Searching

Organizing and retrieving information is at the heart of most computer applications

- Searching is a very frequently performed task
- Search process:
  - ☐ Abstract view: Determine if an element with a particular value is a member of a particular set
  - ☐ Common view: Try to find the record with a record collection that has a particular key value.
- Some of the techniques presented here require material from chapter 8.
  - ☐ Assigned reading: Chapter 8, section 8.3 and all of Chapter 9

Buffers and Buffer Pools (sec 8.3)

The general idea is to use a RAM buffer to hide latency.

- **Caching** or **buffering**: the act of storing in RAM a piece of data from a faster or slower device
  - ☐ allows the faster device to do something else while the slower device reads from or writes to the buffer
- Examples
  - ☐ CPU cache is a buffer for RAM
  - ☐ RAM is a buffer for disks of various types
  - ☐ Disk can buffer for tape
- Associated concepts:
  - ☐ **Buffer pool**: a set of multiple buffers
  - ☐ **Page**: a piece of memory large enough to fill a buffer

Buffer Pools

An example is virtual memory.

- A hard disk is used to simulate a very large RAM memory
- System RAM is the buffer pool
- A **page** is a block of memory (usually some multiple of 512 bytes)
  - □ Address space of a process can be broken into multiple pages
  - □ At any given time, some pages may be on disk and some in RAM
    - ◦ Requesting a memory address currently on disk causes a **page fault**
  - □ Two options for page fault:
    - ◦ Find an "empty" page in RAM and transfer the page from disk
    - ◦ No empty pages in RAM: follow a page replacement strategy
  - □ Page replacement strategies:
    - ◦ FIFO
    - ◦ LFU
    - ◦ LRU

Searching

- Formal definition:
  - □ Suppose k_1, k_2, ... k_n are distinct keys
  - □ Given a collection C of n records of the form

$$(k_1, I_1), (k_2, I_2), \ldots, (k_n, I_n)$$

  - □ I_j is information associated with key k_j for $1 \leq j \leq n$.
- **Search problem**: given key value K, locate the record (k_j, I_j) in C such that $k_j = K$
  - □ **successful** search: record with $k_j = K$ is found
  - □ **unsuccessful** search: no record with $k_j = K$ is found
- Queries:
  - □ **Exact-match query**: search for a record whose key matches a specific key value
  - □ **Range query**: search for all records whose key values fall within a specified range

Searching Categorization

- Three general approaches
  - ☐ Sequential and list methods
    - ∘ Works well for sequences (duplicate keys allowed)
    - ∘ Appropriate for data stored in RAM
  - ☐ Direct access by key value (hashing)
    - ∘ Doesn't work well for sequences
    - ∘ Works well for data on disk or in RAM
  - ☐ Tree indexing methods (chapter 10, not covered)

Searching Sorted Arrays

- Sequential search
  - ☐ $\Theta(n)$, average and worst case
  - ☐ Unacceptable for large data sets
- Binary search
  - ☐ $\Theta(\log n)$, average and worst case
  - ☐ Works only for previously sorted data
- Dictionary search
  - ☐ a "computed" binary search
  - ☐ based on knowledge about key distribution
  - ☐ also called interpolation search

Lists Ordered by Frequency

Instead of ordering by key value, a list may be ordered by frequency of access.

- Lists ordered by frequency: the *expected* frequency of occurrence determines ordering strategy
  - ☐ A sequential search is performed
    - ◦ Cost to access i^{th} record is i
    - ◦ Order in decreasing order of probability: p_i is the probability that record i will be accessed
    - ◦ That is,

$$p_1 \geq p_2 \geq \dots p_n$$

(Note: $\sum_{i=1}^{n} p_i = 1$ must be true)

    - ◦ The cost to access each element is (position of element) × (probability of element)
    - ◦ Then the overall expected search cost is

$$\overline{C_n} = 1p_1 + 2p_2 + \cdots + np_n$$

Lists Ordered by Frequency (cont.)

- Example: all records have equal probability
  - ☐ $p_i = 1/n$
  - ☐ Then

$$\overline{C_n} = 1 \times 1/n + 2 \times 1/n + \cdots + n \times 1/n$$

$$= \sum_{i=1}^{n} i/n = \frac{1}{n} \sum_{i=1}^{n} i$$

$$= \frac{1}{n} \times \frac{n(n+1)}{2} = \frac{n+1}{2}$$

- Example: exponential frequency
  - ☐ Probabilities:

$$p_i = \begin{cases} 1/2^i & \text{if } 1 \leq i \leq n-1 \\ 1/2^{n-1} & \text{if } i = n \end{cases}$$

  - ☐ Thus,

$$\overline{C_n} \approx \sum_{i=1}^{n} \frac{i}{2^i} \approx 2$$

The 80/20 Rule

Many real access patterns follow this rule of thumb.

- The **80/20 rule**: 80
  - ☐ 80 and 20 are estimates (applications have their own values)
  - ☐ This behavior justifies caching techniques
  - ☐ When the rule applies, then reasonable search performance can be expected

- Example: **Zipf distribution**
  - ☐ A pattern followed by some naturally occurring distributions, including:
    - ∘ Distribution for frequency of word usage
    - ∘ Distribution for city populations
  - ☐ Related to the Harmonic series (chapter 2) as follows:
    - ∘ Zipf frequency for item i is $1/i\mathcal{H}_n$
    - ∘ (here $\mathcal{H}_n = \sum_{i=1}^{n} 1/i \approx \log_e n$)
    - ∘ Then

$$\begin{aligned}\overline{C_n} &= \sum_{i=1}^{n} i/i\mathcal{H}_n \\ &= n/\mathcal{H}_n \\ &\approx n/\log_e n\end{aligned}$$

Self-Organizing Lists

This is why we studied the section on buffer pools.

- A **self organizing list** is a list that starts out unordered, but the access policy includes procedures to impose an order based on actual pattern of record access
  - ☐ Use rules called **heuristics** to determine how to reorder the list
  - ☐ The heuristics are similar to the buffer pool management strategies (buffer pools are like a form of self-organizing list)
  - ☐ Heuristics:
    - ∘ **Count**: Count the frequency of access. When a record is found, increment its count and move it up if the count is greater than preceding record(s)
    - ∘ **Move-to-front**: when a record is found, move it to the front of the list
    - ∘ **Transpose**: when a record is found, swap it with the record ahead of it

Self-Organizing Lists, Examples

- Initial list is A, B, C, D, E, F, G, H
- Access pattern is F D F G E G F A D F G E
  - ☐ Count heuristic:
  - ☐ Move-to-front heuristic:
  - ☐ Transpose heuristic:

Self-Organizing Lists, Examples

- Application: text compression
  - ☐ Keep a table of words previously seen
  - ☐ Use the move-to-front heuristic
  - ☐ If a word is not yet seen, then send the word
  - ☐ If a word has been seen, then send its current table index
  - ☐ Example: The car on the left hit the car I left
  - ☐ becomes: The car on 3 left hit 3 5 I 5
  - ☐ Similar in spirit to Ziv-Lempel coding

Searching in Sets

Determining whether a value is a member of a set is a special case of searching for keys in a sequence of records.

- Any of the prior search methods can be used
- This problem allows us to speed up the process:
  - □ **Bit vector** or **bitmap** representation: use an array of n bits corresponding to n potential set members
    - ∘ $i = 1$ means that member i is present
    - ∘ $i = 0$ means that member i is not present
  - □ Application: document retrieval: find all documents in a set containing certain keywords
    - ∘ For each keyword, the system stores a bit vector (one bit for each document)
    - ∘ A '1' means that the document contains the keyword
    - ∘ Searching for three words is a logical AND of 3 bit vectors.

Hashing

A completely different approach in which search is by direct access based on the key value.

- **Hashing** is the process of accessing a record by mapping a key value to a position in a table.
- The mapping process requires a (normally $\Theta(1)$) mathematical function called the **hash function**, denoted by **h**
- The **Hash table** is an array that stores all of the records, denoted **HT**
- A record's position in the hash table is its **slot**
- The number of slots is denoted by M, numbering is from 0 to $M - 1$
- The mapping function **h** must work as follows:
  - □ For any value K in the key range, $h(K) = i$, $0 \leq i < M$ such that key($\mathbf{HT}(i)$) $= K$

Hashing (cont.)

Hashing answers the specific question "what record, if any, has key value K?"

- Works well for sets (no duplicates)
- Not suitable for range queries
- Works well for in-memory and disk-based applications
- Example:
  - ☐ Store the n records with key values in the range 0 to $n-1$
  - ☐ Hash function $h(K) = K$
  - ☐ This is not a practical example (Why?)
- Example:
  - ☐ Store about 1000 records having keys in the range 0 to 16,383
  - ☐ Impractical to keep a hash table with 16,383 slots
  - ☐ We need a hash function that maps the key range to a smaller table

Collisions

- Given a hash function $h(k)$ and keys k_1 and k_2:
  - ☐ If $h(k_1) = h(k_2) = \beta$, then k_1 and k_2 have a **collision** at β under h.
- Collisions are inevitable in most applications
  - ☐ Example: birthday sharing
- Minimizing collisions requires good hash functions
- Finding a record (or a place in which to insert) requires a two-step procedure:
  1. Compute table location $h(k)$
  2. Starting with slot $h(k)$, search for the record containing key k (or an empty location where it may be inserted)
- The search procedure is the collision resolution technique. There are two major classes:
  - ☐ **Open hashing**, also called **Separate chaining**
  - ☐ **Closed hashing**, also called **Open addressing**

Hash Functions

- Requirement:
  - ☐ A hash function must compute a slot index within the hash table's range; thus, it computes (some value) $\bmod M$
- Goals:
  - ☐ A practical hash function evenly distributes the records stored among the hash table slots
  - ☐ Ideally, the even distribution is to all slots with equal probability
    - ◦ Success at this depends on the data's distribution
  - ☐ It should also be fast (probably the easiest goal to accomplish)
- Two situations normally faced:
  - ☐ We know nothing about the incoming key distribution: attempt to evenly distribute the key range over the hash table, trying to avoid clustering
  - ☐ We know something about the incoming key distribution: use a distribution-dependent hash function.

Examples

- A Simple hash function:

```
int h (int x) {
  return (x % 16);
}
```

  - ☐ The $\bmod 16$ operation makes the function dependent on the lower 4 bits of the key
- Mid-square method: square the key value, taking the middle r bits from the result for a hash table having 2^r slots
- Folding method: sum the ASCII values of all letters, taking the result $\bmod M$:

```
int h(char *x) {
  int i = 0; int sum = 0;
  while (x[i] != NULL) {
    sum += (int) x[i];
    i++;
  }
  return (sum % M);
}
```

Examples

- Executable and Linking Format (ELF) hash, Unix Sys/V Release 4:

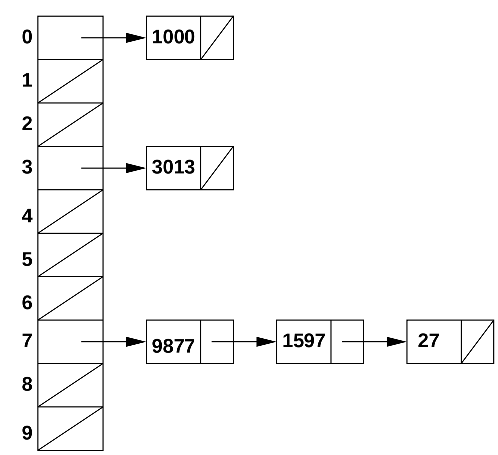
```
int ELFhash(char *key) {
  unsigned long h = 0;
  while (*key) {
    h = (h << 4) + *key++;
    unsigned long g = h & 0xF0000000L;
    if (g) h ^= g >> 24;
    h &= ~g;
  }
  return h % M;
}
```

  - ☐ Works well with short and long strings
  - ☐ Every letter of the string has equal effect
  - ☐ Even distribution into hash table slots is very likely

Open Hashing

This is also called **separate chaining**.

- A collision resolution technique, in which:
  - ☐ The hash table is not an array of records; rather, it is an array of pointers
  - ☐ Each slot is treated as a bin so that collisions do not really occur
  - ☐ For a given record with key k and $h(k) = \beta$:
    - ∘ hash table slot β is the head of a linked list
    - ∘ Insert into slot β becomes a linked list insert



Closed Hashing

This is also called **open addressing**

- All records are stored directly in the hash table
  - ☐ Each record i has a **home position** defined by $h(k_i)$
    - ◦ If record i is inserted and another record already occupies i's home position, then another slot must be found to store i.
    - ◦ The search procedure to find a new slot is the **collision resolution policy**

Bucket Hashing

One implementation of closed hashing in which the extra list space is stored in the table.

- Divide the hash table into buckets
  - ☐ M slots are divided into B buckets, with $M >> B$
  - ☐ Include overflow bucket with large capacity at end
  - ☐ Records hash to the first slot of the bucket, then fill it sequentially
  - ☐ Overflow is used if a given bucket is full
  - ☐ Search: check bucket then check overflow (using linear search in both)

Collision Resolution Policies

- Goal is to find a free slot in the table
- Search proceeds by following a **probe sequence**: the series of slots visited during insert/search after a collision occurs
  - ☐ Whether inserting or searching, the probe sequence must be the same every time
  - ☐ Basic idea: follow probe sequence until one of the following is true:
    - ○ record with key $= k$ is found
    - ○ an empty slot is found (no record with key k exists in the hash table)
  - ☐ Insert with Probing:

```
void insert(item R) {
  int home, pos, i;
  home = h(key(R));
  if (Table[home] == EMPTY)
    Table[home] = R;
  else {
    for (i = 1; Table[pos] != EMPTY; i++) {
      pos = (home + probe(key(R),i)) % M;
      if (key(T[pos]) == key(R)) ERROR;
    }
    Table[pos] = R;
  }
}
```


Linear Probing

From a given position, linear probing searches the next available slot in the table.

- Probe function:

```
int probe(int Key, int i) { return i; }
```

  - ☐ If the end of the table is reached, it wraps around to the top (see code on previous page)
  - ☐ At least one slot must always be empty in the table. Why?
- Linear probing suffers from **primary clustering**:
  - ☐ “Clusters” of occupied cells form
  - ☐ Any key hashing into a cluster requires several attempts to resolve the collision and then will add to the cluster

Primary Clustering

- Probabilities for which slot to use next are not the same
  - ☐ $h(k) = k \mod 11$
  - ☐ $1003 \mod 11 = 2$, $1924 \mod 11 = 10$, $3071 \mod 11 = 2$, $2071 \mod 11 = 3$, $4752 \mod 11 = 0$

Insert in the following order: 1003, 1924, 3071, 2071, 4752

0	**4752**
1	
2	**1003**
3	**3071**
4	**2071**
5	
6	
7	
8	
9	
10	**1924**

Better Linear Probing

- Use a constant c to skip by, instead of going to the next slot on every probe
  - ☐ probe$(h(k), i) = h(k) + c \times i$
  - ☐ M and c should be relatively prime (Why?)

- Clustering can still exist
  - ☐ Example: $c = 3$, $h(k_1) = 3$, $h(k_2) = 9$
  - ☐ Probe sequences for k_1 and k_2 are linked together

Pseudo Random Probing

An ideal probe function selects the next slot in the probe sequence at random

- Why can a real probe function not act randomly?

- Pseudo random probing:
  - □ Select a random permutation of the numbers from 1 to $M-1$: $r_1, r_2, \ldots, r_{M-1}$
  - □ All searches and insertions use the same permutation:
  - □ $p(K,i) = Perm[i-1]$
  - □ that is, the i^{th} value in the probe sequence is $(h(k) + r_i) \mod M$

- Example:
  - □ $M = 101$
  - □ $r_1 = 2, r_2 = 5, r_3 = 32$
  - □ $h(k_1) = 30, h(k_2) = 28$
  - □ Probe sequence for k_1:
  - □ Probe sequence for k_2:

Quadratic Probing

- The i^{th} probe sequence function is i^2

- That is, the ith value in the probe sequence is $(h(k) + i^2) \mod M$

- Example:
  - □ $M = 101$
  - □ $h(k_1) = 23, h(k_2) = 24$
  - □ Probe sequence for k_1:
  - □ Probe sequence for k_2:

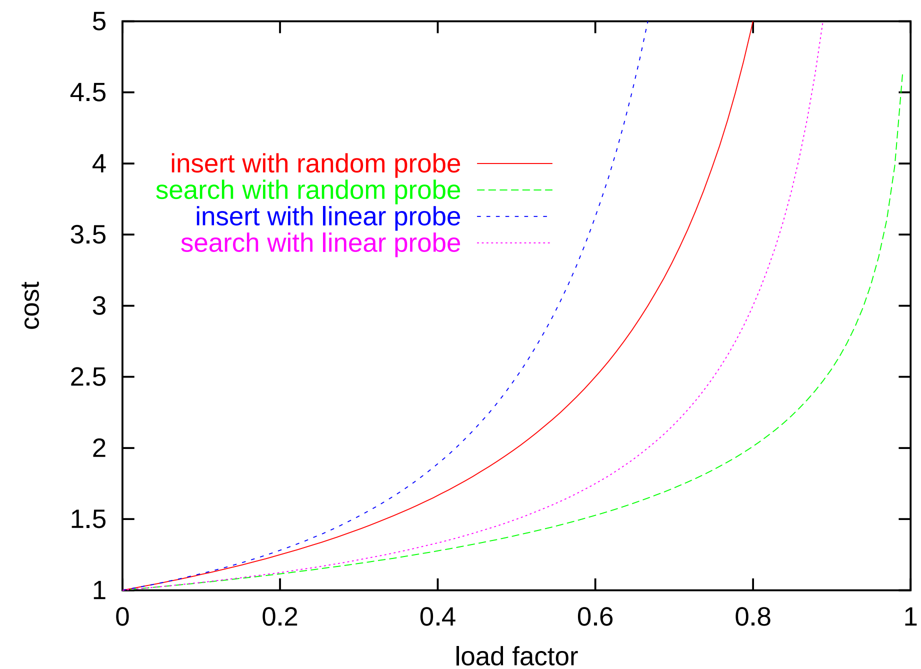
Double Hashing

Prior probing methods can reduce or eliminate primary clustering.

- **Secondary clustering** occurs when two keys hash to the same slot, thus following the exact same probe sequence
- Desirable: the probe sequence is a function of both the key and the home position
- Double hashing adds a second hash function to the probe sequence:
  - ☐ $p(k, i) = i \times h_2(k)$ for $0 \le i \le M - 1$
  - ☐ Poor choice of $h_2(k)$ results in poor ("disastrous") performance
  - ☐ Make sure all cells can be probed by ensuring that all probe sequence constants are relatively prime to M
    - ∘ One method: make M prime
    - ∘ Another method: set $M = 2^m$ and make h_2 return an odd value between 1 and 2^m

Analysis of Closed Hashing

- Visualizing the expected performance of hashing based on load factor
- Load factor $\alpha = N/M$ where N is the number of records stored



- Note: "random probe" is only a theoretical measure

Rehashing

- Consequences of a hash table that is too full:
  - ☐ Running time for operations start to take too long
  - ☐ Insertions might fail for certain collision resolution strategies
- Solution: build a bigger table
  - ☐ Find a prime number at least twice as large as current value of M
  - ☐ Allocate a new hash table (array)
  - ☐ Scan through the old hash table, inserting all elements into the new hash table
  - ☐ Delete the old hash table
- Operation is expensive but occurs relatively infrequently
- Strategies:
  - ☐ Rehash when the table is half full
  - ☐ Rehash when an insertion fails
  - ☐ Rehash when the table reaches a certain **load factor**

Deletion

Deletion is tricky with hashing for the following reasons:

- Deleting a record must not hinder later searches (an empty slot means "stop the search"
- Positions should also not be made unusable due to deletions (avoid a "zombie slot?")
- Solution:
  - ☐ Add a special mark in place of the deleted record.
  - ☐ Mark is called the **tombstone**
  - ☐ Tombstones do not stop search but do add to average search time
  - ☐ Solutions to that added time:
    - ∘ Local reorganizations to try to shorten it
    - ∘ Periodically rehash the table