Examples of Repeated Roots Case (Homogeneous)

EX: Find the general solution of $y'' - 6y' + 9y = 0$

Step 1: Char eq: $a=1, b=-6, c=9$ $\rightarrow$ $r^2 - 6r + 9 = 0$, $(r-3)(r-3)=0$, $r=3$

Step 2: $y = c_1 e^{r_1 t} + t c_2 e^{r_1 t}$

$y = c_1 e^{3t} + t c_2 e^{3t}$

EX: Solve the IVP: $9y'' + 6y' + 1 = 0$; $y(0)=1$, $y'(0)=0$

Step 1: Char eq: $9r^2 + 6r + 1 = 0$

$(3r+1)^2 = 0$

$r = -1/3$

Step 2: General form: $y = c_1 e^{r_1 t} + c_2 t e^{r_1 t}$

$\hookrightarrow y = c_1 e^{-1/3 t} + c_2 t e^{-1/3 t}$

$\rightarrow$ initial values: $y(0)=1$, $y'(0)=0$

$y = c_1 e^{-1/3 t} + c_2 t e^{-1/3 t}$

$1 = c_1 e^0 + c_2 \cdot 0$

$1 = c$

$y' = -1/3\, c_1 e^{-1/3 t} + c_2 e^{-1/3 t} + c_2 t e^{-1/3 t}(-1/3)$

$0 = -1/3\, c_1 e^0 + c_2 e^0 + 0$

$0 = -1/3\, c_1 + c_2$

$0 = -1/3 (1) + c_2$

$1/3 = c_2$

The solution is $y = e^{-1/3 t} + 1/3\, t e^{-1/3 t}$