Kinematic Model

A drone can be represented using:

$$r = [x, y, z, \phi, \theta, \psi, v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]$$

Where:

- $[x, y, z]$ is the world frame position.
- $[\phi, \theta, \psi]$ is the roll, pitch and yaw Euler angles.
- $[v_x, v_y, v_z]$ are the world frame velocities.
- $[\omega_x, \omega_y, \omega_z]$ are the body frame angular velocities.

Using this we are able to calculate the acceleration in x, y and z as:

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \frac{1}{m} \begin{bmatrix} \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi \\ \cos\phi\sin\theta\sin\psi - \cos\psi\sin\phi \\ \cos\theta\cos\phi \end{bmatrix} u_1$$

This allows us to update the velocities of the drone $[v_x, v_y, v_z]$ using the rotation matrix and u_1 which is:

$$u_1 = k_f \times w_1^2 + k_f \times w_2^2 + k_f \times w_3^2 + k_f \times w_4^2$$

Where:

- w_i is the angular speed of each rotor
- k_f is the proportionality constant for thrust

We are able to update the roll pitch and yaw using:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

To update the body frame angular velocities $[\omega_x, \omega_y, \omega_z]$ we need to calculate:

$$\begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}\omega_y\omega_z \\ \frac{I_{zz}-I_{xx}}{I_{yy}}\omega_x\omega_z \\ \frac{I_{xx}-I_{yy}}{I_{zz}}\omega_x\omega_y \end{bmatrix} + \begin{bmatrix} \frac{1}{I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

Where:

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}.$$

And :

$$\begin{bmatrix} F \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \kappa_f & \kappa_f & \kappa_f & \kappa_f \\ 0 & d\kappa_f & 0 & -d\kappa_f \\ -d\kappa_f & 0 & d\kappa_f & 0 \\ \kappa_m & -\kappa_m & \kappa_m & -\kappa_m \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

Where:

- d is the arm length
- k_m is the proportionality constant for moment

Using the above equations I will need:

- d is the arm length
- k_m is the proportionality constant for moment
- k_f is the proportionality constant for thrust
- w_i (max) the maximum angular speed of each rotor

We assume that the angular speed of each rotor can change instantaneously. We also assume that drag is negligible.