Modern Stellar Astrophysics – Prof. Agüeros
Homework 4, Due Tuesday, Dec 9

Feel free to write or type your answers, and to submit either a hardcopy or electronic version of your answers. You may discuss the homework with your classmates, but please write up your homework independently.

Problem 1 - (15 points) - *Estimating the maximum stellar luminosity*
Consider an optically thin blob of material at the surface of a star of radius R and mass M.

(a) Recalling that $L \equiv F(R)$, the outward radiation force per unit mass is given by

$$\frac{\kappa L}{4\pi R^2 c}.$$

By balancing this with the gravitational force per unit mass, show that the criterion for the blob to be ejected from the star by radiation pressure is

$$\frac{M}{L} < \frac{\kappa}{4\pi c G}.$$

(b) If $\kappa = 0.3$ cm^2 g^{-1}, calculate the maximum L that stars of mass M can have without ejecting material by radiation pressure. Express your results in units of $(M/M_\odot)$ erg s^{-1}. What is this luminosity called?

(c) Given a relationship between between L and M

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^4$$

what is the mass (in solar units $M_\odot$) of a star at the maximum luminosity you defined above? For a fixed L in part (b) what would need to change for stars more massive than this to be stable?

Problem 2 - (20 points) - *Core mass-luminosity relation for RGB stars*
Low-mass stars on the RGB obey a core mass-luminosity relation, which is approximately given by

$$L \approx 2.3 \times 10^5 L_\odot \left(\frac{M_c}{M_\odot}\right)^6 \tag{1}$$

where M_c is the core mass. The luminosity is provided by hydrogen shell burning.

(a) Derive a relation between luminosity L and the rate at which the core grows dM_c/dt. Use the energy released per gram in hydrogen shell burning.

(b) Derive how the core mass evolves in time, i.e, $M_c = M_c(t)$.

(c) Assume that a star arrives to the RGB when its core mass is 15% of the total mass, and that it leaves the RGB when the core mass is 0.45 $M_\odot$. Calculate the total time a 1 $M_\odot$ star spends on the RGB and do the same for a 2 $M_\odot$ star. Compare these to the main sequence (MS) lifetimes of these stars.

(d) What happens when the core mass reaches 0.45 $M_\odot$? Describe the following evolution of the star (both its interior and the corresponding evolution in the HR diagram).

(e) What is the difference in evolution with stars more massive than 2 $M_\odot$?

Problem 3 - (25 points) - *Mass loss in AGB stars*
One of the most popular parametrizations of mass loss in asymptotic giant branch (AGB) stars, developed by Dieter Reimers, is given by

$$\dot{M} = -4 \times 10^{-13} \eta \frac{L/L_\odot}{(g/g_\odot)(R/R_\odot)} \ M_\odot \ \mathrm{yr}^{-1},$$

where L, g, and R are the luminosity, surface gravity, and radius of the star, expressed in solar units. η is a free parameter whose value is expected to be near unity. Note that the minus sign has been explicitly included, indicating that the mass of the star is decreasing.

(a) Explain qualitatively why L, g, and R enter the equation in the way they do.

(b) Estimate the mass-loss rate of a 1 $M_\odot$ AGB star that has $L = 7000\ L_\odot$ and $T = 3000$ K. For reference, $g_\odot = 2.74 \times 10^4$ cm s^{-2}.

(c) Re-write the equation in terms of L, R, and M, retaining the solar units.

(d) Assuming (incorrectly) that L, R, and η do not change with time, derive an expression for the mass of the star as a function of time. Let $M = M_0$ when the mass-loss phase begins.

(e) How long does it take for the 1 $M_\odot$ AGB star to lose its envelope, leaving a degenerate core of about 0.6 $M_\odot$? What evolutionary phase does this correspond to?

Problem 4 - (40 points) - *Yet more STATSTAR fun*
Use the data from your model 1.0 $M_\odot$ star to answer the following questions.

(a) In `STATSTAR`, the opacity is calculated based on several functional forms that include a Kramers opacity law and electron scattering. Let's investigate whether we can approximate the opacity that results in the model star using a commonly used power-law functional form. We can express the opacity as $\kappa = \kappa_0 \rho^\alpha T^\beta$, but it is more tractable to use the logarithm of the opacity:

$$log\kappa = \alpha log\rho + \beta logT + log\kappa_0,$$

which we can investigate by plotting log κ vs. log T or log κ vs. log ρ. If we assume a Kramers opacity law, then $\alpha = 1.0$ and $\beta = -3.5$. By estimating the constant term (which should be log $\kappa_0 \sim 24$) does the slope of this law appear to provide a good fit? Compare to a modified functional form discussed in class ($\alpha = 0.5$ and $\beta = -2.5$, with constant term log $\kappa_0 \sim 17$). Does the slope of this law appear to provide a better fit? (It should.)

(b) Show quantitatively (using data from the model output) that the center of the 1 $M_\odot$ star is convective and that the midpoint the outer radii are radiative (choose any two mid and outer points). This requires verifying whether the physical conditions in the star satisfy the convective instability.

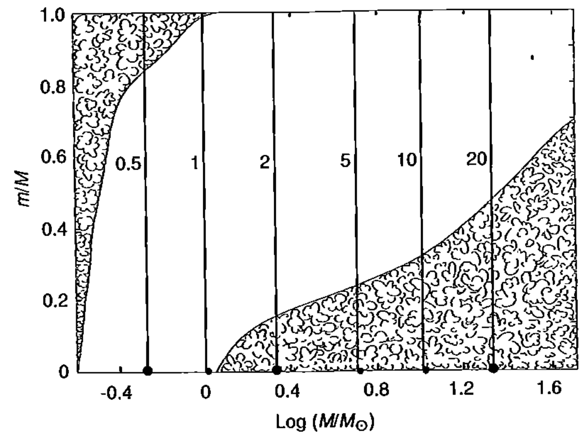

Figure 9.4 The extent of convective zones (shaded areas) in main-sequence star models as a function of the stellar mass (adapted from R. Kippenhahn and A. Weigert (1990), *Stellar Structure and Evolution*, Springer-Verlag).

(c) Compare the results of this model with the figure above. Do your results agree or disagree with where convection zones are expected for a 1 $M_\odot$ main-sequence star? Can you think of any reason why it might agree or might not? In this respect, is the 0.75 $M_\odot$ model that you calculated any different?

(d) Explain the location of the convective zones in the figure. When and why will the outer envelope be convective? When and why will a core become convective?

(e) Explain why the helium burning core of a star is likely to be convective.