Graph Search

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This reading is relatively new, and your feedback will help us improve it! If you notice mistakes (big or small), if you have questions, if anything is unclear, if there are things not covered here that you'd like to see covered, or if you have any other suggestions; please get in touch during office hours or open lab hours, or via e-mail at `6.101-help@mit.edu`.

Table of Contents

1) Introduction

In last week's reading, we looked at two related problems: *flood fill* (an operation on images for recoloring a contiguous region of similarly colored cells) and *pathfinding* (finding and returning a path through a maze, which we arrived at by modifying flood fill to keep track of additional information). In this reading, we'll be building on some of those ideas and formalizing them a little bit, looking at an interesting category of algorithms called *graph search* algorithms, which can be used to solve a wide variety of different problems.

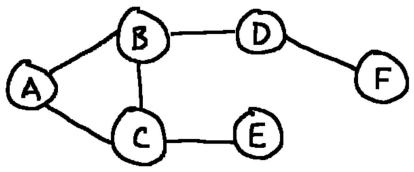
This reading will start at a high level, introducing and discussing graph search in the abstract; but, by the end, we will be talking in detail about code for graph search and about some of the work involved in bringing these approaches to bear on new problems.

2) What is a Graph?

Graph search algorithms are all about systematically exploring structures called *graphs*; but, before we can talk too much about exploring graphs, we will need to define what a graph is. Graphs are structures that represent sets of objects and relationships between pairs of those objects. In 6.101, we'll talk about graphs as consisting of two things in an abstract sense:

- a set of *vertices*, one for each object we're interested in; and
- a set of *edges*, which represent relationships between those objects.

We will often represent graphs using pictures, like so:



In this kind of drawing, the circles represent vertices in our graph, and each corresponds to an object (with which we label each vertex). Here, we have six objects (A, B, C, D, E, and F), each of which is represented by a single vertex.

The lines that connect the circles in the drawing represent edges, each of which corresponds to a relationship between a pair of objects. For example, B and C are related in some way according to this graph (since there is an edge connecting them), but C and D are not related in that way. In this drawing, each line represents a *bidirectional* edge: a symmetric relationship between vertices; if the relationship only goes one way, we will draw it as an arrow (a *directed edge* or *unidirectional edge*) instead.

That's still quite abstract for now, but that's our notion of a graph. Our focus in this reading will be on a specific category of algorithms called *graph search*, which involve systematically "exploring" a graph in some way, such that we can answer questions about it. There are a number of questions that we might be interested to ask about a graph, for example:

- What are all of the objects that are directly connected to A?
- What are all of the objects that I can reach by traversing exactly two connections?
- What if I am willing to traverse an arbitrary number of connections?

All of these questions can be solved using graph search approaches, but our focus in 6.101 is going to be on a particular subcategory of graph search algorithms, pathfinding algorithms, which we will use to answer the question: given a starting object and a goal object, what is a path that connects the two?

If you recall last week's reading, this idea is very similar to the last problem we tried to solve, where we were trying to find a path from one location in a maze to another. To think about a maze as a graph, we treat each location in the maze as a vertex in the graph, with an edge between adjacent locations whenever there is no wall blocking the way. So rather than focusing on images in particular, we're generalizing this notion to finding a path in a graph: a sequence of vertices (or a sequence of edges) leading from our starting vertex to our goal vertex.

2.1) Uses

So far, this might all seem very abstract. But the idea of a graph, and the algorithms we're going to talk about today, are not just interesting made-up things to think about (although they *are* kind of interesting in their own right); these ideas have tremendous practical utility in a broad variety of contexts. Lots of real-world domains can be thought of in terms of objects and their relationships to each other; this allows us to construct graphs corresponding to these domains, and graph search approaches allow us to answer questions about those contexts. So, it turns out that a wide variety of problems can be solved by: constructing a graph related to that problem, using some sort of graph search approach to answer some questions about that graph, and then

reinterpreting those answers in terms of the original problem that we started with. There are a large number of examples, but to name a few:

- **Web search** involves relating web pages to each other and ranking them relative to each other. One of the early approaches to ranking search results, PageRank, ranks pages using a graph whose vertices represent individual web pages and whose edges represent hyperlinks between them.
- **Robot navigation** is about planning actions that a robot can take to affect the world in some way (e.g., move to a different location, bake a cake, deliver a package, etc.). Determining what actions the robot should take to accomplish its goal often involves exploring a graph where vertices represent possible states of the world, and edges reflect actions that the robot can take.
- **VLSI circuit layout** involves arranging millions of transistors on a chip and connecting those components with wires, subject to constraints on the physical locations of the various components (which may affect each other), the total area of the circuit, the total length of wire, or other constraints. This is a complicated process, but it is often solved using graph search techniques on multiple graphs related to the components and the connections between them.
- **Social-network graphs** represent connections between different people (or other entities), based on relationships or past communications. Exploring these graphs can answer complicated questions about these relationships, such as finding groups of people connected by common interests for purposes of targeted advertising, discovering new relationships based on other connections, and more.
- **Route planners** like the GPS-powered systems in most modern cars find optimal paths between physical locations using common modes of transport. These systems often work using graphs of location data (locations connected by ways to move between them) and finding paths between locations that are optimal in some sense (e.g., minimizing travel time, optionally accounting for tolls and traffic, etc.).
- Many kinds of **puzzles and games** can be solved using graph search techniques, often by constructing a graph where each vertex represents the state of the game at a certain point and the edges represent moves that players can make. By finding paths through those graphs, we can often figure out the optimal (or nearly optimal) move to make based on the current state of the game. These techniques are often used when a game is played against artificial intelligence or when games otherwise contain agents that are not controlled by human players.

This is just a small subset of the kinds of problems that can be solved using graph search, but hopefully it gives a sense of the power of some of these approaches. It's also worth tempering expectations a little bit; many of the specific problems mentioned here involve humongous graphs and require more advanced techniques than we're going to be able to cover in 6.101 (we'll focus our attention on a small subset of graph search algorithms). That said, the techniques will serve as an introduction to graph search techniques and as a foundation on which you can build later on if you're interested (particularly in classes like 6.121$_{[6.006]}$ Introduction to Algorithms and 6.122$_{[6.046]}$ Design and Analysis of Algorithms).

2.2) Another Example: The 15 Puzzle

Let's take a look at a concrete example of a puzzle that can be solved using graph search: the 15 puzzle. The 15 puzzle consists of a board split into a 4-by-4 grid, with each cell except one occupied by a numbered tile. The goal is to get the board from an initial scrambled state (example shown on the left below) into an ordered state (shown on the right below) by repeatedly sliding tiles into the open position. From each board position, we have between two and four options for moves; for example, considering the board on the left, we have three options for how to proceed: we could move the "5" tile down into the open space, the "12" tile to the right into the open space, or the "7" tile up into the open space.

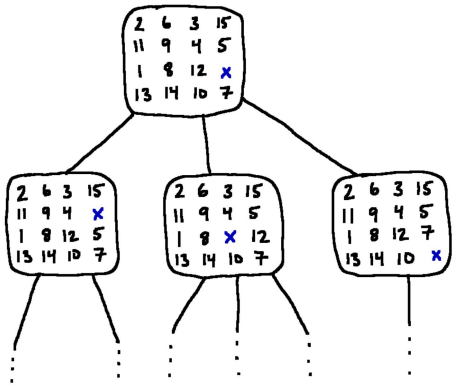
Start (Scrambled)

2	6	3	15
11	9	4	5
1	8	12	
13	14	10	7

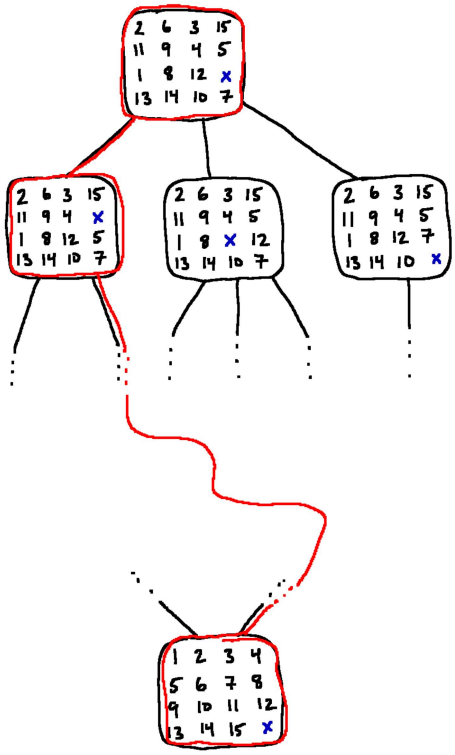
Goal (Ordered)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The reason to mention this puzzle, aside from it just being kind of a fun puzzle, is that it is one example of a kind of problem we will be able to solve by finding a path through a related graph. Consider creating a graph where each vertex represents the state of the board, and these vertices are connected by edges representing moves that we can take to move between the states. Graphically, we might draw such a graph like so (here only showing a small portion of the graph):



Notice that each vertex here corresponds to one possible state that the game board could be in (an arrangement of the tiles) and that the edges connecting the vertices represent moves that we could make (tiles that we could slide) to move between the states. Now, we can imagine that, if we were to draw out the full graph, *somewhere* in that graph would be a state corresponding to our solved puzzle, and a path (a sequence of game states) starting from our original configuration and ending with the goal configuration would represent a solution to the puzzle and tell us what moves we would need to make in order to solve it:


2 6 3 15
11 9 4 5
1 8 12 X
13 14 10 7
2 6 3 15
11 9 4 X
1 8 12 5
13 14 10 7
2 6 3 15
11 9 4 5
1 8 X 12
13 14 10 7
2 6 3 15
11 9 4 5
1 8 12 7
13 14 10 X
1 2 3 4
5 6 7 8
9 10 11 12
13 14 15 X

Before we take a look at some results, let's think a little bit about how big of a problem this is. How many possible distinct arrangements of the tiles are there on the board?

- ○ 15
- ○ 16
- ○ $15^2 = 225$
- ○ $16^2 = 256$
- ○ $15^4 = 50,625$
- ○ $16^4 = 65,536$
- ○ $15! = 1,307,674,368,000$ (around 1.3 trillion)
- ◉ $16! = 20,922,789,888,000$ (around 21 trillion)

Submit

You have submitted this assignment 1 time.

Solution: $16! = 20,922,789,888,000$ (around 21 trillion)

Explanation:

We can think about this like so:

There are 16 possibilities for what could be in the upper-left square (1, 2, 3, 4, ..., 13, 14, 15, or the empty space). Then, for each of those possibilities, there are 15 possibilities for what could go in the next space. So, there are 16×15 possibilities for the first two tile locations.

For each of those possibilities, there are 14 options for what to put in the next space. Following this pattern, we find that there are $16!$ possible layouts of the board.

It turns out that only half of these states can actually be reached by sliding tiles around (the rest require breaking the board by pulling tiles out and sticking them back in elsewhere), but, even then, we are looking at around 10 trillion vertices in our graph; this is gigantic!

We'll return to this specific example a little later in this reading, but, for now, let's look at one solution to the problem (you can use the buttons below to step through it):

<< First Step | < Previous Step | Next Step > | Last Step >>

CONFIGURATION 1

2	6	3	15
11	9	4	5
1	8	12	
13	14	10	7

CONFIGURATION 2

2	6	3	15
11	9	4	
1	8	12	5
13	14	10	7

CONFIGURATION 3

2	6	3	
11	9	4	15
1	8	12	5
13	14	10	7

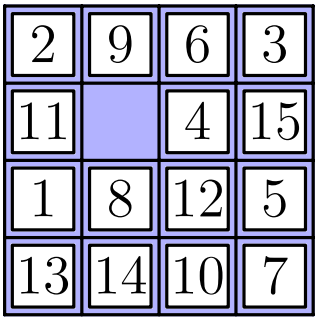
CONFIGURATION 4

2	6		3
11	9	4	15
1	8	12	5
13	14	10	7

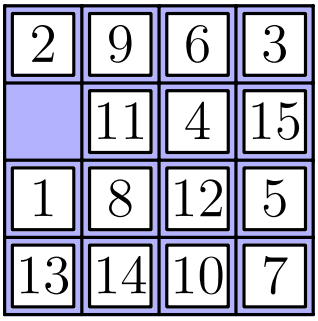
CONFIGURATION 5

2		6	3
11	9	4	15
1	8	12	5
13	14	10	7

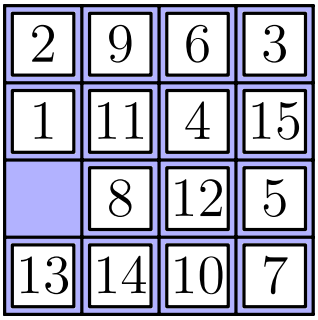
CONFIGURATION 6



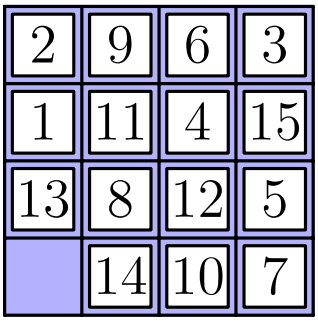
CONFIGURATION 7



CONFIGURATION 8



CONFIGURATION 9



CONFIGURATION 10

2	9	6	3
1	11	4	15
13	8	12	5
14		10	7

CONFIGURATION 11

2	9	6	3
1	11	4	15
13	8	12	5
14	10		7

CONFIGURATION 12

2	9	6	3
1	11	4	15
13	8		5
14	10	12	7

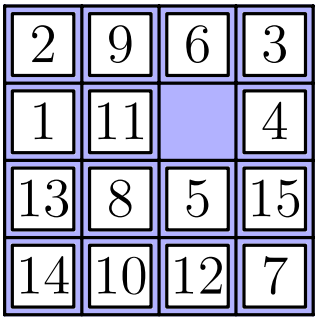
CONFIGURATION 13

2	9	6	3
1	11	4	15
13	8	5	
14	10	12	7

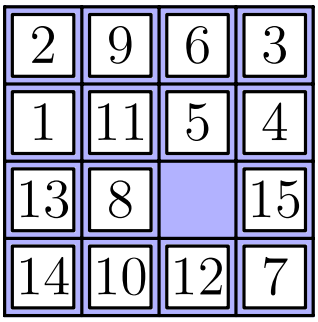
CONFIGURATION 14

2	9	6	3
1	11	4	
13	8	5	15
14	10	12	7

CONFIGURATION 15



CONFIGURATION 16



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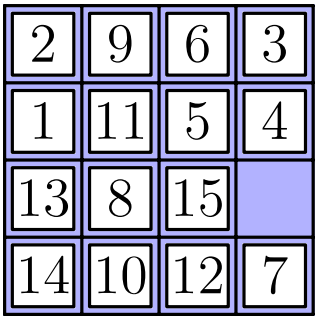
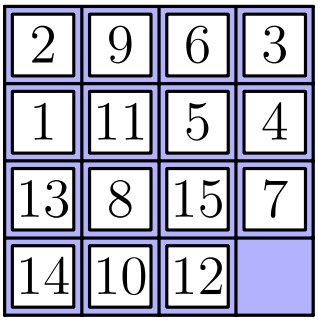


CONFIGURATION 18



CONFIGURATION 19

2	9	6	3
1	11	5	4
13	8	15	7
14	10		12

CONFIGURATION 20

2	9	6	3
1	11	5	4
13	8		7
14	10	15	12

CONFIGURATION 21

2	9	6	3
1	11	5	4
13		8	7
14	10	15	12

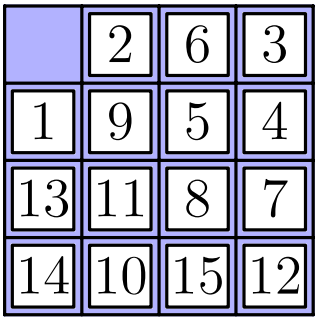
CONFIGURATION 22

2	9	6	3
1		5	4
13	11	8	7
14	10	15	12

CONFIGURATION 23

2		6	3
1	9	5	4
13	11	8	7
14	10	15	12

CONFIGURATION 24



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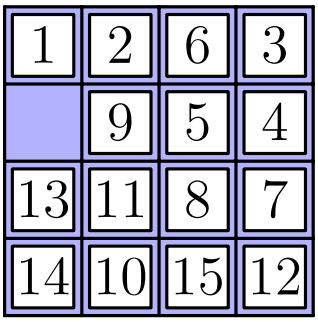

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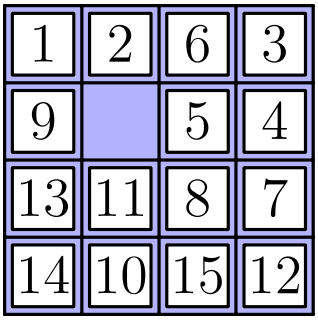

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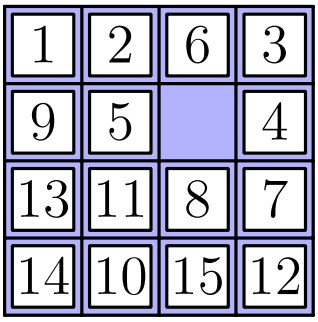


CONFIGURATION 28

1	2		3
9	5	6	4
13	11	8	7
14	10	15	12

CONFIGURATION 29

1	2	3	
9	5	6	4
13	11	8	7
14	10	15	12

CONFIGURATION 30

1	2	3	4
9	5	6	
13	11	8	7
14	10	15	12

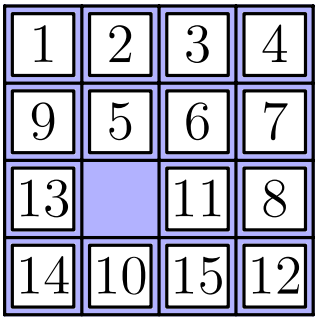
CONFIGURATION 31

1	2	3	4
9	5	6	7
13	11	8	
14	10	15	12

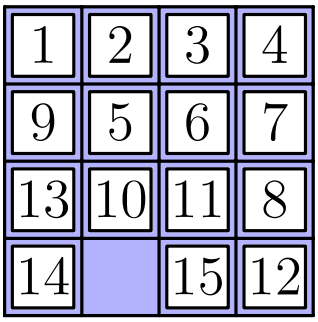
CONFIGURATION 32

1	2	3	4
9	5	6	7
13	11		8
14	10	15	12

CONFIGURATION 33



CONFIGURATION 34



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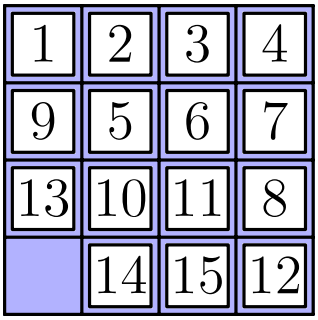

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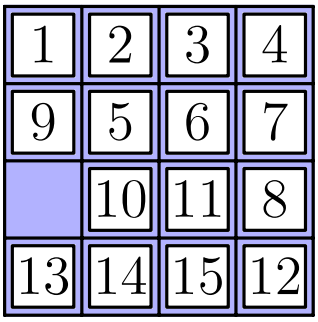

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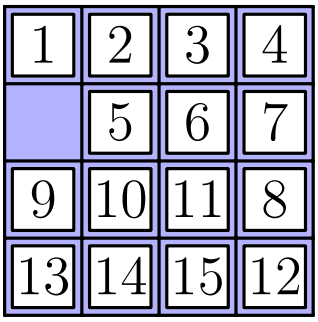
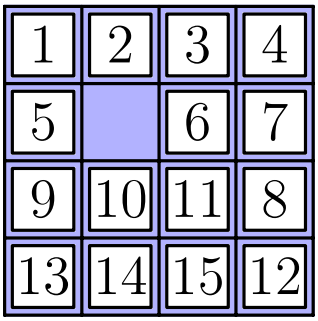


CONFIGURATION 38



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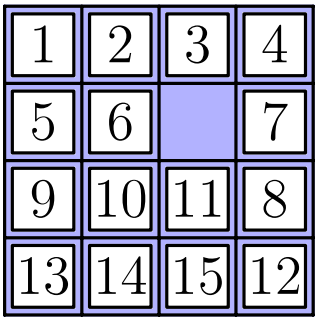
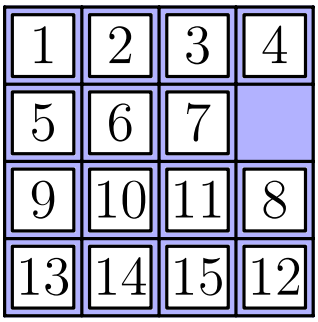
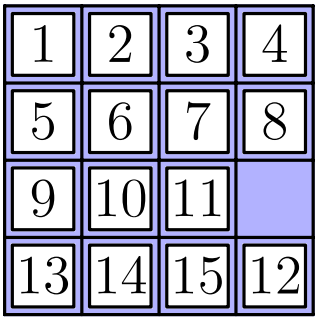


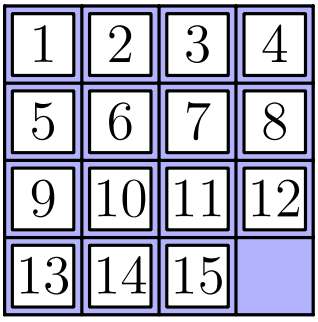
CONFIGURATION 40



CONFIGURATION 41



CONFIGURATION 42



This solution consisted of 42 board states (41 moves). Given this result, and given the scale of the problem, there are some questions we might ask ourselves:

- Is this solution optimal? That is, is there a solution out there that requires fewer than 41 moves?
- How many of the ~10 trillion possible board states do I need to consider in order to find a solution? Could we do things more efficiently?

For the rest of this reading, we're going to build on these ideas by building on the things we started talking about in last week's reading and formalizing some of those ideas to develop general-purpose pathfinding algorithms, and then we'll also spend a little bit of time thinking about how those algorithms perform.

3) Pathfinding in the Abstract

Our approach to pathfinding will be very similar to our code for flood fill from last week. In that example, we maintained an "agenda" (or "work queue") of pixels that we eventually needed to color in, and then we repeatedly pulled one pixel off of the agenda, colored it in, and then added neighboring pixels of the right color back to the agenda. Here, we will do something similar, but our agenda will consist of *paths* from some starting state to some other state. But, we'll still work our way through this process in the same way: we'll repeatedly pull a path out of the agenda, consider it, and add its children (new, longer paths!) onto the agenda. Here is our approach outlined in pseudocode:

To find a path from a state t to some other state:

- Initialize an **agenda** (list of paths to consider) containing a single path that consists of only our starting state
- Initialize a **visited set** (all states that have ever been added to the agenda) to contain only the starting state
- Repeat the following:
    - Remove one path ($t \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_N$) from the agenda
    - For each "neighbor" state n of s_N (for each state n directly connected to s_N via an edge):
        - If n is in the visited set, skip it
        - Otherwise, if n satisfies our goal condition, return the path ($t \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_N \rightarrow n$)
        - Otherwise, add n to the visited set and add the path ($t \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_N \rightarrow n$) to the agenda

  until either we find a path or the agenda is empty (in which case no path exists)

So, there's our high-level approach, and it maybe doesn't sound like much: we're going to keep track of all the paths we need to consider, and then consider them one after the other until we find one that satisfies the goal condition we're looking for. But, while this approach doesn't sound too impressive on the face of it ("just keep looking at new paths until you find one that works"), it's taking advantage of two useful properties of computers: they're good at doing a lot of things really quickly, and they're good at remembering lots of things. Putting that all together, these ideas will end up giving rise to programs that feel quite smart and that can compute some impressive results.

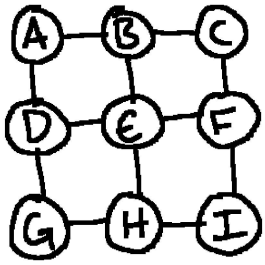
3.1) Order Matters!

The approach outlined above is almost complete, but it turns out that one key piece of information is missing.

As we work through this process, we'll generally have multiple paths in the agenda (often, a whole lot of paths!). But, the first step in the loop in our algorithm simply says to "remove one path from the agenda;" it says nothing about which path we should remove! And, similarly, it talks about "adding [new paths] to the agenda," but it doesn't say anything about where we should put those new paths!

It turns out that the order in which we consider the paths has a big effect on the way that this approach "explores" the graph it's given and also on the nature of the paths that we ultimately return. So, it will be worth taking some time to explore these effects.

Let's see how this process plays out on a small example graph:



How many vertices does this graph contain?

9

Submit

You have submitted this assignment 1 time.

Solution: 9

How many edges does this graph contain?

12

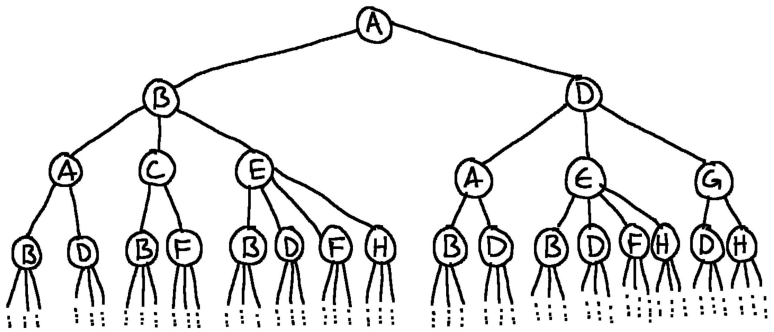
Submit

You have submitted this assignment 1 time.

Solution: 12

Let's now consider the process of finding a path from A to I in this graph, using the approach we outlined above, and we'll try it with a few different rules about ordering. All of these processes are going to start the same way: our agenda will contain a single path (the path containing only A); then we will remove that path and add its children (two paths, one representing AB and one representing AD). After that point, we will start to see the differences that arise from different choices about the order in which we consider the paths from the agenda.

To help us with this, we'll visualize things in a slightly different way, using a structure called a "rooted tree," which we'll use to try to represent all possible paths through the little graph from above. Here is an example showing the paths starting at A (note that we're only showing a portion of the whole tree):



Unlike our original graph drawing, the diagram here represents *paths* starting at A (the "root vertex"); and we explicitly diagram the idea that paths are sequences of states, and, in theory, a path can contain the same state more than once.

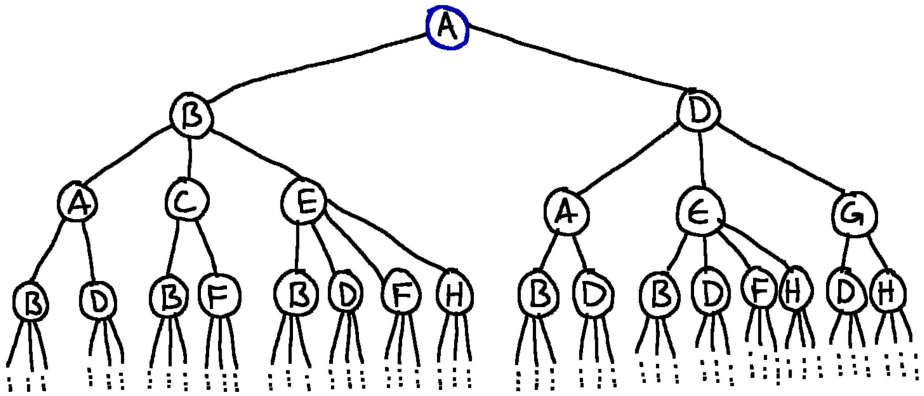
3.1.1) Example One: Replace First Path With Its Children

Let's start by considering the following rule for our agenda: each time we want to remove a path from the agenda, we will remove the element from the *front* of the agenda; and, when we add neighbors on to the agenda, we'll also add them to the *front* of the agenda.

The example below shows the start of how this process evolves when searching from A to I in the graph above.

Use the buttons below to step through this example, paying careful attention to how the agenda and the visited set evolve, as well as the order in which we're considering paths. In these graphs, we will color paths that have been added to the agenda but not yet considered in blue; and the path currently being considered will be shown in red.

<< First Step | < Previous Step | Next Step > | Last Step >>



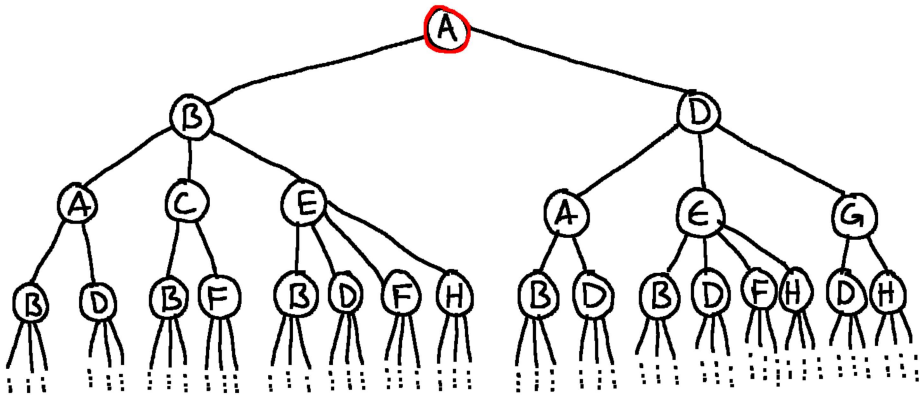
STEP 1

Agenda: (A)

Visited: A

At the start of this process, we have a single path on our agenda, the path containing only our starting state A. Since we have added a path containing A to the agenda, we also mark A as visited.

Our next step is to remove the first path off of the agenda. In this case, we don't have much choice: we'll remove the path (A).

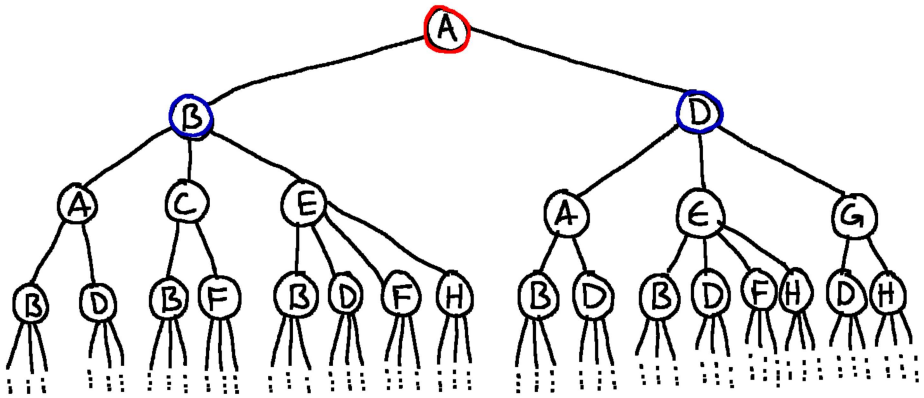


STEP 2

Agenda:

Visited: A

Now we have removed the path (A) from the agenda. Our next step will be to add its children to the front of the agenda. How many paths will we add to the agenda, and what will those paths be? How will our visited set change?



STEP 3

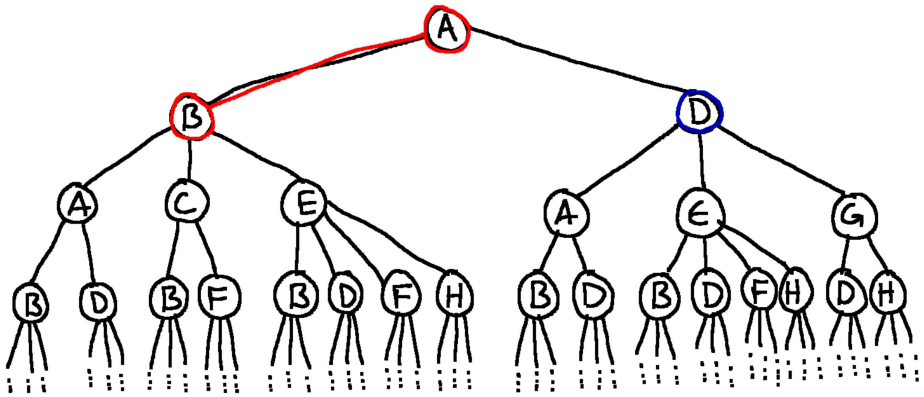
Agenda: $(A \rightarrow B)$, $(A \rightarrow D)$

Visited: A, B, D

Here, we've added two new paths to the agenda, $(A \rightarrow B)$ and $(A \rightarrow D)$. We determined these paths by looking at A in our original graph and noticing that B and D are directly connected to A by edges. There is a question of the order in which we add these children to the agenda, but for now, let's keep them in alphabetical order.

Since we have added paths containing B and D to the agenda, we also add B and D to our visited set.

Since we've not yet found the goal, we'll continue on by pulling the next path out of the agenda. According to our rule, which path will we select?

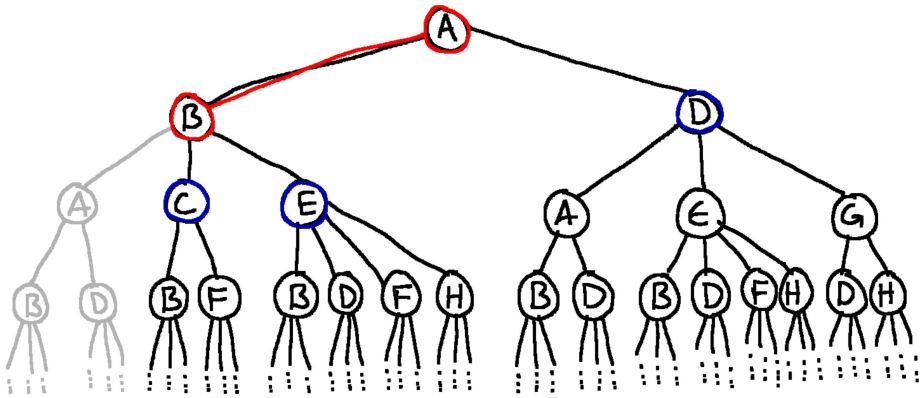


STEP 4

Agenda: $(A \rightarrow D)$

Visited: A, B, D

Our next path to consider is $(A \rightarrow B)$, since it was the first path in our agenda. We have removed it from the agenda, and our next step is to add its children to the agenda. How will the agenda and the visited set change when we do this?



STEP 5

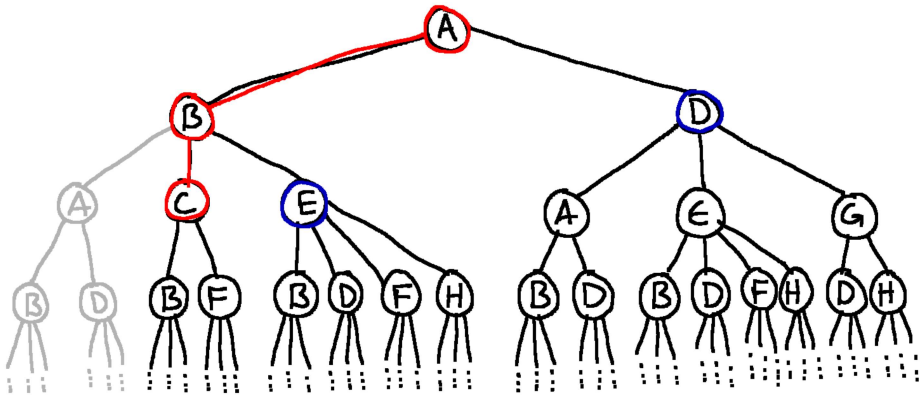
Agenda: $(A \rightarrow B \rightarrow C)$, $(A \rightarrow B \rightarrow E)$, $(A \rightarrow D)$

Visited: A, B, D, C, E

Recall that our rule here is that we are adding new paths to the front of the agenda (to the same side that we are removing from). So $(A \rightarrow D)$ stays at the end of the agenda, and we added our two new paths to the front.

Before moving on, let's take a moment to discuss what happened here. B is directly connected to three other states in our original graph, so why did we only add two paths? The key lies in the fact the A was already in our visited set before considering each of these new paths; as such, we do *not* add the path $(A \rightarrow B \rightarrow A)$ on to the agenda. We've greyed out part of the tree structure here to show that we will never consider those paths.

But there are two neighbors of B that we've not yet considered; each of those generates a new path that we add to the agenda, and we update our visited set accordingly. Next, we'll continue by again pulling the first (left-most) path off of the agenda.

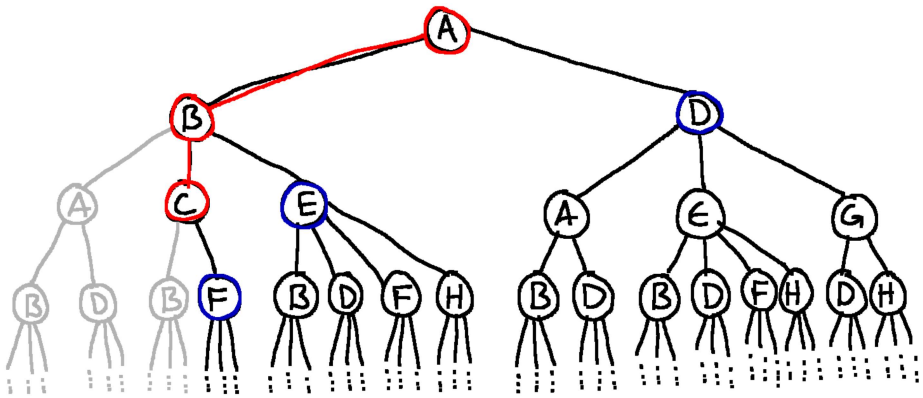


STEP 6

Agenda: $(A \rightarrow B \rightarrow E)$, $(A \rightarrow D)$

Visited: A, B, D, C, E

We have pulled the first path ($A \to B \to C$) off the agenda. Next, we'll consider its children. What new paths will be added to the agenda, and how will the visited set be updated?



STEP 7

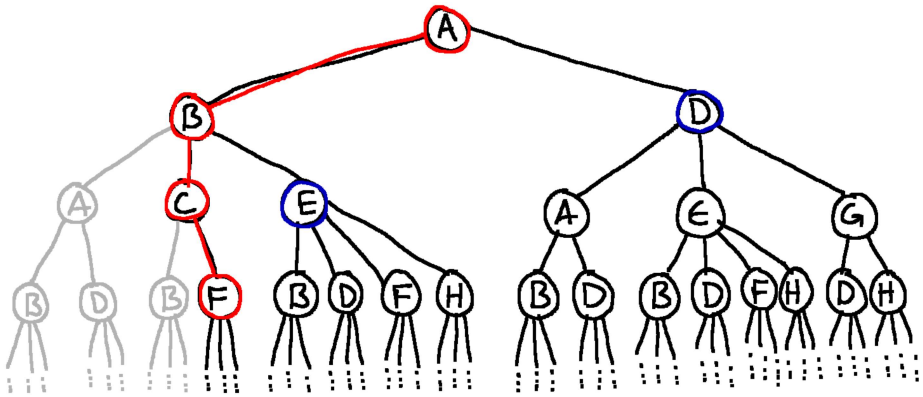
Agenda: ($A \to B \to C \to F$), ($A \to B \to E$), ($A \to D$)

Visited: A, B, D, C, E, F

Importantly, we only get one new path added to the agenda here, ($A \to B \to C \to F$). Note that ($A \to B \to C \to B$) was another possibility, but we did not add it because we have already visited B (and our algorithm says that we shouldn't add any new paths ending in a state that we've already visited).

Because we added a path containing F to the agenda, we also add F to our visited set.

Let's carry out one more step here, pulling one more path off the agenda.



STEP 8

Agenda: ($A \to B \to E$), ($A \to D$)

Visited: A, B, D, C, E, F

According to our rule, we pull the first (left-most) path off of the agenda each time. In this case, we pull ($A \to B \to C \to F$). We'll stop the updates here, but our next step would be to loop over F's neighbors; what will

happen then, following the algorithm from above?

What path from A to I will ultimately be found by this approach when it finishes? Enter your answer as a sequence of state names with no punctuation, e.g. `DCBA`:

ABCFI

Submit

You have submitted this assignment 1 time.

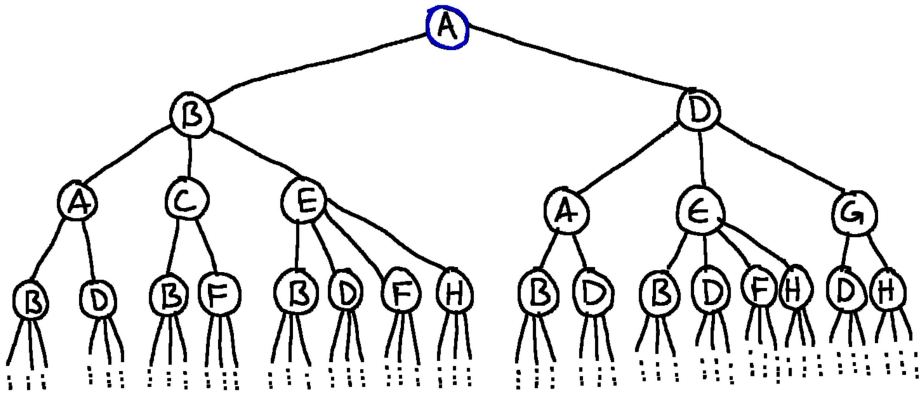
Solution: ABCFI

But, let's think not only about the solution here; let's look also at the way in which this algorithm explored the graph. This approach seemed to be going all the way down one path as far as it could, before coming back to consider other paths. Graphically, this approach is growing a single path as deeply as it can before going back and looking at other paths. This idea (of exploring the entire depth of the tree before really considering any of the breadth of the tree) is referred to as a **depth-first search** (or **DFS** for short).

3.1.2) Example Two: Replace Last Path With Its Children

Now, let's take a look at another approach. Here, we are again looking for a path from A to I, but, instead of replacing the *first* (left-most) path in the agenda with its children, we're instead going to replace the *last* (right-most) path in the agenda with its children. Before clicking through the examples below, try walking through this example on paper in its entirety. You don't need to draw the whole tree structure, but try to work through how the agenda and the visited set will evolve as we perform this search, as well as the path we ultimately return using this approach. Once you have an idea of what you're expecting, click through the example below to verify:

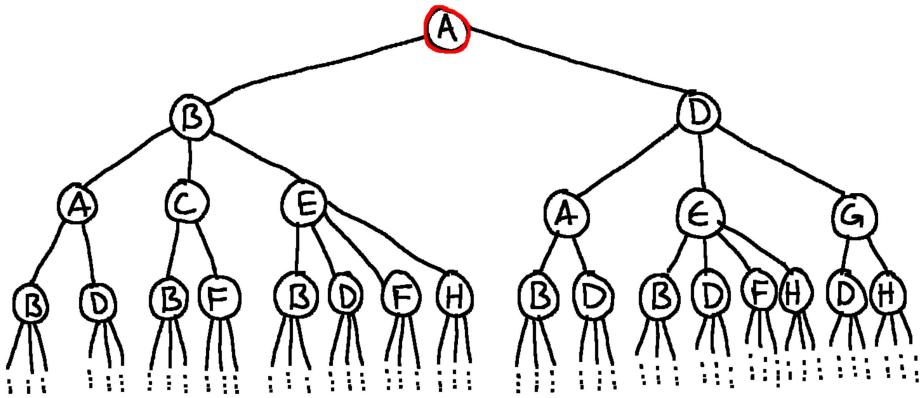
<< First Step | < Previous Step | Next Step > | Last Step >>



STEP 1

Agenda: (A)

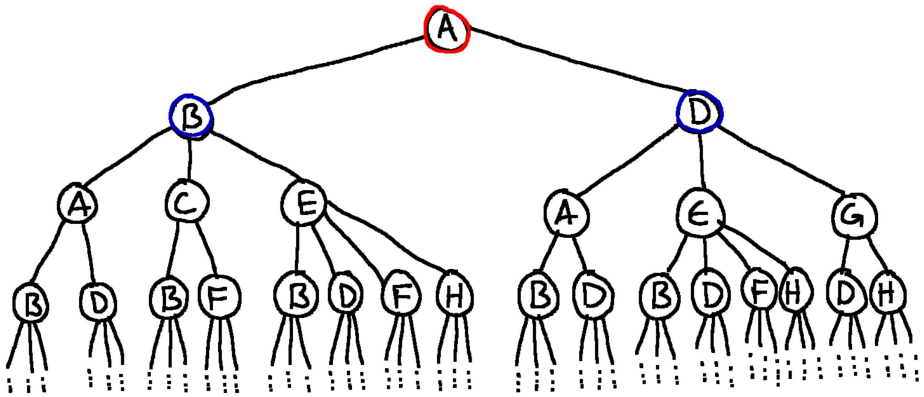
Visited: A



STEP 2

Agenda:

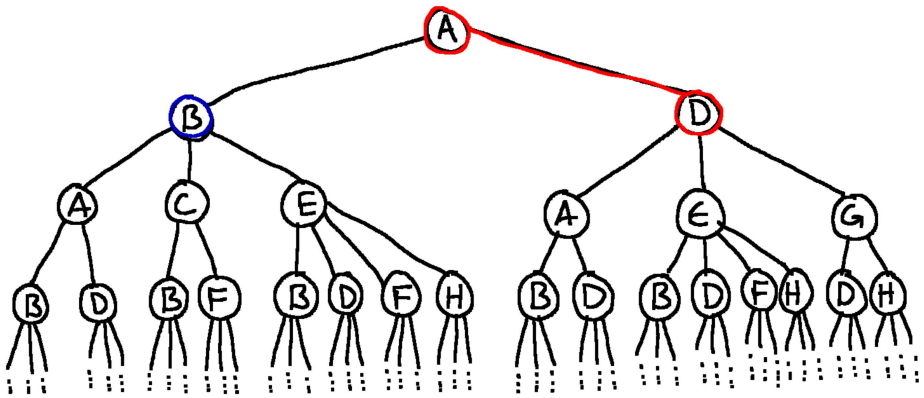
Visited: A



STEP 3

Agenda: $(A \rightarrow B)$, $(A \rightarrow D)$

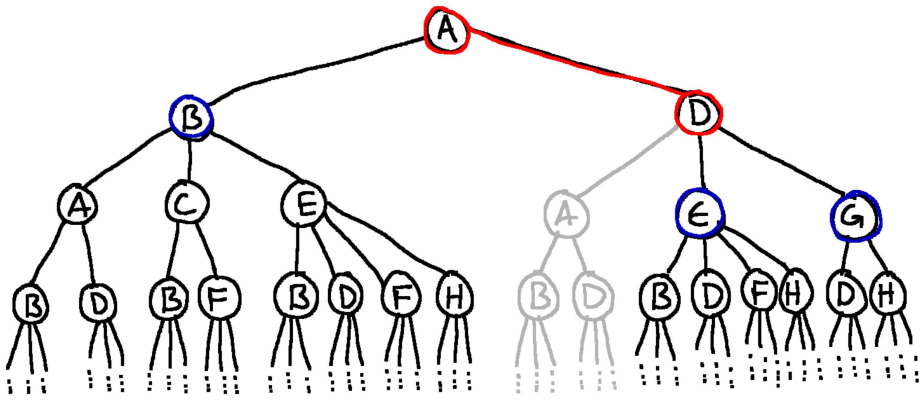
Visited: A, B, D



STEP 4

Agenda: $(A \rightarrow B)$

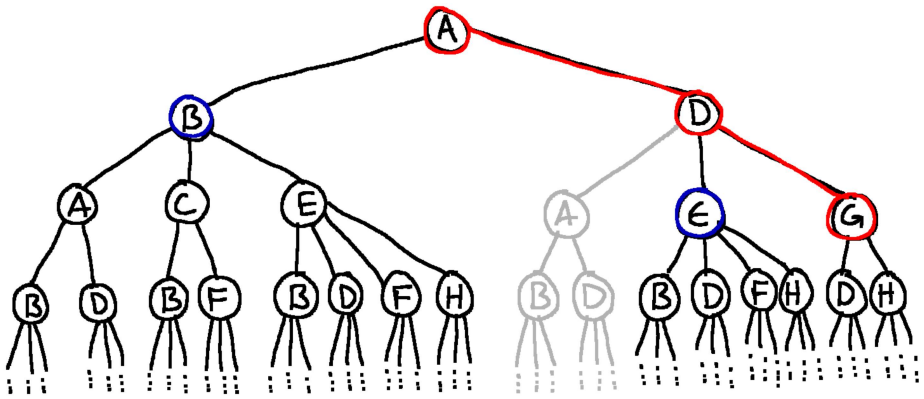
Visited: A, B, D



STEP 5

Agenda: $(A \rightarrow B)$, $(A \rightarrow D \rightarrow E)$, $(A \rightarrow D \rightarrow G)$

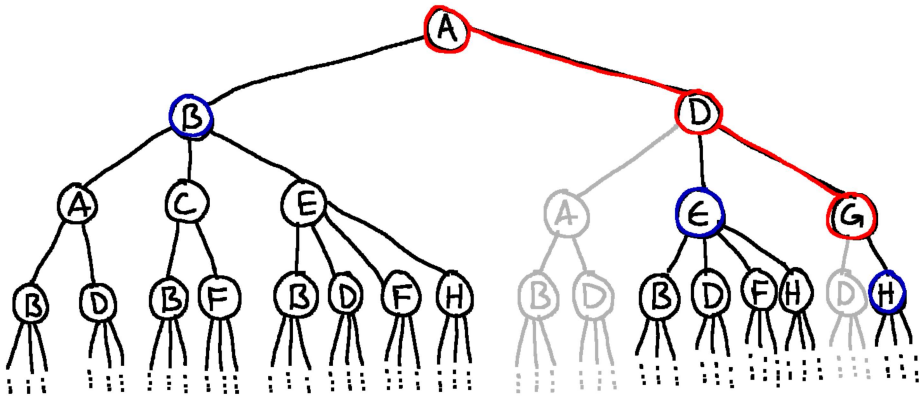
Visited: A, B, D, E, G



STEP 6

Agenda: $(A \rightarrow B)$, $(A \rightarrow D \rightarrow E)$

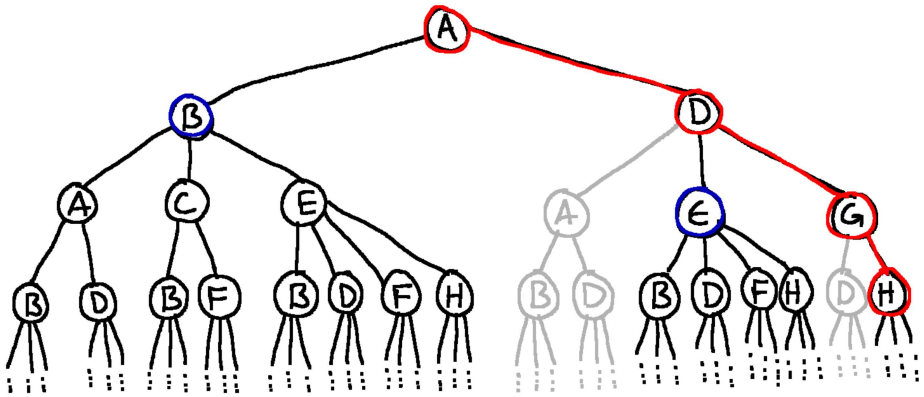
Visited: A, B, D, E, G



STEP 7

Agenda: $(A \rightarrow B)$, $(A \rightarrow D \rightarrow E)$, $(A \rightarrow D \rightarrow G \rightarrow H)$

Visited: A, B, D, E, G, H



STEP 8

Agenda: $(A \rightarrow B)$, $(A \rightarrow D \rightarrow E)$

Visited: A, B, D, E, G, H

What path will ultimately be found by this approach? Enter your answer as a sequence of state names with no punctuation, e.g. `DCBA`:

ADGHI

Submit

You have submitted this assignment 1 time.

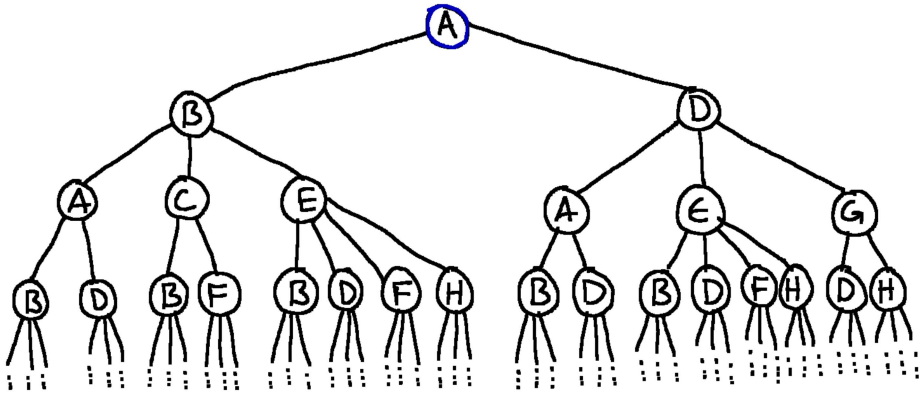
Solution: ADGHI

How can we characterize this approach? Well, this one, too, is going down a single path as far as it can, exploring the entire depth of the tree before coming back to explore any of its breadth; so, despite the change, this is still a **depth-first search**, it just happens to explore the right side of our tree instead of the left.

3.1.3) Example Three: Remove First Path, Add Children to End

Now, one more time, lets look for a path from A to I, but let's consider yet another different kind of ordering we could try: let's try removing paths from the *front* of the agenda but adding children to the *end*. Once again, try to work through this example on your own before using our answers below as a self check.

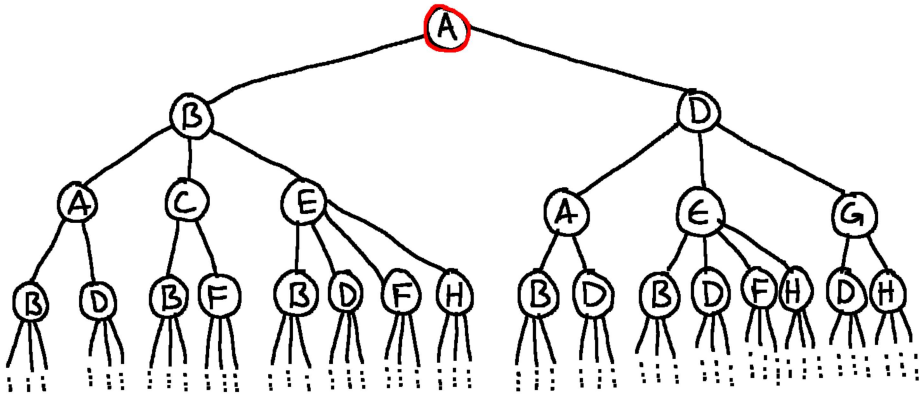
<< First Step | < Previous Step | Next Step > | Last Step >>



STEP 1

Agenda: (A)

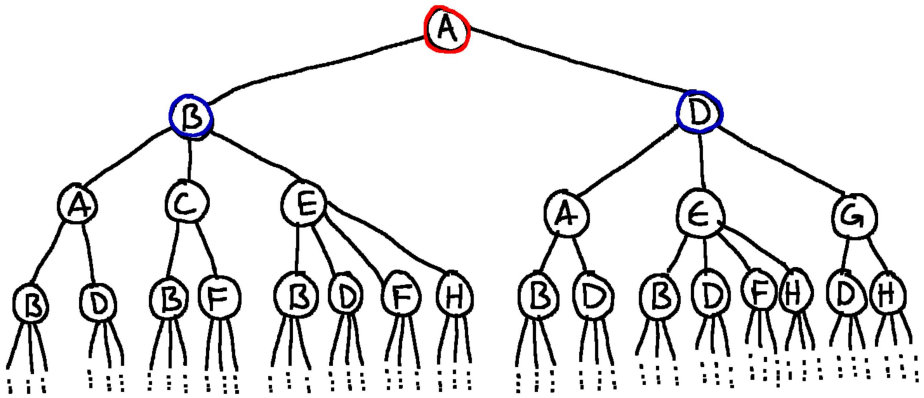
Visited: A



STEP 2

Agenda:

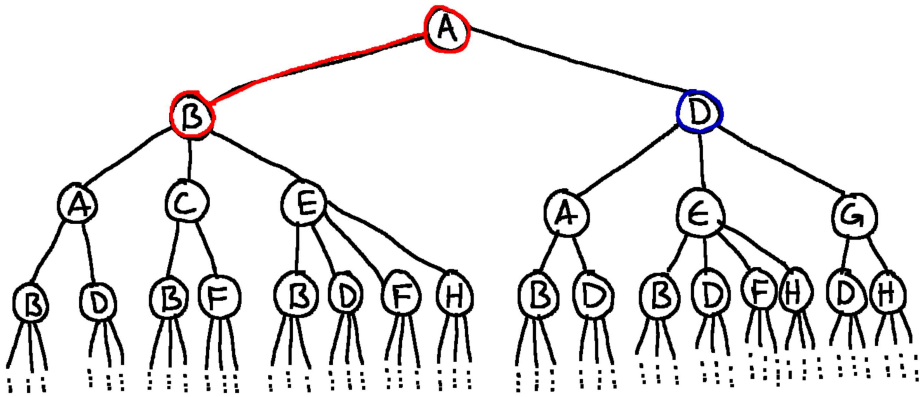
Visited: A



STEP 3

Agenda: $(A \rightarrow B)$, $(A \rightarrow D)$

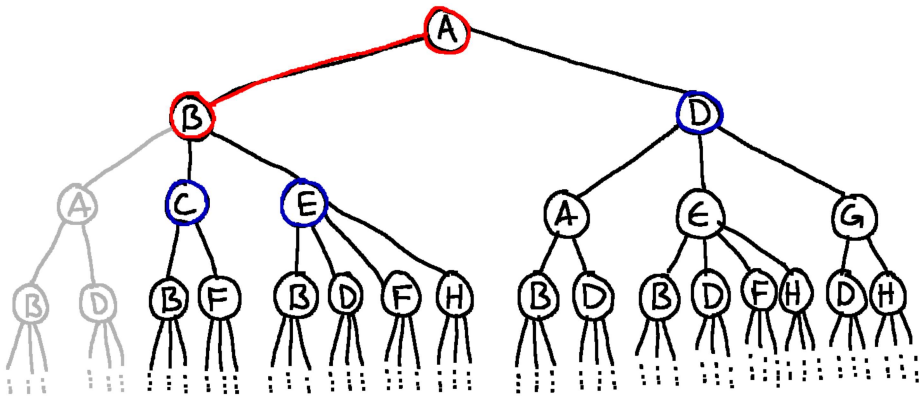
Visited: A, B, D



STEP 4

Agenda: $(A \rightarrow D)$

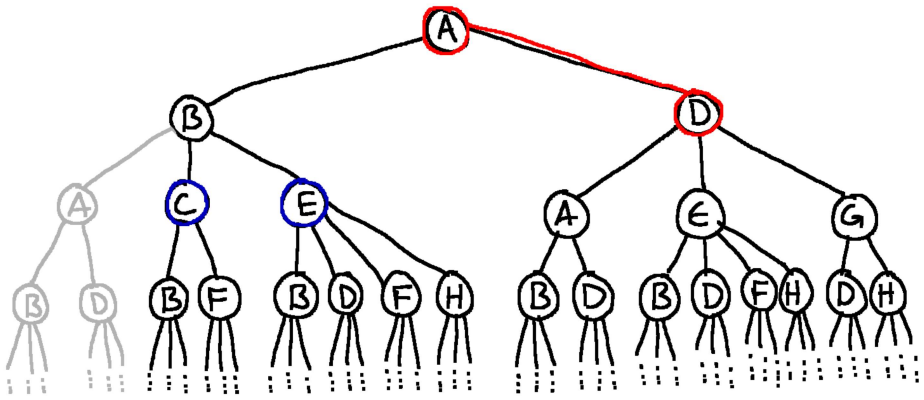
Visited: A, B, D



STEP 5

Agenda: $(A \rightarrow D)$, $(A \rightarrow B \rightarrow C)$, $(A \rightarrow B \rightarrow E)$

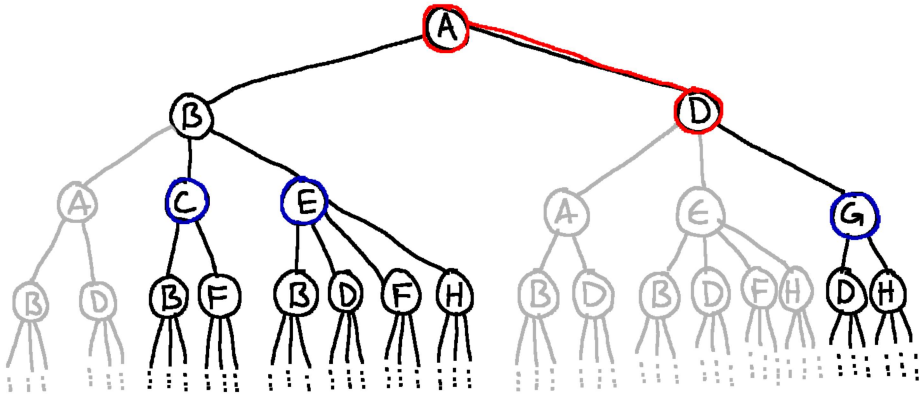
Visited: A, B, D, C, E



STEP 6

Agenda: $(A \rightarrow B \rightarrow C)$, $(A \rightarrow B \rightarrow E)$

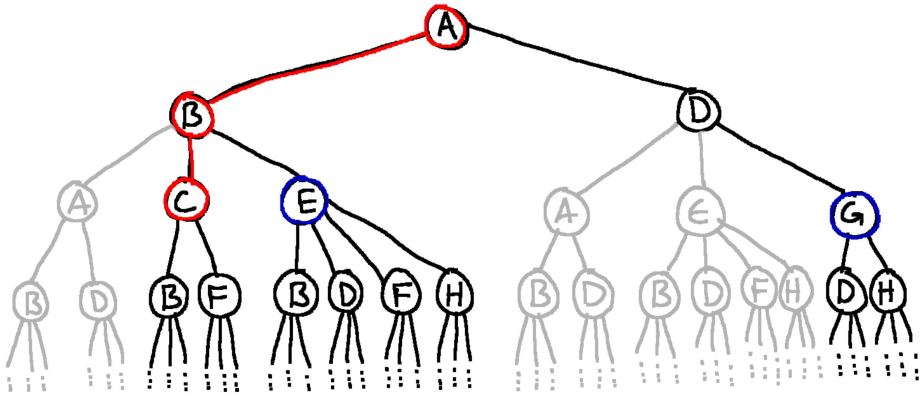
Visited: A, B, D, C, E



STEP 7

Agenda: $(A \rightarrow B \rightarrow C)$, $(A \rightarrow B \rightarrow E)$, $(A \rightarrow D \rightarrow G)$

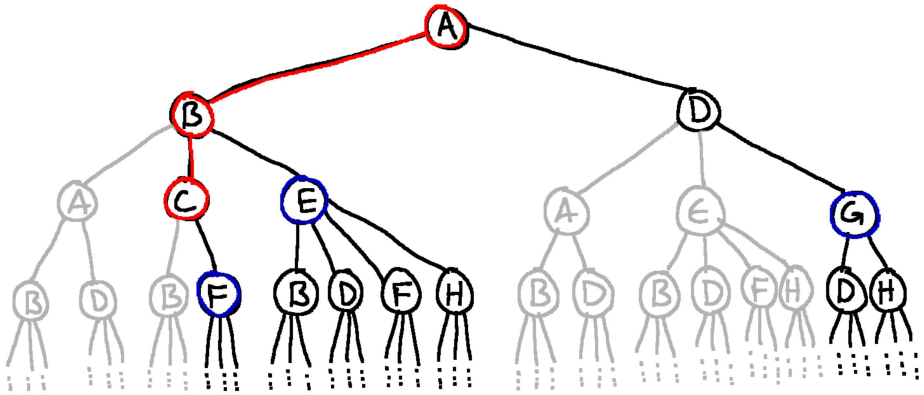
Visited: A, B, D, C, E, G



STEP 8

Agenda: $(A \to B \to E)$, $(A \to D \to G)$

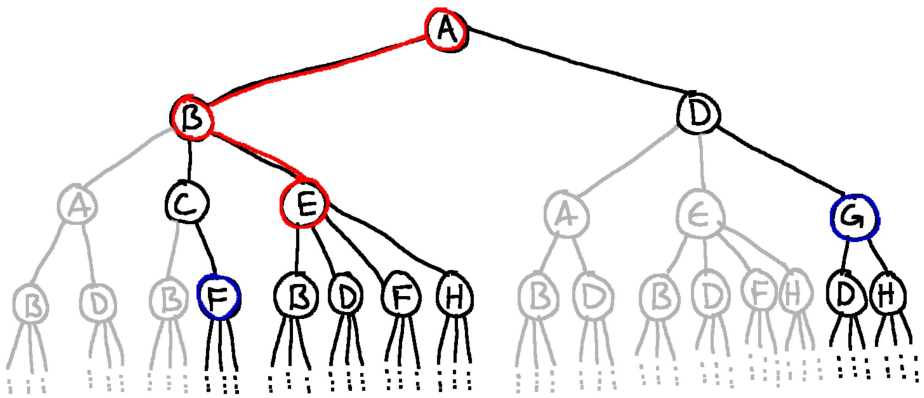
Visited: A, B, D, C, E, G



STEP 9

Agenda: $(A \to B \to E)$, $(A \to D \to G)$, $(A \to B \to C \to F)$

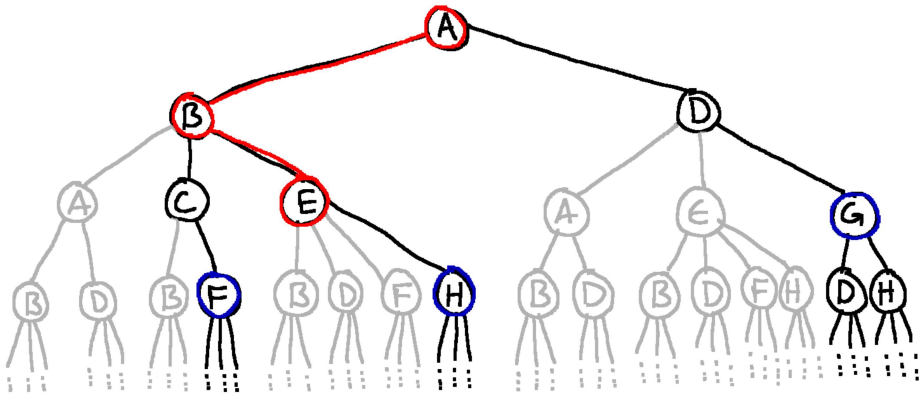
Visited: A, B, D, C, E, G, F



STEP 10

Agenda: $(A \rightarrow D \rightarrow G)$, $(A \rightarrow B \rightarrow C \rightarrow F)$

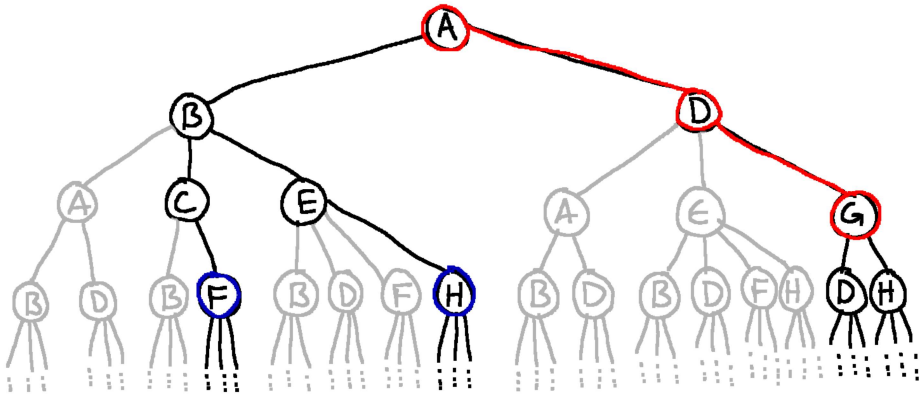
Visited: A, B, D, C, E, G, F



STEP 11

Agenda: $(A \rightarrow D \rightarrow G)$, $(A \rightarrow B \rightarrow C \rightarrow F)$, $(A \rightarrow B \rightarrow E \rightarrow H)$

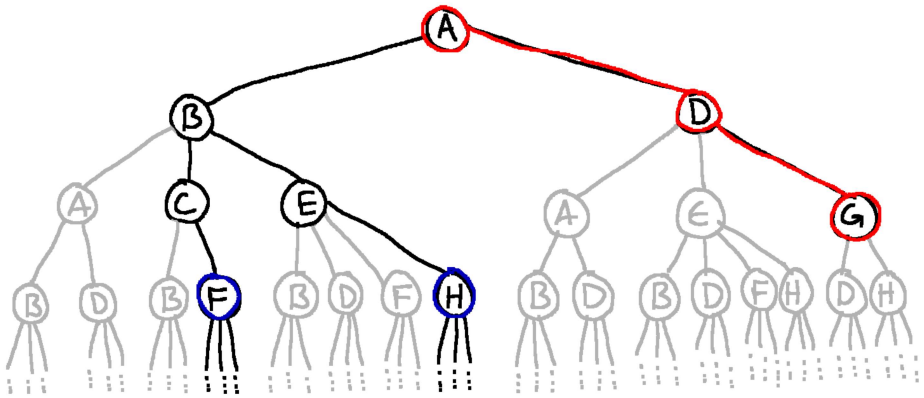
Visited: A, B, D, C, E, G, F, H



STEP 12

Agenda: $(A \rightarrow B \rightarrow C \rightarrow F)$, $(A \rightarrow B \rightarrow E \rightarrow H)$

Visited: A, B, D, C, E, G, F, H



STEP 13

Agenda: $(A \rightarrow B \rightarrow C \rightarrow F)$, $(A \rightarrow B \rightarrow E \rightarrow H)$

Visited: A, B, D, C, E, G, F, H

What path will ultimately be found by this approach? Enter your answer as a sequence of state names with no punctuation, e.g. `DCBA`:

Submit

You have submitted this assignment 0 times.

Once again, we cut off the example a little bit before the end, but, hopefully, we can see that this approach is characteristically different from the first two orderings we tried. This approach is called **breadth-first search** (or **BFS** for short) because we are exploring the entire breadth of the tree before considering paths deeper down the tree. Said another way, this approach considers all paths of a given length n before considering *any* paths of length $n + 1$.

3.1.4) Order Matters: Summary

In this section, we've looked at a couple of *very slight* variations of the algorithm we presented above. The only thing we changed was how we decide which path to consider next when there are multiple paths on our agenda. But, this slight difference in ordering had a big effect on the way in which our algorithm explored the graph; and, ultimately, it will have a big effect on the kinds of paths we return in the end.

The big takeaway here is that, if we always consider the *newest* path that was added to the agenda (i.e., the path that was added to the agenda most recently), we end up performing a *depth-first search*. If we instead consider the *oldest* path (i.e., the one that was added earliest), we instead perform a *breadth-first search*.

Another place that order matters is the order we loop over the neighbors of a vertex. In these examples, we always loop over the neighbors in alphabetical order. But different neighbor ordering may also produce different paths -- particularly for depth-first search, where if we are unlucky in what order we add neighbors to the agenda, we may end up exploring much more of the graph before reaching the goal.

3.2) Another Example

In the examples above, we saw these differences play out in a lot of detail. Next, let's take a step back and look at a related example to see if we can get more of a sense of how DFS and BFS differ in terms of the way they explore a graph. Watch the following brief video for another example:

0:00 / 6:31

Speed: 0.75x 1.00x 1.25x 1.50x 2.00x 2.50x 3.00x

3.3) Check Yourself: USA

In the video below, each light-grey dot represents an intersection of two or more highways in the United States. The video shows various paths as they are considered as part of planning a path from Smith Center, Kansas (the geographic center of the US) to Cambridge, Massachusetts. Watch the video below to answer: which kind of search is being performed here?

0:00 / 0:51

DFS

Submit

You have submitted this assignment 2 times.

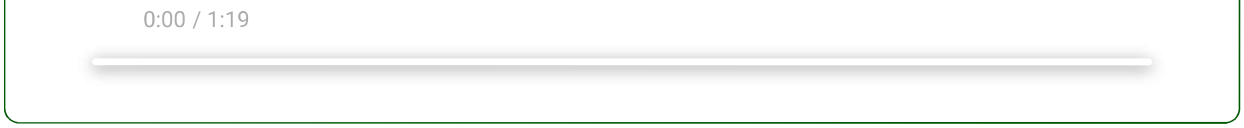
Solution: DFS

Explanation:

The key distinguishing feature here is that the search appears to be considering only a single path, expanding it out as far as possible and only backing up and trying an alternative path when there are no options left for expanding the current path. This behavior is characteristic of a **depth**-first search.

This search ends up finding a path between those two locations that is more than 50,000 miles long; it's great if you want to take the scenic route, but it's far from optimal!

If we were instead to perform a **breadth**-first search, the result would look like the following (which has been cut off before the end because it takes a really long time to show the whole process):

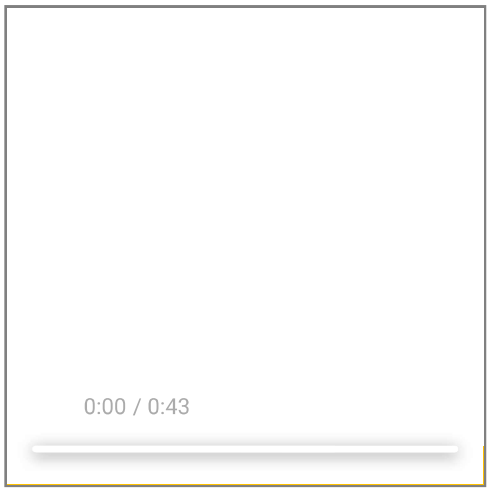


3.4) Check Yourself: Back to Flood Fill

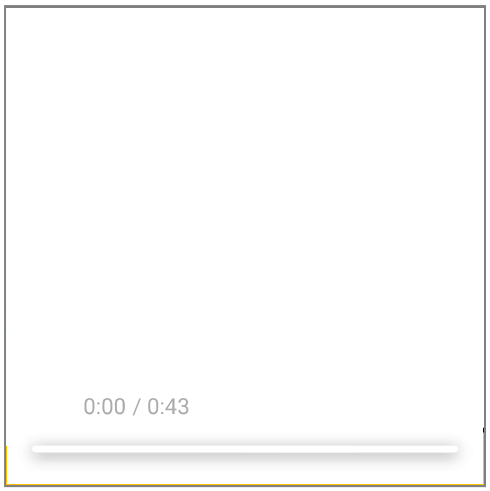
We can also see these kinds of differences play out in our flood-fill example from last week. Even though we weren't finding paths to start with last week, we were still keeping an agenda of pixels that we eventually wanted to color in, and, by changing the order in which we consider elements from the agenda, we can affect the order in which we color the pixels in. This doesn't affect the end result of the flood fill, but it does affect the intermediate results.

Consider the following three videos, each of which shows the evolution of a flood fill process:

Method 1



Method 2



Method 3

One of the flood fill implementations used a breadth-first approach: the pixel we chose to color in at any given point was the one that had been in the agenda the longest. Which of the videos corresponds to this approach?

Video 2

Submit

You have submitted this assignment 1 time.

Solution: Video 2

One of the flood fill implementations used a depth-first approach: the pixel we chose to color in at any given point was the one that had been added to the agenda most recently. Which of the videos corresponds to this approach?

Video 3

Submit

You have submitted this assignment 2 times.

Solution: Video 3

One of the flood fill implementations used a different approach: choosing a pixel from the agenda at random each time. Which of the videos corresponds to this approach?

Video 1

Submit

You have submitted this assignment 2 times.

Solution: Video 1

4) Summary of BFS vs. DFS

So far, we have been thinking fairly abstractly, but we have introduced the notion of graph search, with a specific focus on pathfinding. And we've also seen several examples of the difference between **breadth-first** and **depth-first** approaches. Before we move on to turning these abstract ideas into working Python code, let's take a moment to discuss some of the differences between BFS and DFS when applied to pathfinding.

BFS

- To implement a breadth-first search, we add and remove elements from *opposite sides of the agenda*. This approach is known as "first-in, first-out" (commonly written/pronounced as "FIFO") since the element we remove is always the one that was added first.
- BFS is guaranteed to return a shortest path to a goal vertex if such a path exists, regardless of the structure of our graph. Because we consider all paths of length n before considering any paths of length $n + 1$, we know that, when we first

encounter a state that satisfies our goal condition, the path we're considering must be optimal (in the sense that there is no shorter path to the goal).

- BFS can run forever if it is being applied to an infinite graph with no solution, but it will always terminate in a finite graph or in an infinite graph where a solution exists.

DFS

- To implement a depth-first search, we add and remove elements from *the same side of the agenda*. This approach is known as "last-in, first-out" (commonly written/pronounced as "LIFO") since the element we remove is always the last one that was added.
- DFS is guaranteed to find a path to the goal (but not necessarily an optimal one) if such a path exists and if the graph is finite.
- DFS may run forever on an infinite graph, even if a solution exists.

4.1) DFS = Bad?

In all of the examples we have seen so far, DFS gives us a pretty gnarly path in the end, something that is kind of ugly and far from optimal. And, in the section above, we've stated that the guarantees that DFS offers us are weaker than those that BFS offers us. So, at this point, you may be thinking: why did we even bother introducing DFS? When would you ever want to use it?

The answer is a little bit nuanced, but the short version is that DFS tends to use less memory than BFS. If we consider a graph with a "branching factor" of b (i.e., every state is connected to b other states), then there will be around b^n paths of length n. This means that there will be around b^n elements in the agenda when we're considering the last path of length $n-1$. By contrast, the agenda for a DFS will have around $b \times n$ elements in the agenda when considering a path of length n; and, as b and n increase, $b \times n$ is substantially smaller than b^n. This is especially useful in cases where we don't care about finding the optimal path to a state (sometimes any path will do, or sometimes we're just looking for a particular state and don't care about the path at all!), and, in those cases, DFS can sometimes be the right choice.

5) Pathfinding in Python

Now that we've spent some time thinking about graph search as an abstract process, let's think about how to turn it into Python code so that we can realize the power of these techniques.

As always, when we think about turning an abstract idea into something concrete in Python, an important question has to do with our choice of data structures. That is to say, we have several things that we're going to need to represent in Python, as well as operations that we'll need to perform on those things. In some sense, we have infinitely many choices for how to represent each of those things concretely within our program, but, as we've seen in earlier readings and recitations, the data structures we choose to represent those things can have a dramatic effect on the code that we end up writing in terms of clarity, the ease with which we can avoid common bugs, and efficiency. So, let's spend some time here thinking about representation. In particular, there are at least 5 distinct things we need to be able to represent in order to write code to solve paths using the approach outlined above:

- the graph itself (the various states and the connections between them)
- the start state and goal condition
- a candidate path (a sequence of states)
- the agenda
- the visited set

Check Yourself:

Take a moment to think about different ways that we could represent each of these things. There are multiple options for each; what are some of the tradeoffs?

Show/Hide

We'll revisit some of these choices a little bit further down the page, but, for now, here's one option we can go with:

- The **graph** needs to represent all of the available states as well as the connections to them, and we need to be able to quickly answer the question of which states are neighbors of a given state. For this purpose, we can use an "adjacency dictionary." Each key in our dictionary will be a single state, and the associated value will be a list of the states that share an edge with that state. This representation affords us an efficient way to look up the neighbors of any given state, which is an important action that we'll repeat quite a lot.
- We'll assume a well-defined **start position**. The exact form of this will depend on the problem we're trying to solve, but we will assume that our starting state is a state in whatever graph we're searching over.
- The **goal condition** is a little more complicated to think about. It is tempting to say that it should also just be a state in our graph. We're going to use a slightly more general representation, though: a *function* which takes a state as input and returns a Boolean value indicating whether that state satisfies our goal condition or not. This will allow us to use a single search to find paths through a graph where any of a number of states are OK as goals (for example, we might want to ask our GPS navigation system to take us to "any gas station" rather than to a particular one).
- A **path** consists of an ordered sequence of states. This limits our choice of representation somewhat, but, in our code moving forward, we will represent a path as a `tuple` containing the states in the path in order (with index 0 corresponding to our starting state). The form of the states themselves will vary based on the problem we're solving, but we'll always represent a path as a tuple containing one or more states.
- Our **agenda** needs to store elements in a given order, and we'll need to be able to remove elements from either side of it. The fact that the agenda needs to maintain order suggests that either a list or a tuple would be appropriate; and, because we're going to need to be adding and removing things from it, we'll go with the mutable option: a `list`.
- Our **visited set** does not need to maintain order, and the only thing we really do with it is add elements to it and check whether elements are already in it. As we've seen in recitation, if we were to use a `list` for this, the amount of time that that membership test would take would grow roughly linearly with the length of the list. A Python `set` is a great choice so that we can do that check efficiently. But, we'll need to be careful to remember that we can only add *hashable* elements to it.

So here's the function we want to write:

```
def find_path(graph, start, goal_test):
    """
    Find a path through the given state graph, starting from the `start` state
    and reaching a state satisfying `goal_test`.

    Note that all state representations must be hashable.
```

```
    graph: adjacency dictionary, where each key is a state,
           and its value is the list of the state's immediate neighbors
    start: starting state
    goal_test: function that takes a state and returns True (or truthy)
            if and only if the state satisfies the goal condition

    returns the path of states from start to a goal state,
    or None if no path exists
    """
```

Check Yourself:

With those decisions in mind, try your hand at writing `find_path`. Note that the structure is very similar to what we saw with flood fill last week, so you may wish to use that as inspiration.

Show/Hide

Show/Hide Line Numbers

```
1  def find_path(graph, start, goal_test):
2      if goal_test(start):
3          return (start, )
4
5      agenda = [(start, )]
6      visited = {start}
7
8      while agenda:
9          this_path = agenda.pop(0)
10         terminal_state = this_path[-1]
11
12         # calling get(terminal_state,[]) on the graph dictionary
13         # is the same as graph[terminal_state] if the key is found,
14         # otherwise returns the default value [] (avoiding KeyError)
15         for neighbor in graph.get(terminal_state, []):
16             if neighbor not in visited:
17                 new_path = this_path + (neighbor,)
18
19                 if goal_test(neighbor):
20                     return new_path
21
22                 agenda.append(new_path)
23                 visited.add(neighbor)
24     return None
```

Double-check that **every line of code here** makes sense to you. If not, please reach out and ask for help (during open lab hours, instructor office hours, or via the `6.101-help@mit.edu` mailing list).

Is the code example in the box above performing a BFS, a DFS, or something else?

BFS

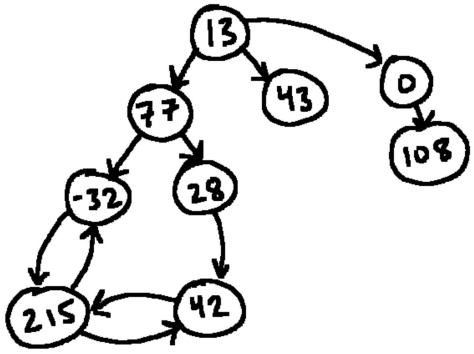
Submit

You have submitted this assignment 1 time.

Solution: BFS

6) A Small Example

With this code in hand, let's walk through a few example programs. Here is a small, contrived example graph that we can start with:



Given the choice of representation we discussed above, how would we represent this graph in Python? Store your answer in a variable called `test_graph`. Note that the states here are integers (and should be represented by Python `int` objects), note also that the edges in this graph are *directed* (one-way), and make sure that your representation accurately reflects this.

```
1 test_graph = {13:[77,43,0], 77:[-32,28], 0:[108], -32:[215], 28:[42], 215:[-32,42], 42:[215]
```

Submit

You have submitted this assignment 3 times.

Here is the solution we wrote:

```
test_graph = {
    13: [77, 43, 0],
    77: [-32, 28],
    28: [42],
    0: [108],
    -32: [215],
    42: [215],
    215: [42, -32],
}
```

Try to answer the questions below without using a Python interpreter.

Consider using the `find_path` function from above *exactly as it is written*. How many states will be in the path we return from calling `find_path(test_graph, 13, lambda state: state == -32)`?

3

Submit

You have submitted this assignment 2 times.

Solution: 3

What would happen if we instead called `find_path(test_graph, 13, lambda state: -32)`? If the function would return a value, type the resulting value in the box below. If it would produce an exception, write `error`. If it would enter an infinite loop, write `infinite`.

(13,)

Submit

You have submitted this assignment 5 times.

Solution: (13,)

Explanation:

The sneaky part of this code is that the goal-test function now returns `-32`. If we look at how this function is being used inside of `find_path`, it is always being used as `if goal_test(some_state):`. Given that function, regardless of what state is passed in, the result will be `-32`. So, we will end up, on the very first line, doing something like `if -32: return (start, )`. Since `-32` is truthy, we won't ever even start the main search process: we'll always return `(state,)` right away!

What would happen if we instead called `find_path(test_graph, 13, -32)`? If the function would return a value, type the resulting value in the box below. If it would produce an exception, write `error`. If it would enter an infinite loop, write `infinite`.

error

Submit

You have submitted this assignment 2 times.

Solution: error

Explanation:

Here, we get an exception right away when trying to to `if goal_test(start)`; we'll get a `TypeError` because the value we passed in as `goal_test` is not callable. Specifically, we'll get an error message like the following:

```
TypeError: 'int' object is not callable
```

What would happen if we instead called `find_path(test_graph, 0, lambda x: x == 215)`? If the function would return a value, type the resulting value in the box below. If it would produce an exception, write `error`. If it would enter an infinite loop, write `infinite`.

None

Submit

You have submitted this assignment 4 times.

Solution: None

In the example from the previous question, `find_path(test_graph, 0, lambda x: x == 215)`, what elements were in the visited set by the end of the function call? Enter your result as a Python `set` below.

{0,108}

Submit

You have submitted this assignment 4 times.

Solution: `{0, 108}`

Check Yourself:

Even though this function currently implements a BFS, it is possible to change a single line such that the function performs a DFS instead. What line could we change to make that happen, and what would we need to change it to?

Show/Hide

In order to change this code to a DFS, we need to set things up so that we are removing paths from the *same side of the agenda we're adding them to*. The lines that govern this behavior are line 9 and line 19. And, indeed, changing *either* of those lines can work. We could either change line 9 to say `this_path = agenda.pop(-1)` (or even `this_path = agenda.pop()`), *or* we could change line 19 to say `agenda.insert(0, new_path)`.

In general, the change to line 9 should be preferred for efficiency reasons (adding and removing from the end of a list is generally more efficient than adding and removing from the beginning of a list).

7) Revisiting Representation of Graph: 15-Puzzle

Let's return now to a problem we discussed earlier in the reading and try to bring this code to bear on the 15-puzzle. To start with, any time we are thinking about using search to solve a new problem, an important question is: how do I represent the *states* (the

vertex labels) for this problem, such that my code can be made to work with them? So, let's start there.

Check Yourself:

Take a moment and think about how you might represent the states in this graph, keeping in mind that, in the graph we drew all the way back in section 2.1, each vertex was labeled with a representation of a possible layout of the game board.

Show/Hide

We have *many* options here, but the code in `find_path` limits us somewhat. Our choice of state representation is used in two places in the code: one is that each path is represented as a sequence of states, and the other is that we store states inside of our visited set. So, unless we want to change our visited set into a visited list (why might we not want to do that?), we are limited to *hashable* representations of the game board.

We still have options here, but, when we note that the board is a collection of elements and that the order of those elements matters, we might lean toward using tuples for our representation. This is not the only option (and there are multiple ways to use tuples), but we could imagine using a 2-d array of numbers (stored as tuples of tuples) to represent the board.

Even once we have settled on that choice, there is still the question of: how do we represent the empty square? The most important feature that this representation needs to have is that it should be *impossible to mistake it for a regular spot occupied by a tile*. There are options here. For example, any non-integer would work, or maybe a negative integer. But, it is often nice to have this representation really stand out; a good choice in this case might be to put `None` in the place representing the location of the empty tile. So, an example board using this representation might look like:

```
((2, 6, 3, 15), (11, 9, 4, 5), (1, 8, 12, None), (13, 14, 10, 7))
```

From here, it's worth asking ourselves: can we imagine using our `find_path` function from above to solve this puzzle? Here, we're confronted with a limitation of an early design choice: for the 15-puzzle, generating the dictionary to represent the graph would take a *long* time, and, even worse, the resulting structure would be far too big to fit in memory! It doesn't make sense to spend the time building up that representation just to then go *back* through the whole structure looking for a path. What we'll look at next is an approach to making this whole process more feasible.

Before we dig in, it's worth mentioning that this kind of revision (which often requires deleting and rewriting code!) is completely normal. It takes a lot of practice to make informed decisions about these kinds of choices, and, even with that experience, it's common not to notice until later that we should have structured things differently. So, the process of going back and changing code to use a better internal representation is not an uncommon occurrence.

Check Yourself:

A place to start when thinking about different ways to represent graphs is to look at our search code and ask ourselves: what questions are we asking about the input graph?

Show/Hide

The *only* place we're using `graph` in our search code is to look up the neighbors of a given state. And what's more, in many graph search problems, we probably won't end up asking for the neighbors of the majority of states in the graph. So, now, we can ask ourselves: can we think of a *different* structure that we could use to answer that same question, but which doesn't require precomputing the entire graph structure?

Show/Hide

This might seem a little bit strange, but a great choice here is a *function*! If we can define a function that takes a state as input and returns a list of directly connected states, we can use that function to answer the question we need to answer by changing line 12 in `find_path` so that, instead of looking up the `terminal_state` in a dictionary to find the list of neighbors, we instead *call a function* to find those neighbors. We will also need to change our function signature (and its docstring if you've written one), so that it is clear that our input representing the graph is now a function (along those lines, `graph` was not a great name for that variable to begin with; perhaps `graph_dict` would have been better, and a different name probably makes sense after this change).

The beauty here is that this allows us to represent the structure of the graph without storing an explicit representation of every vertex and every edge in the graph; we discover and store information about the relevant connections in the graph only when our search process comes across them, which saves us the trouble of constructing the whole graph and also helps us avoid wasting memory on parts of the graph that our search process might never explore.

Check Yourself:

Now, try your hand at making the necessary changes to `find_path` to handle this new design, before looking at our code below.

Show/Hide Our Code

Here is our code, with the modified lines highlighted in yellow. This is a small change, but there is a lot of power in it!

Show/Hide Line Numbers

```
def find_path(neighbors_function, start, goal_test):
    """
    Find a path through a state graph defined by `neighbors_function`,
    starting from the `start` state and reaching a state satisfying
    `goal_test`.

    Note that all state representations must be hashable.

    neighbors_function: function that takes a state and returns its
                        neighbors (as an iterable)
    start: starting state
    goal_test: function that takes a state and returns True (or truthy)
        if and only if the state satisfies the goal condition

    returns the path of states from start to a goal state,
    or None if no path exists
    """
    if goal_test(start):
        return (start, )

    agenda = [(start, )]
    visited = {start}

    while agenda:
        this_path = agenda.pop(0)
        terminal_state = this_path[-1]

        for neighbor in neighbors_function(terminal_state):
            if neighbor not in visited:
                new_path = this_path + (neighbor,)

                if goal_test(neighbor):
                    return new_path

                agenda.append(new_path)
                visited.add(neighbor)
```

This change allows us to solve things like the 15-puzzle, where constructing the whole graph preemptively would be prohibitively expensive. We'll leave the task of writing the code to solve the 15-puzzle as something to do on the weekend if you're bored and looking for something to do; but let's see if we can make use of this new structure to solve a different problem.

8) Another Example: Word Ladders

Let's use our new function-based pathfinding implementation to solve a different kind of puzzle, known as a Word Ladder. In this puzzle, we are given two words, and our task is to find a sequence of words that connect those two words to each other, such that everything in the sequence is a word, and each word in the sequence differs from the one before it and the one after it by exactly one letter.

For example, a puzzle might be given as something like the following (it's common for the starting and ending words to be opposites of each other):

```
cold
...
warm
```

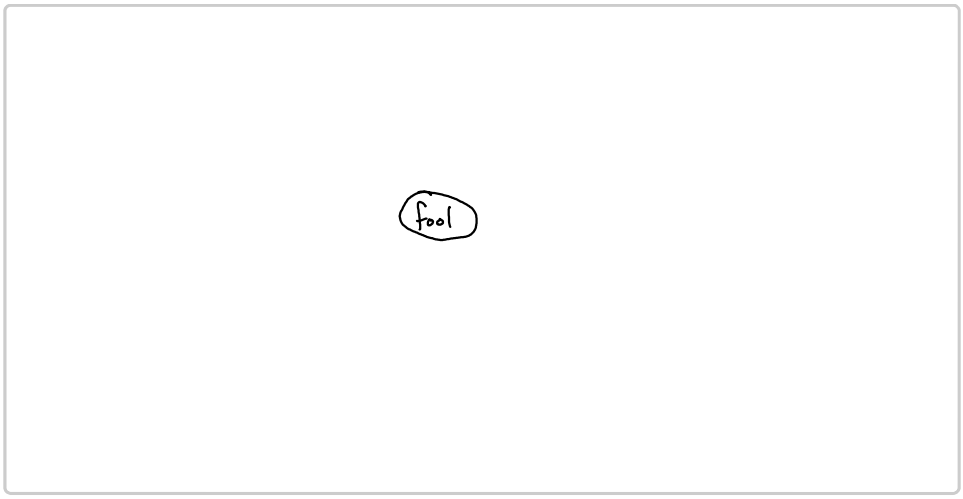
And a valid answer to the puzzle is:

Show/Hide Solution

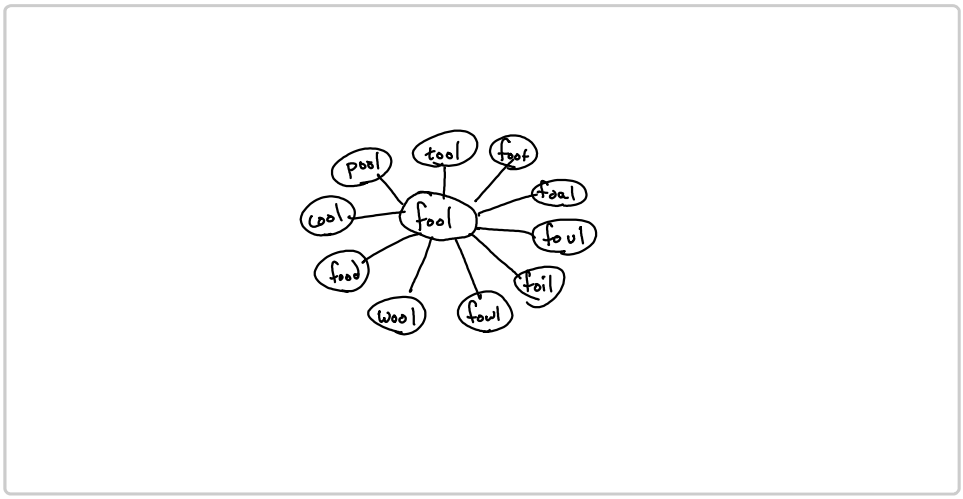
```
cold
cord
card
ward
warm
```

Note that every element in the sequence is a valid word and that each varies from the words preceding and succeeding it by only a single letter.

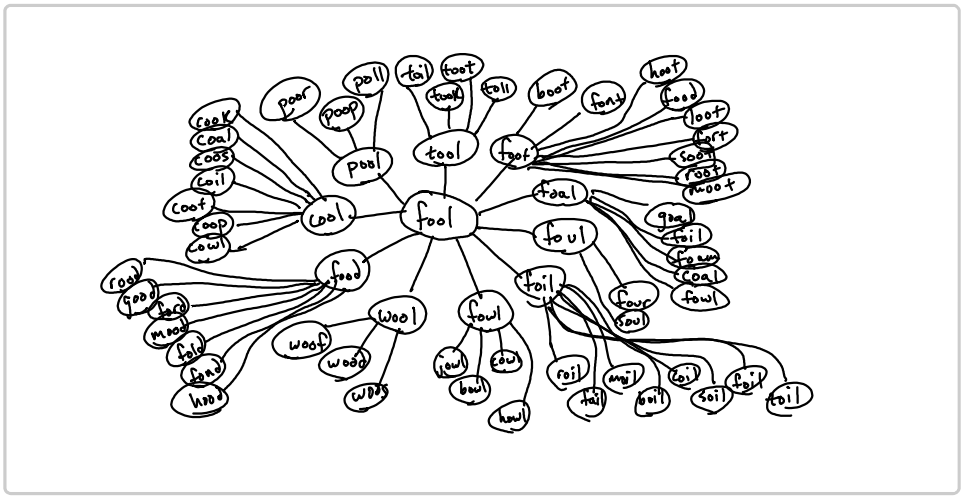
This problem is a prime candidate for our pathfinding approaches, and a natural representation is to use the words themselves as states and to have edges connect words that differ only by a single letter. But, like the 15-puzzle, this has a problem. Let's take a look at just a portion of the graph for solving a common puzzle where the starting word is `"fool"` and the ending word is `"sage"`. Let's start by looking at just the single vertex representing the word `"fool"`:



Isn't it beautiful? But, it's not all that exciting. Let's take a look at a slightly larger piece of the graph, showing all of the vertices that are one step away from `"fool"`:



and we can carry things further by looking at all vertices that are up to two steps away:



Argh! And as you can imagine, the graph gets ever bigger and more complex the farther we move away; so this is a great example for testing out our new implementation of `find_path`! Try your hand at setting up this problem by filling in the code box below, to set up a search from `"patties"` to `"foaming"` in the graph we've been talking about.

You can replicate the `ALL_WORDS` variable on your own machine (if you want to test things out there first) by copying the relevant code from the box below and downloading `words.txt` to the same directory where you're writing your code.

```python
# ALL_WORDS is a set containing all strings that should be considered valid
# words (all in lower-case)
with open('words.txt') as f:
    ALL_WORDS = {i.strip() for i in f}

# replace the following with the starting state
start_state = "patties"
end_state = "foaming"

# replace this neighbors function:
def word_ladder_neighbors(state):
    """
    takes a state as input
    returns all neighboring states (valid words that differ in one letter)
    """
    neighbors = []
    for i in range(len(state)):
        for letter in 'abcdefghijklmnopqrstuvwxyz':
            new_word = state[:i] + letter + state[i+1:]
            if new_word in ALL_WORDS and new_word != state:
                neighbors.append(new_word)
    return neighbors

# replace this goal test function:
def goal_test_function(state, end_state):
    """
    takes a state as input
    returns True if and only if state matches the goal (the target word)
    """
    if state == end_state:
        return True
    return False

# ultimately, these variables will be passed as arguments to the find_path
# function to solve for the path between "patties" and "foaming"
def find_path(neighbors_function, start_state, goal_test_function):
    if (goal_test_function == start_state):
        return(start_state)

    agenda =[(start_state, )]
    visited = {start_state}

    while agenda:
        this_path = agenda.pop(0)
        terminal_state = this_path[-1]
        for neighbor in word_ladder_neighbors(terminal_state):
            if neighbor not in visited:
                new_path = this_path + (neighbor,)
                if(goal_test_function(neighbor, end_state)):
                    return new_path

                else:
                    agenda.append(new_path)
                    visited.add(neighbor)
```

```
55
56 if __name__ == "__main__":
57     output = find_path(word_ladder_neighbors, start_state, goal_test_function)
58
```

Submit

You have submitted this assignment 4 times.

Here is the solution we wrote:

```
with open('words.txt') as f:
    ALL_WORDS = {i.strip() for i in f}

start_state = "patties"

LETTERS = 'abcdefghijklmnopqrstuvwxyz'
def word_ladder_neighbors(word):
    different_in_one_letter = {
        word[:ix] + l + word[ix+1:]
            for ix in range(len(word))
                for l in LETTERS
                    if l != word[ix]
    }
    return different_in_one_letter & ALL_WORDS

def goal_test_function(state):
    return state == 'foaming'
```

9) Summary

Once again, it feels like we have come a long way. We started by thinking back on last week's example problem of flood fill, a d then we expanded and formalized it, arriving at the idea of *graph search*.

Throughout today's reading, we explored several facets of graph search algorithms. We started by talking about graph searc n an abstract sense, including discussing the dramatic differences in behavior that arise from changing the order in which we cons ler elements in the agenda. We also saw several examples of taking problems from the real world and representing them as gra s.

Which of the following statements are true?

- [ ] In order to be a BFS, it's important that new paths are added to (and removed from) the *front* of the agenda, rather than the *end* of the agenda.
- [x] If BFS finds a path, that path is guaranteed to be optimal (in terms of length).
- [ ] DFS might enter an infinite loop even if the search domain is finite.
- [x] It is possible that DFS and BFS could return the same path for some problem.
- [x] It is possible that DFS and BFS could return different paths for some problem.
- [x] BFS is guaranteed to find a path if one exists, even in an infinite domain.

Submit **100.00%**

You have submitted this assignment 8 times.

Solution:

- [ ] In order to be a BFS, it's important that new paths are added to (and removed from) the *front* of the agenda, rather than the *end* of the agenda.
- [x] If BFS finds a path, that path is guaranteed to be optimal (in terms of length).
- [ ] DFS might enter an infinite loop even if the search domain is finite.
- [x] It is possible that DFS and BFS could return the same path for some problem.
- [x] It is possible that DFS and BFS could return different paths for some problem.
- [x] BFS is guaranteed to find a path if one exists, even in an infinite domain.

We then moved to implementing a pathfinding algorithm in Python, and we spent a lot of time thinking about how to represent different structures related to graph search in Python, eventually ending up at a 20-line implementation of a pathfinding algorithm. There is a lot of interesting stuff going on in those 20 lines and a lot of power from relatively little code! We also worked through examples of using that code to solve a few different kinds of problems, and we used those problems as a guide as we sought to make improvements to the code.