Determining the viscosity of water

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Date of experiment: 11/11/2016, Date of report: 20/11/2016

The Newton's Rings interference pattern is a consequence of the wave-like nature of light. By performing analysis of measurements of the diameters of the interference fringes it is possible to calculate the radius of curvature of a planoconvex lens. The value obtained through initial experimentation is $R_{IE} = (15 \pm 3)$m, initially not taken as valid at first due to the large uncertainty. It is not until after subsequent investigations have been made and other values are obtained, $R_{TM} = (24 \pm 2)$m and $R_{RE} = (18 \pm 5)$m, that the initial value should be accepted.

I. INTRODUCTION

Derived experimentally by Poiseuille in 1838 and Hagen in 1839 [1], the volume flow rate $\frac{dV}{dt}$ of a fluid passing through a tube can be expressed as a function of the density of the fluid ρ, the value for acceleration due to gravity g, the height the fluid leaves the tube h, the radius a and length L of the tube, and the viscosity of the fluid η,

$$\frac{dV}{dt} = \frac{\pi}{8} \frac{\rho g h}{\eta} \frac{a^4}{L}. \tag{1}$$

Noting that the group $\rho g h$ can be collectively termed the pressure difference ΔP between the two ends of the tube [2].

If we consider that a fluid is flowing through said tube, it experiences both friction with the inner wall and internal friction within itself. The latter can be defined more readily as the viscosity of the fluid η, and this results in shear stress when two adjacent layers (laminas) of fluid move relative to each other.

We find that as η increases, the volume flow rate decreases, the shear stress between two laminas becomes greater and so restricts the movement of the fluid's molecules trying to flow through the tube.

Generally we can say that the lamina flow streamlines are smooth, top layers sliding over other laminas without the system having any turbulent motion - this condition is required for equation [1] to be valid.

Using the relation derived by Hagen and Poiseuille it is possible to experimentally calculate a value for the viscosity of water, η_{water}.

II. METHOD

A flow of water was created by attaching three different capillary tubes to a water tank. The

III. RESULTS

Shown also in the table are values for d_0.

Experiment	**Radius of Curvature, R** $[m]$	$\boldsymbol{d_0} [\times 10^{-7}, m]$
IE	15 ± 3	7 ± 2
TM	24 ± 2	4.6 ± 0.9
RE	18 ± 5	8 ± 3

TABLE I: Values for radius of curvature of the planoconvex lens, and their respective values for d_0. IE denotes the 'Initial Experiment', TM for 'Travelling Microscope', and RE for 'Repeated Initial Experiment'.

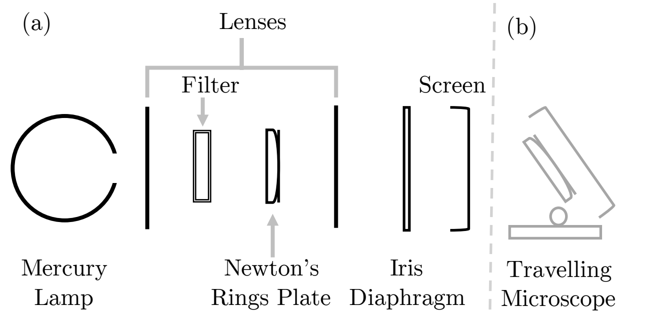

FIG. 1: (a) A schematic of the experimental set-up used to collect the initial set of data. Entire set-up was placed on an optical bench, allowing accurate positioning of the lenses. (b) Modified set-up used in subsequent investigations - Newton's Rings Plate is moved, the screen is adjusted, and a Travelling Microscope is added.

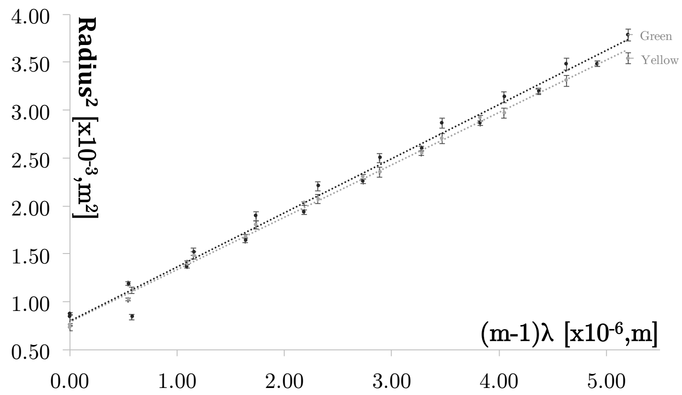

FIG. 2: The fringe radius squared plotted as a function of $(m-1)\lambda$. Horizontal error bars are too small to be seen.

IV. DISCUSSION

V. CONCLUSIONS

In conclusion, through measurements of Newton's Rings it has been possible to determine a value for the radius of curvature of a planoconvex lens, $R_{IE} = (15 \pm 3)$m. A value which at first was doubted, however after additional investigation it has been possible to compare and verify it is valid along with it's uncertainty. However, through modifying the experimental set-up it could be possible to reduce certain sources of uncertainties, and to remove some of the experimental issues experienced.

[1] Salvatore P. Sutera and Richard Skalak *The History of Poiseuille's Law*. Annu. Rev. Fluid Mech., 1993.
[2] Raymond A. Serway, Chris Vuille, and Jerry S. Faughin *College Physics, 8th Edition*. Brooks/Cole, Belmont, CA, USA, 2009.
[3] Hugh D. Young and Roger A. Freedman. *University Physics with Modern Physics, 13th Edition*. Pearson Education Limited, Harlow, Essex, 2015.

Appendix

The uncertainty calculated for the radius of curvature R was found by propagating other uncertainties related to quantities found in equation (3). The majority of those uncertainties are calculated using equations based off formulae found in *Measurements and their Uncertainties* [I. G. Hughes and T. P. A. Hase, *Measurements and their Uncertainties*, Oxford University Press, Oxford, United Kingdom, 2010].

Firstly for the square of the magnified radius of a bright fringe, ρ_m^2, the error is,

$$\alpha_{r^2} = \alpha_r |2 \times r| \tag{2}$$

where r is the value for the magnified radius of a bright fringe, and α_r is it's respective uncertainty. In our experiment this value arises due to a 30 cm ruler being used to measure the diameter, so the highest precision achievable would have been limited at half a division, therefore α_r=0.5 cm.

The uncertainty on the magnification squared, M^2, is found by,

$$\alpha_{M^2} = M^2 \left| 2 \times \frac{\alpha_M}{M} \right| \tag{3}$$

with α_M as the error on the magnification. For the initial and repeated experiment this value was taken to be 0.5 due to the difficulties in measuring the width of one of the tick-marks with a 30cm ruler. In the Travelling Microscope experiment $\alpha_M = 0.005$ due to the precision to which we could measure with the vernier scale.

To calculate the uncertainty on the radius of curvature R, the following equation is thus used,

$$\alpha_R = R \times \sqrt{\left(\frac{\alpha_n}{n}\right)^2 + \left(\frac{\alpha_{M^2}}{M^2}\right)^2}, \tag{4}$$

where the term $n = (m-1)\lambda$, and the uncertainty on that, α_n, was found by using a least squares fitting function in the software Microsoft® Excel. M^2 is again, the magnification squared with it's uncertainty α_{M^2} found above in equation (5).

The uncertainty on d_0 is found by further propagating the uncertainties found above, the resulting equation is,

$$\alpha_{d_0} = d_0 \times \sqrt{\left(\frac{\alpha_c}{c}\right)^2 + \left(\frac{\alpha_{M^2}}{M^2}\right)^2 + \left(\frac{\alpha_R}{R}\right)^2} \tag{5}$$

where c is the intercept of the least squares fitting, and α_c is it's respective uncertainty.

With the case of the Travelling Microscope there is also the uncertainty on the vernier scale used, we shall take the highest precision it can measure to be $\alpha_{vernier} =$ 0.5mm.