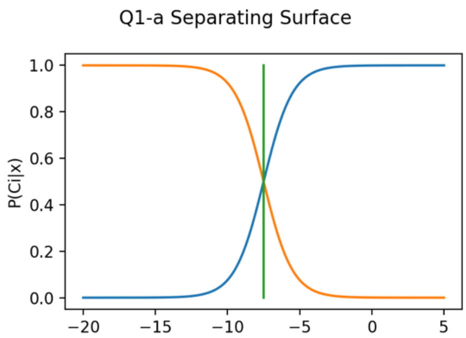
Homework 1

Q1: How to run python code: “python hw1_q1.py”

Q1-A



Naïve Bayes discriminant function is used for classification:

$$g_i(x) = -\frac{1}{2}\log 2\pi - \log \sigma_i - \frac{(x-m_i)^2}{2\sigma_i^2} + \log P(C_i)$$

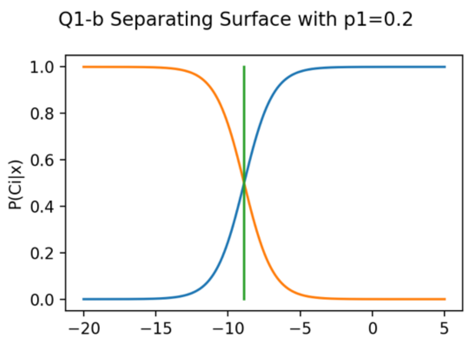
To find separating surface:

$$g_1(x) = g_2(x)$$
$$-\frac{1}{2}\log 2\pi - \log \sigma - \frac{(x-m_1)^2}{2\sigma^2} + \log P(C_1) = -\frac{1}{2}\log 2\pi - \log \sigma - \frac{(x-m_2)^2}{2\sigma^2} + \log P(C_2)$$
$$-(x-m_1)^2 = -(x-m_2)^2$$
$$x = \frac{m_1 + m_2}{2}$$
$$x = \frac{-5 + -10}{2}$$
$$x = -7.5$$

Since prior probabilities are equal, they are ignored.

Homework 1

Q1-B



However, in this case prior probabilities are different.

To find separating surface:

$$-\frac{1}{2}\log 2\pi - \log(\sigma) - \frac{(x-m_1)^2}{(2\sigma^2)} + \log(P_1) = -\frac{1}{2}\log 2\pi - \log(\sigma) - \frac{(x-m_2)^2}{(2\sigma^2} + \log(P_2)$$

$$-\frac{(x-m_1)^2}{(2\sigma^2)} + \log(P_1) = -\frac{(x-m_2)^2}{(2\sigma^2} + \log(P_2)$$

$$-\frac{(x+5)^2}{2\sigma^2} + \log(0.8) = -\frac{(x+10)^2}{2\sigma^2} + \log(0.2)$$

$$2\sigma^2 \log\frac{0.8}{0.2} = (x+5)^2 - (x+10)^2$$

$$x = -8.886$$

It can be seen that separating surface shifted in the left direction.

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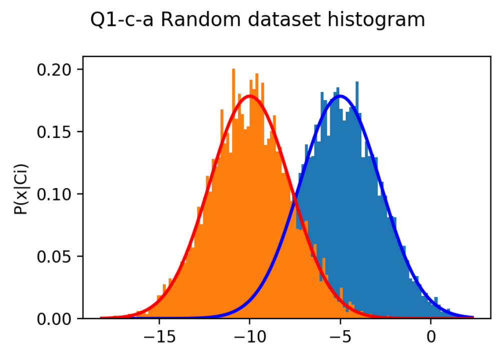
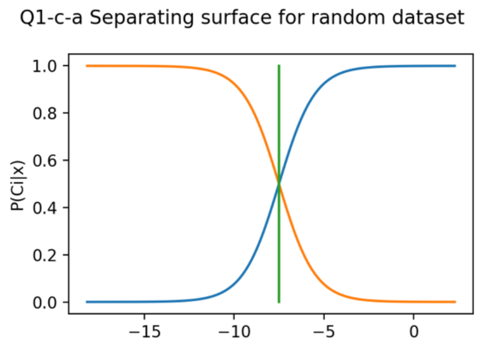
Homework 1

Q1-C

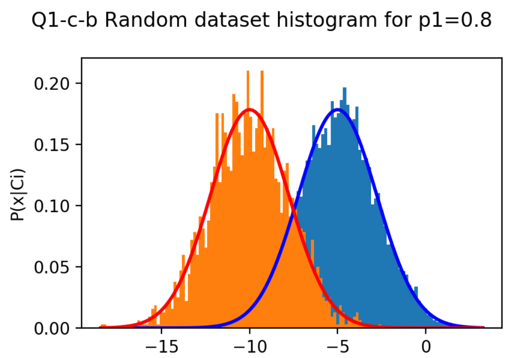


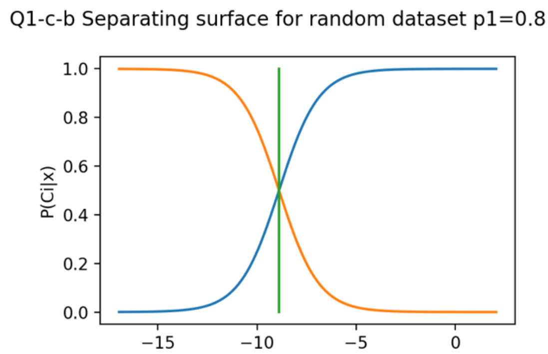


10000, random samples are generated in this case. According to the probabilities of, each distribution has 5000 samples. Mean and variance values can be changed after each run, and it is also related with the sample number. In this case, values are as follows:

	P1 (0.5)	P2 (0.5)
Mean	~-5 (4,977)	~-10 (-10,01)
Variance	~5 (4,962)	5 (4,989)

$$g_1(x) = g_2(x)$$
$$-\log \sigma_1 - \frac{(x-m_1)^2}{2\sigma_1^2} = -\log \sigma_2 - \frac{(x-m_2)^2}{2\sigma_2^2}$$
$$(x-m_1)^2 = (x-m_2)^2$$
$$(x+5)^2 = (x+10)^2$$
$$x = -7.5$$





10000, random samples are generated in this case. According to the probabilities of, p1 has 8000 samples on the other hand p2 has 2000.

	P1 (0.8)	P2 (0.2)
Mean	~-5 (5,021)	~-10 (-9,943)
Variance	~5 (5,015)	~5 (4,901)

$$-\frac{1}{2}\log 2\pi - \log(\sigma) - \frac{(x-m_1)^2}{(2\sigma^2)} + \log(P_1) = -\frac{1}{2}\log 2\pi - \log(\sigma) - \frac{(x-m_2)^2}{(2\sigma^2} + \log(P_2)$$
$$-\frac{(x-m_1)^2}{(2\sigma^2)} + \log(P_1) = -\frac{(x-m_2)^2}{(2\sigma^2} + \log(P_2)$$
$$-\frac{(x+5)^2}{2\sigma^2} + \log(0.8) = -\frac{(x+10)^2}{2\sigma^2} + \log(0.2)$$
$$2\sigma^2 \log \frac{0.8}{0.2} = (x+5)^2 - (x+10)^2$$
$$x = -8.886$$

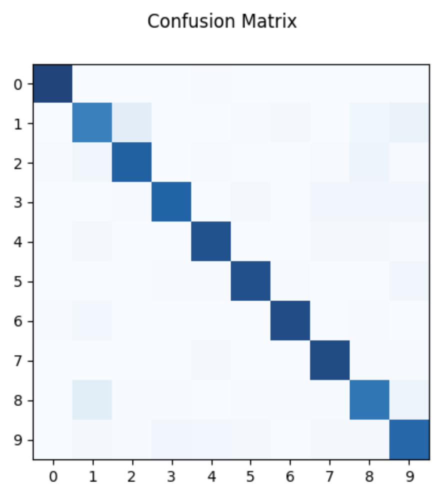
Homework 1

Q2: How to run python code: “python hw1_q2.py”

Q2-

Naïve bayes discriminant function is as follows:

$$g_i(x) = -\frac{1}{2}\sum_{j=1}^{d}\left(\frac{x_j^t - m_{ij}}{s_j}\right)^2 + \log \hat{P}(C_i)$$



Test and training data accuracies and errors are:

	Accuracy	Error
Test Data	89.316 %	10.684 %
Training Data	91.42 %	8.58 %

Accuracy for each class is:

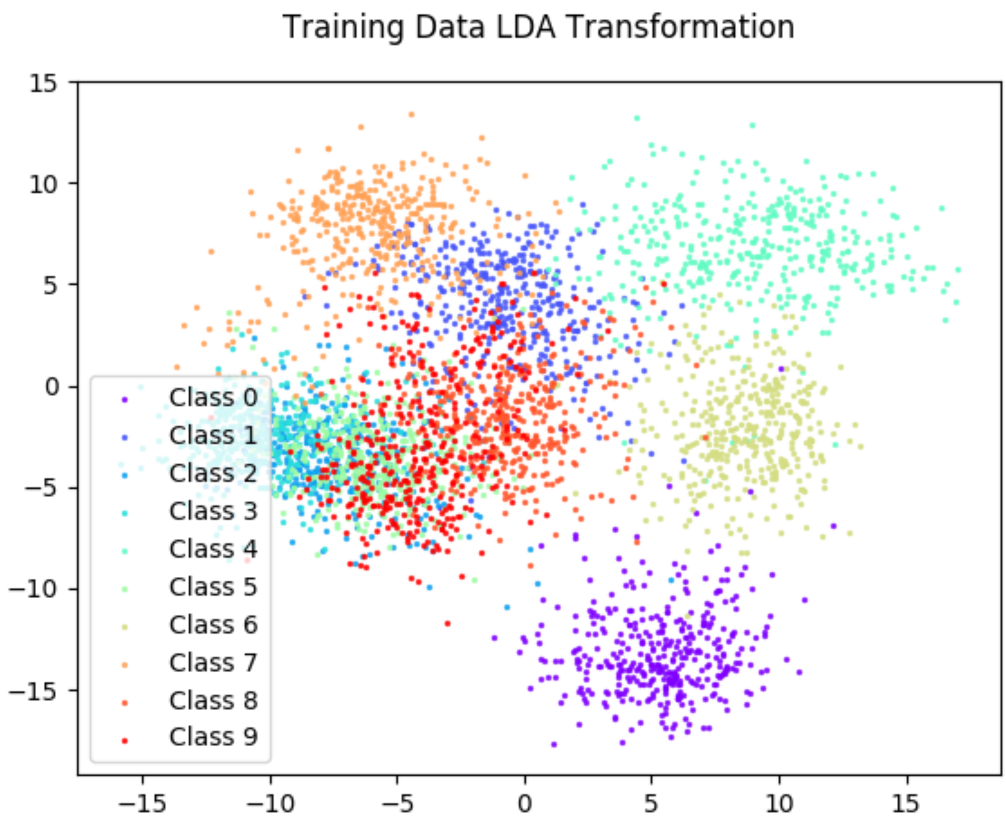
Class	Accuracy
0	99.438 %
1	74.176 %
2	88.136 %
3	86.339 %
4	93.923 %
5	94.505 %
6	96.133 %
7	96.648 %
8	78.736 %
9	85.0 %

Homework 1

Q3: How to run python code: "python hw1_q3.py"

Q3-A

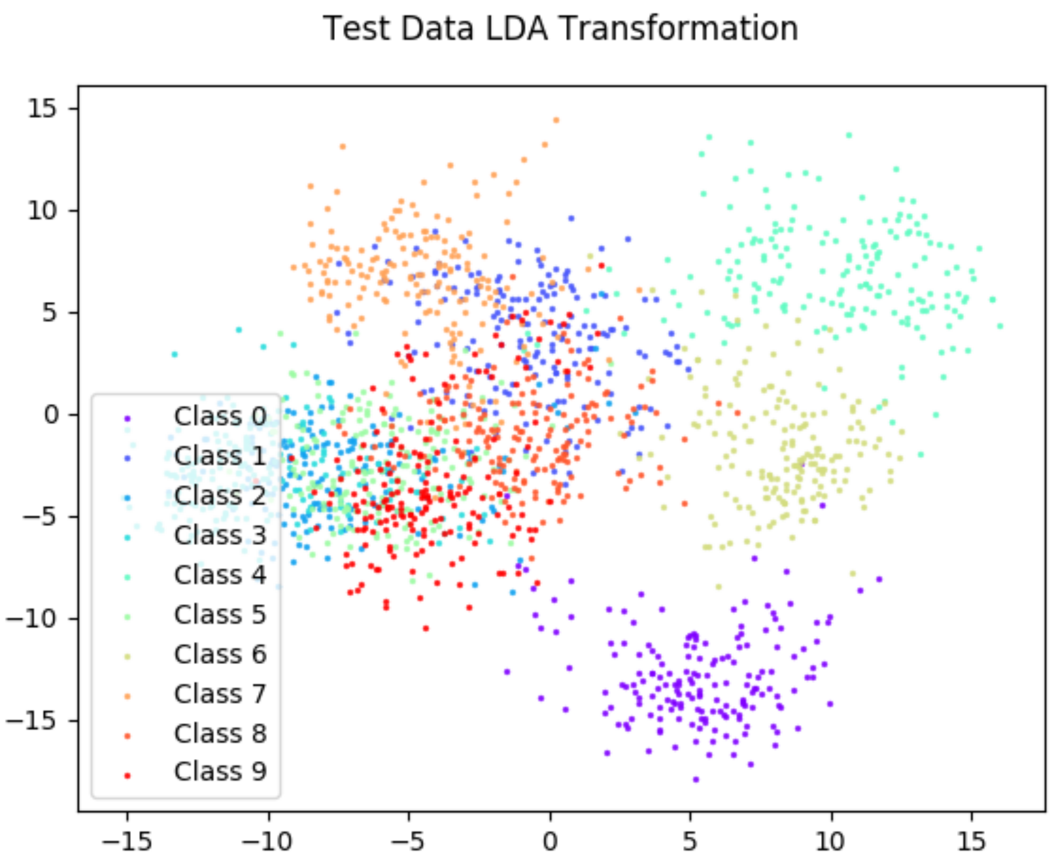
Training data projection to two dimensional space:



Homework 1

Q3-B

Test data projection to two dimensional space:



Homework 1

Q3-C

When covariance matrix is diagonal, the correlation between classes are hidden and untraceable. As it can be seen from the projected scattering images, in the diagonalized versions classes are less distinguishable, e.g. for the training data class 0, 6 and 4 barely separated from the remaining classes whereas after covariance matrix is diagonalized distance between them reduced.