INT201 Decision, Computation and Language

Tutorial 9

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1. Prove that CFGs are decidable.

2. Prove that CFLs are decidable.

3. Prove that the language L_{TM} is undecidable.

Solution

1.

Proof

On input $\langle G, w \rangle \in \Omega$, where G is a CFG and w is a string,

0. Check if $\langle G, w \rangle$ is proper encoding of CFG and string; if not, *reject*.
1. Convert G into equivalent CFG G' in Chomsky normal form.
2. If $w = \varepsilon$, check if $S \to \varepsilon$ is a rule of G'.
   If so, *accept*; otherwise, *reject*.
3. If $w \neq \varepsilon$, list all derivations with $2n - 1$ steps, where $n = |w|$.
4. If any generates w, *accept*;
   otherwise, *reject*.

Solution

2.

Proof

- Let L be a CFL
  - G' be a CFG for language L
  - S be a TM from Theorem 4.7 that decides

$$A_{\text{CFG}} = \{ \langle G, w \rangle \mid G \text{ is a CFG that generates string } w \}$$

- Construct TM $M_{G'}$ for language L having CFG G' as follows:

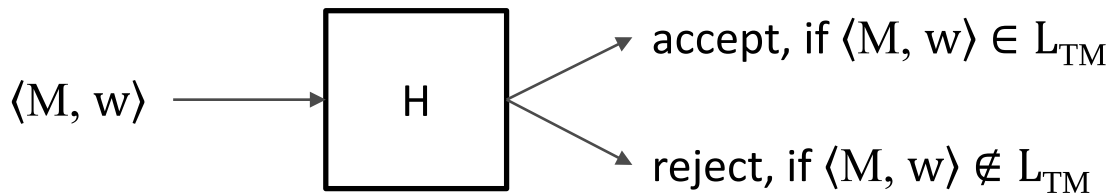
  $M_{G'} =$ "On input w:

  1. Run TM decider S on input $\langle G', w \rangle$.
  2. If S accepts, *accept*;
     otherwise, *reject*."

3.

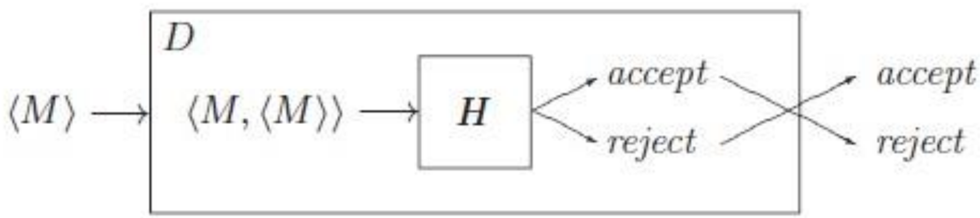
Proof

Suppose L_{TM} is decided by a TM H, with input $\langle M, w\rangle \in \Omega$



- If $\langle M, w\rangle \in L_{TM}$, then H terminates in its accept state.
- If $\langle M, w\rangle \notin L_{TM}$, then H terminates in its reject state.

Use H as a subroutine to construct a new TM D



- If H terminates in its accept state, then D terminates in its reject state.
- If H terminates in its reject state, then D terminates in its accept state.
- If $\langle M, w\rangle \in L_{TM}$, then D terminates in its reject state.
- If $\langle M, w\rangle \notin L_{TM}$, then D terminates in its accept state.

3.

Proof

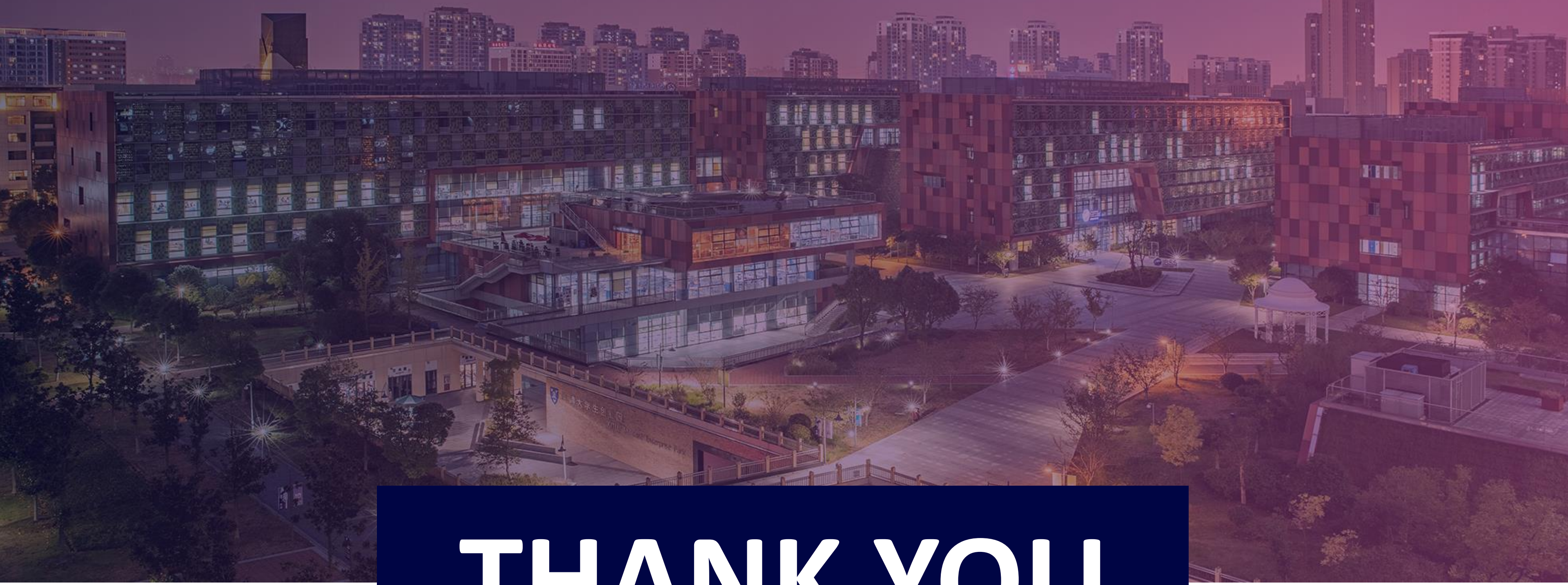
For any string $\langle M \rangle$

- If M accept $\langle M \rangle$, then D rejects $\langle M \rangle$
- If M rejects $\langle M \rangle$ or does not terminate on it, then D accepts $\langle M \rangle$

If we input the string $\langle D \rangle$ and take $M = D$

- If D accept $\langle D \rangle$, then D rejects $\langle D \rangle$
- If D rejects $\langle D \rangle$ or does not terminate on it, then D accepts $\langle D \rangle$

Clearly a contradiction.



THANK YOU

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