axes i and i + 1).

2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the ith axis, assign the link-frame origin.
3. Assign the z_i axis pointing along the ith joint axis.
4.Assign the x_i axis pointing along the common perpendicular, or, if the axes intersect, assign x_i to be normal to the plane containing the two axes.

5. Assign the y_i axis to complete a right-hand coordinate system(optional step).

6. Assign frame {0} to match frame {1} when the first joint variable is zero. For N, choose an origin location and x_n direction freely, but generally so as to cause as many linkage parameters as possible to become zero.



3 Parameters for dh table

The DH table is a method for organizing parameters from a system of known joints, links, and frames. These parameters give us general formulas for constructing the transformations matrices required for computations.

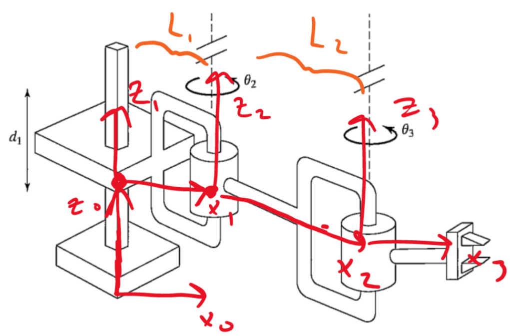
a_i=Distance from z_i to z_{i+1} along the x_i(this is the link length, and mutual perpendicular between frame {i} and {i+1})

α_i=link twist angle, angle from z_i to z_{i+1} measured along x_i (angle between axis i and i+1)

d_i=link offset, the distance from x_{i-1} to x_i measured along z_i (the offset distance of one link to the next)

θ_i=the angle from x_{i-1} to x_i measured along z_i (rotation of link w.r.t. its neighbor along common axis)

DH table				
Frame {i}	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	d_1	$\theta_1 = 0$
2	L_1	0	0	θ_2
3	L_2	0	0	θ_3



4 dh table to transformation matrix

Now that we have filled out the DH table, we can make all our transformation matrices using the matrix formula, Note the transformation uses rotation basis: $(R_z(\theta_i)R_x(\alpha_{i-1}))^T$. Tip - Each row corresponds to one transformation matrix, so row 1 has all the variables need for ${}^0_1T$, row 2 has all the variables for ${}^1_2T$ and so on.

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Note: Use c0=1 and s0=0 to simplify.

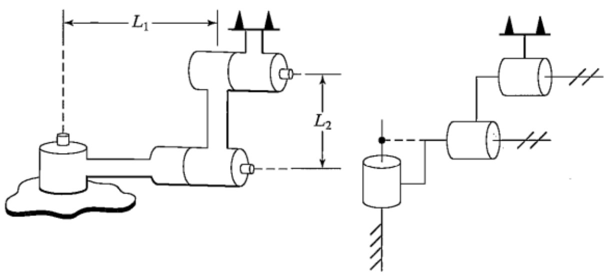
$$
{}^{0}_{1}T = \begin{bmatrix} c0 & -s0 & 0 & a_0 \\ s0c0 & c0c0 & -s0 & -s0d_1 \\ s0c0 & s0c0 & c0 & c0d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 c0 & c\theta_2 c0 & -s0 & -s0d_2 \\ s0c0 & s0c0 & c0 & c0d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_1 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

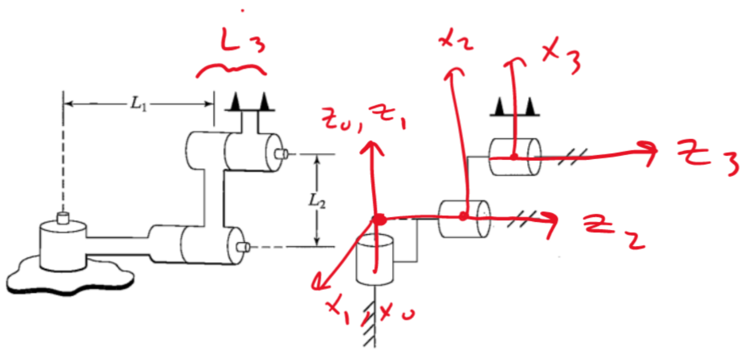
$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 c0 & c\theta_3 c0 & -s0 & -s0d_3 \\ s0c0 & s0c0 & c0 & c0d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The whole system can be manipulated using the transformation:
${}^{0}_{3}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T$

5 2nd Example



(a) (b)



(a) (b)

$a_0 = dist(z_0, z_1)$ in $x_0 = 0$
$a_1 = dist(z_1, z_2)$ in $x_1 = 0$
$a_2 = dist(z_2, z_3)$ in $x_2 = L_2$

$\alpha_0 = angle(z_0, z_1)$ in $x_0 = 0$
$\alpha_1 = angle(z_1, z_2)$ in $x_1 = -90°$
$\alpha_2 = angle(z_2, z_3)$ in $x_2 = 0$

$d_1 = dist(x_0, x_1)$ in $z_1 = 0$
$d_2 = dist(x_1, x_2)$ in $z_2 = L_1$
$d_3 = dist(x_2, x_3)$ in $z_3 = L_3$

$\theta_1 = angle(x_0, x_1)$ in $z_1 = \theta_1$
$\theta_2 = angle(x_1, x_2)$ in $z_2 = \theta_2 - 90°$
$\theta_3 = angle(x_2, x_3)$ in $z_3 = \theta_3$

DH table				
Frame {i}	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	$-90°$	L_1	$\theta_2 - 90°$
3	L_2	0	L_3	θ_3

Using ${}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$ we get:

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_2T = \begin{bmatrix} c(\theta_2 - 90°) & -s(\theta_2 - 90°) & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ s(\theta_2 - 90°) & c(\theta_2 - 90°) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$