CANSU YANIK
150170704

BLG 202E Numerical Methods in CE
2018 – 2019 Spring
Homework 4

① Data were acquired: (0,100), (7,98), (14,101), (21,50), (28,51), (35,50)

ⓐ Points with abscissae:

Lets say $a=7$ $b=14$

Linear interpolant $\Rightarrow$ $f(x) = f(a) + \left(\frac{f(b)-f(a)}{b-a}\right)(x-a)$

$$= f(7) + \left(\frac{f(14)-f(7)}{14-7}\right)(x-7)$$

$$= 98 + \left(\frac{101-98}{7}\right)(x-7)$$

$$= 98 + \frac{3x}{7} - \frac{21}{7}$$

$$\boxed{f(x) = 95 + \frac{3x}{7}}$$

– Adding the value at 0 (First quadratic interpolant :)):

Now, we have 0, 7, 14

Lets say $x_0:0$, $x_1:7$, $x_2=14$

$$L_0(x) = \frac{(x-7)(x-14)}{(0-7)(0-14)} = \frac{x^2-21x+98}{98}$$

$$L_1(x) = \frac{(x-0)(x-14)}{(7-0)(7-14)} = -\frac{(x^2-14x)}{49}$$

$$L_2(x) = \frac{(x-0)(x-7)}{(14-0)(14-7)} = \frac{x^2-7x}{98}$$

$$P(x) = L_0(x)f(0) + L_1(x)f(7) + L_2(x)f(14)$$

$$= \frac{(x^2-21x+98)}{98}.100 + \frac{(14x-x^2)}{49}98 + \frac{(x^2-7x)}{98}.101$$

$$P_1(x) = \frac{5x^2}{98} - \frac{9x}{14} + 100$$

– Adding the value at 21 (second quadratic interpolant):

Now, we have 7, 14, 21

Lets say $x_\phi:7$, $x_2=14$, $x_3=21$.

$$L_0(x) = \frac{(x-14)(x-21)}{(7-14)(7-21)} = \frac{x^2-35x+294}{98}$$

$$L_1(x) = \frac{(x-14)(x-7)}{(21-14)(21-7)} = \frac{x^2-21x+98}{98}$$

$$L_2(x) = \frac{(x-7)(x-21)}{(14-7)(14-21)} = \frac{-(x^2-28x+147)}{49}$$

$$P_2(x) = \frac{(x^2-35x+294)}{\cancel{98}}\cdot\cancel{98} + \frac{(x^2-21x+98)}{98}\cdot 50 - \frac{(x^2-28x+147)}{49}\cdot 101$$

$$P_2(x) = -\frac{27x^2}{49} + 12x + 41$$

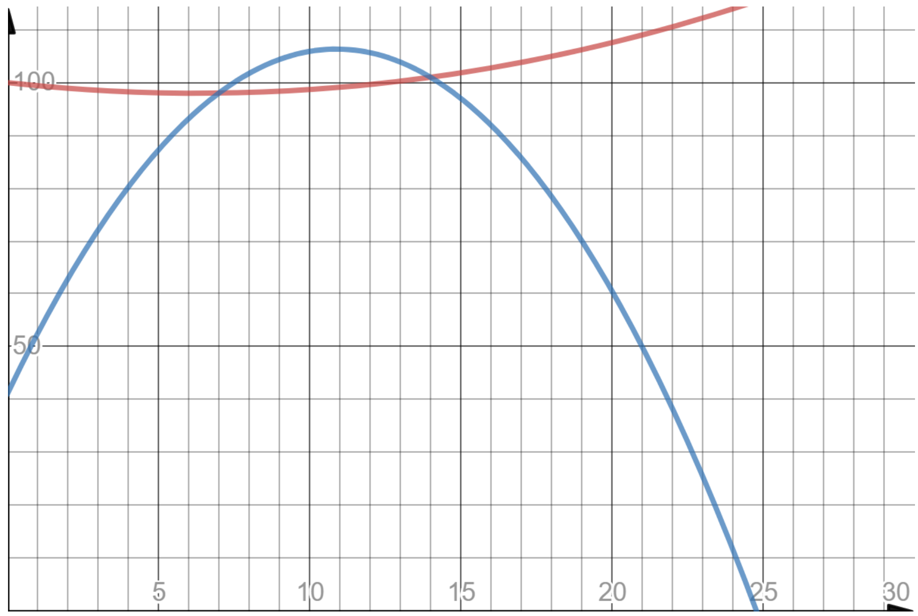
At $x=12$:

$$P_1(12) = \frac{5(12)^2}{98} - \frac{9\cdot 12}{14} + 100 = \underline{99{,}63}$$

$$P_2(12) = -\frac{27(12)^2}{49} + 12\cdot 12 + 41 = 105{,}65$$

$f(12) = 95 + \frac{3.72}{7} = \underline{100.14}$ $\Rightarrow$ Since $P_1(12)$ is closer to the actual result $(f(12))$, $P_1(x)$ is the most accurate.

⑥ This graph shows that this product is not stable and it is risky.



② $f(x) = e^x$ on $[0,1]$

points = $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$

ⓐ Lets find $P_2(x)$ (interpolating polynomial)

Apply Lagrange interpolation method.

$$P_2(x) = \frac{(x-x_1)}{(x_0-x_1)} y_0 + \frac{(x-x_0)}{(x_1-x_0)} y_1 \quad \Big\} \text{ we have two points}$$

Interpolate $\sin x \implies$ $\sin 0 = 0$
$\sin \frac{1}{2} = 0{,}48$

$$P_2(x) = \frac{\cancel{(x-0{,}5)}}{0-\cancel{0{,}5}} 0 + \frac{x-0}{0{,}5-0} 0{,}48 = \frac{0{,}48x}{0{,}5}$$

we need to calculate error at a point $(n=1)$ $\longrightarrow$ $E(x) = \frac{f^{n+1}(a)}{(n+1)!} \prod_{i=0}^{n} (x-x_i)$

$$E(x) = \frac{-\sin(a)}{2!} (x-0)(x-0{,}5) = \frac{-\sin(a) \times (x-0{,}5)}{2}$$

$$E(0{,}4) = \frac{-\sin(a)\ 0.4\ (0{,}4-0{,}5)}{2} = 0.02 \sin(a) \longrightarrow 0{,}02 \sin(0{,}5) = 0{,}0096$$

the actual error is $\sin(0{,}4) - \frac{0{,}48\,(0{,}4)}{0{,}5} = 0{,}0059$ $\leftarrow$ less

Lets try $\implies$ $E(x) = \frac{-\cos(a)}{3!} (x)(x-0{,}5)(x-1) = 0{,}004$
$\uparrow$ $0{,}4$ $\uparrow$ $0{,}4$ $\uparrow$ $0{,}4$

$$P_2(x) = \frac{(x-0{,}5)(x-1)}{(0-0{,}5)(0-1)} 0 + \frac{(x-0)(x-1)}{(0{,}5)(0{,}5-1)} 0{,}48 + \frac{(x-0)(x-0{,}5)}{1(1-0{,}5)} \underbrace{0{,}84}_{\sin(1)}$$

$P_2(0{,}4) = 0{,}39$

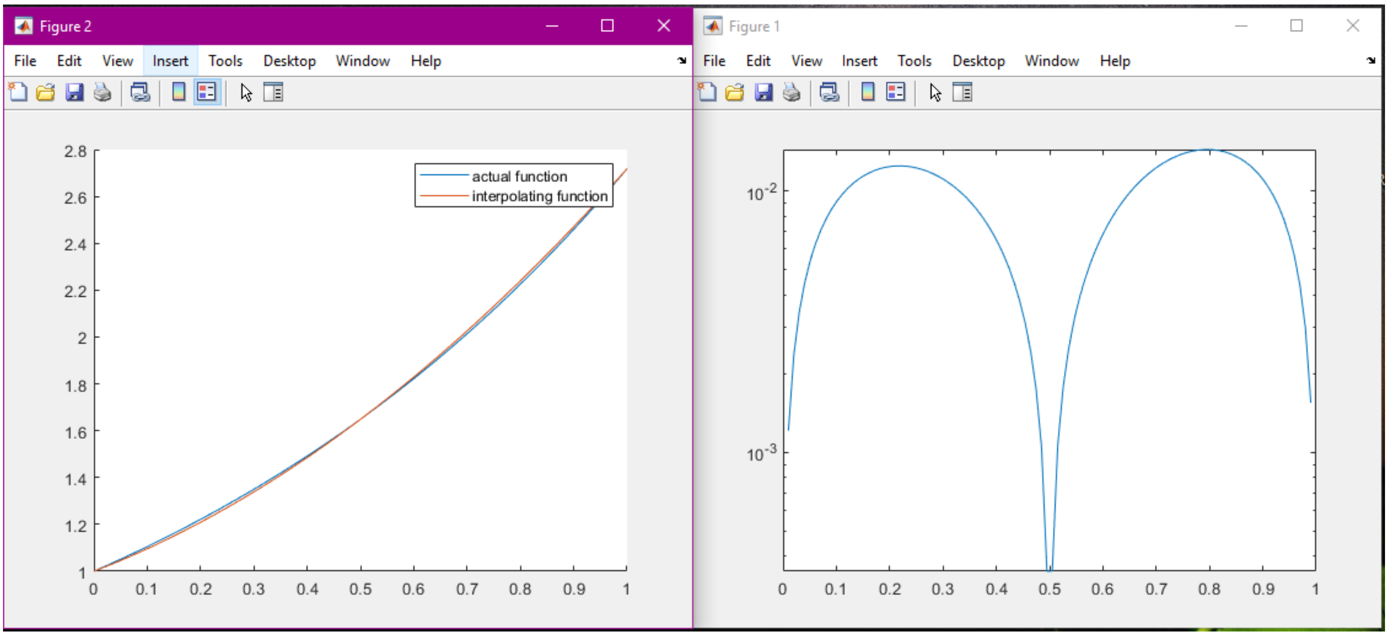
the actual error is $\sin(0{,}4) - P_2(0{,}4) = -0{,}0035$

ⓑ Apply Lagrange interpolation polynomial

$$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \underset{e^0}{f(x_0)} + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot \underset{e^{1/2}}{f(x_1)} + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \underset{e^1}{f(x_2)}$$

$$P_2(x) = \frac{(x-1/2)(x-1)}{1/2} e^0 + \frac{x(x-1)}{-1/4} e^{1/2} + \frac{x(x-1/2)}{1/2} e^1$$

c. and d. Parts:



For the c and d part, I wrote this code given below:

```
f = @(x) exp(x);
f2 = @(x) 2.*(x-1/2).*(x-1).*exp(0) - 4.*x.*(x-1).*exp(1/2) + 2.*x.*(x-1/2).*exp(1);

x1 = linspace(0, 1, 100);
y1 = f(x1);
y2 = f2(x1);

figure(1);
hold on
plot(x1, y1);
plot(x1, y2);
legend('actual function', 'interpolating function');
hold off

figure(2);
error = abs(y1-y2);
semilogy(x1,error);
```

(3) Let's expand Taylor expression for 4 points

$$f(x_0+h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0) + \frac{h^3}{3!}f'''(x_0) + \frac{h^4}{4!}f^{(4)}(x_0)$$

$$f(x_0-h) = f(x_0) - hf'(x_0) + \frac{h^2}{2!}f''(x_0) - \frac{h^3}{3!}f'''(x_0) + \frac{h^4}{4!}f^{(4)}(x_0)$$

$$f(x_0+2h) = f(x_0) + 2hf'(x_0) + \frac{4h^2}{2!}f''(x_0) + \frac{2^3h^3}{3!}f'''(x_0) + \frac{2^4h^4}{4!}f^{(4)}(x_0)$$

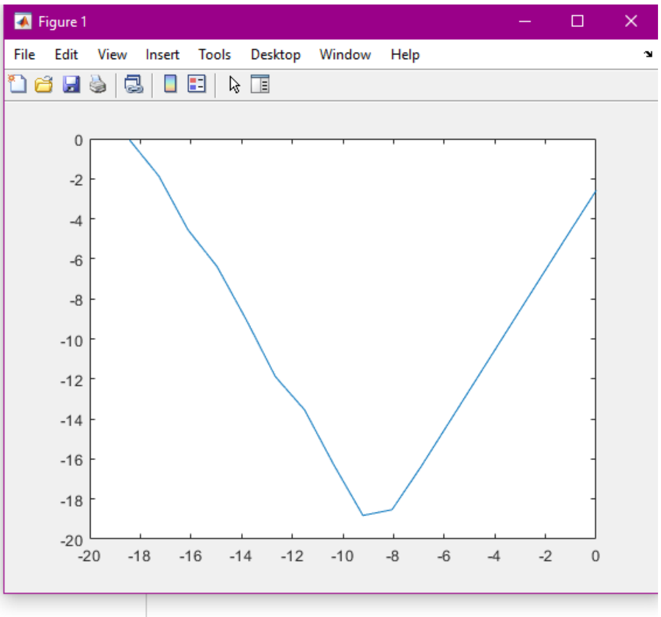
$$f(x_0-2h) = f(x_0) - 2hf'(x_0) + \frac{4h^2}{2!}f''(x_0) - \frac{2^3h^3}{3!}f'''(x_0) + \frac{2^4h^4}{4!}f^{(4)}(x_0)$$

To be able to remove first, second and third derivatives, 3 points will be needed.

4.

```
k = 0: 0.5: 8;
h = 10.^ (-k);
p1 = 1.2 - h;
p2 = 1.2 + h;
f1 = sin(p1);
f0 = sin(1.2);
fminus1 = sin(p2);
fpp = -sin(1.2);
for m=1:length(k)
    fpp0(m) = (f1(m) - 2.* f0 + fminus1(m))/(h(m).^2);
    err(m) = abs(fpp0(m) - fpp);
end

h = log(h);
err = log(err);
figure(1);
plot(h, err);
```



The approximately observed optimal h value is obtained at log(h)=-9.21, log(error) = -18,82