IE 400: Principles of Engineering Management Project
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Group 6

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*To automate the data reading process, in the code, we have divided the "data.xlsx" file into two excels: "distance.xlsx" which only has the distance matrix and "products.xlsx" which has the Product, Need, Price, Satisfaction, Amount per Packet, Market table. We included these two files on the zip file. Then, we saved these tables to pandas DataFrames for ease of use.

Please install the packages written on "requirements.txt" to your virtual environment if you want to run the code.
You can use the following command:
"pip install -r requirements.txt"*

Part a)

Parameters

Ci: unit cost of product i (one dimensional array, [45])

Ai: amount per packet of product i (one dimensional array, [45])

Si: unit satisfaction of product i (one dimensional array, [45])

Dn: demand for each need n (one dimensional array, [4])

B: budget

E: unit satisfaction for extra money saved

Dkl: distance between k and l (two dimensional matrix, [5, 5])

T: distance limit

New Parameters

MPij: shows whether product i is sold at market j (two dimensional matrix, [45, 4])

CPij: shows whether product i is in category j (two dimensional matrix, [45, 4])

Decision Variables:

L: left money (values ranging from [0, infinity), integer)

Xi: shows how much of product i we buy (values ranging from [0, infinity), integer)

Tij: shows whether we take the path from i to j (values ranging from [0, 1], binary)

z1: shows whether we go to market A (values ranging from [0, 1], binary)

z2: shows whether we go to market B (values ranging from [0, 1], binary)

z3: shows whether we go to market C (values ranging from [0, 1], binary)

z4: shows whether we go to market D (values ranging from [0, 1], binary)

Objective Function:

$$\left(\sum_{i=1}^{45} \text{Xi} * \text{Si}\right) + \text{L} * \text{E}$$

Constraints:

Demand Constraint:

- Beverage Demand (first column in category(CP) matrix):

$$\sum_{i=1}^{45} \text{Ai} * \text{CPi1} * \text{Xi} >= 2$$

- Carbohydrates Demand (second column in category(CP) matrix):

$$\sum_{i=1}^{45} \text{Ai} * \text{CPi2} * \text{Xi} >= 2.5$$

- Cheese Demand (third column in category(CP) matrix):

$$\sum_{i=1}^{45} Ai * CPi3 * Xi >= 0.4$$

- Breakfast Foods Demand (fourth column in category(CP) matrix):

$$\sum_{i=1}^{45} Ai * CPi4 * Xi >= 0.3$$

Cost Constraint:

$$\sum_{i=1}^{45} Xi * Ci + L <= 150$$

Distance Constraint:

$$\sum_{i=5}^{5} \sum_{j=5}^{5} Tij * dij <= 1600$$

Market Constraint (If we buy products from market, then we should go to it):

In order to construct this constraint, we first came up with an if statement, then converted it to an either-or statement and linearized it to have a linear constraint. The steps of constructing the if statement and converting it to either-or statement is only given for Market A, since the steps are the same for all markets. The linearized constraint is given for all four markets. It should be also noted that, house is given the index 1 while Market A is given the index 2, B is given the index 3, C is given the index 4 and D is given the index 5.

If

$$\sum_{i=1}^{45} Mi2 \times Xi > 0$$

Then

$$1 - \sum_{k=1}^{5} Tk2 <= 0$$

- Turn it into either or

either

$$\sum_{i=1}^{45} Mi2 * Xi <= 0$$

or

$$1 - \sum_{k=1}^{5} Tk2 <= 0$$

- Linearize

While linearizing the either or statement, a sufficiently large big M value is chosen. We chose this value as budget, since we observed that there was no product that had a price lower than 1, so we can’t buy budget many of any product.

  - For market A:

$$\sum_{i=1}^{45} Mi2 * Xi <= budget * z1$$

$$1 - \sum_{k=1}^{5} Tk2 <= budget * (1 - z1)$$

- For market B:

$$\sum_{i=1}^{45} Mi3 * Xi <= budget * z2$$

$$1 - \sum_{k=1}^{5} Tk3 <= budget * (1 - z2)$$

- For market C:

$$\sum_{i=1}^{45} Mi4 * Xi <= budget * z3$$

$$1 - \sum_{k=1}^{5} Tk4 <= budget * (1 - z3)$$

- For market D:

$$\sum_{i=1}^{45} Mi5 * Xi <= budget * z4$$

$$1 - \sum_{k=1}^{5} Tk5 <= budget * (1 - z4)$$

House Constraint (make sure we leave from and return to house):

$$\sum_{i=1}^{5} (T1i + Ti1) = 2$$

Going to a market at most one time constraint:

$$\sum_{i=1}^{5} (Ti1 + T1i) <= 2$$

$$\sum_{i=1}^{5} (Ti2 + T2i) <= 2$$

$$\sum_{i=1}^{5} (Ti3 + T3i) <= 2$$

$$\sum_{i=1}^{5} (Ti4 + T4i) <= 2$$

$$\sum_{i=1}^{5} (Ti5 + T5i) <= 2$$

If we enter, then we should we leave it constrain:

$$\sum_{i=1}^{5} Ti1 - \sum_{i=1}^{5} T1i = 0$$

$$\sum_{i=1}^{5} Ti2 - \sum_{i=1}^{5} T2i = 0$$

$$\sum_{i=1}^{5} Ti3 - \sum_{i=1}^{5} T3i = 0$$

$$\sum_{i=1}^{5} Ti4 - \sum_{i=1}^{5} T4i = 0$$

$$\sum_{i=1}^{5} Ti5 - \sum_{i=1}^{5} T5i = 0$$

Don’t go from same place to same place constraint:

T11 = 0

T22 = 0

T33 = 0

T44 = 0

T55 = 0

Don't take the same path twice constraint:

T12 + T21 <= 1

T13 + T31 <= 1

T14 + T41 <= 1

T15 + T51 <= 1

T23 + T32 <= 1

T24 + T42 <= 1

T25 + T52 <= 1

T34 + T43 <= 1

T35 + T53 <= 1

T45 + T54 <= 1

Part b)

For the provided data the distance limit constraint was irrelevant as traversing through all the markets required less distance traveled than the distance limit:

House -> Market A -> Market B -> Market C -> Market D -> House

75 + 390 + 645 + 60 + 390 = 1560 < 1600

Objective value: 4534

Which products are bought in which amount in optimal solution:

one of 4th product (sold in A),

one of 22nd product (sold in C),

one of 26th product (sold in C),

two of 31st product (sold in B),

one of 41st product (sold in D)

Part c)

For every product i bought one of product j is also bought thus $X_j >= X_i$. This can be written as a linear constraint:

$X_i - X_j <= 0$

House -> Market B -> Market D -> Market A -> House

465 + 705 + 315 + 75 = 1560 < 1600

Objective value: 4416

Which products are bought in which amount in optimal solution:

one of 4th product (sold in A),

two of 25th product (sold in B),

two of 31st product (sold in B),

one of 41st product (sold in D)

Due to product 31 providing such high satisfaction the optimum result includes two of product 25 to satisfy the constraints. This causes the products bought from market C to be irrelevant as two of product 25 satisfies the Carbohydrates demand.

Part d)

This constraint can be expressed as an either or statement:

Either $X_i <= 0$

Or $X_j <= 0$

Which can then be handled by defining a binary variable, say Z_d:

$X_i <= M Z\_d$

$X_j <= M(1 – Z\_d)$

As a linear constraint:

$X_i - M Z\_d <= 0$

$X_j + M Z\_d <= M$

As mentioned in Part a, we took M as budget as it is sufficiently large for the given scenario.

House -> Market B -> Market A -> Market C -> Market D -> House

465 + 390 + 255 + 60 + 390 = 1560 < 1600

Objective value: 4528

Which products are bought in which amount in optimal solution:

one of 4th product (sold in A),

one of 22nd product (sold in C),

one of 24th product (sold in D),

two of 31st product (sold in B),

one of 41st product (sold in D)

Product 22 has a satisfaction to price ratio of 420 / 14 = 30

Product 26 has a satisfaction to price ratio of 200 / 8 = 25

Therefore the model prefers buying product 22 over buying product 26 given the extra constraint of part d.

Part e)

In order to formulate this constraint we need to find the sum of two elements; time spent on the roads and time spent in the markets.
Time spent on the roads is simple as we have a decision variable that specifies which roads are taken:

$$\sum_{i=1}^{5} \sum_{j=1}^{5} Tij * Dij$$

Time spent in the markets can be thought as whether we took a road that leads to that market in binary times the time to be spent in that market. We do not travel back to the same market again so we can define, for example time spent in Market A as:
(T12 + T22 + T32 + T42 + T52) * 35 minutes.

Combining the two elements together the linear constraint is:

$$\sum_{i=1}^{5} \sum_{j=1}^{5} Tij * Dij + \sum_{i=1}^{5} \sum_{j=2}^{5} Tij * time\_spent\_market(j-1) <= 300$$

House -> Market A -> Market B -> House

Distance: 75 + 390 + 465 = 930 < 1600

Time: 266 < 300

Objective value: 4424

Which products are bought in which amount in optimal solution:

one of 4th product (sold in A),

one of 21st product (sold in B),

one of 25th product (sold in B),

two of 31st product (sold in B),

one of 42nd product (sold in A)