Probability and Statistics (2023)

Homework1

Name : 이풍현　　　　　　Student ID : 201924548

Precautions

1. Please write your name and student ID correctly.

2. When writing the answer, write the final calculated answer, including the solving process, as a reduced fraction or a decimal.

3. If you only write the answer without a solving process, we cannot give you a partial score if the answer is wrong.

4. If you share or copy the answer with another student, you will get 0 points.

1. (1 point) bowl contains 16 chips, of which 6 are blue, 7 are red, and 3 are white. If four chips are taken at random and without replacement, find the probability for each of the following cases.

   (a) (0.3 point) each of the four chips is blue.

다시넣지않으므로
파란칩확률연산

$$\frac{6\times5\times4\times3}{16\times15\times14\times13} = \frac{3}{364}$$

   (b) (0.3 point) none of the four chips is blue.

파란칩이안날확률
1- 파란칩확률

$$\frac{10\times9\times8\times7}{16\times15\times14\times13} = \frac{3}{26}$$

   (c) (0.4 point) there is at least one chip of each color.

$$1 - \frac{10\times9\times8\times7 + 9\times8\times7\times6 + 13\times12\times11\times10}{16\times15\times14\times13}$$

$$13 \overline{)221} = 17, \quad 13, \quad 91$$

1 − 파란칩안나올확률 − 빨간칩안나올확률 − 흰색칩안나올확률

$$1 - \frac{11160}{\ldots} = \frac{26520}{\ldots} = \frac{17}{28}$$

1− 43680 43680

Probability and Statistics (2023)

2. (1 point) Three plants, A1, A2, and A3, produce respectively, 10%, 50%, and 40% of a company's output. Although plant A1 is a small plant, its manager believes in high quality and only 1% of its products are defective. The other two, A2 and A3, are worse and produce items that are 3% and 4% defective, respectively. All products are sent to a central warehouse. One item is selected at random and observed to be defective, say event B. Find the conditional probability that this defective product was produced in plant A1.

$$\frac{0.1\times 0.01}{0.1\times 0.01+0.5\times 0.03+0.4\times 0.04}=\frac{1}{32}$$

3. (1 point) Bill and George go target shooting together. Both shoot at a target at the same time. Suppose Bill hits the target with probability 0.7, whereas George, independently, hits the target with probability 0.4.

   (a) (0.5 point) Given that exactly one shot hit the target, what is the probability that it was George's shot?

$$\frac{\text{george만 맞출}}{\text{Bill만 맞출}+\text{George만 맞출}}=\frac{0.3\times 0.4}{0.1\times 0.6+0.3\times 0.4}=\frac{2}{3}$$

   (b) (0.5 point) Given that the target is hit, what is the probability that George hit it?

$$\frac{\text{George가 맞출확률}}{1-\text{못맞출확률}}=\frac{0.4}{1-0.3\times 0.6}=\frac{20}{41}$$

4. (1 point) Urn 1 has five white and seven black balls. Urn 2 has three white and twelve black balls. We flip a fair coin. If the outcome is heads, then a ball from urn 1 is selected, while if the outcome is tails, then a ball from urn 2 is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

$$\frac{\text{2에서흰공}}{\text{흰공}}=\frac{\frac{1}{2}\times\frac{12}{15}}{\frac{1}{2}\times\frac{5}{12}+\frac{1}{2}\times\frac{12}{15}}=\frac{48}{113}$$

Probability and Statistics (2023)

5. (1 point) Each bag in a large box contains 25 tulip bulbs. It is known that 60% of the bags contain bulbs for 5 red and 20 yellow tulips, while the remaining 40% of the bags contain bulbs for 15 red and 10 yellow tulips. A bag is selected at random and a bulb taken at random from this bag is planted.

   (a) (0.5 point) What is the probability that it will be a yellow tulip?

   1에서노랑+2에서노랑 $= 0.6 \times \frac{20}{25} + 0.4 \times \frac{10}{25} = \frac{16}{25}$

   (b) (0.5 point) Given that it is yellow, what is the conditional probability it comes from a bag that contained 5 red and 20 yellow bulbs?

   $$\frac{0.6 \times \frac{20}{25}}{0.6 \times \frac{20}{25} + 0.4 \times \frac{10}{25}} = \frac{3}{4}$$

6. (1 point) Consider an urn that contains slips of paper each with one of the numbers 1, 2, . . ., 100 on it. Suppose there are i slips with the number i on it for i = 1, 2, . . ., 100. For example, there are 25 slips of paper with the number 25. Assume that the slips are identical except for the numbers. Suppose one slip is drawn at random. Let X be the number on the slip.

   (a) (0.3 point) Find the pmf(probability mass function) of X.

   각숫자 나올 확률은 숫자에 비례하므로 $f(x) = \frac{x}{\sum_{k=1}^{100} k} = \frac{x}{5050}$

   (b) (0.3 point) Compute $P(X \leq 50)$.

   $$\frac{\text{50이하 숫자카드수}}{\text{전체카드수}} = \frac{\sum_{1}^{50} k}{\sum_{1}^{100} k} = \frac{\cancel{1275}}{\cancel{5050}} = \frac{51}{202}$$

   (c) (0.4 point) Find the cdf(cumulative distribution function) of X.

   $$F(x) = \frac{\sum_{1}^{x} k}{\sum_{1}^{100} k} = \frac{\frac{x(x+1)}{2}}{5050} = \frac{x(x+1)}{10100}$$

Probability and Statistics (2023)

7. (1 point) A coin, having probability p of landing heads, is flipped until a head appears for the rth time. Let N denote the number of flips required. Calculate $E(N)$.

$$f(n) = \binom{n-1}{r-1} p^{r-1} (1-p)^{n-r} \cdot p$$

$$E(N) = \sum_{n=1}^{\infty} n \binom{n-1}{r-1} p^{r-1} (1-p)^{n-r} \cdot p$$

8. (1 point) If X is uniform over (0, 1), calculate $E(X^n)$ and $Var(X^n)$.

$$E(X^n) = \int_0^1 x^n (1-0) dx = \frac{1}{n+1} x^{n+1} \Big|_0^1 = \frac{1}{n+1}$$

$$Var(X^n) = E((X^n)^2) - E(X^n)^2 = \frac{1}{2n+1} - \frac{1}{(n+1)^2}$$

9. (1 point) Let the probability density function of X be given by

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) (0.3 point) What is the value of c?

$$\int_0^2 c(4x - 2x^2) dx = 1 \qquad c\left[2x^2 - \frac{2}{3}x^3\right]_0^2 = 1$$

$$c\left(8 - \frac{16}{3}\right) = \frac{8}{3}c = 1, \quad c = \frac{3}{8}$$

(b) (0.3 point) $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$?

$$\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{3}{8}(4x - 2x^2) = \frac{3}{8}\left[2x^2 - \frac{2}{3}x^3\right]_{\frac{1}{2}}^{\frac{3}{2}} = \left(\left(\frac{18}{4} - \frac{9}{4}\right) - \left(\frac{1}{2} - \frac{1}{12}\right)\right)\frac{3}{8}$$

$$= \frac{11}{16}$$

(c) (0.4 point) $E(3X + 6X^2)$?

$$E[X] = \int_0^2 x \cdot f(x) dx = \int_0^2 \frac{3}{2}x^2 - \frac{3}{4}x^3 dx = \frac{3}{4}\int_0^2 2x^2 - x^3 dx$$

$$= \frac{3}{4}\left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^2 = 1$$

$$E[X^2] = \int_0^2 x^2 f(x) dx = \int_0^2 \frac{3}{2}x^3 - \frac{3}{4}x^4 dx = \frac{3}{4}\int_0^2 2x^3 - x^4 dx$$

$$\frac{3}{4}\left[\frac{1}{2}x^4 - \frac{1}{5}x^5\right]_0^2 = \frac{3}{4}\left(8 - \frac{32}{5}\right) = \frac{6}{5}$$

5(

$= 4 [\frac{x^2}{2} - \frac{\quad}{5}]_0 \quad 4$

$E[3X + 6X^2] = 3E(X) + 6E(X^2) = \frac{\quad}{5}$

Probability and Statistics (2023)

10. (1 point) The joint density function of X, Y is

$$f(x,y) = 120xy(1-x-y), \qquad x \geq 0, y \geq 0, x+y \leq 1$$

(a) (0.2 point) Find $E(XY)$.

$\int_0^1 \int_0^{1-y} 120x^2y^2(1-x-y) \; dx\,dy = \int_0^1 120y^2 \int_0^{1-y} x^2(1-x-y)\,dx\,dy$

$= \frac{2}{21}$

(b) (0.3 point) Find $E(X)$.

$\int_0^1 \int_0^{1-y} x \cdot f(x)\,dx\,dy = \frac{1}{3}$

(c) (0.2 point) Find $Cov(X,Y)$.

$Cov(X,Y) = E(XY) - E(X)E(Y) = -\frac{1}{63}$

$\frac{2}{21} - \frac{1}{3} \cdot \frac{1}{3}$

(d) (0.3 point) Find $Var(X)$.

$Var(X) = E[X^2] - (E[X])^2$

$= \frac{1}{7} - \frac{1}{9} = \frac{2}{63}$