Case Summary:

Consider a production facility with 2 identical machines producing a single item. Each machine produces an item at an exponential rate of μ. Machines work independently.

Customer demand occurs according to a Poisson Process with a rate of λ per day. Unsatisfied demand is backordered, until the backorder level is less than or equal to optimal maximum allowable backorder level, B. Once backorder level reaches the level of B, incoming customers are rejected, i.e., their demand is lost.

The following production/inventory control policy is applied:

When the inventory level of the item drops to a level of s1, a setup for the production facility is made. The setup time is negligible and production immediately begins. As long as inventory level is in between s1 and s2, a single machine produces the item. If the inventory level further drops to the level less than $s2<s1$, both machines produce the item until the inventory level reaches the level of s2 again. Production stops when the inventory level reaches the level of S. The next setup is made when the inventory level drops to the level of s1 again. Assume $-B<s2<s1<S$.

Your objective is to find optimal value for S, s1, s2 and B to maximize expected daily profit of the system in the steady state for given problem parameters. Unit selling price of the item is p TL. Fixed production setup cost is k TL and unit production cost is cp TL. Daily unit inventory holding cost for the item is h TL. Unit lost sale cost is cl TL and daily unit backorder cost is cb TL

a) Provide a system state description under which the system can be modelled as a birth and death process. Give the state space, specify birth and death rates at each state, and draw the rate diagram.
   1. Write down the flow balance equations in the steady state.
   2. Express the following performance measures in the steady state as a function of steady state probabilities:
      a. Fraction of the time that there is no production
      b. Fraction of the time that only one machine is working
      c. Average utilization of machines
      d. Expected inventory level
      e. Expected backorder level
      f. Expected daily profit

Note that your answers should be valid for any values of S, s1, s2 and B that satisfy the condition –B<s2<s1<S.

b) Considering the following problem parameters, find the optimal value of S, s1, s2 and B that maximize expected daily profit of the system in the steady state. Note that, Smax and Bmax are the upper bounds for S and B respectively.

Parameter	***Value***
S_{max}	50
B_{max}	20
λ	16
μ	12
k	100

Parameter	***Value***
h	9
c_b	8
c_l	20
c_p	20
p	80