INT201 Decision, Computation and Language

Tutorial 6

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1.

Consider language A with CFG $G = (V, \Sigma, R, S)$
Variables $V = \{S, C, D\}$
Terminals $\Sigma=\{a, b\}$
Rules:
$S \rightarrow CDa \mid CD$
$C \rightarrow aD$
$D \rightarrow Sb \mid b$

Derivation for string $s = ababbba$:

$S \Rightarrow CDa \Rightarrow aDDa \Rightarrow abDa \Rightarrow abSba \Rightarrow abCDba \Rightarrow abaDDba \Rightarrow ababDba \Rightarrow ababbba$

(1) Construct a parse tree based on the given CFG and derivation

(2) Apply the constructed parse tree to split string "ababbba".

2. Prove that $L = \{ a^n b^n c^n \mid n \geq 0 \}$ is non-CFL. (using Pumping Lemma)

Solution

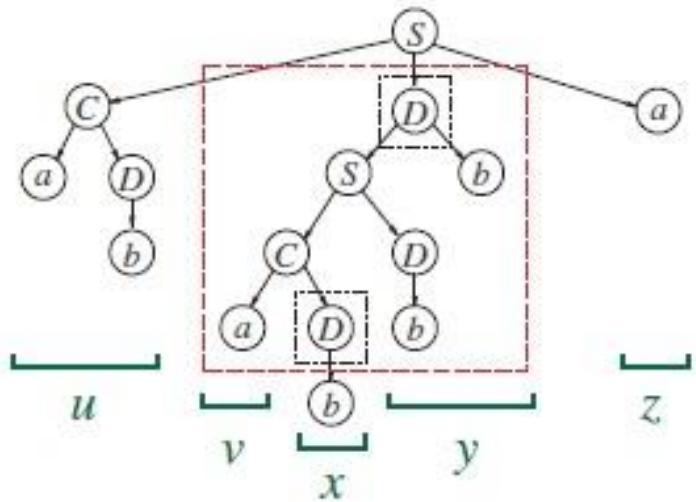
1.

- Split string $s \in A$ into

  $$s = \underbrace{ab}_{u}\underbrace{a}_{v}\underbrace{b}_{x}\underbrace{bb}_{y}\underbrace{a}_{z}$$

  using repeated variable D.

- In depth-first traversal of tree
  - $u = ab$ is before D-D subtree
  - $v = a$ is before second D within D-D subtree
  - $x = b$ is what second D eventually becomes
  - $y = bb$ is after second D within D-D subtree
  - $z = a$ is after D-D subtree



Solution

2.

Recall Pumping Lemma for CFL

Let L be a context-free language. Then there exists an integer $p \geq 1$, called the pumping length, such that the following holds: Every string s in L, with $|s| \geq p$, can be written as $s = uvxyz$, such that

1. $|vy| \geq 1$ (i.e., v and y are not both empty),

2. $|vxy| \leq p$, and

3. $uv^ixy^iz \in L$, for all $i \geq 0$.

Step 1. Suppose we could construct some CFG G for L

$$S \rightarrow CC \mid BC \mid a$$
$$B \rightarrow CS \mid b$$
$$C \rightarrow SB \mid c$$

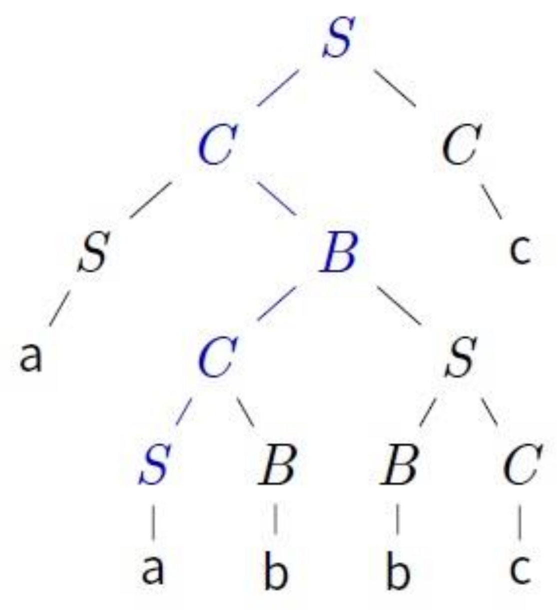
Step 2. Set string $s = aabbcc$

$$S \Rightarrow CC \Rightarrow SBC \Rightarrow SCSC \Rightarrow SSBSC \Rightarrow SSBBCC \Rightarrow aSBBCC \Rightarrow aaBBCC \Rightarrow aabBCC \Rightarrow aabbCC \Rightarrow aabbcC \Rightarrow aabbcc$$

Solution

Step 3. Parse tree and splitting

Taking B as the repeated variable, get $s = uv^i xy^i z = a(a)^i b(bc)^i c$



Step 4. Contradiction

Satisfies property 1 and 2 of Pumping Lemma, but does not satisfy property 3: $a(a)^i b(bc)^i c \notin L$

THANK YOU

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