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CPT205 Computer Graphics

3D Modelling

Lecture 07
2022-23

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Topics for today

➤ 3D modelling techniques

- Wireframe
- Surface
- Solid

➤ Constructive solid geometry (CSG)

- CSG tree
- Unambiguous but not unique

➤ Boundary representation (B-Rep)

- Geometry (shape and size) and topology (connectivity)
- Types of B-Rep models – manifold and nonmanifold
- Validity of B-Rep models – Euler's law
- Implementation of B-Rep models
- **Romulus** and **ACIS** modellers

Wireframe modelling (1)

- It is the oldest and simplest approach.
- The model consists of a finite set of points and curves.
- Parametric representation of a space curve

$$x = x(t), \quad y = y(t), \quad z = z(t)$$

- Implicit representation of a space curve

$$s1(x,y,z) = 0, \quad s2(x,y,z) = 0$$

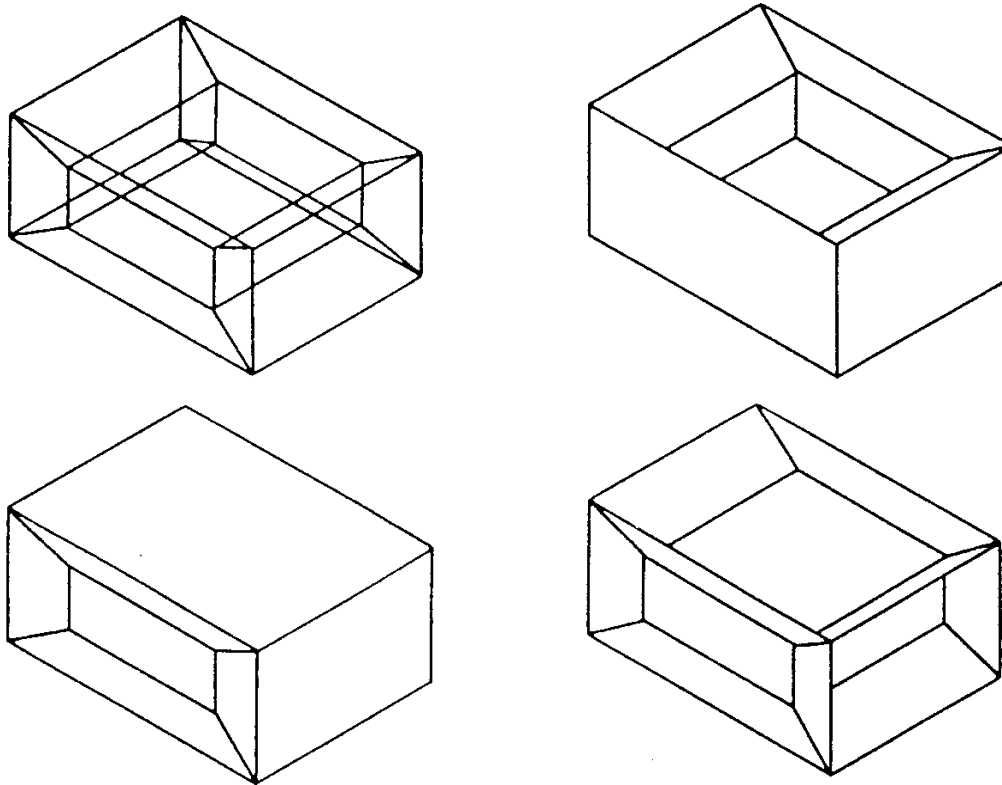
Wireframe modelling (2)

	Parametric representation	Implicit representation
Straight line	$\begin{aligned}x &= t + 1 \\ y &= 2t + 1\end{aligned}$	$y = 2x - 1$
Circle	$\begin{aligned}x &= r\cos\theta + a \\ y &= r\sin\theta + b\end{aligned}$	$(x - a)^2 + (y - b)^2 = r^2$

Wireframe modelling (3)

- Combined use of curves can represent 3D objects in the space.
- Its disadvantages are the ambiguity of the model and the severe difficulty in validating the model.
- Furthermore, it does not provide surface and volume-related information.

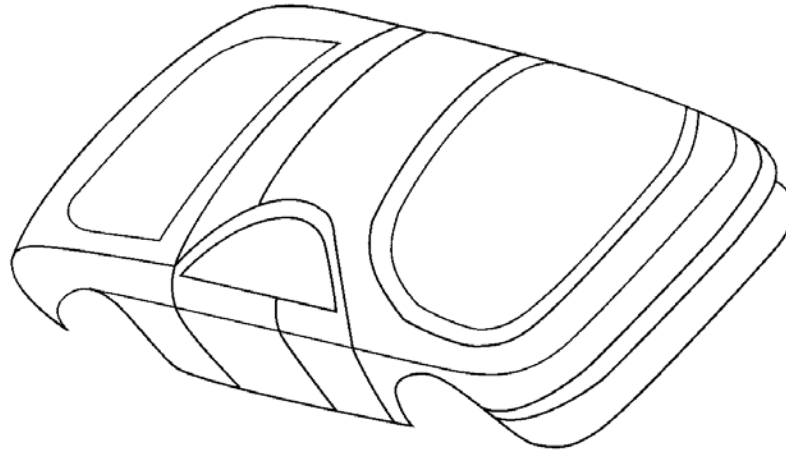
Wireframe modelling (4)



Ambiguous wireframe models

Surface modelling (1)

- It generates objects with a more complete and less ambiguous representation than its wireframe model.
- It is obvious that surface models are suitable for more applications, for example, design and representation of car bodies.



Surface modelling (2)

- Surfaces can be 2D and 3D represented by a closed loop of curves with skin on it, the simplest form being a plane. Surfaces are built from points and curves.
- They are very important in modelling objects, and in many situations, used to represent 3D models to a large variety of satisfaction.
- Modelling packages usually provide a range of useful surface creation functions, some of which are similar to those for curves (but the geometric characteristics are different).

Surface modelling (3)

- Despite their similar appearance, there are differences between surface and solid models.
- Apart from the lack of volume-related information, surface models normally define the geometry of their corresponding objects.



Surface model - hollow model:
volume, mass, centroid?

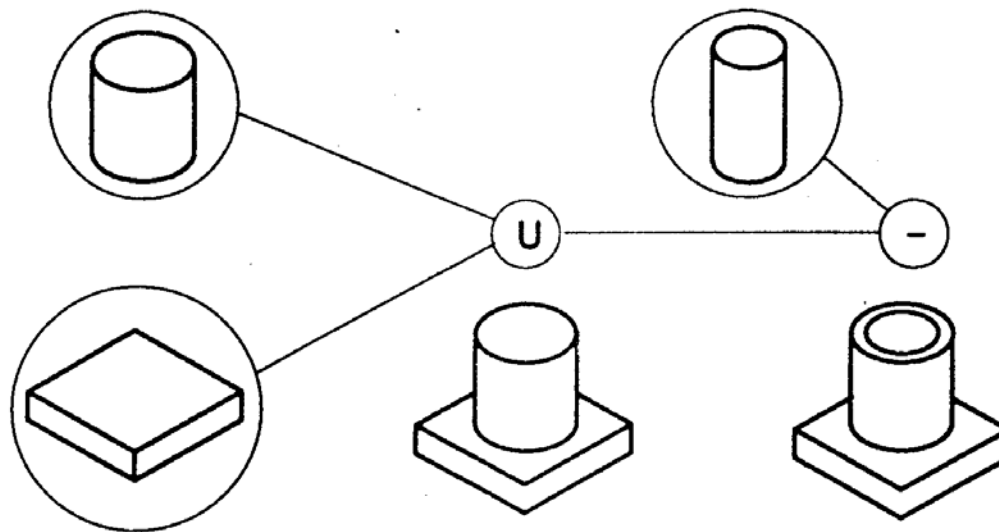
Solid modelling

- Solid modelling represents both the geometric properties (e.g. points, curves, surfaces, volume, centre of shape) and physical properties (e.g. mass, centre of gravity and inertia) of solid objects.
- There is a number of schemes, namely primitive instancing, cell decomposition, constructive solid geometry (CSG) and boundary representation (B-Rep).
- CSG and B-Rep are the most popular.

Constructive solid geometry (1)

- The CSG model is an ordered binary tree where the non-terminal nodes represent the operators and the terminal nodes are the primitives or transformation leaves.
- The operators may be rigid motions or regular Boolean operations.
- The primitive leaf is a primitive solid in the modelling space while the transformation leaf defines the arguments of rigid motion.

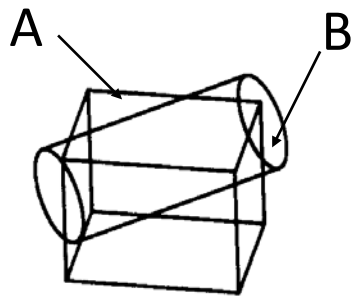
Constructive solid geometry (2)



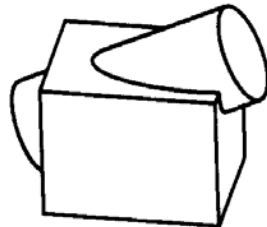
CSG tree

Constructive solid geometry (3)

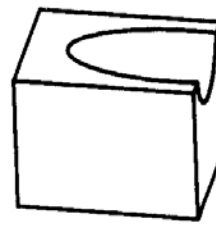
- Boolean operations include Boolean Union, Boolean Difference and Boolean Intersection.
- It should be noticed that the resultant solid of a Boolean operation depends not only on the solids but also on their location and orientation.



Objects



Union
 $A \cup B$



Subtraction
 $A - B$



Intersection
 $A \cap B$

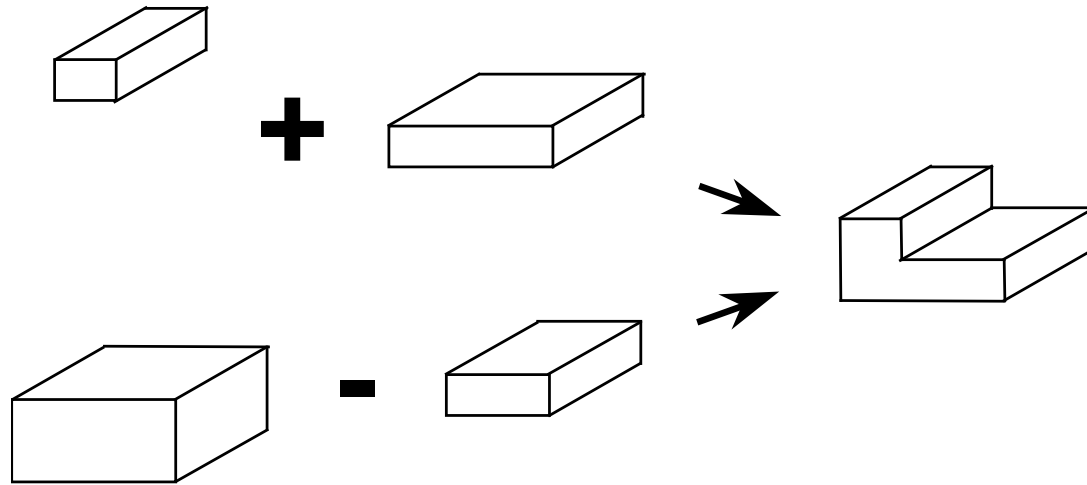
Constructive solid geometry (4)

- Each solid usually has its local co-ordinate frame specified relative to the world co-ordinate frame.
- Before a Boolean operation is performed, it may be necessary to translate and/or rotate the solids in order to obtain the required relative location and orientation relationship between them.

Constructive solid geometry (5)

- If an object can be represented by a unique set of data, the representation is said to be unique.
- The representation scheme for some applications (e.g. geometric reasoning) should ideally be both unambiguous and unique.
- Solid representations are usually unambiguous but few of them can be unique, and it is not feasible to make CSG representation unique.

Constructive solid geometry (6)

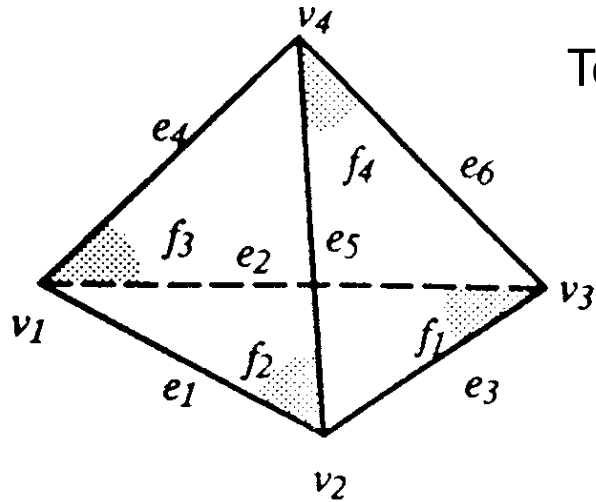


Nonuniqueness of CSG model

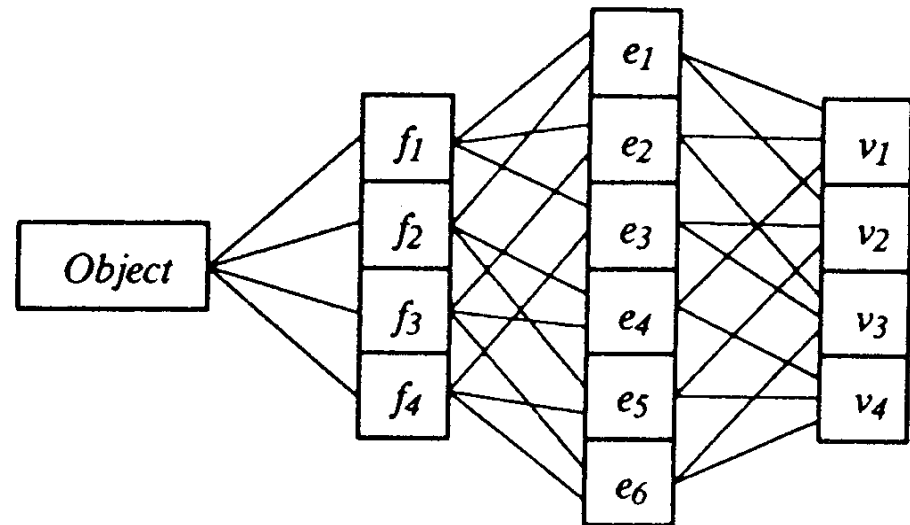
Boundary representation (1)

- The boundary representation (B-Rep) model represents a solid by segmenting its boundary into a finite number of bounded subsets.
- It is basically a topologically explicit representation. Both geometric and topological information is stored in the data structure.
- The **geometry** is about the shape and size of the boundary entities called *points*, *curves* and *surfaces* while the topology keeps the **connectivity** of the boundary entities referred as *vertices*, *edges* and *faces* (corresponding to points, curves and surfaces).

Boundary representation (2)



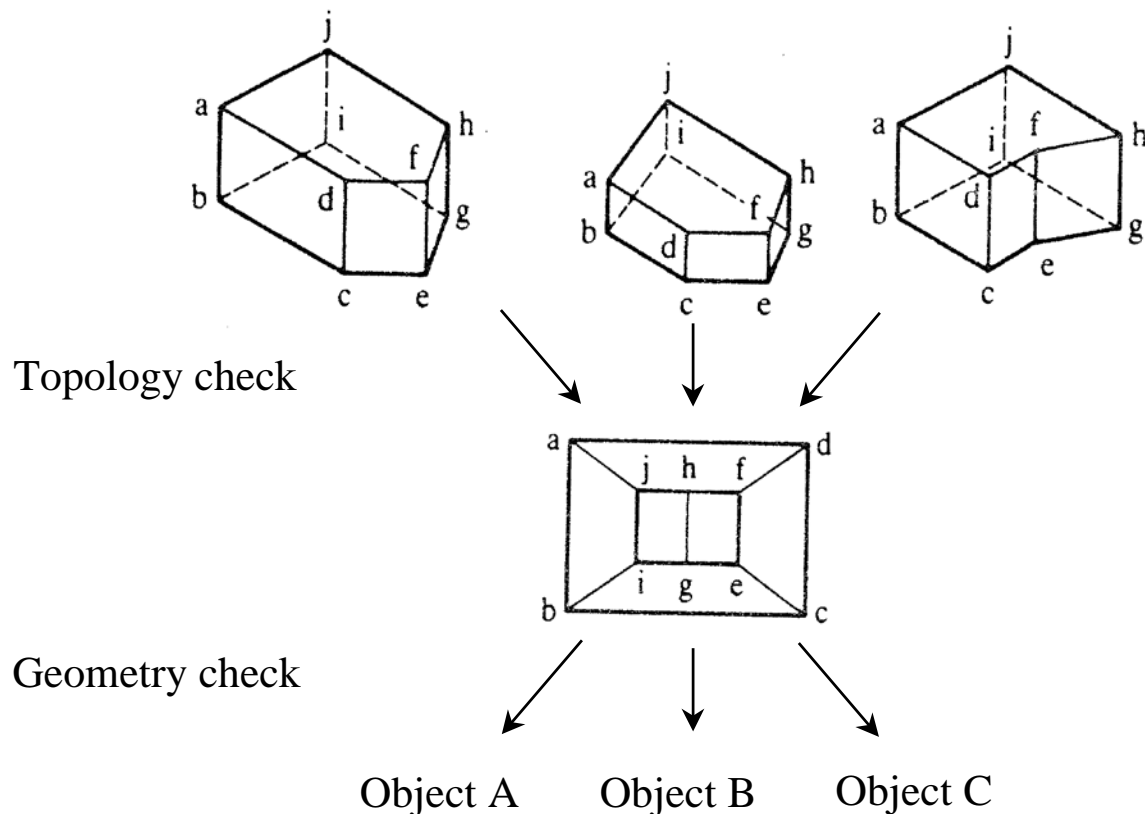
Tetrahedron



Topology of tetrahedron

Boundary representation (3)

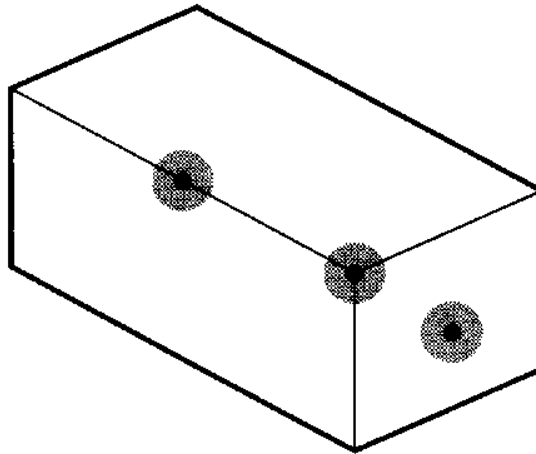
The same topology may represent different geometric shapes and therefore both topological and geometric data is necessary to fully and uniquely define an object.



Types of B-Rep

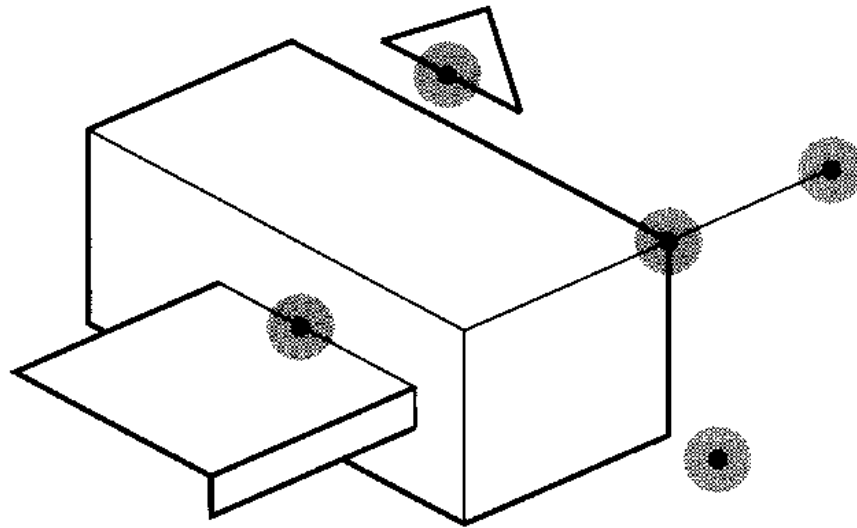
- B-Rep models can be divided into two types: *manifold* and *nonmanifold*.
- In a manifold model, an edge is connected exactly by two faces and at least three edges meet at a vertex.
- A nonmanifold model may have dangling faces, edges and vertices, and therefore represent a nonrealistic object.

Manifold B-Rep model



Two faces meet exactly at one edge,
and at least three edges meet at a vertex

Nonmanifold B-Rep model



Dangling faces, edge and vertices

Euler's law for manifold B-Rep (1)

To ensure the topological validity for a solid (i.e. manifold model), a manifold model must satisfy the following Euler (Leonhard Euler, 1707-1783) formula,

$$V - E + F - R + 2H = 2S$$

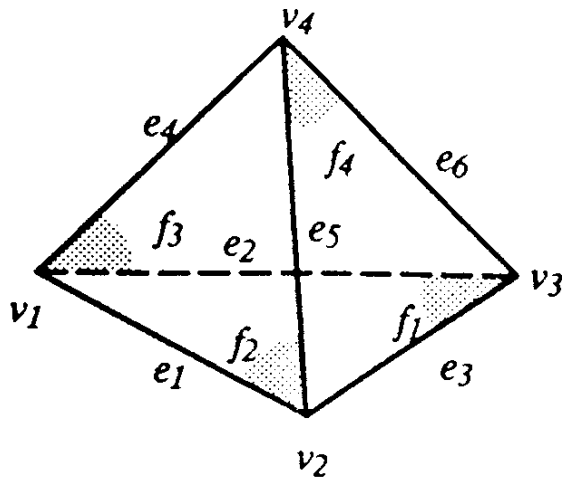
where V , E , F , R , H and S are the number of vertices, edges, faces, rings (inner loops on faces), passages/holes (genus) and shells (disjoint bodies), respectively.

Euler's law for manifold B-Rep (2)

The Euler's law in its simplest form is

$$V - E + F = 2$$

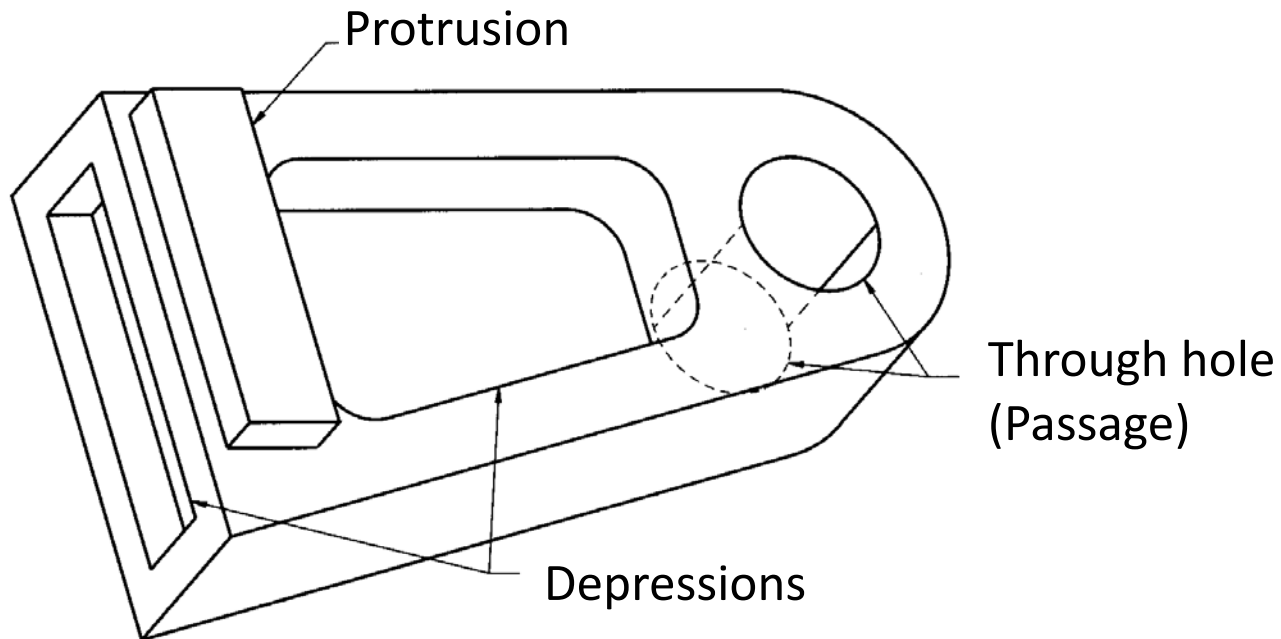
which can be applied to simple polyhedra (i.e. objects without inner loops of edges and passages).



For the tetrahedron

$$4 - 6 + 4 = 2$$

Euler's law for manifold B-Rep (3)



Euler's law for manifold B-Rep (4)

	V	E	F	R	H	S
Basic shape	8	12	6			1
Protrusion	8	12	5	1		
Sharp corner depression	8	12	5	1		
Round corner depression	16	24	9	1		
Hole	4	6	2	2	1	
Total	44	66	27	5	1	1

$$V - E + F - R + 2H = 2S$$

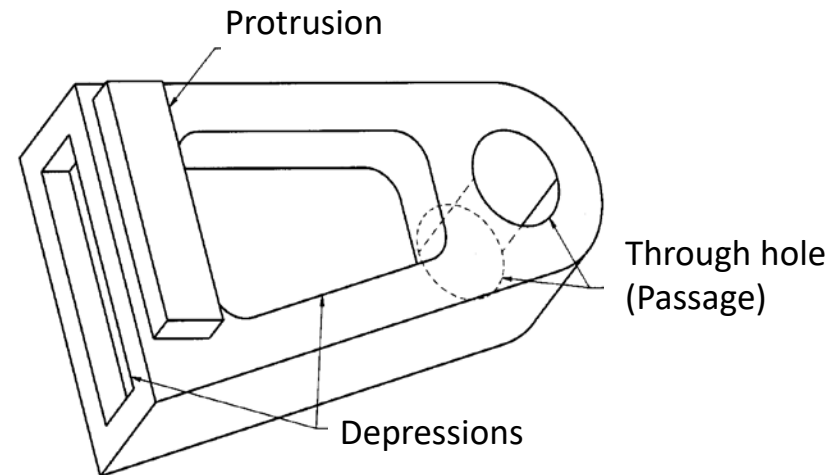
$$44 - 66 + 27 - 5 + 2 \times 1 = 2 \times 1$$

Euler's law for manifold B-Rep (4)

	V	E	F	R	H	S
Basic shape	8	12	6			1
Protrusion	8	12	5	1		
Sharp corner depression	8	12	5	1		
Round corner depression	16	24	9	1		
Hole	4	6	2	2	1	
Total	44	66	27	5	1	1

$$V - E + F - R + 2H = 2S$$

$$44 - 66 + 27 - 5 + 2 \times 1 = 2 \times 1$$



Euler's law for manifold B-Rep (5)

- The cylindrical hole in the previous slide is represented by two edges along its axis, resulting in 4 vertices, 6 edges and 2 faces.
- Hence, a cylinder body can be represented by 4 vertices, 6 edges and 4 faces, satisfying the Euler's law.
- It is worth noting that the Euler's law also applies to curved objects represented by patches, curve segments and vertices.

Implementation of B-Rep (1)

- B-Rep models can be conveniently implemented on computers by representing the topology as pointers and the geometry as numerical information in the data structure for extraction and manipulation using object-oriented programming techniques (e.g. C++).
- The latest B-Rep modellers also provide facilities to tag attributes (such as colour, tolerance and surface finish) on the boundary elements.

Implementation of B-Rep (2)

Faces

Face	Edge
1	1
	2
	3
	4
2	.
	.

Edges

Edge	Vertex
1	1
	2
2	5
3	6
	.
	.
	.

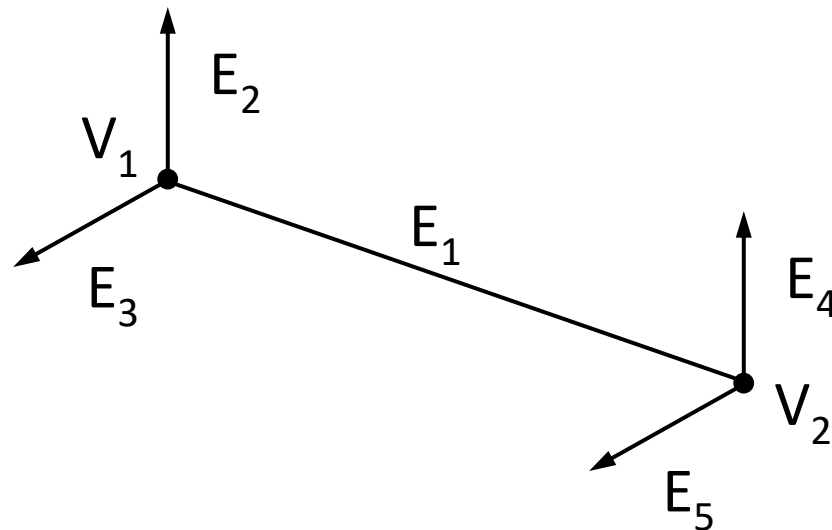
Vertices

Vertex	X	Y	Z
1	2	1	3
2	4	2	5
3	2	3	6
4	.	4	.
5	.	.	.
.	.	.	.
.	.	.	.

The three table-structure

Implementation of B-Rep (3)

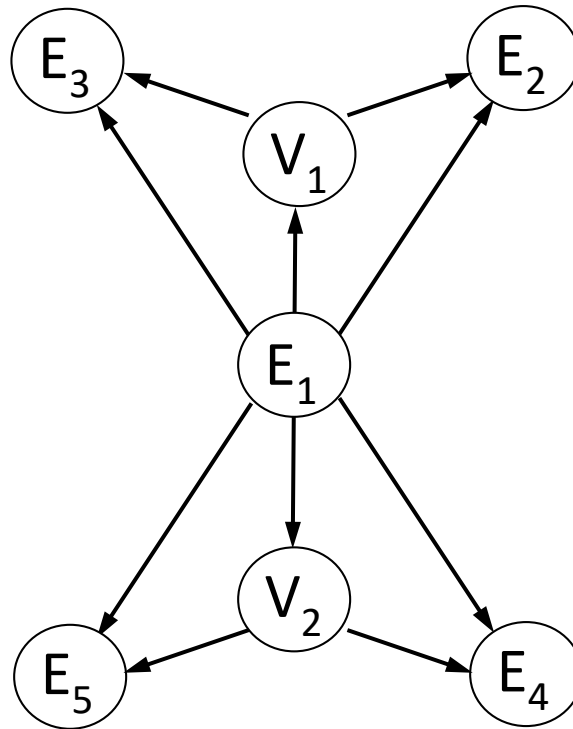
- The figure shows a single edge, ending in two vertices, which then each has two other edges leading off from them.
- The edge might, for example, be an edge of a cuboid. This is how the edge might be represented in the computer.



Implementation of B-Rep (4)

- The edge has *pointers* to the vertices at its ends, and to the next edges. A pointer is essentially the address in the computer's memory where something is stored.
- The vertices have pointers to their coordinates (x, y, z) and so on.
- This structure is called *Baugmart's winged edge* data structure named after its inventor. The 'winged edge' phrase refers to the edge and its adjoining faces, which look like a dihedral wing.

Implementation of B-Rep (5)

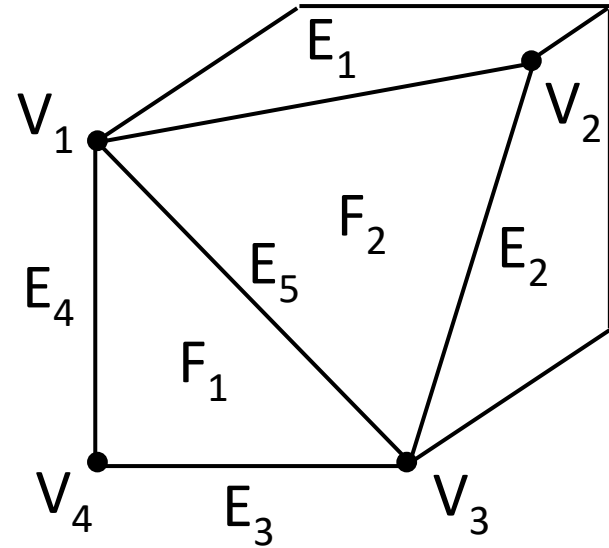
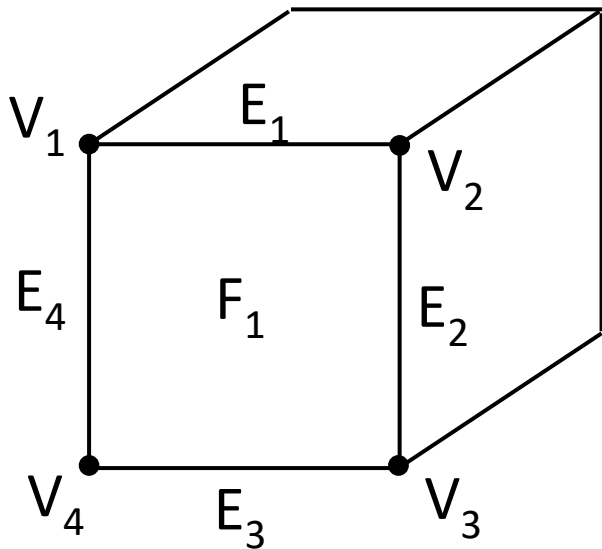


Baugmart's winged edge data structure

Implementation of B-Rep (6)

- Many B-Rep modelling systems have procedures called Euler-operators.
- These modify the face-edge-vertex pointer structure in such a way that the Euler formula is always kept true.
- An example is: *make_edge_and_face(f1, v1, v3)*.
 - This would take the cuboid on the left, and split the face into two along its diagonal, making a new object.
 - Note that in the original cuboid, v1, v2, v3 and v4 must all lie in the same plane, but in the new object v2 is free to move along the top-right edge.

Implementation of B-Rep (7)



ACIS B-Rep Geometric Modeller

- **Romulus** (modeling kernel), known as Romulus (b-rep solid modeler), Romulus (disambiguation), Romulus b-rep

The Romulus b-rep solid modeler (or simply Romulus) was released in 1978 by Ian Braid, Charles Lang, Alan Grayer, and the Shape Data team in Cambridge, England. It was the first commercial solid modeling kernel designed for straightforward integration into CAD software.

[https://www.semanticscholar.org/topic/Romulus-\(modelling-kernel\)/589274](https://www.semanticscholar.org/topic/Romulus-(modelling-kernel)/589274)

- **ACIS**, known as .sab, **A**lan, **C**harles and **I**an's **S**ystem (disambiguation)
The 3D ACIS Modeler (ACIS) is a geometric modeling kernel developed by Spatial Corporation (formerly Spatial Technology), part of Dassault Systemes. ACIS is used by many software developers in industries such as computer-aided design (CAD), computer-aided manufacturing (CAM), computer-aided engineering (CAE), architecture, engineering and construction (AEC), coordinate-measuring machine (CMM), 3D animation and shipbuilding. ACIS provides software developers and manufacturers with the underlying 3D modeling functionality.

<https://www.semanticscholar.org/topic/3D-ACIS-Modeler/3712130>

Summary

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- Surface
- Solid

➤ Constructive solid geometry (CSG)

- CSG tree
- Unambiguous but not unique

➤ Boundary representation (B-Rep)

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