HW 4

1 derive $P(X)$ and $P(\theta|X)$ for $\theta \sim U[0,1]$, $X \sim U[0,\theta]$

$$P(\theta|x) = \pi(\theta) \cdot P(x|\theta) / P(x)$$

$$P(x) = \int_\theta \pi(\theta) \cdot P(x|\theta)\, d\theta$$

$0 \le \theta \le 1$ $\quad \pi(\theta) = \frac{1}{1-0} = 1$

$0 \le x \le \theta$ $\quad P(x|\theta) = \frac{1}{\theta - 0} = \frac{1}{\theta}$

$$P(x) = \int_0^x \pi(\theta) \cdot P(x|\theta)\, d\theta$$

$$\downarrow$$

$0 \le x \le \theta$ $\quad P(x) = \int_0^x \frac{1}{\theta}\, d\theta = \ln(x)$