Applied Mathematics
001 – Introduction

Course introduction and policies

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Introduction

- Who am I?

- How can you contact me?
  - email: christian.wallraven+AMF2023@gmail.com
  - I do not have fixed office hours – please contact me via email anytime

What are we trying to do?

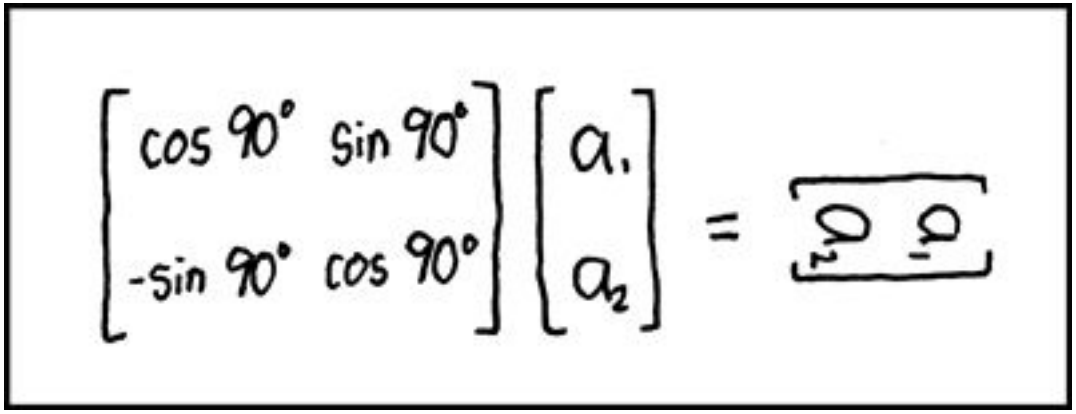
$$\frac{1}{n}\sin x = ?$$

$$\frac{1}{n}\sin x =$$

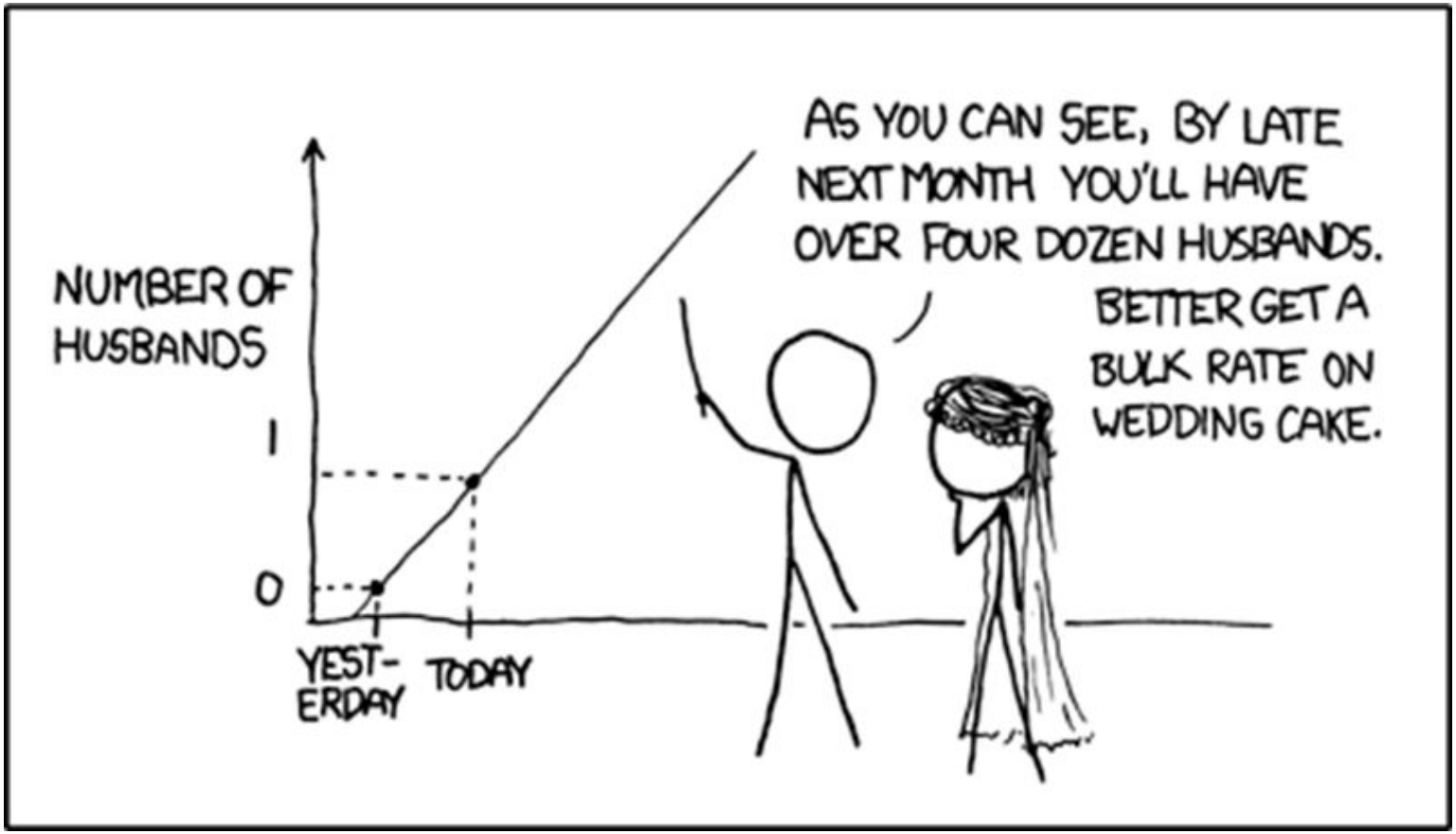
$$six = 6$$

$$\frac{\sin(\text{gerine})}{\cos(\text{gerine})} =$$

What are we trying to do?

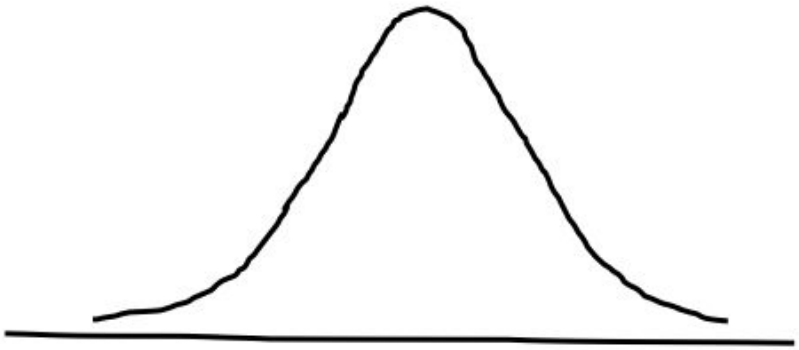


What are we trying to do?



xkcd.com

What are we NOT trying to do?



Normal Distribution

Paranormal Distribution

What are we doing this for?

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- A good grasp of mathematics lies at the core of any career in the natural sciences
- This is especially true for neuroscience and cognitive science in which vast amounts of data need to be analyzed efficiently in order to come up with viable models and predictions
  - typical size of fMRI dataset from one session is ~100MB per person
  - typical size of image datasets for object recognition is ~1000000 images

Big numbers

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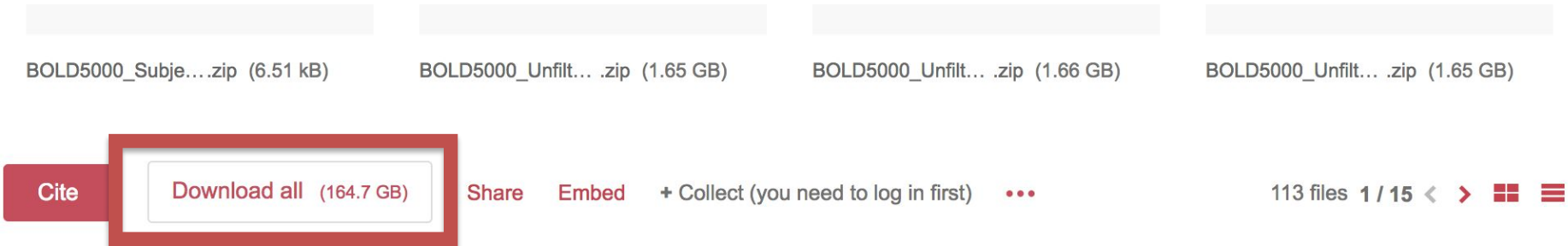
- fMRI datasets are huge
  - "fMRI dataset consisting of 11 subjects performing 4 simple motor tasks: left finger-to-thumb, left squeeze, right finger-to-thumb, and right squeeze"
  - "Each volume after standard preprocessing (realignment, normalization, and smoothing, described in the Image preprocessing section) consisted of 79 × 95 × 69 voxels. Thus, the dataset was most easily represented as a 6th order tensor of dimensionality 79 × 95 × 69 × 120 × 4 × 11, of which the first four modes were spatiotemporal and the remaining two (task and subject) were categorical. Our total dataset's size was thus **2,734,221,600 voxels, occupying approximately 9.3 GB of space**."

Michael Barnathan, Vasileios Megalooikonomou, Christos Faloutsos, Scott Faro, Feroze B. Mohamed. TWave: High-order analysis of functional MRI. NeuroImage, Volume 58, Issue 2, 15 September 2011, Pages 537–548

Big numbers

- fMRI datasets are getting more huge – BOLD5000
  - The experiment including 4,803 images presented on a single trial throughout the experiment, and 112 images repeated four times, and one image repeated three times, throughout the experiment, yielding a total of 5,254 stimuli trials. The stimuli were drawn from three datasets: 1) 1000 images from Scene Images (250 scene categories, based on SUN categories, with four exemplars each); 2) 2000 images from the COCO dataset; and 3) 1916 images from the ImageNet dataset. In the experiment, images were presented for 1 second, with 9 seconds of fixation between trials. Participants were asked to judge whether they liked, disliked, or were neutral about the image.

    Chang, Pyles, Gupta, Tarr, Aminoff, BOLD5000.org



Goal of this course

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- To get an understanding of some core concepts in scientific computing
- To connect theoretical with practical knowledge
- To get an introduction into programming
- To design, implement, and present a course project

Introduction

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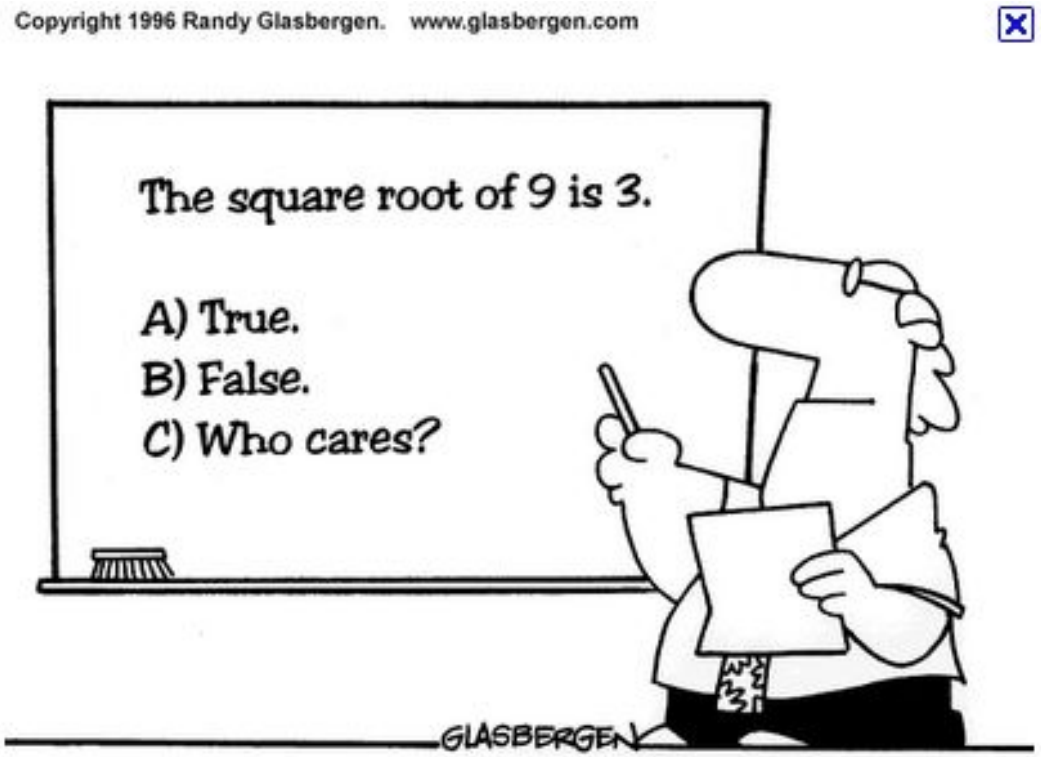
- After a brief math refresher, this course gives an introduction into applied computational mathematics (also called "scientific computing") covering numerical methods for solving linear equations, function minimization and approximation, Fourier analysis, as well as programming neural networks.
- One of the main topics at the end of this class is to use your programming skills to work on a few topics related to Exploratory Data Analysis
- It also works through a set of examples, and gives you the opportunity to design, implement, and present a small mathematical course project of your own.

Course content

- The lecture part of the course will provide a theoretical introduction into each topic that focuses on building up an intuitive understanding of the underlying mathematics.
- We will start fairly basic in the first few weeks with a refresher on calculus and linear algebra.



Many students actually look forward to Mr. Atwadder's math tests.

Course content

- Then we will dive into more in-depth topics, such as function optimization, equation solving, Fourier analysis, and more.
- The course will end with an introduction into exploring and describing data, laying foundations for
  - statistics
  - pattern recognition

Course content

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- The accompanying application part of the course will focus on working through a set of representative examples such as:
  - decoding of speech sounds
  - fitting psychophysical data
  - **initial** basics of neural networks and deep learning
  - image processing
  - exploring data
  - time series data…

Course content

- All examples and applications in the course will be based on Python
- Accordingly, a brief introduction into Python (and programming in general) will be given during the first weeks of the course.

Course content

- Finally, the last part of the course will be devoted to presentations of the course projects. These projects should be related somehow to methods introduced in the course, but can also be analyses of your own data, an implementation of a game, etc.

Teams

- Assignments and projects will be handled in teams of 2-3 students. Teams will have to be formed by the **beginning of the 3rd week of the semester**
- **Each team member is FULLY responsible for ALL the work of ALL other members. If ONE person in the team fails, the WHOLE team fails!**

Assignments

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- You will get a total of 6-7 assignments containing programming questions
- You will have one week to complete each assignment
- The assignment needs to be handed in **via email** with a strict deadline of **one hour** before class starts.
- The email should contain a commented piece of code and should be self-contained, that is, I should be able to run it (based on the documentation in the code) on my computer.

Assignments

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- If there are any problems meeting this deadline, **email me beforehand** – failure to deliver an assignment more than once on the deadline without being excused by me will result in the team failing the course.
- **Do not copy & paste other peoples' code, but understand and solve the problem yourself – you will need to be able to explain each line of the code to me when asked.**
- **If you copy code from ANY source, I will have to fail you for this course!!**

Course project

- You will have to choose a course project one month before the end of the semester.
- The project could be a small data analysis, or a user interface, a game, or can be related to your area of interest.
- You will be required to present your idea to me, so that I can approve it.
- Your project should be presented in class with a demo and an accompanying presentation of about 15 minutes explaining the background, code, and results.

Course project

- The project code will need to be handed in via email before the class session in which you are to present it.
- Again, the code needs to be able to run on my computer without modifications and needs to be well-commented and documented.

Course policies

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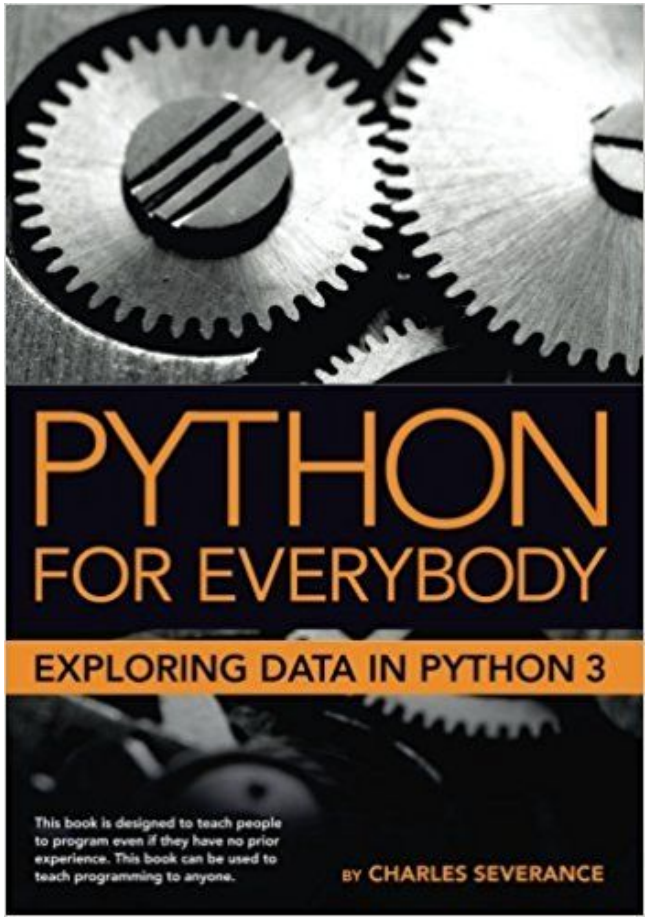
- I used to implement that regular class attendance is required and regular attendance determined 10% of the grade
- For me, whether you come to the class or not – in the end – is a matter of **your** motivation!
- I promise to do my best as a teacher to teach you well
- I promise to listen to your concerns and take them seriously
- I also hope that you can promise to do YOUR best for this class and to make it worthwhile!

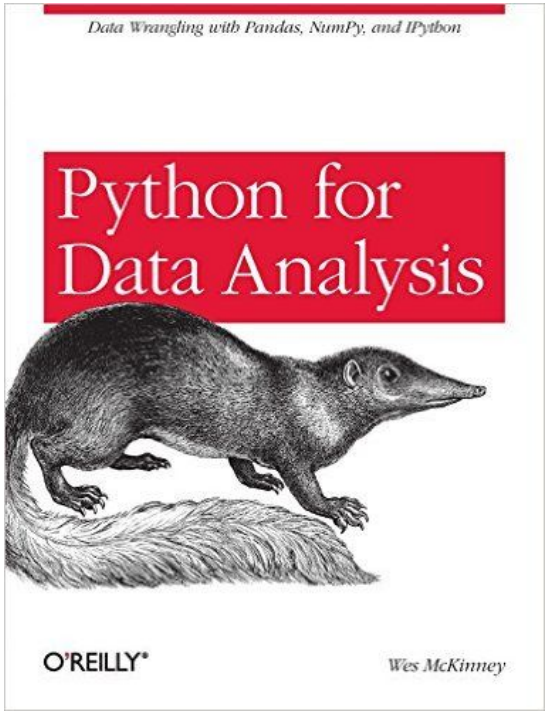
Suggested reading

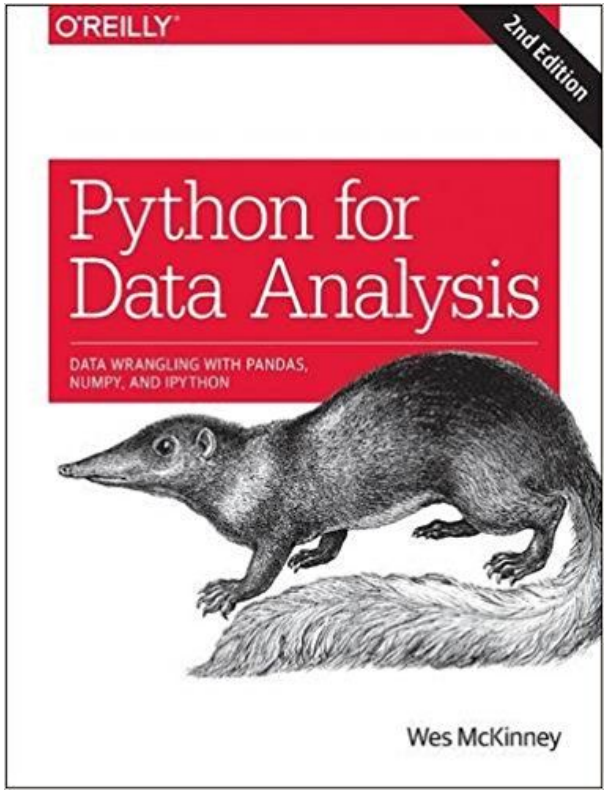
- Python for Everyone
- Good focus on data handling
- Free ebook at
  - http://do1.dr-chuck.com/pythonlearn/EN_us/pythonlearn.pdf



Suggested reading

- Python for Data Analysis
- Nice exploration of core algorithms in data analysis, which we will talk about also in this class





The Plan™

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Day	Date	Topic
Day1	Week 1	Course introduction
Day2	Week 1	Intro to Programming
Day1	Week 2	Intro to Programming
Day2	Week 2	Intro to Programming
Day1	Week 3	Intro to Programming
Day2	Week 3	Elementary Functions
Day1	Week 4	Derivatives
Day2	Week 4	Integrals
Day1	Week 5	Vectors
Day2	Week 5	Vectors & Matrices
Day1	Week 6	Matrices
Day2	Week 6	Linear Equations
Day1	Week 7	Linear Equations
Day2	Week 7	Linear Equations
Day1	Week 8	Least Squares / Optimization
Day2	Week 8	Least Squares / Optimization
Day1	Week 9	Least Squares / Optimization
Day2	Week 9	Least Squares / Optimization

The Plan™

Day1	Week 10	Neural Networks
Day2	Week 10	Neural Networks
Day1	Week 11	Neural Networks
Day2	Week 11	Neural Networks
Day1	Week 12	Fourier Analysis
Day2	Week 12	Fourier Analysis
Day1	Week 13	Fourier Analysis
Day2	Week 13	Exploratory Data Analysis – Introduction
Day1	Week 14	Exploratory Data Analysis – Dimensionality Reduction
Day2	Week 14	Exploratory Data Analysis – Cluster Analysis
Day1	Week 15	**Project Presentations**
Day2	Week 15	**Project Presentations**

NOTES:

There will be NO class during the mid-term period
Final project presentations will be in the final-exam period

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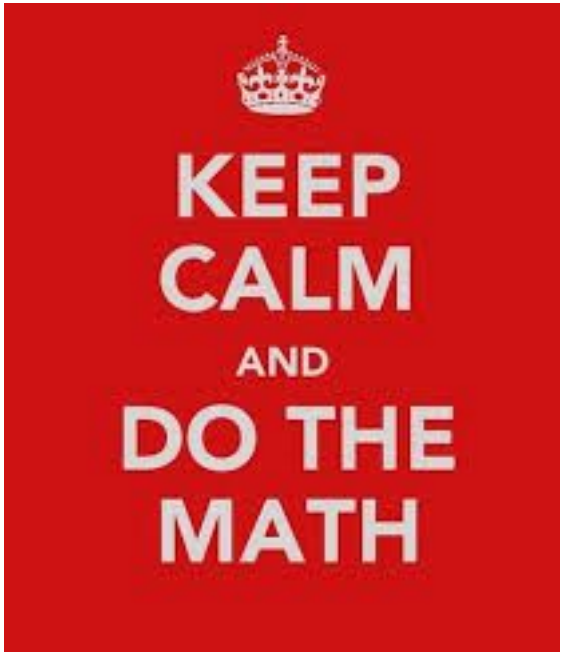
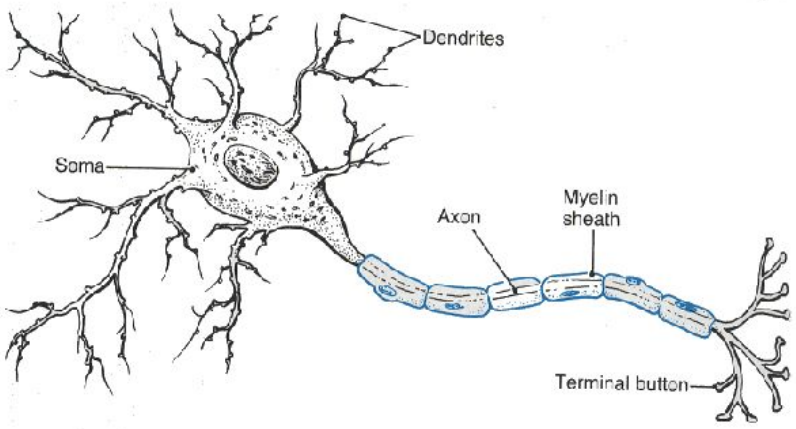
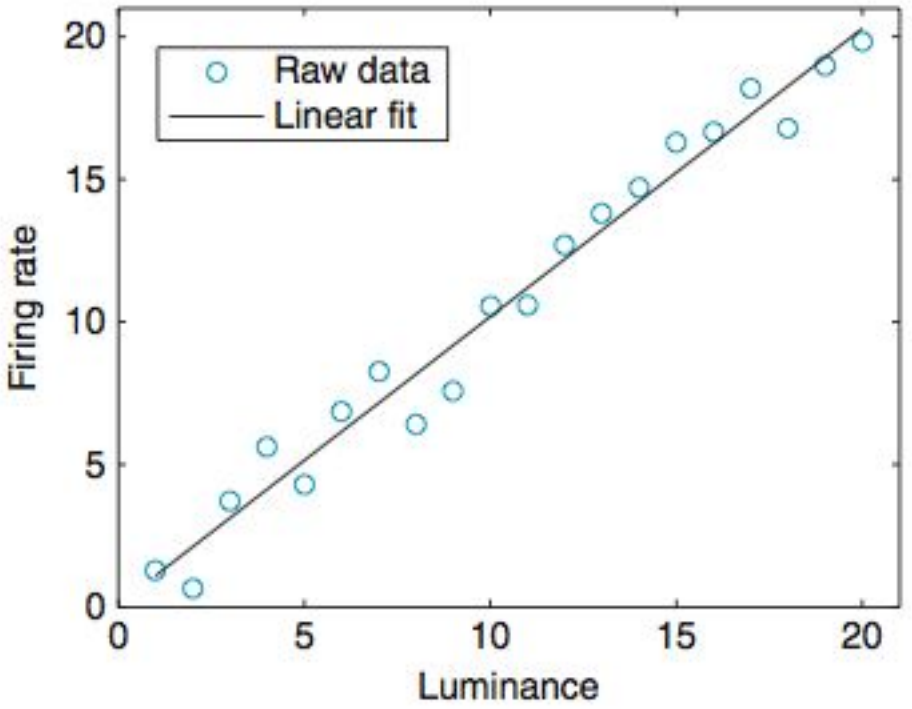
- Questions on organizational issues?



Fitting neural firing rates

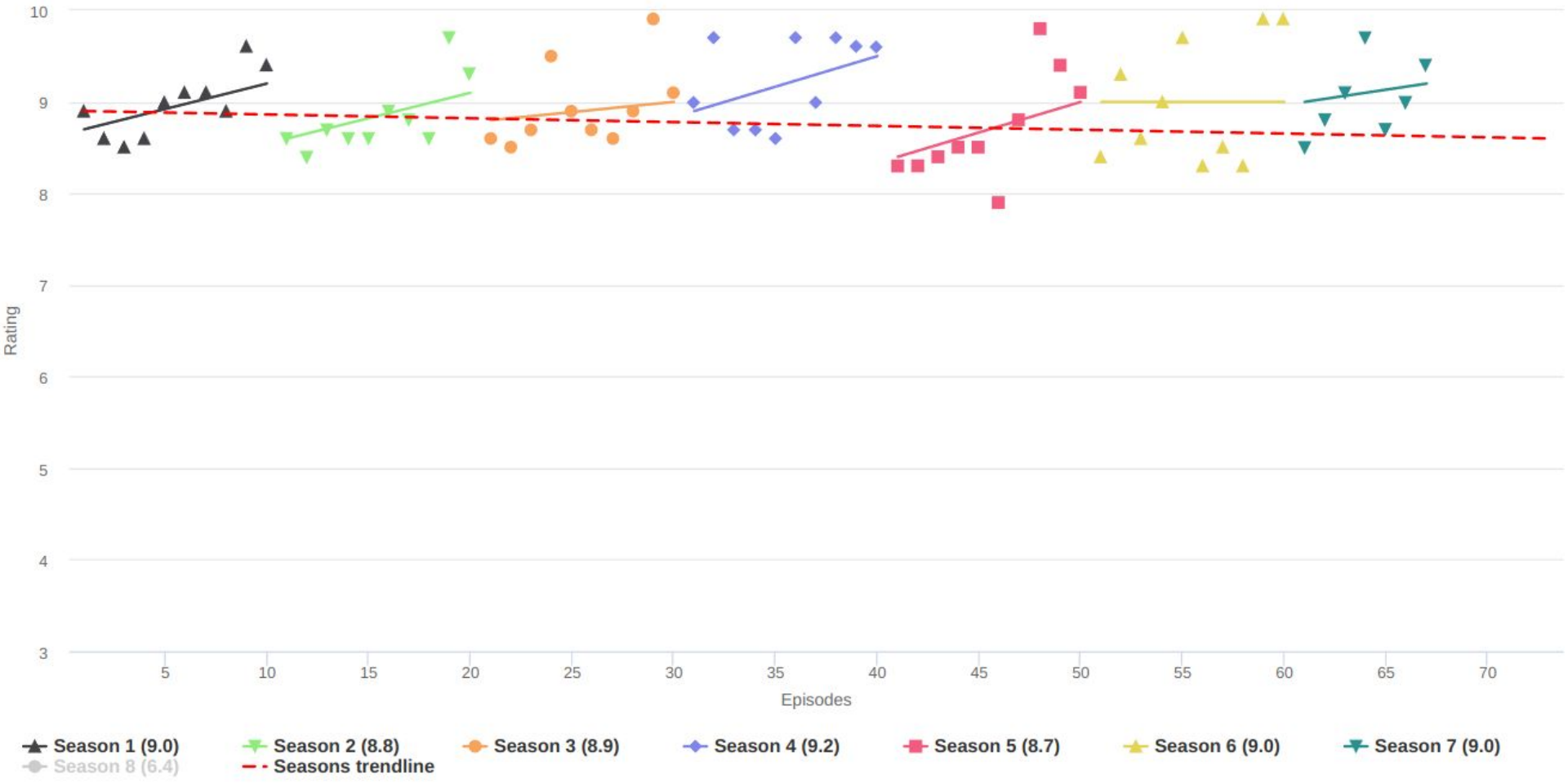


http://www2.cedarcrest.edu/academic/bio/hale/bioT_EID/lectures/tetanus-neuron.gif



Fitting patterns in television

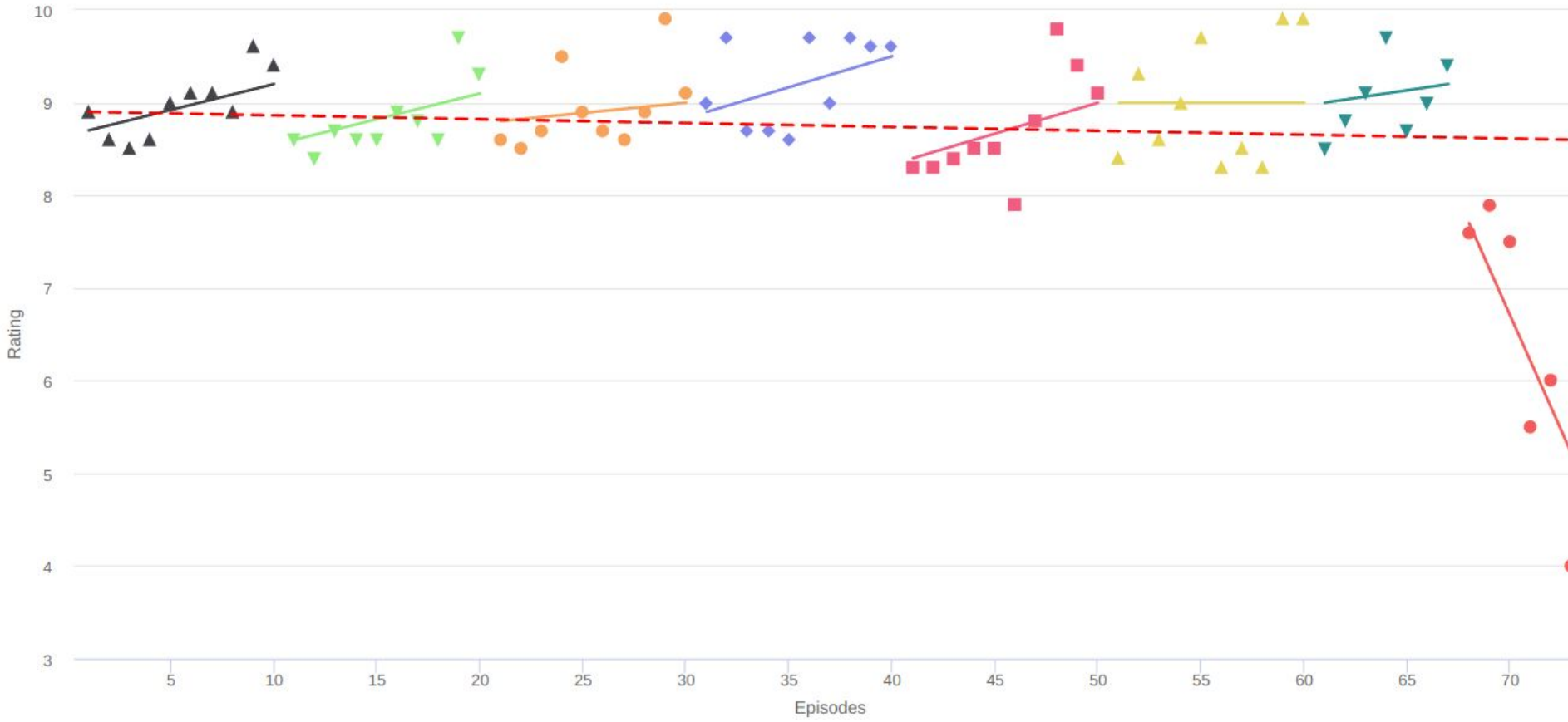
- Ratings for the show "Game of Thrones" Seasons 1-7



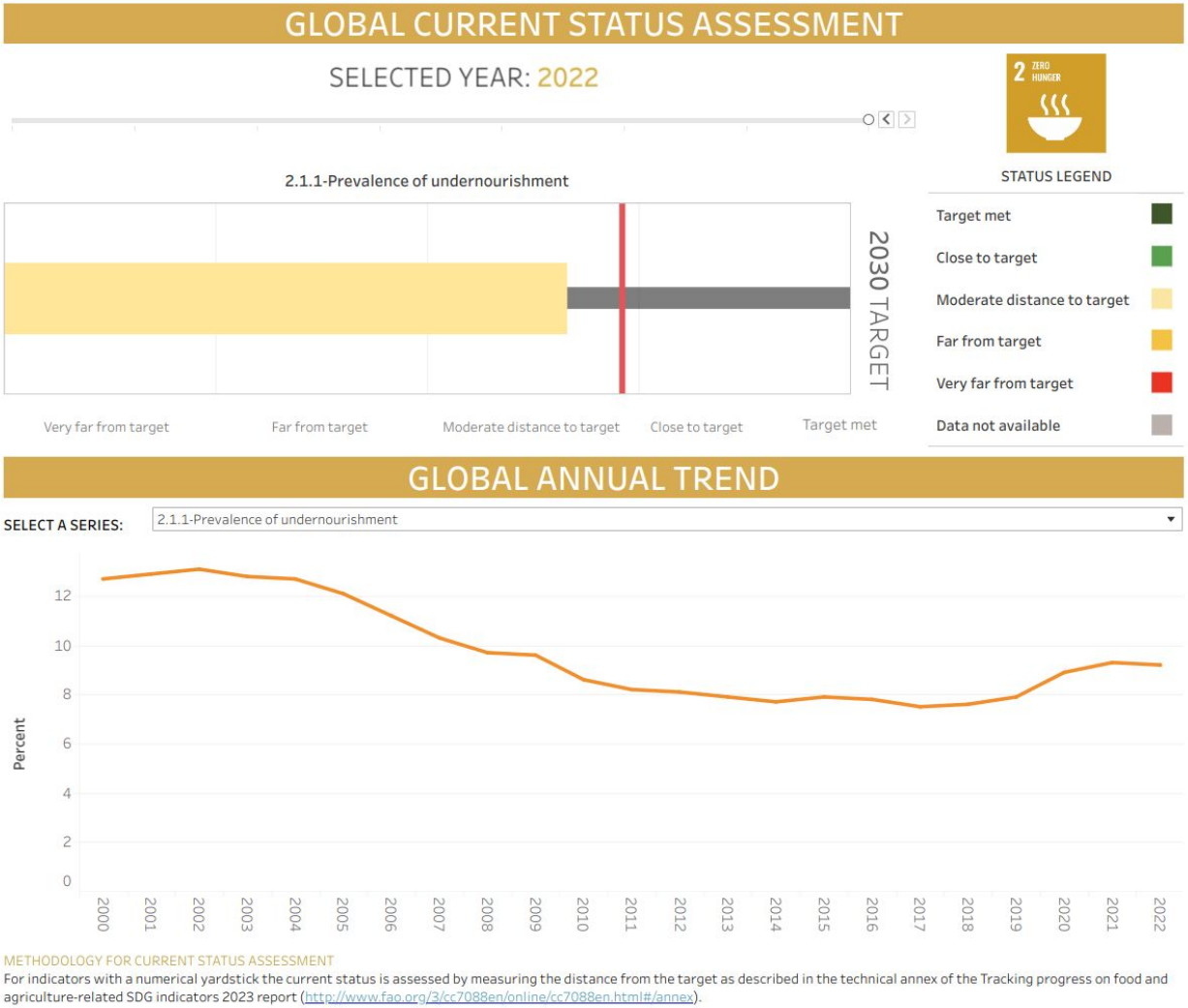
https://www.ratingraph.com/tv-shows/game-of-thrones-ratings-26649/

Fitting patterns in television

- Ratings for the show "Game of Thrones" Seasons 1-8(!)

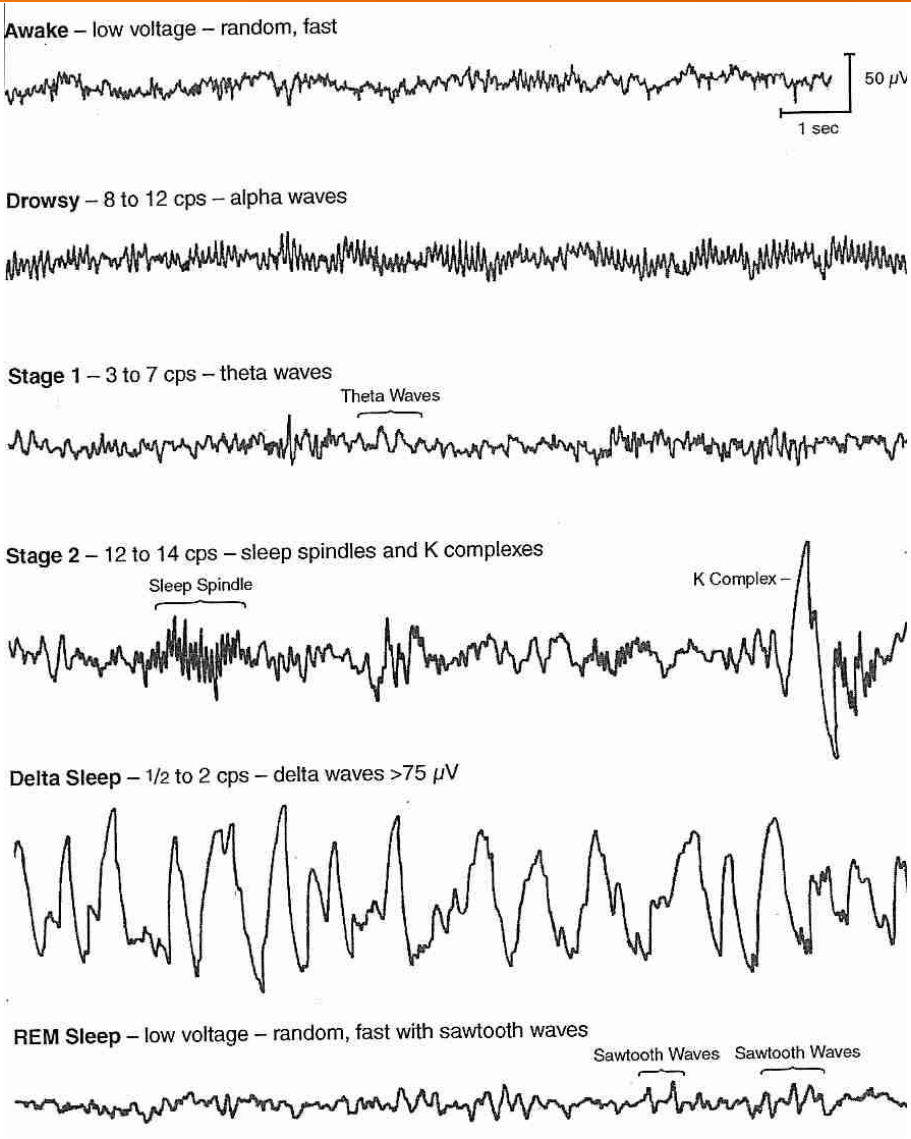


Finding patterns in data



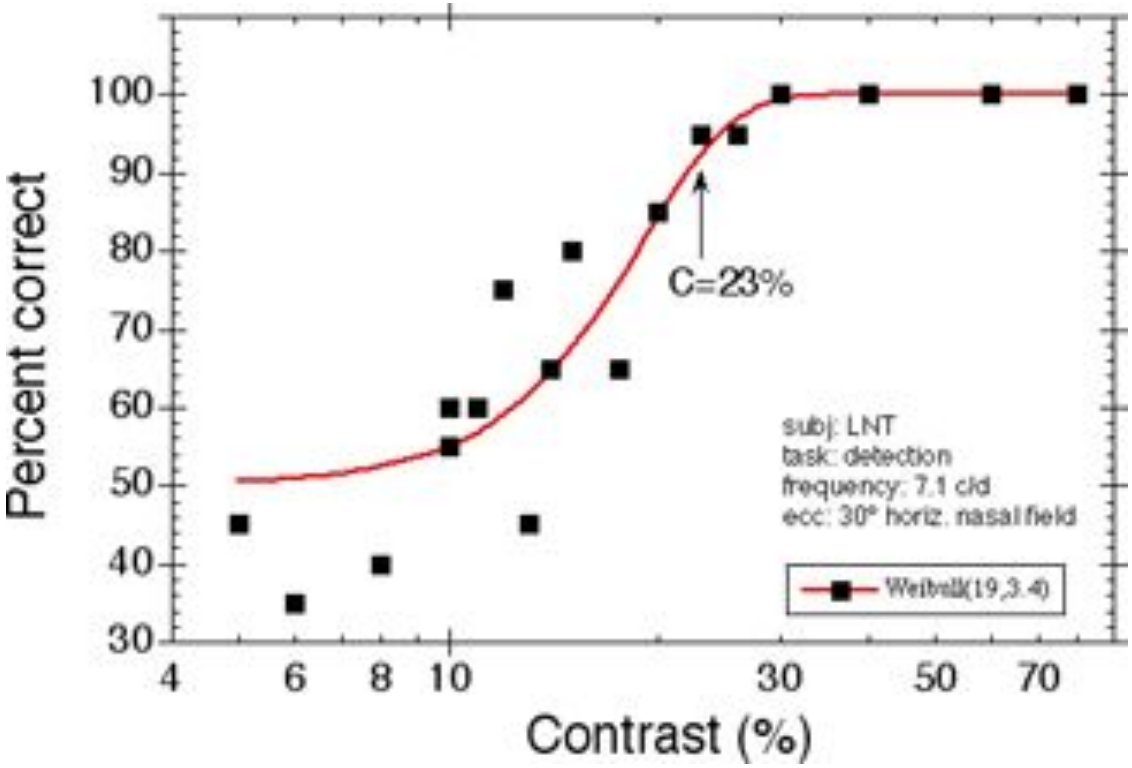
https://www.fao.org/sustainable-development-goals-data-portal/data/indicators/2.1.1-prevalence-of-undernourishment/en

Analysis of EEG patterns



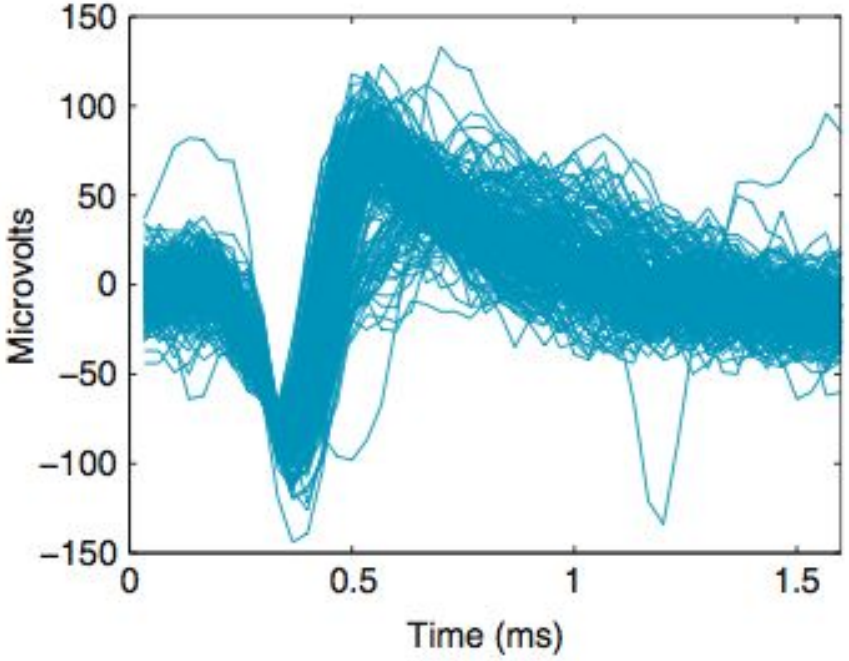
http://www.benbest.com/health/EEGs.jpg

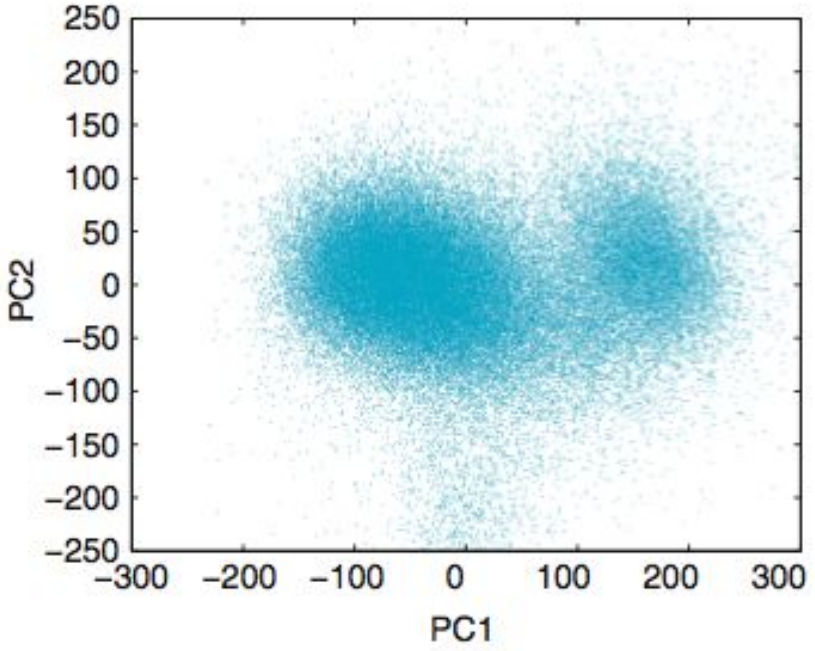
Fitting of psychometric curves



http://research.opt.indiana.edu/Library/SpatialAliasing/figs/fig3.gif

Spike-sorting with PCA

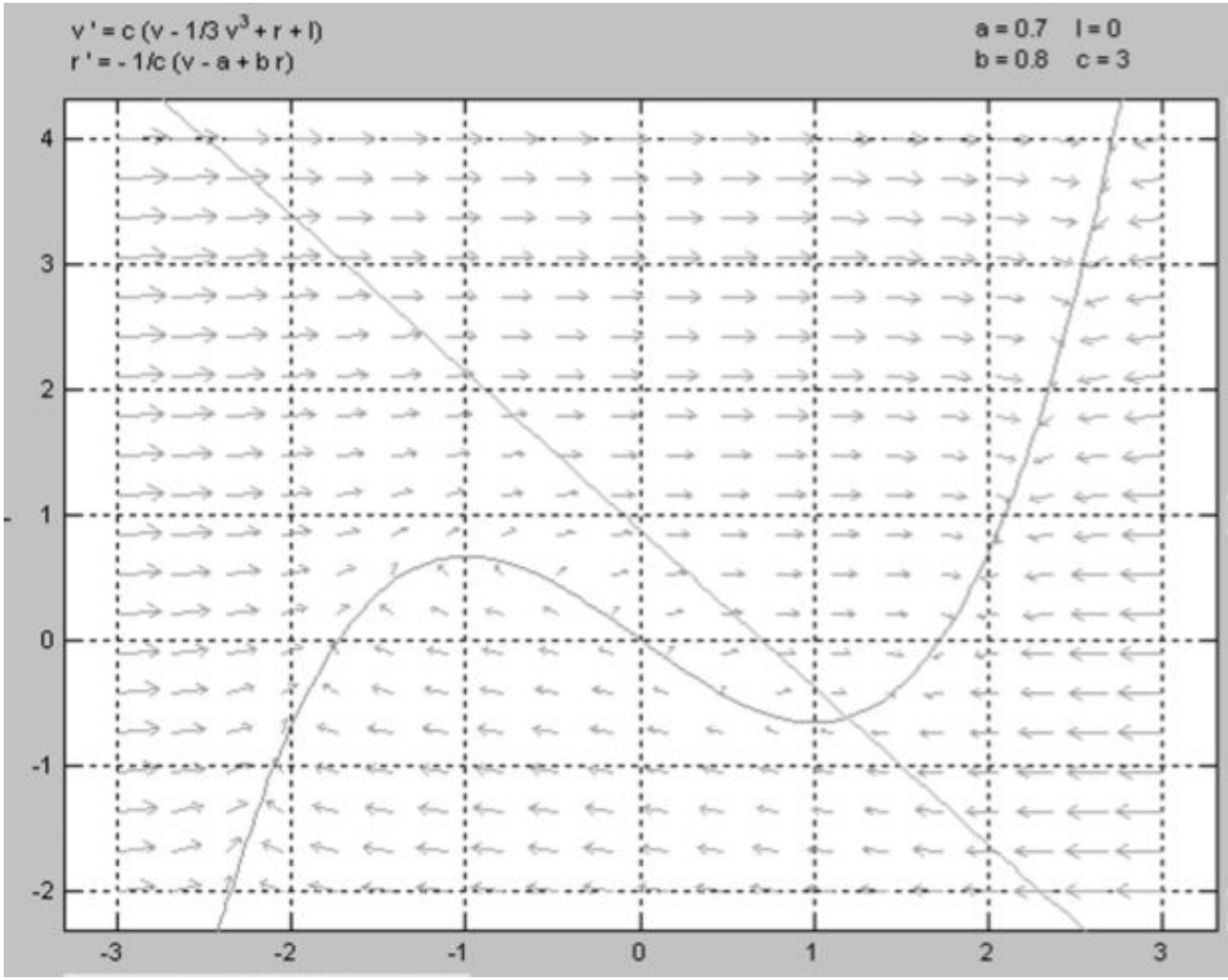




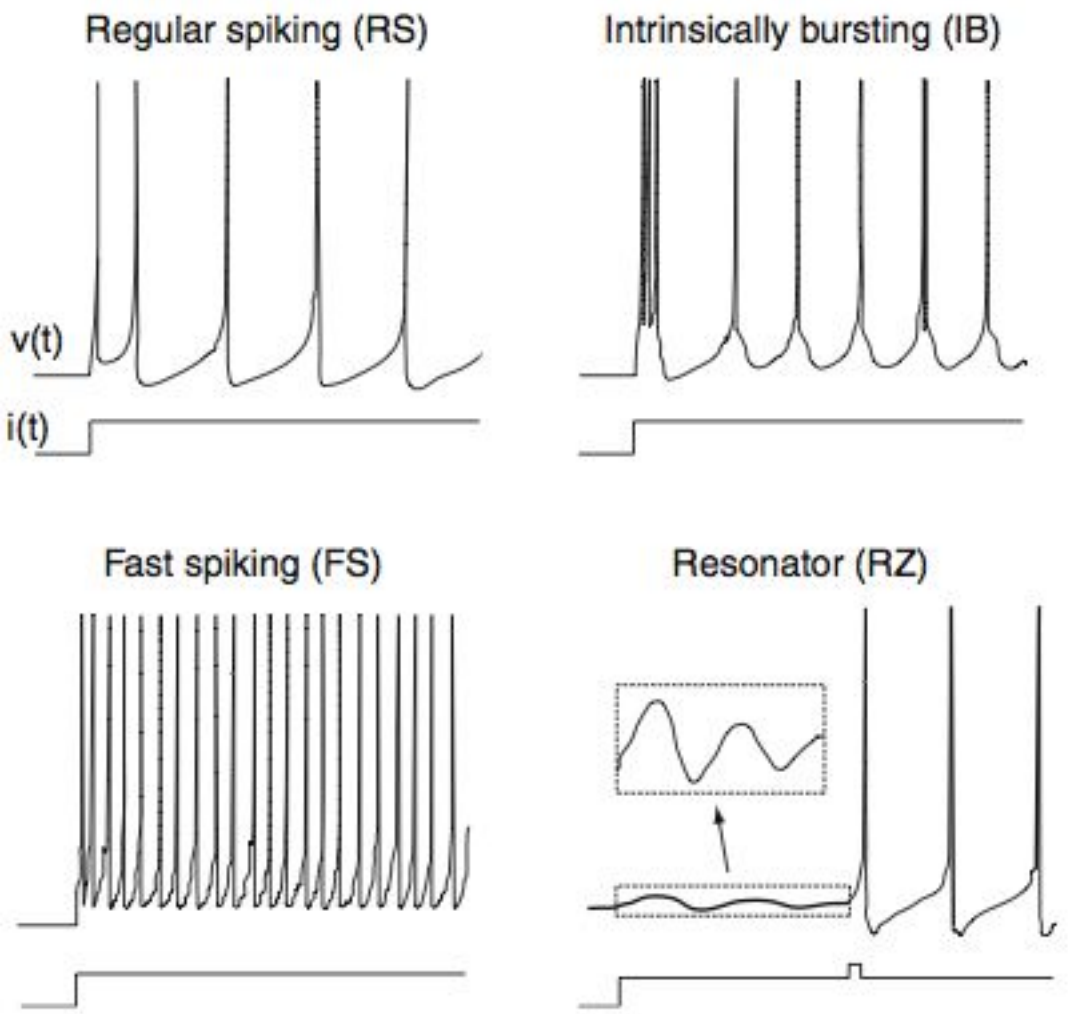
Fitzhugh-Nagumo Model

$$\frac{dv}{dt} = c(v - \frac{1}{3}v^3 + r + I)$$

$$\frac{dr}{dt} = -\frac{1}{c}(v - a + br)$$



Models of Neurons



Hebb's Postulate

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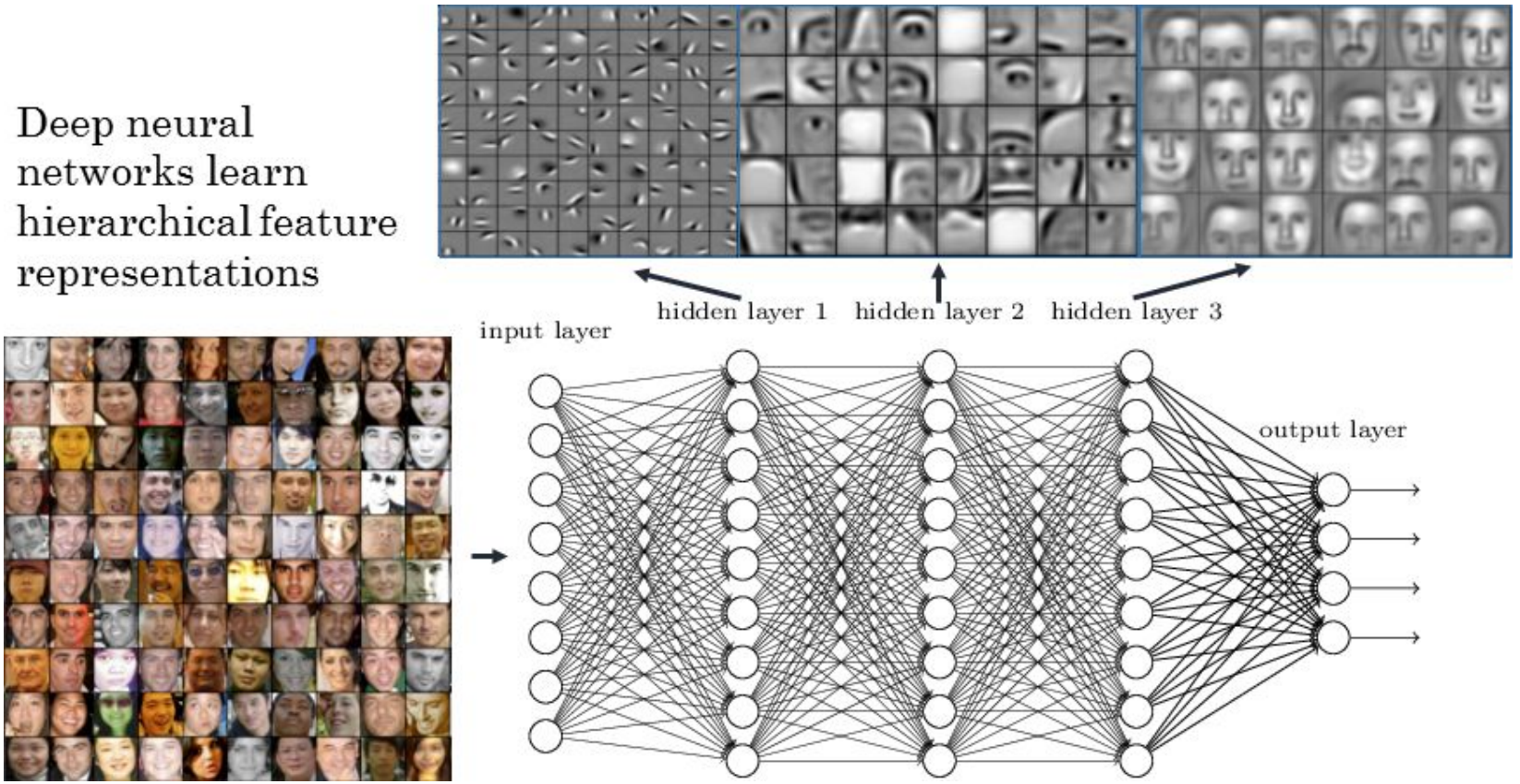
- "When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased."
  - D. O. Hebb, Organization of Behavior, 1949
- Can be summarized as: "**Cells that fire together wire together.**"
- Mathematically, this is equivalent to:

  $\Delta w_{ba} = \varepsilon * s_b * s_a$
  - (ε is the "learning rate", s_b, s_a are the neural firing rates)

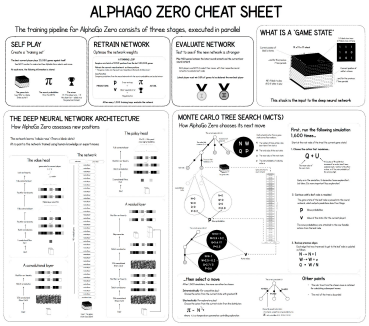
Jump ahead 60 years

- Instead of a single layer of neurons (like in the Hebb's network we just implemented), these deep nets use a lot of massively interconnected layers (just like in the brain)



Beating humans at…

- Chess (IBM 1997), Go (Deepmind, 2016)
- Ms PacMan (with "impossible high score", 2018)
- DotA 2, Grandmaster level in Starcraft 2 (Deepmind, nature 2019)
- More open-ended games like Poker (UoAlberta, science 2015), Diplomacy (Cicero from Meta, science 2022) and Stratego (OpenNash from Deepmind, science 2022) involving bluffing
- Real-time control for drone racing (UZH, nature 2023)



Atari Games on Atari-57