Examples of 2^{nd} Order Homogeneous ODE's with Constant Coefficient

EX: Find the general solution of: $d^2y/dt^2 - 3dy/dt + y = 0.$

$\rightarrow y'' - 3y' + y = 0$

Step 1: Characteristic equation:

$a=1$
$b=-3$
$c=1$

$\hookrightarrow r^2 - 3r + 1 = 0$

Step 2: find roots:

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{+3 \pm \sqrt{9-4}}{2}$$

$$r_{1,2} = \frac{3 \pm \sqrt{5}}{2}$$

Step 3: Plug into solution form:

$$y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

$$y = c_1 e^{\frac{3+\sqrt{5}}{2}t} + c_2 e^{\frac{3-\sqrt{5}}{2}t}$$

EX: Solve the initial value problem: $d^2y/dt^2 - 3dy/dt - 4y = 0$; $y(0)=1$, $y'(0)=0$

Step 1: Char eq:

$a=1, b=-3, c=-4 \longrightarrow r^2 - 3r - 4 = 0$

Step 2: roots:

$r^2 - 3r - 4 = 0$
$(r-4)(r+1) = 0$
$r_1 = 4$
$r_2 = -1$

Step 3: $y = c_1 e^{4t} + c_2 e^{-t}$

$\rightarrow$ initial values: $y(0)=1$, $y'(0)=0$

$\rightarrow y = c_1 e^{4t} + c_2 e^{-t}$
$1 = c_1 e^0 + c_2 e^0$
$1 = c_1 + c_2$
$c_1 = 1 - c_2$

$y' = 4c_1 e^{4t} - c_2 e^{-t}$
$y'(0) = 4c_1 e^0 - c_2 e^0$
$0 = 4c_1 - c_2$
$c_2 = 4c_1$

$c_1 = 1 - 4c_1$
$5c_1 = 1$
$c_1 = 1/5$, then $c_2 = 4/5$

Then the unique solution is: $y = \frac{1}{5} e^{4t} + \frac{4}{5} e^{-t}$