MEASUREMENT EQUATION

Until now, we've dealt with the future. We've derived two Kalman Filter prediction equations:

- State Extrapolation Equation
- Covariance Extrapolation Equation

From now on, we are going to deal with the present. Let's start with the Measurement Equation.

In the "One-dimensional Kalman Filter" section, we denoted the measurement by z_n.

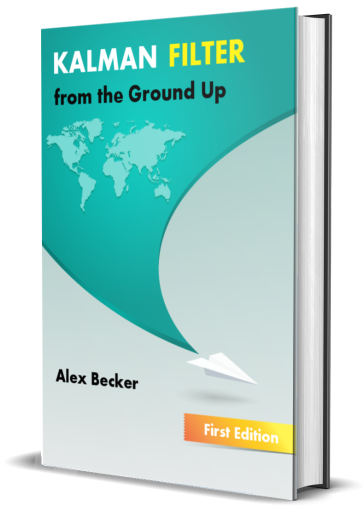
The measurement value represents a true system state in addition to the random measurement noise v_n, caused by the measurement device.

The measurement noise variance r_n can be constant for each measurement - for example, scales with a precision of 0.5kg (standard deviation). On the other hand, the measurement noise variance r_n can be different for each measurement - for example, a thermometer with a precision of 0.5% (standard deviation). In the latter case, the noise variance depends on the measured temperature.

The generalized measurement equation in matrix form is given by:

$$z_n = \boldsymbol{H}\boldsymbol{x}_n + \boldsymbol{v}_n$$

Where:
z_n is a measurement vector
$\boldsymbol{x}_n$ is a true system state (hidden state)
$\boldsymbol{v}_n$ is a random noise vector
$\boldsymbol{H}$ is an **observation matrix**



THE OBSERVATION MATRIX $\boldsymbol{H}$

In many cases, the measured value is not the desired system state. For example, a digital electric thermometer measures an electric current, while the system state is the temperature. There is a need for a transformation of the system state (input) to the measurement (output).

The purpose of the observation matrix $\boldsymbol{H}$ is to convert the system state into outputs using linear transformations. The following chapters include examples of observation matrix usage.

A range meter sends a signal toward a destination and receives a reflected echo. The measurement is the time delay between the transmission and reception of the signal. The system state is the range.

In this case, we need to perform a scaling:

$$z_n = \left[\frac{2}{c} \right] \boldsymbol{x}_n + \boldsymbol{v}_n$$

$$\boldsymbol{H} = \left[\frac{2}{c} \right]$$

Where:

c is the speed of light

$\boldsymbol{x}_n$ is the range

z_n is the measured time delay

STATE SELECTION

Sometimes certain states are measured while others are not. For example, the first, third, and fifth states of a five-dimensional state vector are measurable, while the second and fourth states are not measurable:

$$z_n = \boldsymbol{H}\boldsymbol{x}_n + \boldsymbol{v}_n = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \boldsymbol{v}_n = \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} + \boldsymbol{v}_n$$

COMBINATION OF STATES

Sometimes some combination of states can be measured instead of each separate state. For example, maybe the lengths of the sides of a triangle are the states, and only the total perimeter can be measured:

$$z_n = \boldsymbol{H}\boldsymbol{x}_n + \boldsymbol{v}_n = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \boldsymbol{v}_n = (x_1 + x_2 + x_3) + \boldsymbol{v}_n$$

MEASUREMENT EQUATION DIMENSIONS

The following table specifies the matrix dimensions of the measurement equation variables:

Variable	Description	Dimension
$\boldsymbol{x}$	state vector	$n_x \times 1$

$\boldsymbol{z}$	measurements vector	$n_z \times 1$
$\boldsymbol{H}$	observation matrix	$n_z \times n_x$
$\boldsymbol{v}$	measurement noise vector	$n_z \times 1$

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