

### Question 1

Given a CFG  $G = (V, \Sigma, R, S)$  with set of variables  $V = \{S\}$ , where  $S$  is the start variable; set of terminals  $\Sigma = \{0, 1, (, ), \cup, *, \emptyset, \varepsilon\}$ . ; and rules:

$$S \rightarrow S \cup S | SS | S^* | (S) | 0 | 1 | \emptyset | \varepsilon$$

Using this G, solve the following questions. **(20 marks)**

(1) Give a derivation for the string  $(0 \cup (10)^*1)^*$ . **(11 marks)**

(2) Give the corresponding parse tree for the string  $(0 \cup (10)^*1)^*$ . **(9 marks)**

### Question 2

Consider the following CFG  $G = (V, \Sigma, R, S)$ , where  $V = \{S, T, X\}$ ,  $\Sigma = \{a, b\}$ , the start variable is  $S$ , and the rules  $R$  are:

$$S \rightarrow aTXb$$

$$T \rightarrow XTS | \varepsilon$$

$$X \rightarrow a | b$$

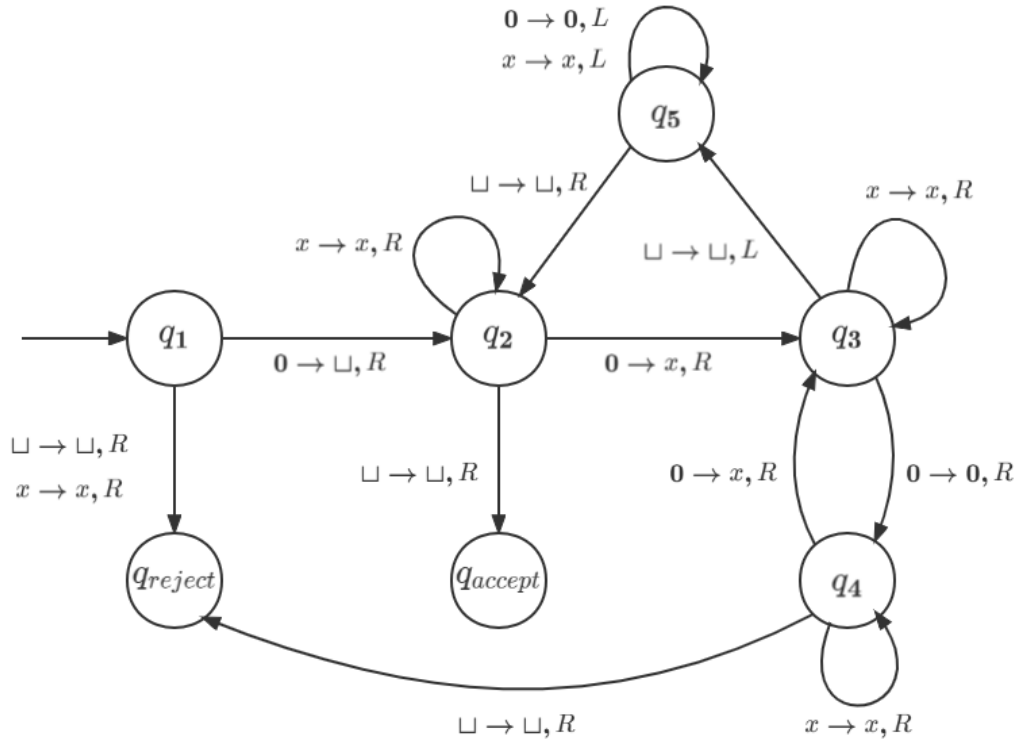
Convert this G to an equivalent PDA. **(20 marks)**

### Question 3

Use the pumping lemma to prove that the language  $A = \{0^{2n}1^{3n}0^n \mid n \geq 0\}$  is not context free. **(20 marks)**

### Question 4

The Turing machine M below recognizes the language  $A = \{0^{2^n} \mid n \geq 0\}$ .



Give the sequence of configurations when the input string is 000000. **(20 marks)**

### Question 5

Given the language  $E_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG with } L(G) = \emptyset\}$ , prove that it is decidable (briefly describe the proof idea). **(20 marks)**