Case Summary:

- Consider a small car repair shop with limited ability. 3 successive operations are required to complete a repair.
- There are 2 single skilled repairmen with distinct skills working in the repair shop. Let rjk be the man hour requirement of operation j=1,2,3 for the skill of repairman k=1,2. Repair shop pay c TL to a repairman per hour. Each repairman works at most 8 hours a day. If the total man hour requirement of the shop for a certain skill exceeds 8 hours in a day, multi-skilled temporary workers are hired for the overload. The repair shop pays c' TL to temporary workers per hour. Assume there is no limit on the number of temporary workers to be hired; therefore each car spends a day under each operation independent from the current workload.
- During each operation an assessment is made on whether the car under repair can be fixed in the repair shop or not. After operation j=1,2,3, with probability j, it is concluded that with the limited ability of the repair shop, the car cannot be repaired and it is sent to its owner, i.e., the repair is failed after operation j=1,2,3 with probability j. Otherwise, the car is sent to the next operation (after operation 3, it is successfully repaired and is sent to the owner in good condition). If a car is successfully repaired, it brings a revenue of p TL, and a car that cannot be fixed costs w TL.
- The number of car arrivals to the repair shop is random. Let qi be the probability that i=0,1,2 cars arrive each morning. After the cars that are sent to their owners, the number of the cars in the system is checked. If it is at most 1, all arriving cars are admitted to the system; otherwise all arrivals are rejected. There is no explicit cost of rejecting an arriving car.

a) Model the system as a Markov chain to calculate the following steady-state performance measures:

1. Expected daily workload of the repairman with skill k=1,2
2. Expected daily workload of temporary workers
3. Expected number of cars in a day under each operation
4. Expected total number of cars in the repair shop in a day
5. Acceptance rate to repair shop
6. Expected time between two admissions (when more than one cars arrive simultaneously and accepted, it is regarded as a single admission)
7. Expected daily profit

Provide a clear state definition, characterize the state space. Give transition probabilities. Provide mathematical expressions for the performance measures above given steady state distribution.

b) Restrict your attention to the following problem instance. Man hour requirement for each operation, failure of repair probability after each operation and the distribution of number of arrivals in a day are given in Table 1, Table 2 and Table 3 respectively.

Table 1. Man Hour Requirements for Each Operation (r_{jk})

	Operation, j		
Skill, k	1	2	3
1	3.0	4.5	5.9
2	4.4	3.8	5.3

Table 2. The failure of repair probability (α_j) after operation j=1,2,3

Operation, j	α_j
1	0.3
2	0.2
3	0.05

Table 3. Probability (q_i) that i=0,1,2 cars arrive each morning

Number of Cars, i	q_i
0	0.3
1	0.4
2	0.3

Repair shop pays 10 TL to both types of single skilled repairmen per hour and 12 TL to multi skilled temporary workers per hour. If a car is successfully repaired, it brings a revenue of 800 TL, and a car that cannot be fixed costs 100 TL.