Appendix H

STATSTAR, A STELLAR STRUCTURE CODE

The FORTRAN computer program listed here (`STATSTAR`) is based on the equations of stellar structure and the constitutive relations developed in Chapters 9 and 10. An example of the output generated by STATSTAR is given in Appendix I.

STATSTAR is designed to illustrate as clearly as possible many of the most important aspects of numerical stellar astrophysics. To accomplish this goal, STATSTAR models are restricted to a fixed composition throughout (i.e., homogeneous main-sequence models).

The four basic stellar structure equations are computed in the functions `dPdr`, `dMdr`, `dLdr`, and `dTdr`. These functions calculate the derivatives dP/dr (Eq. 10.7), dM_r/dr (Eq. 10.8), dL_r/dr (Eq. 10.45), and dT/dr [Eqs. (10.61) and (10.81)].

The density [$\rho(r)$ = `rho`] is calculated directly from the ideal gas law and the radiation pressure equation (Eq. 10.26), given local values for the pressure [$P(r)$ = `P(i)`], temperature [$T(r)$ = `T(i)`], and the mean molecular weight (μ = `mu`, assumed here to be for a completely ionized gas only); note that `i` is the *number* of the mass shell currently being calculated (`i` $\equiv 1$ at the surface, see Fig. 10.11). Once the density is determined, both the opacity [$\overline{\kappa}(r)$ = `kappa`] and the nuclear energy generation rate [$\epsilon(r)$ = `epslon`] may be calculated. The opacity is determined using the bound–bound and bound–free opacity formulae [Eqs. (9.19) and (9.20), respectively], together with electron scattering (Eq. 9.21). The energy generation rate is computed from the equation

for the total pp chain (Eq. 10.49) and for the CNO cycle (Eq. 10.53). Each of these calculations is carried out in the equation-of-state subroutine, `EOS`.[1]

The program begins by asking the user to supply the desired stellar mass (`Msolar`, in solar units), the trial luminosity (`Lsolar`, also in solar units), the trial effective temperature (`Te`, in kelvin), and the mass fractions of hydrogen (`X`) and metals (`Z`). Using the stellar structure equations, the program proceeds to integrate from the surface of the star toward the center, stopping when a problem is detected or when a satisfactory solution is obtained. If the inward integration is not successful, a new trial luminosity and/or effective temperature must be chosen. Recall that the Vogt–Russell theorem states that a unique stellar structure exists for a given mass and composition. Satisfying the central boundary conditions therefore requires specific surface boundary conditions. It is for this reason that a well-defined main sequence exists.

Since it is nearly impossible to satisfy the central boundary conditions exactly by the crude *shooting method* employed by STATSTAR, the calculation is terminated when the core is approached. The stopping criteria used here are that the interior mass $M_r < 0.01 M_s$ and the interior luminosity $L_r < 0.1 L_s$, when the radius $r < 0.02 R_s$, where M_s, L_s, and R_s are the surface mass, luminosity, and radius, respectively. [Within STATSTAR, $M_r =$ `M_r(i)`, $M_s =$ `Ms`, $L_r =$ `L_r(i)`, $L_s =$ `Ls`, $r =$ `r(i)`, and $R_s =$ `Rs`.] Once the criteria for halting the integration are detected, the conditions at the center of the star are estimated by an extrapolation procedure.

Since the pressure, temperature, and density are all assumed to be zero at the surface of the star, it is necessary to begin the calculation with approximations to the basic stellar structure equations. This can be seen by noting that the mass, pressure, luminosity, and temperature gradients are all proportional to the density and are therefore exactly zero at the surface. It would appear that applying these gradients in their usual form implies that the fundamental physical parameters cannot change from their initial values since the density would remain zero at each step!

One way to overcome this problem is to assume that the interior mass and luminosity are both constant through a number of surface zones. In the case of the luminosity, this is clearly a valid assumption since temperatures are not sufficient to produce nuclear reactions near the surfaces of main-sequence stars. For the interior mass, the assumption is not quite as obvious. However, we will see that in realistic stellar models, the density is so low near the surface that

[1] State-of-the-art research codes use much more sophisticated prescriptions for the equations of state.

the approximation is indeed valid. Of course, it is important to verify that the assumption is not being violated to within some specified limit.

Assuming that the surface zone is radiative, and given the surface values $M_r = M_s$ and $L_r = L_s$, dividing Eq. (10.7) by Eq. (10.61) leads to

$$\frac{dP}{dT} = \frac{16\pi ac}{3}\frac{GM_s}{L_s}\frac{T^3}{\overline{\kappa}}.$$

Since relatively few free electrons exist in the atmospheres of stars, electron scattering can be neglected and $\overline{\kappa}$ may be replaced by the bound–free and free–free Kramers' opacity laws [Eqs. (9.19) and (9.20)] expressed in the forms $\overline{\kappa}_{bf} = A_{bf}\rho/T^{3.5}$ and $\overline{\kappa}_{ff} = A_{ff}\rho/T^{3.5}$, respectively. Defining $A \equiv A_{bf} + A_{ff}$ and using Eq. (10.14) to express the density in terms of the pressure and temperature through the ideal gas law (assuming that radiation pressure may be neglected),

$$\frac{dP}{dT} = \frac{16\pi}{3}\frac{GM_s}{L_s}\frac{ack}{A\mu m_H}\frac{T^{7.5}}{P}.$$

Integrating with respect to temperature and solving for the pressure, we find that

$$P = \left(\frac{1}{4.25}\frac{16\pi}{3}\frac{GM_s}{L_s}\frac{ack}{A\mu m_H}\right)^{1/2} T^{4.25}. \tag{H.1}$$

It is now possible to write T in terms of the independent variable r through Eq. (10.61), again using the ideal gas law and Kramers' law, along with Eq. (H.1) to eliminate the dependence on pressure. Integrating,

$$T = GM_s\left(\frac{\mu m_H}{4.25k}\right)\left(\frac{1}{r} - \frac{1}{R_s}\right). \tag{H.2}$$

Equation (H.2) is first used to obtain a value for $T(r)$, then Eq. (H.1) gives $P(r)$. At this point it is possible to calculate ρ, $\overline{\kappa}$, and ϵ from the usual equation-of-state routine, `EOS`.

A very similar procedure is used in the case that the surface is convective. In this situation Eq. (10.81) may be integrated directly if γ is constant. This gives

$$T = GM_s\left(\frac{\gamma - 1}{\gamma}\right)\left(\frac{\mu m_H}{k}\right)\left(\frac{1}{r} - \frac{1}{R_s}\right). \tag{H.3}$$

Now, since convection is assumed to be adiabatic in the interior of our simple model, the pressure may be found from Eq. (10.75). Subroutine `STARTMDL` computes Eqs. (H.1), (H.2), and (H.3).

The conditions at the center of the star are estimated by extrapolating from the last zone that was calculated by direct numerical integration. Beginning

with Eq. (10.7), and identifying $M_r = 4\pi\rho_0 r^3/3$, where ρ_0 is taken to be the average density of the central ball (the region inside the last zone calculated by the usual procedure),[2]

$$\frac{dP}{dr} = -G\frac{M_r\rho_0}{r^2} = -\frac{4\pi}{3}G\rho_0^2 r.$$

Integrating,

$$\int_{P_0}^{P} dP = -\frac{4\pi}{3}G\rho_0^2 \int_0^r r\, dr$$

and solving for the central pressure results in

$$P_0 = P + \frac{2\pi}{3}G\rho_0^2 r^2. \tag{H.4}$$

Other central quantities can now be found more directly. Specifically, the central density is estimated to be $\rho_0 = M_r/(4\pi r^3/3)$, where M_r and r are the values of the last zone calculated. Neglecting radiation pressure, T_0 may be determined from the ideal gas law. Finally, the central value for the nuclear energy generation rate is computed using $\epsilon_0 = L_r/M_r$.

The numerical integration technique employed here is a fourth-order Runge–Kutta algorithm, which is accurate through fourth order in the step size $\Delta r =$ `deltar`. This means that if $\Delta r/r = 0.01$, the solutions for P, M_r, L_r, and T are accurate to approximately a few parts in $0.01^4 = 10^{-8}$, assuming that the results of the previous zone were exact. To accomplish this accuracy, the Runge–Kutta algorithm evaluates derivatives at several intermediate points between mass shell boundaries. Details of the Runge–Kutta method are given in many numerical analysis texts and will not be discussed further here.

STATSTAR execution times vary depending on the machine being used. For instance, on PCs with 486 33MHz chips or higher, a model can be completed in a few seconds; on faster machines only a fraction of a second is required. It should be pointed out that if STATSTAR is compiled on a VAX computer running VMS, the \G_FLOAT option should be invoked. This option provides the large exponent range required of most astrophysical calculations.

[2]You might notice that dP/dr goes to zero as the center is approached. This behavior is indicative of the smooth nature of the solution. Close inspection of the graphs in Section 11.1 showing the detailed interior structure of the Sun illustrates that the first derivatives of many physical quantities go to zero at the center.