ME-442-TERM PROJECT

Cankan Çelik

2226736

PROBLEM STATEMENT

The simplified model of an electromagnetic suspension system is given below, where a metal object is subjected to the magnetic field and is able to float in a vacuum chamber so as not to be affected by air drag. By changing the voltage, the position of this metal object can be controlled. The position of the object is monitored via a sensor attached to the side of the moving platform.

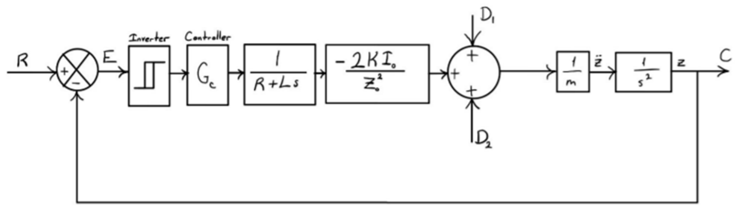

Figure 1. Block Diagram of the System

Table 1. System Properties

N(Turns)	L	R	I_0	$m_{platform}$	$m_{passenger}$	μ	Area	Z_0
100	10 H	500 Ω	10 A	100 kg	60 kg	2.9×10^{-4} F/m	20 m^2	1.72 m

a) *Desired Close Loop Pole with 2% t_s =5 s, ζ = 0.6, e_{ss} = 0 for step input*

No Disturbance

First of all Root Locus of the uncompansated system obtained. As seen from Figure 2. it is an unstable system except the gain K=0. Also, Open loop poles are s_1 = -50 and $s_{2,3}$ = 0 which at K=0 system is marginally stable since the coincide dominant poles on the imaginary axis.

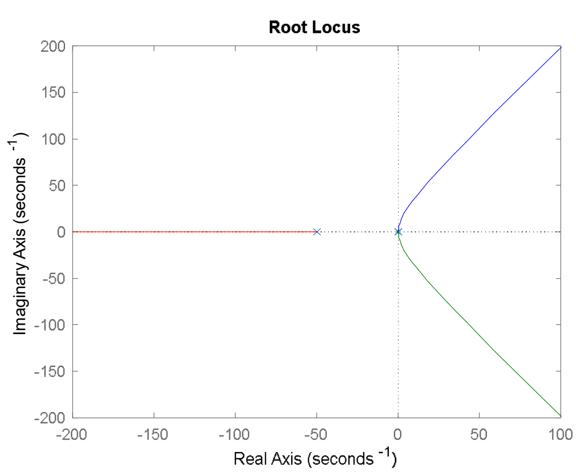

Figure 2. Root Locus of the Uncompansated OLTF

After finding the RL of the uncompansated system angle of deficiency determined and required controller design implemented for the zero disturbance as it follows.

$$\pi - \phi = \pi - 1.6059 = 1.5357\ Rad = 87.98\ ^{\circ}$$

As seen positive angle contribution to system will make system stable for the desired Closed Loop Pole. Using the Anlytical PID method for $K_i = 0$ will provide K_p and K_d and zero location consequently for the desired CLP.

$$K_p = 3.0831 * 10^4 \quad K_d = 3.6816 * 10^3$$

$$z = -\frac{K_p}{K_d} = -8.3743$$

$$Closed\ Loop\ Pole = -8.0000 + 10.6667i$$

For the added zero sytem will compensated. Following figure shows the root locus of compensated OLTF and the RL passes through the desired pole.

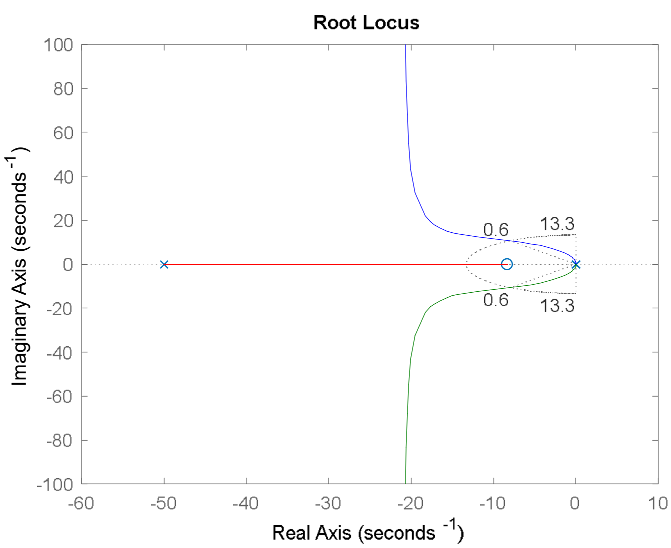

Figure 3. Root Locus of Compensated OLTF

And close loop transfer function of for the compenstaed systems is as it follows.

$$CLTF = \frac{721.8 * s + 6044}{s^3 + 50 * s^2 + 721.8 * s + 6044}$$

As seen on the figure the desired close loop location is satisfied.

$$Desired\ CLP = -8.0000 + 10.6667i$$

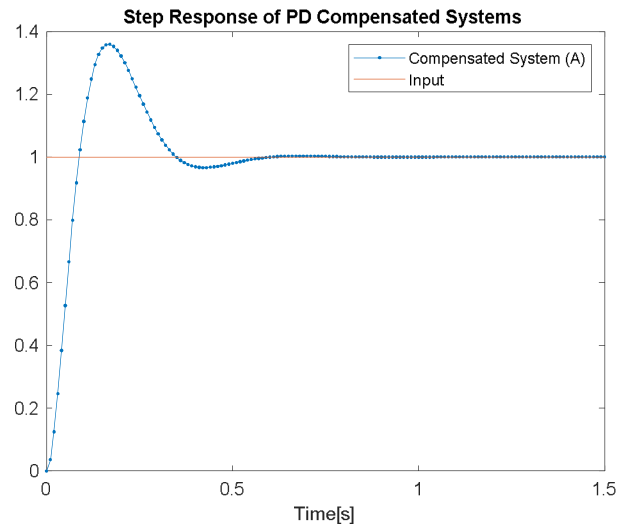

Figure 4. Transient Behaviour of PD Compenstaed System, m = 100 kg

To investigate steady-state error, the type number of the compensated OLTF should be acknowledged.

$$OLTF = \frac{2.135 * 10^6 s + 1.788 * 10^7}{2958 * s^3 + 147920 * s^2}$$

Since type number is 2 steady state at unit-step input is zero. Also, using the FVT where R(s) is equal to unit-step function.

$$e_{ss} = \lim_{s \to 0} s * R(s) * \frac{1}{1 + G_{OL}} = \lim_{s \to 0} s \frac{1}{s} \frac{s^3 + 50 * s^2}{s^3 + 50 * s^2 + 721.8 * s + 6044} = 0$$

b) 2% t_s =5 s, ζ = 0.6, e_{ss} = 0 for step input, m_{total} = 160 kg

In this part, theere will be two changes. The passenger will act on the system as step disturbance and the sytem mass will change. However, for this part disturbances caused by the linearization and weight are not equal to each other since the total weight increased on the other hand K, I_0, Z_0 are constant. Hence, PID controller will be used since there will be steady-state error. The obtained error transfer function of disturbance as it follows. [1]

$$\frac{E(s)}{D(s)} = -\frac{\frac{1}{ms^2}}{1 + \frac{K_d s^2 + K_p s^2 + K_i}{s} * \frac{1}{R + Ls} * \frac{2KI_0}{Z_0^2} * \frac{1}{ms^2}}$$

The first and second disturbance caused by the weight when a passenger on the platform and disturbance caused linearization term respectively.

$$D_1(s) = \frac{1}{s} * 9.81 * 160 = \frac{1}{s} * 1569.6 \qquad D_2(s) = \frac{1}{s} * -\frac{KI_0^2}{Z_0^2} = -\frac{1}{s} * 980.26$$

Total disturbance can be found as below.

$$D(s) = \frac{1}{s} * 589.34$$

One must determine K_i for a decided criteria such as dynamic error approximately e_{ss} = 0.002 for ramp input caused by disturbances since larger disturbances exist in the system. Controller Transfer function obtained.

$$G_c(s) = 1.1697 * 10^5 + \frac{7.5151 * 10^5}{s} + 1.0118 * 10^4 * s$$

OLTF of compensated sytem becomes.

$$OLTF = \frac{1240 * s^2 + 1.433 * 10^4 * s + 9.208 * 10^4}{s^4 + 50 * s^3}$$

And close loop transfer function of for the compenstaed systems is as it follows.

$$CLTF = \frac{1240 * s^2 + 1.433 * 10^4 * s + 9.208 * 10^4}{s^4 + 50 * s^3 + 1240 * s^2 + 1.433 * 10^4 * s + 9.208 * 10^4}$$

As seen on the Table 2. desired pole location s_3 obtained and transient behaviour within the range 2% settling time.

Table 2. Closed Loop Poles of PID Controlled System

CL Poles	Close Loop Pole Locations
s_1	-17.0000 +15.1319i
s_2	-17.0000 -15.1319i
s_3	-8.0000 +10.6667i
s_4	-8.0000 -10.6667i

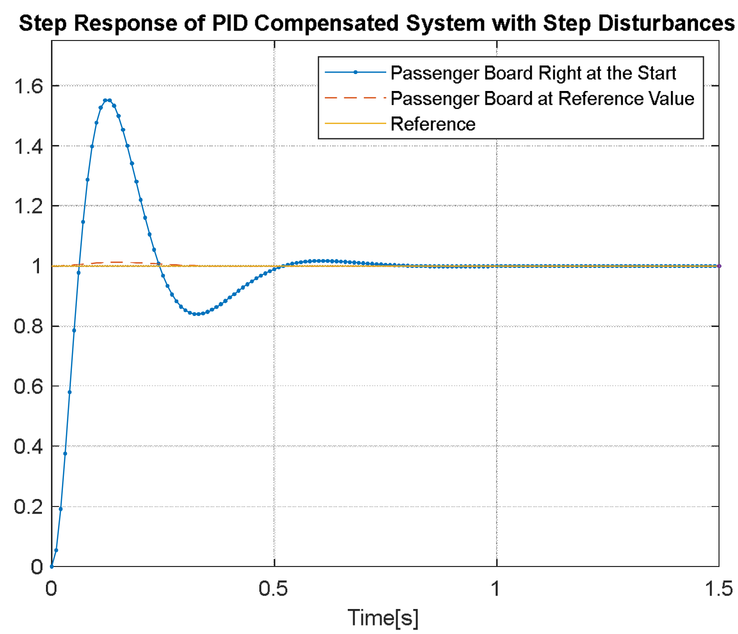

Figure 5. Transient Behaviour of PID Compenstaed System m_{total} = 160 kg

To find the Steady-State error two different error transfer function investigated with the Final Value Theorem. First one is error caused because of the disturbances. Note that unit-step from the disturbances and "s" term in the limit expression cancels out.

$$\frac{E(s)}{D(s)} = \frac{-0.00625 * s^2 - 0.3125s}{s^4 + 50 * s^3 + 1240 * s^2 + 1.433 * 10^4 * s + 9.208 * 10^4}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{-0.00625 * s^2 - 0.3125s}{s^4 + 50 * s^3 + 1240 * s^2 + 1.433 * 10^4 * s + 9.208 * 10^4} 589.34 = 0$$

For the unit-step input, error steady-state obtaines as zero.

$$\frac{E(s)}{R(s)} = \frac{s^4 + 50 * s^3}{s^4 + 50 * s^3 + 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = s\frac{1}{s}\frac{s^4 + 50 * s^3}{s^4 + 50 * s^3 + 1240 * s^2 + 1.433 * 10^4 * s + 9.208 * 10^4} = 0$$

In this part of the project, two different types of control approached considered. One of them is the changing controller after the passenger jumps on the board, second one is the choosing a controller that compensates error and satisfies desired pole location and transient response with or without passenger. Second approach has been chosen since only sensor exist in the sytsem is the position sensor that it is not possible to identify and use adaptive control approach. Error steady-state desired to be zero. Hence, the system can only have zero steady-state error for the PID control approach as mentioned in the part (a). The sytem is designed to have total mass of 160 kg since it is a transportation vehicle and its operation can be considered as the passenger on the board. However, designin controller with this approach cause a larger overshoot that is because of the system 100 kg at the beginning larger gain values causes an overshoot. On the other hand, this is a levitating system unless overshoot is less than 100% it means that it does not hit the floor. In addition it will have better disturbance rejection and stabler operation while transporting the passenger.

c) Time responses before and after the mass change

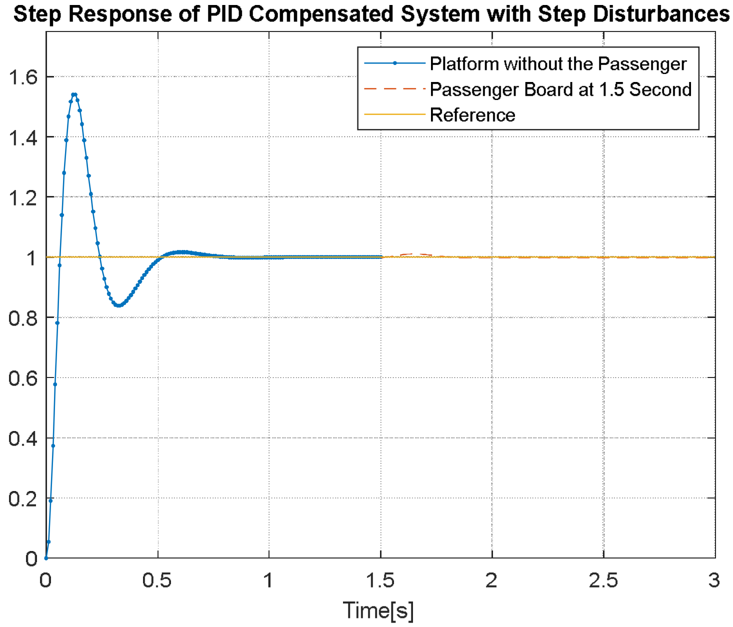

Figure 6. Transient Response Before and After the Mass Change

As mentioned in part (b), PID controller with an appropriate gains are used to control system with or without a passenger. It is exhibited in Figure 6. settling time achieve. Also, eventhough the passenger jumps on the system response in the range of stability. The system was able to compensate the step disturbance with ease since it is designed to have such a mass at the beginning. The only adverse effect can be said is the oscillatory behaviour at the initiation of the system. However, it happens very quickly nearly half of a second and while operating with a passenger the overshoot will be much less since the sytsem designed to operate with a passenger having a 60 kg mass.

d) Find out the steady state error caused by the disturbances D_1 and D_2 Similarly, find the total steady state error.

Error Caused by Disturbances

The obtained transfer function of distrubance for the PD controller in the part ***(a).*** [1]

$$\frac{E(s)}{D_{1,2}(s)} = -\frac{C(s)}{D_{1,2}(s)} = \frac{-0.01s - 0.5}{s^3 + 50 * s^2 + \ 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4}$$

since the system in the vacuum chamber, the one disturbance is the the weight that platform provides at the start. The other disturbance is the caused by the linearization will be taken into account. For the part ***(a),*** the first disturbance caused by the weight

$$D_1(s) = \frac{1}{s} * m_{platform} * g = \frac{1}{s} * 981$$

Error caused by the first disturbance.

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{-0.01s - 0.5}{s^4 + 50 * s^3 + 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4} \frac{981}{s}$$

$$e_{ss} = \ -5.32 * 10^{-3}$$

The second disturbance caused by the linearization

$$D_2(s) = -\frac{1}{s} * \frac{KI_0^2}{Z_0^2} = \ -\frac{1}{s} * 980.26$$

Error caused by the second disturbance.

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{-0.01s - 0.5}{s^4 + 50 * s^3 + 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4} \frac{980.26}{s}$$

$$e_{ss} = \ 5.32 * 10^{-3}$$

As seen total error caused by the disturbances is zero. This is because the linearization at the reference point having defined F as the necessary force to balance the gravity pull on the object at equilibrium. [2]

The obtained transfer function of distrubance for the PID controller in the part ***(b).***

$$\frac{E(s)}{D_{1,2}(s)} = -\frac{C(s)}{D_{1,2}(s)} = \frac{-0.00625 * s^2 - 0.3125 * s}{s^3 + 50 * s^2 + \ 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4}$$

For the part ***(b)***, the first disturbance caused by the weight

$$D_1(s) = \frac{1}{s} * m_{total} * g = \frac{1}{s} * 1569.6$$

The second disturbance caused by the linearization

$$D_2(s) = -\frac{1}{s} * \frac{KI_0^2}{Z_0^2} = -\frac{1}{s} * 980.26$$

Total disturbance can be found as below

$$D(s) = \frac{1}{s} * 589.34$$

Error caused by the total disturbance.

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{-0.00625 * s^2 - 0.3125 * s}{s^4 + 50 * s^3 + 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4} \frac{589.34}{s}$$

$$e_{ss} = 0$$

<u>Error Caused by Reference Inputs (R(s) = 1/s)</u>

In the part ***(a)*** the error transfer function for the reference input obtained as it follows.

$$\frac{E(s)}{R(s)} = \frac{s^3 + 50 * s^2}{s^3 + 50 * s^2 + 721.8 * s + 6044}$$

Zero steady-state error achieved when a unit-step input feed to the system

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{s} \frac{s^3 + 50 * s^2}{s^3 + 50 * s^2 + 721.8 * s + 6044} = 0$$

In the part ***(b)*** the error transfer function for the reference input obtained as it follows

$$\frac{E(s)}{R(s)} = \frac{s^4 + 50 * s^3}{s^4 + 50 * s^3 + 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4}$$

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$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{s} \frac{s^4 + 50 * s^3}{s^4 + 50 * s^3 + 1240 \ * s^2 + \ 1.433 * 10^4 * s + 9.208 * 10^4} = 0$$

The required compensator to obtain zero steady-state error is achieved for both cases. The required compensator to obtain

$$\Sigma e_{ss,(a)} = 0 \quad \Sigma e_{ss,(b)} = 0$$

e) Design a compensator such that the phase margin is greater than 35º and the gain margin is greater than 14 dB.

a. <u>For the vehicle without a passenger</u>

First the uncompansated but gain adjusted sytem Bode Plot Obtained. Since no steady-state error exist in the system, the only specification might be considered as the previously supplied gain for the sytem in the part (a). Large gains iterated to obtain deisred phase and gain margin within %2 settling in 0.5 s. Lead compensation will be used for no disturbance case.

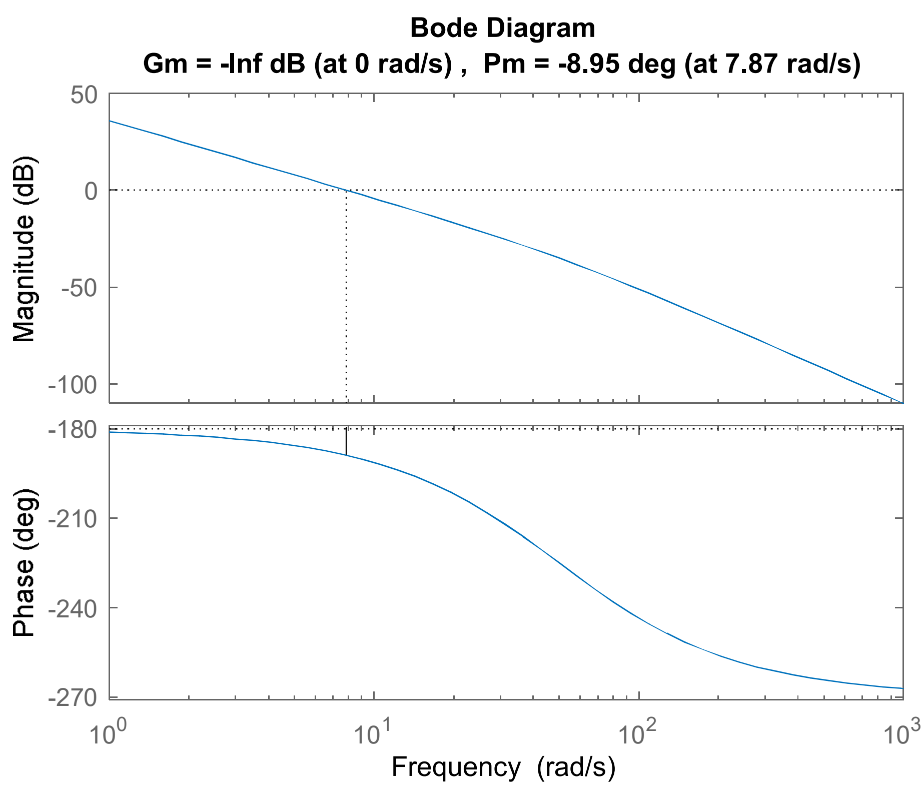


Figure 7. Bode Diagram of Gain Adjusted but Uncompansated System

As seen from the Figure 6. gain adjusted sytem is unstable. Also, Nyquist diagram of the uncompansated system has CW encirclements -1+j0 indicates unstability. (See Appendix A.)

After the lead compensation procedure applied for itarated gain. Desired phase and gain margin obtained by final iteration.

$$K = 16000$$

$$K_g = -\infty\ (dB), \quad \gamma = -8.95º$$

Phase Lead can be found as it follows.

$$\phi_m = 35º - \gamma + 8º = 51.95$$

Alpha term of the Lead Compensator can be found from following formula.

$$sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\alpha = 0.1189 \qquad \frac{1}{\sqrt{\alpha}} = 2.90$$

New gain cross-over determined.

$$w_{gc} = 13.38$$

The compensator zero and pole is able to be derived.

$$\frac{1}{T} = \sqrt{\alpha * w_{gc}} = 4.61, \qquad T = 0.2167$$

$$\frac{1}{\alpha T} = \frac{w_{gc}}{\sqrt{\alpha}} = 38.80, \qquad \alpha T = 0.0257$$

At the end the transfer function of the Lead-Compensator obtained.

$$G_c(s) = 16000\,\frac{0.2167s + 1}{0.0257s + 1}$$

Open Loop Transfer Function of the system.

$$OLTF = \frac{2.638 * 10^4 * s + 1.217 * 10^5}{s^4 + 88.81 * s^3 + \ 1940\ * s^2}$$

Close Loop Transfer Function of the system.

$$CLTF = \frac{2.638 * 10^4 * s + 1.217 * 10^5}{s^4 + 88.81 * s^3 + \ 1940\ * s^2 + 2.638 * 10^4 * s + 1.217 * 10^5}$$

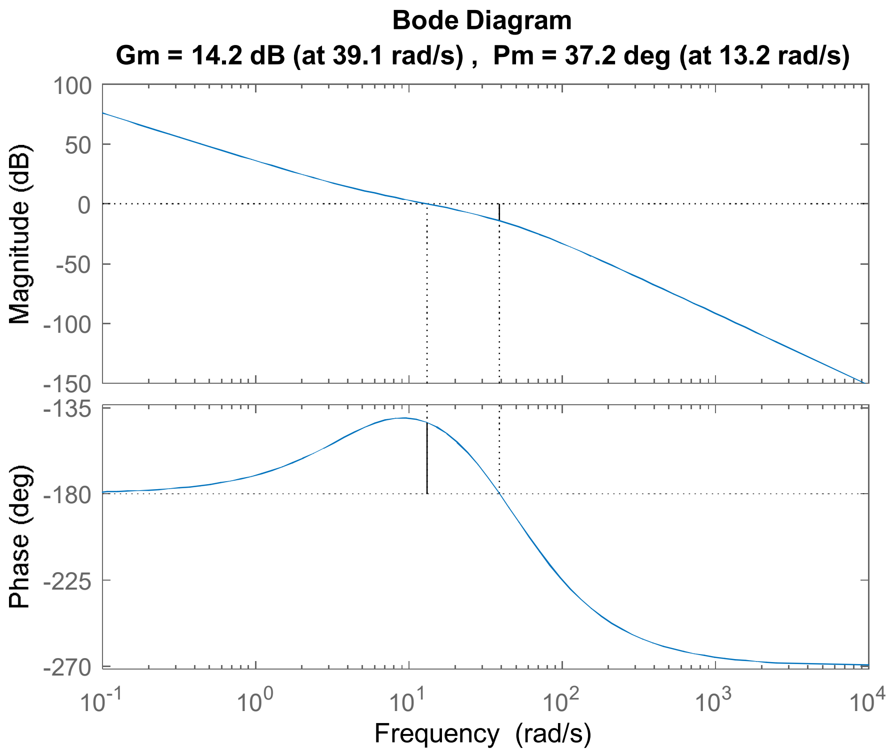


Figure 8. Bode Diagram of Gain Adjusted and Compansated System

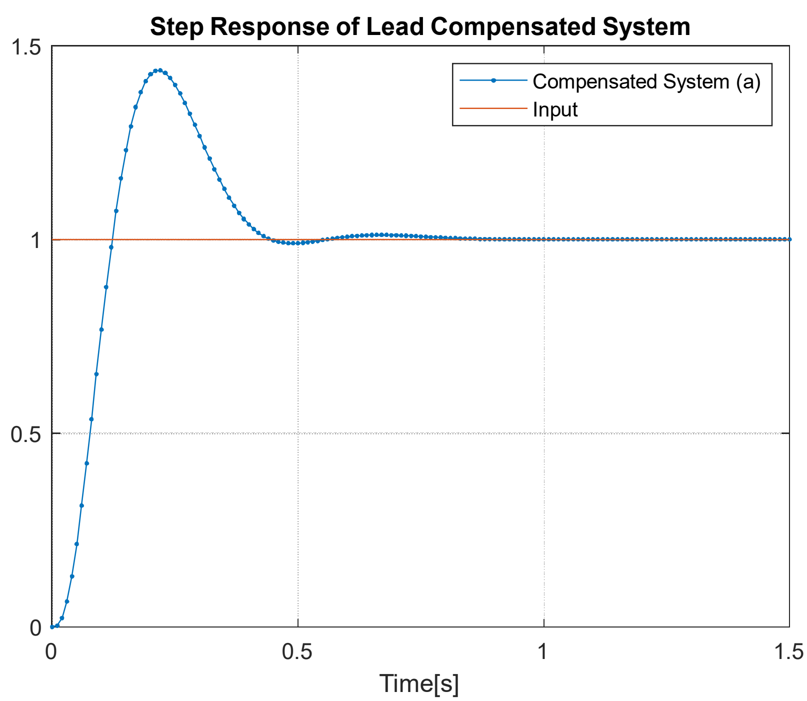


Figure 9. Transient Response of Lead Compensated System

b. For the vehicle with the passenger

First the uncompansated but gain adjusted sytem Bode Plot Obtained. Since no steady-state error exist in the system, the only specification might be considered as the previously supplied gain for the sytem in the part (a). Large gains iterated to obtain deisred phase and gain margin within %2 settling in 0.5 s. Lead compensation will be used for no disturbance case.

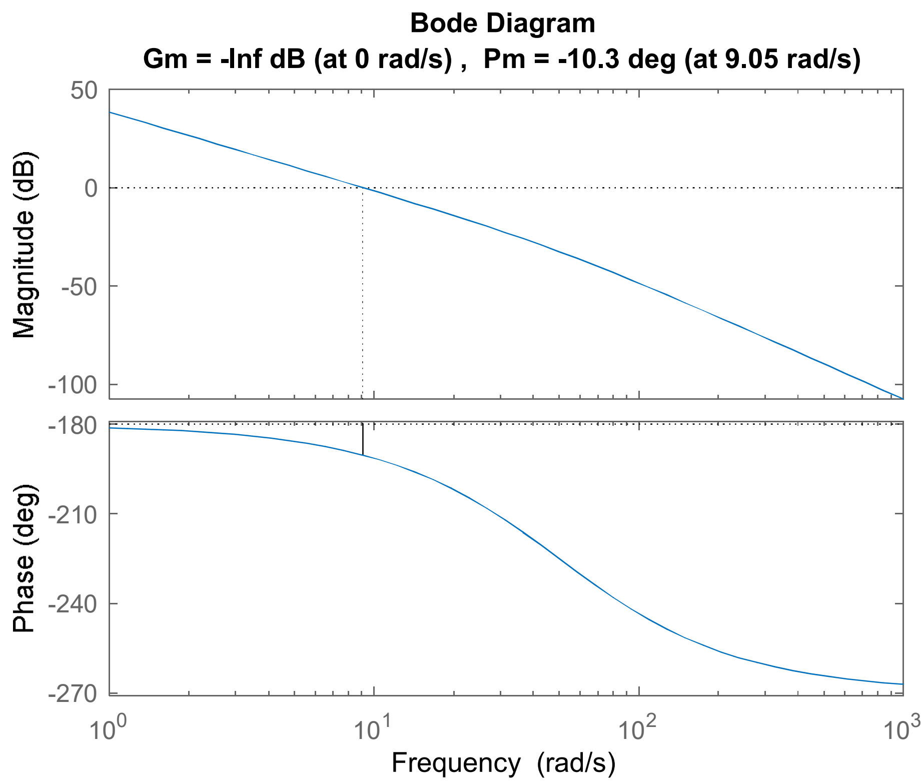


Figure 10. Bode Diagram of Gain Adjusted but Uncompansated System

As seen from the Figure 6. gain adjusted sytem is unstable. Also, Nyquist diagram of the uncompansated system has CW encirclements -1+j0 indicates unstability. (See Appendix B.) After the lead compensation procedure applied for itarated gain. Desired phase and gain margin obtained by final iteration.

$$K = 34000$$

$$K_g = -\infty\ (dB), \quad \gamma = -10.3^{\circ}$$

Phase Lead can be found as it follows.

$$\phi_m = 35^{\circ} - \gamma + 10^{\circ} = 55.3$$

Alpha term of the Lead Compensator can be found from following formula.

$$sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\alpha = 0.0863, \qquad \frac{1}{\sqrt{\alpha}} = 3.4040$$

New gain cross-over determined.

$$w_{gc} = 16.32$$

The compensator zero and pole is able to be derived.

$$\frac{1}{T} = \sqrt{\alpha * w_{gc}} = 4.794, \qquad T = 0.2085$$

$$\frac{1}{\alpha T} = \frac{w_{gc}}{\sqrt{\alpha}} = 38.80, \qquad \alpha T = 0.0180$$

At the end the transfer function of the Lead-Compensator obtained.

$$G_c(s) = 34000\ \frac{0.2085s + 1}{0.0180s + 1}$$

Open Loop Transfer Function of the system.

$$OLTF = \frac{4.83 * 10^4 * s + 2.349 * 10^5}{s^4 + 106.3 * s^3 + 2819 * s^2}$$

Close Loop Transfer Function of the system.

$$CLTF = \frac{4.83 * 10^4 * s + 2.349 * 10^5}{s^4 + 106.3 * s^3 + 2819 * s^2 + 4.83 * 10^4 * s + 2.349 * 10^5}$$

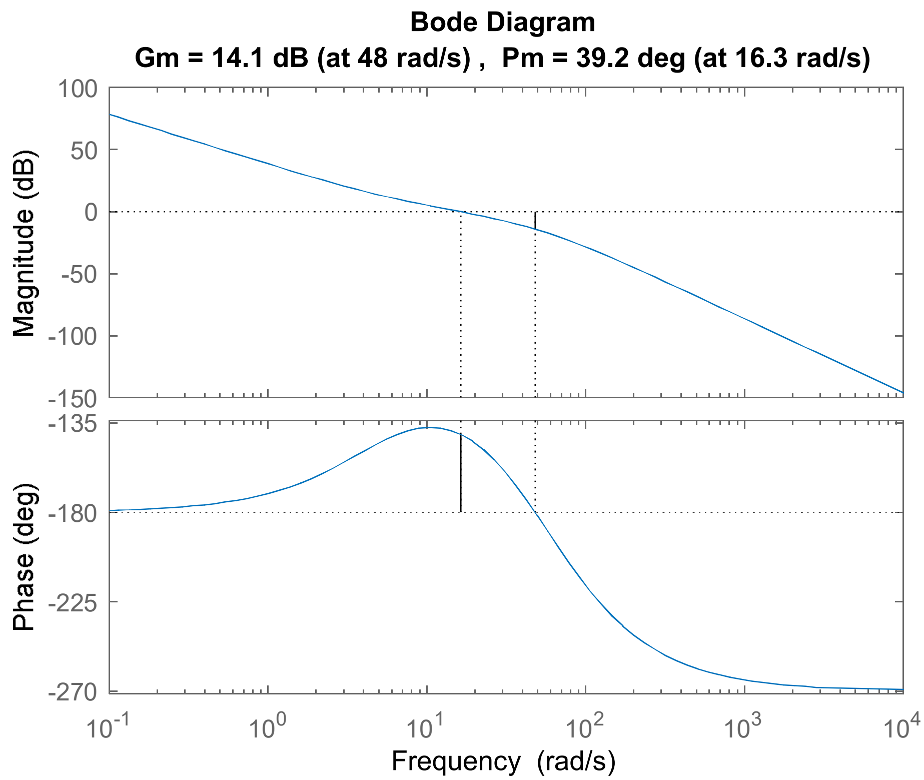


Figure 11.Bode Diagram of Gain Adjusted and Compansated System

Figure 12.Transient Response of Lead Compensated System

Also, error transfer function

$$\frac{E(s)}{D(s)} = \frac{-0.00625\ * s^2 - 0.6649 * s - 17.62}{s^4 + 106.4 * s^3 + \ 2819\ * s^2 + \ 4.83 * 10^4 * s + 2.349 * 10^5}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} s \frac{-0.00625\, s^2 - 0.6649s - 17.62}{s^4 + 106.4s^3 + \ 2819\, s^2 + \ 4.83 * 10^4 s + 2.349 * 10^5} \frac{589.34}{s}$$

$$e_{ss} = 4.42\%$$

Total steady-state error is zero since it is known that lead compensator does not change the type number and from the previous parts steady state error caused by the step input has been found as zero.

$$\Sigma e_{ss,} = 0$$

In this part of the project, lead compensator used for both of the system (with or without passenger). The gain and phase margin requirements achieved in both parts also. However, in the part (e) even though larger gains are used minimize steady-state error it was not possible to eliminate steady-state entirely. This is caused by the elimination of steady state-error requires increase in the type number which can only be satisfied with an I controller. Cascaded or combined controller required to be used to eliminate steady-state error while satisfying desired margin criteria. To have desired gain margin and phase margin lead compensator used instead of the PID since LEAD compensator changes the system dynamics and to have desired characteristics. On the other hand, PID control method is not useful considering both transient response and margin specifications since PID control method modeled from the root locus of the sytem. However, LEAD compensator directly modeled considering frequency response specifications that enables the control such specifications.

REFERENCES

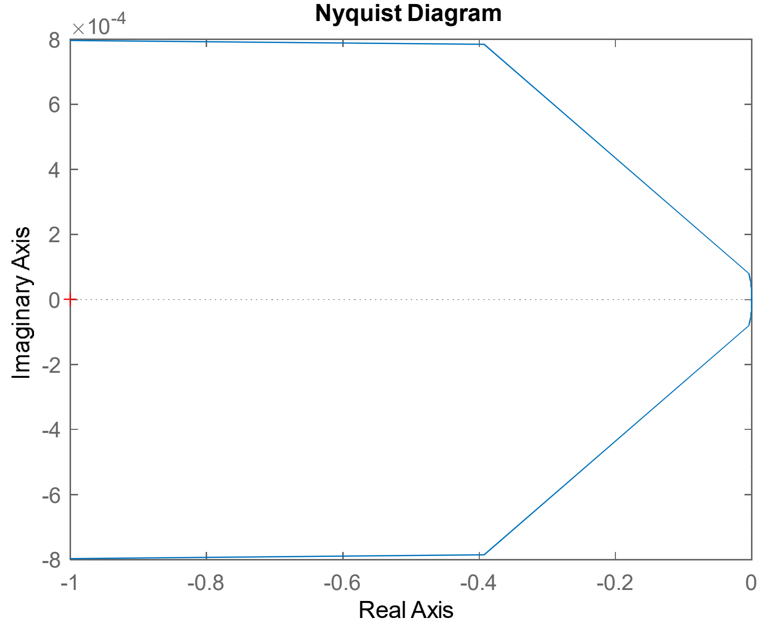
[1] Ogata, K. (2010). Modern Control Engineering. Pearson.

[2] Khan, M. K. A. A., Manzoor, S., Marais, H., Aramugam, K., Elamvazuthi, I., & Parasuraman, S. (2018). PID Controller design for a Magnetic Levitation system. *2018 IEEE 4th International Symposium in Robotics and Manufacturing Automation, ROMA 2018*, (3). https://doi.org/10.1109/ROMA46407.2018.8986710

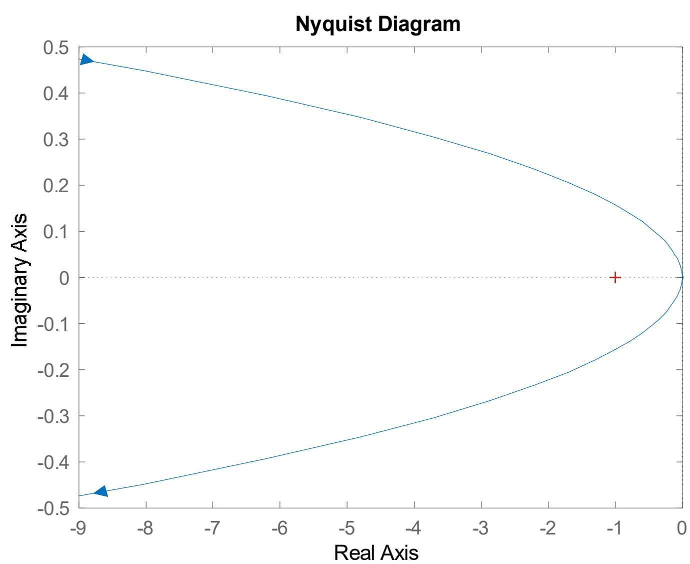
[3] Azgın, K. (2022). *Design of Control Systems*. Lecture.

APPENDIX

A.

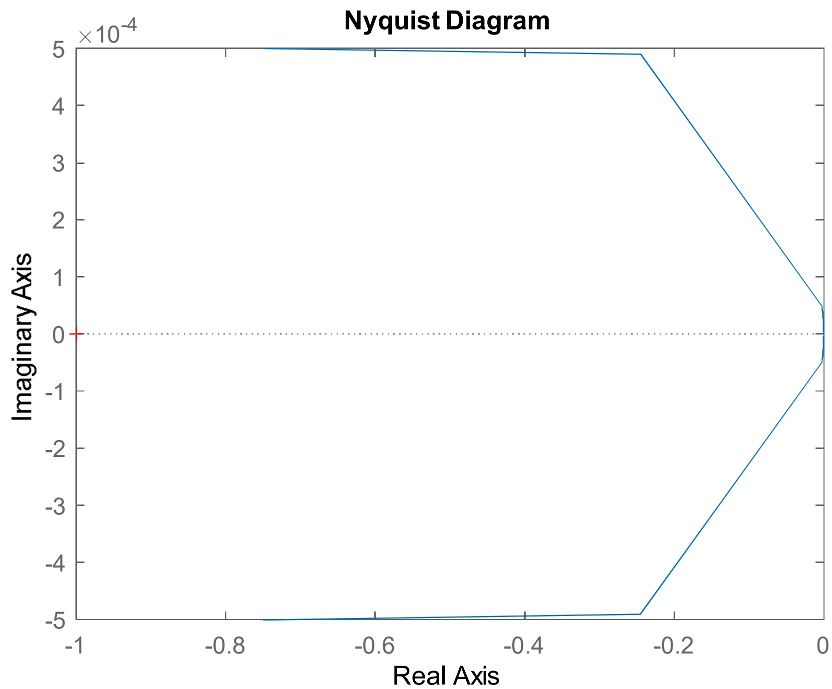


Model 1. Nyquist Diagram of Ucompansated Sytem

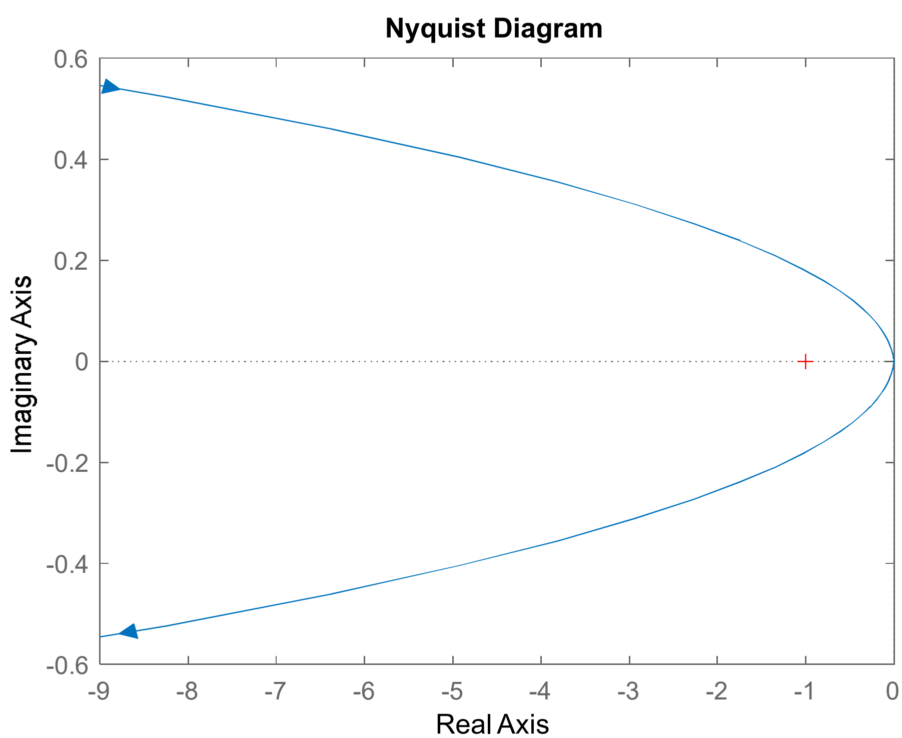


Model 2. Nyquist Diagram of Ucompansated but Gain Adjusted Sytem

B.



Nyquist Diagram
×10^-4
5
4
3
2
1
0
-1
-2
-3
-4
-5
Imaginary Axis
-1
-0.8
-0.6
-0.4
-0.2
0
Real Axis



Nyquist Diagram
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Imaginary Axis
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
Real Axis