Examples of seperable equations:

Ex: Find the general solution of: $dy/dt = 1 + t + y^2 - ty^2$

$*$ Need to put in standard form: $f(y)\, dy/dt = g(t)$

$$dy/dt = 1 - t + (1-t)y^2$$
$$dy/dt = (1-t)(1+y^2)$$
$$dy/dt\left(\frac{1}{1+y^2}\right) = 1-t, \quad \text{where } f(y) = \frac{1}{1+y^2} \text{ and } g(t) = 1-t$$

Step 1: Integrate both sides w/respect to t:

$$\int \frac{1}{1+y^2}\frac{dy}{dt}\, dt = \int 1-t \, dt$$
$$\int \frac{1}{1+y^2}\, dy = t - \frac{1}{2}t^2 + c$$
$$\tan^{-1}(y) = t - \frac{1}{2}t^2 + c$$

Step 2: Solve for y: $y = \tan\left(t - \frac{1}{2}t^2 + c\right)$

Ex: Solve the initial value problem: $dy/dt = 2t/y + yt^2$; $y(2) = 3$

$*$ Put in standard form: $dy/dt = 2t/y(1+t^2)$

$$= \left(\frac{2t}{1+t^2}\right)\left(\frac{1}{y}\right)$$
$$y\left(\frac{dy}{dt}\right) = 2t/1+t^2$$

u-sub: $u = 1+t^2$
$du = 2t\, dt$

Step 1: Integrate: $\int y \frac{dy}{dt}\, dt = \int 2t/1+t^2 \, dt$

$$\int y\, dy = \int \frac{1}{u}\, du$$
$$\frac{1}{2}y^2 = \ln(u) = \ln(1+t^2) + C$$

Step 2: Solve for y: $y^2 = 2\ln(1+t^2) + 2C$

$$y = \sqrt{2\ln(1+t^2) + 2C}$$

$\rightarrow$ initial values: $y(2) = 3$:

$$3 = \sqrt{2\ln(1+4) + 2C}$$
$$9 = 2\ln(5) + 2C$$
$$9 - 2\ln(5) = 2C$$
$$C = \frac{9 - 2\ln(5)}{2}$$

Then the unique solution is: $y = \sqrt{2\ln(1+t^2) + 9 - 2\ln(5)}$

$$y = \sqrt{\ln\left(\frac{(1+t^2)^2}{25}\right) + 9} \quad \text{(simplified)}$$