INT201 Decision, Computation and Language

Tutorial 4

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1. Prove the following language is not regular

$$L = \{w \in \{a, b\}^* \mid w = w^R\}$$

2. Prove that if we add a finite set of strings to a regular language, the result is a regular language.

3. Give context-free grammars that generate the following languages

$$L = \{w \in \{0, 1\}^* \mid w \text{ contains at least three 1s } \}$$

4. Convert the following CFG into an equivalent CFG in Chomsky normal form

$$S \rightarrow BSB \mid B \mid \varepsilon$$
$$B \rightarrow 00 \mid \varepsilon$$

Solution

1.

Answer: Suppose that A_2 is a regular language. Let p be the "pumping length" of the Pumping Lemma. Consider the string $s = a^p b a^p$. Note that $s \in A_2$ since $s = s^{\mathcal{R}}$, and $|s| = 2p + 1 \geq p$, so the Pumping Lemma will hold. Thus, we can split the string s into 3 parts $s = xyz$ satisfying the conditions

i. $xy^i z \in A_2$ for each $i \geq 0$,

ii. $|y| > 0$,

iii. $|xy| \leq p$.

Since the first p symbols of s are all a's, the third condition implies that x and y consist only of a's. So z will be the rest of the first set of a's, followed by ba^p. The second condition states that $|y| > 0$, so y has at least one a. More precisely, we can then say that

$$\begin{aligned} x &= a^j \text{ for some } j \geq 0, \\ y &= a^k \text{ for some } k \geq 1, \\ z &= a^m b a^p \text{ for some } m \geq 0. \end{aligned}$$

Since $a^p b a^p = s = xyz = a^j a^k a^m b a^p = a^{j+k+m} b a^p$, we must have that $j + k + m = p$. The first condition implies that $xy^2 z \in A_2$, but

$$\begin{aligned} xy^2 z &= a^j a^k a^k a^m b a^p \\ &= a^{p+k} b a^p \end{aligned}$$

since $j + k + m = p$. Hence, $xy^2 z \notin A_2$ because $(a^{p+k} b a^p)^{\mathcal{R}} = a^p b a^{p+k} \neq a^{p+k} b a^p$ since $k \geq 1$, and we get a contradiction. Therefore, A_2 is a nonregular language.

Solution

2. Let A be a regular language, and let B be a finite set of strings. We know from class that finite languages are regular, so B is regular. Thus, $A \cup B$ is regular since the class of regular languages is closed under union.

3. $G = (V, \Sigma, R, S)$ with set of variables $V = \{S,X\}$, where S is the start variable; set of terminals $\Sigma=\{0, 1\}$; and rules

$$S \to X1X1X1X$$
$$X \to 0X \mid 1X \mid \varepsilon$$

4.

1st step. introduce new start variable S_0 and the new rule $S_0 \to S$

$$S_0 \to S$$
$$S \to BSB \mid B \mid \varepsilon$$
$$B \to 00 \mid \varepsilon$$

2nd step. remove ε rules

Removing $B \to \varepsilon$

$$S_0 \to S$$
$$S \to BSB \mid BS \mid SB \mid S \mid B \mid \varepsilon$$
$$B \to 00$$

Solution

2nd step. remove ε rules

Removing $S \to \varepsilon$

$$
\begin{aligned}
&S_0 \to S \mid \varepsilon \\
&S \to BSB \mid BS \mid SB \mid S \mid B \mid BB \\
&B \to 00
\end{aligned}
$$

3rd step. remove unit rules

Removing $S \to S$

$$
\begin{aligned}
&S_0 \to S \mid \varepsilon \\
&S \to BSB \mid BS \mid SB \mid B \mid BB \\
&B \to 00
\end{aligned}
$$

Removing $S \to B$

$$
\begin{aligned}
&S_0 \to S \mid \varepsilon \\
&S \to BSB \mid BS \mid SB \mid 00 \mid BB \\
&B \to 00
\end{aligned}
$$

Solution

3rd step. remove unit rules

Removing $S_0 \rightarrow S$

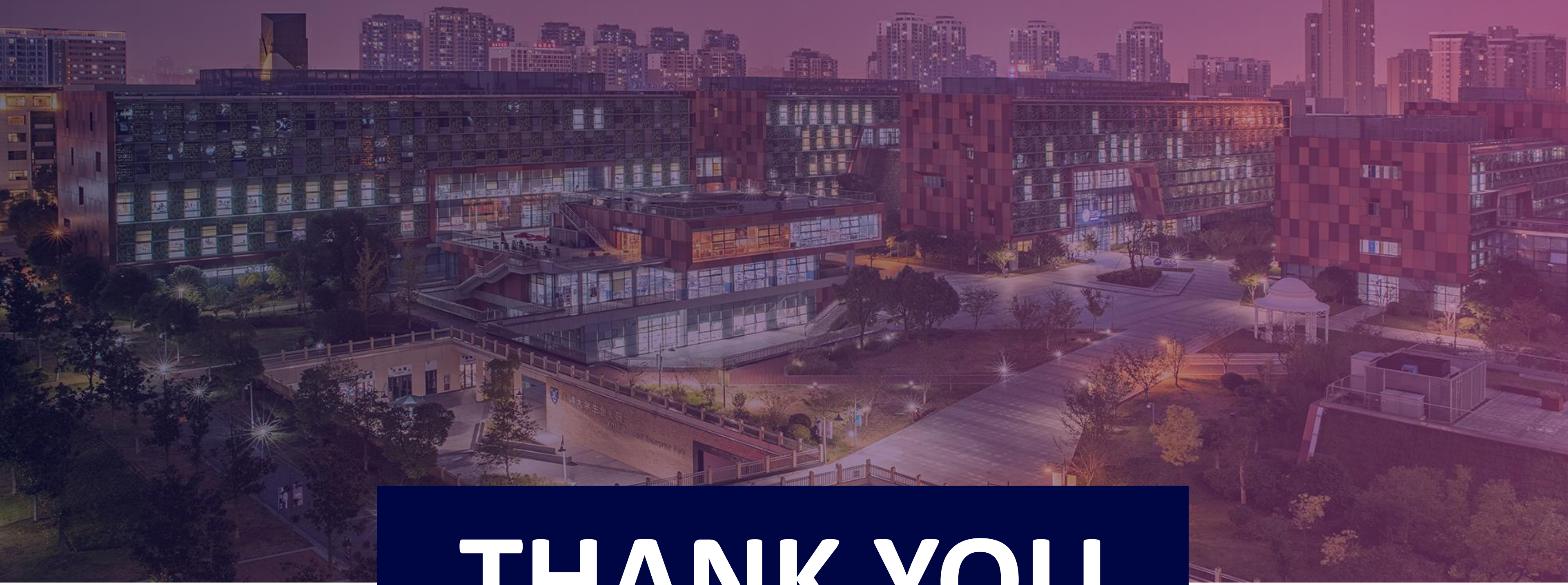
$$S_0 \rightarrow BSB \mid BS \mid SB \mid 00 \mid BB \mid \varepsilon$$
$$S \rightarrow BSB \mid BS \mid SB \mid 00 \mid BB$$
$$B \rightarrow 00$$

4th step. replaced ill-placed terminals 0 by variable U

$$S_0 \rightarrow BSB \mid BS \mid SB \mid UU \mid BB \mid \varepsilon$$
$$S \rightarrow BSB \mid BS \mid SB \mid UU \mid BB$$
$$B \rightarrow UU$$
$$U \rightarrow 0$$

5th step. eliminate all rules having more than two symbols

$$S_0 \rightarrow BA_1 \mid BS \mid SB \mid UU \mid BB \mid \varepsilon$$
$$S \rightarrow BA_2 \mid BS \mid SB \mid UU \mid BB$$
$$B \rightarrow UU$$
$$U \rightarrow 0$$
$$A_1 \rightarrow SB$$
$$A_2 \rightarrow SB$$



THANK YOU

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