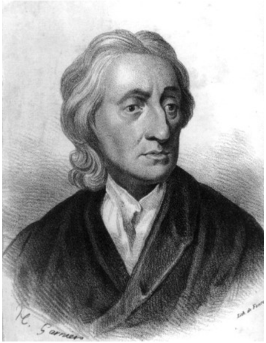
Causality and directed acyclic graphs

Get to know and have some intuition about

- Causality in philosophy (of science)
- Conditional independence relations
- Causal discovery algorithms
- Confounds and back doors



David Hume (1711-1776)

What is a causal graph?

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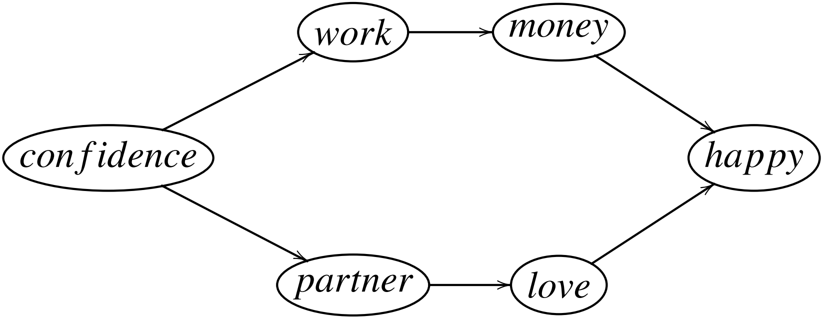

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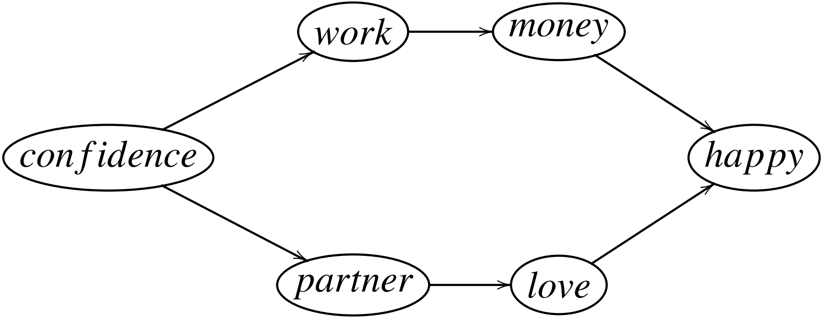

John Stuart Mill

- A (always) co-occurs with B
- A occurs before B
- There is no alternative explanation for the co-occurrence of A and B

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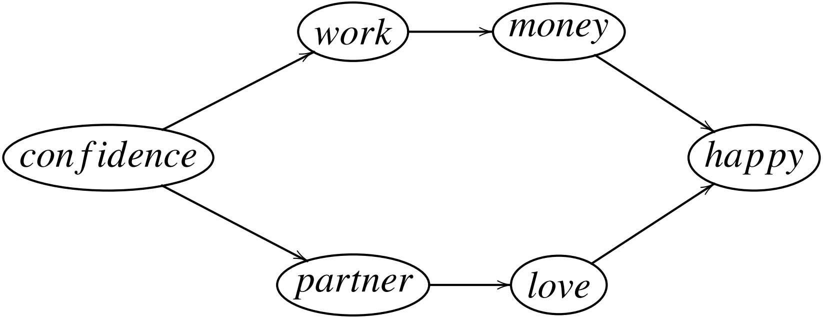

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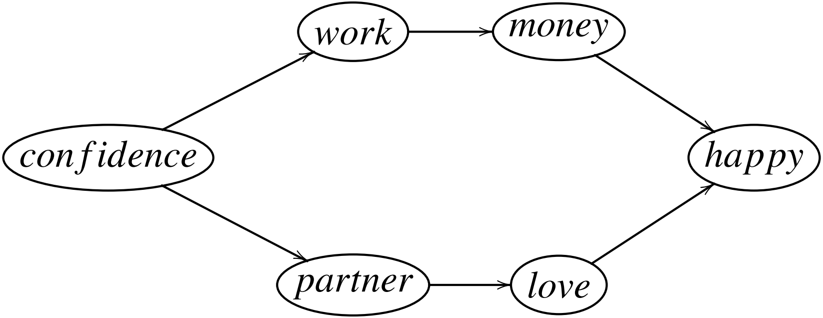

Causality by intervention

- If I wiggle A then B wiggles too (manipulation)
- There is no alternative explanation for the change in B as a result of the change in A

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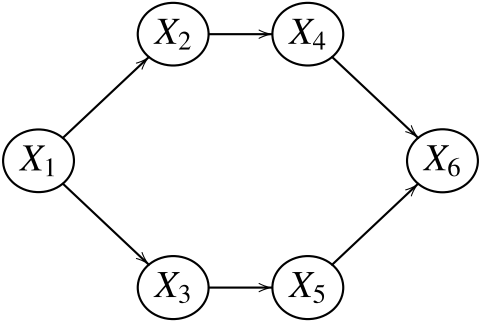

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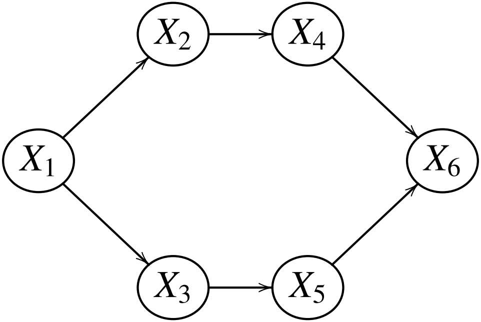

- A directed acyclic graph (DAG)
- A probability distribution over the nodes

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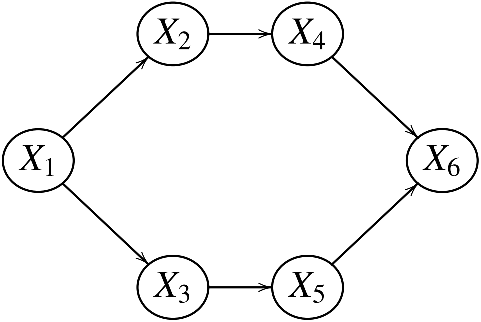

- A directed acyclic graph (DAG)
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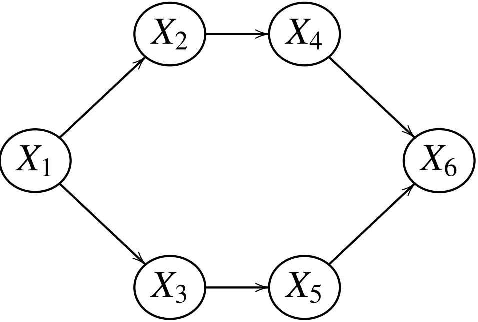
This forms the basis to infer causal relations between variables

Graphs and probability



$$\Longleftrightarrow \quad P(X_1, X_2, X_3, X_4, X_5, X_6)$$

Graphs and probability

Theorem 1.2.5 (Pearl, 2000, p. 18)
For any three nodes (X, Y, Z) in a DAG G and for all probability functions P, we have

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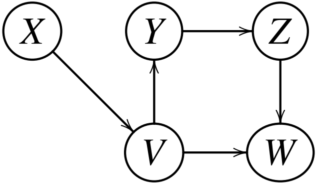
For any three nodes (X, Y, Z) in a DAG G and for all probability functions P, we have

(i) if in the graph X and Y are d-separated given Z, then the conditional independence holds in all distributions that are compatible with G; and

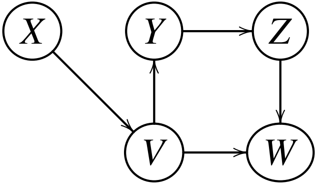
(ii) if X and Y are independent conditional on Z in all distributions compatible with G, then X and Y are d-separated given Z.

d-separation



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

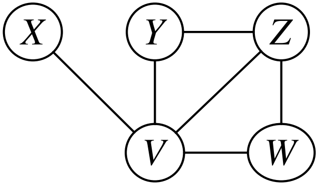
d-separation



Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.

d-separation



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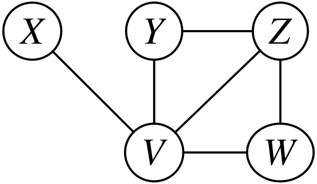

Lauritzen $(X \perp\!\!\!\perp Z \mid YW)$

1. Make the *ancestral graph* (G_{an}): the variables of interest and all variables that have a directed path to those variables $\{W, X, Y, Z\}$.
2. Make the *ancestral graph moral* (G_{an}^{m}): marry all the parents that have a child in common *and* convert all arrows into undirected edges.
3. Consider separating all paths in G_{an}^{m} between X and Z.

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$$(X \perp\!\!\!\perp Z \mid YW)$$

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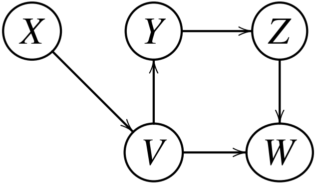
G



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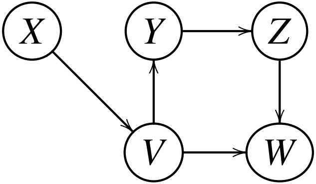
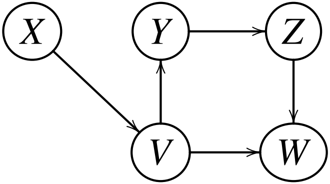


G_{an}



d-separation

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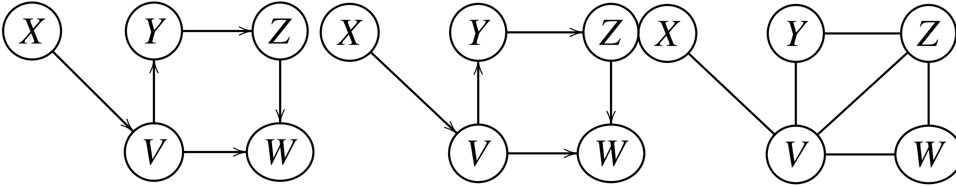
G $\qquad G_{an}$ $\qquad G_{an}^{m}$



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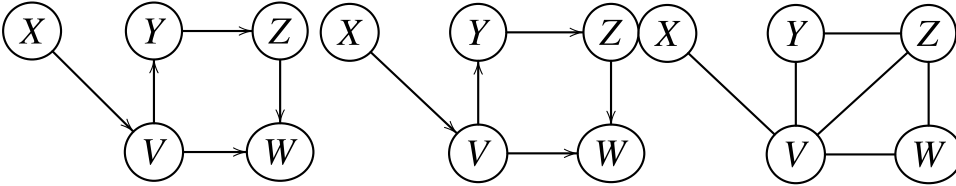

$$\cancel{(X \perp\!\!\!\perp Z \mid YW)}$$

d-separation

$$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$$

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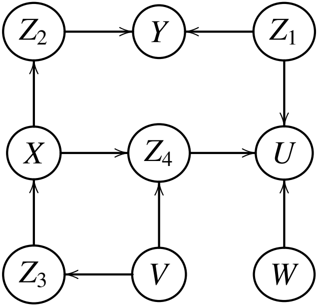

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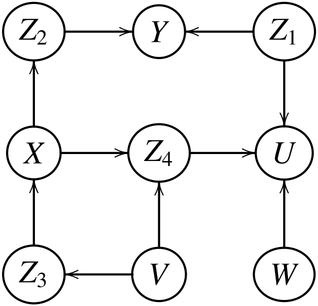

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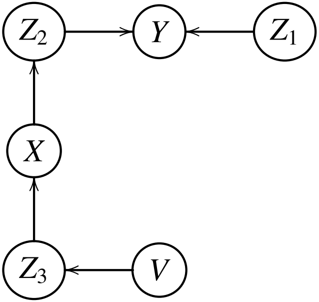

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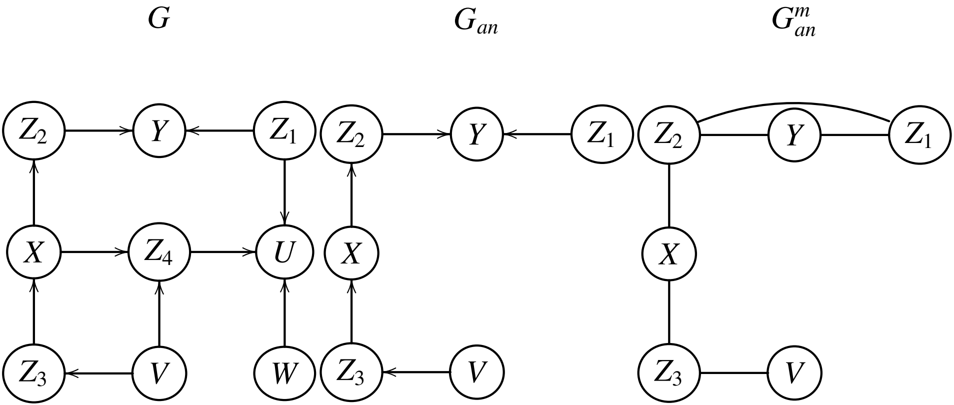

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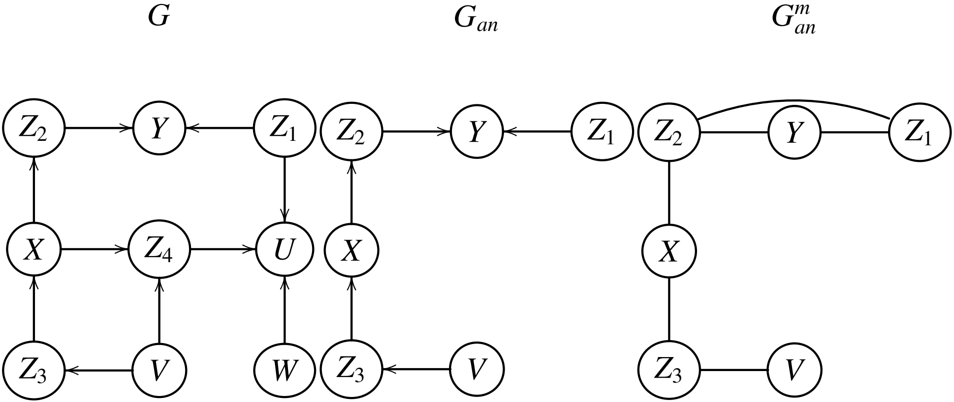

$(X \perp\!\!\!\perp Y \mid Z_2, Z_3)$

When are models the same?

$$X \rightarrow Z \rightarrow Y \qquad\qquad X \leftarrow Z \rightarrow Y$$

When are models the same?

d-separation

When are models the same?

$$X \longrightarrow Z \longrightarrow Y \qquad\qquad X \longleftarrow Z \longrightarrow Y$$

d-separation

$$(X \perp\!\!\!\perp Y \mid Z)$$

When are models the same?

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$X \rightarrow Z \rightarrow Y$ $\quad$ $X \leftarrow Z \rightarrow Y$ $\quad$ $X \rightarrow Z \leftarrow Y$

$X \rightarrow Z \rightarrow Y$	$X \leftarrow Z \rightarrow Y$	$X \rightarrow Z \leftarrow Y$
$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\perp\!\!\!\perp Y \mid Z)$
		$(X \perp\!\!\!\perp Y)$

When are models the same?

$X \rightarrow Z \rightarrow Y$ $\quad$ $X \leftarrow Z \rightarrow Y$ $\quad$ $X \rightarrow Z \leftarrow Y$

$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\perp\!\!\!\perp Y \mid Z)$
$(X \not\perp\!\!\!\perp Y)$	$(X \not\perp\!\!\!\perp Y)$	$(X \perp\!\!\!\perp Y)$

When are models the same?

$X \to Z \to Y$	$X \leftarrow Z \to Y$	$X \to Z \leftarrow Y$
$(X \perp\!\!\!\perp Y \mid Z)$	$(X \perp\!\!\!\perp Y \mid Z)$	$(X \not\perp\!\!\!\perp Y \mid Z)$
$(X \not\perp\!\!\!\perp Y)$	$(X \not\perp\!\!\!\perp Y)$	$(X \perp\!\!\!\perp Y)$

Models are the same when the d-separations are the same!