Determining the viscosity of water

Jacky Cao, Room 205, Friday, Lab Partners: Peter Dorey, Jon Pritchett
Date of experiment: 11/11/2016, Date of report: 20/11/2016

The volume flow rate of a fluid can be expressed functionally as derived by Poiseuille and Hagen. This relationship can be rearranged so that the viscosity of water can be experimentally determined. Using a capillary tube and altering the height of water in a tank, the volume and mass of water which has flowed out in a time period can be measured. Using this data, it is then possible to calculate a value for the viscosity of water. The average value was determined to be 1.00 ± 0.03 mPa s, which does not agree with the literature value, 1.679 mPa s [4].

I. INTRODUCTION

Derived experimentally by Poiseuille in 1838 and Hagen in 1839 [1], the volume flow rate dV/dt of a fluid passing through a tube can be expressed as a function of the density of the fluid ρ, the value for acceleration due to gravity g, the height the fluid leaves the tube h, the radius a and length L of the tube, and the viscosity of the fluid η,

$$\frac{dV}{dt} = \frac{\pi}{8}\frac{\rho g h}{\eta}\frac{a^4}{L}. \tag{1}$$

Noting that the group $\rho g h$ can be collectively termed as the pressure difference ΔP between the two ends of the tube [2].

If we consider that a fluid is flowing through a tube like this, it experiences both friction with the inner wall and internal friction within itself. The latter can be defined more readily as the viscosity of the fluid, η, this results in shear stress when two adjacent layers (laminas) of fluid move relative to each other.

We find that as η increases, the volume flow rate decreases, the shear stress between two laminas becomes greater and so restricts the movement of the fluid's molecules trying to flow through the tube.

Generally we can say that the lamina flow streamlines are smooth, the top layers of fluid are sliding over other laminas without the system having any turbulent motion - this condition is required for equation (1) to be valid.

Using a rearranged form of the relation derived by Hagen and Poiseuille, and making initial assumptions, it is possible to experimentally calculate a value for the viscosity of water. The assumptions being that the fluid is incompressible, the temperature of the water does not change, and the volume flow rate of water follows a linear relationship with height.

II. METHOD

A flow of water was created by fixing a capillary tube to a water tank. The tank was raised to an initial height using an adjustable platform. The tank was then filled up with water from heights 2 cm to 16 cm at 2 cm intervals, these values were measured with the markings on the side of the tank. During this we had to ensure that we did not create parallax between the level of the water and the markings on the side.

Water was allowed to flow out of the tube for a period of 90s for each height of water, this time was measured using a digital stopwatch. As the water flowed out it was caught within a large beaker so that the mass and volume of it could be measured after the allotted time had passed.

The mass was found by having the beaker already on a set of electronic scales and initially zeroed to account for the beaker's mass. The volume on the other hand required the water to be transferred from the beaker to a measuring cylinder, this was performed by using a pipette. Care was again taken so that there was no parallax and that no unaccounted water was left within the beaker. It was also necessary to clear the tube once air bubbles formed along the length of it - this was done using a long piece of copper wire.

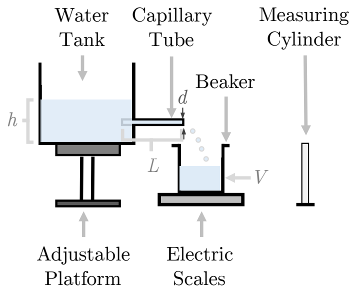

FIG. 1: A schematic of the experimental set-up used to collect data.

An initial value for the density of water was calculated by taking four separate measurements of the volume and mass of the water, then calculating four individual values for ρ, and then averaging them.

One set of data was taken for each of the three capillary tubes that were used. Each tube varied in their internal diameter, d, this was measured using a travelling microscope along the horizontal axis.

The collected data was applied to a least squares fitting to create an initial linear model. This was then used to calculate the flow rate for each different height, and thus allowing us to work out η.

III. RESULTS

The volumetric flow rate of water is plotted against the varying height, as shown in Fig. 2. This flow rate was calculated by dividing each value of the measured volume with the measured time period. Using this data, a value of viscosity could be calculated through a rearranged form of equation (1),

$$\eta = \frac{\pi \rho g a^4}{8Lm}, \tag{2}$$

where m is the gradient of the least squares regression line, calculated with the known data for dV/dt and h.

From preliminary results taking and analysis, our density of water is 1001 ± 1 kgm^{-3}.

The calculated values for the viscosity of water are shown in Table I with which respective tube was used, the radius, and the reduced χ^2 statistic for each of those tubes.

An average value for viscosity can thus be found to be 1.00 ± 0.03 mPa s, which does not agree with the literature value [4], 1.679 mPa s.

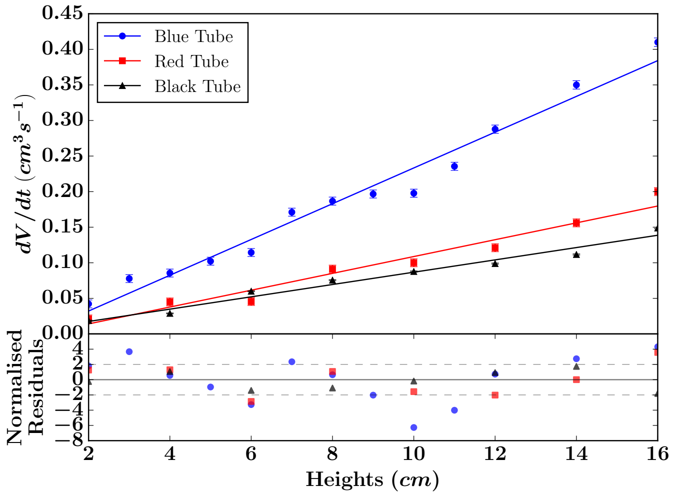

FIG. 2: The volume flow rate of water (dV/dt) as a function of height (h) for three capillary tubes of different diameter. The vertical error bars on dV/dt are too small to be seen.

Tube	**Radius, a [mm]**	η **[mPa s]**	χ^2_ν
Blue	0.55 ± 0.03	1.0 ± 0.2	11.0
Red	0.47 ± 0.03	1.1 ± 0.3	5.31
Black	0.46 ± 0.03	1.0 ± 0.3	1.94

TABLE I: For each tube is shown its radius, their respective calculated value for the viscosity of water η, and the reduced chi-squared statistic, χ^2_ν.

IV. DISCUSSION

From an initial inspection of Fig. 2 we see that the change in dV/dt with h appears to follow a linear regression model. However, the normalised residuals are not contained within ± 2, for the blue tube about 46% of the points are outside this. This suggests that the fit of the blue data is not good and further experimentation may be required.

We can further explore the validity of our data by looking at the calculated reduced chi-squared statistics. A reasonable fit to the model should expect χ^2_ν to be approximately one [6]. After minimising and reducing χ^2, we see that the black tube has the closest χ^2_ν value to unity, so we can say this set of data has the best fit to the linear model, and that the viscosity value calculated here would be the most acceptable. We could suggest this may be because the black tube had the smallest diameter so the flow rate would be slower, thus the actual volume that flowed out would be easier to measure.

From Table I we can see that all three values of η do agree with each other within their experimental errors. Although, the calculated value for the viscosity using the blue tube has the biggest disparity when compared with the literature value, it also has the greatest χ^2_ν value. The data not fitting the model could be due to the volume not being the exact value that had flowed out. With increased flow rate from a greater diameter, it proved more difficult to know when exactly to remove the beaker away from the water flow.

With our experimentation, there were other limitations affecting the measured value for the volume of water. It is more than likely that the V we measured was not the actual value that had flowed out of the tank for each tube. This problem arose due to the fact that there was always some water left within the pipette when transferring from beaker to the measuring cylinder, and also that the capillary tube would not form a perfectly air tight seal with the tank so water inevitably leaked out.

Another experimental limitation was that the diameter of each tube was not uniform throughout. With this, we see that due to the tapering of the tube, the volume flow rate would have been reduced as the flow of the water would have been non-uniform. Thus again, shifting our measured quantities from their true values.

With our analysis, other factors were not taken into account during our calculations such as the variation in the density of water with temperature change. From the Thiesen-Scheel-Diesselhorst equation [5], we see that as the temperature of water increases, the density decreases, meaning that the viscosity will decrease as a result. While we assumed that the fluid was incompressible, our calculations could be altered to account for this and thus potentially produce a result for η which would be closer to the literature value.

For our experiment we took only one set of data for each tube as we were constrained by time, for the red and black tubes we had to reduce the amount of measurements that we were taking. In future experiments it would be highly beneficial to take repeat data sets for each tube. This would allow us to create a clearer picture of whether or not our data is a good fit to the model, and if our viscosities truly disagree with the stated literature value.

V. CONCLUSIONS

In conclusion, from measurements of the volume and mass of water, it was possible to calculate an average value for the viscosity of water. This was found to be 1.00 ± 0.03 mPa s, which is not consistent with the literature value of 1.679 mPa s [4]. Our data did not prove to be a perfect fit to the linear regression model. More experimentation would be required to produce results that would offer a clearer picture, and noting that various improvements would be required in the methodology to improve accuracy.

[1] Salvatore P. Sutera and Richard Skalak *The History of Poiseuille's Law.* Annu. Rev. Fluid Mech., 1993.

[2] Raymond A. Serway, Chris Vuille, and Jerry S. Faughin *College Physics, 8th Edition.* Brooks/Cole, Belmont, CA, USA, 2009.

[3] Hugh D. Young and Roger A. Freedman. *University Physics with Modern Physics, 13th Edition.* Pearson Education Limited, Essex, UK, 2015.

[4] W. M. Haynes. *CRC Handbook of Chemistry and Physics, 92nd Edition.* CRC Press, Florida, USA, 2011.

[5] J. L. Martin and S. C. McCutcheon *Hydrodynamics and Transport for Water Quality Modelling.* CRC Press, Florida, USA, 1999.

[6] I. G. Hughes and T. P. A. Hase *Measurements and their Uncertainties.* Oxford University Press, Oxford, UK, 2010.

Appendix

For the data that was collected using analogue devices, the precision is half a devision [4]. The uncertainty in the measurement of the height h vertically up the side of the water tank was found to be 0.5 cm, and for the volume of water within the measuring cylinder it was 0.5 cm^3.

With the capillary tubes, a 30 cm ruler was used to measure the total length of the tube and the uncertainty was 0.5 cm. The diameter of the tube was measured using a travelling microscope setup with a vernier scale. We could not say the uncertainty was 1×10^{-4} cm as there was error in defining the beginning and end points of the diameter, plus there was also difficult in reading the vernier scale. The uncertainty on the diameter was thus took to be entire division greater, so we estimated a value of 3×10^{-4} cm.

For the measured mass using the electric scales, the uncertainty was taken to be one in the last digit [4], 0.1 g.

The following uncertainties were found using equations based off formulae found in *Measurements and their Uncertainties* [I. G. Hughes and T. P. A. Hase, *Measurements and their Uncertainties*, Oxford University Press, Oxford, United Kingdom, 2010].

For the average value of the density of water, its uncertainty was,

$$\alpha_\rho = \rho\sqrt{\sum_{i=1}^{4}\left(\frac{\alpha_{\rho_i}}{\rho_i}\right)^2}. \quad (3)$$

ρ_i represents the value of density calculated from the ith set out of the four initial data readings. The density was found by dividing the mass m_i by the volume V_i, thus,

$$\alpha_{\rho_i} = \rho_i\sqrt{\left(\frac{\alpha_{m_i}}{m_i}\right)^2 + \left(\frac{\alpha_{V_i}}{V_i}\right)^2}, \quad (4)$$

where m_i is the mass of a specific volume of water V_i, and α_{m_i} and α_{V_i} are their respective uncertainties.

The error in the value of the volume flow rate was calculated with this equation,

$$\alpha_{V'} = \left[\left(\frac{V+\alpha_V}{t} - \frac{V}{t}\right)^2 + \left(\frac{V}{t+\alpha_t} - \frac{V}{t}\right)^2\right]^{0.5}. \quad (5)$$

For the linear model that was used in our data analysis, the uncertainty in the gradient α_m and the gradient m itself was found using the LINEST function on Microsoft Excel. These values were then reduced using Solver on Microsoft Excel during the minimisation of the calculated value for χ^2. The final minimised gradient and its error for each tube is shown below in Table II.

Tube	$\boldsymbol{m}$ **(cm^2s^{-1})**
Blue	0.025 ± 0.001
Red	0.012 ± 0.001
Black	0.009 ± 0.001

TABLE II: The minimised values for the gradient of the linear regression model for each tube. The uncertainties on the gradient is also shown.

The uncertainties on the calculated values for the viscosity of water were calculated using the following equation,

$$\alpha_\eta = \eta\left[\left(\frac{\alpha_{\rho g}}{\rho g}\right)^2 + \left(\frac{\alpha_{ha^4}}{ha^4}\right)^2 + \left(\frac{\alpha_L}{L}\right)^2 + \left(\frac{\alpha_m}{m}\right)^2\right]^{0.5}. \quad (6)$$

with η found from equation (2) and the uncertainty on ρg and ha^4 being,

$$\alpha_{\rho g} = \rho g\sqrt{\left(\frac{\alpha_\rho}{\rho}\right)^2 + \left(\frac{\alpha_g}{g}\right)^2}, \quad (7)$$

$$\alpha_{ha^4} = ha^4\sqrt{\left(\frac{\alpha_h}{h}\right)^2 + \left(\frac{\alpha_{a^4}}{a^4}\right)^2}, \quad (8)$$

respectively. The error on a^4,

$$\alpha_{a^4} = a^4\left|4\frac{\alpha_a}{a}\right|. \quad (9)$$

The uncertainty on our average value for the viscosity of water was found using the standard error,

$$\alpha_{\eta-av} = \frac{\sigma_{N-1}}{\sqrt{N}}, \quad (10)$$

with σ_{N-1} as the standard deviation of the mean, and N as the number of data points.

Our χ^2 values were found using this formula,

$$\chi^2 = \sum_i \frac{(y_i - y(x_i))^2}{\alpha_i^2}, \quad (11)$$

where y_i represents the theoretical linear model of the volume flow rate, $y(x_i)$ the calculated value for the volume flow rate (V') using the measured V and t values, and α_i is the uncertainty on V' as shown in equation (5).

χ^2 was then minimised using the Solver function on Microsoft Excel, this value was then reduced using the following formula,

$$\chi^2_\nu = \frac{\chi^2_{min}}{\nu}, \quad (12)$$

ν being the number of degrees of freedom. For our experiment ν was 11 for the blue tube, and 6 for the red and black tube.

The normalised residuals plotted in Figure 2 are defined as,

$$R_i = \frac{y_i - y(x_i)}{\alpha_i}, \quad (13)$$

with $(y_i - y(x_i))$ representing the difference between the theoretical model and the data, and α_i is the uncertainty in the data ($\alpha_{V'}$).