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Freeway Ramp Metering: An Overview

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Abstract

Recurrent and non-recurrent congestion on freeways may be alleviated if today's "spontaneous" infrastructure utilisation is replaced by an orderly, controllable operation via comprehensive application of ramp metering and freeway-to-freeway control, combined with powerful optimal control techniques. This paper first explains why ramp metering can lead to a dramatic amelioration of traffic conditions on freeways. An overview of ramp metering algorithms is provided next, ranging from early fixed-time approaches to traffic-responsive regulators and to modern sophisticated nonlinear optimal control schemes. Finally, a large-scale example demonstrates the high potential of advanced ramp metering approaches.

Keywords: ramp metering; traffic-responsive regulators; nonlinear optimal control; congestion management.

1. Introduction

Urban and interurban freeways had been originally conceived so as to provide virtually illimited mobility to road users. The on-going dramatic expansion of car-ownership, however, has lead to the daily appearance of recurrent and nonrecurrent freeway congestions of thousands of kilometers in length around the world. Ironically, daily recurrent congestions reduce substantially the available infrastructure capacity at the rush hours, i.e. at the time this capacity is most urgently needed, causing delays, increased environmental pollution, and reduced traffic safety. Similar effects are observed in the frequent case of nonrecurrent congestions caused by incidents, road works, etc. It has been recently realized that the mere infrastructure expansion cannot provide a complete solution to these problems due to economic and environmental reasons or, in metropolitan areas, simply due to lack of space.

The traffic situation on today's freeways resembles very much to the one in urban road networks prior to the introduction of traffic lights: blocked links, chaotic intersections, reduced safety. It seems like road authorities and road users are still chasing the phantom of illimited mobility that freeways were originally supposed to provide. What is urgently needed, however, is to restore and maintain the full utilisation of the freeways' capacity along with an orderly and balanced satisfaction of the occurring demand both during rush hours and in case of incidents. Clearly, the passage from chaotic to optimal traffic conditions is only possible if today's "spontaneous" use of the freeway infrastructure is replaced by suitable control actions aiming at the benefit of all users. Ramp metering is the most efficient means to this end, whereby short delays at on-ramps and freeway-to-freeway intersections is the (relatively low) price to pay for capacity flow on the freeway itself, leading to substantial savings for each individual road user.

This overview paper first explains, based on simple, mathematically sound arguments, the reasons why ramp metering may lead to a substantial amelioration of traffic conditions on freeways (section 2). An overview of ramp metering algorithms is provided in section 3, ranging from early fixed-time approaches to traffic-responsive regulators and to modern, sophisticated nonlinear optimal control schemes. A simulation example of large-scale nonlinear optimal ramp metering is presented in section 4 to demonstrate the high amelioration potential of advanced ramp metering algorithms. Section 5 summarizes the main conclusions.

2. Why ramp metering?

2.1 A basic property

To be able to answer this question, we will first recall a simple fact. Consider any traffic network (Figure 1) with demand appearing at several locations (e.g. at the on-ramps, in case of a freeway network) and exit flows forming at several destinations (e.g. at the freeway off-ramps). Clearly, the accumulated demand over, say, a day will be equal to the accumulated exit flows, because no vehicle disappears or is generated in the network. Let us assume that the demand level and its spatial and temporal

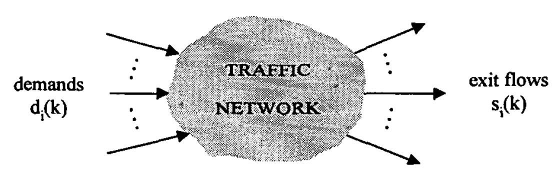

Figure 1: A general traffic network.

distribution are independent of any control measures taken in the network. Then, we are interested to know how much accumulated time will be needed by all drivers to reach their respective destinations at the network exits (network efficiency!). It is quite evident that this **total time spent** by all drivers in the traffic network will be longer if, for any reason (e.g. due to lack of suitable control measures), the exit flows are temporarily lower, i.e. if vehicles are delayed within the network on their way to their destinations. As a consequence, any control measure or control strategy that can manage to increase the early exit flows of the network, will lead to a corresponding decrease of the total time spent.

The above statements may be formalized by use of simple mathematics [1, 2]. For the needs of this paper we will use a discrete-time representation of traffic variables with discrete time index $k = 0, 1, 2, \ldots$ and time interval T. A **traffic volume** or **flow** $q(k)$ (in veh/h) is defined as the number of vehicles crossing a corresponding location during the time period $[kT, (k+1)T]$, divided by T.

We consider a traffic network (Figure 1) that receives demands $d_i(k)$ (in veh/h) at its origins $i = 1, 2, \ldots$ and we define the total demand $d(k) = d_1(k) + d_2(k) + \ldots$ We assume that $d(k)$, $k = 0, \ldots, K-1$, is independent of any control measures taken in the network. We define exit flows $s_i(k)$ at the network destinations $i = 1, 2, \ldots$, and the total exit flow $s(k) = s_1(k) + s_2(k) + \ldots$ We wish to apply control measures so as to minimize the total time spent T_s in the network over a time horizon K, i.e.

$$T_s = T\sum_{k=1}^{K} N(k) \qquad (1)$$

where N(k) is the total number of vehicles in the network at time k. Due to conservation of vehicles

$$N(k) = N(k-1) + T[d(k-1) - s(k-1)] = = N(0) + T\sum_{\kappa=0}^{k-1}[d(\kappa) - s(\kappa)]. \qquad (2)$$

Substituting (2) in (1) we obtain

$$T_s = T\sum_{k=1}^{K}[N(0) + T\sum_{\kappa=0}^{k-1} d(\kappa) - T\sum_{\kappa=0}^{k-1} s(\kappa)]. \qquad (3)$$

The first two terms in the outer sum of (3) are independent of the control measures taken in the network, hence minimization of T_s is equivalent to maximization of the following quantity

$$S = T^2\sum_{k=1}^{K}\sum_{\kappa=0}^{k-1} s(\kappa) = T^2\sum_{k=0}^{K-1}(K-k)s(k). \qquad (4)$$

Thus, minimization of the total time spent in a traffic network is equivalent to maximization of the time-weighted exit flows. In other words, the earlier the vehicles are able to exit the network (by appropriate use of the available control measures) the less time they will have spent in the network.

2.2 First answer

We consider (Figure 2) two cases for a freeway on-ramp, (a) without and (b) with metering control. Let q_{in} be the upstream freeway flow, d be the ramp demand, q_{con} be the mainstream outflow in presence of congestion, and q_{cap} be the freeway capacity. It is well-known that the outflow q_{con} in case of congestion is lower by some 5-10% than the freeway capacity q_{cap}. In Figure 2b, we assume that ramp metering may be used to maintain capacity flow on the mainstream, e.g. by using the control strategy ALINEA [3] (see section 3.2). Of course, the application of ramp metering creates a queue at the on-ramp but, because q_{cap} is greater than q_{con} (increased outflow!), ramp metering leads to a reduction of the total time spent (including the ramp waiting time). It is easy to show [2] that the amelioration ΔT_s (in %) of the total time spent is given by

$$\Delta T_s = \frac{q_{cap} - q_{con}}{q_{in} + d - q_{con}} 100. \qquad (5)$$

As an example, if $q_{in}+d = 1.2\ q_{cap}$ (i.e. the total demand exceeds the freeway capacity by 20%) and $q_{con} = 0.95\ q_{cap}$ (i.e. the capacity drop due to the congestion is 5%) then $\Delta T_s = 20\%$ results from (5), which demonstrates the importance of ramp metering.

2.3 Second answer

We consider (Figure 3) two cases of a freeway stretch that includes an on-ramp and an off-ramp, namely (a) without and (b) with metering control. In order to clearly separate the different effects of ramp metering, we will assume here that $q_{con} = q_{cap}$, i.e. no capacity drop due to congestion. Defining the exit rate γ ($0 \le \gamma \le 1$) as the portion of the upstream flow that exits at the off-ramp, it is easy to show [2] that the exit flow without control is given by

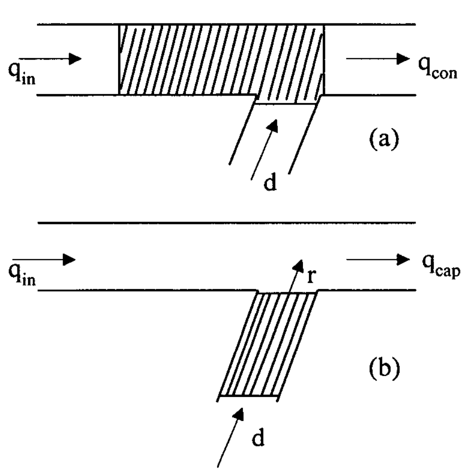

Figure 2: Two cases, (a) without and (b) with ramp metering; grey areas indicate congestion zones.

$$s^{nc} = \frac{\gamma}{1-\gamma}(q_{cap} - d) \qquad (6)$$

while with metering control we have

$$s^{rm} = \gamma \cdot q_{in}. \qquad (7)$$

Because $(1-\gamma)q_{in} + d > q_{cap}$ holds (else the congestion would not have been created), it follows that s^{nc} is less than s^{rm}, hence ramp metering increases the outflow thus decreasing the total time spent in the system. It is easy to show [2] that the amelioration of the total time spent in this case amounts to

$$\Delta T_s = \gamma \cdot 100. \qquad (8)$$

As an example, if the exit rate is $\gamma = 0.05$ then the amelioration is $\Delta T_s = 5\%$. If several upstream off-ramps are blocked by the congestion in absence of ramp metering (which is typically the case in many freeways during rush hours) then the amelioration achievable via introduction of ramp metering is accordingly higher.

Summing up the effects of sections 2.2, 2.3 in a freeway network, overall amelioration of total time spent by as much as 50% (i.e. halving of the average journey time) may readily result (see section 4).

2.4 Further impacts

The road users choose their respective routes towards their destinations so as to minimize their individual travel times. When a control measure (e.g. ramp metering) is introduced that may change the delay experienced in particular network links (e.g. on-ramps), a portion of the drivers will accordingly change their usual route in order to benefit from, or avoid disbene- fits due to the new network conditions. For example, in the case of Figure 2b, the upstream flow q_{in} will probably increase while the ramp demand d will decrease as compared to Figure 2a. Because the route choice behaviour of drivers is predictable to a large extent (Traffic Assignment problem!), ramp metering may also

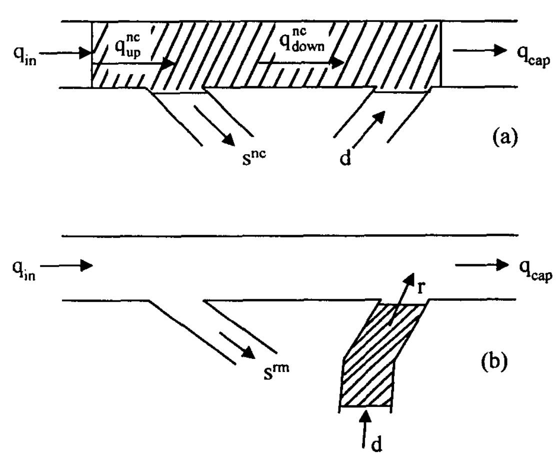

Figure 3: Two cases, (a) without and (b) with ramp metering.

be used so as to impose an operationally desired traffic flow distribution in the overall network, e.g. avoidance of the rat-running phenomenon, increased or decreased utilisation of underutilized or overloaded, respectively, parallel arterials etc. Clearly, the modified routing behaviour of drivers should be taken into account in the design and evaluation phases of ramp metering control strategies.

Several field evaluation results (see e.g. [4]) demonstrate that ramp metering improves the merging behaviour of traffic flow at freeway intersections which may have a significant positive impact on traffic safety due to less lane changes and reduced driver stress. Moreover, the increase of network efficiency related to both answers above, is expected to lead to accordingly improved network traffic safety and reduced pollutant emissions to the environment.

3. Overview of ramp metering strategies

3.1 Fixed-time strategies

Fixed-time ramp metering strategies are derived off-line for particular times-of-day, based on constant historical demands, without use of real-time measurements. They are based on simple static models. A freeway with several on-ramps and off-ramps is subdivided into sections, each containing one on-ramp. We then have

$$q_j = \sum_{i=1}^{j} \alpha_{ij} r_i \qquad (9)$$

where q_j is the mainline flow of section j, r_i is the on-ramp volume (in veh/h) of section i, and $\alpha_{ij} \in [0, 1]$ expresses the (known) portion of vehicles that enter the freeway in section i and do not exit the freeway upstream of section j. To avoid congestion

$$q_j \le q_{cap,j} \qquad \forall j \qquad (10)$$

must hold, where $q_{cap,j}$ is the capacity of section j. Further constraints are

$$r_{j,min} \le r_j \le \min\{r_{j,max}, d_j\} \qquad (11)$$

where d_j is the demand while $r_{j,max}$ is the ramp capacity at on-ramp j. This approach was first suggested by Wattleworth [5]. Other similar formulations may be found in [6-11].

As an objective criterion, one may wish to maximize the number of served vehicles (which is equivalent to minimising the total time spent)

$$\sum_j r_j \to \text{Max} \qquad (12a)$$

or to maximize the total travelled distance

$$\sum_j \Delta_j q_j \to \text{Max} \qquad (12b)$$

(where Δ_j is the length of section j), or to balance the ramp queues

$$\sum_j (d_j - r_j)^2 \to \text{Min.} \qquad (12c)$$

These formulations lead to linear-programming or quadratic-programming problems that may be readily solved by use of broadly available computer codes. An extension of these methods that renders the static model (9) dynamic by introduction of constant travel times for each section, was suggested in [12].

The main drawback of fixed-time strategies is that their settings are based on historical rather than real-time data. This may be a rude simplification because:

- Demands are not constant, even within a time-of-day.
- Demands may vary at different days, e.g. due to special events.
- Demands change in the long term leading to "aging" of the optimized settings.
- The portions α_{ij} are also changing in the same ways as demands; in addition, these portions may change due to the drivers' response to the new optimized signal settings, whereby they try to minimize their individual travel times.
- Incidents and farther disturbances may perturb traffic conditions in a non-predictable way.

Hence, fixed-time ramp metering strategies may lead (due to the absence of real-time measurements) either to overload of the mainstream flow (congestion) or to underutilization of the freeway. In fact, ramp metering is an efficient but also delicate control measure. If ramp metering strategies are not accurate enough, then congestion may not be prevented from forming, or the mainstream capacity may be underutilized (e.g. due to groundlessly strong metering).

3.2 Reactive ramp metering strategies

Reactive ramp metering strategies are employed at a tactical level, i.e. in the aim of keeping the freeway traffic conditions close to pre-specified set values, based on real-time measurements.

Local ramp metering. Local ramp metering strategies make use of traffic measurements in the vicinity of a ramp to calculate suitable ramp metering values. The *demand-capacity strategy* [13], quite popular in North America, reads

$$r(k) = \begin{cases} q_{cap} - q_{in}(k-1) & \text{if } o_{out}(k) \le o_{cr} \\ r_{min} & \text{else} \end{cases} \qquad (13)$$

where (Figure 4) q_{cap} is the freeway capacity downstream of the ramp, q_{in} is the freeway flow measurement upstream of the ramp, o_{out} is the freeway occupancy measurement downstream of the ramp, o_{cr} is the critical occupancy (at which the freeway flow becomes maximum), and r_{min} is a pre-specified minimum ramp flow value. The strategy (13) attempts to add to the measured upstream flow $q_{in}(k-1)$ as much ramp flow r(k) as necessary to reach the downstream freeway capacity q_{cap}. If, however, for some reason, the downstream measured occupancy $o_{out}(k)$ becomes overcritical (i.e. a congestion may form), the ramp flow r(k) is reduced to the minimum flow r_{min} to avoid or to dissolve the congestion. Clearly, (13) does not really represent a closed-loop strategy but an open-loop disturbance-rejection policy (Figure 4a) which is generally known to be quite sensitive to various non-measurable disturbances.

The *occupancy strategy* [13] is based on the same philosophy as the demand-capacity strategy, but it relies on occupancy-based estimation of q_{in}, which may, under certain conditions, reduce the corresponding implementation cost.

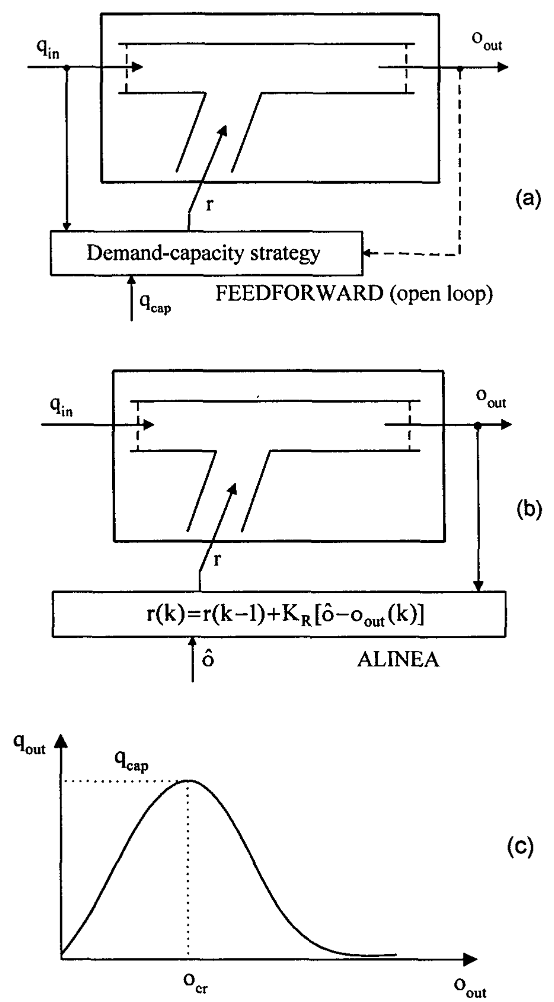

Figure 4. Local ramp metering strategies: (a) Demand-capacity, (b) ALINEA, (c) the fundamental diagram.

An alternative, closed-loop ramp metering strategy (*ALINEA*), suggested in [14], reads

$$r(k) = r(k-1) + K_R[\hat{o} - o_{out}(k)] \qquad (14)$$

where $K_R > 0$ is a regulator parameter and $\hat{o}$ is a set (desired) value for the downstream occupancy (typically, but not necessarily, $\hat{o} = o_{cr}$ may be set, in which case the downstream freeway flow becomes close to q_{cap}, see Figure 4c). In field experiments, ALINEA has not been very sensitive to the choice of the regulator parameter K_R.

Note that the demand-capacity strategy reacts to excessive occupancies o_{out} only after a threshold value (o_{cr}) is exceeded, and in a rather crude way, while ALINEA reacts smoothly even to slight differences $\hat{o} - o_{out}(k)$, and thus it may prevent congestion by stabilizing the traffic flow at a high throughput level. It is easily seen that at a stationary state (i.e. if q_{in} is constant), $o_{out}(k) = \hat{o}$ results automatically from (14), although no measurements of the inflow q_{in} are explicitly used in the strategy.

The set value $\hat{o}$ may be changed any time, and thus ALINEA may be embedded into a hierarchical control system with set values of the individual ramps being specified in real time by a superior coordination level or by an operator.

All control strategies calculate suitable ramp volumes r. In the case of **traffic-cycle realization** of ramp metering, r is converted to a green-phase duration g by use of

$$g = (r/r_{sat}) \cdot c \qquad (15)$$

where c is the fixed cycle time and r_{sat} is the ramp's saturation flow. The green-phase duration g is constrained by $g \in [g_{min}, g_{max}]$, where $g_{min} > 0$ to avoid ramp closure, and $g_{max} \leq c$. In the case of an **one-car-per-green realization**, a constant-duration green phase permits exactly one vehicle to pass. Thus, the ramp volume r is controlled by varying the red-phase duration between a minimum (zero) and a maximum value. Note that ALINEA is also applicable directly to the green or red-phase duration, by combining (14) and (15)

$$g(k) = g(k-1) + K_R' [\hat{o} - o_{out}(k)] \qquad (16)$$

where $K_R' = K_R c/r_{sat}$. Note also that the values r(k−1) or g(k−1) used on the right-hand side of (14) or (16), respectively, should be the *bounded* values of the previous time step (i.e. after application of the g_{min} and g_{max} constraints) in order to avoid the wind-up phenomenon in the regulator.

If the queue of vehicles on the ramp becomes excessive, interference with surface street traffic may occur. This may be detected with suitably placed detectors (on the upstream part of the on-ramp), leading to an override of the regulator decisions to allow more vehicles to enter the freeway and the ramp queue to diminish.

Note that the above specifications and constraints apply in the same way to any ramp metering strategy.

Comparative field trials have been conducted in various countries to assess and compare the efficiency of local ramp metering strategies, see e.g. [3]. These trials have demonstrated the clear superiority of ALINEA over other local strategies and over the no-control case with regard to any performance criterion: total time spent, total travelled distance, mean speed, mean (daily) congestion duration. Typical local improvements of the total time spent (including the waiting time at the ramps) may reach 20%.

Multivariable regulator strategies. Multivariable regulators for ramp metering pursue the same goals as local ramp metering strategies: They attempt to operate the freeway traffic conditions near some pre-specified set (desired) values. While local ramp metering is performed independently for each ramp, based on local measurements, multivariable regulators make use of all available mainstream measurements $o_i(k)$, i = 1,..., n, on a freeway stretch, to calculate simultaneously the ramp volume values $r_i(k)$, i = 1,..., m, for all controllable ramps included in the same stretch [15]. This provides potential improvements over local ramp metering because of more comprehensive information provision and because of coordinated control actions. Multivariable regulator approaches to ramp metering have been reported in [15-26]. The multivariable regulator strategy *METALINE* may be viewed as a generalisation and extension of ALINEA, whereby the metered on-ramp volumes are calculated from (bold variables indicate vectors and matrices)

$$\mathbf{r}(k) = \mathbf{r}(k-1) - \mathbf{K}_1[\mathbf{o}(k) - \mathbf{o}(k-1)] + \mathbf{K}_2[\hat{\mathbf{O}} - \mathbf{O}(k)] \qquad (17)$$

where $\mathbf{r} = [r_1 \ldots r_m]^T$ is the vector of m controllable on-ramp volumes, $\mathbf{o} = [o_1 \ldots o_n]^T$ is the vector of n measured occupancies on the freeway stretch, $\mathbf{O} = [O_1 \ldots O_m]^T$ is a subset of $\mathbf{o}$ that includes m occupancy locations for which pre-specified set values $\hat{\mathbf{O}} = [\hat{O}_1 \ldots \hat{O}_m]^T$ may be given. Note that for control-theoretic reasons, the number of set-valued occupancies cannot be higher than the number of controlled on-ramps. Typically one bottleneck location downstream of each controlled on-ramp is selected for inclusion in the vector **O**. Finally, $\mathbf{K}_1$ and $\mathbf{K}_2$ are the regulator's constant gain matrices that must be suitably designed, see [15, 27] for details.

Field trials and simulation results comparing the efficiency of METALINE versus ALINEA lead to the following conclusions [3]:

- While ALINEA requires hardly any design effort, METALINE application calls for a rather sophisticated design procedure that is based on advanced control-theoretic methods (LQ optimal control).
- For urban freeways with a high density of on-ramps, METALINE was found to provide no advantages

over ALINEA (the later implemented independently at each controllable on-ramp) under recurrent congestion.

- In the case of non-recurrent congestion (e.g. due to an incident), METALINE performs better than ALINEA due to more comprehensive measurement information.

Some system operators hesitate to apply ramp metering because of the concern that congestion may be conveyed from the freeway to the adjacent street network. In fact, a ramp metering application designed to avoid or reduce congestion on freeways may have both positive and negative effects on the adjacent road network traffic. It is easy to see, based on notions and statements made earlier, that, if an efficient control strategy is applied for ramp metering, the freeway throughput will be generally increased. More precisely, ramp metering at the beginning of the rush hour may lead to on-ramp queues in order to prevent congestion to form on the freeway, which may temporarily lead to diversion towards the urban network. But due to congestion avoidance or reduction, the freeway will be eventually enabled to accommodate a higher throughput, thus attracting drivers from urban paths and leading to an improved overall network performance. This positive impact of ramp metering on both the freeway and the adjacent road network traffic conditions was confirmed in a specially designed field evaluation in the Corridor Périphérique in Paris [28].

3.3 Nonlinear optimal ramp metering strategies

Prevention or reduction of traffic congestion on freeway networks may dramatically improve the infrastructure efficiency in terms of throughput and total time spent. Congestion on limited-capacity freeways forms, because too many vehicles attempt to use them in a non-coordinated (uncontrolled) way. Once congestion is built up, the outflow from the congestion area is reduced and the off-ramps and interchanges covered by the congestion are blocked, which may in some extreme cases even lead to fatal gridlocks. Reactive ramp metering strategies may be helpful to a certain extent, but, first they need appropriate set values, and, second, their character is more or less local. What is needed for freeway networks or long stretches is a superior coordination level that calculates, in real time, optimal and fair set values from a proactive, strategic point of view. Such an optimal control strategy should explicitly take into account:

- The current traffic state both on the freeway and on the on-ramps.
- Demand predictions over a sufficiently long time horizon.
- The limited storage capacity of the on-ramps.
- The ramp metering constraints discussed earlier.
- The nonlinear traffic flow dynamics, including the infrastructure's limited capacity.
- Any incidents currently present in the freeway network.

Based on this comprehensive information, the control strategy should deliver set values for the overall freeway network over a future time horizon so as:

- to respect all present constraints
- to minimize an objective criterion such as the total time spent in the whole network (including the on-ramps).

Such a comprehensive dynamic optimal control problem may be formulated and solved with moderate computation time by use of suitable solution algorithms (see section 4).

The nonlinear traffic dynamics may be expressed by use of suitable dynamic models in the form

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{r}(k), \mathbf{d}(k)] \qquad (18)$$

where the state vector $\mathbf{x}$ comprises all traffic densities and mean speeds of 500-m long freeway sections, as well as all ramp queues; the control vector $\mathbf{r}$ comprises all controllable ramp volumes; the disturbance vector $\mathbf{d}$ comprises all on-ramp demands and turning rates (at network bifurcations or at off-ramps). The ramp metering constraints are given by (11) while the queue constraints read

$$l_i(k) \le l_{i,max} \qquad (19)$$

where l_i are queue lengths (in veh). The total time spent in the whole system over a time horizon K may be expressed

$$T_s = T\sum_{k=0}^{K}[\sum_{i=1}^{n}\rho_i(k)\cdot\Delta_i + \sum_{i=1}^{m} l_i(k)]. \qquad (20)$$

where $\rho_i(k)$ is the traffic density (in veh/km) in segment i at time k·T.

Thus, for given current (initial) state $\mathbf{x}(0)$ from corresponding measurements or estimates, and given demand predictions $\mathbf{d}(k)$, k = 0, ..., K−1, the problem consists in specifying the ramp flows $\mathbf{r}(k)$, k = 0, ..., K−1, so as to minimize the total time spent (20) subject to the nonlinear traffic flow dynamics (18) and the constraints (11) and (19).

This problem or variations thereof was considered and solved in various works [1, 29-36]. Although simulation studies indicate substantial savings of travel time and substantial increase of throughput, advanced control strategies of this kind have not been implemented in the field as yet.

3.4 Integrated freeway network traffic control

Modern freeway networks may include different types of control measures. The corresponding control strategies are usually designed and implemented independently, thus failing to exploit the synergistic effects that might result from coordination of the respective control actions. An advanced concept for integrated freeway network control results from suitable extension of the optimal control approach outlined above. More precisely, the dynamic model (18) of freeway traffic flow may be extended to enable the inclusion of further control measures, beyond the ramp metering rates **r**(k). Formally **r**(k) is then replaced in (18) by a general control input vector **u**(k) that comprises all implemented control measures of any type. Such an approach was implemented in the integrated, generic freeway network control tool *AMOC* [37] where ramp metering and route guidance are considered simultaneously with promising results, see also [38, 39].

4. An advanced example

4.1 The freeway network traffic model

The efficiency and the amelioration potential of nonlinear optimal ramp metering strategies may be demonstrated by means of simulation for a large-scale network with the use of the AMOC generic freeway network control tool. In this case AMOC does not consider routing control measures, but only ramp metering control actions.

The macroscopic model employed for control design purposes is suitable for free flow, critical, and congested traffic conditions. It has two distinct modes of operation. When traffic assignment (routing) aspects of the traffic process are not taken under consideration, it operates in the non-destination oriented mode. When traffic assignment is an issue, it operates in the destination oriented mode. When route guidance measures are not included, then the traffic model does not need to operate in the destination oriented mode, although this is not imperative. Since we are interested here only in ramp-metering, the destination oriented mode of operation will not be described (see [40] for details).

The network is represented by a directed graph whereby the links of the graph represent freeway stretches. Each freeway stretch has uniform characteristics, i.e. no on-/off-ramps and no major changes in geometry. The nodes of the graph are placed at locations where a major change in road geometry occurs, as well as at junctions, on-ramps, and off-ramps.

The time and space arguments are discretised. The discrete-time step is denoted by T. A freeway link m is divided into N_m sections of equal length L_m. Each section i of link m at time instant t = kT, k = 0,..., K, is characterised by the following macroscopic quantities: The traffic density $\rho_{m,i}(k)$ (veh/lane/km) is the number of vehicles in section i of link m at time t = kT divided by L_m and by the number of lanes λ_m; the mean speed $\upsilon_{m,i}(k)$ (km/h) is the mean speed of the vehicles included in section i of link m at time kT; and the traffic volume or flow $q_{m,i}(k)$ (veh/h) is the number of vehicles leaving section i of link m during the time period [kT, (k+1)T], divided by T. The basic equations used for their calculation for each section i of link m at each time step, are

$$\rho_{m,i}(k+1)=\rho_{m,i}(k)+\frac{T}{L_m\cdot\lambda_m}\cdot\left[q_{m,i-1}(k)-q_{m,i}(k)\right] \quad (21)$$

$$q_{m,i}(k)=\rho_{m,i}(k)\cdot\upsilon_{m,i}(k)\cdot\lambda_m \quad (22)$$

$$\begin{aligned}\upsilon_{m,i}(k+1)=\upsilon_{m,i}(k)&+\frac{T}{\tau}\cdot\left\{V\left[\rho_{m,i}(k)\right]-\upsilon_{m,i}(k)\right\}\\&+\frac{T}{L_m}\cdot\upsilon_{m,i}(k)\cdot\left[\upsilon_{m,i-1}(k)-\upsilon_{m,i}(k)\right]-\\&-\frac{\nu\cdot T\cdot\left[\rho_{m,i+1}(k)-\rho_{m,i}(k)\right]}{\tau\cdot L_m\cdot\left[\rho_{m,i}(k)+\kappa\right]}\end{aligned} \quad (23)$$

$$V\left[\rho_{m,i}(k)\right]=\upsilon_{f,m}\cdot\exp\left[-\frac{1}{a_m}\cdot\left(\frac{\rho_{m,i}(k)}{\rho_{cr,m}}\right)^{a_m}\right] \quad (24)$$

where $\upsilon_{f,m}$ and $\rho_{cr,m}$ denote the free speed and critical density per lane, respectively, of link m while a_m is a further parameter of the fundamental diagram (eqn. (24)) of link m. Equation (21) is the well-known conservation equation, eqn. (22) is the flow equation to be substituted in (1), eqn. (23) is an empirical dynamic speed equation with (24) to be replaced therein (for more details see [41]). The mean speeds in the network sections are limited from below by υ_{min}. $\upsilon_{f,m}$, $\rho_{cr,m}$, a_m, τ, ν, and κ are constant parameters which reflect particular characteristics of a given traffic system and depend upon street geometry, vehicle characteristics, drivers' behaviour, etc. For a real-life network these parameters are determined by a validation procedure as the one described in [42].

In order for the speed calculation to take into account the speed decrease caused by merging phenomena, the term $-(\delta T/L_m\lambda_m)[q_\mu(k)\upsilon_{m,1}(k)/(\rho_{m,1}(k)+\kappa)]$ is added to the right-hand side of (23), where δ is a further parameter, μ is the merging link and m is the leaving link. In order for the speed reduction due to weaving phenomena, resulting from lane drops in the mainstream, to be considered, the term $-(\phi T/L_m\lambda_m)\ [\Delta\lambda\rho_{m,Nm}(k)/\rho_{cr,m}]\upsilon_{m,Nm}(k)^2$ is also added in (23), where $\Delta\lambda$ is the number of lanes being dropped, and ϕ is a further parameter. For more details on these two additional terms, see [41].

For origin links (e.g. on-ramps), i.e. links that receive traffic demand and subsequently forward it into the freeway network, a simple queue model is used. The outflow $q_o(k)$ of an origin link o is given by

$$q_o(k)=r_o(k)\min\left\{d_o(k)+l_o(k)/T,\,q_{max,o}(k)\right\} \quad (25)$$

where $d_o(k)$ is the demand flow at time period k at origin o, $l_o(k)$ is the length (in vehicles) of a possibly existing waiting queue at time k, and $r_o(k)\in[r_{min},1]$ is the metering rate for the origin link o at period k, i.e. a control variable. If $r_o(k)=1$, no ramp metering is applied; if $r_o(k)<1$, ramp metering becomes active. $q_{max,o}$ is the maximum outflow at the specific time instant. The later depends on the density of the mainstream link μ in the following way

$$q_{max,o}(k)=\begin{cases} Q_o & \text{if } \rho_{\mu,1}(k)<\rho_{cr,\mu} \\ Q_o\cdot p(k) & \text{else} \end{cases} \tag{26}$$

where Q_o is the flow capacity of the origin link and p(k) is the portion of Q_o that can enter link μ if $\rho_{1,\mu}>\rho_{cr,\mu}$, where

$$p(k)=1-\frac{\rho_{\mu,1}(k)-\rho_{cr,\mu}}{\rho_{max}-\rho_{cr,\mu}} \tag{27}$$

with ρ_{max} the maximum possible density in the network's links. Equations (26), (27) express the reduction of the ramp's outflow capacity caused by mainstream congestion. The conservation equation for an origin link yields

$$l_o(k+1)=l_o(k)+T\cdot[d_o(k)-q_o(k)]. \tag{28}$$

Freeway bifurcations and junctions (including on-ramps and off-ramps) are represented by nodes. Traffic enters a node n through a number of input links and is distributed to a number of output links according to

$$Q_n(k)=\sum_{\mu\in I_n} q_{\mu,N_\mu}(k) \tag{29}$$

$$q_{m,0}(k)=\beta_n^m(k)\cdot Q_n(k) \qquad \forall m\in O_n \tag{30}$$

where I_n is the set of links entering node n, O_n is the set of links leaving node n, $Q_n(k)$ is the total traffic volume entering node n at period k, $q_{m,0}(k)$ is the traffic volume that leaves node n via outlink m, and $\beta_n^m(k)$ is the portion of Q_n that leaves the node through link m. $\beta_n^m(k)$ are the turning rates of node n and are assumed known for the entire time horizon. Equations (29) and (30) provide $q_{m,0}(k)$ which is needed in (21) for i = 1.

When a node n has more than one leaving links then the upstream influence of density has to be taken into account in the last section of the incoming link (see (23)). This is provided via

$$\rho_{m,N_m+1}(k)=\sum_{\mu\in O_n}\rho_{\mu,1}^2(k)\Big/\sum_{\mu\in O_n}\rho_{\mu,1}(k) \tag{31}$$

where ρ_{m,N_m+1} is the virtual downstream density of the entering link m to be used in eqn. (23) for i = N_m, and $\rho_{\mu,1}(k)$ is the density of the first section of leaving link μ. This quadratic form is used because one congested leaving link may block the entering link even if there is free flow in the other leaving link.

When a node n has more than one entering links, then the downstream influence of speed has to be taken into account according to equation (23). The mean speed value is calculated from

$$\upsilon_{m,0}(k)=\sum_{\mu\in I_n}\upsilon_{\mu,N_\mu}(k)\cdot q_{\mu,N_\mu}(k)\Big/\sum_{\mu\in I_n}q_{\mu,N_\mu}(k) \tag{32}$$

where $\upsilon_{m,0}$ is the virtual speed upstream of the leaving link m that is needed in (23) for i = 1.

4.2 The constrained optimal control problem

By substituting (22), (29), (30) into (21); (22), (24), (31), (32) into (23); and (25)-(27) into (28), a discrete-time dynamic traffic model in the sense of (18) is formulated for any arbitrary freeway network. The state vector **x** consists of all traffic densities and mean speeds of every section i, and of all queues formed at on-ramps. The control vector **r** consists of the on-ramp metering rates $r_o(k) \in [r_{o,min}, 1]$. The disturbance vector **d** consists of all on-ramp demands and turning rates at the network bifurcations and off-ramps.

The cost criterion to be minimized is the total time spent T_s (see (20)) in the whole system over a time horizon K, plus two penalty terms, one for consideration of the queue constraints (19) and the other to suppress high-frequency oscillations of the control variables. More precisely, the cost criterion is given by

$$J=T\cdot\sum_k\Big\{\sum_m\sum_i\rho_{m,i}(k)\cdot L_m\cdot\lambda_m+\sum_o l_o(k)+ + a_f\sum_o[r_o(k)-r_o(k-1)]^2+a_w\sum_o\psi[l_o(k)]^2\Big\} \tag{33}$$

with

$$\psi[l_o(k)]=\max\{0,\ l_o(k)-l_{o,max}\} \tag{34}$$

where a_w and a_f are appropriately chosen weighting parameters.

The constrained discrete-time optimal control problem described is solved numerically by a powerful feasible-direction optimization algorithm, see [43] for details.

4.3 Site description

The previously described approach to network-wide optimal ramp metering has been applied to the Amsterdam ring-road with the use of AMOC.

A sketch of the Amsterdam Orbital Freeway (A10) is shown in Figure 5. The A10 simultaneously serves local, regional, and inter-regional traffic and acts as a hub for traffic entering and exiting North Holland. There are four main connections with other freeways, the A8 at the North, the A4 at the South-West, the A2 at the South, and the A1 at the South-East. The A10 contains two tunnels, the Coen Tunnel at the North-West of A10 and the Zeeburg Tunnel at the East.

For the purposes of our study we will constrain ourselves to the counter-clockwise direction of the A10, which is about 32 km long. There are 21 on-ramps on this freeway, including the connections with the A8, A4, A2, and A1 freeways, and a total of 20 off-ramps, including the junctions with A4, A2, A1, and A8. It is assumed that ramp metering may be performed at each on-ramp, whereby the maximum permissible queue length for the urban on-ramps is set to 20 vehicles, while storage of a maximum of 90 vehicles is permitted on each of the freeway-to-freeway ramps of A8, A4, A2, and A1.

4.4 The no-control case

The ring-road was studied for a time horizon of 4 hours, from 16:00 until 20:00 using realistic historical demands from the site. This time period includes the evening peak hour. In absence of any control measures, the ring-road is subject to recurrent congestion that is formed downstream of the junctions of A10 with A2 and A1 in A10-South. This congestion propagates backwards causing severe congestion to the A10-West. Figure 6 depicts the density propagation along the freeway sections (section 0 is the first section of A10-West, after the junction of A10 with A8). This congestion causes the formation of large queues at the on-ramps of A10-West as can be seen in Figure 7, where the queues of each on-ramp are depicted (on-ramp 0 corresponds to A8). Note that on-ramp queues may be created even in absence of ramp metering due to high demand or due to reduced ramp outflow caused by mainstream congestion according to (26), (27). Additionally, the congestion blocks the exits of A10 to A4 and A2 as well as the off-ramps of A10-West and A10-South. As a result, the total time spent over the 4h-horizon is equal to 11998 veh·h.

4.5 The optimal-control case

When ramp metering is performed at all on-ramps, including the entrances of A8, A4, A2, and A1 to the A10, the congestion is virtually lifted from the network (Figure

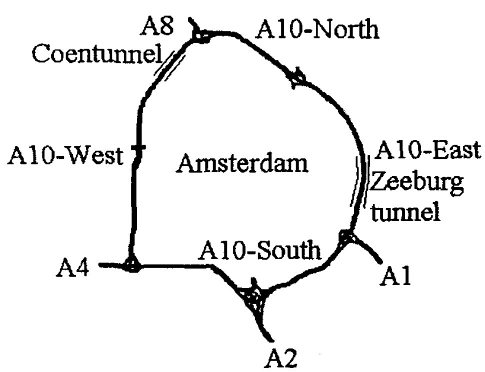

Figure 5. The Amsterdam ring-road.

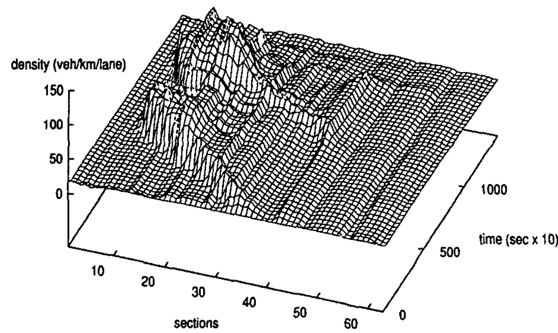

Figure 6. No control: Density.

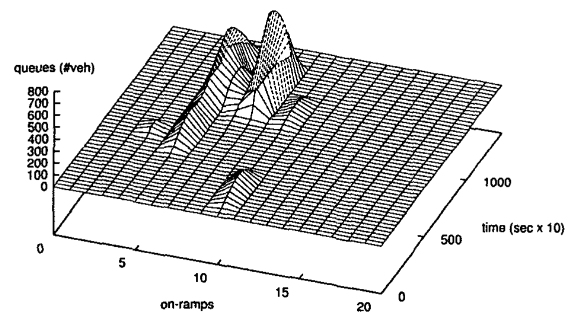

Figure 7. No control: Ramp queues.

8). The control strategy, aiming at maximizing the time-weighted network outflow (4), succeeds in establishing optimal uncongested traffic conditions on the A10-South and A10-West by applying ramp metering mainly at A1 and A2 at an early stage. In Figure 9, the queues are mainly occurring at A2 and A1 because these ramps have larger maximum permissible queues (90 vehicles). But since more storage capacity is required for complete elimination of the congestion, further ramp metering is performed at the on-ramps of A10-South and West and at A4, thereby utilising their storage capacity to the fullest extent. The resulting total time spent is 7609 veh·h which is a 36.6% improvement compared to the no-control case. It has to be noted here that this improvement refers to the overall ring-road (not just to the shorter congested stretch) and, also, that it is calculated for the whole time horizon of 4h (not just for the time period where congestion occurs). Therefore there is a systematic underestimation of the amelioration occurring in the critical stretch at the critical time period. Thus, the travel time of drivers involved in the critical stretch during the critical period are reduced even more, without any significant disbenefit to any network user.

A further improvement to the total time spent could be reached with larger maximum permissible queues. Had there been no queue constraints at all, the density profile of Figure 8 would be completely flat. In fact, the control strategy performs a trade-off between the queue lengths

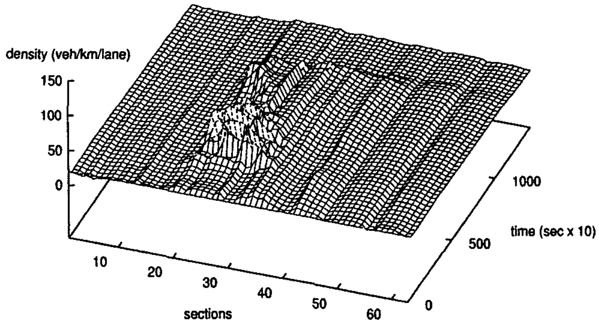

Figure 8. Optimal control: Density.

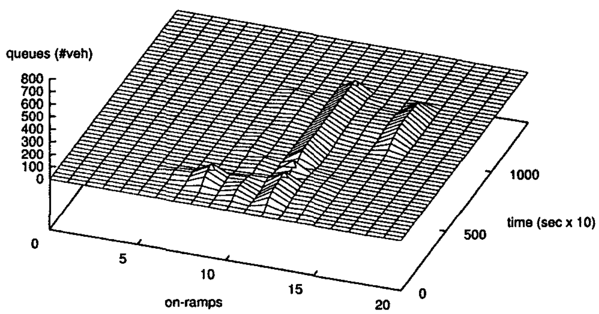

Figure 9. Optimal control: Ramp queues.

and the existence of congestion inside the network. Stricter queue constraints result in more degraded traffic conditions inside the freeway due to accordingly reduced control manoeuvrability.

4.6 Further developments

The results of this approach demonstrate its efficiency and feasibility. The computation time is moderate, since for the 4h time horizon the bulk of the 36.6% improvement (more that 32%) was obtained in 20 min on a Sun Ultra5 with a Sparc IIi-360MHz processor workstation.

Further control measures such as speed control or route guidance may be readily integrated cooperatively due to the flexible nature of the problem formulation.

The control trajectories obtained represent a strategic decision in the sense of providing optimal and fair set values over a long time horizon (e.g. 4h) for subordinate reactive ramp metering (section 3.2), using e.g ALINEA. This strategic role can be further enhanced by use of a rolling horizon framework whereby the optimal control problem is solved repeatedly in real time, with updated (measured) initial condition $\mathbf{x}(0)$, updated demand predictions and turning rates $\mathbf{d}(k)$, and with inclusion of possible incidents.

5. Conclusions

Modern freeway networks' capacity is daily underutilized, particularly during rush hours and at the occurrence of incidents, i.e. when it is most urgently needed, due to:

- reduced congestion outflow (see section 2.2)
- reduced off-ramp flow (see section 2.3)
- uncontrolled flow distribution in the overall network (see section 2.4).

The introduction of ramp metering at some particular ramps or particular freeway stretches within the overall network can help to alleviate some local traffic problems and to improve the local traffic conditions. However, the significant amelioration of the global traffic conditions in the overall traffic network calls for **comprehensive** control of **all or most** of the ramps, including the freeway-to-freeway links, in the aim of optimal utilisation of the available infrastructure. The limitations of **partial** (rather than comprehensive) ramp metering are:

(1) The potential benefits of partial ramp metering (according to Figures 2, 3) may be counterbalanced to some extent by a modified route choice behaviour of drivers who attempt the minimisation of their individual travel times under the new conditions.
(2) Individual on-ramps have a limited storage capacity for waiting vehicles; if the on-ramp queue reaches back to the surface street junction, ramp metering control is typically released in order to avoid interference with surface street traffic.
(3) The freeway network is a common resource for many driver groups with different origins and destinations. Partial ramp metering, by its nature, does not address the strategic problem of optimal utilisation of the overall infrastructure, nor does it guarantee a fair and orderly capacity allocation among the ramps.

Comprehensive ramp metering, on the other hand, does not suffer from these shortcomings, first because of complete control of the network traffic flow and its spatial and temporal distribution, and second because of sufficient available storage capacity. In fact, one or a few particular ramps located at a critical bottleneck area may not have sufficient storage capacity to completely avoid the building up of a congestion. However, in case of comprehensive optimal ramp metering in the sense of section 4, the total available storage space in all ramps and freeway intersections is usually sufficient to effectively and ultimately combat freeway congestion.

It should be emphasized that the implementation and operation cost of a comprehensive ramp metering system is estimated to be rather low as compared to the corresponding infrastructure cost and to the expected benefits in terms of dramatically reduced delays, increased traffic safety, and reduced environmental pollution. It should also be noted that the advanced methodological tools required for efficient operation of such a comprehensive ramp metering system are currently available, see section 4. The major problem to overcome today, is the inertia of political decision-makers which, on its turn, is mainly due to the lack of understanding of the huge potential of comprehensive ramp metering systems.

We believe that freeway networks will have to be operated as completely controllable systems in the near future, similar to the urban traffic networks, because this is the smartest way to avoid further degradation and fatal gridlocks. The sooner this is realized by the road authorities, the better for the road users who will be the major beneficiaries of this evolution.

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