2021-2022 final

2022年10月17日　　16:09

1   Given alphabet A = {a, b, c} answer the following questions.

     a)   Let L be the set of words over A, in which the letters are in alphabetical order and appear at least once, for example abc, aabbbc but not ac, abb, bbbc. Write down a regular expression for L.   **4**

     b)   Give a DFA by diagram that accepts the language.   **4**

2   Let E, F be regular expressions, and E = {0}, F = {1}.

     a)   Give a DFA by diagram that accepts E + F.   **4**

     b)   Give a DFA by diagram that accepts EF.   **4**

     c)   Give a NFA by diagram that accepts (EF)*.   **4**

3   Given the following DFA, find out its equivalent regular expression. (You can do this either by observation or by applying the `DFA to regular expression' algorithm.)   **6**
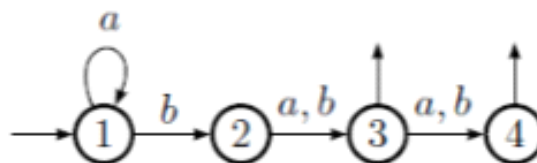
4   For the language L = {$a^k b^l a^m$ | k+l = m}, use the Pumping Lemma to prove it is not regular.   **8**

5   A DFA is given below.
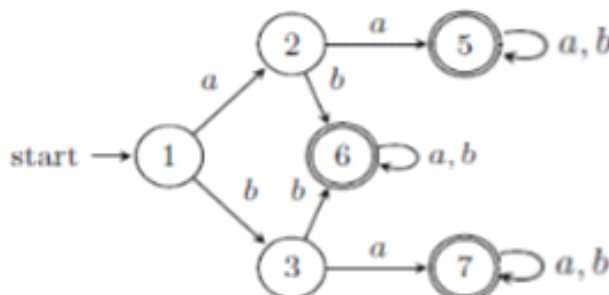
     a)   Are there any inaccessible states? If yes, list all inaccessible states.   **3**

     b)   List all classes of equivalent states in the DFA.   **6**

     c)   Minimise the DFA by merging the equivalent states.   **6**

6 Consider a context-free grammar G = ({0, 1}, {S, A, B}, S, P), where P is the set of productions:

$$S \to 0A \mid 1B \mid \varepsilon$$
$$A \to 1S \mid 0AA$$
$$B \to 0S \mid 1BB$$

a) Give a leftmost derivation of the string 0011. 4
b) Give a rightmost derivation of the string 0011. 4
c) Draw a parse tree for the string 1001. 4
d) Can the string 1110 be derived from the grammar G? If yes, write down the associated derivation tree. If not, give a justification. 4

7 Convert the following grammar to Chomsky Normal Form. (You can do this by first substituting variables for the constants and then breaking apart rules the length of whose right hand side is more than 2.) 8

$$S \to aAS \mid a$$
$$A \to SbA \mid SS \mid ba$$

8 Consider the Turing machine
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, x, y, B\}, \delta, q_0, B, \{q_4\})$$
where δ is defined as follows:

$$\delta(q_0, a) = (q_1, x, R)$$
$$\delta(q_1, a) = (q_1, a, R)$$
$$\delta(q_1, y) = (q_1, y, R)$$
$$\delta(q_1, b) = (q_2, y, L)$$

$$\delta(q_2, y) = (q_2, y, L)$$
$$\delta(q_2, a) = (q_2, a, L)$$
$$\delta(q_2, x) = (q_0, x, R)$$

$$\delta(q_0, y) = (q_3, y, R)$$
$$\delta(q_3, y) = (q_3, y, R)$$
$$\delta(q_3, B) = (q_4, B, R)$$

Assume the head of the machine initially points to the left-most letter.

a) Give the instantaneous descriptions for the input "aabb". Is "aabb" accepted by the machine? 4
b) Give the instantaneous descriptions for the input "abbb". Is "abbb" accepted by the machine? 4
c) What is the language accepted by M? 5

9   **a)** What are recursive and recursively enumerable languages? Which one of the   **4**
two sets stands for decidable problems?

       b) It was shown that in lectures that the halting problem is undecidable. Write   **10**
down a general explanation of what this means, such that it would be useful for
someone who knows about Turing Machine but not about undecidability.

END OF THE PAPER