PREPARING FOR THIS TEXT *Before getting started, read "To the Student" at the front of this text.*

OBJECTIVES
1 Work with Sets (p. 2)
2 Classify Numbers (p. 4)
3 Evaluate Numerical Expressions (p. 8)
4 Work with Properties of Real Numbers (p. 10)

1 Work with Sets

A **set** is a well-defined collection of distinct objects. The objects of a set are called its **elements**. By **well-defined**, we mean that there is a rule that enables us to determine whether a given object is an element of the set. If a set has no elements, it is called the **empty set**, or **null set**, and is denoted by the symbol $\varnothing$.

For example, the set of **digits** consists of the collection of numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. If we use the symbol D to denote the set of digits, then we can write

$$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

In this notation, the braces { } are used to enclose the objects, or **elements**, in the set. This method of denoting a set is called the **roster method**. A second way to denote a set is to use **set-builder notation**, where the set D of digits is written as

$$D = \{\ x \mid x \text{ is a digit}\}$$

Read as "D is the set of all x such that x is a digit."

EXAMPLE 1 **Using Set-builder Notation and the Roster Method**

(a) $E = \{x \mid x \text{ is an even digit}\} = \{0, 2, 4, 6, 8\}$

(b) $O = \{x \mid x \text{ is an odd digit}\} = \{1, 3, 5, 7, 9\}$

Because the elements of a set are distinct, we never repeat elements. For example, we would never write $\{1, 2, 3, 2\}$; the correct listing is $\{1, 2, 3\}$. Because a set is a collection, the order in which the elements are listed is immaterial. $\{1, 2, 3\}$, $\{1, 3, 2\}$, $\{2, 1, 3\}$, and so on, all represent the same set.

If every element of a set A is also an element of a set B, then A is a **subset** of B, which is denoted $A \subseteq B$. If two sets A and B have the same elements, then A **equals** B, which is denoted $A = B$.

For example, $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$ and $\{1, 2, 3\} = \{2, 3, 1\}$.

DEFINITION

If A and B are sets, the **intersection** of A with B, denoted $A \cap B$, is the set consisting of elements that belong to both A and B. The **union** of A with B, denoted $A \cup B$, is the set consisting of elements that belong to either A or B, or both.

EXAMPLE 2 **Finding the Intersection and Union of Sets**

Let $A = \{1, 3, 5, 8\}$, $B = \{3, 5, 7\}$, and $C = \{2, 4, 6, 8\}$. Find:

(a) $A \cap B$ (b) $A \cup B$ (c) $B \cap (A \cup C)$

Solution (a) $A \cap B = \{1,3,5,8\} \cap \{3,5,7\} = \{3,5\}$
(b) $A \cup B = \{1,3,5,8\} \cup \{3,5,7\} = \{1,3,5,7,8\}$
(c) $B \cap (A \cup C) = \{3,5,7\} \cap [\{1,3,5,8\} \cup \{2,4,6,8\}]$
$= \{3,5,7\} \cap \{1,2,3,4,5,6,8\} = \{3,5\}$

Now Work PROBLEM 15

Usually, in working with sets, we designate a **universal set** U, the set consisting of all the elements that we wish to consider. Once a universal set has been designated, we can consider elements of the universal set not found in a given set.

DEFINITION If A is a set, the **complement** of A, denoted $\overline{A}$, is the set consisting of all the elements in the universal set that are not in A.*

EXAMPLE 3 **Finding the Complement of a Set**

If the universal set is $U = \{1,2,3,4,5,6,7,8,9\}$ and if $A = \{1,3,5,7,9\}$, then $\overline{A} = \{2,4,6,8\}$.

It follows from the definition of complement that $A \cup \overline{A} = U$ and $A \cap \overline{A} = \varnothing$. Do you see why?

Now Work PROBLEM 19

It is often helpful to draw pictures of sets. Such pictures, called **Venn diagrams**, represent sets as circles enclosed in a rectangle, which represents the universal set. Such diagrams often help us to visualize various relationships among sets. See Figure 1.

If we know that $A \subseteq B$, we might use the Venn diagram in Figure 2(a). If we know that A and B have no elements in common—that is, if $A \cap B = \varnothing$—we might use the Venn diagram in Figure 2(b). The sets A and B in Figure 2(b) are said to be **disjoint**.

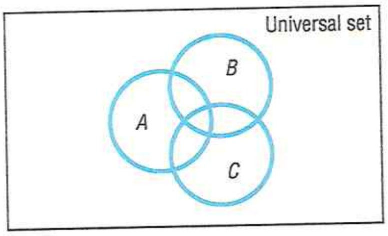

Figure 1 Venn diagram

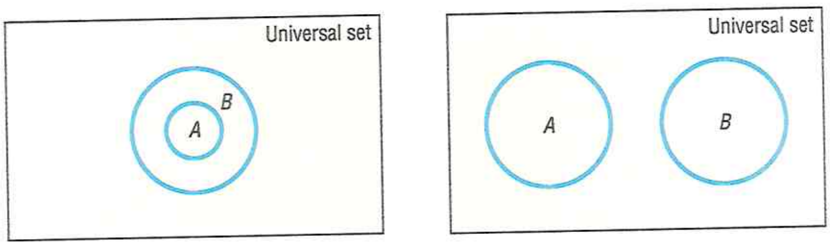

Figure 2 (a) $A \subseteq B$ subset (b) $A \cap B = \varnothing$ disjoint sets

Figures 3(a), 3(b), and 3(c) use Venn diagrams to illustrate the definitions of intersection, union, and complement, respectively.

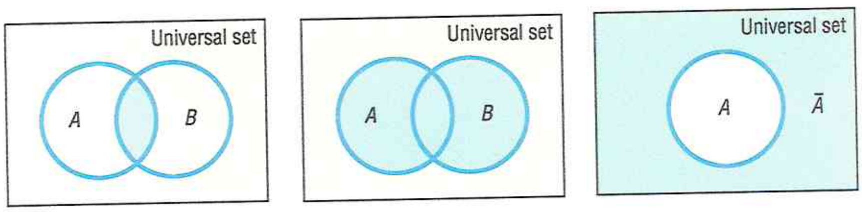

Figure 3 (a) $A \cap B$ intersection (b) $A \cup B$ union (c) $\overline{A}$ complement

*Some texts use the notation A' for the complement of A.

Historical Feature

The real number system has a history that stretches back at least to the ancient Babylonians (1800 BC). It is remarkable how much the ancient Babylonian attitudes resemble our own. As we stated in the text, the fundamental difficulty with irrational numbers is that they cannot be written as quotients of integers or, equivalently, as repeating or terminating decimals. The Babylonians wrote their numbers in a system based on 60 in the same way that we write ours based on 10. They would carry as many places for π as the accuracy of the problem demanded, just as we now use

$$\pi \approx 3\frac{1}{7} \quad \text{or} \quad \pi \approx 3.1416 \quad \text{or} \quad \pi \approx 3.14159$$
$$\text{or} \quad \pi \approx 3.14159265358979$$

depending on how accurate we need to be.

Things were very different for the Greeks, whose number system allowed only rational numbers. When it was discovered that $\sqrt{2}$ was not a rational number, this was regarded as a fundamental flaw in the number concept. So serious was the matter that the Pythagorean Brotherhood (an early mathematical society) is said to have drowned one of its members for revealing this terrible secret. Greek mathematicians then turned away from the number concept, expressing facts about whole numbers in terms of line segments.

In astronomy, however, Babylonian methods, including the Babylonian number system, continued to be used. Simon Stevin (1548–1620), probably using the Babylonian system as a model, invented the decimal system, complete with rules of calculation, in 1585. [Others, for example, al-Kashi of Samarkand (d. 1429), had made some progress in the same direction.] The decimal system so effectively conceals the difficulties that the need for more logical precision began to be felt only in the early 1800s. Around 1880, Georg Cantor (1845–1918) and Richard Dedekind (1831–1916) gave precise definitions of real numbers. Cantor's definition, although more abstract and precise, has its roots in the decimal (and hence Babylonian) numerical system.

Sets and set theory were a spin-off of the research that went into clarifying the foundations of the real number system. Set theory has developed into a large discipline of its own, and many mathematicians regard it as the foundation upon which modern mathematics is built. Cantor's discoveries that infinite sets can also be counted and that there are different sizes of infinite sets are among the most astounding results of modern mathematics.

R.1 Assess Your Understanding

Concepts and Vocabulary

1. The numbers in the set $\left\{x \,\middle|\, x = \frac{a}{b}\text{, where } a, b \text{ are integers and } b \neq 0\right\}$ are called ________ numbers.
2. The value of the expression $4 + 5 \cdot 6 - 3$ is ___.
3. The fact that $2x + 3x = (2 + 3)x$ is a consequence of the __________ Property.
4. "The product of 5 and $x + 3$ equals 6" may be written as __________.
5. The intersection of sets A and B is denoted by which of the following?
   (a) $A \cap B$ (b) $A \cup B$ (c) $A \subseteq B$ (d) $A \varnothing B$
6. Choose the correct name for the set of numbers $\{0, 1, 2, 3, \ldots\}$.
   (a) Counting numbers (b) Whole numbers
   (c) Integers (d) Irrational numbers
7. ***True or False*** Rational numbers have decimals that either terminate or are nonterminating with a repeating block of digits.
8. ***True or False*** The Zero-Product Property states that the product of any number and zero equals zero.
9. ***True or False*** The least common multiple of 12 and 18 is 6.
10. ***True or False*** No real number is both rational and irrational.

Skill Building

In Problems 11–22, use U = *universal set* $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 4, 5, 9\}$, $B = \{2, 4, 6, 7, 8\}$, *and* $C = \{1, 3, 4, 6\}$ *to find each set.*

11. $A \cup B$
12. $A \cup C$
13. $A \cap B$
14. $A \cap C$
15. $(A \cup B) \cap C$
16. $(A \cap B) \cup C$
17. $\overline{A}$
18. $\overline{C}$
19. $\overline{A \cap B}$
20. $\overline{B \cup C}$
21. $\overline{A} \cup \overline{B}$
22. $\overline{B} \cap \overline{C}$

In Problems 23–28, list the numbers in each set that are (a) Natural numbers, (b) Integers, (c) Rational numbers, (d) Irrational numbers, (e) Real numbers.

23. $A = \left\{-6, \frac{1}{2}, -1.333\ldots \text{(the 3's repeat)}, \pi, 2, 5\right\}$
24. $B = \left\{-\frac{5}{3}, 2.060606\ldots \text{(the block 06 repeats)}, 1.25, 0, 1, \sqrt{5}\right\}$
25. $C = \left\{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
26. $D = \{-1, -1.1, -1.2, -1.3\}$
27. $E = \left\{\sqrt{2}, \pi, \sqrt{2} + 1, \pi + \frac{1}{2}\right\}$
28. $F = \left\{-\sqrt{2}, \pi + \sqrt{2}, \frac{1}{2} + 10.3\right\}$

In Problems 29–40, approximate each number (a) rounded and (b) truncated to three decimal places.

29. 18.9526
30. 25.86134
31. 28.65319
32. 99.05249
33. 0.06291
34. 0.05388
35. 9.9985
36. 1.0006
37. $\frac{3}{7}$
38. $\frac{5}{9}$
39. $\frac{521}{15}$
40. $\frac{81}{5}$

In Problems 41–50, write each statement using symbols.

41. The sum of 3 and 2 equals 5.
42. The product of 5 and 2 equals 10.
43. The sum of x and 2 is the product of 3 and 4.
44. The sum of 3 and y is the sum of 2 and 2.
45. The product of 3 and y is the sum of 1 and 2.
46. The product of 2 and x is the product of 4 and 6.
47. The difference x less 2 equals 6.
48. The difference 2 less y equals 6.
49. The quotient x divided by 2 is 6.
50. The quotient 2 divided by x is 6.

In Problems 51–88, evaluate each expression.

51. $9 - 4 + 2$
52. $6 - 4 + 3$
53. $-6 + 4 \cdot 3$
54. $8 - 4 \cdot 2$
55. $4 + 5 - 8$
56. $8 - 3 - 4$
57. $4 + \frac{1}{3}$
58. $2 - \frac{1}{2}$
59. $6 - [3 \cdot 5 + 2 \cdot (3 - 2)]$
60. $2 \cdot [8 - 3(4 + 2)] - 3$
61. $2 \cdot (3 - 5) + 8 \cdot 2 - 1$
62. $1 - (4 \cdot 3 - 2 + 2)$
63. $10 - [6 - 2 \cdot 2 + (8 - 3)] \cdot 2$
64. $2 - 5 \cdot 4 - [6 \cdot (3 - 4)]$
65. $(5 - 3)\frac{1}{2}$
66. $(5 + 4)\frac{1}{3}$
67. $\frac{4 + 8}{5 - 3}$
68. $\frac{2 - 4}{5 - 3}$
69. $\frac{3}{5} \cdot \frac{10}{21}$
70. $\frac{5}{9} \cdot \frac{3}{10}$
71. $\frac{6}{25} \cdot \frac{10}{27}$
72. $\frac{21}{25} \cdot \frac{100}{3}$
73. $\frac{3}{4} + \frac{2}{5}$
74. $\frac{4}{3} + \frac{1}{2}$
75. $\frac{5}{6} + \frac{9}{5}$
76. $\frac{8}{9} + \frac{15}{2}$
77. $\frac{5}{18} + \frac{1}{12}$
78. $\frac{2}{15} + \frac{8}{9}$
79. $\frac{1}{30} - \frac{7}{18}$
80. $\frac{3}{14} - \frac{2}{21}$
81. $\frac{3}{20} - \frac{2}{15}$
82. $\frac{6}{35} - \frac{3}{14}$
83. $\dfrac{\frac{5}{18}}{\frac{11}{27}}$
84. $\dfrac{\frac{5}{21}}{\frac{2}{35}}$
85. $\frac{1}{2} \cdot \frac{3}{5} + \frac{7}{10}$
86. $\frac{2}{3} + \frac{4}{5} \cdot \frac{1}{6}$
87. $2 \cdot \frac{3}{4} + \frac{3}{8}$
88. $3 \cdot \frac{5}{6} - \frac{1}{2}$

In Problems 89–100, use the Distributive Property to remove the parentheses.

89. $6(x + 4)$
90. $4(2x - 1)$
91. $x(x - 4)$
92. $4x(x + 3)$
93. $2\left(\frac{3}{4}x - \frac{1}{2}\right)$
94. $3\left(\frac{2}{3}x + \frac{1}{6}\right)$
95. $(x + 2)(x + 4)$
96. $(x + 5)(x + 1)$
97. $(x - 2)(x + 1)$
98. $(x - 4)(x + 1)$
99. $(x - 8)(x - 2)$
100. $(x - 4)(x - 2)$

Explaining Concepts: Discussion and Writing

101. Explain to a friend how the Distributive Property is used to justify the fact that $2x + 3x = 5x$.
102. Explain to a friend why $2 + 3 \cdot 4 = 14$, whereas $(2 + 3) \cdot 4 = 20$.
103. Explain why $2(3 \cdot 4)$ is not equal to $(2 \cdot 3) \cdot (2 \cdot 4)$.
104. Explain why $\frac{4 + 3}{2 + 5}$ is not equal to $\frac{4}{2} + \frac{3}{5}$.

105. Is subtraction commutative? Support your conclusion with an example.

106. Is subtraction associative? Support your conclusion with an example.

107. Is division commutative? Support your conclusion with an example.

108. Is division associative? Support your conclusion with an example.

109. If $2 = x$, why does $x = 2$?

110. If $x = 5$, why does $x^2 + x = 30$?

111. Are there any real numbers that are both rational and irrational? Are there any real numbers that are neither? Explain your reasoning.

112. Explain why the sum of a rational number and an irrational number must be irrational.

113. A rational number is defined as the quotient of two integers. When written as a decimal, the decimal will either repeat or terminate. By looking at the denominator of the rational number, there is a way to tell in advance whether its decimal representation will repeat or terminate. Make a list of rational numbers and their decimals. See if you can discover the pattern. Confirm your conclusion by consulting books on number theory at the library. Write a brief essay on your findings.

114. The current time is 12 noon CST. What time (CST) will it be 12,997 hours from now?

115. Both $\frac{a}{0}(a \neq 0)$ and $\frac{0}{0}$ are undefined, but for different reasons. Write a paragraph or two explaining the different reasons.

R.2 Algebra Essentials

OBJECTIVES
1 Graph Inequalities (p. 19)
2 Find Distance on the Real Number Line (p. 20)
3 Evaluate Algebraic Expressions (p. 21)
4 Determine the Domain of a Variable (p. 22)
5 Use the Laws of Exponents (p. 22)
6 Evaluate Square Roots (p. 24)
7 Use a Calculator to Evaluate Exponents (p. 25)
8 Use Scientific Notation (p. 25)

The Real Number Line

Real numbers can be represented by points on a line called the **real number line**. There is a one-to-one correspondence between real numbers and points on a line. That is, every real number corresponds to a point on the line, and each point on the line has a unique real number associated with it.

Pick a point on a line somewhere in the center, and label it O. This point, called the **origin**, corresponds to the real number 0. See Figure 10. The point 1 unit to the right of O corresponds to the number 1. The distance between 0 and 1 determines the **scale** of the number line. For example, the point associated with the number 2 is twice as far from O as 1. Notice that an arrowhead on the right end of the line indicates the direction in which the numbers increase. Points to the left of the origin correspond to the real numbers -1, -2, and so on. Figure 10 also shows the points associated with the rational numbers $-\frac{1}{2}$ and $\frac{1}{2}$ and with the irrational numbers $\sqrt{2}$ and π.

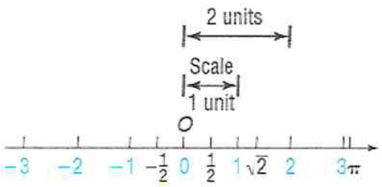

Figure 10 Real number line

DEFINITION The real number associated with a point P is called the **coordinate** of P, and the line whose points have been assigned coordinates is called the **real number line**.

Now Work PROBLEM 13