Determining the radius of curvature of a planoconvex lens through measurements of the Newton's Rings interference pattern

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The radius of curvature of a planoconvex lens can be calculated through measurements of the diameters of the fringes in the Newton's Rings interference pattern. The value obtained through initial experimentation is $R_{IE} = 15 \pm 3$m, this does not appear to be valid at first especially with the large uncertainty. It is not until after subsequent investigations have been made and other values are obtained, $R_{IE} = 15 \pm 3$m and $R_{RP} = 18 \pm 5$, that the initial value should be accepted.

I. INTRODUCTION

In nature, light can behave in two states, through wave-like and through particle-like properties. It is possible to observe the wave-like behaviour when rays of light are reflected or transmitted from different surfaces of a thin film. They interfere with one another when outgoing rays overlap, creating interference patterns.

For two waves to cause a steady interference pattern, the waves must be coherent. The usage of a thin film allows for coherency as the two reflected waves are part of the same initial burst. We can achieve this thin film (of air) between two surfaces by having the convex surface of a lens in contact with a plane glass plate. Then if we shine monochromatic light through the lens we can create an interference pattern of concentric rings progressively getting further apart from each other - known as Newton's Rings [1]. These were first observed by Robert Hooke in 1665, and then later studied by Sir Isaac Newton in 1775 [3].

By observing and measuring this phenomena it is possible to calculate the radius of curvature of a planoconvex lens.

We can derive a relationship between the radius of curvature, R, and other quantities. Initially, through using geometry, R can be shown to be related to the length of the lens divided by two (r), the greatest distance between lens and glass plate (d), and the spacing between the lens and the plate due to a non perfect contact (d_0),

$$R = \frac{r^2}{2(d + d_0)}. \tag{1}$$

Since interference is involved, $\Delta = m\lambda$ (constructive interference with light of wavelength λ), and $\Delta = 2d + \lambda$ (path difference due to reflections) can be used to find the square of the radius of a bright fringe with fringe number m,

$$r_m^2 = \lambda R((m - 1) + 2d_0). \tag{2}$$

Generally to observe the fringes the image is magnified by a factor of M so we get,

$$\rho_m^2 = M^2 R\lambda(m - 1) + 2M^2 R d_0, \tag{3}$$

this can be used accordingly with data collected from experimentation.

II. METHOD

The light source used in this experiment was provided by a mercury lamp. That light was then passed through

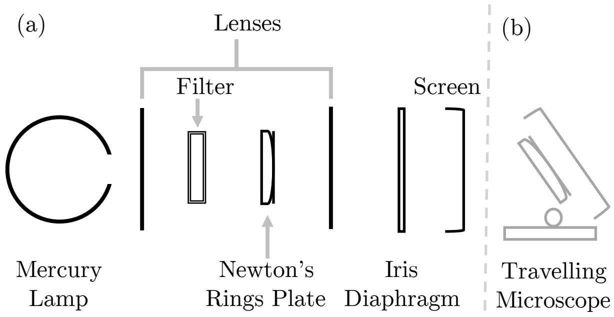

FIG. 1: (a) A schematic of the experimental set-up used to collect the initial set of data. Entire set-up was placed on an optical bench, allowing accurate positioning of the lenses.
(b) Modified set-up used in subsequent investigations - Newton's Rings Plate is moved, the screen is adjusted, and a Travelling Microscope is added.

a set-up as shown in Fig. 1(a), this allowed for the interference pattern to be projected onto a screen along with a horizontal scale.

The two main filters used varied by colour, yellow and green, this created the monochromatic light needed to view the interference fringes. It is worth noting that during set-up the positions of the lenses and rings plate often had to be adjusted so that the pattern and the horizontal scale could be both focussed to the best that they both could be. An iris diaphragm was added to the setup so that there could be the greatest contrast between the light and dark fringes. Once this was done an initial measurement of the magnification was performed by comparing the distance of one of the projected tick-marks on the scale to the actual given value.

Two sets of data were taken for each filter. In both cases the positions of the centre of the bright fringes along a horizontal axis were marked on a piece of paper attached to the screen. This was done for the 10 innermost bright fringes. The diameter was then measured for each fringe and a radius value was obtained. A least squares fitting was applied to the fringe radius squared, ρ_m^2, along with $(m - 1)\lambda$, where m is the ring number and λ is the wavelength of light used. For the yellow and green filters respectively, $\lambda_Y = 578 \pm 0.5$ nm and $\lambda_G = 546 \pm 0.5$ nm [3].

III. RESULTS

The measured fringe radius squared, ρ_m^2, and $(m - 1)\lambda$ (with m as fringe number and λ as wavelength of light), is shown in Fig. 1. Through using a least squares fitting and equation (3) a value obtained for the radius of curvature, R_{IE}, turns out to be 15 ± 3m.

In Table 1 we can see other values calculated for the radius of curvature from subsequent investigations using different methods but the same calculations. For example using a Travelling Microscope instead of direct screen measurements (see Fig. 1(b)) to measure fringe radius, and also repeating data collection.

Shown also in the table are values for d_0.

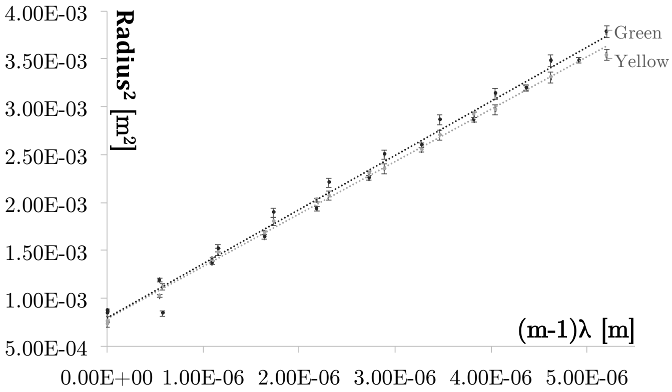

FIG. 2: The fringe radius squared plotted as a function of $(m-1)$ multiplied by wavelength. Horizontal error bars are too small to be seen.

Experiment	**Radius of Curvature, R** $[m]$	$\mathbf{d_0}[\times 10^{-7}, m]$
IE	15 ± 3	7 ± 2
TM	24 ± 2	4.6 ± 0.9
RP	18 ± 5	8 ± 3

TABLE I: Values for radius of curvature of the planoconvex lens, and values for d_0. IE denotes the 'Initial Experiment', TM for 'Travelling Microscope', and RP for 'Repeated'.

IV. DISCUSSION

In analysing our data we find that there are no official manufacturer's or literature value for the radius of curvature of the lens. We must compare the initial data and data collected in subsequent weeks of investigation in order to be able to draw some form of conclusion on what the radius of curvature should be.

If we look at the plotted data in Fig. 2. we see that while the points do generally show the linear relationship between ρ_m^2 and $(m-1)\lambda$, more than half do not lie either on or within their respective trendlines with their uncertainties. This could be due to the fact that during data collection, the bright and dark fringes blur into one another so distinguishing where the centre of the bright fringe lay was difficult to do, this would inevitably lead to distances being incorrectly measured. Furthermore, random errors may have arisen due to varying eyesight and the interference pattern not being focussed.

However, when we use this data to find a value for the radius of curvature, $R_{IE} = 15 \pm 3$m, we at first doubt it's validity - the uncertainty is very large along with the actual value. But, after collecting data with a Travelling Microscope and using the same analysis, we get $R_{TM} = 24 \pm 2$m, which is similar to R_{IE}. Given some consideration, we find that the large uncertainty could be due to how we measured the magnification, through measurement of only one of the projected tick-marks, if we had in fact measured ten of the distances individually, the uncertainty would have reduced.

This is the case for the repeated experiment, $R_{RP} = 18 \pm 5$, comparable values for the radius of curvature although the uncertainty has increased. This is because a different magnification was used, $\times 4$ instead of $\times 6$ (noting TM's $\times 1$).

There are a range of possible reasons for the differing values of R. For example, different filters were used in the investigations - what appeared to be the same colour each time could have been different. This would have lead to different wavelengths of monochromatic light being used. So the value for the radius of curvature would change accordingly, as can be seen by rearranging equation (3). This can also be caused by the non-perfect contact between the lens and glass plate, d_0. If the thickness of the film increases then the wavelength of the light in the film increases, so the radius of curvature will decrease.

We can see from Table 1 that when we used the Travelling Microscope set-up as shown in Fig. 1(b), the value for d_{0-TM} is smaller then compared to d_{0-IE} and d_{0-RP}. This could be due to the geometric path difference of the interfering light waves changing. The value would be decreasing after being reflected at an angle from the screen thus resulting in a decrease of the value for d_{0-TM}.

There are other ways in which the radius of curvature of a lens can be measured precisely using Newton's Rings, one is to use a digital camera and directly image the interference pattern produced - then digital software could be used to measure the fringe radius, avoiding any unnecessary random errors. On the other hand, we could coat the convex lens and the glass plate with high reflecting transparent silverings, this would allow the rings to appear sharper to avoid the issue of focussing the rings, moreover high precision could be attained here as well [2].

V. CONCLUSIONS

In conclusion, through measurements of Newton's Rings it has been possible to determine a value for the radius of curvature of a planoconvex lens, $R_{IE} = 15 \pm 3$. A value which at first was doubted, however after additional investigation it has been possible to compare and verify it is valid along with it's uncertainty. However, through modifying the experimental set-up it could be possible to reduce certain sources of uncertainties, and to remove some of the experimental issues experienced.

[1] Hugh D. Young and Roger A. Freedman. *University Physics with Modern Physics, 13th Edition.* Pearson Education Limited, Harlow, Essex, 2015.

[2] S. Tolansky. *New Interference Phenomena with Newton's Rings.* Nature, Vol. 153, 1944, p. 314.

[3] P. Swift. *Discovery Skills in Physics 2015/2016, Epiphany Term 2016, Full Experiments.* Department of Physics, Durham University, Durham, 2016

Appendix

The uncertainty calculated for the radius of curvature R was found by propagating other uncertainties related to quantities found in equation (3). The majority of them are based off formulae found in *Measurements and their Uncertainties* [I. G. Hughes and T. P. A. Hase, *Measurements and their Uncertainties*, Oxford University Press, Oxford, United Kingdom, 2010].

Firstly for the square of the magnified radius of a bright fringe, ρ_m^2, the error is,

$$\alpha_{r^2} = \alpha_r |2 \times r| \tag{4}$$

where r is the value for the magnified radius of a bright fringe, and α_r is the uncertainty on that. This arises due to a 30 cm ruler being used to measure the diameter, so the highest precision achievable would have been limited at half a division, therefore α_r=0.5 cm.

The uncertainty on the magnification squared, M^2, is found by,

$$\alpha_{M^2} = M^2 \left| 2 \times \frac{\alpha_M}{M} \right| \tag{5}$$

with α_M as the error on the magnification. For the combined term $(m-1)\lambda = n$, the uncertainty on this was found by using a least squares fitting function in the software Microsoft® Excel.

To calculate the error on the radius of curvature R, the following equation is thus used,

$$\alpha_R = R \times \sqrt{\left(\frac{\alpha_n}{n}\right)^2 + \left(\frac{\alpha_{M^2}}{M^2}\right)^2}. \tag{6}$$

The uncertainty on d_0 is found by further propagating the uncertainties found above, the resulting equation is,

$$\alpha_{d_0} = d_0 \times \sqrt{\left(\frac{\alpha_c}{c}\right)^2 + \left(\frac{\alpha_{M^2}}{M^2}\right)^2 + \left(\frac{\alpha_R}{R}\right)^2} \tag{7}$$

where c is the intercept of the least squares fitting, and α_c is it's respective uncertainty.