

BOOLEAN
ALGEBRA

AND

LOGICAL

CIRCUITS

USUAL ALGEBRA

$$C = a + b$$

\swarrow \swarrow \searrow
 ∞ ∞ ∞

BOOLEAN ALGEBRA

$$C = a + b$$

\swarrow \swarrow \searrow
 $0, 1$ $0, 1$ $0, 1$

1 Logic

True

Open

~~Low~~

High

Yes

Open Switch

0 Logic

False

Closed

Low

No

Closed Switch

Numerical representation of voltage changes, is the most commonly usage of the boolean algebra.

Such as

0 \longrightarrow 0 - 2.5 V

1 \longrightarrow 2.51 - 5 V

BASIC BOOLEAN ALGEBRA OPERATIONS & GATES

AND OR NOT

AND OPERATOR and GATE

$$X = A \cdot B$$

$$X = AB$$

X, A and B are Boolean Variables.
Their values can be only
1 or 0.

If both A and B is 1
the result is 1, in all other
cases result is 0.

All possible results of
a Boolean equation are showed
on the truth table.

A	B	$X = A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

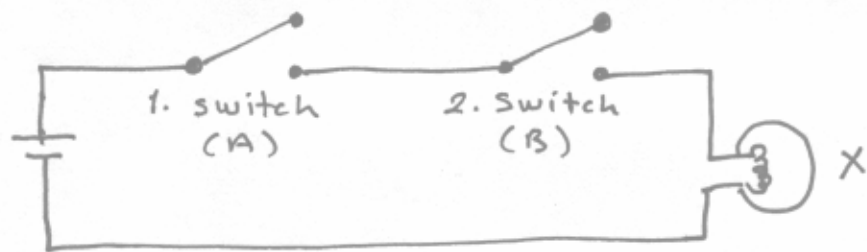
it looks like
multiplication
operation in
usual algebra

Truth Table of $X = A \cdot B$

AND Gate

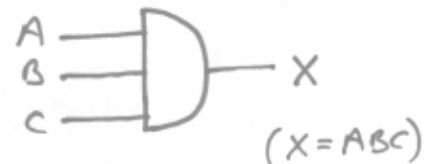


AND operation looks like



$$X = ABC$$

A	B	C	X = ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



If there are N variables in a boolean equation, there are 2^N possible results.

$$N = 3 \quad 2^3 \rightarrow 8$$

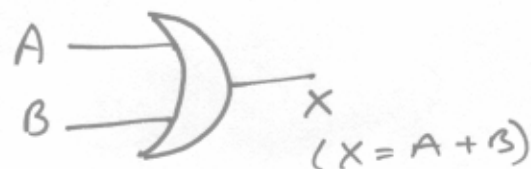
OR OPERATOR and GATE

$$X = A + B$$

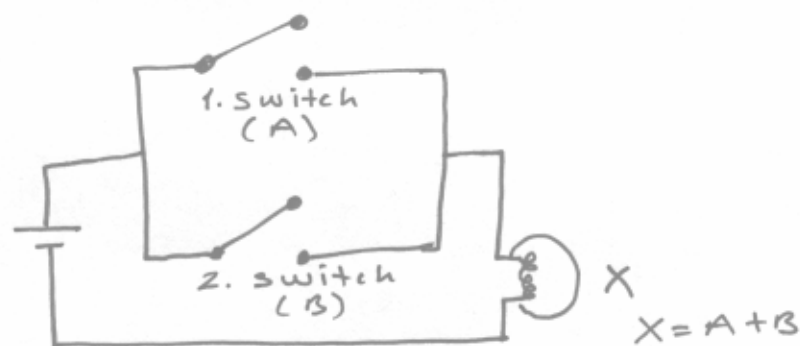
Truth
Table
of
 $X = A + B$

A	B	$X = A + B$
0	0	0
0	1	1
1	0	1
1	1	1

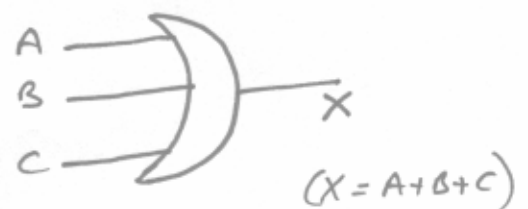
(like addition
in usual
algebra)



OR Gate



A	B	C	$X = A + B + C$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



NOT Operator & GATE

NOT OPERATOR REVERSE A BOOLEAN VALUE.

IF IT IS 1 WITH NOT OPERATOR
IT BECOMES 0. OR IF 0 IT IS 1.

$$X = \bar{A}$$

A	X = \bar{A}
0	1
1	0



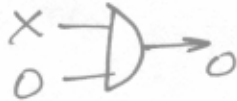
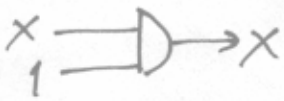
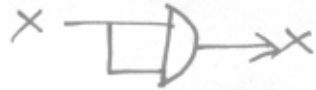






NOT GATE HAS ALWAYS
ONE INPUT, AND OUTPUT
IS REVERSE OF IT.

RULES OF BOOLEAN ALGEBRA

A Boolean Algebra equation/function is made up AND, OR and NOT. There can be any number of boolean variables.

When a lot of variables are used, there are some simplification rules to get small but equivalent equations instead of long equations.

a) Rules for Single Variable

- 1) $X \cdot 0 = 0$ 
- 2) $X \cdot 1 = X$ 
- 3) $X \cdot X = X$ 
- 4) $X \cdot \bar{X} = 0$ 
- 5) $X + 0 = X$ 
- 6) $X + 1 = 1$ 
- 7) $X + X = X$ 
- 8) $X + \bar{X} = 1$ 
- 9) $\bar{\bar{X}} = X$ 

They can be tested with giving 0 and 1. Also, these rules can be used for more than variables. Example:
 $AB \cdot (\bar{A}\bar{B}) \quad X = AB \rightarrow X \cdot \bar{X} \rightarrow 0 \text{ (4. Rule)}$