

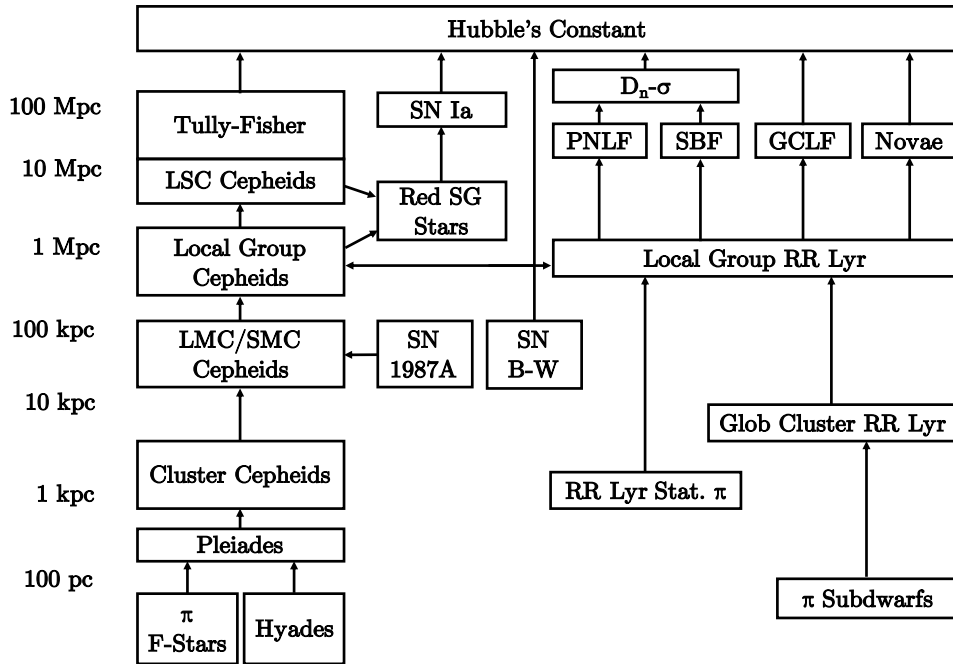
# The measurement of the Hubble Constant: beyond the cosmic ladder

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A precisely determined Hubble's constant  $H_0$  would have an overarching effect on any feature of cosmological theory: the age or critical density of the Universe, or with the formation of cosmic structure. Producing a conclusive value for  $H_0$  is difficult as absolute distances on the cosmic scale are difficult to measure. Inhomogeneous gravitational acceleration generates motion which does not follow the simple expansion as described by Hubble's Law  $v = H_0 d$ . An uncertainty arises due to the discrepancy between the methods to connect local distances to the smooth large-scale Hubble flow (Fukugita et al. 1993).

Several approaches for cosmic distance measurement should therefore be used to reduce systematic errors. These measurements can form the “rungs” of a *cosmic distance ladder*, where large extragalactic distances ( $> 1000$  Mpc) are informed and calibrated by techniques which have smaller ranges (Carroll and Ostlie 2007). Astronomers may employ a variety of methods in tandem, therefore the ladder could instead be expressed as several pathways (Figure 1).



**Figure 1:** Adapted from Jacoby et al. (1992), this diagram illustrates the various approaches to calculate  $H_0$ , each technique is roughly placed at the approximate range it operates at. One can see that there is not one strict “cosmic ladder”, rather multiple pathways. For reference, the acronyms used are: B-W - Baade-Wessenlink; GCLF - Globular-Cluster Luminosity Function; LSC - Local Super Cluster; PNLF - Planetary Nebula Luminosity Function; SBF - Surface-Brightness Fluctuations; SG - Super Giant; SN - Supernovae;  $\pi$  - parallax.

The Hubble Space Telescope (HST)  $H_0$  Key Project was an effort in the early 2000s to determine  $H_0$  by calculating distances to Cepheid variables in local galaxies ( $\leq 20$

Mpc) then applying them as a calibration to 5 secondary independent distance indicators. Described by Freedman et al. (2001), four of the methods (Type Ia supernovae, Tully-Fisher relation, surface-brightness fluctuations, and Type II supernovae) were able to produce  $70 \leq H_0 \leq 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and the remaining technique (fundamental plane for elliptical galaxies)  $H_0 \approx 82 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Over the next decade, the methodology would be refined and the sample of Cepheids and Type Ia supernovae improved (better observations and other data types)  $H_0 = 73.48 \pm 1.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Riess et al. 2018, 2011, 2016). These results set a standard benchmark for  $H_0$ , they were found by taking steps along the cosmic ladder and whilst they have high accuracy, it would be beneficial to directly calculate  $H_0$  at large distances without the need for Cepheid-based calibration.

One alternative is measuring Cosmic Microwave Background (CMB) anisotropies. Through analysing all-sky temperature and polarisation maps,  $\Lambda$ CDM cosmology models can be fitted which constrain cosmological parameters. Surveys of the CMB have included those performed by the spacecraft COBE, WMAP, and more recently, Planck. The results of the latter include  $H_0 = 67.5 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Planck Collaboration et al. 2018), this is of particular importance as it is discrepant when compared to the most recent HST-based result (Riess et al. 2018). Investigations of potential systematics in either methods have concluded that some arise due to the modelling of the Cepheids (Follin and Knox 2018) and residual systematics from certain spectra used in the Planck likelihood calculation (Spergel et al. 2015). However the tension between the  $H_0$  values still exists, therefore it would be beneficial to explore other methods for calculating the constant, especially those which are able to function immediately at large cosmic distances.

Still considering the CMB, the Sunyaev-Zel'dovich effect (SZE) leads to a change in the apparent brightness of the CMB towards a cluster of galaxies or for any reservoir of hot plasma (Birkinshaw 1999, Carlstrom et al. 2002). Combined with X-ray emission from intracluster gas, the SZE can be used as a tracer for cosmological parameters. Birkinshaw (1999) describes the technique to be a comparison of the angular size of a galaxy cluster with a measure of the line-of-sight size of the cluster. With spectra data at hand, the emission of gas in a galaxy cluster can be described by the X-ray surface brightness, and the gas absorption by the measurement of the thermal SZE (an intensity change). The surface brightness and intensity change can be re-expressed in terms of physical constants and angular structure factors, this then leads to a single expression for calculating the angular diameter distance,

$$d_A = \left( \frac{N_{SZ}^2}{N_X} \right) \frac{\Lambda_{e0}}{4\pi(1+z)^3 [I_0 \Psi_0 \sigma_T]^2}. \quad (1)$$

*The full derivation and definitions for Equation 1 can be found in Holzappel et al. (1997).*

Employing this equation with values for cluster redshifts  $z$  and the deceleration parameter  $q_0$ ,  $H_0$  can be obtained in a direct and alternate way which is independent of the chain of distance estimators. The South Pole Telescope Sunyaev-Zel'dovich (SPT-SZ) survey was a programme which made use of the SZE to detect galaxy clusters (Chang et al. 2009). Multiple analyses have been performed to constrain cosmological parameters using the resulting SPT-SZ datasets, the majority of which tested variations of the  $\Lambda$ CDM model (de Haan et al. 2016, Hou et al. 2014). Hou et al. (2014) examined models

which were constrained single or double parameters, for example constraining neutrino mass. They combined external data sets (WMAP7, BAOs and previous calibrations for  $H_0$ ) with their SPT-SZ data and they were able to achieve  $H_0 = 68.3 \pm 1.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$  which agrees with HST and Planck results, therefore showing the validity of the method.

The CMB can therefore be utilised in a variety of ways for the calculation of  $H_0$ . In a similar vein, the effects of the gravity can be explored and employed as a method for measuring Hubble’s constant without the need for the cosmic ladder. The manifestation of which is in the form of gravitational lensing and gravitational waves.

Optical observations of the night sky may sometimes reveal multiple arcs of light surrounding a central object, this effect is more commonly known as *gravitational lensing*. It occurs when light travelling towards us from a distant bright object, for example a quasar, is curved by the space-time of a much massive object, say a galaxy cluster or hypothetical MACHOs, in the foreground which acts as a lens and produces the arcs of light (Carroll and Ostlie 2007). If the initial source, such as an active galactic nucleus or a supernovae, varies in luminosity then this variability can be viewed from the aberrations albeit with a time delay as a result of the different light paths (Suyu 2017). This time delay can be related to the distribution of the lens mass and the “time-delay distance”  $D_{\Delta t}$ , where the latter is multiplicative combination of three angular diameter distances: observer-source distance  $D_s$ , observer-lens distance  $D_d$ , and lens-source distance  $D_{ds}$  (Shajib et al. 2018, Suyu 2017). The application to cosmology arises as  $D_{\Delta t}$  is inversely proportional to  $H_0$  plus weakly dependent on other cosmological constants.

Koopmans et al. (2003) presents mass models which have been developed to determine a value for Hubble’s constant. They aimed to reduce known systematics such as the radial mass profile, dust extinction, etc. Three particular mass models (SIE, SPLE1, SPLE2) were tested which were based solely on the gravitational lensing constraints so no stellar dynamics were considered. This resulted in  $H_0$  ranging from 71–74  $\text{km s}^{-1} \text{ Mpc}^{-1}$  and a best lensing-only value of  $H_0 = 74^{+10}_{-11} \text{ km s}^{-1} \text{ Mpc}^{-1}$ . It is clear then that whilst the uncertainties are large, the general value for  $H_0$  agrees with the Cepheid result and with additional constraints it would become more precise. Courbin et al. (2011) demonstrate that parameters such as the baryonic fraction in the Einstein radius and the velocity dispersion of the lensing galaxy can be found by combining spatially deconvolved HST F160W images with VLT spectroscopic data.

Gravitational lensing is therefore a highly viable method in the measurement of  $H_0$ . However it has the limitation of requiring long-periods of photometric observations on objects of interest to produce the time delays. A more suitable strategy would be to utilise gravitational waves as a standard siren where observatories have been purposely built for their detection. Waves originating from the decaying orbit of an ultra-compact, binary neutron star system would be the most likely to be registered by Earth based detectors (Schutz 1986).

LIGO Scientific Collaboration et al. (2017) describes the approach they used to calculate  $H_0$  for object GW170817 detected by the Advanced Laser Interferometer Gravitational-wave Observatory (LIGO) (LIGO Scientific Collaboration et al. 2015) and the Virgo detector (Acernese et al. 2015). The gravitational wave (GW) data is used to infer the distance  $d$  to the source through the constraining of a posterior probability in a Bayesian framework model. An initial posterior distribution for the observed data

$x_{GW}$  can be converted into a posterior on the inclination angle  $\cos i$  and  $H_0 = v_H/d$ , where  $v_H$  is the Hubble flow velocity.  $\cos i$  is of importance as it was found that the distance measurement is strongly correlated with the inclination of the binary orbital plane. Additionally, to obtain  $v_H$  for the source, the host galaxy’s measured recessional velocities can be corrected for local peculiar motions. This method is used as it does not require the Hubble flow velocities of any local calibrating galaxies which have been estimated using the distance ladder.

Applying  $v_H = 3017 \pm 166 \text{ km s}^{-1}$  to the Bayesian model produces a maximum a posteriori value of  $H_0 = 70.0^{+12.0}_{-8.0} \text{ km s}^{-1} \text{ Mpc}^{-1}$ , with this particular analysis, the value produced is consistent with those found with and without the cosmic distance ladder. The precision of the GW method could be improved by employing more detectors in the observation network. KAGRA is a Japanese GW telescope which is currently under construction, the main features are that it is being installed underground and that it will use cryogenic cooling (Akutsu and collaboration 2015), both of these will reduce the seismic and thermal noise and as a result the data signal-to-noise ratio should hopefully improve. Alternatively, the precision could be refined simply by incorporating alternative data-sets. Recent work by Hotokezaka et al. (2018) demonstrate that the uncertainty in the LIGO Scientific Collaboration et al. (2017)  $H_0$  value is dominated by the degeneracy in the GW signal between the source distance and the “weakly constrained” viewing angle. They provide an alternative analysis which makes use of a collection of radio images for GW170817’s superluminal jets in order to constrain the inclination angle. Employing analytical modelling, full hydrodynamic numerical simulations and semi-analytic calculations of synthetic jet models, the new constraints resulted in an improved measurement of  $H_0 = 68.9^{+4.7}_{-4.6} \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

Method	$H_0 \text{ (km s}^{-1} \text{ Mpc}^{-1})$
HST Key Project with Cepheids+secondary	$73.48 \pm 1.66$
Planck observations of the CMB	$67.5 \pm 0.5$
Sunyaev-Zel’dovich Effect and SPT-SZ data	$68.3 \pm 1.0$
Gravitational lensing and mass modelling	$74^{+10}_{-11}$
Gravitational waves with LIGO and Virgo	$70.0^{+12.0}_{-8.0}$
Gravitational waves plus radio data for jets	$68.9^{+4.7}_{-4.6}$

**Table I:** Summarised results from the various methods used to calculate Hubble’s Constant. All values are in general agreement, however there is tension between the results found from Cepheid calibrated values and the CMB Planck results. The alternative methods appear to support the Planck value for  $H_0$ .

In conclusion, this discussion has explored three alternative methods which can be employed to measure and calculate Hubble’s constant  $H_0$  without the need to climb the cosmic ladder in the local Universe. Exploiting and measuring the Sunyaev-Zel’dovich effect, gravitational lensing, and gravitational waves leads to accurate values for  $H_0$  which agree with those found with the traditional standard candle techniques. If used in tandem, one could potentially produce an extremely accurate and precise value. Measuring Hubble’s constant beyond the cosmic ladder is a promising field of research, with more observations and analysis there is no doubt that eventually a single unifying value for  $H_0$  will be found.

*Dissertation final word count: 1600*

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