INT201 Decision, Computation and Language

Tutorial 10

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1. Prove the below theorem

If $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.

2. Prove the below theorem

The cost of combining two descriptions leads to a bound that is greater than the sum of the individual complexities. And this theorem can be described as:

$$\exists c \ \forall x, y \ [K(xy) \leq 2K(x) + K(y) + c]$$

Solution

1.

Proof

- Let M_B be a TM that recognizes B
- Let f be a reducing function from A to B.
- Define a new TM as follows:

  $M_A =$ "On input w:
  1. Compute $f(w)$.
  2. Run M_B on input $f(w)$ and give the same result."
- Since f is a reducing function, $w \in A \iff f(w) \in B$.
  - If $w \in A$, then $f(w) \in B$, so M_B and M_A accept.
  - If $w \notin A$, then $f(w) \notin B$, so M_B and M_A reject or loop.
- Thus, M_A recognizes A.

Solution

2.

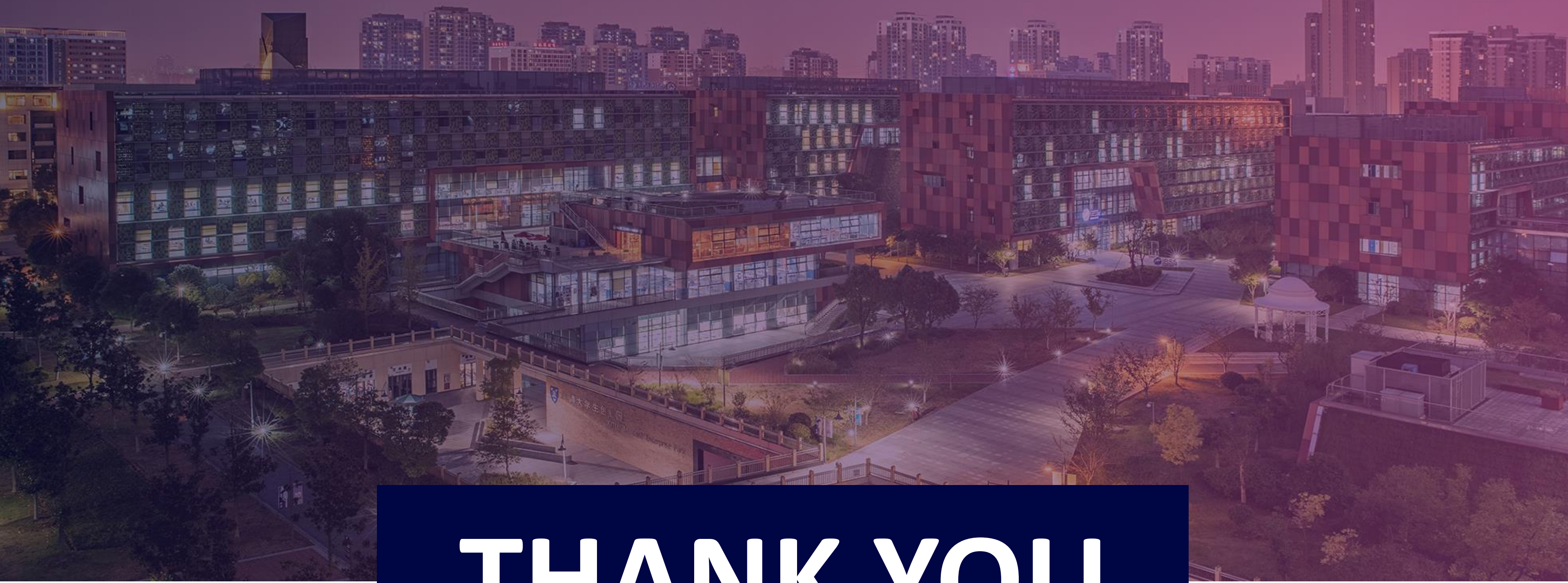
Proof

We construct a TM M that breaks its input w into two separate descriptions. The bits of the first description d(x) are all doubled and terminated with string 01 before the second description d(y) appears, as presented by the following figure. Once both descriptions have been obtained, they are run to obtain the strings x and y and the output xy is produced. The length of this description of xy is clearly twice the complexity of x plus the complexity of y plus a fixed constant for describing M. This sum is

$$2K(x) + K(y) + c,$$

and the proof is complete.

$$\langle M, w\rangle = \underbrace{11001111001100\cdots1100\overbrace{01}^{\text{delimiter}}}_{\langle M\rangle}\underbrace{01101011\cdots010}_{w}$$



THANK YOU

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