CMPE480 - 2023-2024 Fall - HW3

Implement the following algorithms, following the recursive formulations from the lecture. We use rain, umbrella domain.

- Filtering
- Likelihood of evidence
- Smoothing
- Most likely explanation

Submit a single python file with the following format:
<last-name>-<first-name>.py

It should run with python3.10. Do not use external modules or libraries. You need to follow the input/output rules, below, otherwise might lose significant points, as they will be evaluated automatically.

Input

Your program accepts a single line from the user prompt:
<prob0> <prob1> <prob2> <prob3> <prob4> <type> <query>

where

- prior probability of rain: <prob0>
- transition probability from rain to rain: <prob1>
- transition probability from no-rain to rain: <prob2>
- emission probability from rain to umbrella: <prob3>
- transition probability from no-rain to umbrella: <prob4>
- the query type: <type>
- the query: <query>

where

- <type> might be F or L or S or M for filtering, likelihood of evidence, smoothing and most likely explanation, respectively
- <query> might be
    - [T T F F.. T] for filtering where T and F corresponds to Umbrella=True and Umbrella=False, respectively.
    - [T T F F.. T] for likelihood of evidence where T and F corresponds to Umbrella=True and Umbrella=False, respectively.
    - [T T F F.. T] <k> for smoothing where T and F corresponds to Umbrella=True and Umbrella=False, respectively. <k> is a number between 1 and the evidence count.
    - [T T F F.. T] for the most likely explanation

Output

Your program outputs:

- Probability distribution of rain at time point t for filtering.
- Probability of the evidence for the likelihood of evidence.
- Probability distribution of rain at time point k for smoothing
- The sequence of the most likely path and the corresponding probability distributions for each time step.
- The probabilities should be printed out with two decimal points

Example Input:

```
0.5 0.7 0.3 0.9 0.2 F [T T]
```

Example Output:

```
<0.82, 0.18>
```

Example Input:

```
0.5 0.7 0.3 0.9 0.2 S [T T] 1
```

Example Output:

```
<0.88, 0.12>
```

Example Input:

```
0.5 0.7 0.3 0.9 0.2 M [T T F T]
```

Example Output:

```
[T T F T] [<0.82, 0.18>, <0.52, 0.05>, <0.04, 0.12>, <0.03, 0.02>]
```