Constraining the geometry of the Universe using Type Ia supernovae and statistical methods

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Type Ia supernovae have the unique trait of being standard candles, their light curves can be used in cosmology to calculate and constrain cosmological parameters. In observing Type Ia supernovae and fitting model light curves to such data one can attempt to derive such values. We have monitored, collected, and analysed data for supernova explosions over a period of 34 days. A 16″ and a 0.5 m telescope situated in Durham and La Palma was used for this project. After calculating the magnitudes for a Type Ia (2017hhz) and Type Ia-91bg (2017hle) supernova object, we fitted template light curves with the Python program, *SNooPy*. The quality of fit for the program's `fit()` function was deemed to be acceptable in accordance to the average reduced χ^2 values calculated for the B and V photometric bands, $\chi^2_{\nu,B} \approx 1.38$ and $\chi^2_{V,\nu} \approx 2.95$ - a good fit requiring $\chi^2_\nu \approx 1$. The distance modulus to the supernova 2017hhz was calculated by SNooPy to be 36.121 ± 0.106 mag, using this value we were able to compute $H_0 = 70 \pm 20$ km s^{-1} Mpc^{-1}. However, the quoted error negates the meaning of H_0 as it is too large of an uncertainty. In improving the accuracy and uncertainties we suggest that more observations of the supernovae were required, and constraining values should be used for the parameters in SNooPy's templates.

I. INTRODUCTION AND THEORY

In cosmology, one can argue that one of the most important observable events are supernova explosions. As a massive main sequence star runs out of nuclear fuel to burn, the equilibrium configurations which initially provided structure will cease to exist. What follows is a cataclysmic supernova explosion [6].

We can generally class supernova explosions into two separate groups, Type I and Type II supernovae. The main distinction arises due to the fact that Type I's have an optical spectra which contains no Balmer hydrogen features, whilst Type II supernovae do contain this hydrogen feature [2].

Within these two subclasses there are further divisions which can be characterised through their spectra and through features as found in their light curves [3]. Light curves are a way to show the evolution of a supernova's magnitude as time passes. For example, with two of the subclasses of Type II supernovae, Type II-L and Type II-P, they can be classed from features of their light curves. For a Type II-L supernova there is a rapid linear decrease in magnitude, and with a Type II-P we see a constant magnitude after maximum [2].

For the application of supernovae in cosmology, we must turn to the variety of Type Ia.

a. Type Ia Supernovae

It is accepted that the light curves of Type Ia supernovae are generally homogeneous [14], this means that they can be utilised as standard candles therefore allowing us a measure of cosmic distance.

Their homogeneity arises due to the mechanism behind the explosion. The progenitor system for Type Ia's consist of a binary system with a primary white dwarf and secondary star which has a mass close towards the Chandrasekhar limit of $1.4M_\odot$ [2, 14]. The white dwarf will accrete matter from it's companion until it itself reaches the critical mass limit. After this point we would then expect the primary star to collapse into a neutron star, however this is not what happens. Instead, we find that a supernova explosion occurs. An event which arises due to a massive disruption to the star's internal structure by a large amount of energy. [[give a rough value]]

It is currently understood that as the white dwarf accretes matter it heats up and produces thermonuclear energy. This energy is achieved in the stellar interior at a temperature of 10^9 K, as a result a disruption to the electron degeneracy pressure takes place. The star becomes gravitationally unstable from the nuclear energy and a total collapse into a neutron star is prevented [6, 14].

This particular mechanism can be used to explain the standard profile of Type Ia light curves. Using these plots, the maximum B band photometric magnitude can be obtained with the following equation,

$$M_{\max}(B) = -21.726 + 2.698\Delta m_{15}(B), \qquad (1)$$

where $\Delta m_{15}(B)$ is the decline rate parameter or Phillips' parameter [11]. This is defined as the mean rate of decline of the B band light curve from peak light to 15 days after the maximum [10]. This parameter relates the apparent magnitude to time and provides a way to compare and contrast different Type Ia supernovae.

With values for the absolute magnitude of Type Ia supernovae and with values of their redshift obtained from spectroscopic observations, the distance to these objects can then be calculated using the following equation,

$$\mu = 5\log_{10}(d) - 5, \qquad (2)$$

where μ is the distance modulus and can be defined as the absolute magnitude of supernova subtracted from it's apparent magnitude, and we define d is the distance to the supernova in parsecs [2].

From data sets of Type Ia supernovae we can easily employ them to calculate the geometry of the universe. But before that, we must first understand what cosmological parameters are and how they can be constrained.

b. Cosmological Parameters

At large cosmological distances, the appearance of objects will be affected by the spacetime which it travels through. If we therefore wanted to describe the geometrical properties of the universe, we would require to solve Einstein's general theory of relativity. From *An Introduction to Modern Astrophysics* [2] we can build the framework required to calculate and compute cosmological parameters.

If we solve Einstein's field equations for an isotropic, homogenous universe a description of the evolution of the universe can be obtained in the form of a differential equation which is also known as the Friedmann equation. In the 1922 form of the equation, a non-static universe is accounted for,

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2, \tag{3}$$

where $R(t)$ is the scale factor, G the gravitational constant, ρ is the mass density, k is a parameter which describes the shape of the universe, and c is the speed of light in a vacuum.

k can be defined as a constant which is equal to -1 for a negatively curved universe, 0 for a spatially flat universe, and $+1$ for a positively curved universe [6].

Separate work performed by Einstein eventually led to a cosmological constant Λ being introduced in the Friedmann equation. This was included in the form of $\frac{1}{3}\Lambda c^2$ within the square brackets of the left hand side of Equation 3. Whilst Einstein included this term to account for his static and closed universe, as a consequence of observations which implied an accelerating universe, astronomers in the 1990s eventually related the cosmological constant to dark energy.

Taking the Friedmann equation, it can be rewritten so that it considers a three component universe composed of baryonic and dark matter, relativistic photons and neutrinos, and dark energy,

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G(\rho_M + \rho_{\mathrm{rel}} + \rho_\Lambda)\right]R^2 = -kc^2, \tag{4}$$

where ρ_M is the density of matter, ρ_{rel} is the density of the radiation, and ρ_Λ is the density of the dark energy. Combining these densities with a critical density we can adapt the Friedmann equation so that it includes *cosmological parameters*, these are values which describe the content makeup of the universe in terms of matter and energy. This critical density is defined as the value of the density that will result in a flat ($k = 0$) universe. We see that the Friedmann equation becomes,

$$H^2[1 - (\Omega_M + \Omega_{\mathrm{rel}} + \Omega_\Lambda)]R^2 = -kc^2, \tag{5}$$

where H is the Hubble parameter and is related to the scale factor and expansion factors of the universe, Ω_M is the matter density parameter, Ω_{rel} is density contribution from radiation, and Ω_Λ is defined as the dark energy density parameter.

With this in hand, it is now possible to calculate the proportions of the universe which amounts to matter and the amount which is energy. In these calculations, cosmologists normally choose to omit the contribution from radiation as it is a negligible amount. Results from the Wilkinson Microwave Anisotropy Probe (WMAP) show that $\Omega_{\mathrm{rad}} = 8.25 \times 10^{-5}$, an insignificant value when compared to $\Omega_\Lambda = 0.74 \pm 0.04$ and $\Omega_M = 0.27 \pm 0.04$ [2].

Using these cosmological variables we can additionally define the total density parameter,

$$\Omega \equiv \Omega_M + \Omega_\Lambda + (\Omega_{\mathrm{rel}}). \tag{6}$$

Inputting the WMAP data into Equation 6 we see that a value of $\Omega \sim 1$ is produced, indicating that the universe is flat with $k = 0$ and that dark energy is the dominant factor in the expansion of the universe [2].

This result was obtained through measurements of the cosmic microwave background, however it would be especially useful if data from other cosmic objects could be used. We therefore return to Type Ia supernovae as they are the perfect candidate to help us constrain the geometry of the universe. They are more readily accessible and observations can be more easily made.

c. Cosmological Parameters from Type Ia Supernovae

To constrain the cosmological parameters using supernovae we do not look at the Friedmann equation directly. Instead, the main methodology is to use a Least Squares test to select the best model which would then correspond to cosmological parameters. We can define this test as the χ^2 statistic,

$$\chi^2 = \sum \frac{(f_{\mathrm{obs}} - f_{\mathrm{model}})^2}{\sigma^2_{\mathrm{obs}}}, \tag{7}$$

where f_{obs} is the peak observed flux of the supernova, f_{model} is the model flux for the supernova calculated using the corresponding redshift and with certain defined cosmological parameters, and σ_{model} is the uncertainty on the observational flux [1].

Obtaining f_{obs} is a simple matter, we can find it by using a rearranged form of,

$$m_{\mathrm{obs}} = m_0 - 2.5\log_{10}(f_{\mathrm{obs}}), \tag{8}$$

that is, m_{obs} is the effective peak magnitude of the supernova, m_0 is the instrumental zero point constant with a value of -20.45, and f is the supernova flux in units of erg $\mathrm{cm}^{-2}\mathrm{s}^{-1}\text{Å}^{-1}$ [1].

On the other hand, to find f_{model} we begin by defining the flux equation for a certain model,

$$f_{\mathrm{model}} = \frac{L_{\mathrm{peak}}}{4\pi[R_0 S(\eta)]^2(1+z)^2}, \tag{9}$$

where L_{peak} is the peak luminosity value for Type Ia supernovae, $R_0 S(\eta)$ is the comoving distance between the observer and where the supernova exploded in space, and z is the redshift for the supernova [1].

To find L_{peak}, the low-redshift ($z < 0.1$) Type Ia supernovae objects can be selected out of a data set and then Equation 9 can be used by taking the approximation $R_0 S(\eta) \approx cz/H_0$. Optimum L_{peak} can then found by using Equation 7, the smallest value for χ^2 corresponds to the best luminosity. This value can then be applied to calculations involving supernovae which have redshifts higher than 0.1. For this data set the comoving distance cannot be simply approximated so we must use the following definition,

$$R_0\eta = c\int_0^z \frac{dz'}{H(z')}, \tag{10}$$

we find that within this integral we need to integrate a form of the Friedmann equation between a value for the redshift of the supernova z and no redshift. We can derive this by using Equation 5 and with the assumption that if the universe is flat then $\Omega_M = 1 - \Omega_\Lambda$, reducing the cosmological parameters which we have to find [1].

We now arrive at the following result for the Hubble Parameter,

$$H^2 = H_0^2[(1+z)^3 - \Omega_\Lambda((1+z)^3 - 1)], \quad (11)$$

where H_0 is a value for the Hubble's constant, z is the redshift for a supernova, and Ω_Λ is a cosmological constant.

To find the optimum value for Ω_Λ we can select a range from 0.0 to 1.0 and then calculate f_{model} and χ^2 for each number. Identical to finding L_{peak}, the best value for the cosmological parameter is found by choosing the minimum value for χ^2. Through the usage of this least squares method one is able to explore cosmological parameters using Type Ia supernova data sets.

d. Project Aims

In this paper we discuss the experimental work performed to constrain cosmological parameters using Type Ia supernova data. Least Squares and Bayesian methods are initially explored, then details of experiments involving models which contain more than two parameters are analysed.

II. RESULTS AND DISCUSSION

a. Least Square Statistics

To calculate cosmological parameters using supernovae we must first obtain data sets of Type Ia supernovae which contain redshift, effective magnitude and the uncertainty on that magnitude. We specify effective magnitude as this means that it has been correct for light curve stretching and galactic dust extinction amongst other factors [1]. This means that the magnitudes used in our calculations will be more accurate and thus produce more accurate parameters.

In our first experiments we chose to use a data of set of 42 high-redshift ($z > 0.1$) Type Ia supernovae from the Supernova Cosmology Project (SCP) and 18 low-redshift ($z < 0.1$) ones from the Calán Tololo Survey (CTS) [9]. As indicated in our previous discussion of the experimental theory we require the low-redshift supernovae to 'calibrate' the Type Ia data so that Equation 9 can be used. From Table I we see that we find $L_{\text{peak}} = (3.1 \pm 0.1) \times 10^{35}$ W for the peak luminosity using the SCP and CTS data.

With this we found a corresponding Ω_Λ to be 0.86 ± 0.04. Comparing it to a literature value of $0.761^{+0.017}_{-0.018}$ [13] we find that there is only a $\sim 10\%$ difference between them. One can gather that whilst our result is consistent with known values, within it's own uncertainties it does not include the accurate value. This potentially could be the case because we calculated our χ^2 values in what is known as *flux*-space. Equation 7 uses the observational and model fluxes of the supernovae to calculate the χ^2 values. Instead of this, magnitudes could be used which would shift us into *magnitude*-space. In our investigations we attempted to recalculate Ω_Λ and found 0.71 ± 0.04, we see that this is more consistent to the given literature value.

Whilst we could have performed the rest of our experiment in magnitude-space, we chose not to as there would be additional implications and uncertainties.

In converting from fluxes into magnitudes using Equation 8 we find that the transformation modifies the input flux distribution and the shape of it will be biased towards fainter magnitudes [7]. However this does not account for the disparity between our calculated cosmological constants.

Investigating this further in literature, we have not been able to find the reason for this disparity. Although we could suggest that it may arise due to the conversion of flux into magnitude for the model in magnitude-space. If we consider observational data for supernovae, we find that multiple steps are required to convert an image into a magnitude value. First of all, frames are taken by CCDs and then aperture or point-spread function photometry is used to measure the amount of flux being received from the object. This flux then has to be converted into a magnitude using Equation 8, where the zero point is found by using objects from the same frame.

In our work we have kept $m_0 = -20.45$, a value which is constant no matter which supernova we are working with. If we were to change it to a more suitable value, would this produce more accurate results? Whilst this possibly could be the case, in our later research with larger data sets we found that working in flux-space returns cosmological parameters which are consistent to measured values. This discrepancy between flux and magnitude-space could just be a consequence of the method requiring a large data set for it to easily constrain the variables.

From Table I we find our value for Ω_Λ which was calculated using a larger data set, the value of which is 0.74 ± 0.01. We see that it is a $\sim 3\%$ difference between it and the literature value suggesting that using a larger collection of Type Ia supernovae helps to produce a more accurate result. The data set used is the Union2.1 Compilation from the SCP [12], it contains 753 Type Ia supernovae which is able to provide us with a more accurate calculation when we use the Least Squares method.

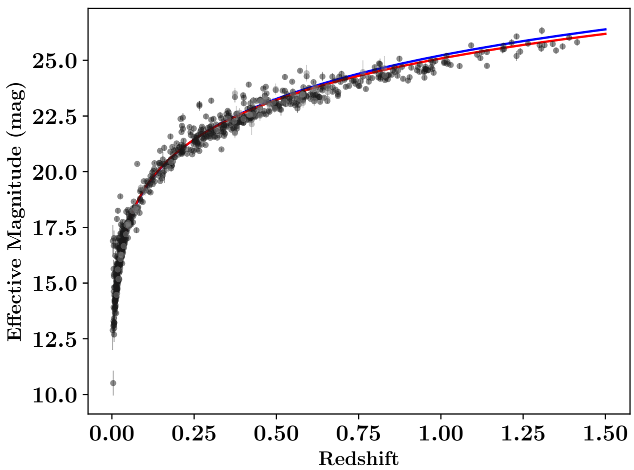

FIG. 1: Hubble diagram showing the Union2.1 Type Ia supernova data plotted alongside two models. The blue line represents a universe with $\Omega_\Lambda = 0.86 \pm 0.04$ and the red line 0.74 ± 0.01. We gather that the latter cosmological parameter is the more accurate one as more of the data lies close to it within their magnitude uncertainties. We note that there are too many points for the majority of magnitude error bars to be seen.

Parameter	$\chi_1^2(f)$	$\chi_1^2(m)$	χ_2^2	MCMC	Literature
$L_{\rm peak}$ (W)	$(3.1 \pm 0.1) \times 10^{35}$	$(3.3 \pm 0.1) \times 10^{35}$	$(2.94 \pm 0.04) \times 10^{35}$	$(3.4 \pm 0.1) \times 10^{35}$	-
Ω_Λ	0.86 ± 0.04	0.71 ± 0.04	0.74 ± 0.01	0.73 ± 0.01	$0.761^{+0.017}_{-0.018}$
Ω_k	-	-	-	-0.0029 ± 0.009	$-0.0030^{+0.0095}_{-0.0102}$
Ω_M	-	-	-	0.20 ± 0.02	$0.239^{+0.018}_{-0.017}$
$\Omega_{\rm rad}$	-	-	-	$(4.4 \pm 0.7) \times 10^{-6}$	$(4.16) \times 10^{-6}$
w	-	-	-	-0.912 ± 0.01	$-0.941^{+0.087}_{-0.101}$

TABLE I: Cosmological parameters calculated using different data sets and methods. χ_1^2 are the parameters which have been calculated using the SCP and Calán Tololo Survey data. χ_2^2 are the parameters found with the larger Union2.1 SCP data set. MCMC corresponds to the parameters calculated using the Markov Chain Monte Carlo method from Bayesian statistics. We also provide 'Literature' values obtained from work performed by M. Tegmark et al (2006) [13].

In Figure 1 we have plotted the Union2.1 data set alongside our two models which have been produced by using the calculated cosmological parameters from flux-space. To assess the quality of fit between the data and the model we have calculated reduced χ^2 values. For the model which makes use of $\Omega_{\Lambda,1} = 0.86 \pm 0.04$ (the blue line) we find $\chi^2_{\nu,1} = 2.4539$, and for the secondary model which uses $\Omega_{\Lambda,2} = 0.71 \pm 0.01$ we proceed to calculate $\chi^2_{\nu,2} = 2.4547$. The criteria for a model to be a good fit to the data is to produce a reduced χ^2 which is ~ 1 [4]. We therefore would be justified to conclude that $\Omega_{\Lambda,1}$ should be the accepted cosmological constant as it is a closer value to unity. However, between both χ^2_ν's there is only a 0.03% difference and we can see from the figure that they are extremely similar. For that reason, we could not form a convincing conclusion based upon this analysis, instead we decided to explore Bayesian statistics in an attempt to see if the same results could be replicated and whether the accuracy could be improved.

b. Bayesian Statistics

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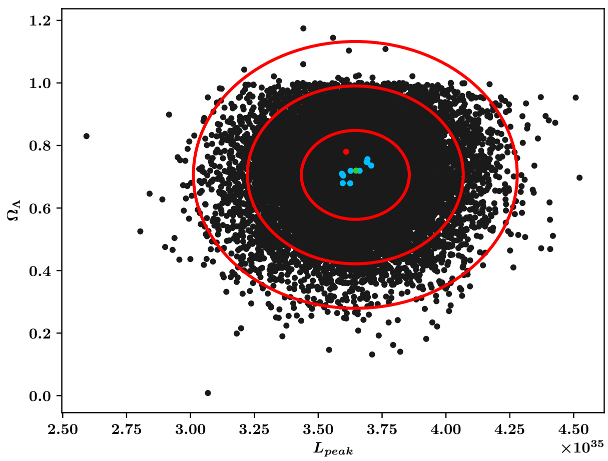

FIG. 2: Plot showing the general forms of Type Ia supernovae in the B and V bands. Archival data was translated along the time and magnitude axes to produce the general forms of the light curves. The B band has been offset by a magnitude of 3 to allow us to see both graphs clearly. The supernovae which were used to plot this graph are: *1994ae, 1994S, 1995bd, 1996bo,* and *1998aq* [5, 8].

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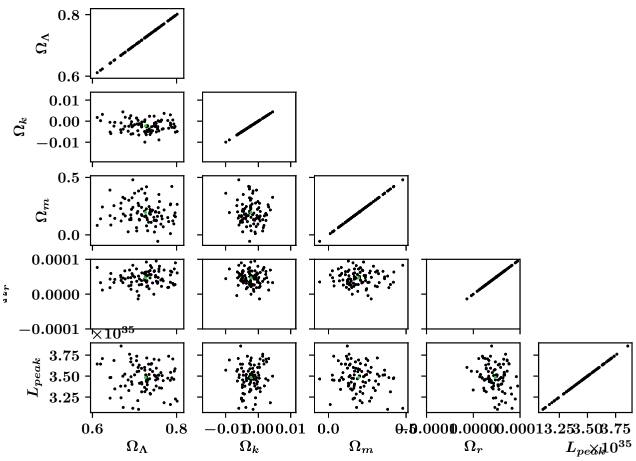

FIG. 3: Plot showing the general forms of Type Ia supernovae in the B and V bands. Archival data was translated along the time and magnitude axes to produce the general forms of the light curves. The B band has been offset by a magnitude of 3 to allow us to see both graphs clearly. The supernovae which were used to plot this graph are: *1994ae, 1994S, 1995bd, 1996bo,* and *1998aq* [5, 8].

c. Further Investigation

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III. CONCLUSIONS

In conclusion, we have found that it is possible to constrain cosmological parameters of the universe using Type Ia supernova data.

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Appendix A: Uncertainties