b) Rules for Multiple Variables

10) Changing Rule (commutative)

$$X+Y=Y+X$$
$$X.Y=Y.X$$

11) Union Rule

$$X+(Y+Z)=(X+Y)+Z=X+Y+Z$$
$$X(YZ)=(XY)Z=XYZ$$

12) Distribution Rule

$$X(Y+Z)=XY+XZ$$
$$X+(YZ)=(X+Y).(X+Z)$$

13) Absorption Rule

$$X+XY=X$$
$$X.(X+Y)=X$$

14) De Morgan Rule

$$(\overline{X+Y})=\bar{X}.\bar{Y}$$
$$(\overline{X.Y})=\bar{X}+\bar{Y}$$

15) $X+\bar{X}Y=X+Y$

This rules can be proven by giving 0 or 1 values to the variables.

Samples

1) $X = ACD + \bar{A}BCD$ will be simplified.

$= CD(A + \bar{A}B)$ 15. rule $A + \bar{A}B \rightarrow A + B$

$= CD(A + B)$

$= ACD + BCD$

2) $Y = ABC + AB\bar{C}$

$= AB(C + \bar{C})$ Distribution Rule

$= AB.1$

$= AB$

3) $Y = \overline{(\bar{A} + C)(B + \bar{D})} + AC$

$= (\overline{\bar{A} + C}) + (\overline{B + \bar{D}}) + AC$ De Morgan

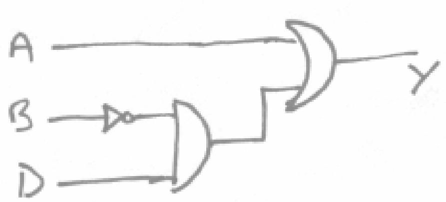
$= \overline{\bar{A}\,C} + \overline{B\bar{D}} + AC$ De Morgan

$= A\bar{C} + \bar{B}D + AC$

$= A(\bar{C} + C) + \bar{B}D$ Distribution

$= A.1 + \bar{B}D$

$= A + \bar{B}D$



NOT-AND, NOT-OR GATES

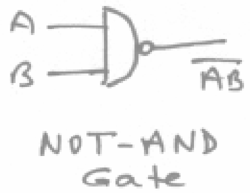
instead of following gates of
AND, OR and NOT gates.

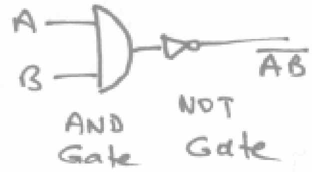
instead of using two different gates
use single special gate
NOT-AND NOT-OR

Examples

1)

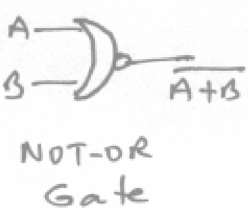
A	B	AB	$\overline{AB}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

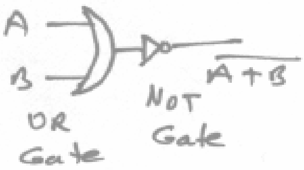




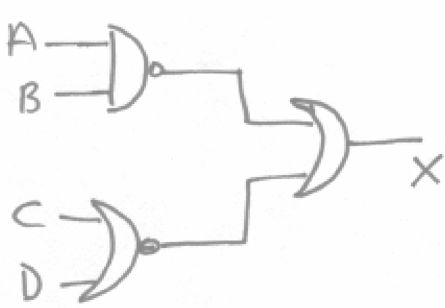
2)

A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0





3) $X = \overline{AB} + \overline{(C+D)}$



BOOLEAN FUNCTIONS AND LOGICAL DESIGN

To handle a Boolean function:

1) Prepare a truth table for variables
2) Decide to which cases will be 1 in logically, make it AND operation
3) If there are more than one AND operation, collect them in OR operation (Addition of multiplications)
4) Simplify the equation in 3.
5) Draw the logical circuit.

Example

1)

A	B	C
0	0	0
0	1	1 → $C = \bar{A}B$
1	0	0
1	1	0

2)

A	B	C
0	0	0
0	1	1 → $\bar{A}B$
1	0	1 → $A\bar{B}$
1	1	0

$C = \bar{A}B + A\bar{B}$

3)

A	B	C	Y	
0	0	0	0	
0	0	1	1	$\rightarrow \bar{A}\bar{B}C$
0	1	0	0	
0	1	1	1	$\rightarrow \bar{A}BC$
1	0	0	0	
1	0	1	1	$\rightarrow A\bar{B}C$
1	1	0	0	
1	1	1	0	

$Y = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$ (Addition of multiplication)

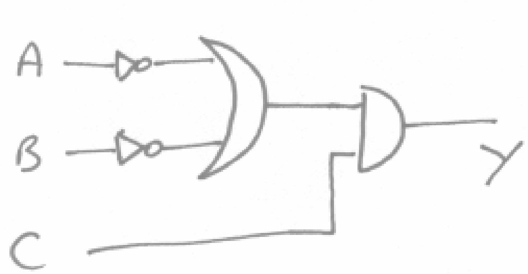
$= C(\bar{A}\bar{B} + \bar{A}B + A\bar{B})$

$= C(\bar{A}\bar{B} + \bar{A}\bar{B} + \bar{A}B + A\bar{B})$ (if we suppose $AB = X$, $X + X = X$ so, adding $\bar{A}\bar{B}$ will not change the result.

$= C(\bar{A}(B+\bar{B}) + \bar{B}(\bar{A}+A))$

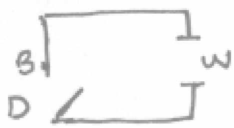
$= C(\bar{A}.1 + \bar{B}.1)$

$= C(\bar{A} + \bar{B})$



Question

An alarm system will be prepared to a house. The house has a door and a window.



There is a button near the door. If this button is pressed first, and later door or window is open, alarm will not work. Other cases alarm works.

Door is closed 0
" " open 1
Window is closed 0
" " open 1
Button is pressed 1
" is not pressed 0

D	W	B	A	
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{D}W\bar{B}$ (Button is not pressed but the door is open)
0	1	1	0	
1	0	0	1	$D\bar{W}\bar{B}$
1	0	1	0	
1	1	0	1	$DW\bar{B}$
1	1	1	0	

$$
\begin{aligned}
A &= \bar{D}W\bar{B} + D\bar{W}\bar{B} + DW\bar{B} \\
&= \bar{B}(\bar{D}W + D\bar{W} + DW) \\
&= \bar{B}(\bar{D}W + D\bar{W} + DW + DW) \\
&= \bar{B}(W(\bar{D}+D) + D(\bar{W}+W)) \\
&= \bar{B}(W+D)
\end{aligned}
$$