Homework #03

인공지능

정보컴퓨터공학부

For all the questions, show your work.

1. Given the full joint distribution shown below, calculate the following:

a. $P(toothache)$.

0.110+0.010+0.012+0.068 = 0.2

b. **P**(*Catch*). <0.302,0.698>

$\langle 0.110, 0.012 \rangle + \langle 0.010, 0.68 \rangle + \langle 0.060, 0.120 \rangle + \langle 0.020, 0.600 \rangle$

$= \langle 0.302, 0.648 \rangle$

c. **P**(*Cavity* | *catch*).

$\alpha[P(cavity, catch, toothache) + P(cavity, catch, \neg toothache)]$

$= \alpha[\langle 0.110, 0.012 \rangle + \langle 0.060, 0.120 \rangle] = \alpha\langle 0.17, 0.132 \rangle = \langle \frac{170}{302}, \frac{132}{302} \rangle$

d. **P**(*Cavity* | *toothache* ∨ *catch*).

$\alpha[\langle 0.110, 0.012 \rangle + \langle 0.010, 0.008 \rangle + \langle 0.06, 0.12 \rangle]$

$= \alpha\langle 0.18, 0.2 \rangle = \langle \frac{18}{38}, \frac{20}{38} \rangle$

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.110	0.010	0.060	0.020
¬cavity	0.012	0.068	0.120	0.600

A full joint distribution for the *Toothache*, *Cavity*, *Catch* world.

2. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 97% accurate (i.e., the probability of testing positive when you do have the disease is 0.97, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 100,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?

$P(disease \mid positive) = \frac{P(disease, positive)}{P(positive)} = \frac{\frac{1}{100,000} \times \frac{97}{100}}{\frac{1}{100,000} \times \frac{97}{100} + \frac{3}{100} \times \frac{99999}{100,000}} = \frac{97}{300,994}$

P(disease) 의 확률이 매우 낮기에 P(disease,positive) 확률도 매우 낮아짐

3. It is quite often useful to consider the effect of some specific propositions in the context of some general background evidence that remains fixed, rather than in the complete absence of information. The following questions ask you to prove more general versions of the product rule and Bayes' rule, with respect to some background evidence **e**:

a. Prove the conditionalized version of the general product rule:

$$\mathbf{P}(X, Y \mid \mathbf{e}) = \mathbf{P}(X \mid Y, \mathbf{e})\mathbf{P}(Y \mid \mathbf{e}) .$$

$$P(X,Y|e) = \frac{P(X,Y,e)}{P(e)} = \frac{P(X,Y,e)}{P(e)} \times \frac{P(Y,e)}{P(Y,e)} = \frac{P(X,Y,e)}{P(Y,e)} \times \frac{P(Y,e)}{P(e)}$$

b. Prove the conditionalized version of Bayes' rule:

$$\mathbf{P}(Y \mid X, \mathbf{e}) = \mathbf{P}(X \mid \mathrm{Y}, \mathbf{e})\mathbf{P}(Y \mid \mathbf{e}) / \mathbf{P}(X \mid \mathbf{e}).$$

$$P(Y|X,e) = \frac{P(Y,X,e)}{P(X,e)} = \frac{P(X,Y,e)}{P(Y,e)} \times \frac{P(Y,e)}{P(X,e)} = \frac{P(X,Y,e)}{P(Y,e)} \times \frac{\frac{P(Y,e)}{P(e)}}{\frac{P(X,e)}{P(e)}} = \frac{P(X|Y,e) \cdot P(Y|e)}{P(X|e)}$$

4. Show that the statement of conditional independence

$$\mathbf{P}(X, Y \mid Z) = \mathbf{P}(X \mid Z)\,\mathbf{P}(Y \mid Z)$$

is equivalent to each of the statements

$$\mathbf{P}(X \mid Y, Z) = \mathbf{P}(X \mid Z) \text{ and } \mathbf{P}(Y \mid X, Z) = \mathbf{P}(Y \mid Z) .$$

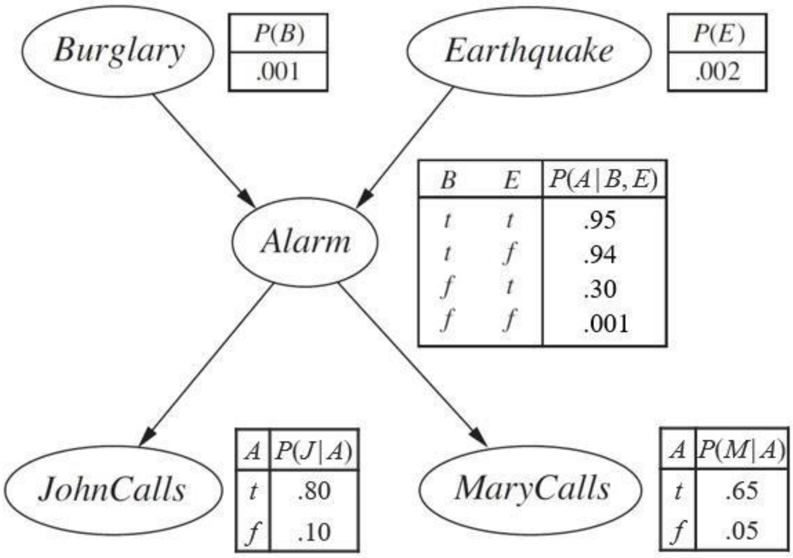
$$\frac{P(X,Y,Z)}{P(Z)} = \frac{P(X,Z)}{P(Z)} \cdot \frac{P(Y,Z)}{P(Z)} \Rightarrow \frac{P(X,Y,Z)}{P(Z)} \times \frac{P(Z)}{P(Y,Z)} = \frac{P(X,Z)}{P(Z)}$$

$$\Rightarrow P(X|Y,Z) = P(X|Z)$$

$$\frac{P(X,Y,Z)}{P(Z)} = \frac{P(X,Z)}{P(Z)} \cdot \frac{P(Y,Z)}{P(Z)} \Rightarrow \frac{P(X,Y,Z)}{P(Z)} \cdot \frac{P(Z)}{P(X,Z)} = \frac{P(Y,Z)}{P(Z)}$$

$$\Rightarrow P(Y|X,Z) = P(Y|Z)$$

5. The following graph is Bayesian network which includes probability of alarm ringing by burglary and/or earthquake. Compute the probability $P(j \wedge \neg m \wedge a \wedge \neg b \wedge e)$ from the below Bayesian network
(j: johncalls, m: marrycalls, a: alarm, b: burglary, e: earthquake). Show your work.



$$P(j|a) \times P(\neg m|a) \times P(a|\neg b,e) \times P(\neg b) \times P(e)$$

$$= 0.8 \times 0.35 \times 0.3 \times 0.999 \times 0.002$$

$$= 0.000167832$$