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- 1** Complete the following tasks. Indicate **true or false** of the following statements and briefly justify your answer.
- a)** For any language  $L$ ,  $L\emptyset = L\{\epsilon\}$ , where  $\emptyset$  is the empty language. 3
  - b)** Let  $L = \{11, 01\}$ , then  $110101011101 \in L^*$  3
  - c)** The language defined by the grammar  $S \rightarrow aS \mid bS \mid a \mid b$  is  $(a+b)^*$ . 3
  - d)** If  $L_1$  and  $L_2$  are regular language, then  $L_1 \cap L_2$  is a regular language. 3
  - e)** If  $L_1$  and  $L_2$  are context-free language, then  $L_1 \cap L_2$  is a context-free language. 3
  - f)** The relationship among class of languages are  
regular languages  $\subset$  context-free languages  $\subset$  context-sensitive languages  $\subset$  recursively enumerable languages. 3
  - g)** All the languages listed in **f)** are decidable. 3
- 2** Let  $L$  be the language of all strings over  $\{0,1\}$  containing 11.
- a)** Give a regular expression that defines  $L$ . 5
  - b)** Give a NFA with 3 states that accepts  $L$  by completing the following table: 5

	0	1
$\rightarrow i$		
$q$		
$t \rightarrow$		

- c)** Convert the NFA to an equivalent DFA using subset construction by completing the following table (indicate the initial state and accepting states): 5

	0	1
$\{\}$		
$\{i\}$		
$\{q\}$		
$\{t\}$		
$\{i, q\}$		
$\{i, t\}$		
$\{q, t\}$		
$\{i, q, t\}$		

- d)** Is the DFA obtained by the subset construction in c) a minimum-state DFA? If yes, justify it. If not, minimise it. 5

- 3      a) Explain how to use the Pumping Lemma to prove that a language is not regular. 4  
        b) Use the Pumping Lemma to prove the language  $L = \{a^m b^n \mid m < n\}$  is not a regular language. 6
- 4      Consider the following grammar and answer the questions.  
           $E \rightarrow a \mid E + E \mid E^* E$
- a) Give a leftmost derivation of  $a^* a + a$ . Write down also the associated derivation trees. 4  
        b) Give a rightmost derivation of  $a^* a + a$ . Write down also the associated derivation trees. 4  
        c) Is it an ambiguous grammar? Justify your answer. 4
- 5      Consider the following context free grammar
- $$\begin{aligned} S &\rightarrow aAS \mid a \mid BS \\ A &\rightarrow SbA \mid ba \mid AB \mid \varepsilon \end{aligned}$$
- a) Find all nullable variables and eliminate  $\varepsilon$ -productions. 3  
        b) Find all unit pairs and eliminate all unit productions in the resulting grammar of a). 3  
        c) Find all useless symbols and eliminate them in the resulting grammar of b). 3  
        d) Put the resulting grammar in c) into Chomsky normal form. 3
- 6      Consider the Turing machine with double infinite tape  
           $M = (\{q_0, q_1, q_2, q_3\}, \{1\}, \{1, x, B\}, \delta, q_0, B, \{q_3\})$   
        where  $\delta$  is defined as follows:
- $$\begin{aligned} \delta(q_0, 1) &= (q_0, x, R) \\ \delta(q_0, B) &= (q_1, B, L) \\ \delta(q_1, x) &= (q_2, 1, R) \\ \delta(q_2, 1) &= (q_2, 1, R) \\ \delta(q_2, B) &= (q_1, 1, L) \\ \delta(q_1, 1) &= (q_1, 1, L) \\ \delta(q_1, B) &= (q_3, B, R) \end{aligned}$$
- For each of the following initial inputs on the tape given below what will be left on the tape after the machine halts? (Assume the head of the machine initially points to the left-most 1.)
- a) 111 5  
        b) For any input  $w \in \{1\}^+$ , what is left on the tape after the machine halts? 5

- 7      a) What is a reduction? Briefly explain how this technique can be used to prove that certain problems are undecidable. 5
- b) Given 10
- $L_{\text{accept}} = \{ \langle M \rangle w \mid M \text{ is a Turing Machine that accepts input } w \}$
- and
- $L_{\text{halt}} = \{ \langle M \rangle w \mid M \text{ is a Turing Machine that halts on input } w \}$
- In the lecture, we have presented a reduction from  $L_{\text{halt}}$  to  $L_{\text{accept}}$ . Here assume that  $L_{\text{accept}}$  is undecidable, show that  $L_{\text{halt}}$  is undecidable by informally describing a reduction from  $L_{\text{accept}}$  to  $L_{\text{halt}}$
- The end**

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