Dynamic Programming

Homework #2

1. Consider the following probability space:

$$\Omega = \{(i, j) \text{ such that } i, j \text{ are positive or negative integers or zero}\}$$

Assume associated probabilities p_{ij}. Define the random variables

$$x_1 = i * j, \quad x_2 = \max(|i|, |j|)$$

(a) Show that Ω is countable

(b) Derive expressions for the following

i. $P(x_1 < 0)$

ii. $P(x_2 \leq 1)$

iii. $P(x_1 < 0 \quad | \quad x_2 \leq 1)$

(c) Suppose

$$p_{00} = 1/2$$

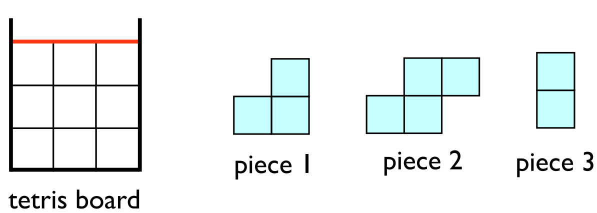
$$p_{ij} = \frac{(1/2)^{(\rho+1)}}{8\rho}, \quad (i, j) \neq (0, 0)$$

where

$$\rho = \max(|i|, |j|)$$

Determine whether x_1 and x_2 are independent.

2. Tetris: Consider the tetris game highlighted below. Suppose at each stage you know the state of the board and the current piece. You do not have knowledge of the next piece. The goal is to place the piece (orientation, location) to optimize the number of rows that you can successfully eliminate without exceeding the red line.



(a) What is the state space? How many total states?

(b) What is the state transition function? Give example.

(c) What are the stage costs? Give example.

(d) Given an example of a policy for two of the states.

(e) Suppose the tetris game has a fixed number of stages $N = 100$ and the selection of the pieces is deterministic according to the sequence $1, 2, 3, 1, 2, 3, 1, 2, 3, ...$, i.e., the piece at stage 1 is 1, the piece at stage 2 is 2, etc. Write a Matlab program to solve for the optimal cost to go? Note that if you ever exceed the red line then the game immediately terminates. How does the optimal policy compare to your policy for the two states in (b)?

(f) What is the optimal cost to go $J^*(x_0)$ where x_0 represents the initial empty board and we start with piece 1? piece 2? piece 3?

(g) Simulate your optimal policy on the above situation. How many rows did you eliminate when you started with piece 1? piece 2? piece 3?

(h) Suppose the deterministic sequences of pieces was unknown and you were forced to doing a worst-case analysis. Now, what is the optimal cost to go $J^*(x_0)$ where x_0 represents the initial empty board and we start with piece 1? piece 2? piece 3? Does this depend on the sequence of pieces? How do these answers compare to the ones in (f)?

(i) Simulate your optimal policy for the situation highlighted above, i.e., $N = 100$ and the deterministic selection of pieces. How many rows did you eliminate when you started with piece 1? piece 2? piece 3? How do the answers compare to (g)?