CONSTRAINING THE GEOMETRY OF THE *UNIVERSE*

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AN INTRODUCTION TO STELLAR-BASED COSMOLOGY

Type Ia Supernovae

To probe the geometry of the Universe, Type Ia supernovae can be utilised. Known for their curious homogenous nature, they are used as standard candles, a feature which can be taken advantage of to calibrate cosmic distances.

Through measuring their magnitude as time evolves, a 'light-curve' can be plotted and a maximum B-band magnitude obtained. With this and a value for the supernova redshift, z, we can use Hubble's Law and the Friedmann equation to find a value for the dark energy density, Ω_Λ.

In our model, the following equation was utilised in the Friedmann equation as the **Hubble Parameter**,

$$H=H_0[(1+z)^3-\Omega_\Lambda((1+z)^3-1)]^{1/2}.$$

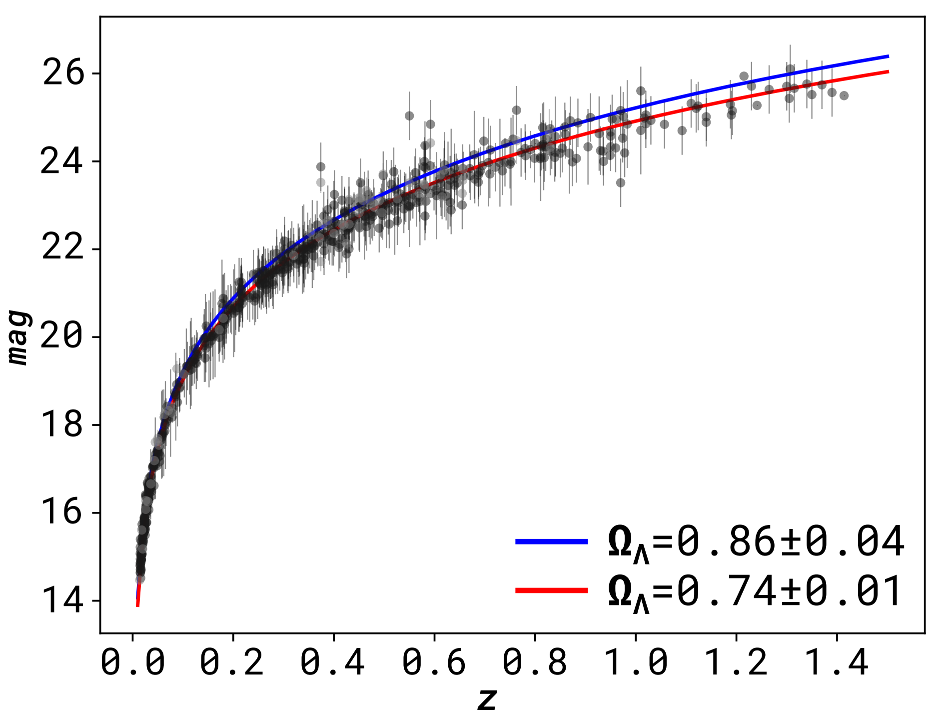

FIG. 1: Hubble's diagram plotted with data from the Supernova Cosmology Project and using data provided for our initial research – uncertainties have been plotted as well. Two models were also fitted, one using Ω_Λ calculated with the initial data (blue), and a second model with the extended data set (red). The given magnitudes (mag) are in the B band and the redshifts (z) are unit-less.

A Least-Squares Fit

For our first model cosmic model, we only had to find an optimum value for the dark energy density parameter, Ω_Λ. To find this we chose to employ X^2 statistics. With,

$$X^2=\Sigma(f_{obs}-f_{model})^2/\sigma^2,$$

where f_{obs} is the flux of the observed supernovae, f_{model} is the flux for the model corresponding to the same redshift, and σ is uncertainty on the supernova data.

Each model used a different Ω_Λ, the smallest value for X^2 overall would correspond to the best parameter.

We found a value of $\mathbf{\Omega_\Lambda=0.86\pm0.04}$ in our initial experimentation. Compared to a literature value of $\mathbf{\Omega_\Lambda=0.761\pm0.018}$, we find that it is similar, but more work is required to improve the accuracy.

AN EXPLORATION OF BAYESIAN STATISTICS

UNION2.1 Data Set

Using a larger data set improves the accuracy of our results, with the *Supernova Cosmology Project*'s Union2.1 data set and with the X^2 method we were able to produce $\mathbf{\Omega_\Lambda=0.74\pm0.01}$ which is more consistent to literature than previously found.

In Figure 1 we can see the fitting of this larger data set to our two models which made use of Ω_Λ.

Bayesian Statistics

To work with a model with multiple parameters, MCMC analysis was chosen. It allowed us to easily sample and manipulate a parameter space to obtain accurate parameters.

This method simply assigns a probability to the model produced and to the corresponding parameters used, the system with the highest likelihood would then be accepted.

We attempted this for the single Ω_Λ model and obtained **0.720±0.008**. In Figure 2 we see the multiple spaces attempted until optimum was found.

A larger model

We chose to implement the full **Lambda Cold Dark Matter** model (ΛCDM). With this, the Hubble Parameter became,

$$H=H_0[X(z)\Omega_\Lambda+(1+z)^2\Omega_k+(1+z)^3\Omega_m+(1+z)^4\Omega_r)]^{1/2},$$

where $X(z)$ is the dark energy density, Ω_k the spatial curvature, Ω_m the matter density, and Ω_r as the radiation from photons and neutrinos.

In Table I we provide the results that we achieved if $X(z)$ is set to one. We see that they are in strong agreement with literature values.

An extra parameter

To build on using ΛCDM, our future research would involve setting $X(z)$ equal to $(1+z)^{3(1+\omega)}$, where ω is the dark energy equation of state. This would incorporate **quintessence** into our model and allow us to constrain even more parameters together. But, we must be aware that there may be a large correlation between some parameters and this would definitely need to be corrected for.

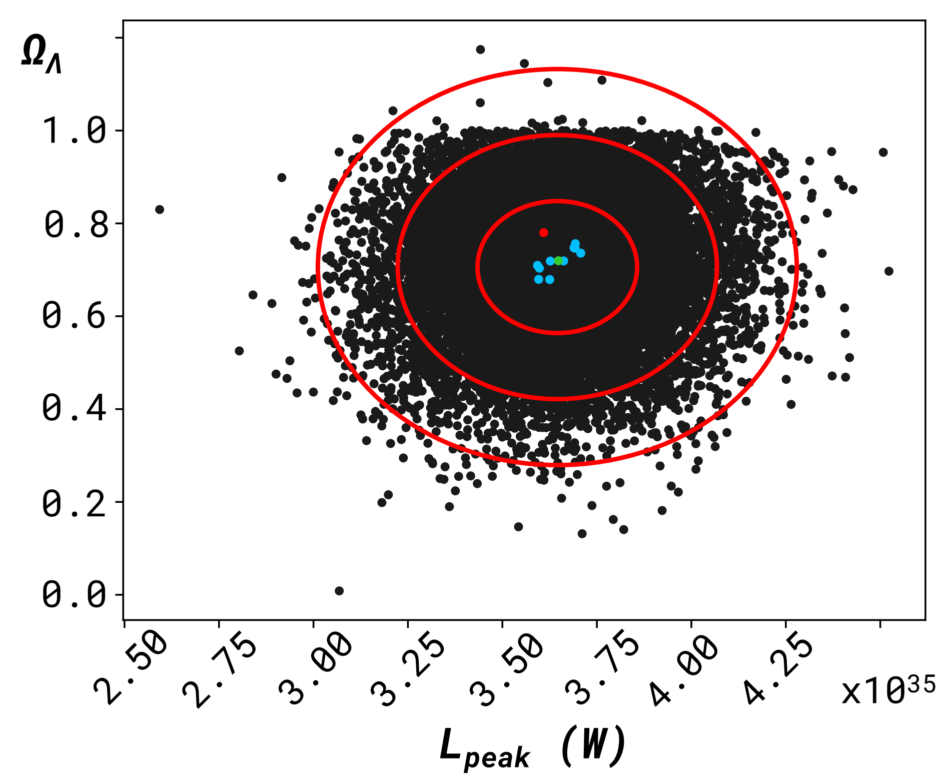

FIG. 2: Plot showing the MCMC sample space. Ω_Λ was fitted with L_{peak}, an extra variable in the supernova flux model. 30,000 parameter values were tested over 10 runs of 1500 tests each. The red dot shows the initial search, the blue shows the most likely from each run, and the green is the final averaged values. The red ellipses show the standard deviation spread (1σ, 2σ, and 3σ). We constrained Ω_Λ <1.0 as anything above would not be physically possible.

REFERENCES

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ΛCDM Results

	MCMC Analysis	*Literature*
Ω_Λ	0.76±0.05	$0.761^{+0.017}_{-0.018}$
Ω_k	-0.005±0.001	$-0.0030^{+0.0095}_{-0.0102}$
Ω_m	0.33±0.02	$0.239^{+0.018}_{-0.017}$
Ω_r	$(3\pm1)\times10^{-5}$	4.16×10^{-5}

TABLE I: Cosmological parameters obtained through MCMC Analysis acting on the Friedmann equation. Results are consistent with literature values.