INT201 Decision, Computation and Language

Lecture 7 – Context-Free Languages (2)

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Pushdown Automata (PDAs)

The class of languages that can be accepted by pushdown automata is exactly the class of context-free languages (finite automata are for regular languages).

- The input for a pushdown automaton is a string w in Σ^*.
- PDA accepts or doesn't accept w.
- Different from finite automata, PDAs have a single stack.
- Stack have 2 different operations:

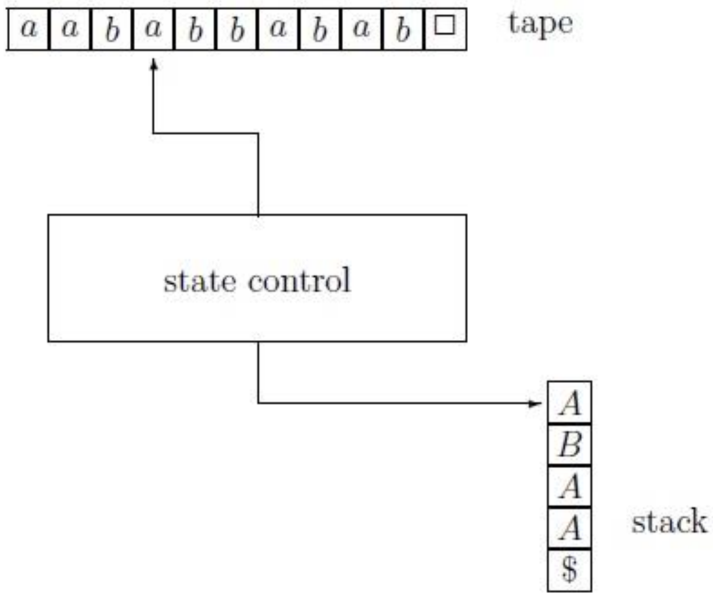
  (1) push – adds item to top of stack

  (2) pop – removes item from top of stack

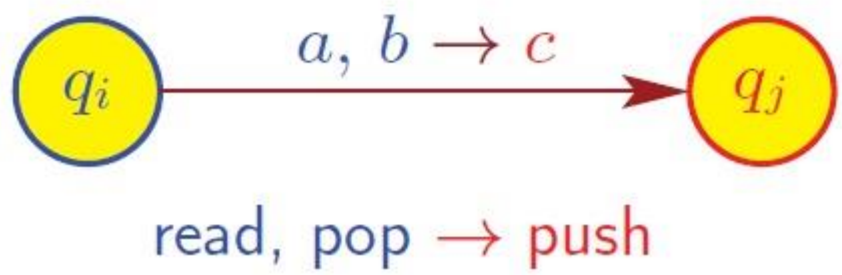
Pushdown Automata (PDAs)

A PDA consists of: a tape, a stack and a state control

- **Tape**: divided into cells that store symbols belonging to $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$.
- **Tape head**: move along the tape, one cell to the right per move.
- **Stack**: containing symbols from a finite set Γ, called the stack alphabet. This set contains a special symbol $\$$ (often mark bottom of stack).
- **Stack head**: reads the top symbol of the stack. This head can also pop the top symbol, and it can push symbols of Γ onto the stack.
- **State control**: can be in any one of a finite number of states. The set of states is denoted by Q. The set Q contains one special state q, called the start state.



PDA Transition



If PDA

- in state q_i
- reads $a \in \Sigma_\varepsilon$
- pops $b \in \Gamma_\varepsilon$ off the stack

If $a = \varepsilon$, then no input symbol is read.

If $b = \varepsilon$, then nothing is popped off stack.

then PDA

- moves to state q_j
- push $c \in \Gamma_\varepsilon$ onto top of stack

If $c = \varepsilon$, then nothing is pushed onto stack.

If $c = u_1u_2. . . u_k$ with $k \geq 1$ and $u_1, u_2, . . . , u_k \in \Gamma$, then b is replaced by c, and u_k becomes the new top symbol of the stack .

PDA Definition

Definition

A **pushdown automaton** is a 5-tuple $M = (Q, \Sigma, \Gamma, \delta, q, F)$:

- Q is finite set of states
- Σ is (finite) input (tape) alphabet
- Γ is (finite) stack alphabet
- δ is the transition function: $Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow Q \times \{N, R\} \times \Gamma_\varepsilon{}^*$
- q is start state, $q \in Q$
- F is set of accept states, $F \subseteq Q$

Let $r' \in Q$, $\sigma \in \{N, R\}$, and $w \in \Gamma^*$

$$\delta(r, a, b) = (r', \sigma, c).$$

The tape head moves according to σ:

- if $\sigma = R$, it moves one cell to the right.
- if $\sigma = N$, it does not move.

Example

Given a PDA M = $(Q, \Sigma, \Gamma, \delta, q_1, F)$

- $Q = \{q_1, q_2, q_3, q_4\}$
- $\Sigma = \{0, 1\}$
- $\Gamma = \{0, \$\}$
- q_1 is start state
- $F = \{q_1 , q_4\}$
- $\delta: Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow Q \times \{N, R\} \times \Gamma_\varepsilon{}^*$

Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$	$\{(q_3, \varepsilon)\}$					
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

Example

Draw the corresponding PDA diagram

Example

Process string 000111

Example

Example

Example

What’s the complete evolution (state, stack) of this PDA?

Example

How about 0111?

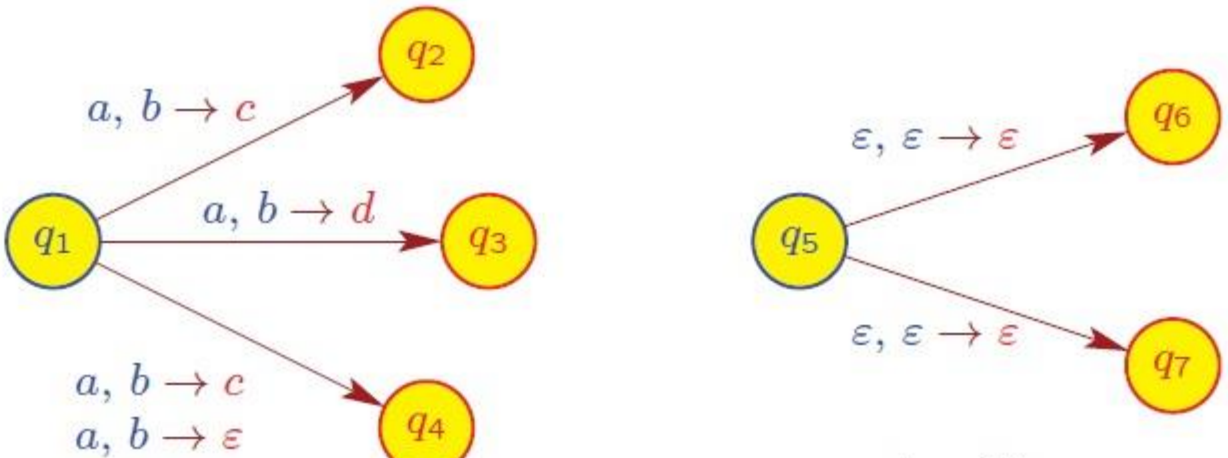
Example

Which language this PDA accept?

Nondeterministic PDA

PDA transition function allows for nondeterminism

$$\delta: Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow P(Q \times \Gamma_\varepsilon)$$



ε-transitions

Language accepted by PDA

Definition

The set of all input strings that are accepted by PDA M is the language recognized by M and is denoted by $L(M)$.

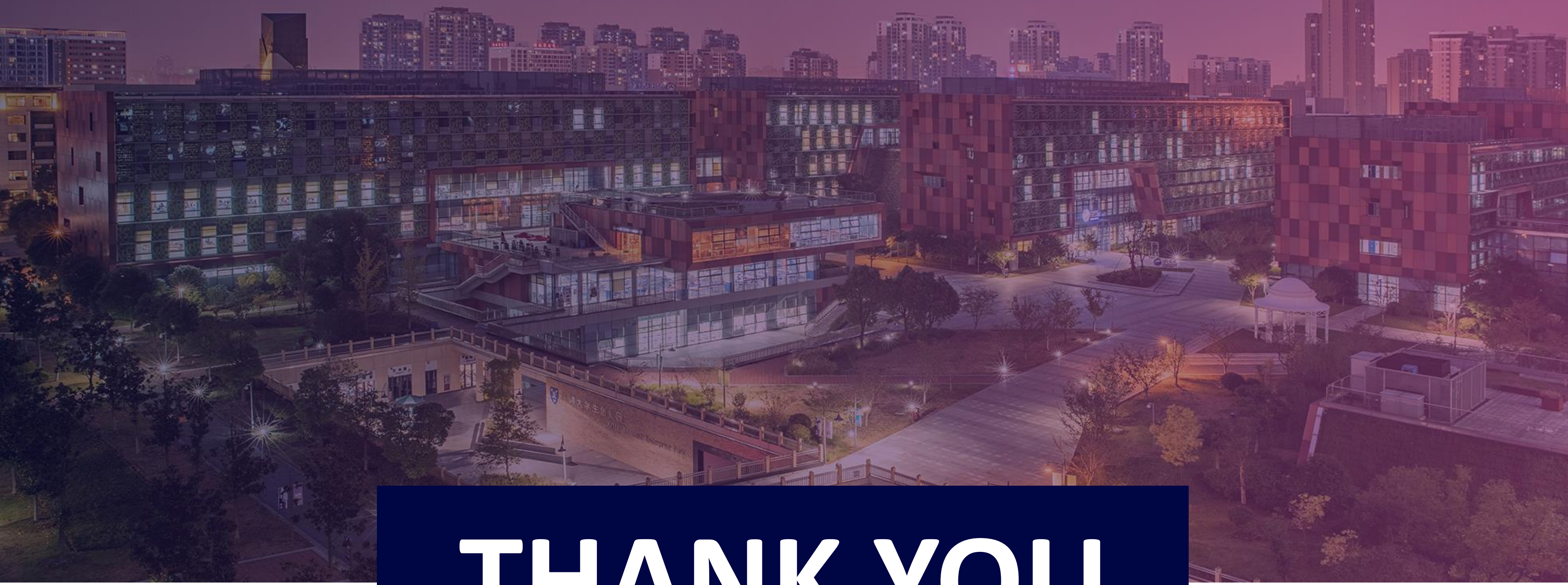
Example

PDA for language $\{ww^R \mid w \in \{0, 1\}^*\}$

Language accepted by PDA

Example

PDA for language $\{ a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k \}$



THANK YOU

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