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CPT205 Computer Graphics

Transformation Pipeline and Geometric Transformations

Lecture 04
2022-23

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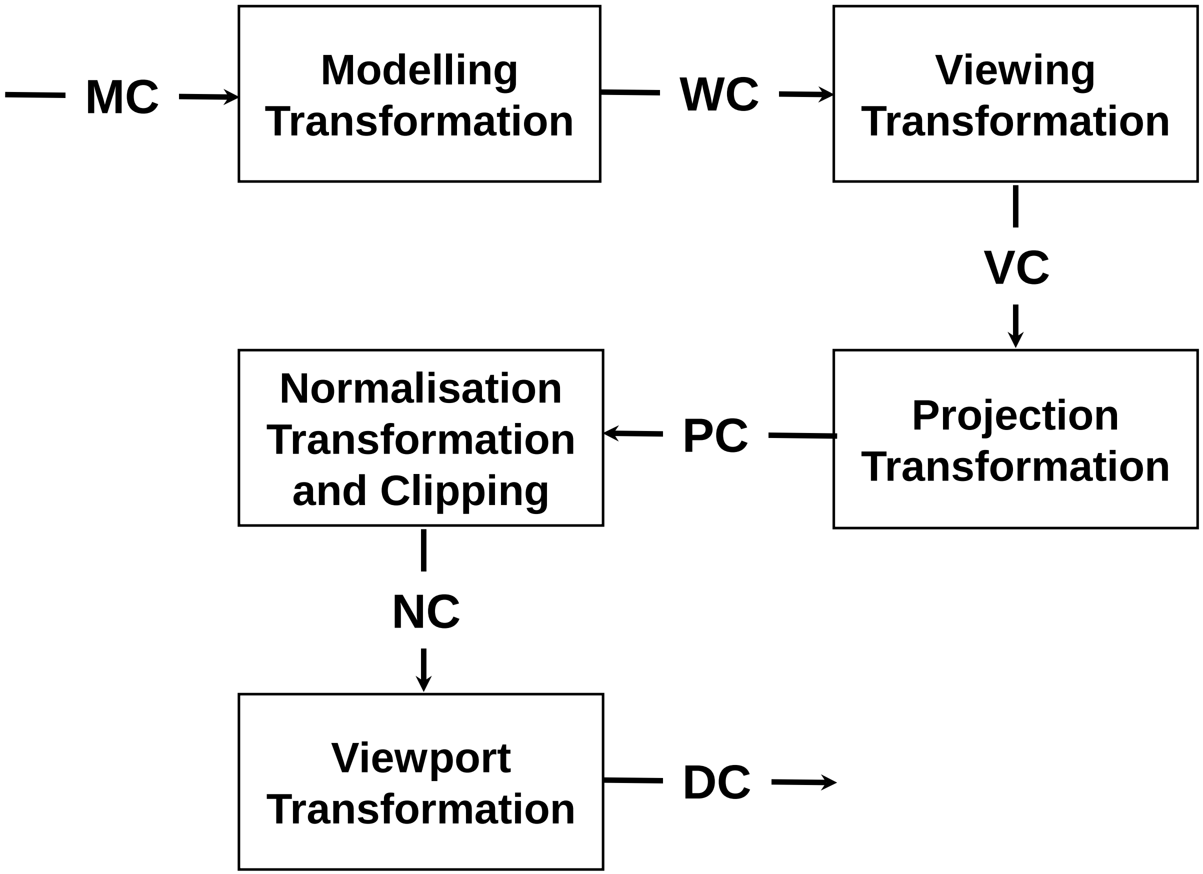
Topics for today

- Transformation pipeline
- Standard transformations
  - Translation
  - Rotation
  - Scaling
  - Reflection
  - Shearing
- Homogeneous co-ordinate transformation matrices
- Composite (arbitrary) transformation matrices from simple transformations
- OpenGL functions for transformations

Transformation pipeline (1)

- The Transformation Pipeline is the series of transformations (alterations) that must be applied to an object before it can be properly displayed on the screen.
- The transformations can be thought of as a set of processing stages. If a stage is omitted, very often the object will not look correct. For example if the *projection* stage is skipped then the object will not appear to have any depth to it.
- Once an object has passed through the pipeline it is ready to be displayed as either a wire-frame item or as a solid item.

Transformation pipeline (2)



Transformation pipeline (3)

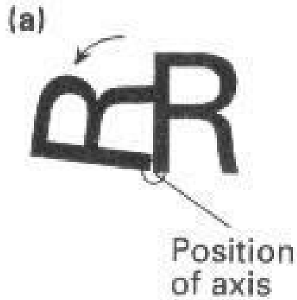
- Modelling Transformation - to place an object into the Virtual World.
- Viewing Transformation - to view the object from a different vantage point in the virtual world.
- Projection Transformation - to see depth in the object.
- Viewport Transformation - to temporarily map the volume defined by the "window of interest" plus the front and rear clipping planes into a unit cube. When this is the case, certain other operations are easier to perform.
- Device Transformation - to map the user defined "window of interest" (in the virtual world) to the dimensions of the display area.

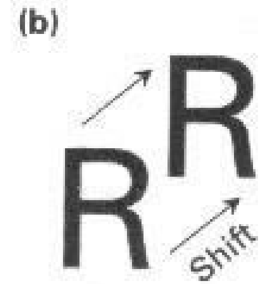
Transformation pipeline (4)

- We start when the object is loaded from a file and is ready to be processed.
- We finish when the object is ready to be displayed on the computer screen.
- You should be able to draw a picture of a simple object, say a cuboid, and show visually what happens to it as it passes through each pipeline stage.

Types of geometric transformation

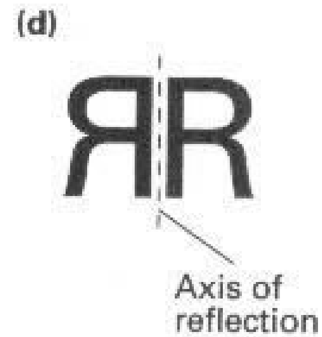
Rotation



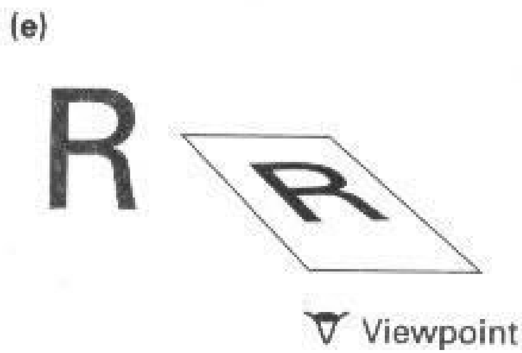


Translation

Scaling



Reflection

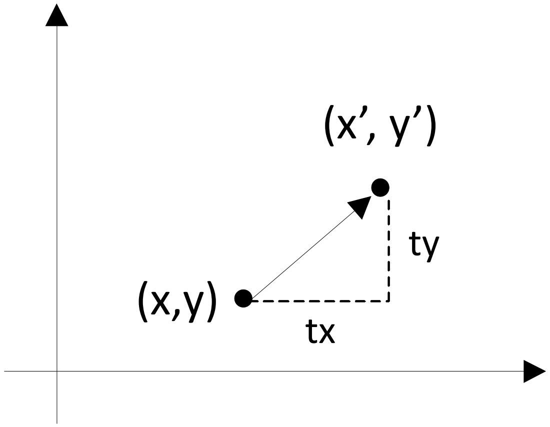


Shearing

2D translation (1)

- Translating a point from P(x, y) to P'(x', y') along vector T
- Importance in computer graphics – we need to only transform the two endpoints of a line segment and let the implementation draw the line segment between the transformed endpoints

$$\begin{cases} x' = x + t_x \\ y' = y + t_y \end{cases}$$



2D translation (2)

P = $\begin{bmatrix} x \\ y \end{bmatrix}$, **P'** = $\begin{bmatrix} x' \\ y' \end{bmatrix}$, **T** = $\begin{bmatrix} t_x \\ t_y \end{bmatrix}$

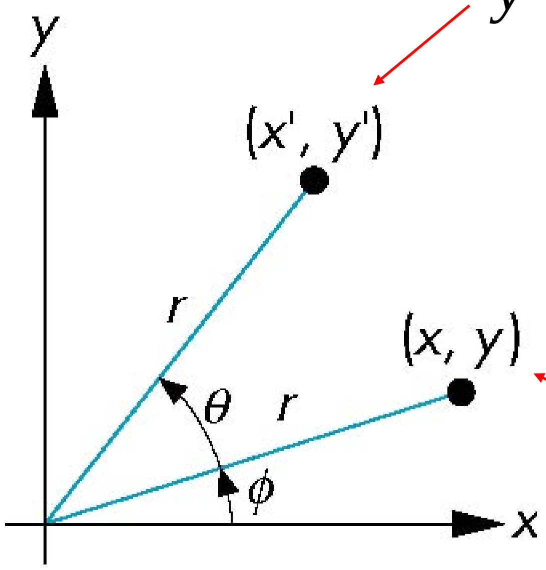
where **P**(x, y) and **P'**(x', y') are the original and new positions, and **T** is the distance translated.

P' = P+T or $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

2D rotation (1)

Rotating a point from P(*x*, *y*) to P'(*x'*, *y'*) about the origin by angle θ - radius stays the same, and angle increases by θ.

$$x' = r\cos(\phi + \theta) = x\cos\theta - y\sin\theta$$
$$y' = r\sin(\phi + \theta) = x\sin\theta + y\cos\theta$$



$$x = r\cos\phi$$
$$y = r\sin\phi$$

2D rotation (2)

$$\begin{cases} x' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{cases}$$

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$$

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \end{cases}$$

2D rotation (3)

P' = R · P

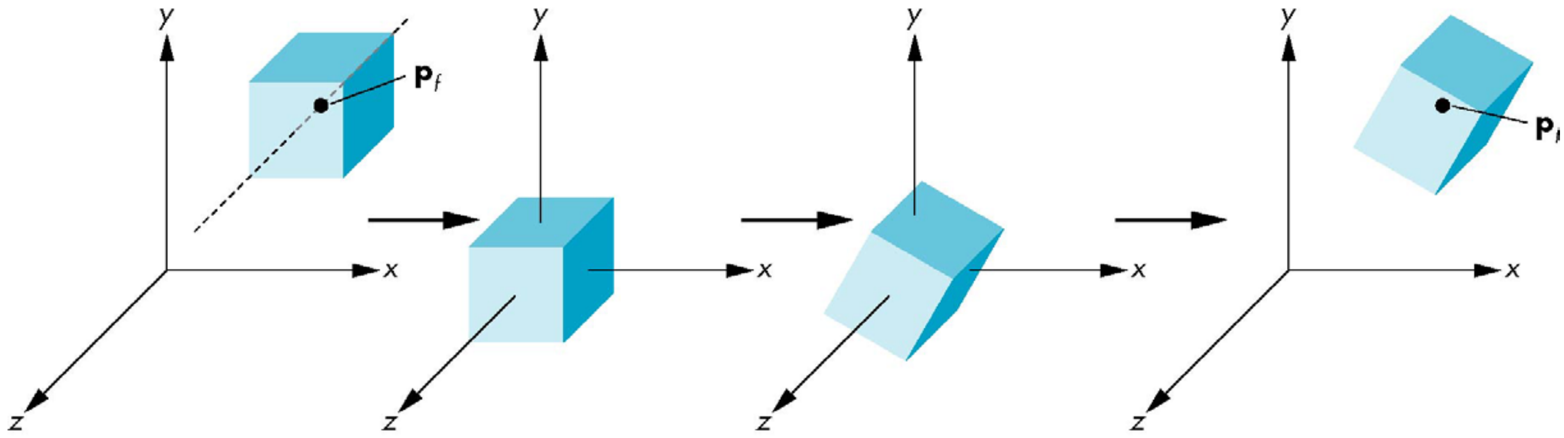
$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \qquad \text{so} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix}$$

where θ is the rotation angle and ϕ is the angle between the *x*-axis and the line from the origin to (x, y).

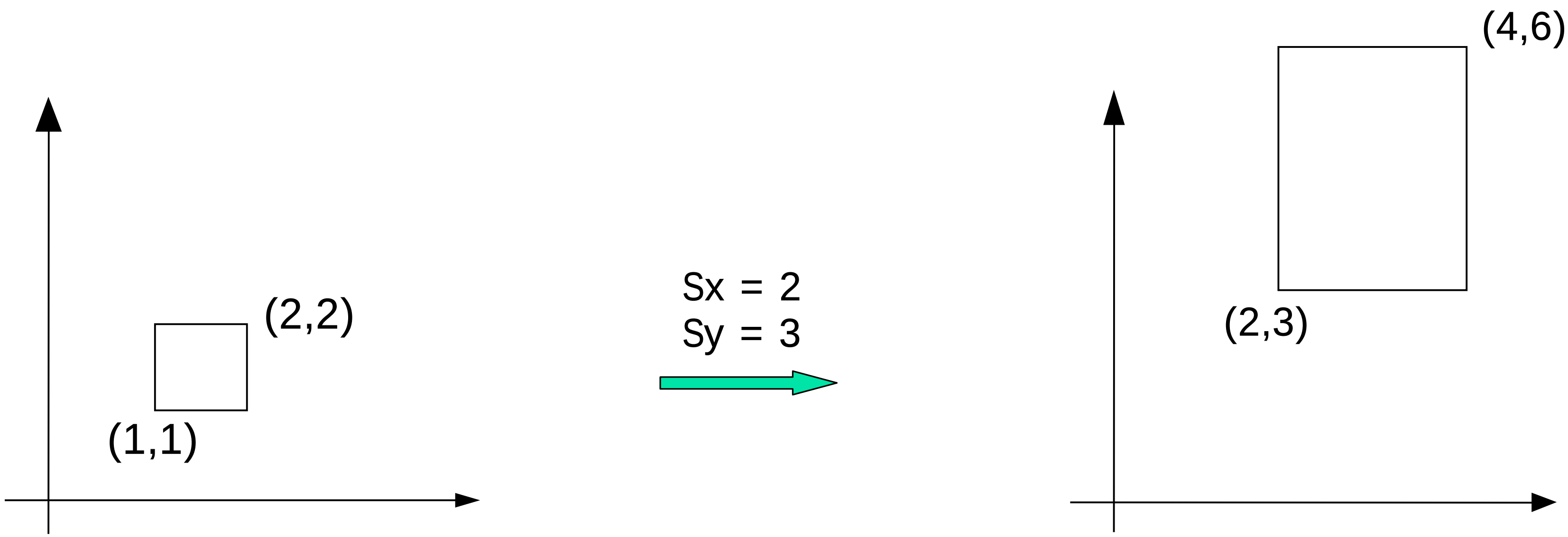
Notice that the rotation point (or pivot point) is <u>the co-ordinate origin</u>.

Rotation about a fixed point rather than the origin

- Move the fixed point to the origin
- Rotate the object
- Move the fixed point back to its initial position
- $\mathbf{M} = \mathbf{T}(p_f)\,\mathbf{R}(\theta)\,\mathbf{T}(-p_f)$



2D scaling (1)



When an object is scaled, <u>both the size and location</u> change.

2D scaling (2)

$$\begin{cases} x' = x \cdot s_x \\ y' = y \cdot s_y \end{cases}$$

P’ = **S** · **P** so $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$

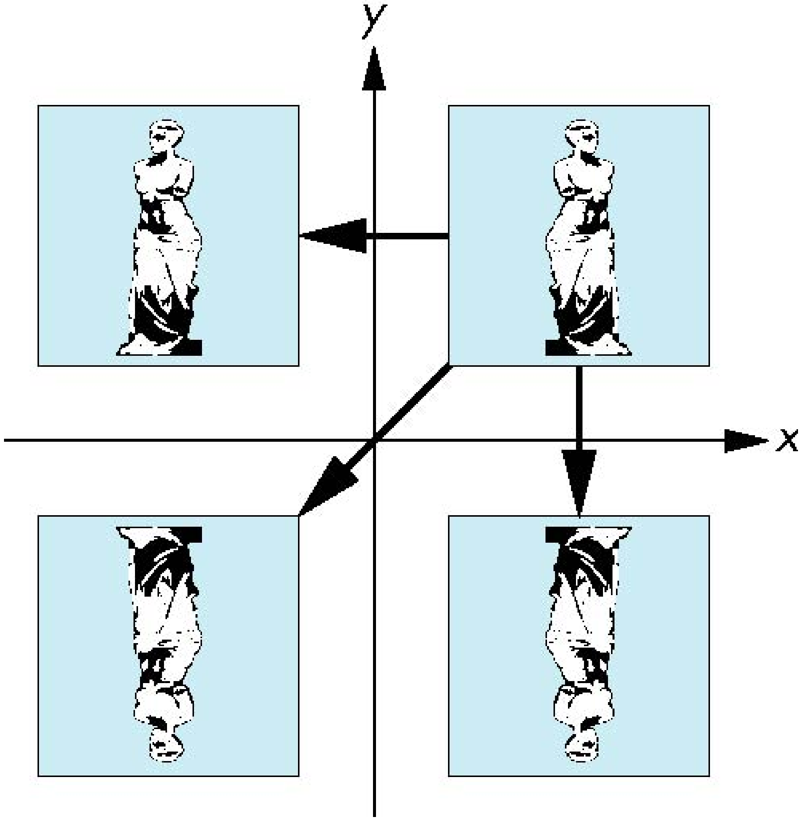
where **P**, and **P’** are the original and new positions, and s_x and s_y are the scaling factors along the x- and y-axes.

2D reflection

Special case of scaling - corresponding to negative scale factors.

$s_x = -1$
$s_y = 1$

original

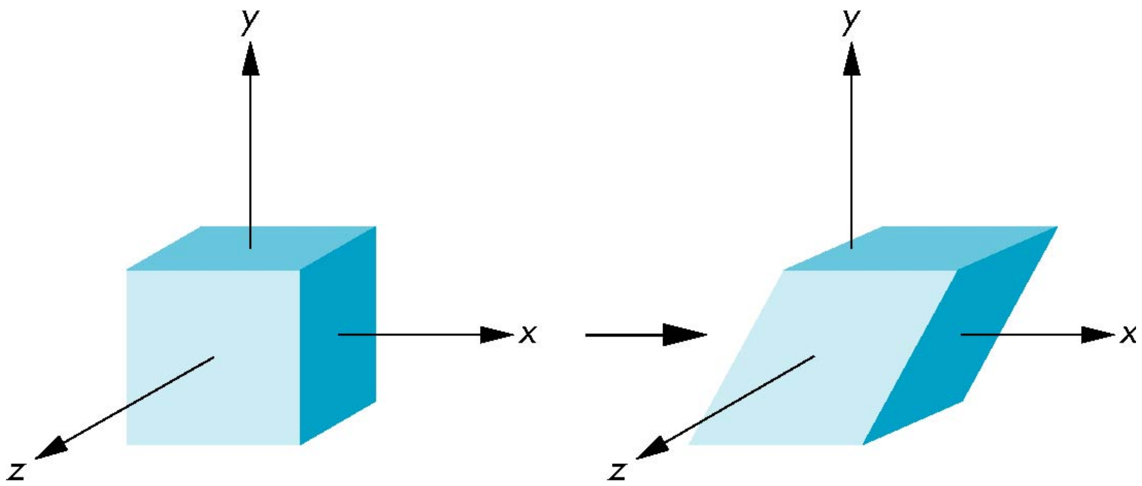


$s_x = -1$
$s_y = -1$

$s_x = 1$
$s_y = -1$

2D shearing (1)

Equivalent to pulling faces in opposite directions.

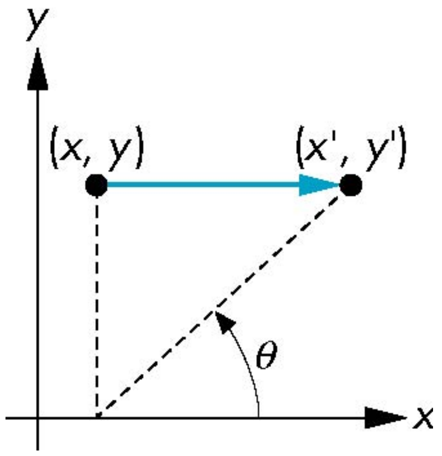


2D shearing (2)

Consider simple shearing along the *x* axis.

$$x' = x + y\cot\theta$$
$$y' = y$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \cot\theta \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$



2D homogeneous co-ordinates

- By expanding 2x2 matrices to 3x3 matrices, homogeneous co-ordinates are used.

- A homogeneous parameter is applied so that each 2D position is represented with homogeneous co-ordinates ($h \cdot x$, $h \cdot y$, h).

- The homogeneous parameter has a non-zero value, and can be set to 1 for convenient use.

2D homogeneous matrices (1)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(2D translation)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(2D rotation)

2D homogeneous matrices (2)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(2D scaling)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \cot\theta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(2D shearing)

2D composite transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} rs_{xx} & rs_{xy} & trs_x \\ rs_{yx} & rs_{yy} & trs_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where elements *rs* are the multiplicative rotation-scaling terms in the transformation (which involve only rotation angles and scaling factors);

elements *trs* are the translation terms, containing combination of translation distances, pivot-point and fixed-point co-ordinates, rotation angles and scaling parameters.

3D translation

3D translations and scaling can be simply extended from the corresponding 2D methods.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D co-ordinate axis rotations (1)

- The extension from 2D rotation methods to 3D rotation is less straightforward (because this is about an arbitrary axis instead of an arbitrary point).
- Equivalent to rotation in two dimensions in planes of constant z (i.e. about the origin).

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(About z-axis)

3D co-ordinate axis rotations (2)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(About y-axis)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(About x-axis)

General rotation about the origin

A rotation by q about an arbitrary axis can be decomposed into the concatenation of rotations about the x, y, and z axes.

$$\mathbf{R}(q) = \mathbf{R}_z(q_z)\, \mathbf{R}_y(q_y)\, \mathbf{R}_x(q_x)$$

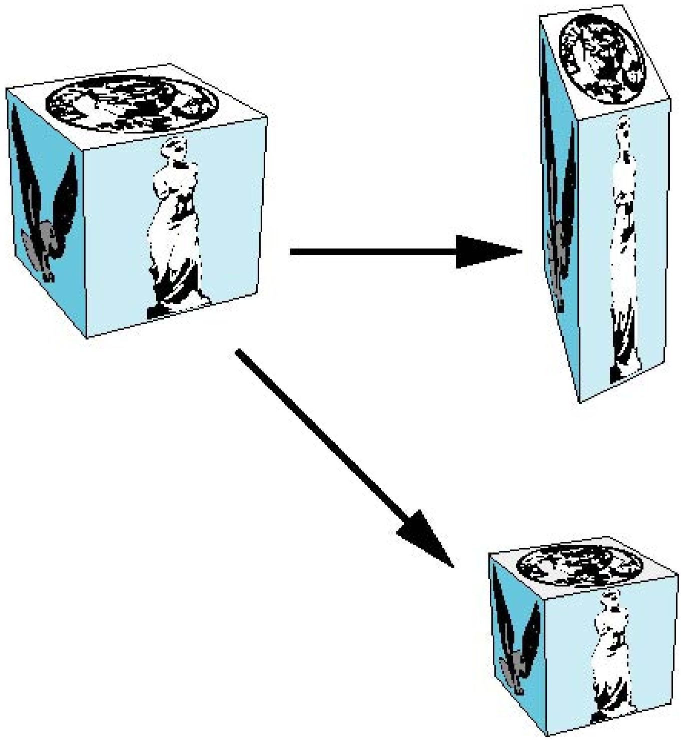
where q_x , q_y and q_z are called the Euler angles.

Note that rotations do not commute though we can use rotations in another order but with different angles.

3D scaling

$x' = s_x x$
$y' = s_y y$
$z' = s_z z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D composite transformation (1)

- As with 2D transformation, a composite 3D transformation can be formed by multiplying the matrix representations for the individual operations in the transformation sequence.
- There are other forms of transformation, namely reflection and shearing which can be implemented with the other three transformations.
- Translation, scaling, rotation, reflection and shearing are all <u>affine</u> transformations in that transformed point P'(x', y', z') is a linear combination of the original point P(x, y, z).

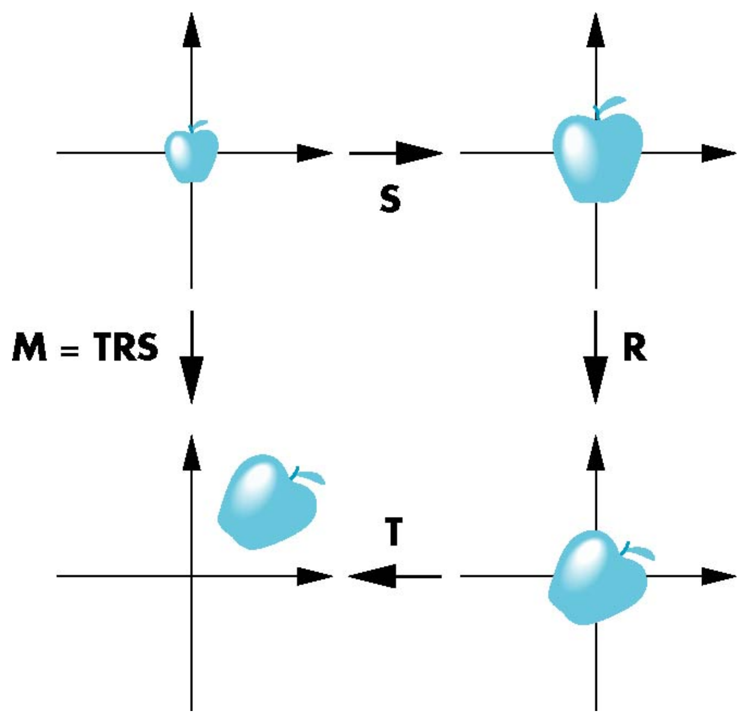
3D composite transformation (2)

- Matrix multiplication is associative
- $M3 \cdot M2 \cdot M1 = (M3 \cdot M2) \cdot M1 = M3 \cdot (M2 \cdot M1)$
- Transformation products are
  not always commutative

  $A \cdot B \neq B \cdot A$

Instancing

- ➢ In modelling, we often start with a simple object centred at the origin, oriented with an axis, and of a standard size.
- ➢ We apply an *instance transformation* to its vertices to
  - Scale
  - Orient
  - Locate



OpenGL matrices

- In OpenGL matrices are part of the state.
- Multiple types
  - Model-View (**`GL_MODELVIEW`**)
  - Projection (**`GL_PROJECTION`**)
  - Texture (**`GL_TEXTURE`**)
  - Color (**`GL_COLOR`**)
- <u>Single</u> set of functions for manipulation
- Select which to be manipulated by
  - **`glMatrixMode(GL_MODELVIEW);`**
  - **`glMatrixMode(GL_PROJECTION);`**

Current transformation matrix

- Conceptually there is a 4x4 homogeneous coordinate matrix, the *current transformation matrix* (CTM) that is part of the state and is applied to all vertices that pass down the pipeline.
- The CTM is defined in the user program and loaded into a transformation unit.
- The CTM can be altered either by loading a new CTM or by postmutiplication.
- OpenGL has a model-view and a projection matrix in the pipeline which are concatenated together to form the CTM.
- The CTM can manipulate each by first setting the correct matrix mode.

CTM operations

The CTM can be altered either by loading a new CTM or by postmutiplication

Load an identity matrix: $\mathbf{C} \leftarrow \mathbf{I}$
Load an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{M}$

Load a translation matrix: $\mathbf{C} \leftarrow \mathbf{T}$
Load a rotation matrix: $\mathbf{C} \leftarrow \mathbf{R}$
Load a scaling matrix: $\mathbf{C} \leftarrow \mathbf{S}$

Postmultiply by an arbitrary matrix: $\mathbf{C} \leftarrow \mathbf{CM}$
Postmultiply by a translation matrix: $\mathbf{C} \leftarrow \mathbf{CT}$
Postmultiply by a rotation matrix: $\mathbf{C} \leftarrow \mathbf{CR}$
Postmultiply by a scaling matrix: $\mathbf{C} \leftarrow \mathbf{CS}$

Arbitrary Matrices

- We can load and multiply by matrices defined in the application program
  - **`glLoadMatrixf(m)`**
  - **`glMultMatrixf(m)`**
- Matrix **`m`** is a one-dimension array of 16 elements which are the components of the desired 4 x 4 matrix stored by <u>columns.</u>
- In **`glMultMatrixf`**, **`m`** multiplies the existing matrix on the right.

Matrix stacks

- CTM is not just one matrix but a matrix stack with the "current" at top.
- In many situations we want to save transformation matrices for use later
  - Traversing hierarchical data structures
  - Avoiding state changes when executing display lists
- Pre 3.1 OpenGL maintains stacks for each type of matrix
  - Access present type (as set by **`glMatrixMode)`** by

    ```
    glPushMatrix()
    glPopMatrix()
    ```
- Right now just 1-level CTM.

Matrix stacks

- We can also access matrices (and other parts of the state) with *query* functions
  - **`glGetIntegerv`**
  - **`glGetFloatv`**
  - **`glGetBooleanv`**
  - **`glGetDoublev`**
  - **`glIsEnabled`**

- For matrices, we use as
  - **`double m[16];`**
  - **`glGetFloatv(GL_MODELVIEW, m);`**

OpenGL transformation functions (1)

- **`glTranslate*`**
  Specify translation parameters
- **`glRotate*`**
  Specify rotation parameters for rotation about any axis through the origin
- **`glScale*`**
  Specify scaling parameters with respect to the co-ordinate origin
- **`glMatrixMode`**
  Specify current matrix for geometric-viewing, projection, texture or colour transformations
- **`glLoadIdentity`**
  Set current matrix to identity
- **`glLoadMatrix*(elems)`**
  Set elements of current matrix

OpenGL transformation functions (2)

- **`glMultMatrix*(elems)`**
  Post-multiply the current matrix by the specified matrix
- **`glGetIntegerv`**
  Get max stack depth or current number of matrices in the stack for selected matrix mode
- **`glPushMatrix`**
  Copy the top matrix in the stack and store copy in the second stack position
- **`glPopMatrix`**
  Erase top matrix in stack and move second matrix to top stack
- **`glPixelZoom`**
  Specify 2D scaling parameters for raster operations

Example of rotation

- Rotation about the z axis by 30 degrees with a fixed point of (1.0, 2.0, 3.0)

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
glTranslatef(1.0, 2.0, 3.0);
glRotatef(30.0, 0.0, 0.0, 1.0);
glTranslatef(-1.0, -2.0, -3.0);
```

- Note that the last matrix specified in the program is the first applied.

Summary

- Transformation pipeline
- Standard transformations
  - Rotation
  - Translation
  - Scaling
  - Reflection
  - Shearing
- Homogeneous co-ordinate transformation matrices
- Composite (arbitrary) transformation matrices from simple transformations
- OpenGL functions for transformations