Feel free to write or type your answers, and to submit either a hard copy or electronic version of your answers. You may discuss the homework with your classmates, but please write up your homework independently.

Problem 1 - (15 points) - *Luminosity variation from stellar pulsation*
In this problem we will make a crude estimate of how the luminosity variation of a pulsating star is related to the variation in its radius. This provides a way of estimating the amount by which the radius is changing for a star that we observe to pulsate. We will consider a star whose unperturbed state consists of a luminosity L_0, a radius R_0, and a surface temperature T_0.

(a) The star's luminosity is related to its radius and surface temperature by $L = 4\pi R^2\sigma T^4$. Suppose that, as a result of pulsation, the radius changes by an amount δR and the surface temperature changes by an amount δT. Estimate the resulting change in luminosity δL. You may assume that $\delta R/R$, $\delta T/T$, and $\delta L/L$ are all small, so the equations can be linearized.

(b) Assume that the star is composed of an adiabatic, ideal gas with adiabatic index γ_a, and that the expansion and contraction of the star are homologous. Using these assumptions, derive a relationship between δR and δT.

(c) Use the result of part (b) to eliminate δT from your answer to part (a), and arrive at an estimate for the relationship between δL and δR. Based on this result, does the peak luminosity of a pulsating star with $\gamma_a = 5/3$ occur when its radius is at its maximum value or its minimum value?

Problem 2 - (35 points) - *Generating the T vs. ρ diagram*
For this problem you will end up a figure like Figure 1, which we have used many times in class. Use cgs units throughout. Also insert the numerical values for all the physical constants used and give the derived relationships with all their proper numerical constants.

(a) Assume a fully ionized gas that by mass is 75% hydrogen and 25% helium. Calculate μ_{I}, μ_{e}, and μ.

(b) Solve for the boundary between the ideal gas zone and the non-relativistic degenerate gas zone. *Hint: To answer, begin by balancing the pressure equations. In this case that means setting*

$$K_0\rho T = K_1\rho^{5/3}.$$

Then calculate K_0 and K_1, and finally solve for ρ (g cm^{-3}) as a function of T (K). You should end up with an expression of the form

$$log\rho = \alpha logT - \text{constant},$$

where α is a number.

(c) Solve for the boundary between the non-relativistic and relativistic degenerate gas zones.

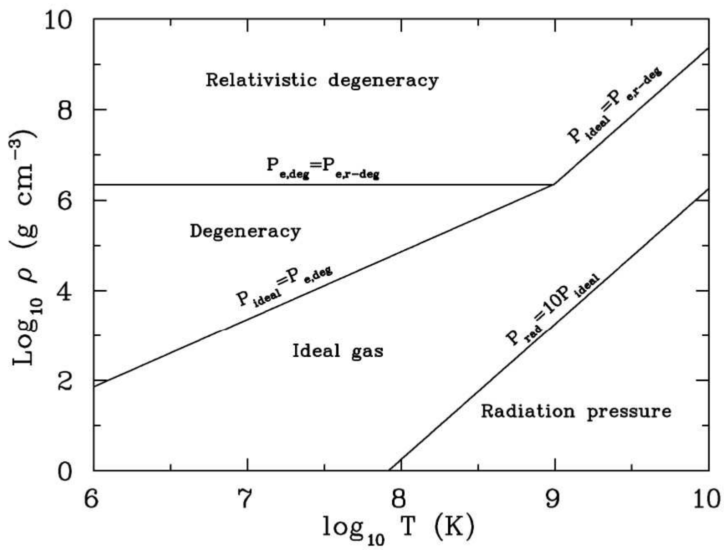

Fig. 1.— The temperature-density diagram.

(d) Solve for the boundary between the ideal gas zone and the relativistic degenerate gas zone.

(e) Solve for the value of (T, ρ) where the ideal gas pressure, non-relativistic degenerate gas pressure, and relativistic gas pressure are all equal.

(f) Solve for the boundary between the ideal gas zone and the radiation pressure taking the boundary as $P_{rad} = 10P_{gas}$.

(g) Using the values generated in (a)-(e), plot the boundaries between the different zones for $\log T(\mathrm{K}) = 6-10$ and $\log \rho(\mathrm{g\ cm^{-3}}) = 0-10$. The general structure of your figure should match Figure 1, though it will differ in the exact details.

Problem 3 - (20 points) - Convection

(a) Show that the envelope of a star that has a Kramers Law opacity (with a = 1, b = −3.5) is stable against convection if the equation of state is that of an ideal gas with $\gamma = 5/3$.

(b) Use this result to predict which main-sequence stars should have convective envelopes.

Problem 4 - (30 points) - *More STATSTAR fun*
(a) Use the stellar structure code STATSTAR to calculate homogeneous, main-sequence models for the parameters in the table on the next page. All models have X = 0.7, Y = 0.292, and thus Z = 0.008. Remember to rename the output file after each model run, because future runs will overwrite this file.

(b) After obtaining satisfactory models, overplot these masses on a P vs. R, a M vs. R, a L vs. R, and a T vs. R graph. You may find that you need to plot only a subset of these masses (say, 0.5, 1.0, 4.0, 8.0, and 13.5 $M_\odot$) rather than every single one. *Plot as many masses as needed to show the differences in structure.* Note that to this point we are just doing the same work you did last time with the code — we are just making sure we remember how to make things work!

Mass ($M_\odot$)	Luminosity ($L_\odot$)	Temperature (K)
0.5	0.02150	2331.45
1.0	0.86071	5500.20
2.0	22.61200	11218.40
4.0	341.09998	17904.00
6.0	1375.34998	22310.00
8.0	3421.51978	25613.60
10.0	6641.59961	28263.60
13.5	15246.32910	32149.90

(c) For each of the masses, plot log $(L/L_\odot)$ vs. log $(M/M_\odot)$. Then, using an approximate power-law relation (formally known as the mass-luminosity relation) of the form $(L/L_\odot) = (M/M_\odot)^\nu$, find an appropriate value for ν. Do you find that ν varies with mass? How does your value for ν compare to those found in the literature (see Figure 2)?

(d) How close is the model 1.0 $M_\odot$ star to being adiabatic? To determine this, first fit a line to the relation $\log P = \gamma_a \log \rho + \log K_a$, which is obtained from taking the log of the adiabatic equation of state. Then evaluate the goodness of fit (is the line fit within a few percent of the model value?). Comment on the corresponding adiabatic index.

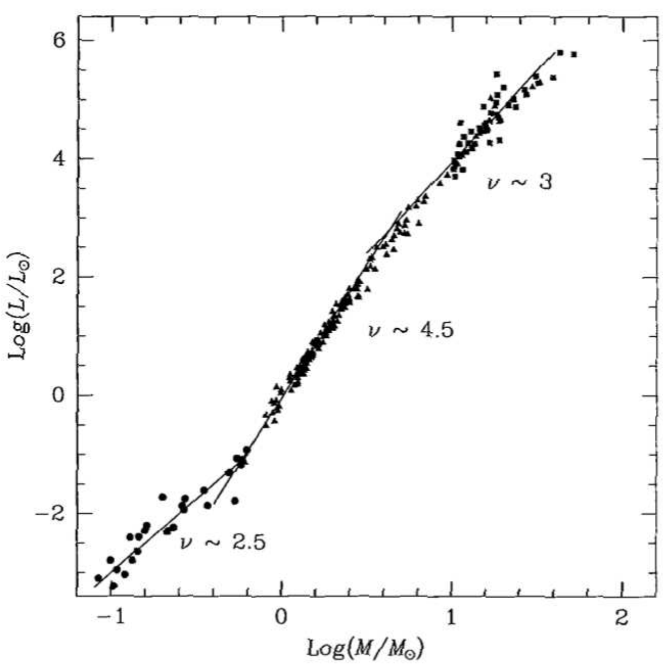

Fig. 2.— From lecture 13. Figure from Prialnik, *Theory of Stellar Structure and Evolution.*